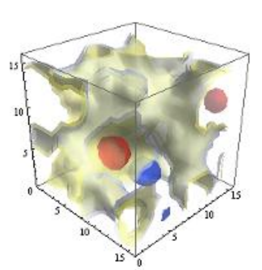
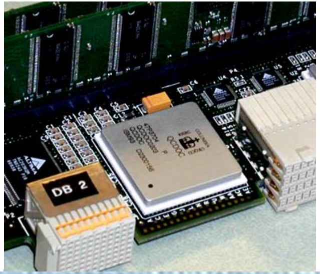
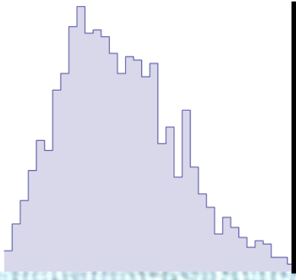
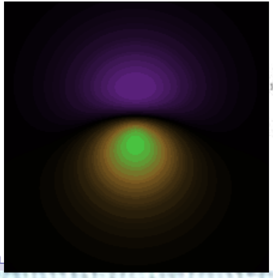
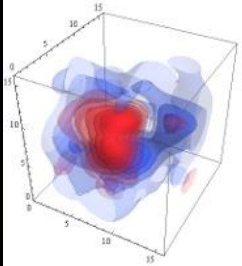
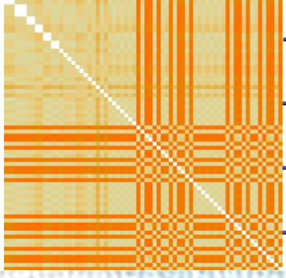
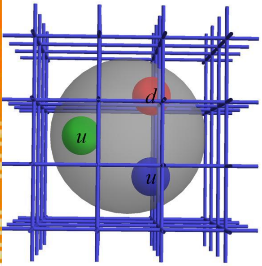



The Trouble with the Lattice Axial Charge

Huey-Wen Lin

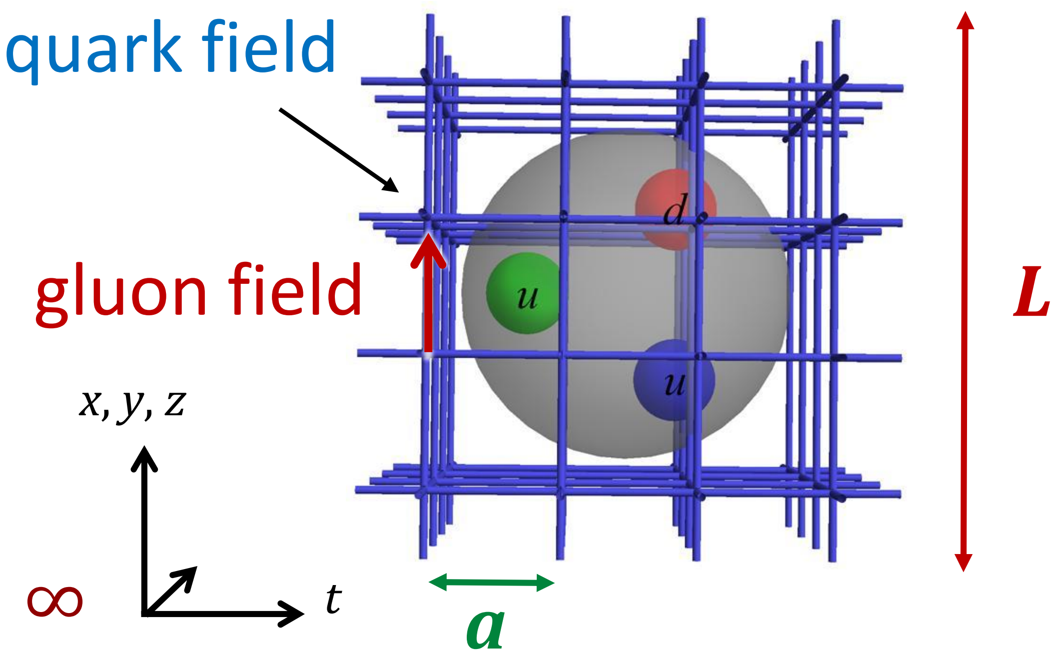
QCD in a Box

- § Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories
- § Physical observables are calculated from the path integral

$$\langle 0 | O(\bar{\psi}, \psi, A) | 0 \rangle = \frac{1}{Z} \int \mathcal{D}A\, \mathcal{D}\bar{\psi}\, \mathcal{D}\psi\, e^{iS(\bar{\psi}, \psi, A)} O(\bar{\psi}, \psi, A)$$

in **Euclidian** space

- Quark mass parameter (described by m_π)
- Impose a UV cutoff
  discretize spacetime
- Impose an infrared cutoff
  finite volume

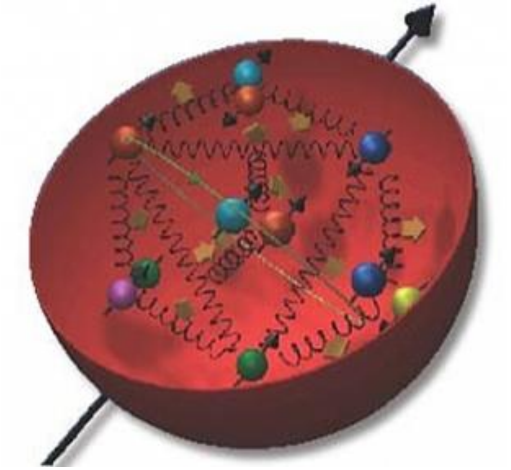


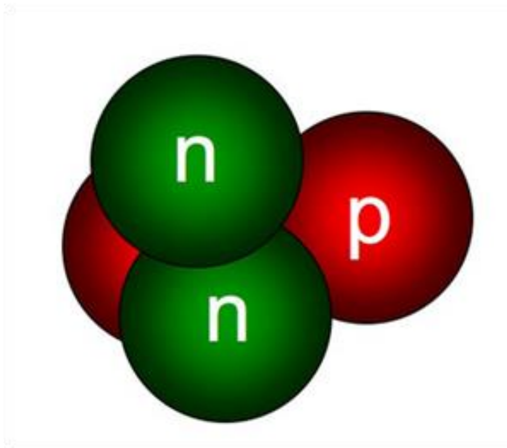
- § Recover physical limit

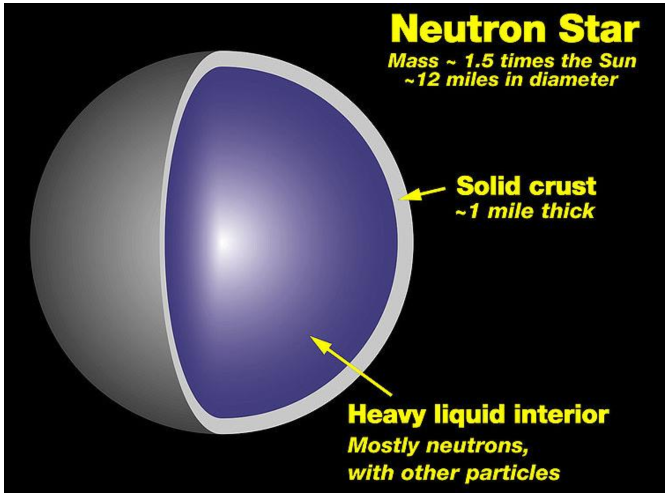
$$m_\pi \to m_\pi^{\text{phys}},\ a \to 0,\ L \to \infty$$

Wide-Scale Applications

§ What can we learn from it?







10^{-15} m

10^{4} m

Parton distribution functions
Properties for
new-physics searches

HWL et al, 1402.1462, 1506.06411; 1506.04196 …

Nuclei and why we exist

1409.3556, 1206.5219, 1109.2889, 1012.3812
…

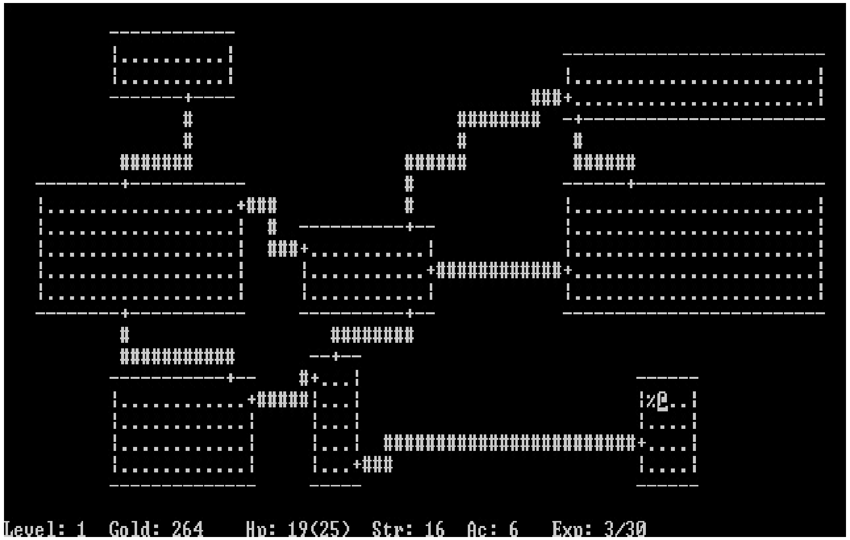
Neutron matter
Neutron-star evolution

1204.3606

Are We There Yet?

- Lattice gauge theory was proposed in the 1970s by Wilson
  - Why haven't we solved QCD yet?
- Progress is limited by computational resources

1980s



Today

- Greatly assisted by advances in algorithms
  - Physical pion-mass ensembles are not uncommon!

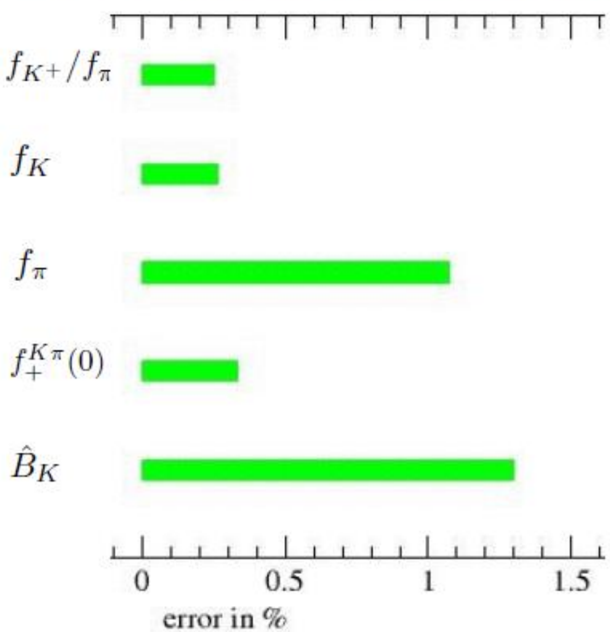
Successful Examples

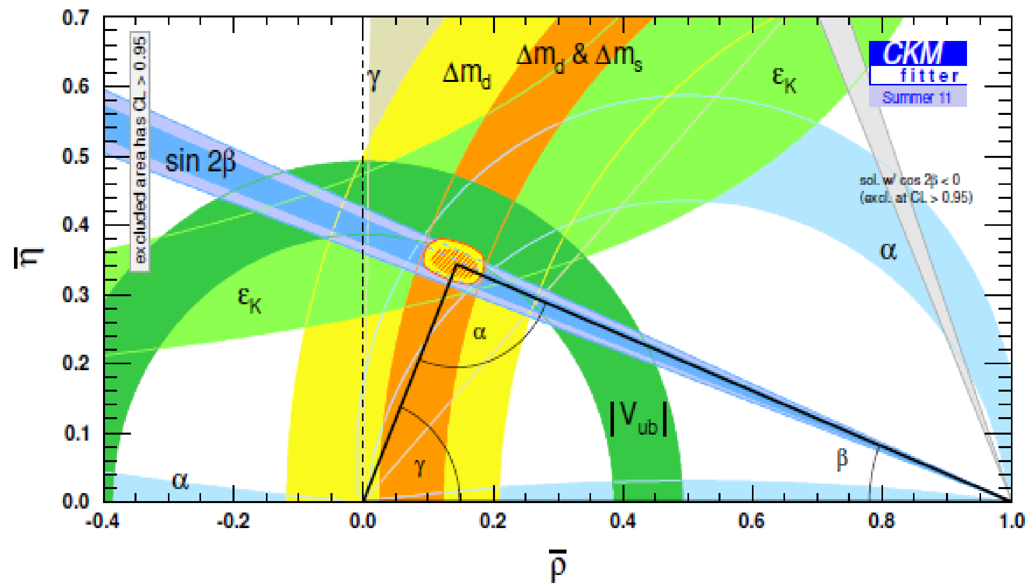
- § Lattice flavor physics provides precise inputs from the SM

  A. El-Khadra, Sep. 2015, INT workshop "QCD for New Physics at the Precision Frontier"

  - Very precise results in many meson systems

errors (in %) **(preliminary) FLAG-3 averages**



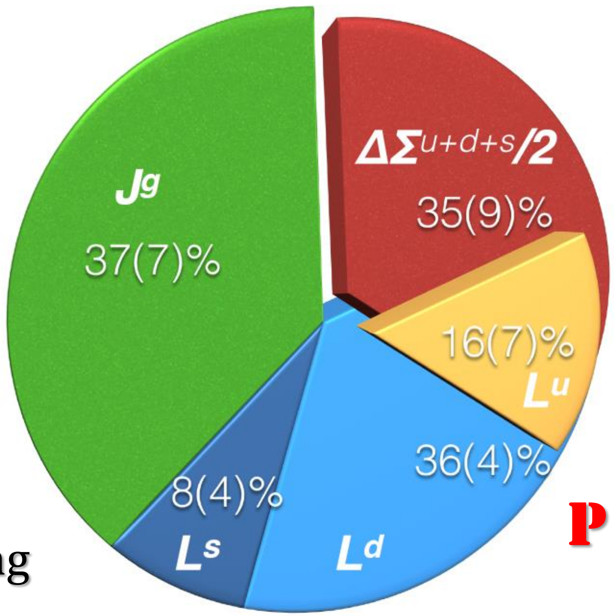


- § We are beginning to do precision calculations in nucleons

Origin of Proton Spin

§ What is the makeup of the nucleon?

- Decomposition using Ji's GPD moment connection
- Preliminary result from χQCD (2+1f ov/DWF 400 MeV)



$$J^q = \frac{1}{2}(A_{20}^q + B_{20}^q)$$

ΔΣ: quark spin

$L = J - \Delta\Sigma$: orbital angular momentum

Plots by Yi-Bo Yang

PRELIMINARY

- ETMC (2f TMF 130 MeV) $M_\pi L = 3$ Preliminary

$$\Delta\Sigma^{u+d+s} = 0.214(61),\ L^{u+d+s} = 0.168(60),\ J^g = 0.118(57)$$

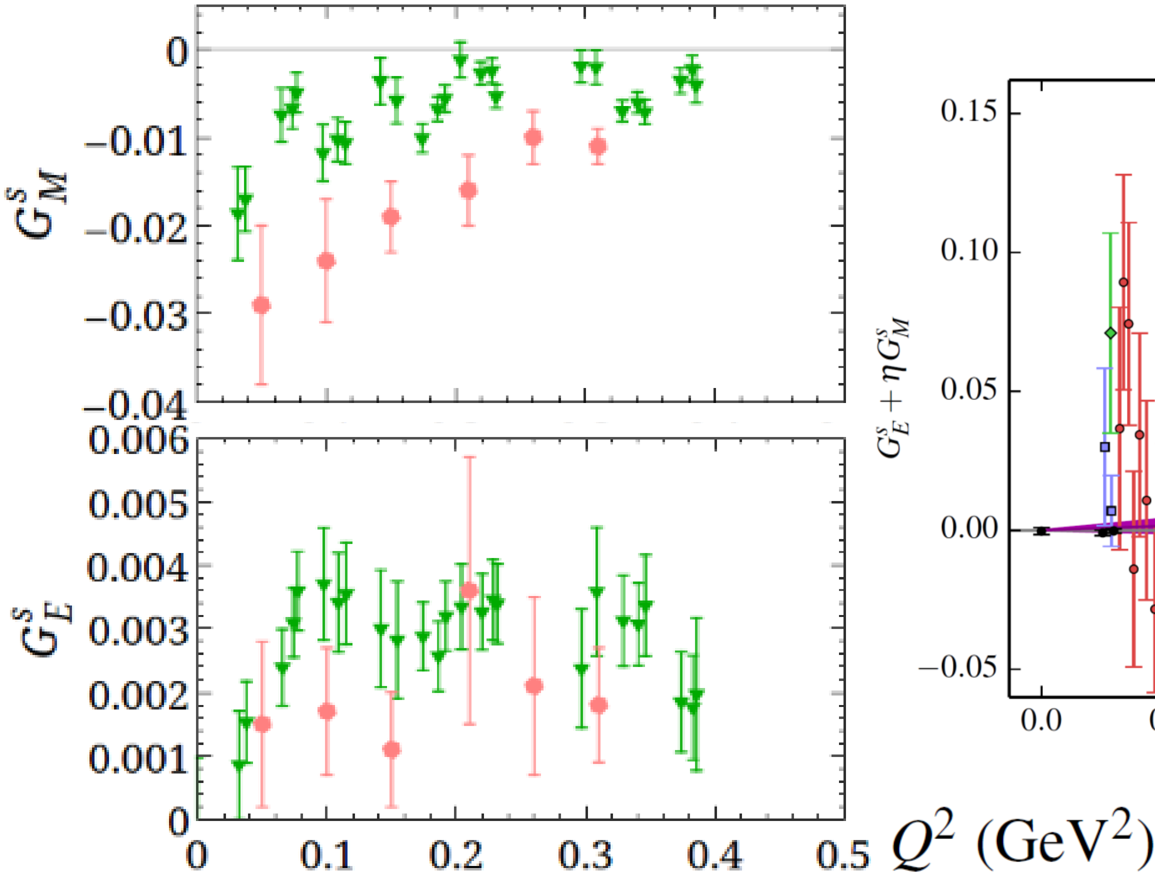
M. Constantinou, Spin 16

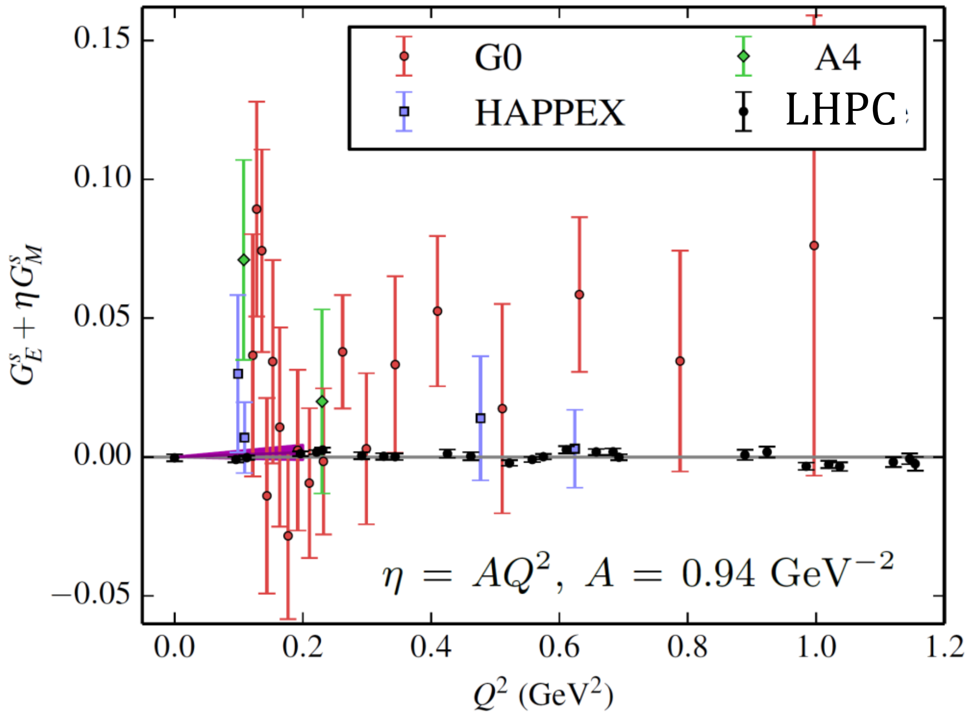
Strange Form Factors

§ Better determined strange form factors

K. Orginos/R. Sufian, Spin 16

- LHPC (2+1f): clover $M_\pi = 317$ MeV, $a = 0.11$ fm
- χQCD (2+1f): ov/DWF $M_\pi = 207{,}140$ MeV, $a = 0.11$ fm

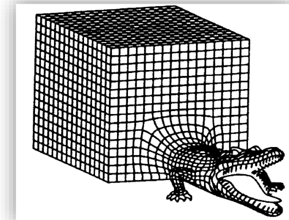




Sea Flavor Asymmetry

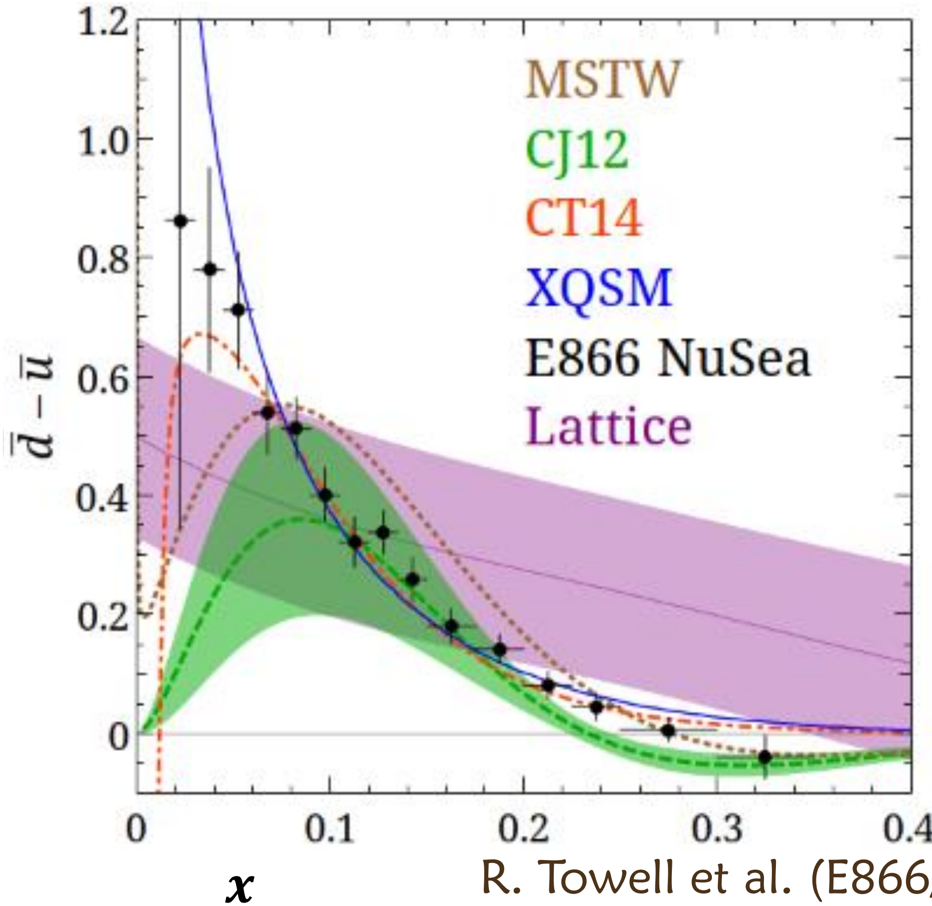
§ First time in LQCD history to study antiquark distribution!

- $M_\pi \approx 310$ MeV



$$\bar{q}(x) = -q(-x)$$

HWL et al. 1402.1462



Lost resolution in small-x region

Future improvement: larger lattice volume

$$\int dx\,(\bar{u}(x) - \bar{d}(x)) \approx -0.16(7)$$

Experiment	x range	$\int_0^1[\bar{d}(x)-\bar{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
HERMES	$0.020<x<0.30$	0.16 ± 0.03

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Nucleons and BSM

Many opportunities to probe BSM with nucleon inputs

- Parton distribution functions for SM background 1402.1462
  - Especially less known intrinsic strange/charm contribution
- Dark matter detection 1306.6939
  - Popular candidates (e.g. SuSy neutralinos) exchange Higgs
- Electric dipole moment 1506.04196
  - CP-violating effect , extremely small: in SM $\approx 10^{-30}$ e-cm
- Neutron beta decay 1110.6448; 1506.06411
  - Non-V−A interactions to probe the existence of new particles (mediating new forces) with masses in the multi-TeV range
- Nucleon (transition) axial form factor 0803.3020, 1003.3387
  - First-principles inputs into Monte Carlo event generators for precision neutrino physics

Many of these are supported by P5 recommendations

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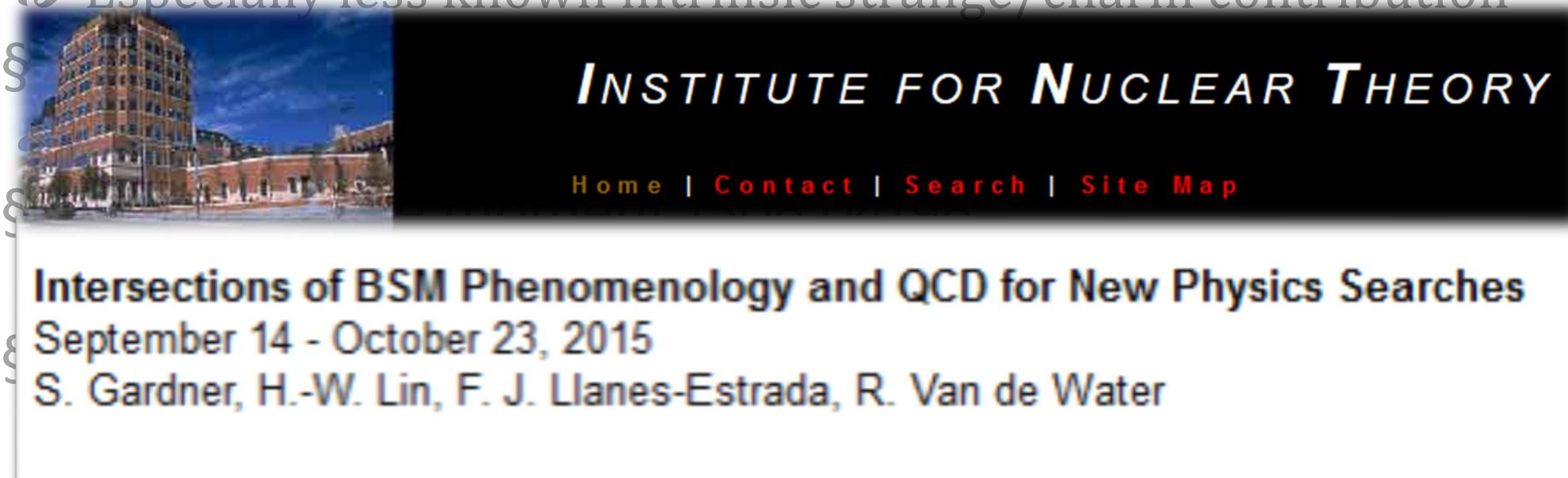

  (mediating new forces) with masses in the multi-TeV range

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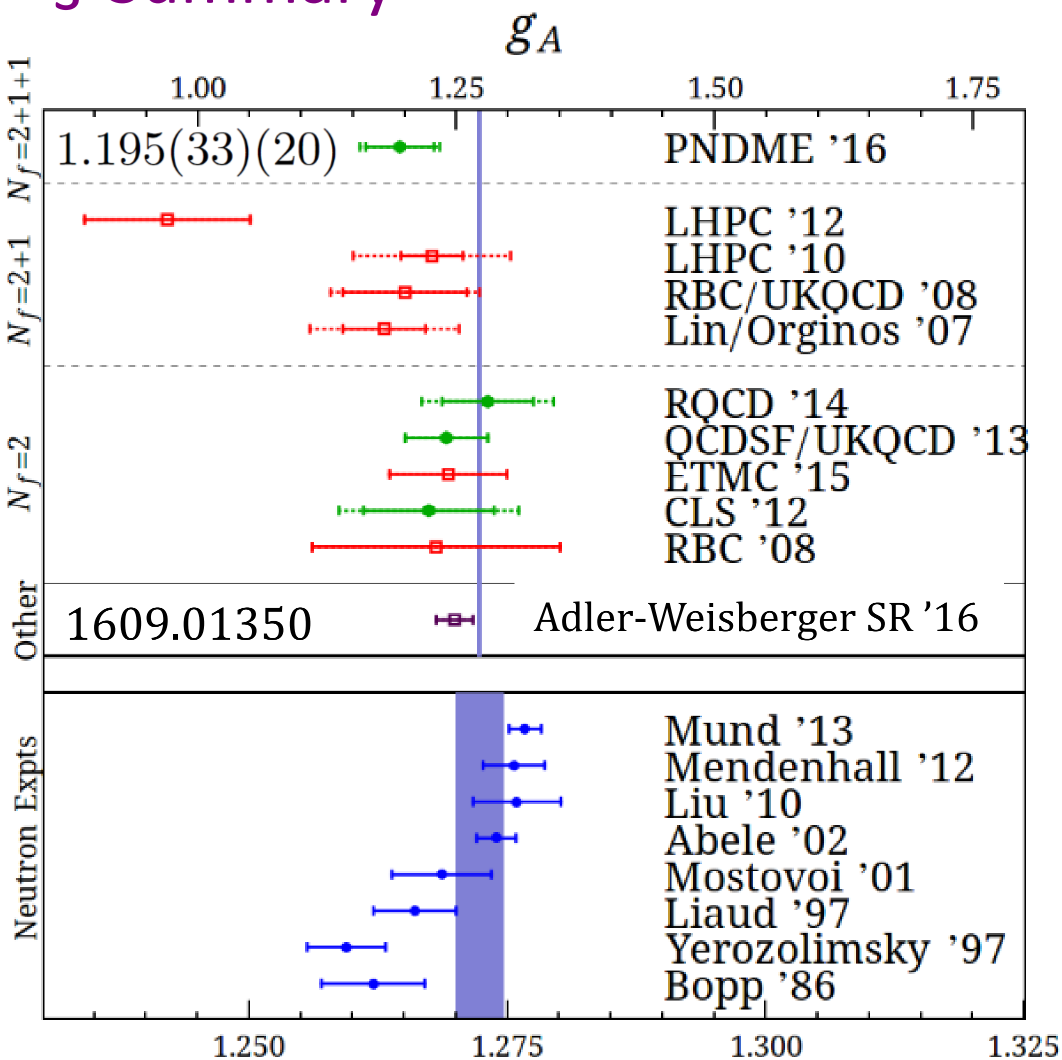
Nucleon Axial Charge

§ Summary



Nucleon Axial Charge

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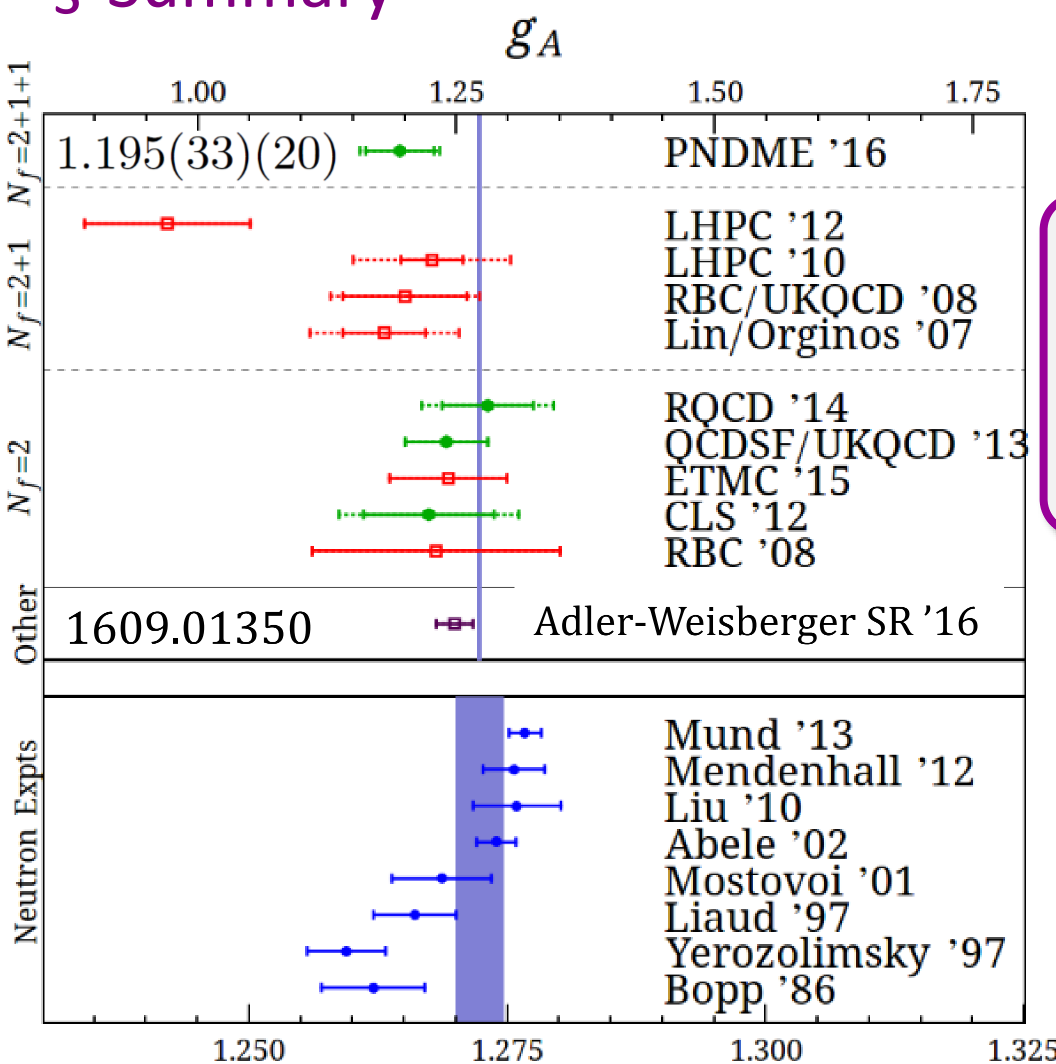


§ Implications?

- 2σ might go away with greater statistics

Lattice 2016 Prelim.

- RBC* 2+1f 1.15(4)
- PACS* 2+1f 1.18(4)

§ What's going on?

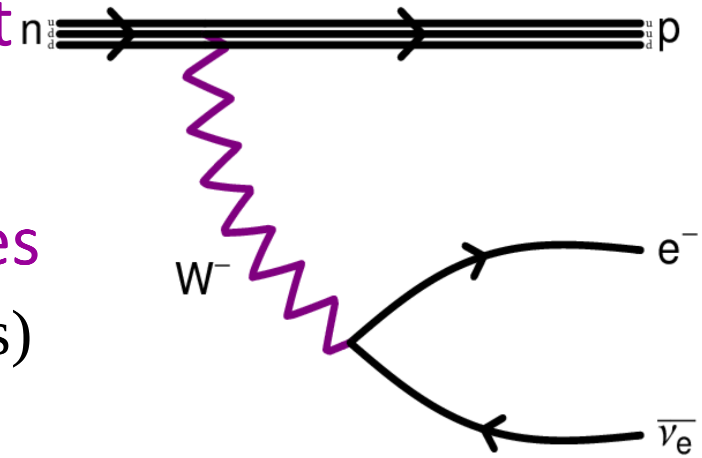
Outline

- § What do we really know about axial charge?
  - Revisit the experiment
- § Does LQCD calculation control ALL systematics?
  - Issues and problems
  - The tale of a 6-year quest
- § Conclusions(?)

Nucleon Axial Charge

- A fundamental measure of nucleon structure
- Axial-vector–current matrix element

$$g_A = G_A^{u-d}(Q^2{=}0)$$



- Important to many nuclear processes
  - The rate of *pp* fusion (as in Sun-like stars)
  - $0\nu\beta\beta$ searches, "quenching" g_A^4
  - V_{ud} values through *n*-lifetime measurements
  - New-physics searches such as right-handed neutrinos

- In lattice QCD, it was long called

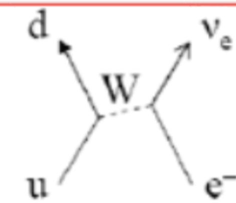
  "A benchmark for nucleon structure"

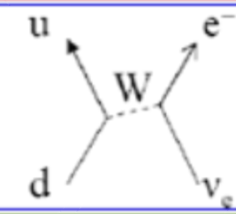
Nucleon Axial Charge

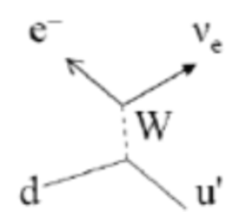
§ A fundamental measure of nucleon structure

Processes governed by lifetime and g_A/g_V

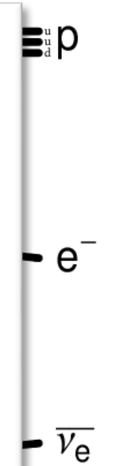
Cosmology	Primordial element formation	$n + e^+ \to p + \bar{\nu}_e$	$\sigma_\nu \sim 1/\tau$
	(^{2}H, ^{3}He, ^{4}He, ^{7}Li, …)	$p + e^- \to n + \nu_e$	$\sigma_\nu \sim 1/\tau$
		$n \to p + e^- + \bar{\nu}_e$	τ
Astronomy	Solar cycle	$p + p \to {}^2H + e^+ + \nu_e$	
		$p + p + e^- \to {}^2H + \nu_e$ etc.	$\sim (g_A/g_V)^5$
	Neutron star formation	$p + e^- \to n + \nu_e$	
Particle Physics	Pion decay	$\pi^- \to \pi^0 + e^- + \bar{\nu}_e$	
	Neutrino detectors	$\bar{\nu}_e + p \to e^+ + n$	
	Neutrino forward scattering	$\nu_e + n \to e^- + p$ etc.	
	W and Z production	$\bar{u} + d \to W^- \to e^- + \bar{\nu}_e$ etc.	







from D. Dubbers



Nucleon Axial Charge

§ Ask somebody what they know about the axial charge…

- The PDG number has errorbars so tiny, we just drop the error!

Particle Data Group

- Ask somebody what they know about the axial charge…
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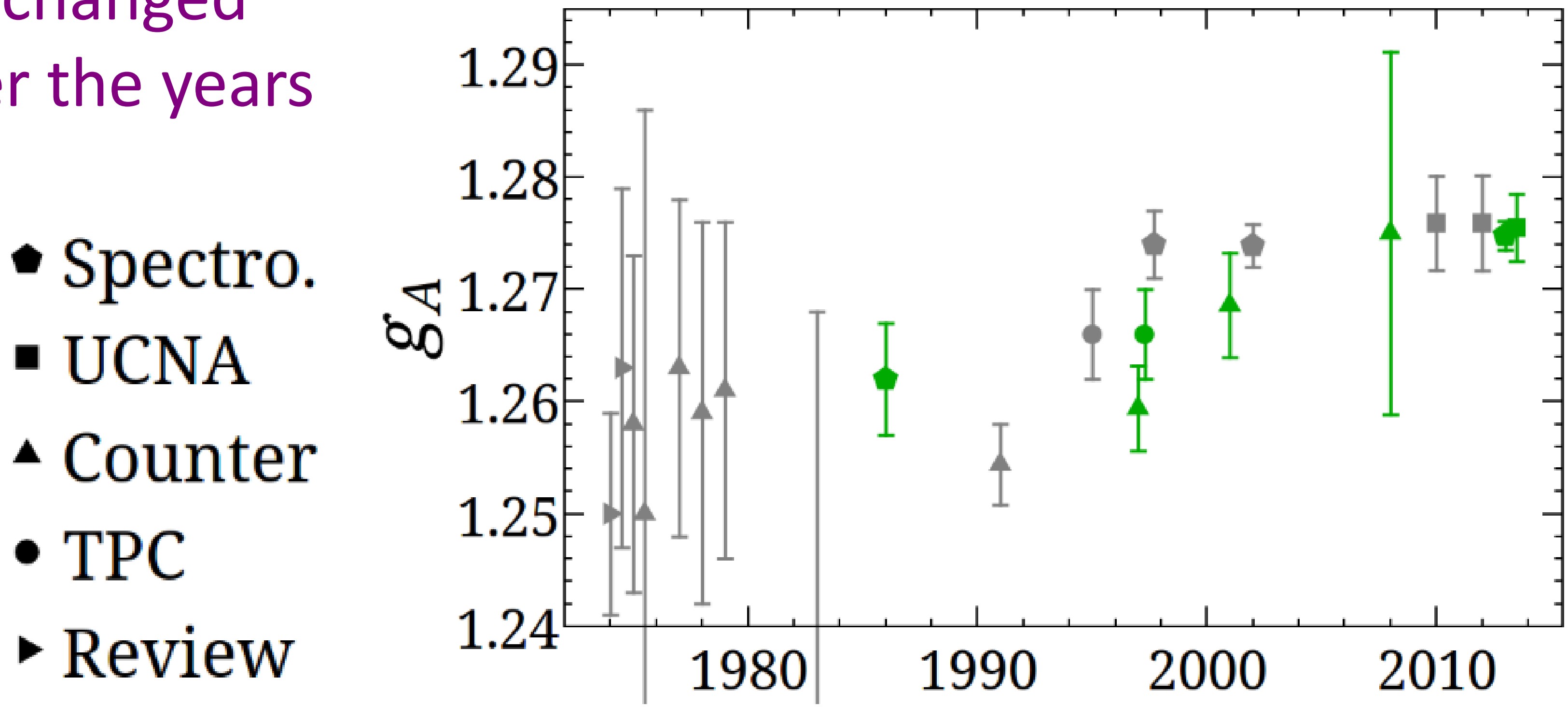
- If you look closer, it's changed over the years



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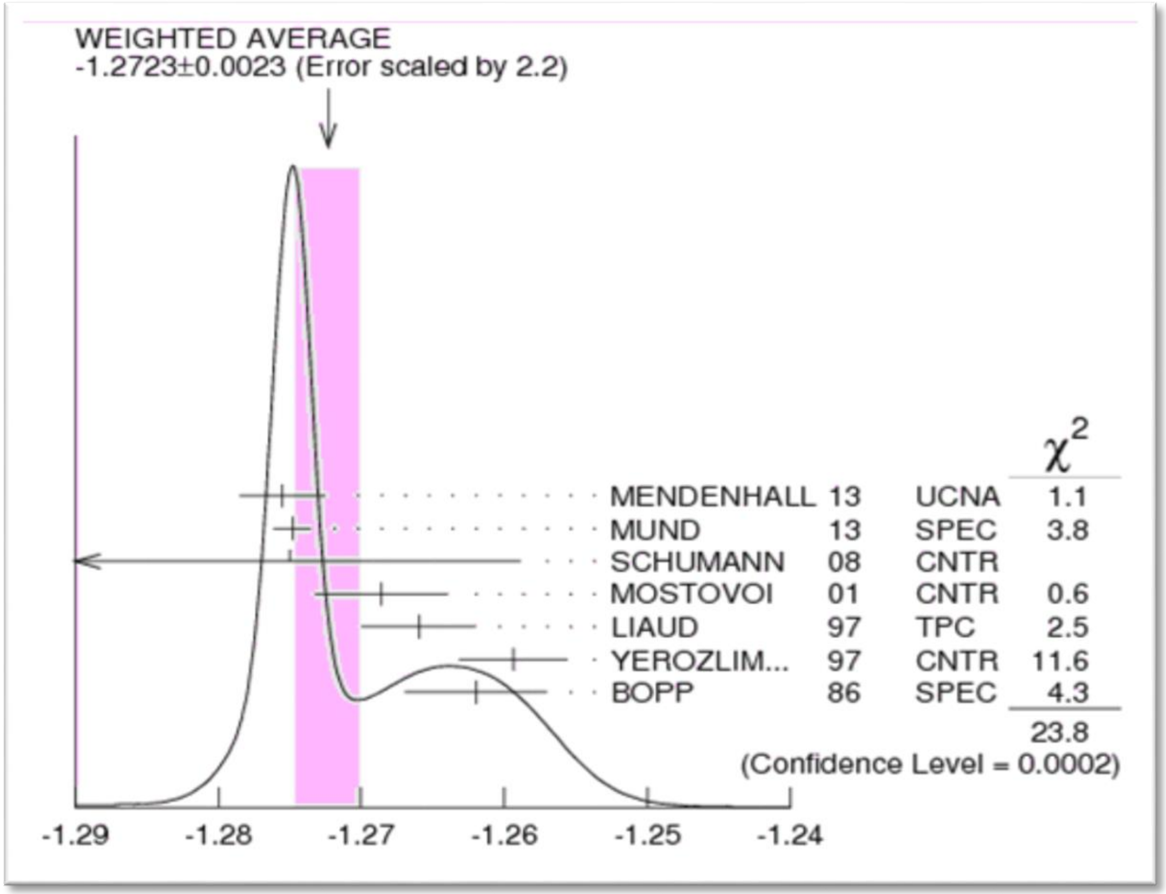

Weak Experiments

§ Let us look closely at how g_A is determined experimentally

§ Two main types of experimental input

- Asymmetry in neutron differential decay rate (by UCN)

$$d\Gamma \propto F(E_e)\left(1 + a\frac{\overrightarrow{p_e}\cdot\overrightarrow{p_\nu}}{E_e E_\nu} + A\frac{\overrightarrow{\sigma_n}\cdot\overrightarrow{p_e}}{E_e} + \cdots\right) \qquad A_0 = \frac{-2(\lambda^2 - |\lambda|)}{1+3\lambda^2}$$

$$\lambda = G_A/G_V = 1.2755(30) \quad \text{UCNA 13}$$

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$$\lambda = G_A/G_V = 1.2755(30) \text{ UCNA 13}$$

  - n-lifetime decay (requires additional input V_{ud})

$$\tau_n^{\text{ave}} = 880.2(1.0) \text{ sec} \qquad |V_{ud}|^2 = \frac{4908.7(1.9) \text{ sec}}{\tau_n(1+3g_A^2)}$$

    - V_{ud} from…
      - nuclear $0^+ \to 0^+$ superallowed: $0.97417(21) \Rightarrow g_A = 1.2749(10)$
      - $\text{BR}(0^+ \to e^+\nu_e(\gamma))$: $0.9728(30) \Rightarrow g_A = 1.2771(44)$

Weak Experiments

- Let us look closely at how g_A is determined experimentally
- Two main typ

❧ Asymmetry i

$d\Gamma \propto F(E_e)\Big(1 +$

λ

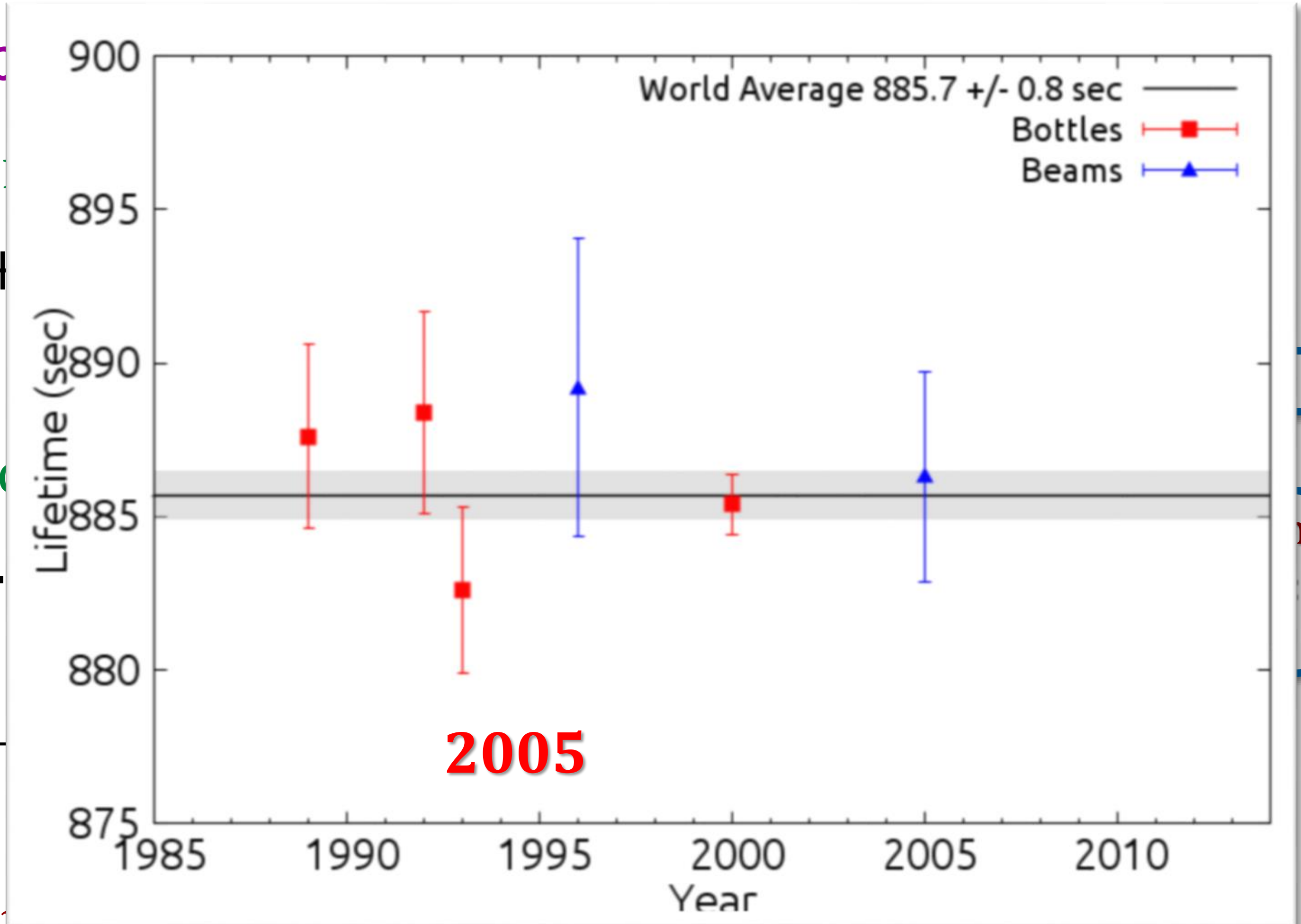
❧ n-lifetime de

$\tau_n^{\text{ave}} = 880.$

❧ V_{ud} from...

❧ nuclear 0^+ –

❧ BR($0^+ \rightarrow e^+$



g_A

Weak Experiments

- § Let us look closely at how g_A is determined experimentally
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  - Asymmetry i
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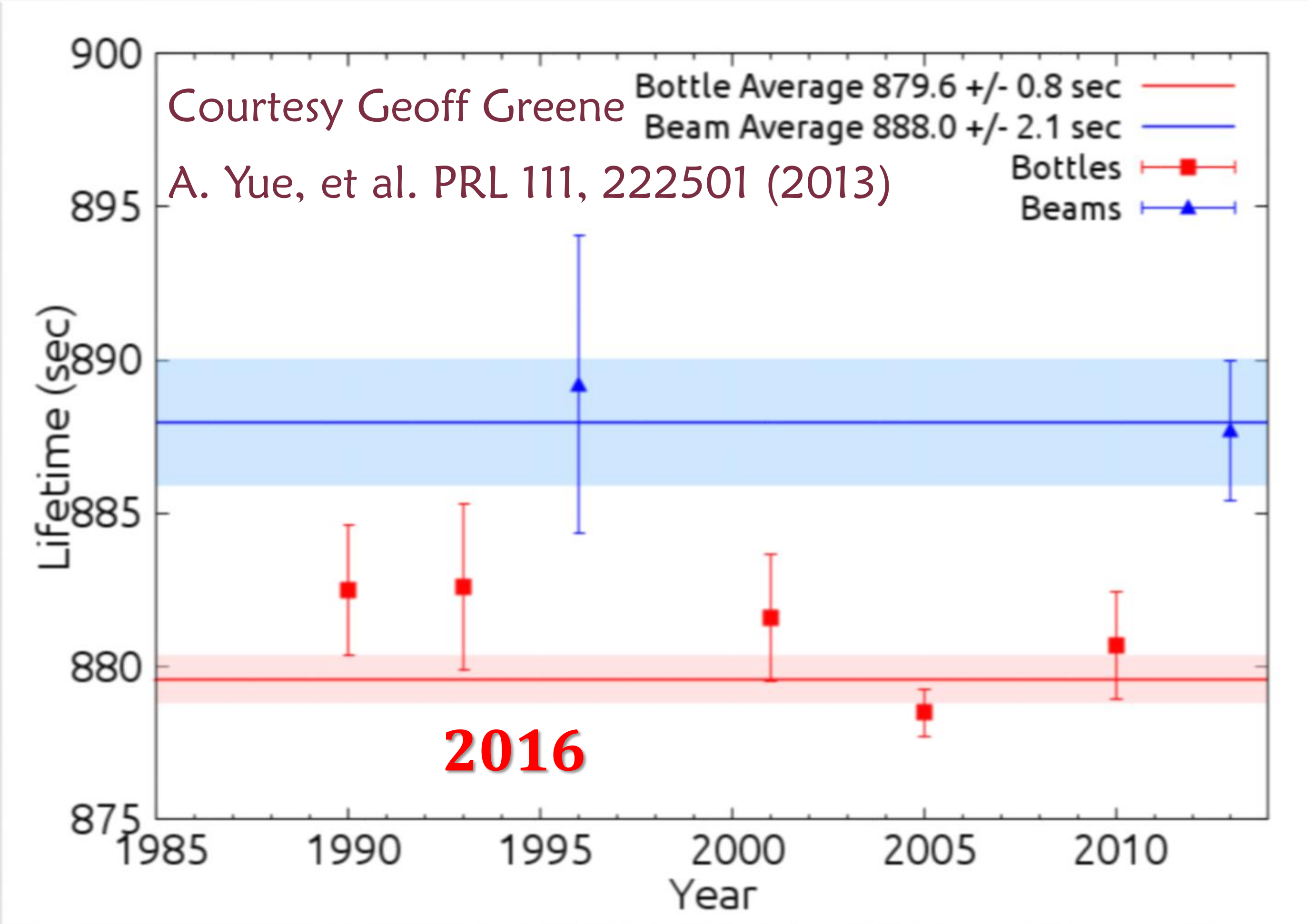

Experiments

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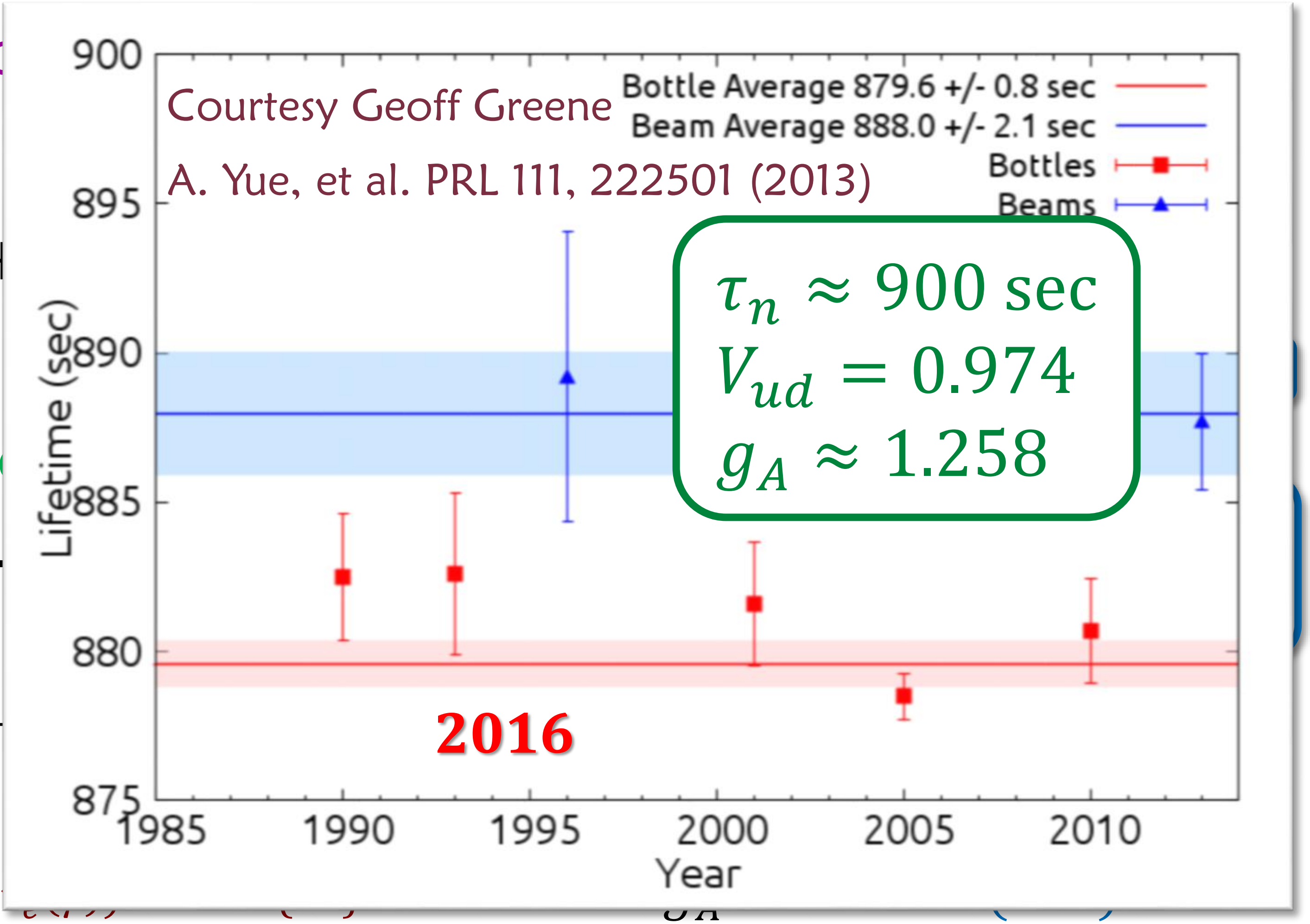
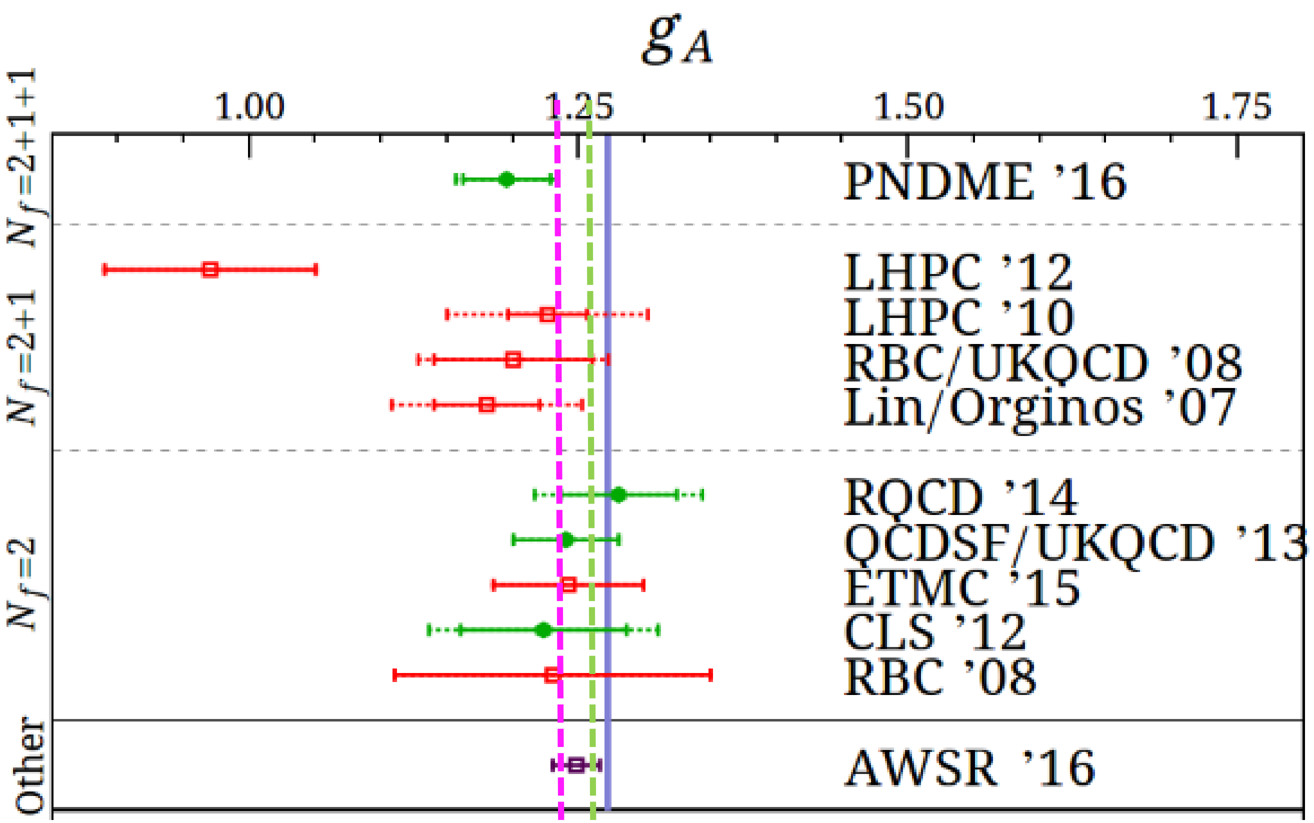


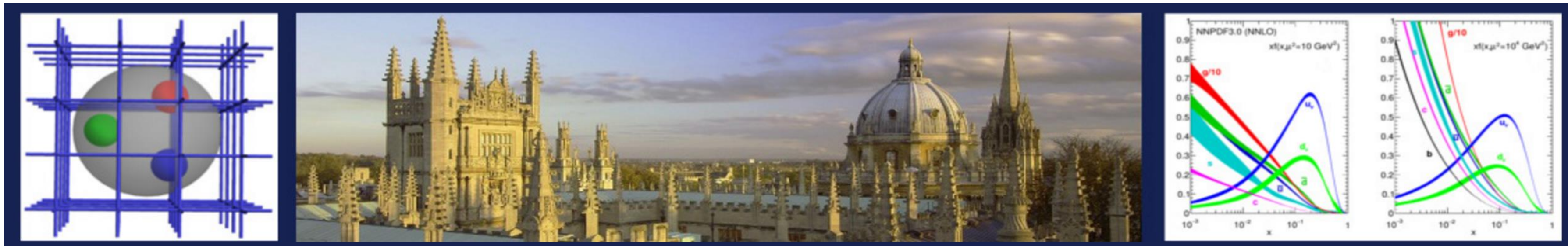
Experiments

- What can we infer about g_A from other observables?
- Constraints from V_{ud} experiments (must be ≤ 1)
  - The allowed region is $g_A \geq 1.23524(98)$



QCD Experiment

- How about QCD experiments?
  - With a polarized target or polarized beam, one can find the helicity distribution and get g_A
  - Global analysis? g_A is used as a constraint
- LQCD currently is the only reliable QCD source for g_A
- Does LQCD g_A agree with QCD experiments?



Parton Distributions and Lattice Calculations in the LHC era
(PDFLattice 2017)
22-24 March 2017, Oxford, UK

  - First workshop with global-fit community to address LQCD
    - http://www.physics.ox.ac.uk/confs/PDFlattice2017

Lattice Aspects

The Trouble with Nucleons

Nucleons are more complicated than mesons because…

- Noise issue
  - Signal diminishes at large t_E relative to noise
  - Gets worse when quark mass decreases
- Excited-state contamination
  - Nearby excited state: Roper(1440)
- Hard to extrapolate in pion mass
  - Δ resonance nearby; multiple expansions, poor convergence…
  - Less an issue in the physical pion-mass era
- Requires larger volume and higher statistics
  - Ensembles are not always generated with nucleons in mind
  - **High-statistics**: large measurement and long trajectory

The Trouble with Nucleons

Nucleons are more complicated than mesons because…

- § Noise iss
  - Signal di
  - Gets wo
- § Excited-s
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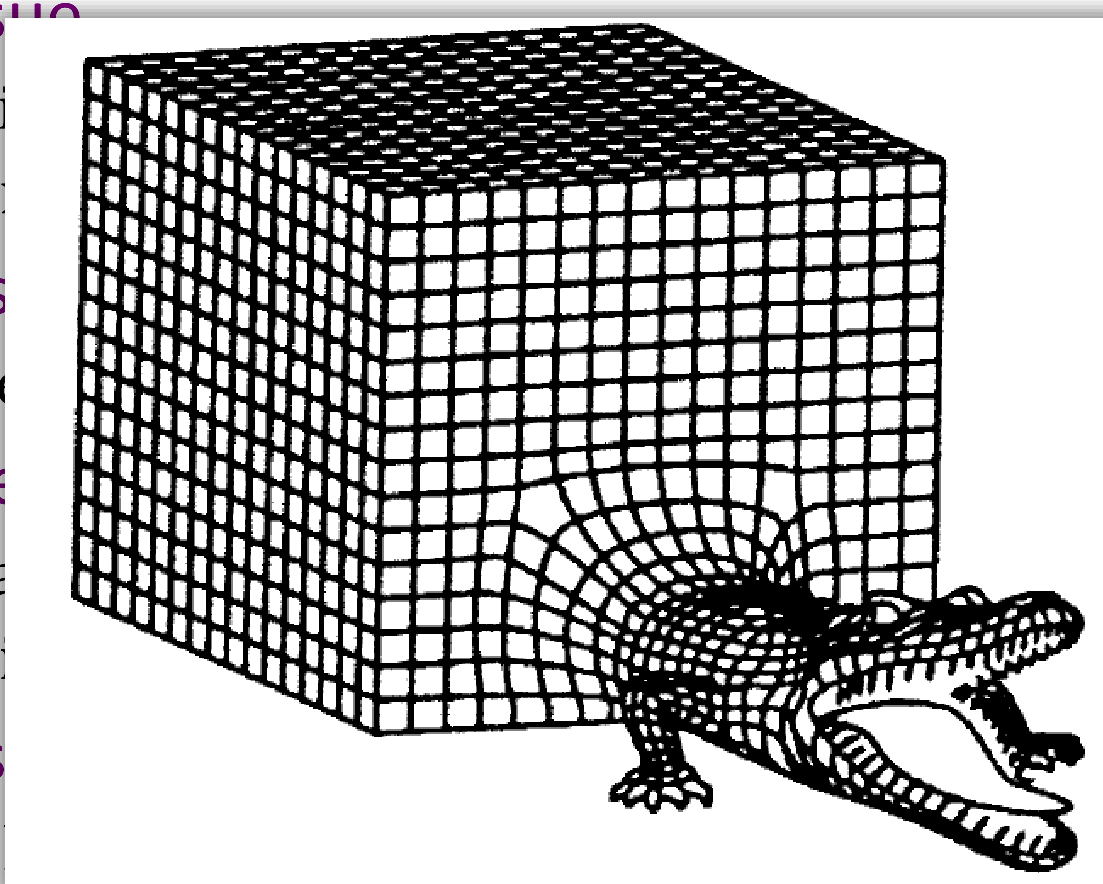
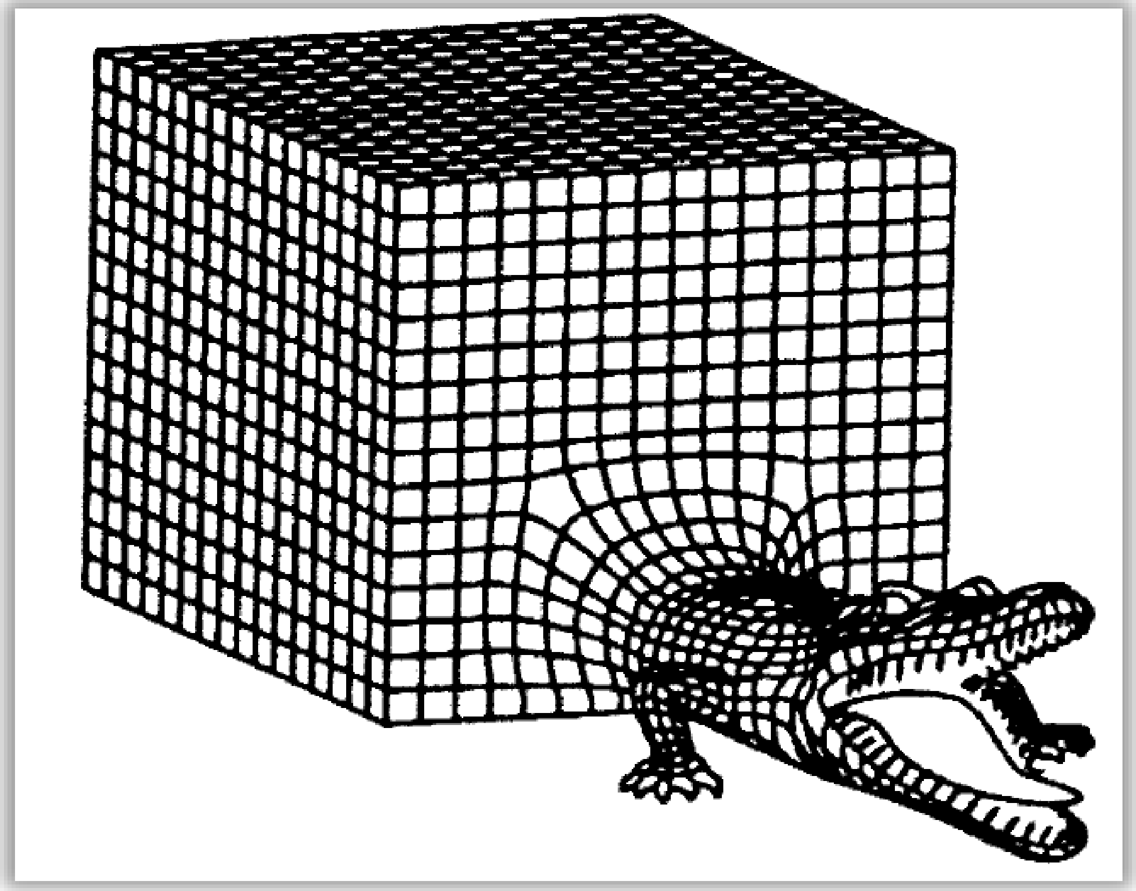


The Trouble with Nucleons

"Welcome to the lattice and its dangerous animals."

Karl Jansen







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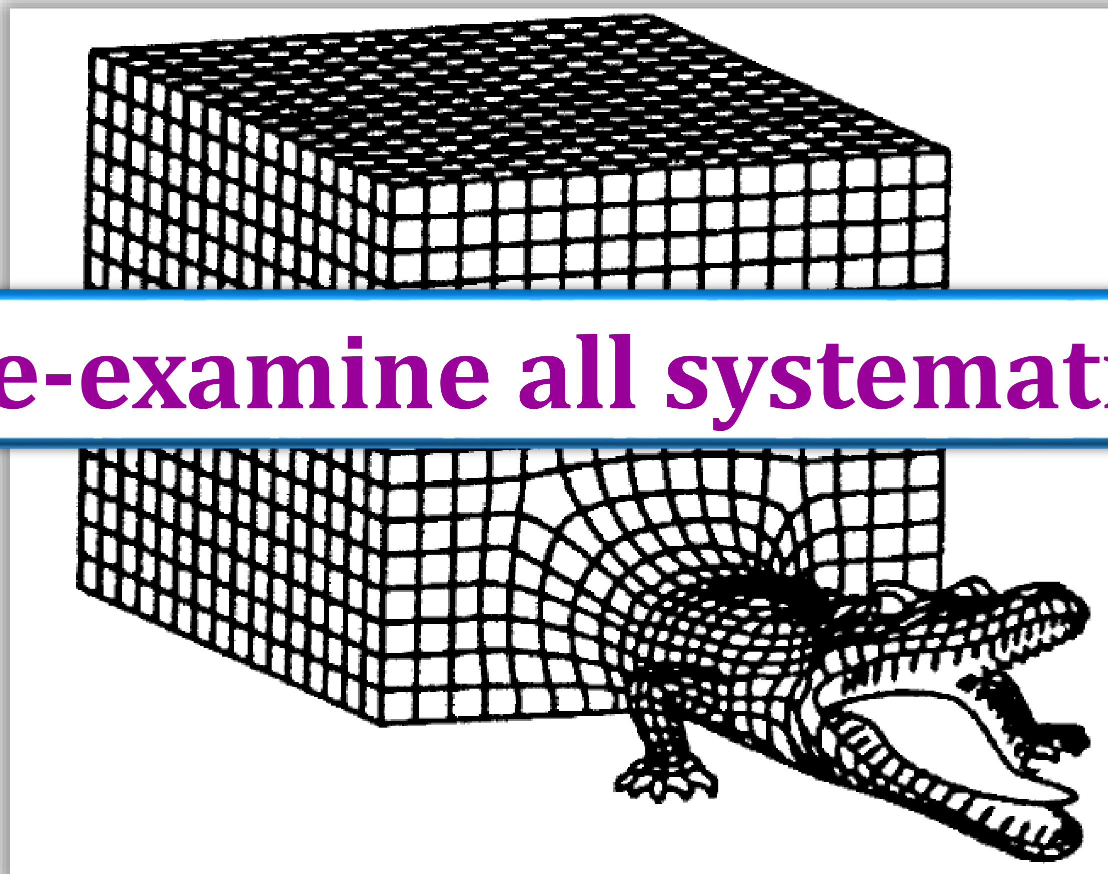
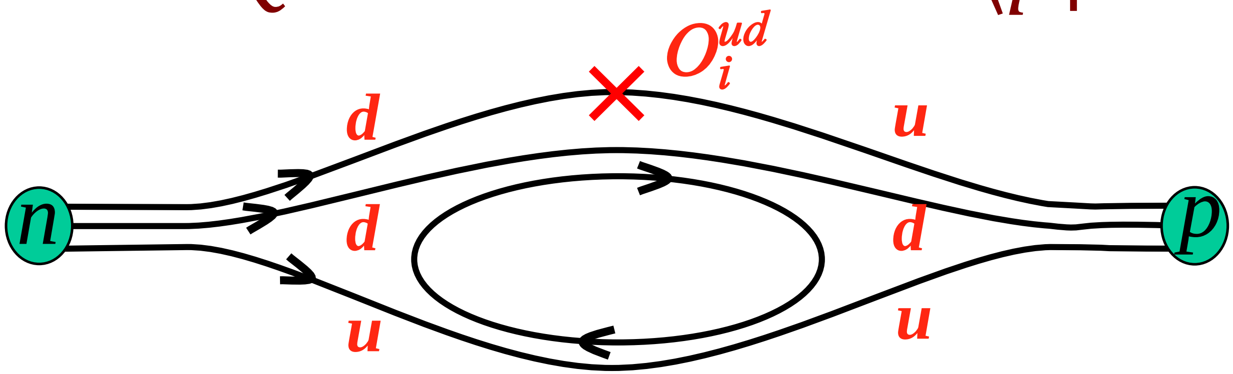


Re-examine all systematics!

PROCEED WITH CAUTION

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle p|\bar{u}\Gamma d|n\rangle$



§ Control all systematic errors:

- Finite-volume effects
- Chiral extrapolations to physical u and d quark masses
- Extrapolation to the continuum limit (lattice spacing $a \to 0$)
- Nonperturbative renormalization using the RI/SMOM scheme
- Contamination from excited states
- Statistical effects

PNDME

Precision Neutron-Decay Matrix Elements

https://sites.google.com/site/pndmelqcd/

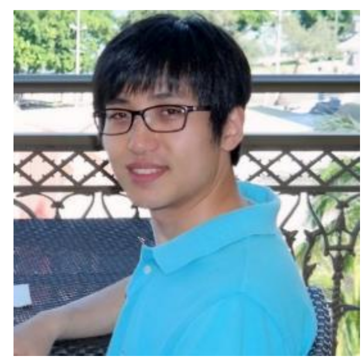
Tanmoy Bhattacharya | Rajan Gupta | HWL | Vincenzo Cirigliano




+







Saul Cohen | Anosh Joseph | Yong-Chull Jang | Boram Yoon

Precision Nucleon Couplings

- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME)

a (fm)	V	$M_\pi L$	M_π (MeV)	t_{sep}	# Meas.
0.12	$24^3 \times 64$	4.55	310	8,10,12	64.8k
0.12	$24^3 \times 64$	3.29	220	8,10,12	24k
0.12	$32^3 \times 64$	4.38	220	8,10,12	7.6k
0.12	$40^3 \times 64$	5.49	220	8,10,12,14	64.6k
0.09	$32^3 \times 96$	4.51	310	10,12,14	7.0k
0.09	$48^3 \times 96$	4.79	220	10,12,14	7.1k
0.09	$64^3 \times 96$	3.90	130	10,12,14	56.5k
0.06	$48^3 \times 144$	4.52	310	16,20,22,24	64.0k
0.06	$64^3 \times 144$	4.41	220	16,20,22,24	41.6k
0.06	$96^3 \times 192$	3.80	130		On-going

Excited-State Contamination

§ Trade off: signal-to-noise versus contamination

- Noise issue (P. Lepage; D. Kaplan)
- Consider a baryon correlator $C = \langle O \rangle = \langle qqq(t)\bar{q}\bar{q}\bar{q}(0) \rangle$
- Variance (noise squared) of $C \propto \langle O^\dagger O \rangle - \langle O^2 \rangle$

What you want:

Signal falls exponentially as
$\boldsymbol{e^{-m_N t}}$

Excited-State Contamination

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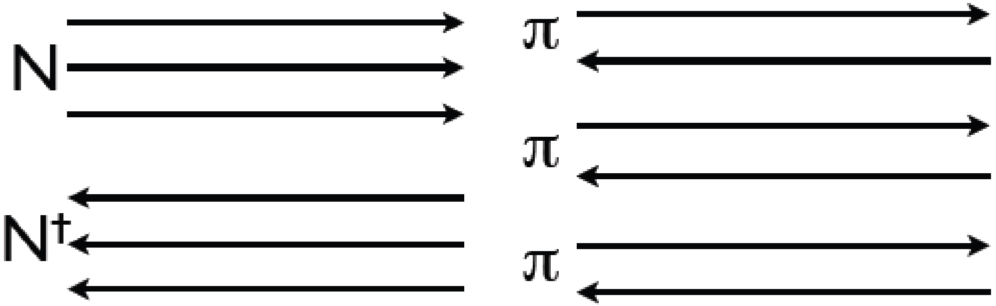
What you want: What you get:



Signal falls exponentially as $e^{-m_N t}$

Noise falls as $e^{-\frac{3}{2}m_\pi t}$

§ Difficulties in Euclidean space

- True ground state (nucleon in this case) at large Euclidean time

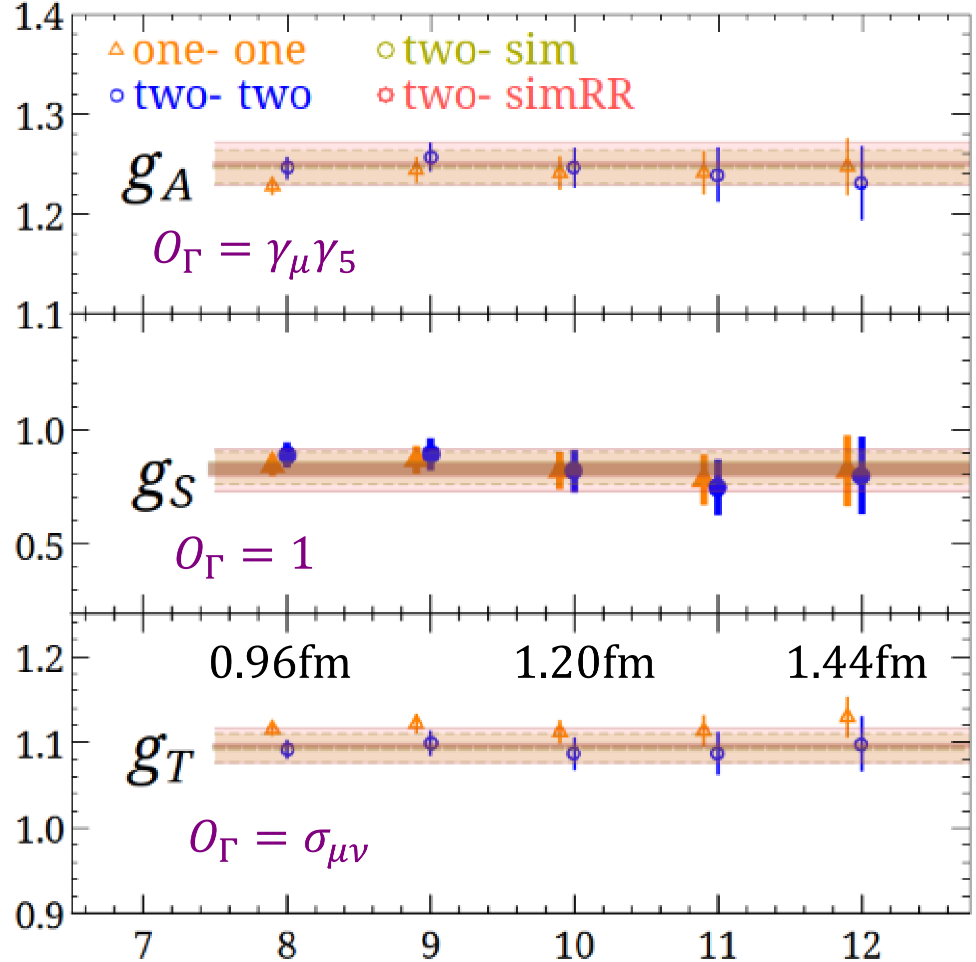
Systematic Control

- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME) $a = \mathbf{0.12}$ **fm**, 310-MeV pion

- Move the **excited-state systematic** into the statistical error

$$C^{3\text{pt}}(t_f,t,t_i) = |\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$
$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0|\mathcal{O}_\Gamma|1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)}$$
$$+\mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e^{-M_1(t-t_i)}e^{-M_0(t_f-t)}$$
$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f-t_i)}$$

- No obvious contamination between 0.96 and 1.44 fm separation



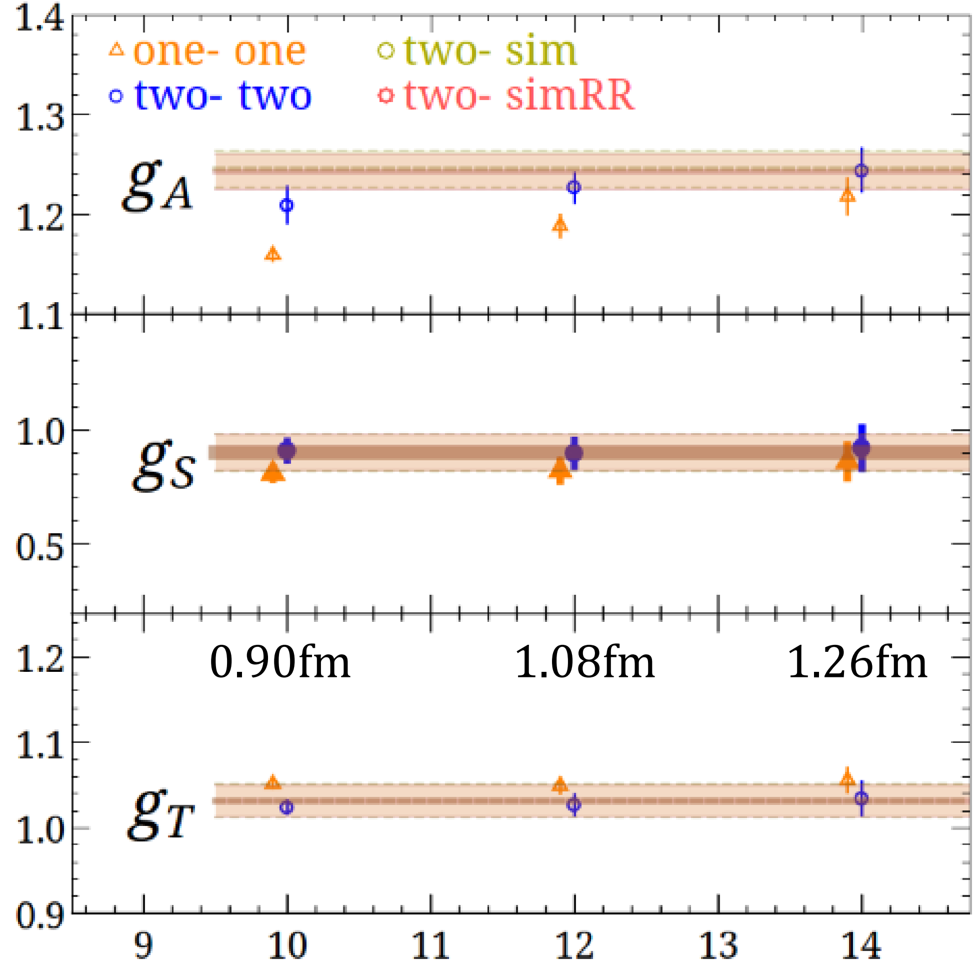
Systematic Control

- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME) $a = 0.09$ fm, 310-MeV pion

  - Move the **excited-state systematic** into the statistical error

$$C^{\rm 3pt}(t_f,t,t_i) = |\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$
$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0|\mathcal{O}_\Gamma|1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)}$$
$$+\mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e^{-M_1(t-t_i)}e^{-M_0(t_f-t)}$$
$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f-t_i)}$$

  - Much stronger effect at finer lattice spacing!
  - Needs to be studied case by case



Systematic Control

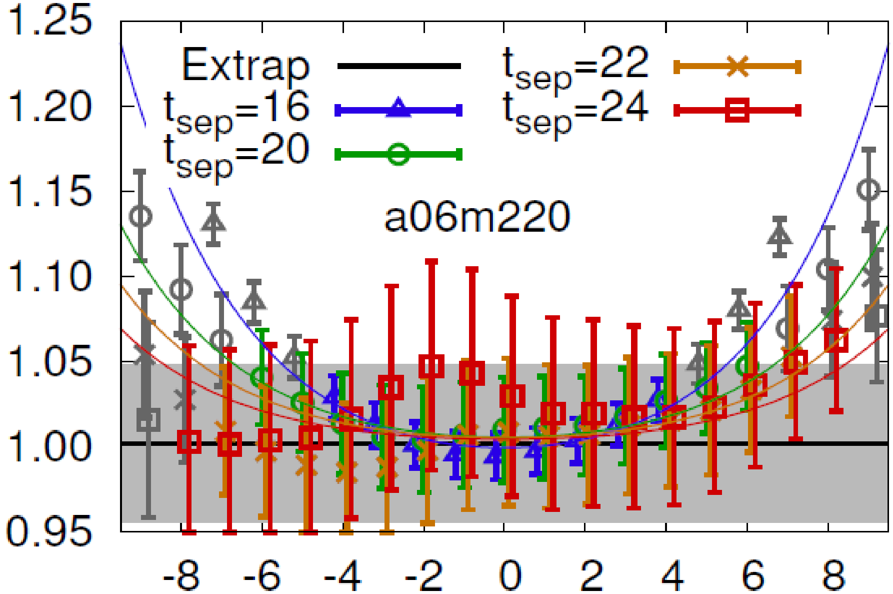
- § Much effort has been devoted to controlling systematics
- § A state-of-the art calculation (PNDME)

❧ Statistical effect

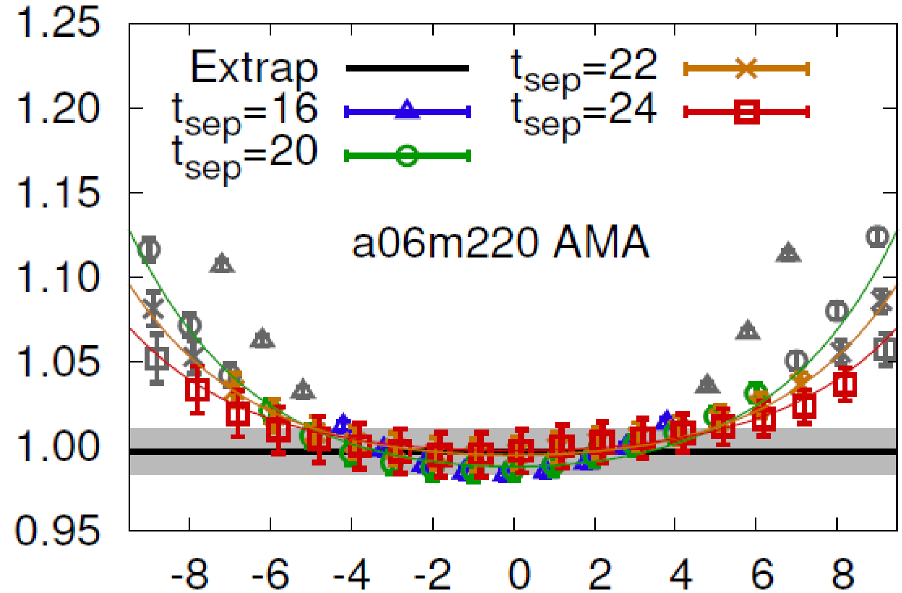
$a = 0.06$ fm, 220-MeV pion

g_T^{bare}

2.6k



41.6k



Plots by Boram Yoon

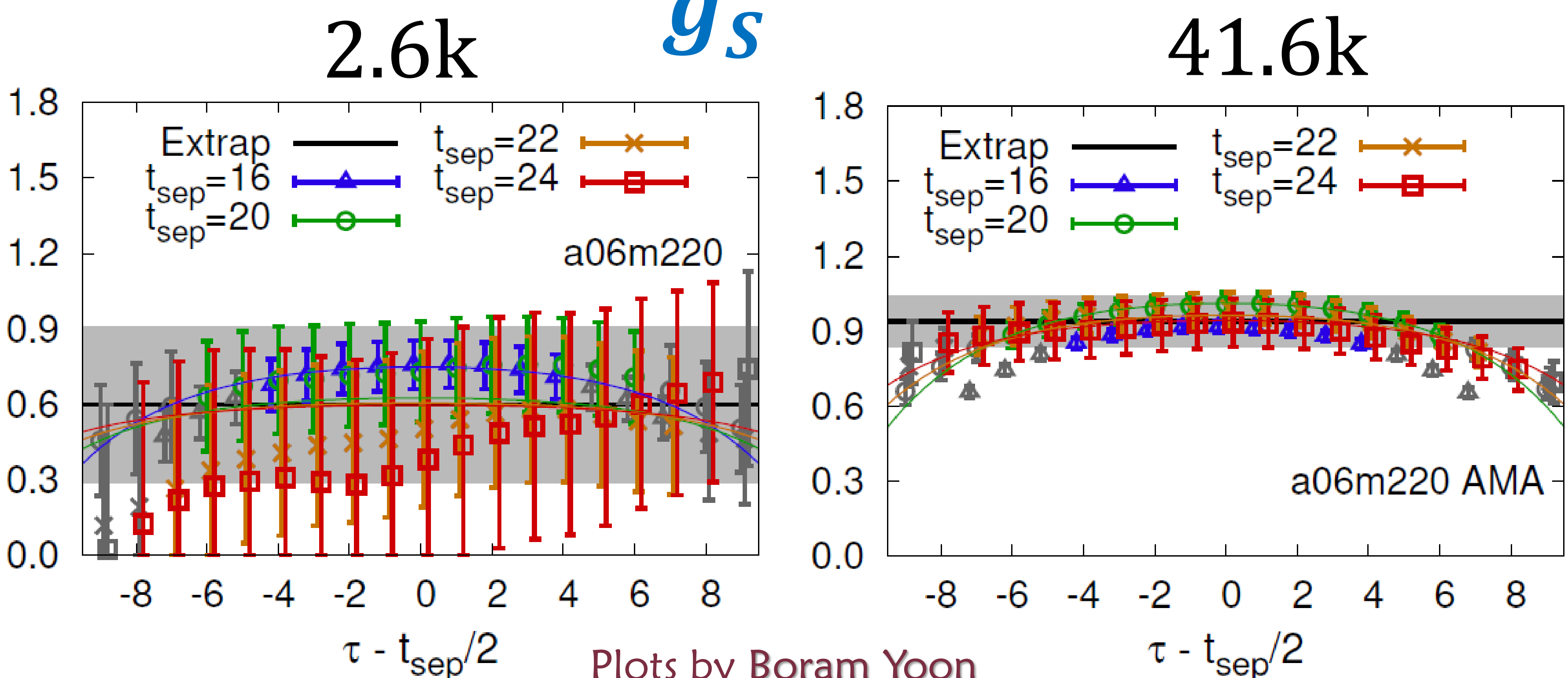
Systematic Control

- § Much effort has been devoted to controlling systematics
- § A state-of-the art calculation (PNDME)

❧ Statistical effect

$a = 0.06$ fm, 220-MeV pion

g_S^{bare}



Plots by Boram Yoon

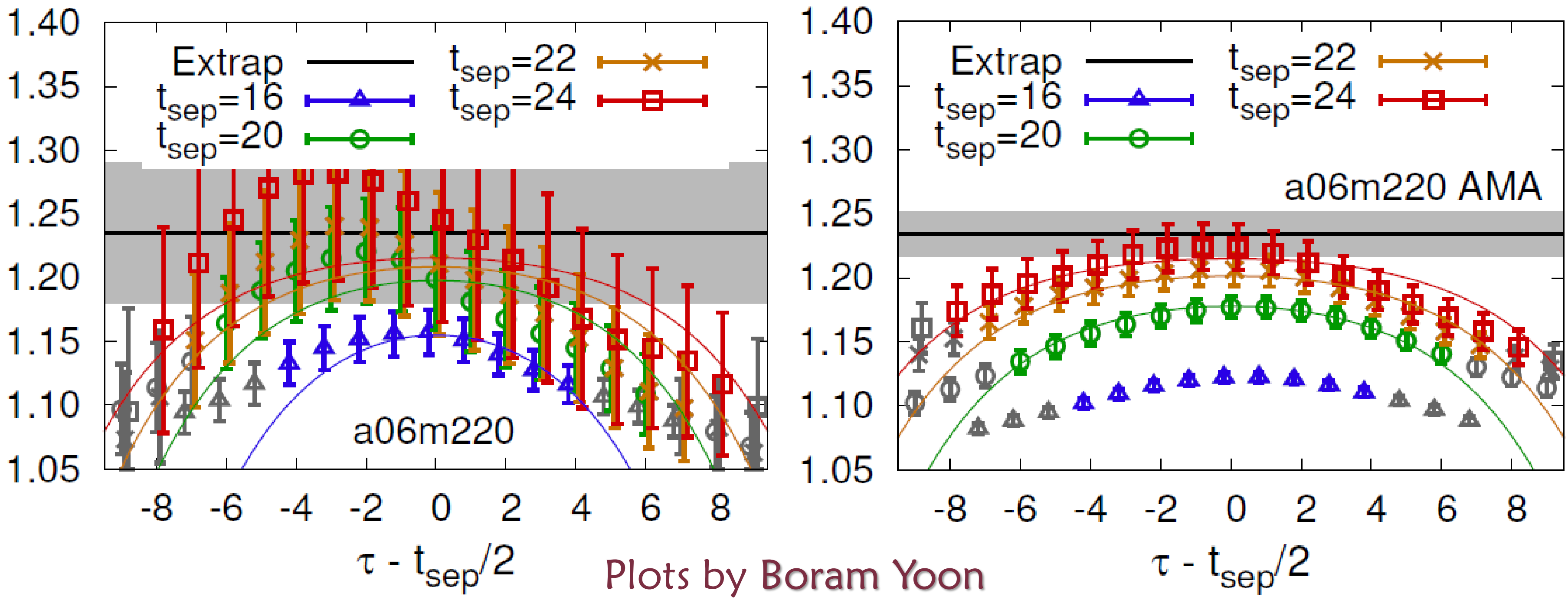
Systematic Control

- § Much effort has been devoted to controlling systematics
- § A state-of-the art calculation (PNDME)

❧ Statistical effect (worst case) $a = 0.06$ fm, 220-MeV pion

$g_A^{\mathbf{bare}}$

2.6k 41.6k



Plots by Boram Yoon

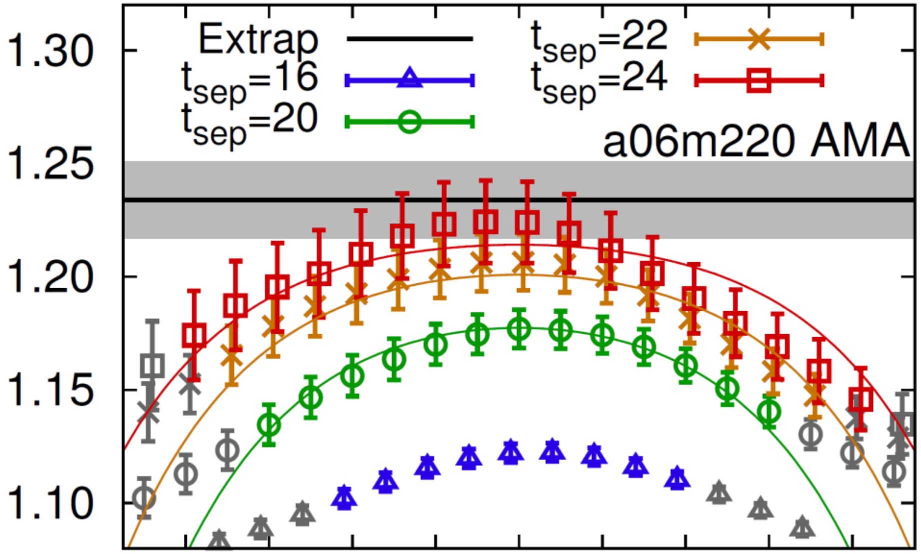
Systematic Control

- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME)

Robustness of the 2-state fit $a = 0.06$ fm, 220-MeV pion

g_A^{bare}

2.6k



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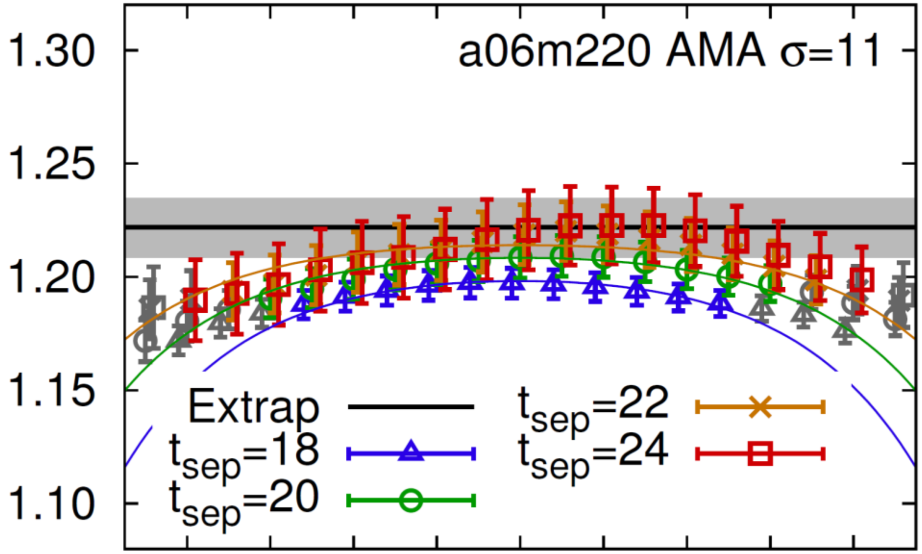
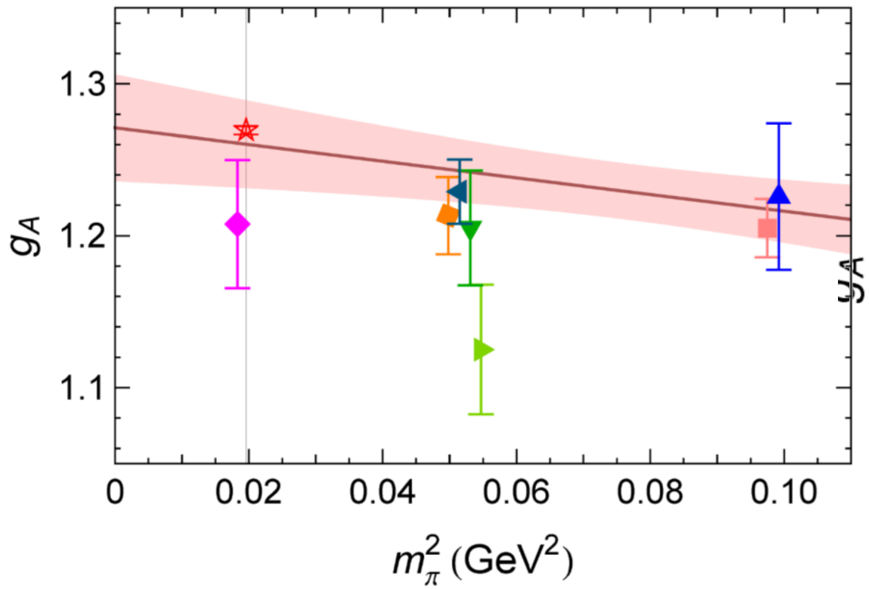
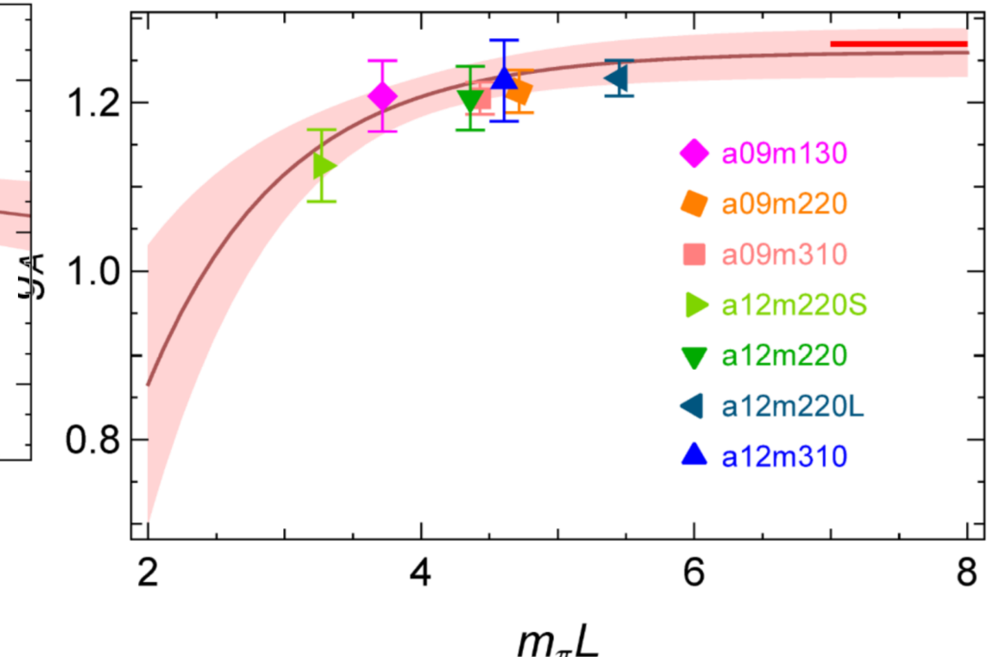


Plots by Boram Yoon

Extrapolations

§ Finite-volume/statistical effects



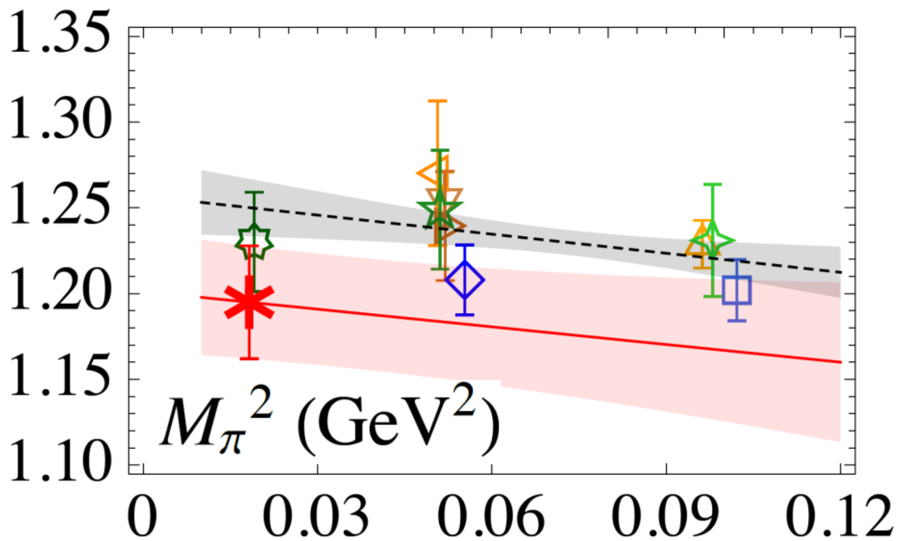


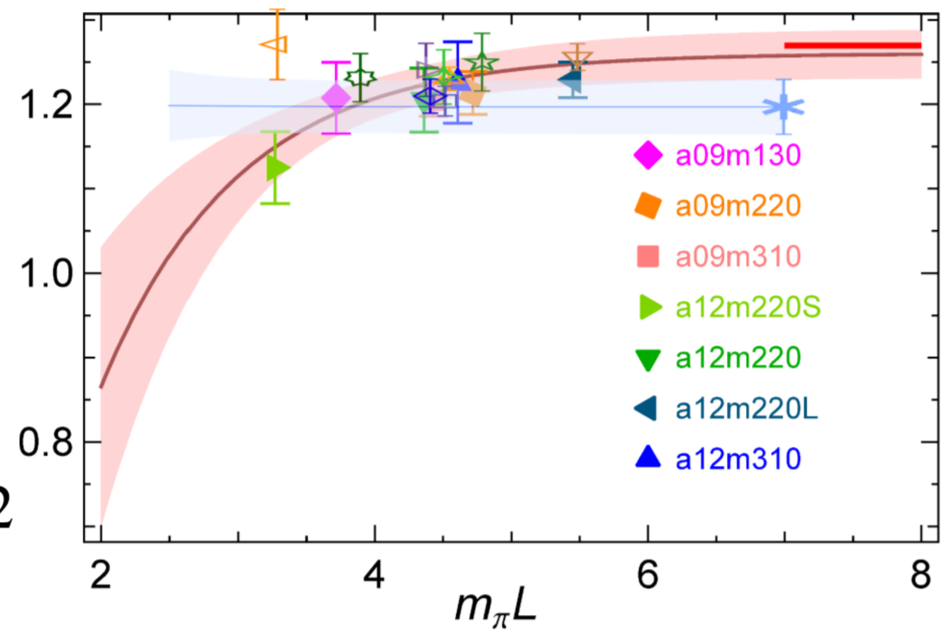
2013 Results

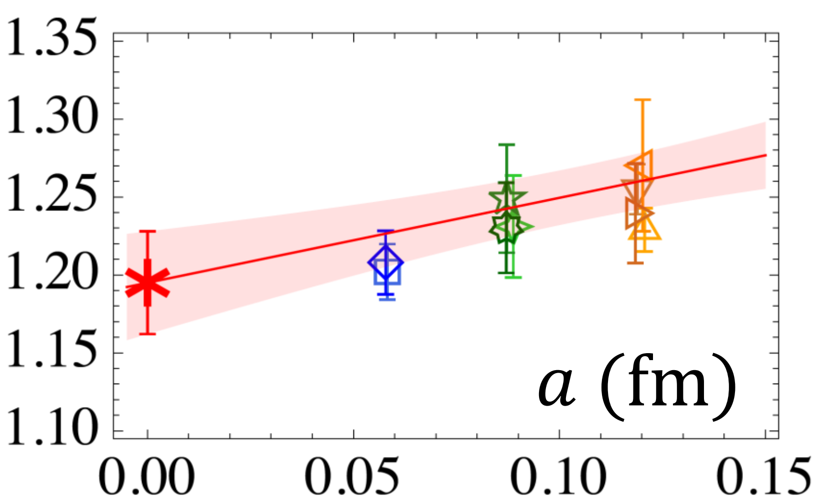
$$g_T(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$

Extrapolations

§ Finite-volume/statistical effects



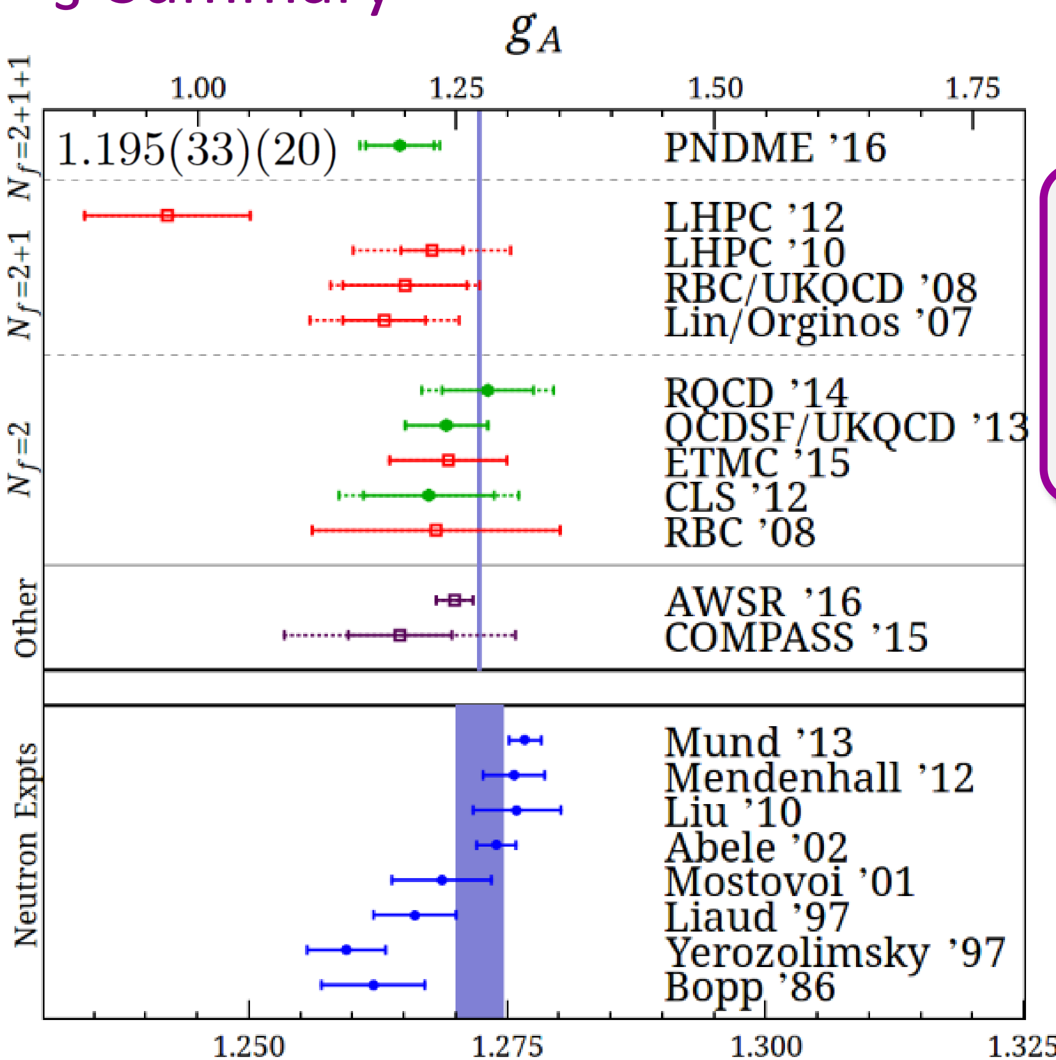




2016 Results

Here we are

§ Summary



§ Implications?

- 2σ might go away with greater statistics

Lattice 2016 Prelim.

- RBC* 2+1f 1.15(4)
- PACS* 2+1f 1.8(4)

§ New physics?

- $\lambda = g_A / g_V \, f_{NP}$

$$A_0 = \frac{-2(\lambda^2 - |\lambda|)}{1 + 3\lambda^2}$$

Conclusions(?)

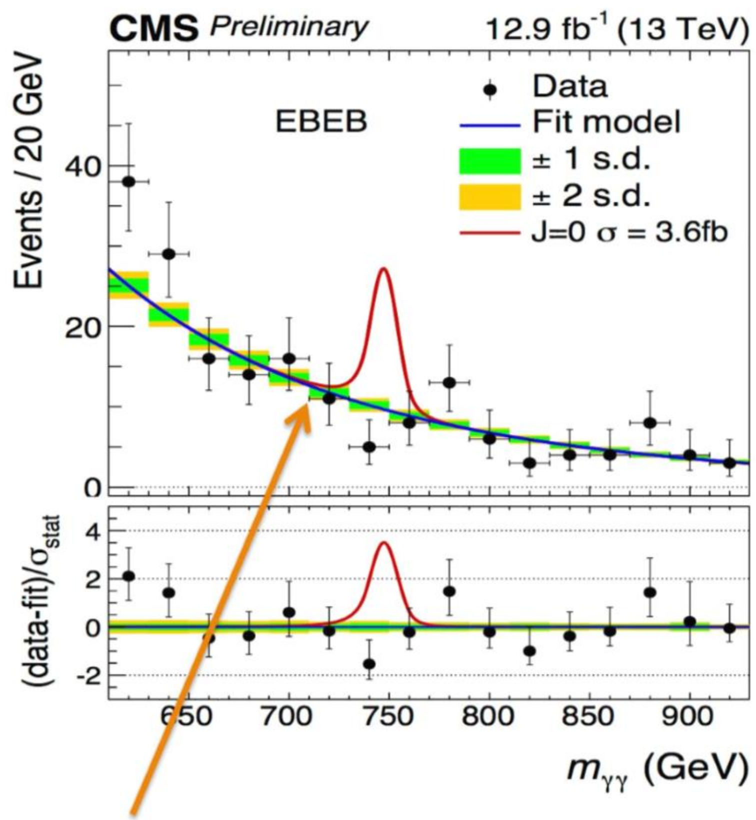
- § g_A is not a gold-plated quantity
  - Early idea that g_A would be easy underestimated systematics
- § High-statistics and large-volume studies are needed!
- § Can you trust other lattice calculations?

  …from groups who do due diligence for every ensemble and carefully study systematics
- § Disappointment?
  - Certainly not.

  We are just entering into the precision era to explore these issues…
- § Difficulties = opportunities
  - Getting g_A to subpercent precision will be very hard
- § New physics?
  - $\lambda = g_A / g_V f_{NP}$ $\qquad A_0 = \frac{-2(\lambda^2 - |\lambda|)}{1+3\lambda^2}$ $\qquad$ Stay tuned…

Can We Trust LQCD?

The disappearance of X(750)



A signal with cross-section as the largest excess in 2015+8TeV would look like this

Can we trust LHC?

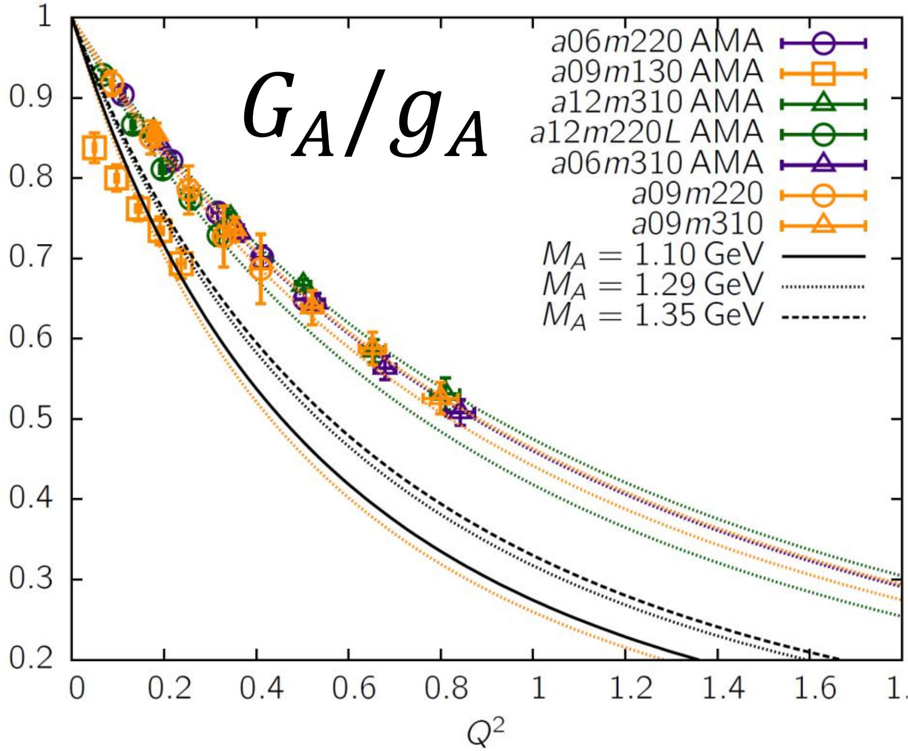
http://resonaances.blogspot.co.uk

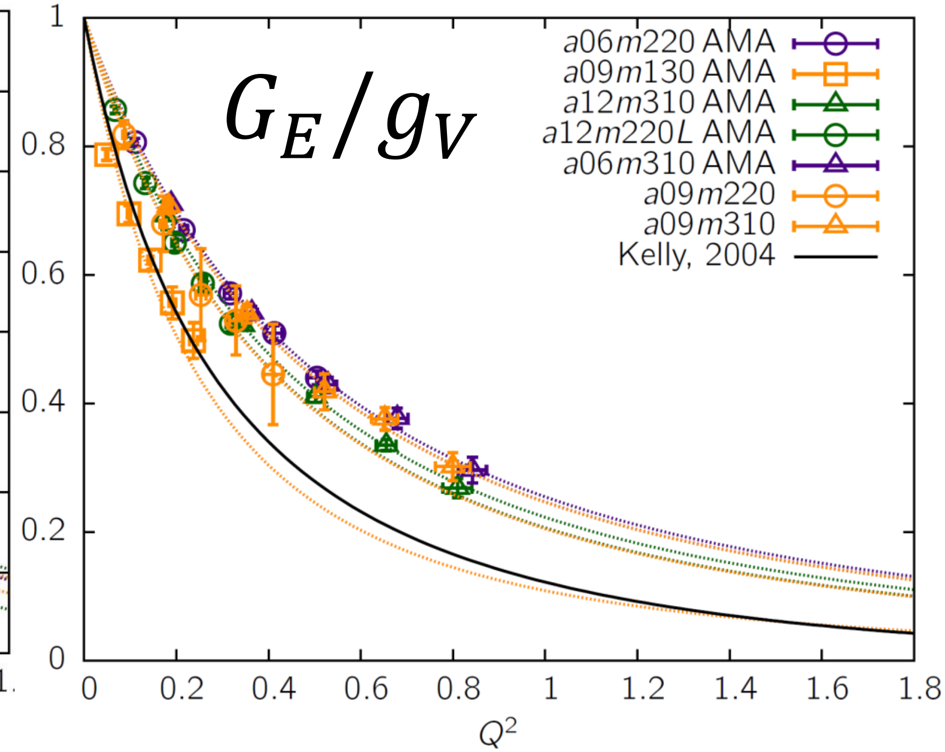
Backup Slides

Other Results

§ Isovector form factors

Plots by Yong-Chull Jang

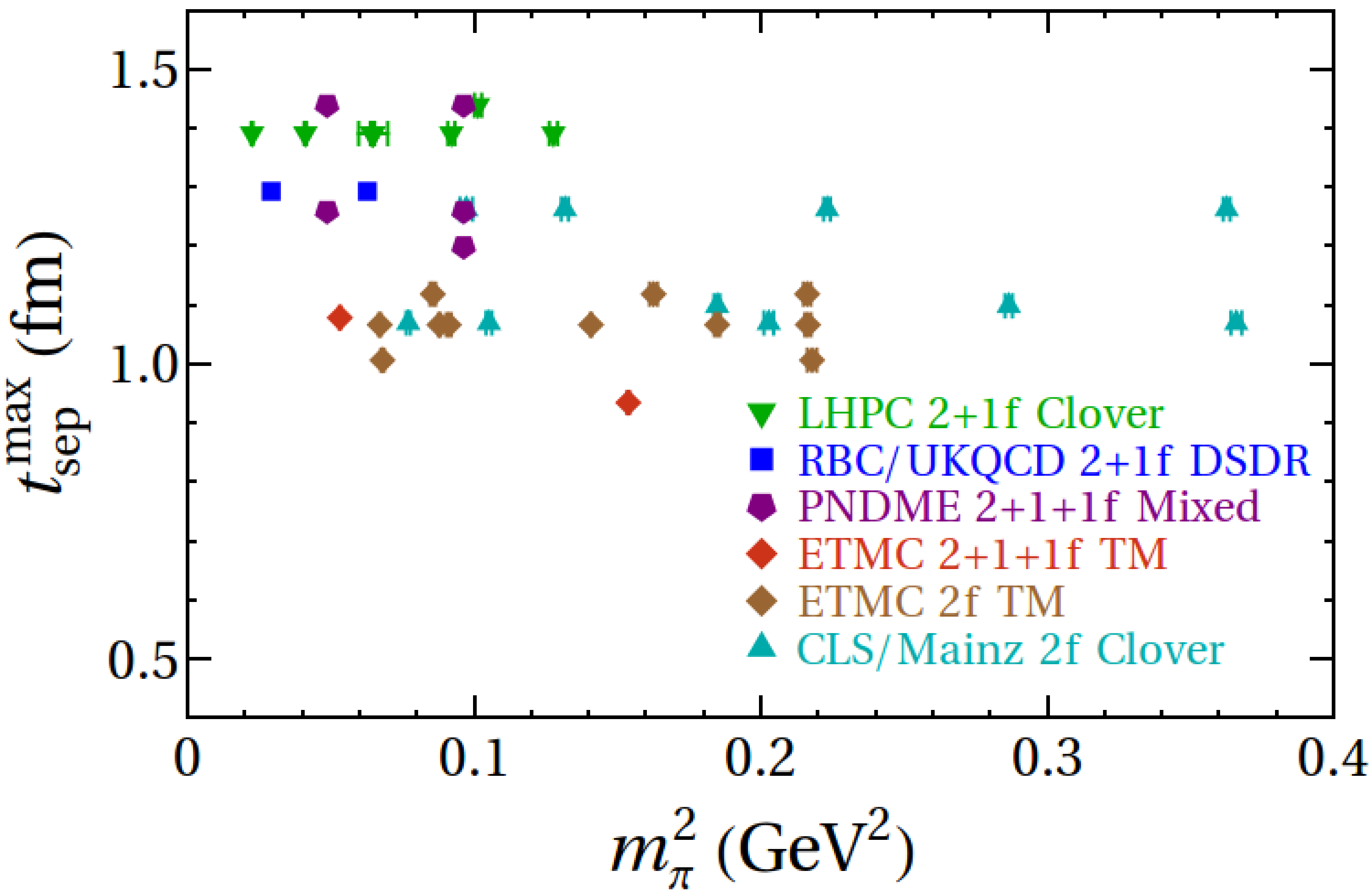




§ Flavor-dependent couplings, 1st moments of PDFs, …

- qEDM by Cirigliano (this afternoon)

Available Time Separations



Excited-State Contamination

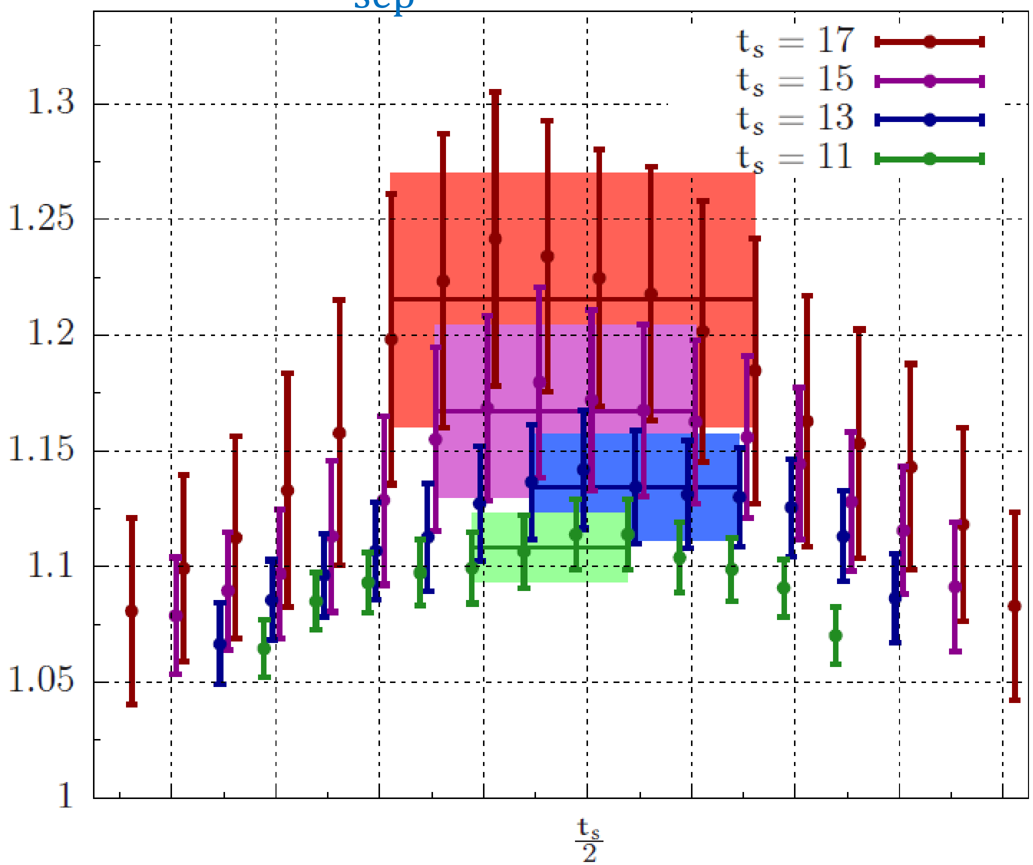
§ Tradeoff: signal-to-noise versus contamination

- Noise issue (P. Lepage; D. Kaplan 2011)
- For example, CLS/Mainz

  2f NP clover,
  $M_\pi \approx 320$ MeV
  $a \approx 0.063$ fm
  Fix $N_{\text{meas}} = 200$

1205.0180 &
private communication

t_{sep} 0.69–1.07 fm



Summation Method

§ Tradeoff: signal-to-noise versus contamination

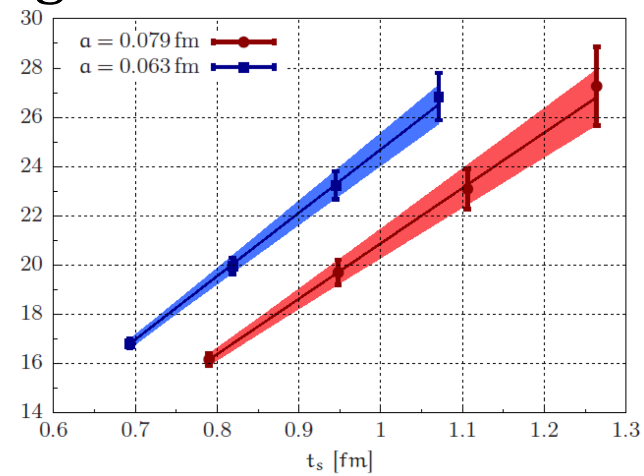
- Noise issue (P. Lepage; D. Kaplan 2011)

§ Options

- Stay at large t_{sink}: RBC/UKQCD (must check smaller pion mass)
- Include excited-state degrees of freedom
  - Multistate fitting or variational method from 3pt correlator matrix
  - HWL (Lat 2008); ETMC/LHPC/Mainz-CLS (2011); CSSM 2012 (mesons)
- Extend to small tsink to pick up better signal and apply "summation" method

$$S(t_s) \coloneqq \sum_{t=0}^{t_s} R(t, t_s) \xrightarrow{t_s \gg 0} c + t_s\left(g_A^{\text{bare}} + O\left(e^{-\Delta t_s}\right)\right)$$

  - g_A obtained from slope

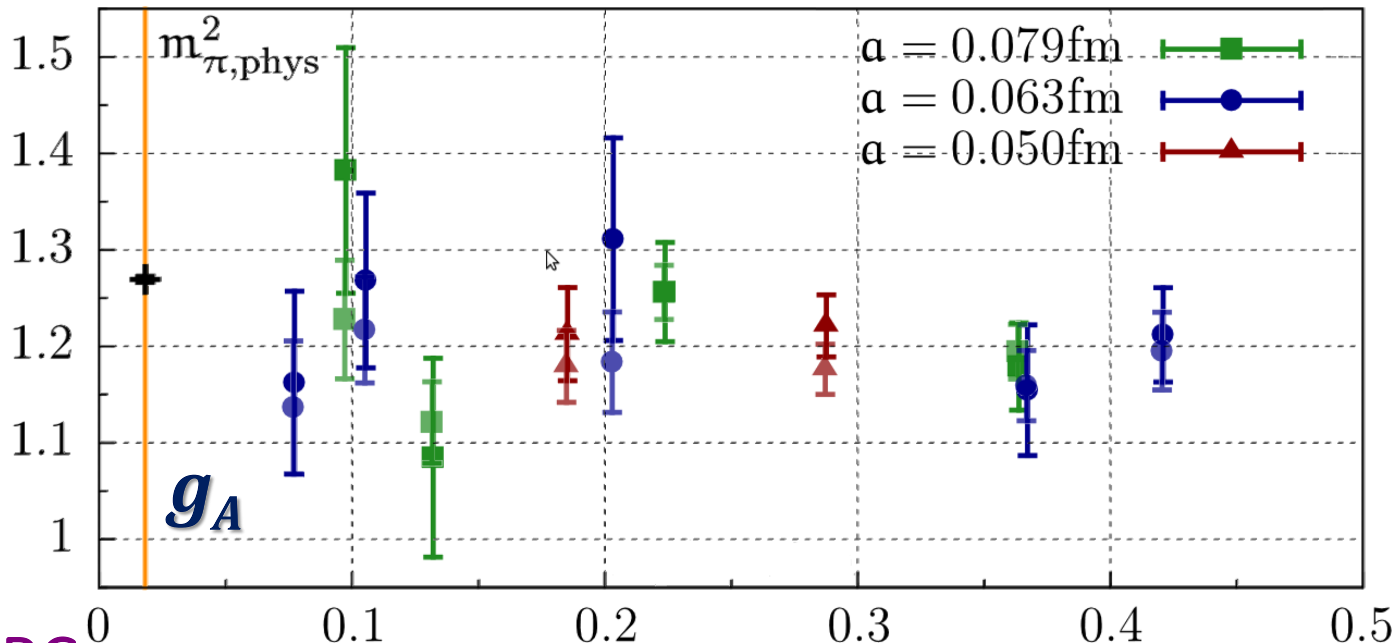


Summation Method

§ CLS/Mainz

S. Capitani et al. 1205.0180

- 2f, NP Clover ("summation" vs "plateau" $t_{sep} = 1.1$ fm)



§ LHPC

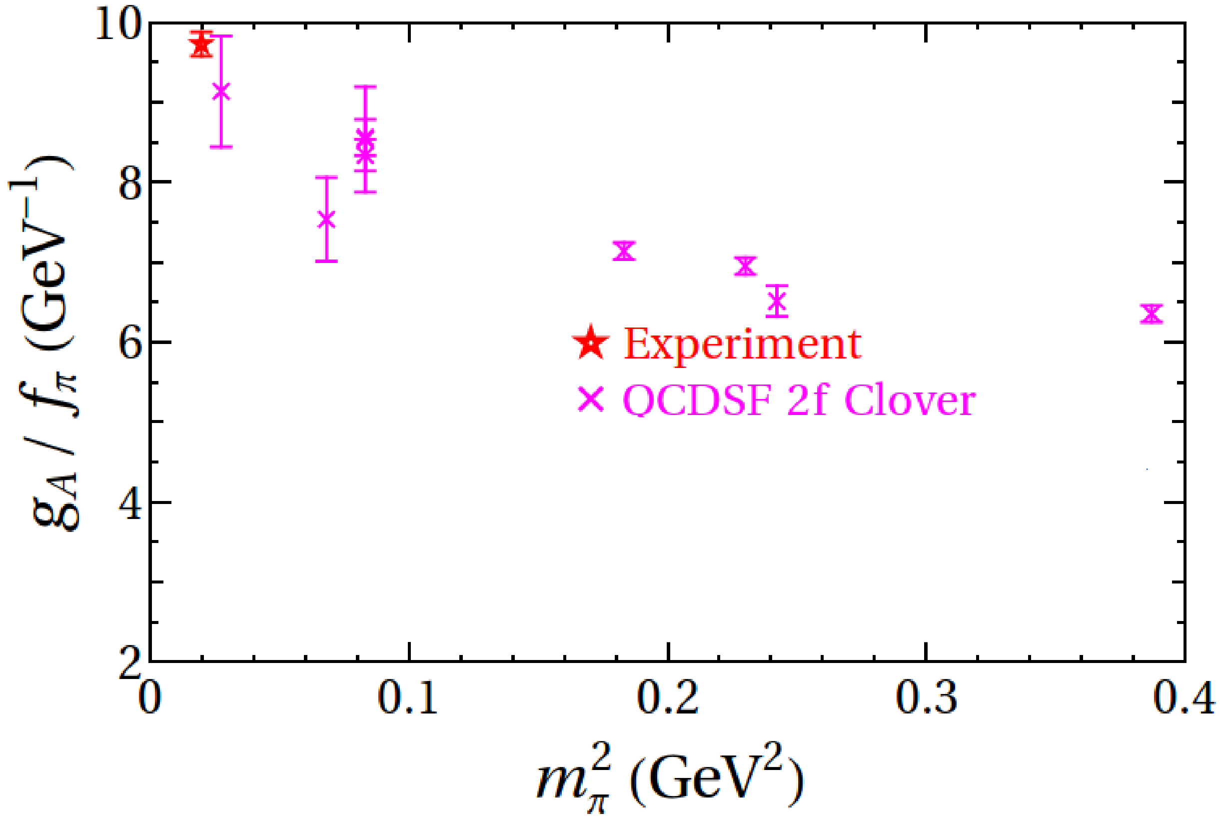
consistent results using largest t_{sep} and summation

- My two cents: Not clearly superior

Renormalization

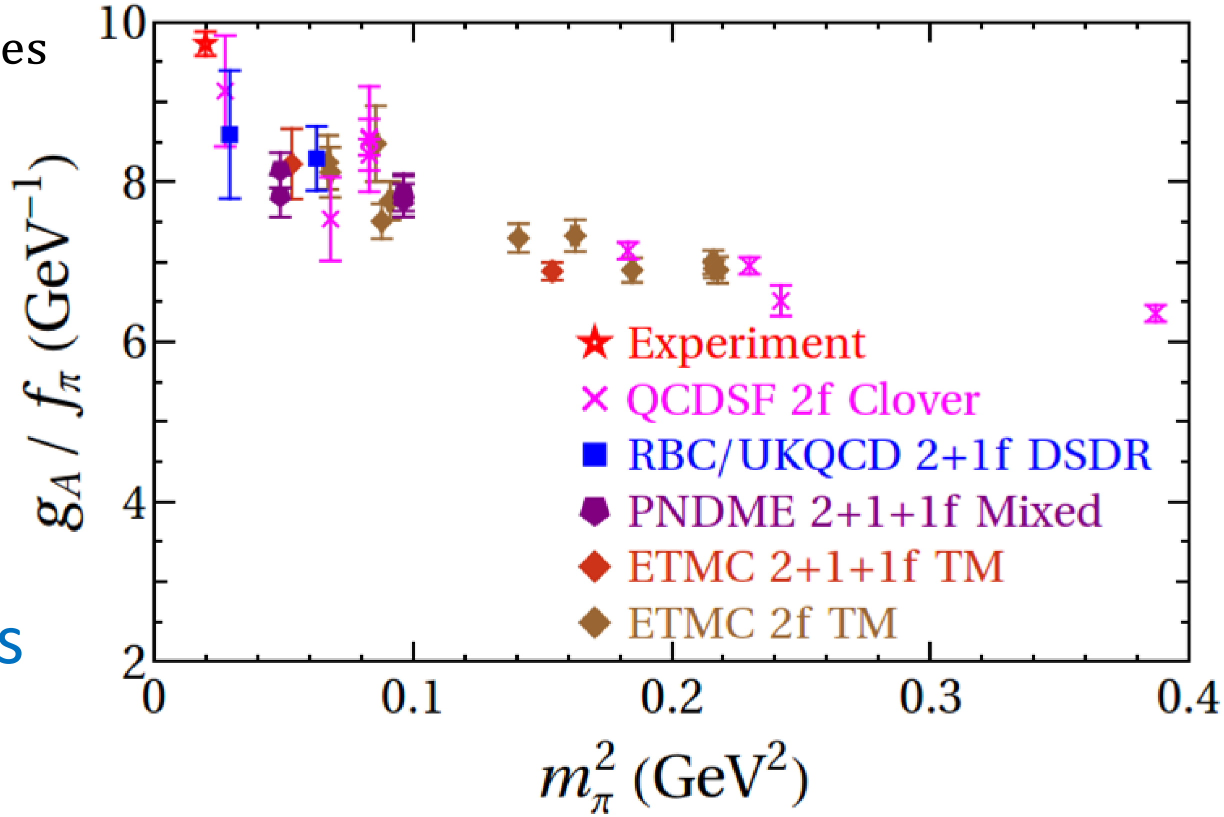
Renormalization

§ QCDSF hypothesis: Z_A might be a problem?



Renormalization

- § QCDSF hypothesis: Z_A might be a problem?
- § Residual O(a) artifacts in
  - Clover: $A_\mu^{\mathrm{R}} = Z_A(1 + b_A a m_q)(A_\mu + a c_A \partial_\mu P)$
  - Chiral fermions: m_{res}



§ Other systematic cancellations (such as volume, …)

Need higher statistics to be conclusive

Renormalization

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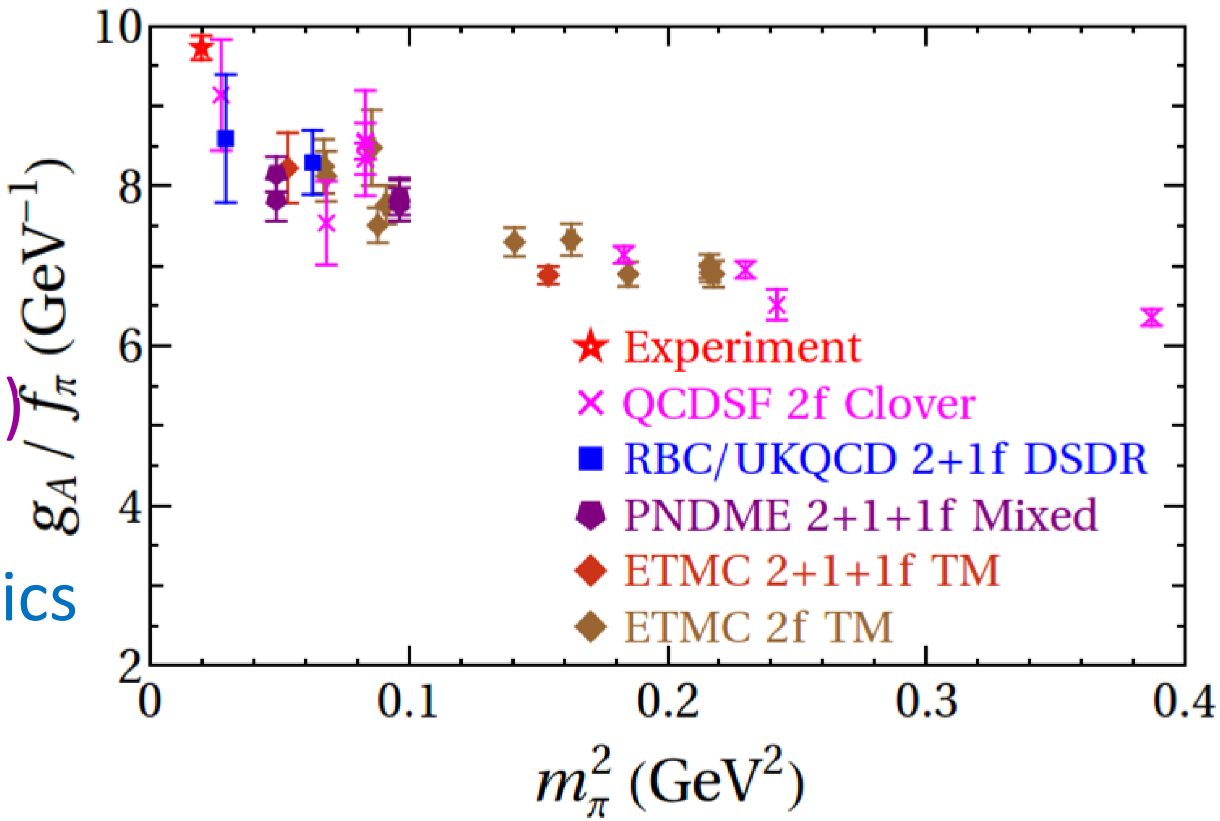

- Other systematic
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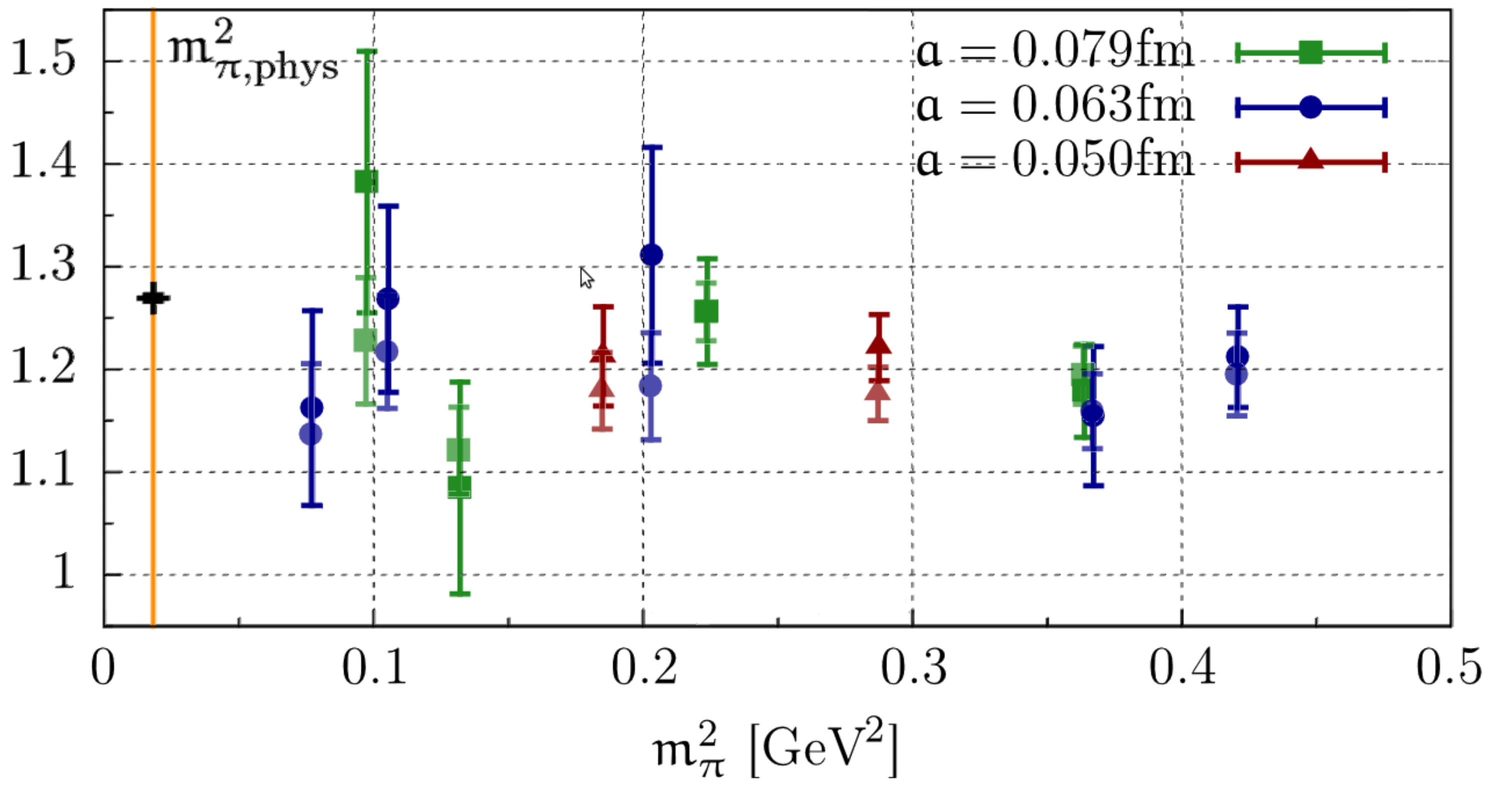
Chiral Extrapolation

Chiral Extrapolation

- Chiral extrapolation
- Small shift matters?

CLS/Mainz, 1205.0180



Chiral Extrapolation

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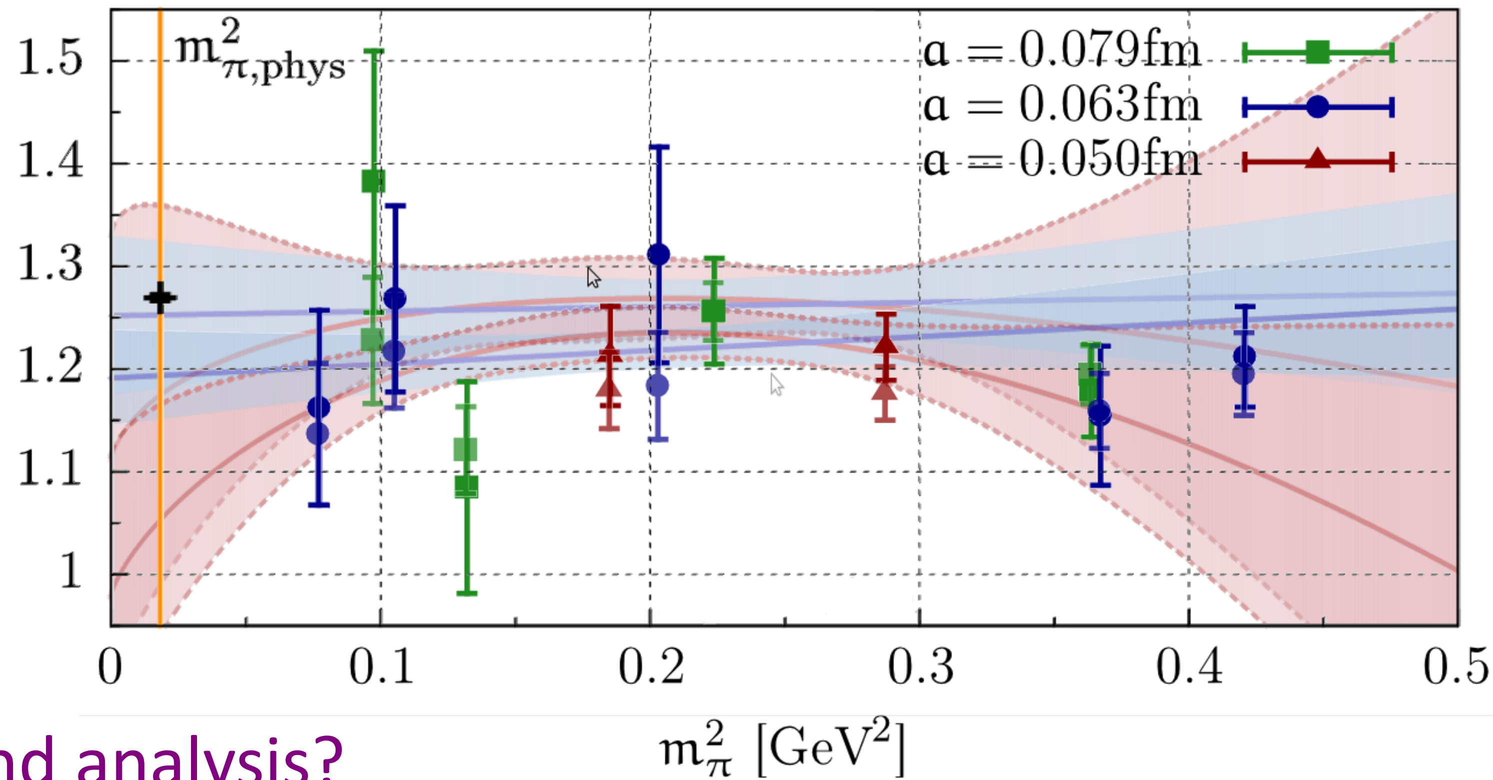
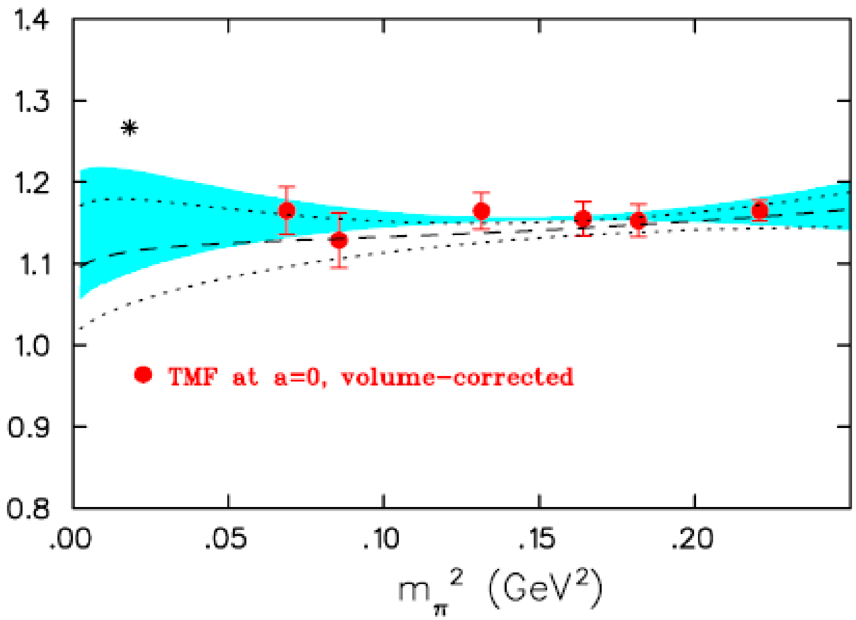
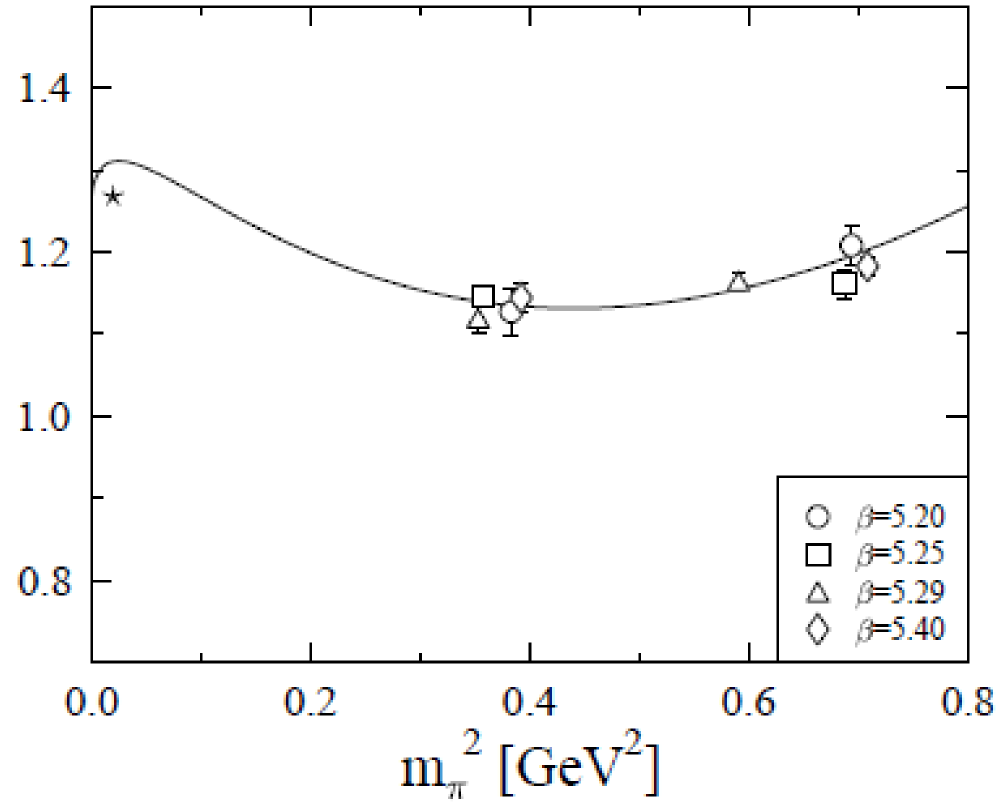


- Blind analysis?
- More precise studies are needed

Chiral Extrapolation

- Chiral extrapolation
- Same formula, similar LECs fixed, different ChPT behavior





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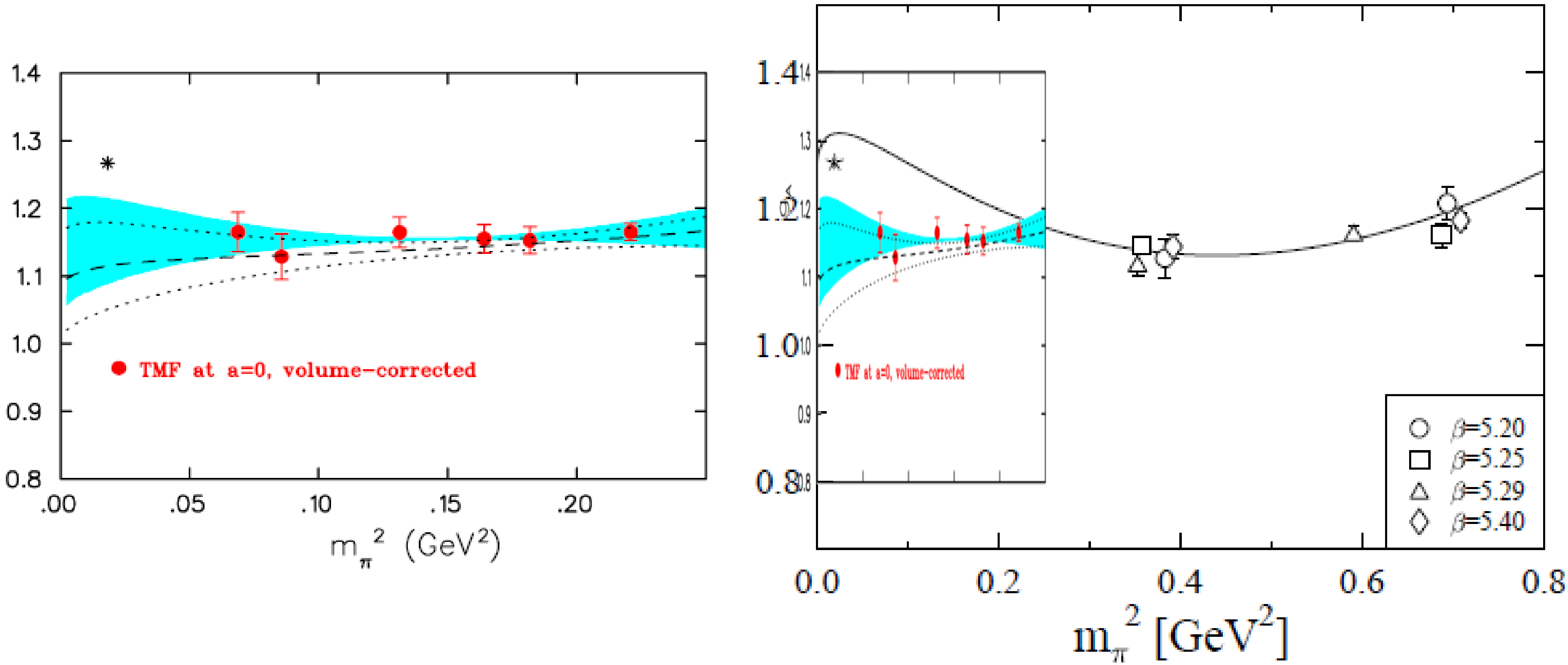
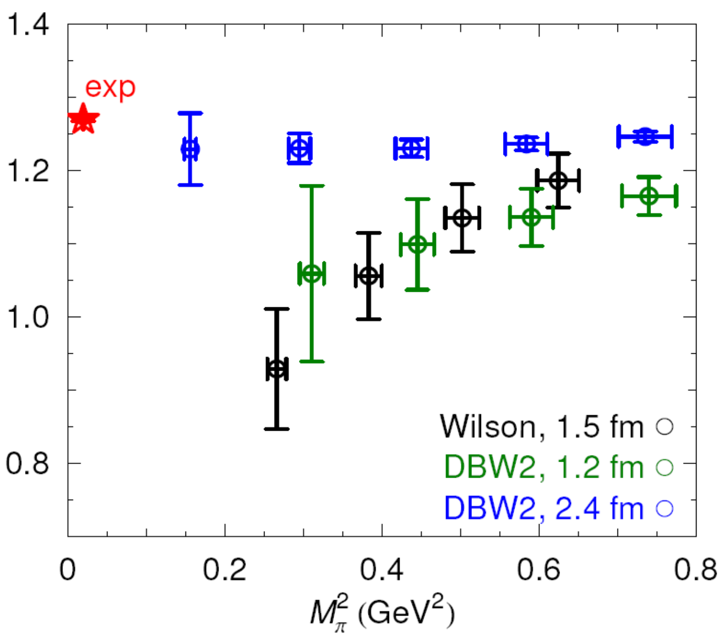


Finite-Volume Effects

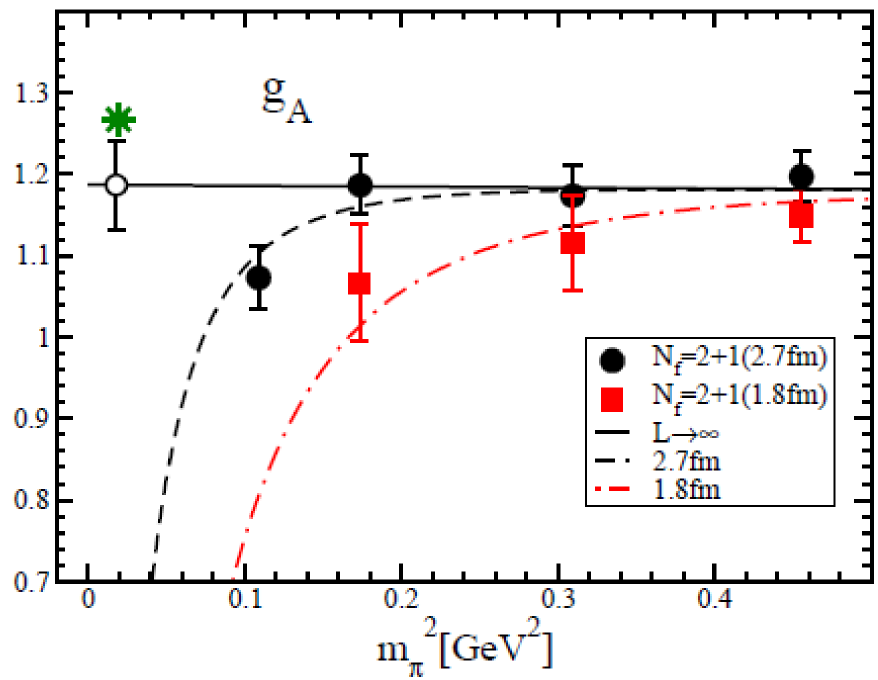
Finite-Volume Effects

- How big $M_\pi L$ is required?
- ChPT volume correction/used to estimate systematics
  - ETMC, QCDSF, CLS/Mainz: possibly underestimated?
- Example study (RBC/UKQCD)

$$A + B m_\pi^2 + C f_V(m_\pi L)$$



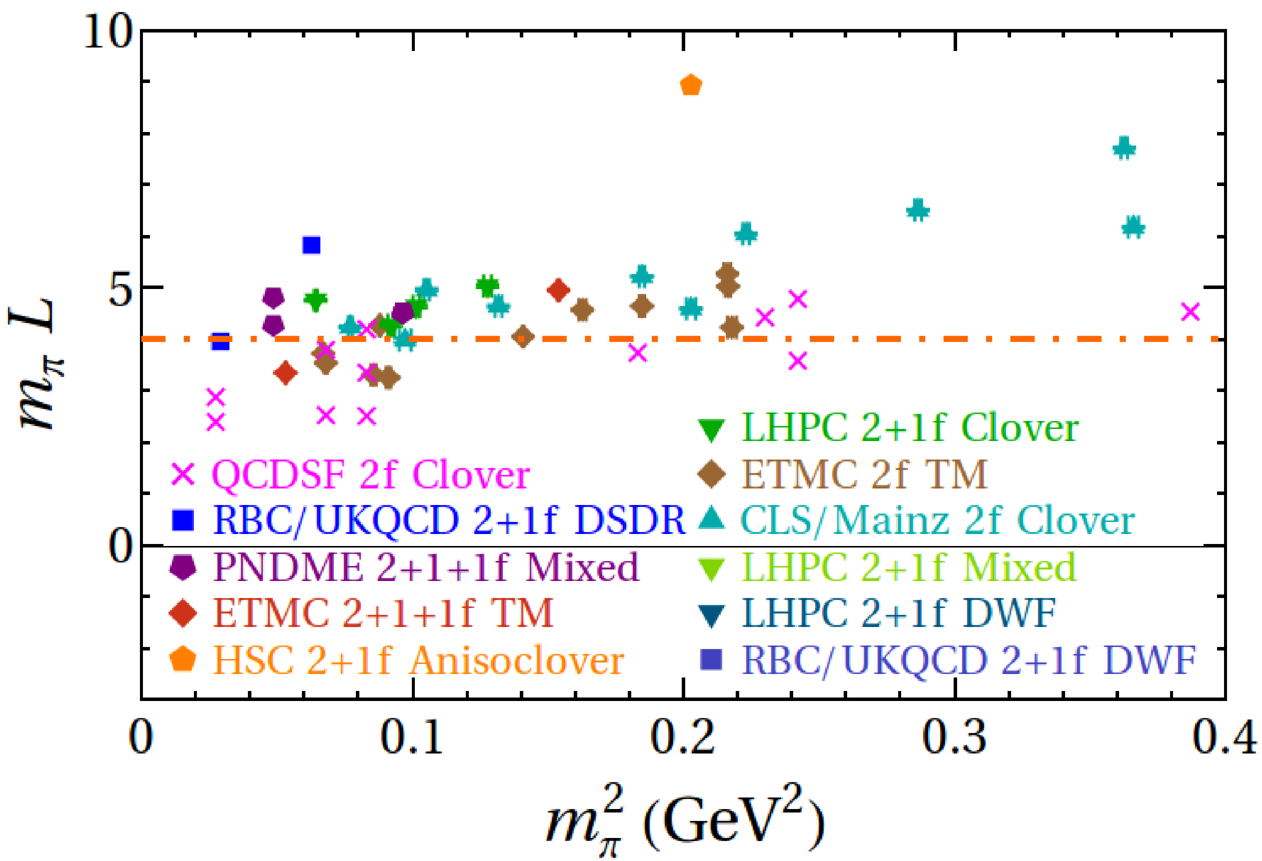
RBC, Phys.Rev.D68:054509 (2003)



RBC/UKQCD, Phys. Rev. Lett. 100:171602 (2008)

Available Volumes

§ How big $M_\pi L$ is required?



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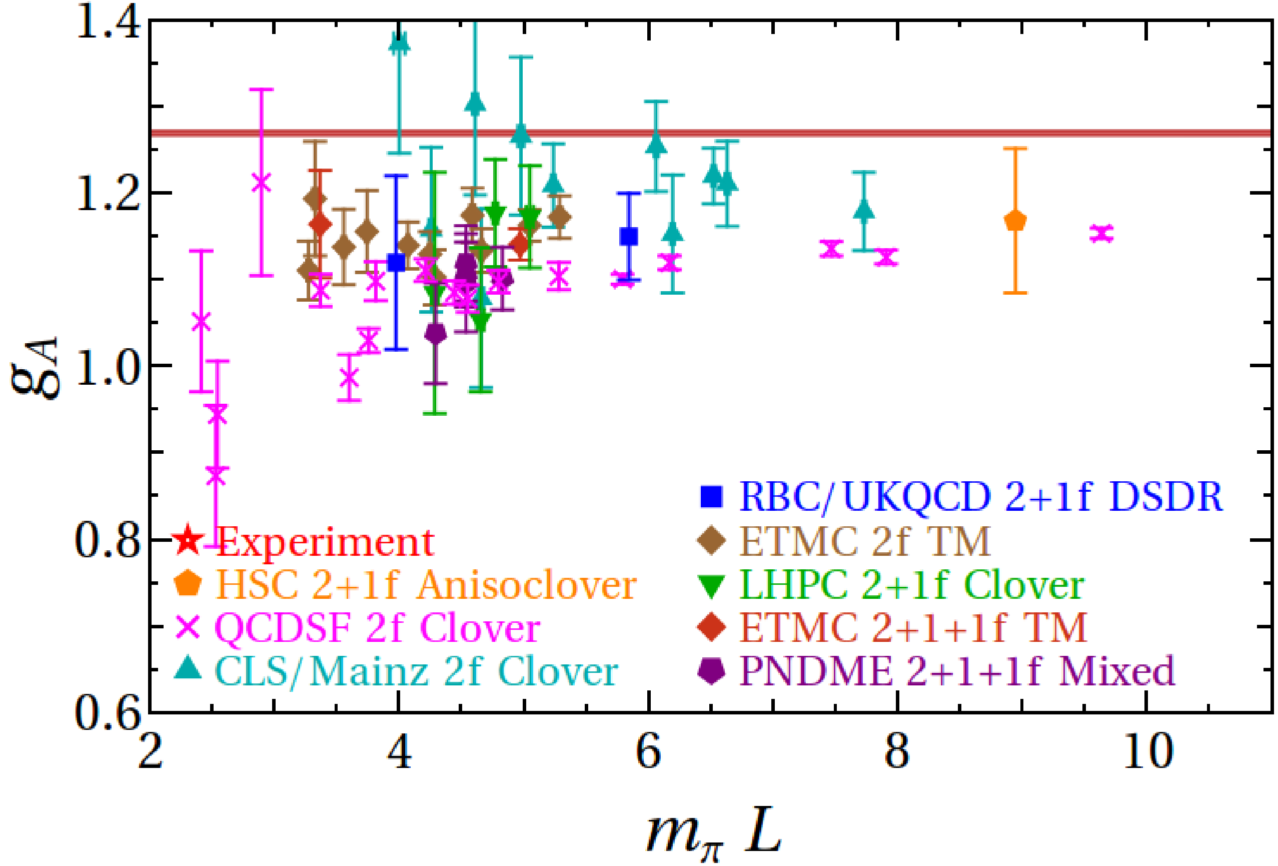


Finite-Volume Effects

§ How big $M_\pi L$ is required?

§ ChPT volume correction/used to estimate systematics

- ETMC, QCDSF, CLS/Mainz: possibly underestimated?

Highly sensitive to what parameters used in ChPT

$$\begin{aligned}
\Delta g_A(L) =& -\frac{g_A^0 m_\pi^2}{4\pi^2 F_\pi^2}\sum_{\vec{n}}{}' \frac{K_1(L|\vec{n}|m_\pi)}{L|\vec{n}|m_\pi} \\
&+\frac{(g_A^0)^3 m_\pi^2}{6\pi^2 F_\pi^2}\sum_{\vec{n}}{}' \left[K_0(L|\vec{n}|m_\pi) - \frac{K_1(L|\vec{n}|m_\pi)}{L|\vec{n}|m_\pi}\right] \\
&+\frac{25 c_A^2 g_1}{81\pi^2 F_\pi^2}\int_0^\infty dy\, y \sum_{\vec{n}}{}' \left[K_0(L|\vec{n}|f(m_\pi,y)) - \frac{L|\vec{n}|f(m_\pi,y)}{3} K_1(L|\vec{n}|f(m_\pi,y))\right] \\
&-\frac{c_A^2 g_A^0}{\pi^2 F_\pi^2}\int_0^\infty dy\, y \sum_{\vec{n}}{}' \left[K_0(L|\vec{n}|f(m_\pi,y)) - \frac{L|\vec{n}|f(m_\pi,y)}{3} K_1(L|\vec{n}|f(m_\pi,y))\right] \\
&+\frac{8 c_A^2 g_A^0}{27\pi^2 F_\pi^2}\int_0^\infty dy \sum_{\vec{n}}{}' \frac{f(m_\pi,y)^2}{\Delta_0}\left[K_0(L|\vec{n}|f(m_\pi,y)) - \frac{K_1(L|\vec{n}|f(m_\pi,y))}{L|\vec{n}|f(m_\pi,y)}\right] \\
&-\frac{4 c_A^2 g_A^0}{27\pi F_\pi^2}\frac{m_\pi^3}{\Delta_0}\sum_{\vec{n}}{}' \frac{1}{L|\vec{n}|m_\pi} e^{-L|\vec{n}|m_\pi} + \mathcal{O}(\epsilon^4) \qquad (18)
\end{aligned}$$

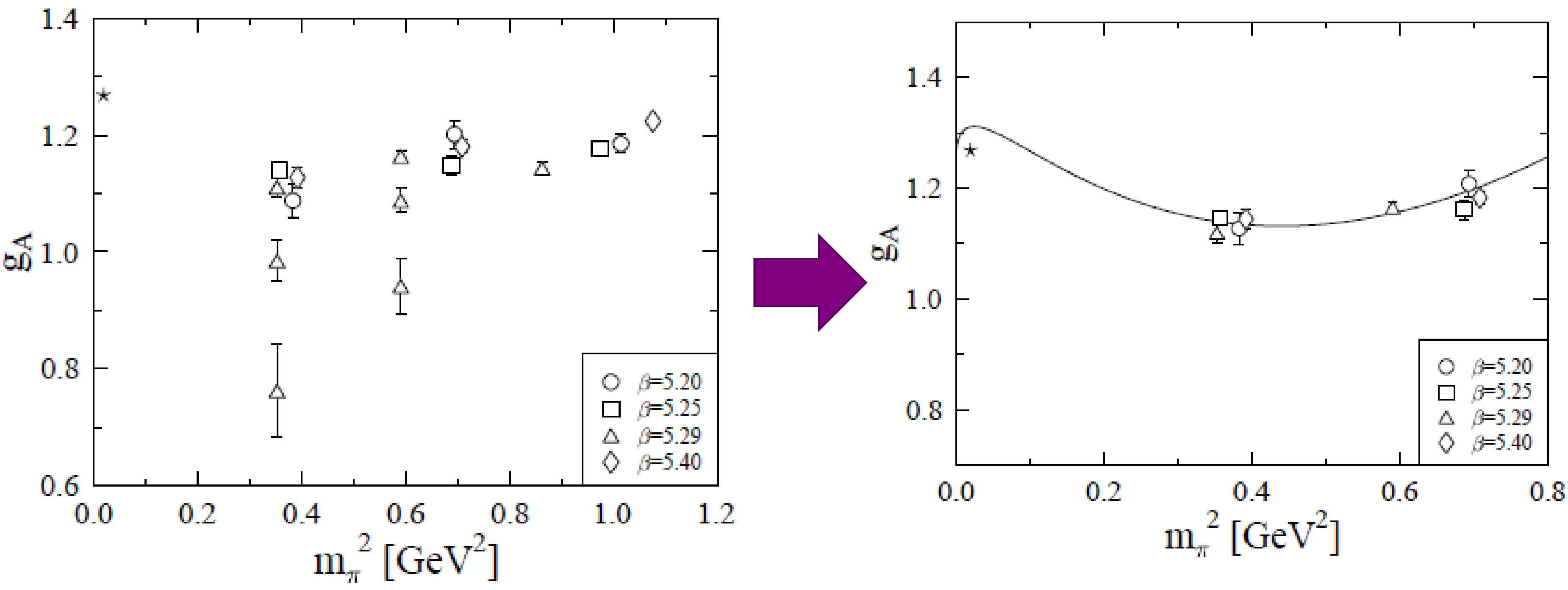
$$\text{fix } \Delta_0 = 0.271\,\text{GeV},\ c_A = 1.5,\ F_\pi = 86.2\,\text{MeV}$$

Finite-Volume Effects

§ How big $M_\pi L$ is required?

§ ChPT volume correction/used to estimate systematics

- ETMC, **QCDSF**, CLS/Mainz: possibly underestimated?



Finite-Volume Effects

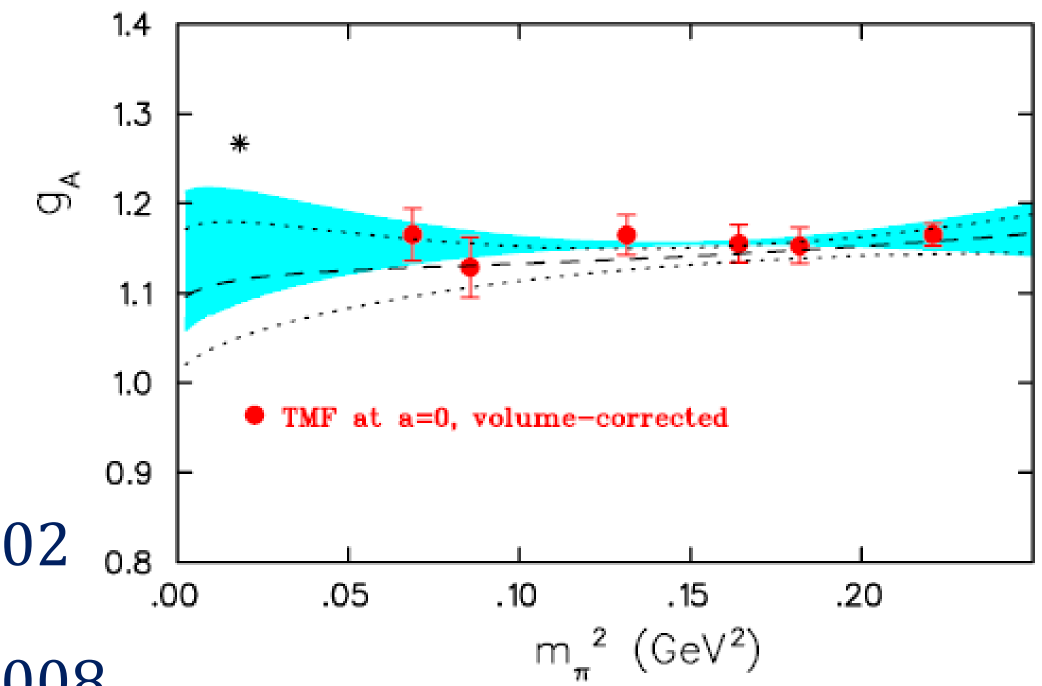
§ How big $M_\pi L$ is required?

§ ChPT volume correction/used to estimate systematics

- **ETMC**, QCDSF, CLS/Mainz: possibly underestimated?

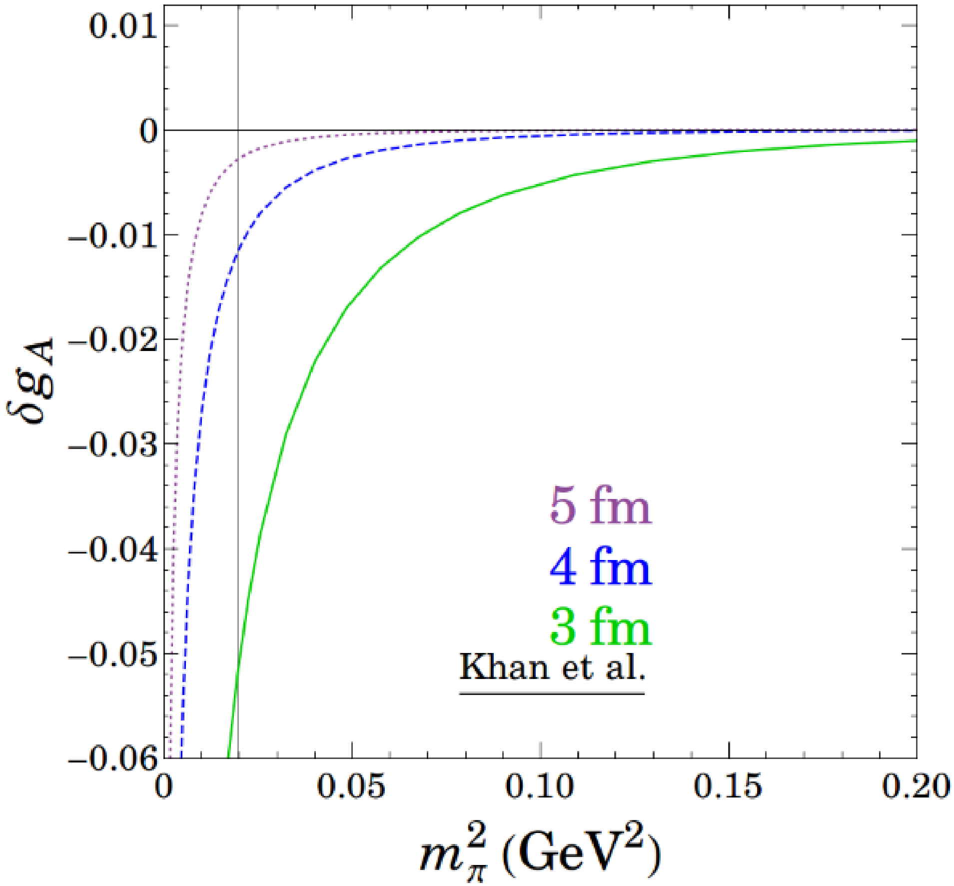
ETMC $N_f = 2$ example

m_π	Lm_π	g_A	$g_A(L \to \infty)$
$\beta = 3.9$			
0.4675	5.04	1.163(18)	1.167
0.4319	4.66	1.134(25)	1.140
0.3770	4.06	1.140(27)	1.150
0.3032	3.27	1.111(34)	1.133
0.2978	4.28	1.103(32)	1.106
0.2600	3.74	1.156(47)	1.162
$\beta = 4.05$			
0.4653	5.28	1.173(24)	1.177
0.4035	4.58	1.175(31)	1.182
0.2925	3.32	1.194(66)	1.218 ← 0.02
$\beta = 4.2$			
0.4698	4.24	1.130(26)	1.144
0.2622	3.55	1.138(43)	1.146 ← 0.008



Finite-Volume Effects

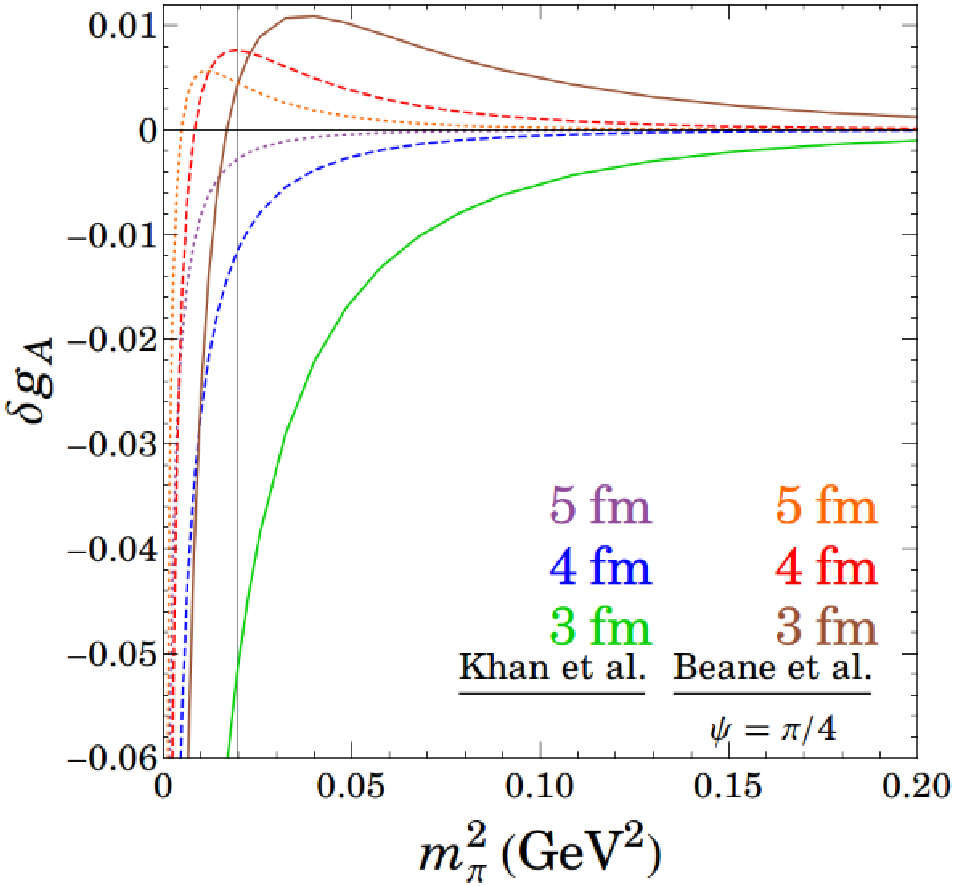
§ Sensitivity to the parameters chosen in ChPT



Ref. [20] using a variety of constraints to $F_\pi = 86.2$ MeV, $c_A = 1.5$, $g_1 = 2.6$, $g_A^0 = 1.15$,
Ref. [33] use SU(6) relations to derive $g_A = 1 + (2/3)\cos^2\psi$, $g_{\Delta N} = -2\cos\psi$, $g_{\Delta\Delta} = -3$.

Finite-Volume Effects

§ Sensitivity to the parameters chosen in ChPT

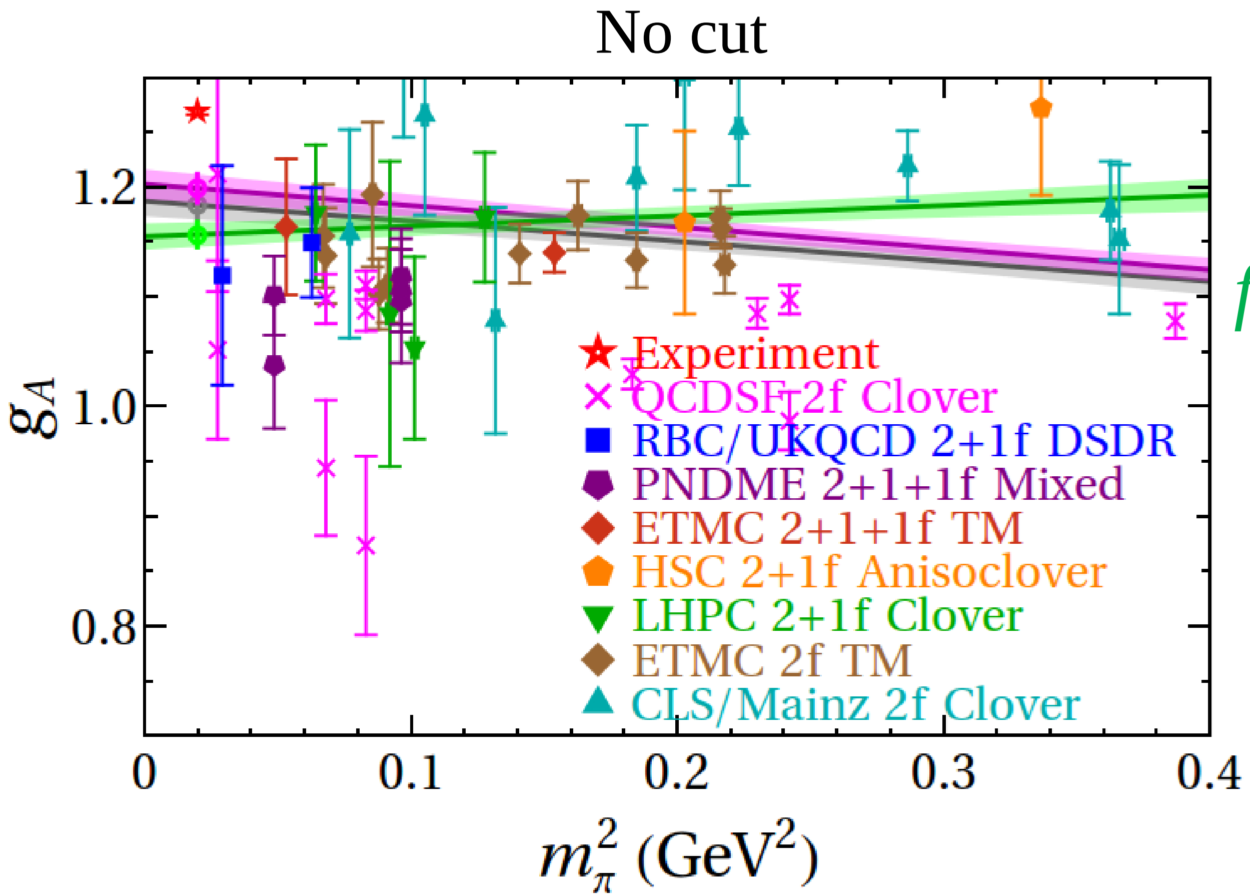


Ref. [20] using a variety of constraints to $F_\pi = 86.2$ MeV, $c_A = 1.5$, $g_1 = 2.6$, $g_A^0 = 1.15$,

Ref. [33] use SU(6) relations to derive $g_A = 1 + (2/3)\cos^2\psi$, $g_{\Delta N} = -2\cos\psi$, $g_{\Delta\Delta} = -3$.

Finite-Volume Effects

- How big $M_\pi L$ is required?
- How global data changes with a cut $A + B m_\pi^2 + C f_V(m_\pi L)$



$f_V \sim e^{-m_\pi L}$

$f_V \sim (m_\pi L)^{-3}$

$f_V \sim m_\pi^2 e^{-m_\pi L} (m_\pi L)^{-0.5}$

Finite-Volume Effects

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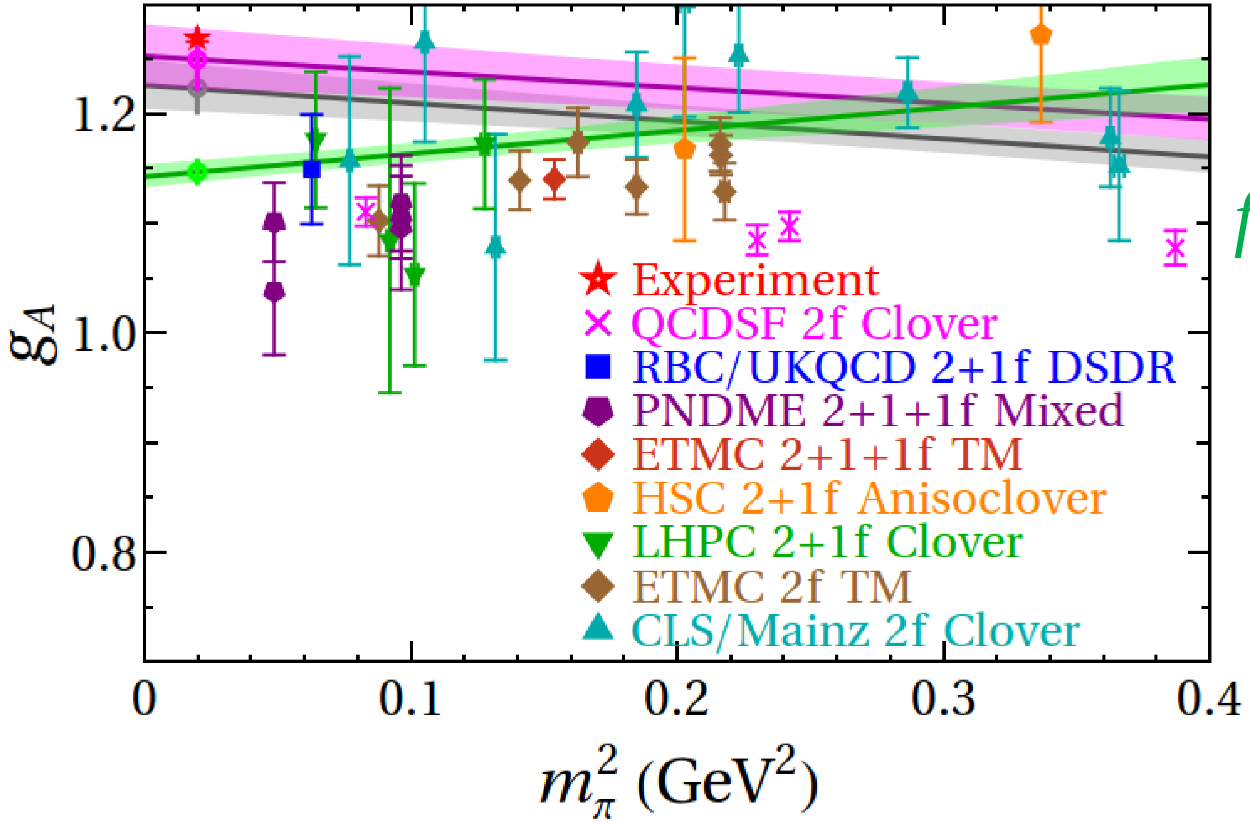
Cut by $m_\pi L > 4$



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Finite-Volume Effects

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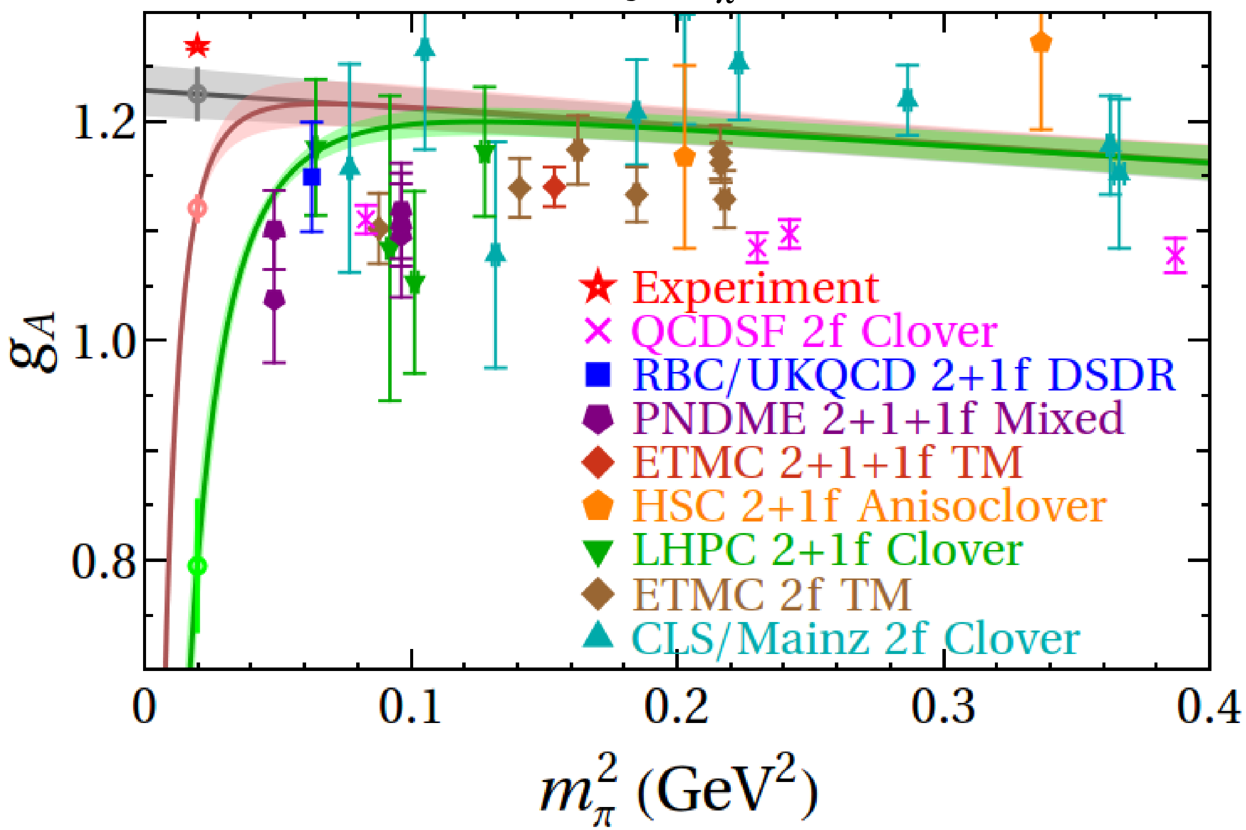
Cut by $m_\pi L > 4$



$f_V \sim e^{-m_\pi L}$

6-fm box

4-fm box