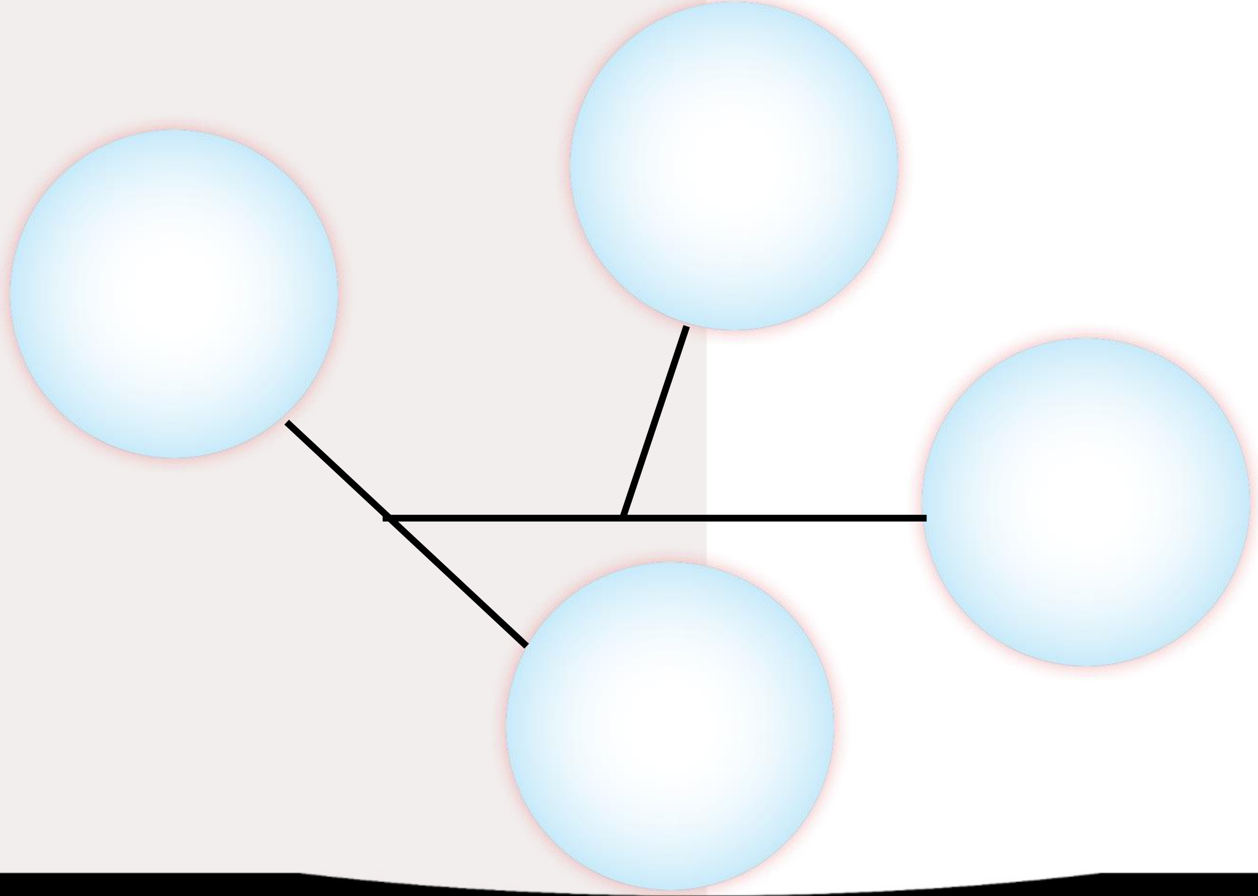
Description of few-nucleon collisions by FY eq's approach



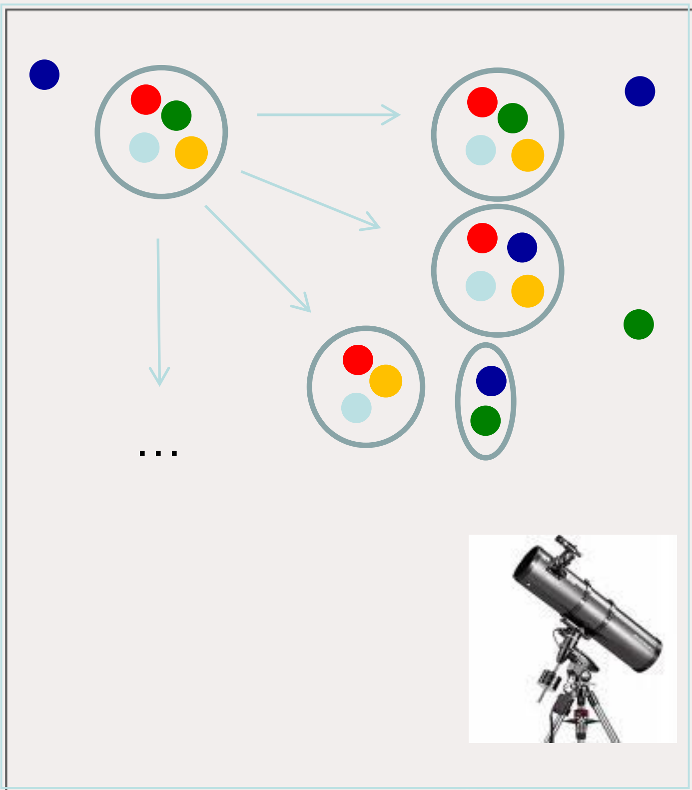
R. Lazauskas

Contents

- Solution of the 3-5 body Faddeev-Yakubovsky equations for nuclear systems
- Application of the complex-scaling method to solve complicated scattering problems using trivial boundary conditions

Collisions

- In configuration space wave functions extend to infinity!
- Increasingly complex asymptotic behaviour for A>2 systems!!



How to take care of the boundary condition?

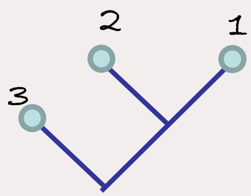
✓ Conceptual difficulties to uncouple different particle channel, to constrain assymptotes of the solutions in all directions and thus get unique (physical) solution to the Schrodinger eq.

- It is ok, as long as there is single particle channel (elastic plus target excitations)
- Mathematically ill-conditioned problem when several particle channels are open

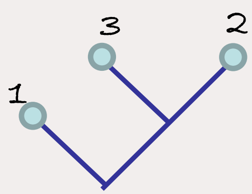
✓ Faddeev-Yakubovsky equations efficiently separates asymptotes of the binary channels

L. D. Faddeev, Zh. Eksp. Teor. Fiz. 39, 1459 (1960). [Sov. Phys. JETP 12, 1014(1961)].
O. A. Yakubovsky, Sov. J. Nucl. Phys. 5, 937 (1967).

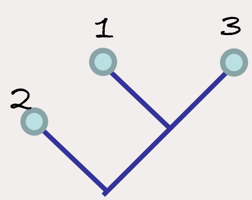
3-body
(Faddeev eq.)



$$\phi_{12} = G_0 V_{12} \Psi$$



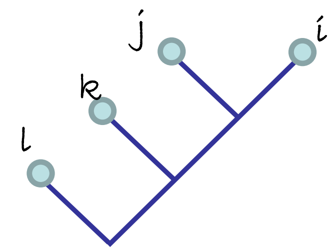
$$\phi_{23} = G_0 V_{23} \Psi$$



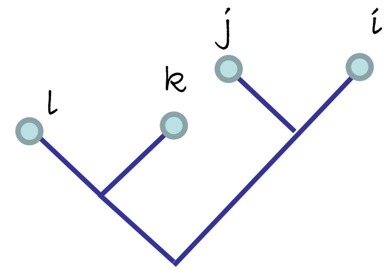
$$\phi_{31} = G_0 V_{31} \Psi$$

$$\Psi = \phi_{12} + \phi_{23} + \phi_{31}$$

4-body
(Faddeev-Yakubovsky eq.)



K-type
(12 components)



H-type
(6 components)

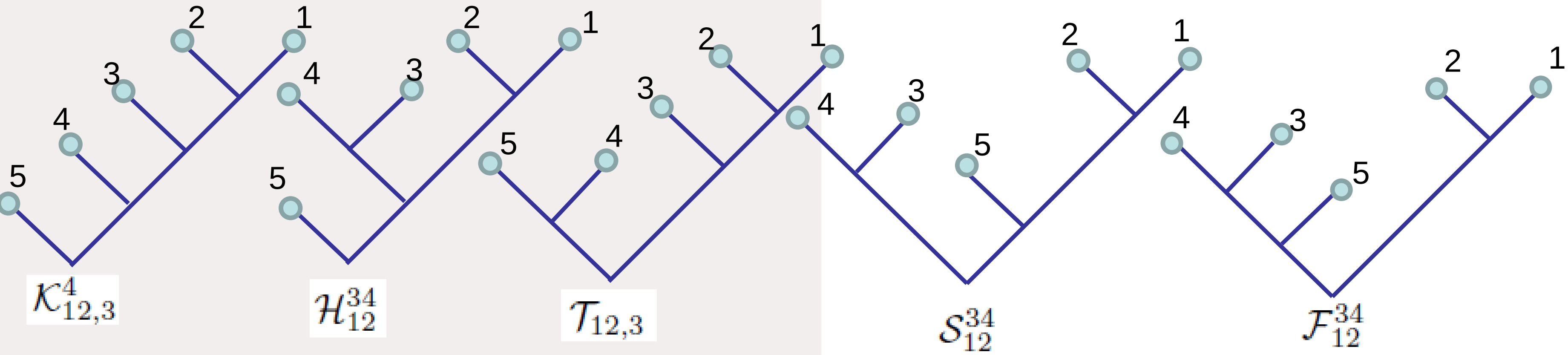
$$\phi_{jk} = G_0 V_{jk} \Psi$$

$$K_{ij,k}^{l} = G_{ij} V_{ij} (\phi_{jk} + \phi_{ik}); \qquad H_{ij}^{kl} = G_{ij} V_{ij} \phi_{kl}$$

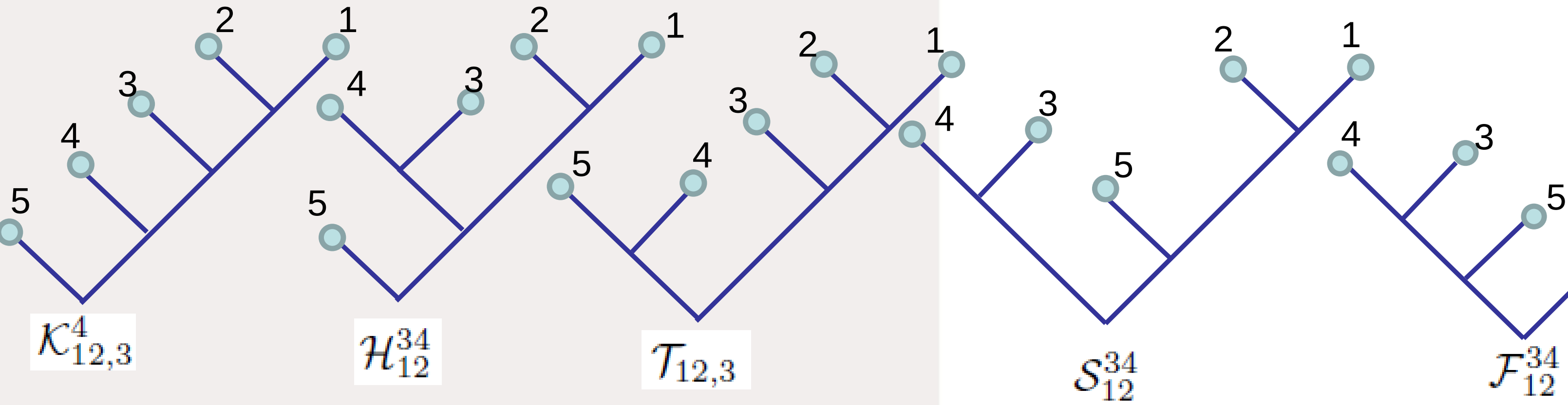
$$\Psi = \sum_{i<j} \phi_{ij} = \sum_{i<j} (K_{ij,k}^{l} + K_{ij,l}^{k} + H_{ij}^{kl})$$

Equations for short-ranged pairwise interactions

5-body Faddeev-Yakubovski eq



$$
\begin{aligned}
(E-H_0-V_{12})\,\mathcal{K}^4_{12,3} &= V_{12}\,(\mathcal{K}^4_{13,2}+\mathcal{K}^4_{23,1}+\mathcal{K}^5_{13,4}+\mathcal{K}^5_{23,4}+\mathcal{K}^2_{13,4}+\mathcal{K}^1_{23,4} \\
&\quad +\mathcal{T}_{13,4}+\mathcal{T}_{23,4} \\
&\quad +\mathcal{H}^{24}_{13}+\mathcal{H}^{14}_{23}+\mathcal{S}^{24}_{13}+\mathcal{S}^{14}_{23}+\mathcal{F}^{24}_{13}+\mathcal{F}^{14}_{23}) \\
(E-H_0-V_{12})\,\mathcal{H}^{34}_{12} &= V_{12}\,(\mathcal{H}^{12}_{34}+\mathcal{K}^2_{34,1}+\mathcal{K}^1_{34,2}+\mathcal{K}^5_{34,1}+\mathcal{K}^5_{34,2} \\
&\quad +\mathcal{T}_{34,1}+\mathcal{T}_{34,2}) \\
(E-H_0-V_{12})\,\mathcal{T}_{12,3} &= V_{12}\,(\mathcal{T}_{13,2}+\mathcal{T}_{23,1} \\
&\quad +\mathcal{H}^{45}_{13}+\mathcal{H}^{45}_{23}+\mathcal{S}^{45}_{13}+\mathcal{S}^{45}_{23}+\mathcal{F}^{45}_{13}+\mathcal{F}^{45}_{23}) \\
(E-H_0-V_{12})\,\mathcal{S}^{34}_{12} &= V_{12}\,(\mathcal{F}^{12}_{34}+\mathcal{S}^{15}_{34}+\mathcal{S}^{25}_{34} \\
&\quad +\mathcal{F}^{15}_{34}+\mathcal{F}^{25}_{34} \\
&\quad +\mathcal{H}^{15}_{34}+\mathcal{H}^{25}_{34}) \\
(E-H_0-V_{12})\,\mathcal{F}^{34}_{12} &= V_{12}\,(\mathcal{S}^{12}_{34}+\mathcal{K}^1_{34,5}+\mathcal{K}^2_{34,5}+\mathcal{T}_{34,5})
\end{aligned}
$$



Merits:

- ✓ Handling of symmetries
- ✓ Boundary conditions for binary channels
- ✓ Easy reduction to subsystems

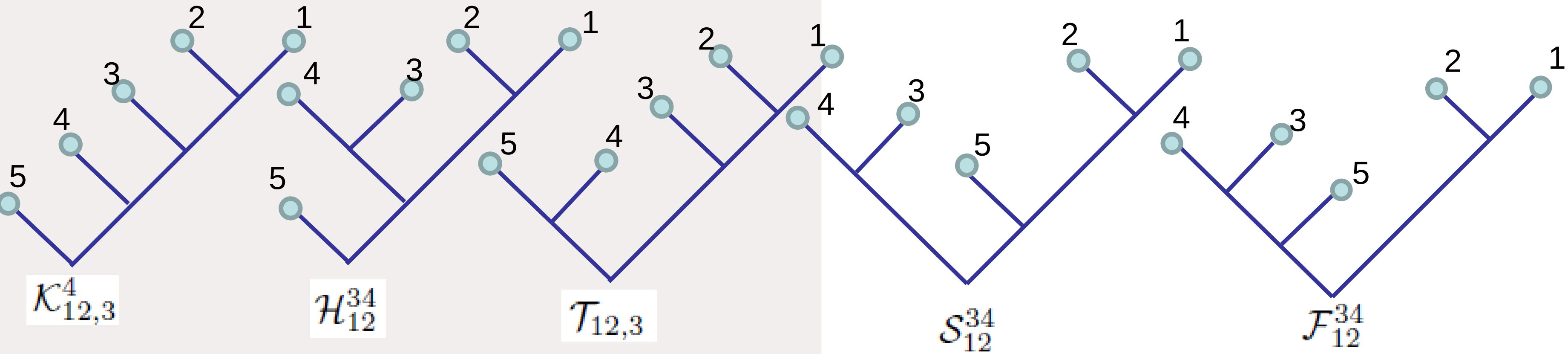
Price

- ✓ Overcomplexity

Problem	Number eq. (identical particles)	Number eq. (different particles)
A=2	1	1
A=3	1	3
A=4	2	18
A=5	5	180
A=6	15	2700
A=N		$\frac{N!\,(N-1)!}{2^{N-1}}$

J.W. Waterhouse : « Pandora »

5-body Faddeev-Yakubovski eq



$$\mathcal{K}_{12,3}^{4}\left(\vec{x},\vec{y},\vec{z},\vec{w},S,L,T\right)=\sum_{\alpha_K=(l_{..},s_{..},t_{..})}\frac{f_{\alpha_K}(x,y,z,w)}{xyzw}\left[\left\{(l_x l_y)_{l_{xy}}(l_z l_w)_{l_{zw}}\right\}_L\{...\}_S\right]_{JM}\{...\}_T$$

NUMERICAL SOLUTION

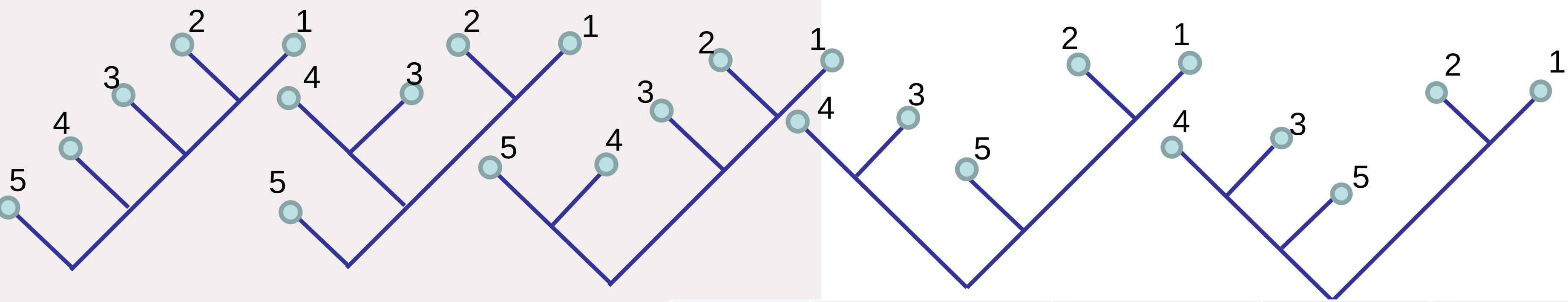
*R.L., PhD Thesis, Université Joseph Fourier, Grenoble (2003).

- PW decomposition of the components K,H,T,S,F
- Radial parts expanded using Lagrange-mesh method

D. Baye, Physics Reports 565 (2015) 1

- Resulting linear algebra problem solved using iterative methods
- Observables extracted using integral relations

Numerical costs



Problem	Number eq. (ident particles)	Number eq. (diff. particles)	PW basis.	Radial disc.
2N	1	1	2	~N
3N	1	3	~100	$\sim N^2$
4N	2	18	$\sim 10^4$	$\sim N^3$
5N	5	180	$\sim 10^6$	$\sim N^4$

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D. Baye, Physics Reports 565 (2015) 1

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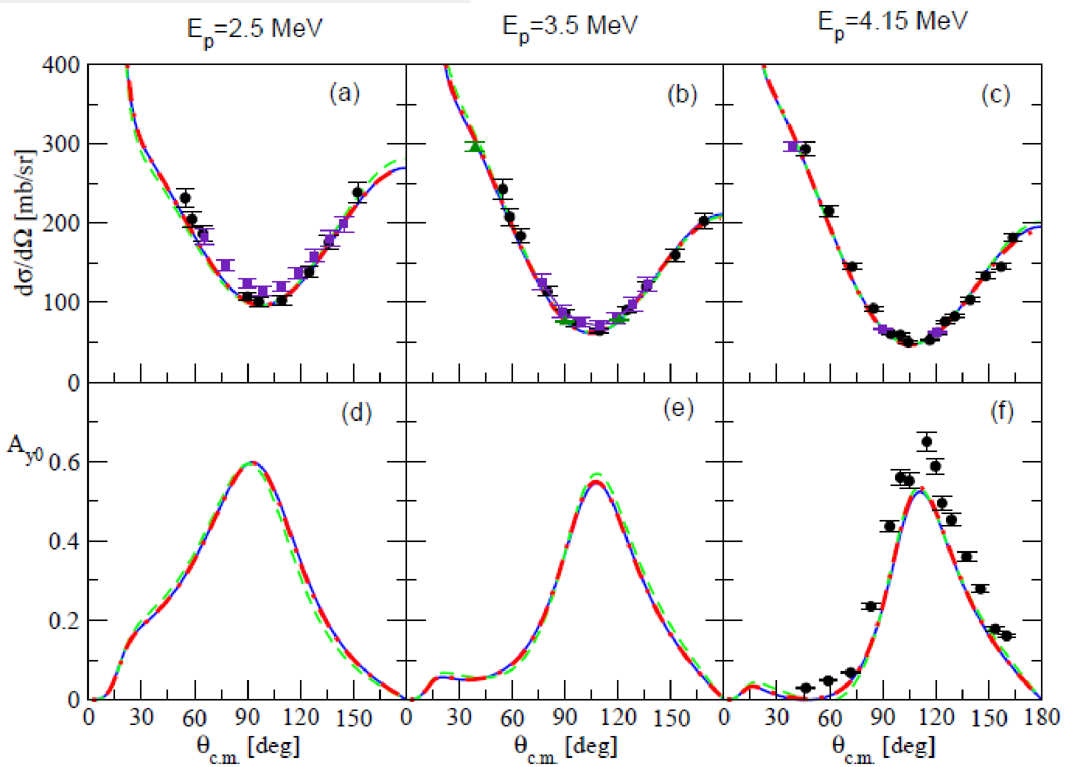
Short overview of nuclear problems by FY eq's

3N-probl...
(Faddeev ...

1st solution: ...
G. Gignoux, ...
Review: W. G...

4N-pro...
(Faddeev...

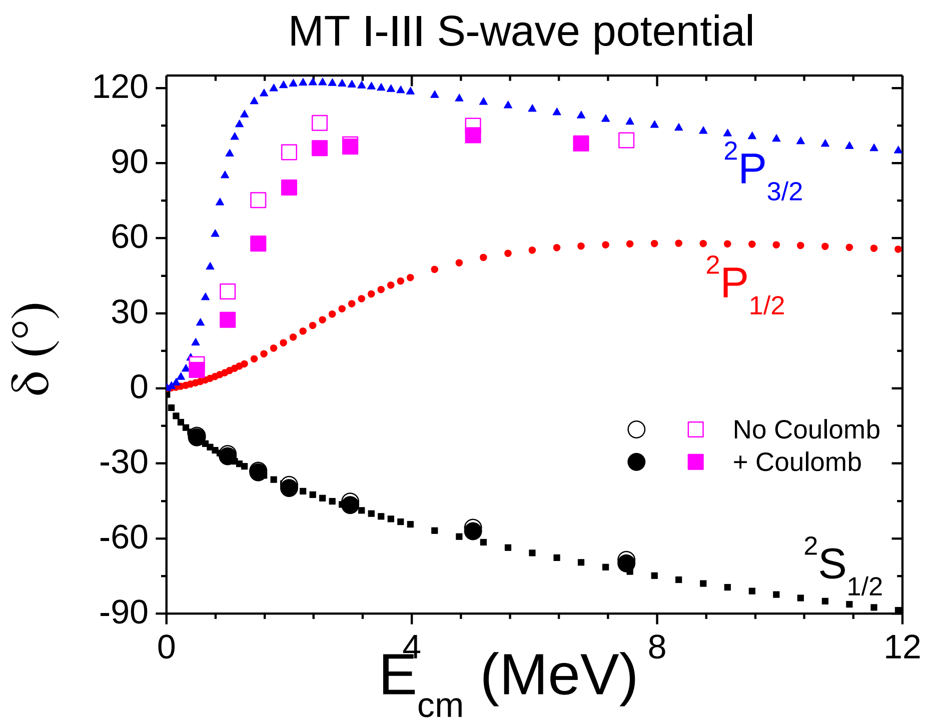
1st solution: ...
(1984) 125...
Benchmarks: ...
state)
R. Lazauskas ...
M. Viviani et ...
M. Viviani et ...



$p +^3 H \leftrightarrow n +^3 He$

FIG. 5: (Color online) Differential cross sections (upper panels) and proton analyzing powers A_{y0} (lower panels) for p-^{3}H elastic scattering at $E_p = 2.5$, 3.5, and 4.15 MeV proton energies obtained using the N3LO500 potential. The lines show the results obtained using the AGS (blue solid lines), FY (red dot-dash lines), and the HH (green dashed lines) methods. In many cases, the curves overlap and cannot be distinguished. The experimental data in panel (a) are from Refs. [36] (circles) and [35] (squares), in panel (b) from Refs. [36] (circles), [39] (squares), and [40] (triangles), in panel (c) from Refs. [41] (circles) and [40] (squares), and finally in panel (f) from Ref. [41] (circles).

5N problem: n-^{4}He scattering



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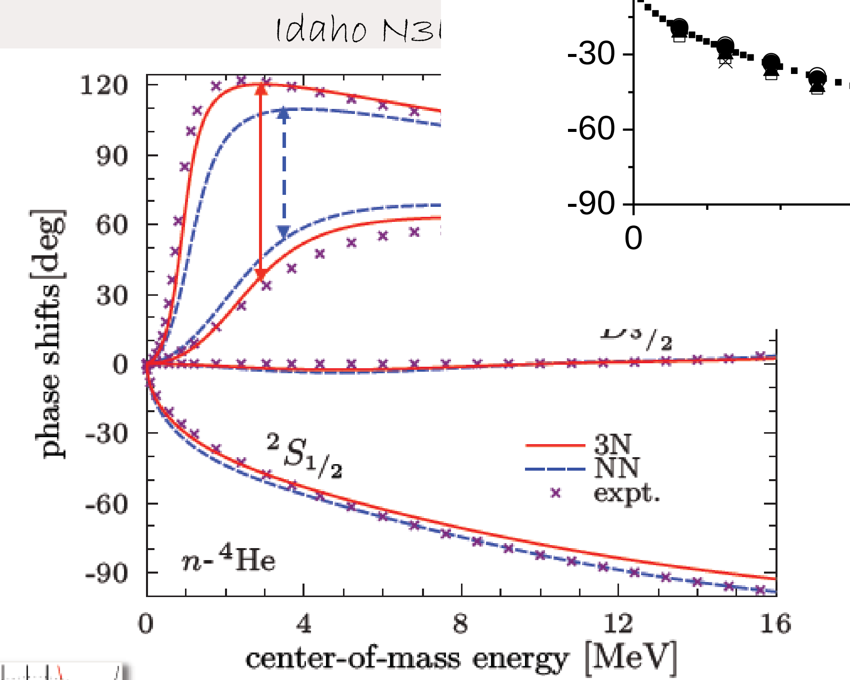
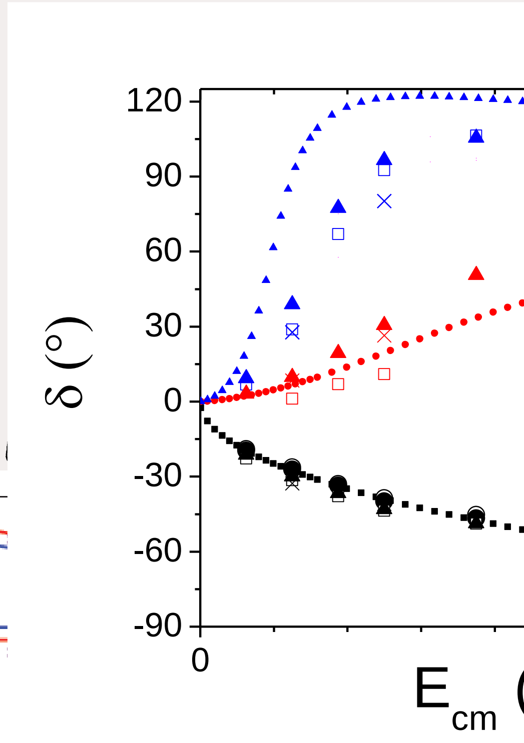
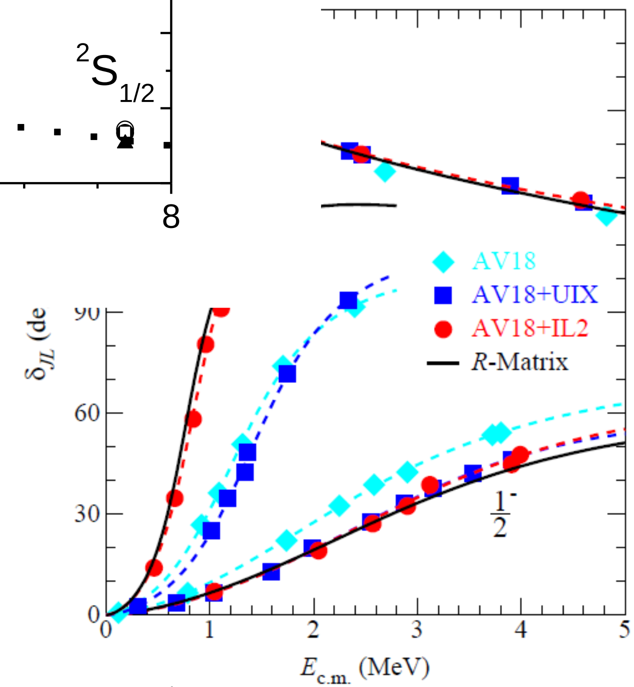
n-^{4}He scattering



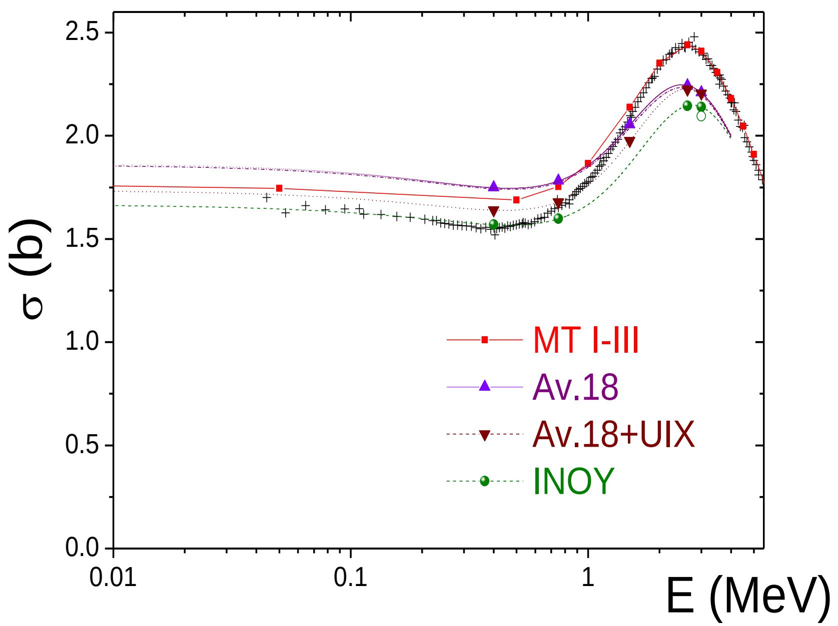


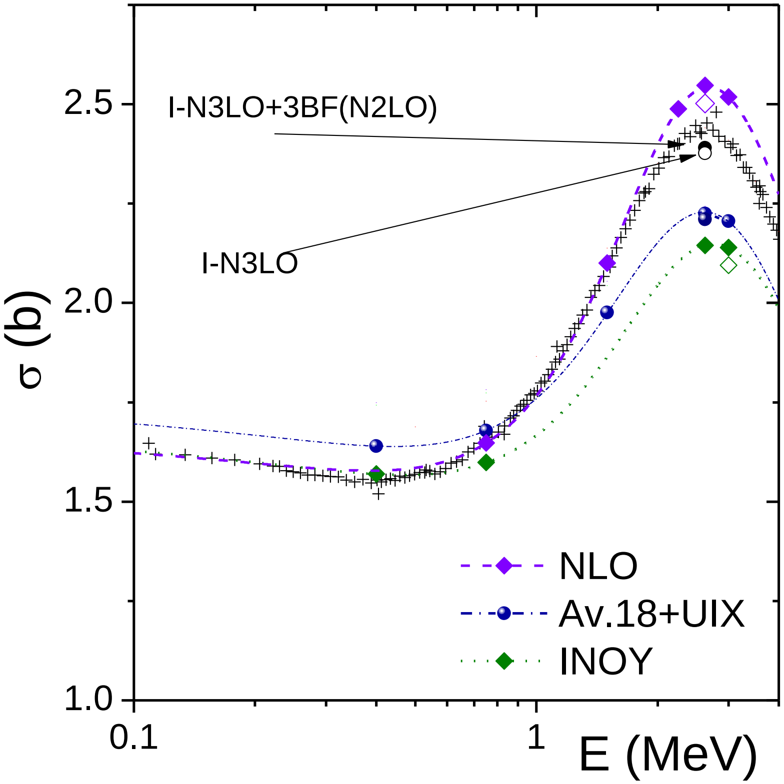


P. Navratil, INT, Nuclear Physics from Lattice QCD, March 21 - May 27, 2016

K.M. Nollett et. al., Phys.Rev.Lett.99:022502,2007

n-^{3}H total cross section





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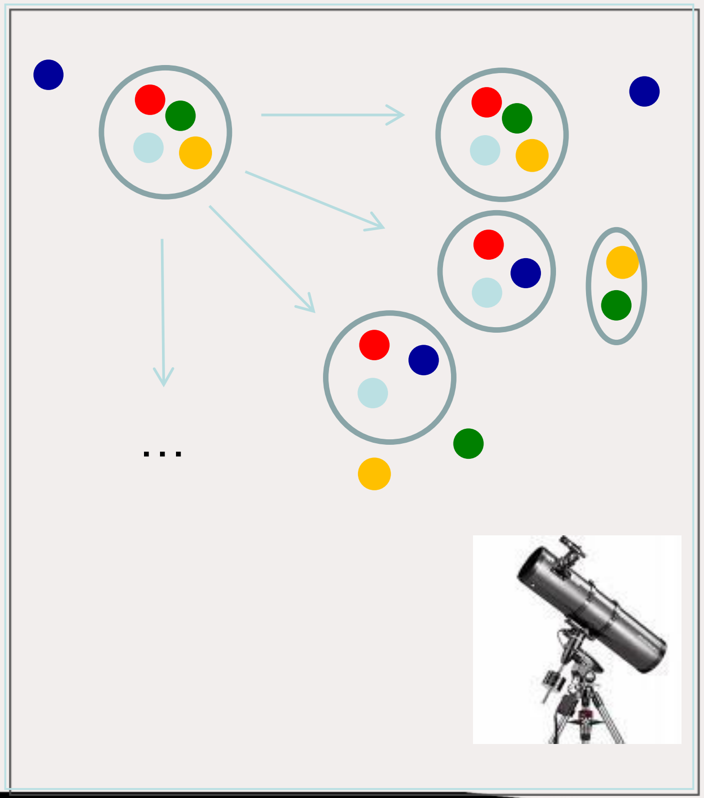


What to do?

Take care of the boundary condition

✓ Faddeev-Yakubovsky equations efficiently separates assymptotes of the binary channels

BUT

✓ Number of the reaction channels increases rapidly with A

✓ Even for 3-body systems there may exist infinite number of binary channels $e+H \rightarrow e+H(n=1,2,\ldots,\infty)$

✓ Complex behavior of the breakup assymptotes

FIND SOME TRICKS TO AVOID PROBLEMS AT BOUNDARY!

A dream: to solve scattering problems with a similar ease as bound state one (avoiding complex singularities or boundary conditions)

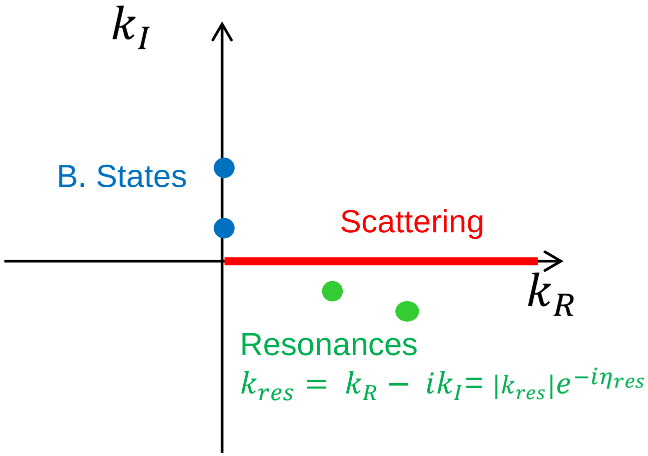
A. Deltuva, R.L. et al.,PPNP 74 (2014)

- "Calculable" R-matrix E.P. Wigner, Phys. Rev. 70 (1946) 15; P. Descouvemont, D. Baye, RPP 73 (2010) 036301.
- Lorentz integral transform V. D. Efros, W. Leidemann, and G. Orlandini, Phys. Rev. Lett. 78, 4015 (1997).
- Complex energy method F. A. McDonald and J. Nuttal, PRL 23 ,361 (1969) (config. space); E. Uzu, H. Kamada, and Y. Koike, Phys. Rev. C 68, 061001 (2003).; A. Deltuva and A. C. Fonseca, Phys. Rev. C 86, 011001(2012).
- Momentum lattice technique O. A. Rubtsova, V. N. Pomerantsev, and V. I. Kukulin, Phys. Rev. C 79, 064602 (2009).
- Continuum discretization A. Kievsky, M. Viviani, and L. E. Marcucci, Phys. Rev. C 85, 014001 (2012).
- Complex scaling method B. Giraud, K.Kato and A. Ohnishi, J. of Phys. A37 , 11575 (2004) (passing via spectral function); Y. Suzuki, W.Horiuchi, D.Baye, PTP,123 (2010) (passing via spectral function); A. T. Kruppa et al., Phys. Rev. C 75 (2007); R.L. & J. Carbonell, Phys. Rev. C 84, 034002 (2011).

Complex scaling: resonances, reactions & bound states in an unified formalism

In Quantum-Mechanics problem of N-particle dynamics may be often formulated in time-independent formalism and for the wave functions whose asymptotes contain ***only nontrivial outgoing waves***

$$\left(E_{sc} - H_0 - \sum_{i>j} V_{ij}(r_{ij})\right)\psi(\boldsymbol{r_i}) = S(\boldsymbol{r_i})$$



- Bound state problem: $\boldsymbol{S(r_i) \equiv 0}$
- Resonances: $\boldsymbol{S(r_i) \equiv 0}$
- Reactions due to external probe: $\boldsymbol{S(\rho \to \infty) = 0}$

$$\psi(\rho \to \infty) \propto e^{ik_x r_x}$$

- Collisions: $\boldsymbol{S(r_i) \equiv 0}$

$$\psi_{sc}(\rho \to \infty) \propto \psi_{in}(\boldsymbol{r_i}) + \sum_c A_c(\hat{k}_c)\, e^{i|k_c|r_c}$$

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Complex scaling: resonances, reactions & bound states in an unified formalism

R. Hartree , J.G. L. Michel, P. Nicolson (1946)
J. Nuttal and H. L. Cohen, Phys. Rev. **188** (1969) 1542

Complex scaling kills outgoing waves in the assymptote

$$r \longrightarrow re^{i\theta}$$

$$\tilde{\psi}\,(\rho \to \infty) \propto e^{ik_x r_x e^{i\theta}} = e^{ik_x r_x cos\theta} e^{-k_x r_x sin\theta}$$

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only outgoing waves!

Diverges

R. Lazauskas

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exp. bound if $0<\theta<\pi$

- Schrödinger equation in a driven form:

$$r\Psi(r) = F_l^{\,in}(r) + F_l^{\,sc}(r)$$

$$\frac{\hbar^2}{2\mu}\left(-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} - k^2\right) F_l^{\,in}(r) = 0$$

$$\frac{\hbar^2}{2\mu}\left(-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} + V_l(r) - k^2\right) F_l^{\,sc}(r) = -V_l(r)\, F_l^{\,in}(r)$$

Complex scaling $r \longrightarrow re^{i\theta}$

$$\frac{\hbar^2}{2\mu}\left(-\frac{d^2}{e^{2i\theta}dr^2} + \frac{l(l+1)}{e^{2i\theta}r^2} + V_l(re^{i\theta}) - k^2\right) \tilde{F}_l^{\,sc}(r) = -V_l(re^{i\theta})\, F_l^{\,in}(re^{i\theta})$$

exp. bound for the short range pot. term V^s

Complex scaling: guide for pedestrians

- Solution (using standard bound state techniques)

1. Expand c.s. outgoing wave in your favorite basis

$$\widetilde{\Psi}^{sc}(r) \approx \sum_{i=1}^{N} c_i \, \psi_i(r)$$

Complex coefficients

square integrable basis functions

2. Convert c.s. Schrödinger equation into linear algebra problem
3.

$$\int \psi_j(r)dr : \quad j = 1,..N \qquad \frac{\hbar^2}{2\mu}\left(-\frac{d^2}{e^{2i\theta}dr^2}+\frac{l(l+1)}{e^{2i\theta}r^2}+V_l(re^{i\theta})-k^2\right)\tilde{\Psi}_l^{sc}(r) = -V_l^s(re^{i\theta})\Psi_l^{in}(re^{i\theta})$$

$$\widetilde{H}_{j,i} c_i = b_j$$

4. Solve linear algebra problem to determine coefficients c_i
   i. Spectral expansion (*Y.Suzuki, W.Horiuchi, K. Kato*): *determine all eigenvalues/eigenvectors* — *limit* 10^5 *basis st.*
   ii. *(Iterative) linear algebra methods* — *upto* 10^{10} *basis st.*

5. Extract scattering observables from obtained solution(s) : multiple choice, most efficient way via integral expressions

Review of the method*

- Complex scaling method is very functional, adaptable to almost any b.s. technique. Already successfully applied using:
  - ✓ Spline basis
  - ✓ Laguerre, HO, Gaussian, correlated Gaussian, Sturmian basis functions
  - ✓ Lagrange mesh method

T. Myo, Y. Kikuchi, and K. Katō, Phys. Rev. C 85, 034338 (2012) , Attila Csótó, Phys. Rev. C 49, 2244 (1994), B. Gyarmati and A. T. Kruppa, Phys. Rev. C 34, 95 (1986)

- External probes
  - ✓ 2-body, 3-body,4-body systems, including repulsive Coulomb

T.M, K. Kato, S. Aoyama and K. Ikeda PRC63(2001)054313, T.Myo, K. Kato, H. Toki, K. Ikeda, PRC76(2007) 024305

- **Collisions, demonstrated to work for:**
  - ✓ 2-body collisions including Coulomb interaction, Optical potentials, $\frac{1}{r^n}$ potentials with $n \geq 4$
  - ✓ 3-body scattering including the break-up
  - ✓ 3-body scattering including non-local, Optical potential, attractive/repulsive Coulomb interactions
  - ✓ 3-body break-up amplitude for n-d & p-d scattering
  - ✓ 4-body scattering in 4N systems, including Coulomb

…

R.L. & J. Carbonell, Phys. Rev. C 84, 034002 (2011); A. Deltuva, R.L., A.C. Fonseca, "Clusters in Nuclei Vol.3 –LNP 875, 1 (2013); A. Deltuva, R.L. et al.,PPNP 74 (2014) ; R.L., Phys. Rev. C 91, 041001(R) (2015),..

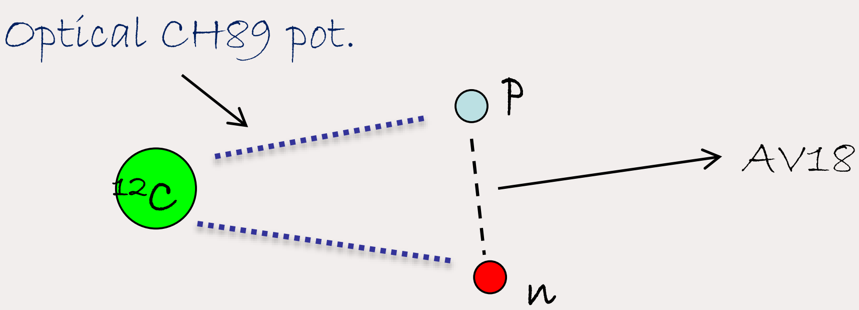
n-d scattering for MT I-III potential

TABLE V: Neutron-deuteron $^3S_1$ break-up amplitude calculated at E_{lab}=42 MeV as a function of the break-up angle ϑ.

	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
This work Re(^{3}S$_1$)	1.49[-2]	8.84[-4]	-3.40[-2]	-3.33[-2]	7.70[-2]	2.52[-1]	4.47[-1]	6.47[-1]	6.30[-1]	1.62[-1]
This work Im(^{3}S$_1$)	1.69[0]	1.74[0]	1.87[0]	1.92[0]	1.80[0]	1.68[0]	1.70[0]	1.96[0]	2.23[0]	3.17[0]
Ref. [23] Re(^{3}S$_1$)	1.48[-2]	9.22[-4]	-3.21[-2]	-3.09[-2]	7.70[-2]	2.52[-1]	4.51[-1]	6.53[-1]	6.93[-1]	1.05[-1]
Ref. [23] Im(^{3}S$_1$)	1.69[0]	1.74[0]	1.87[0]	1.92[0]	1.80[0]	1.67[0]	1.70[0]	1.95[0]	2.52[0]	3.06[0]

Ref.[23] J. L. Friar et al.:, Phys. Rev. C 51 (1995) 2356.

$^{2}H+^{12}C \leftrightarrow p+^{13}C$ scattering within 3-body model



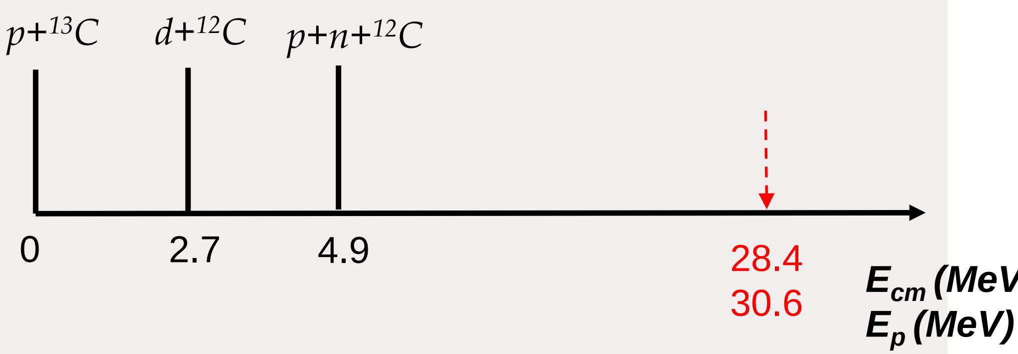


Fig. 4 Comparison of momentum- (solid curves) and configuration-space (dashed-dotted curves) results for the deuteron-^{12}C scattering at 30 MeV deuteron lab energy. Differential cross sections for elastic scattering and stripping are shown, the former in ratio to the Rutherford cross section $d\sigma_R/d\Omega$. The experimental data are from Refs. [41, 40].

$$^{12}C+d \Leftrightarrow \begin{cases} ^{12}C+d \\ ^{13}C+p \\ ^{12}C+n+p \end{cases}$$

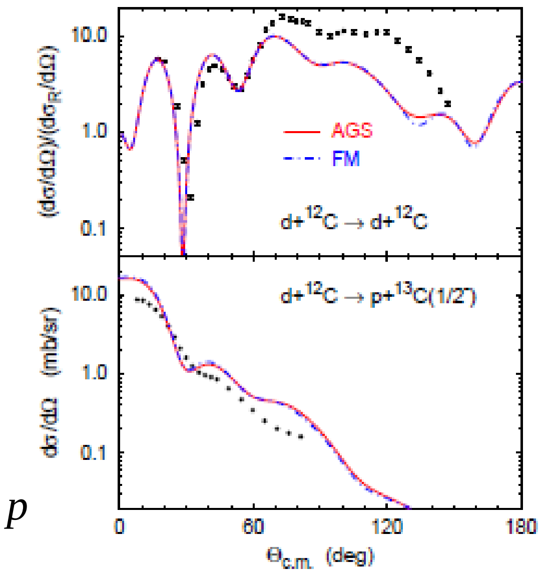
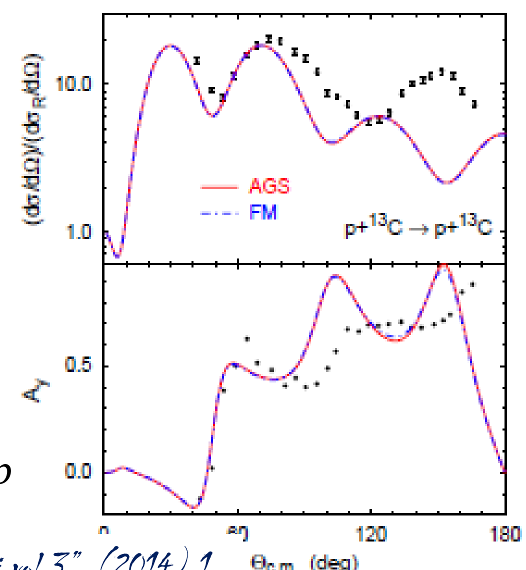


Fig. 5 Comparison of momentum- (solid curves) and configuration-space (dashed-dotted curves) results for the proton-^{13}C elastic scattering at 30.6 MeV proton lab energy. Differential cross section divided by the Rutherford cross section and proton analyzing power are shown. The experimental data are from Ref. [25].

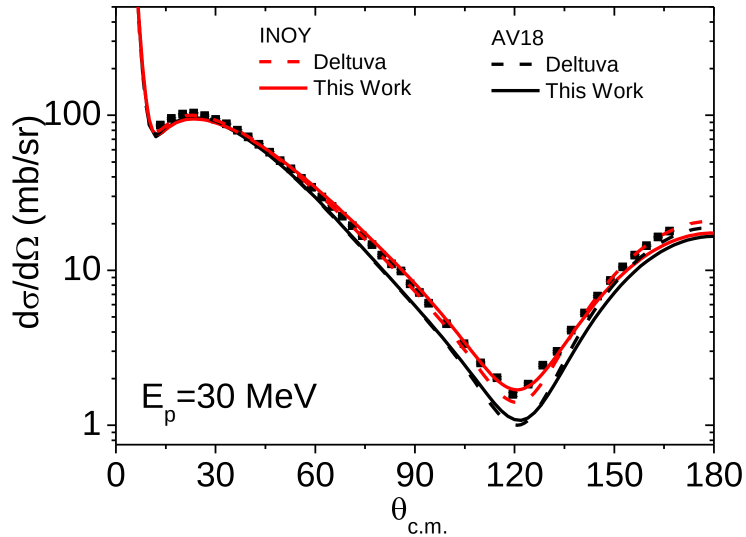
$$^{13}C+p \Leftrightarrow \begin{cases} ^{12}C+d \\ ^{13}C+p \\ ^{12}C+n+p \end{cases}$$

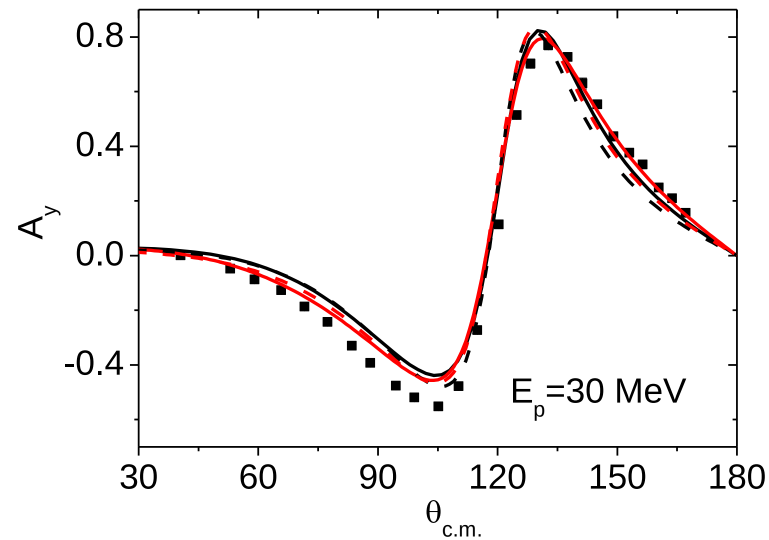


A. Deltuva, A.C. Fonseca and R.L., "Clusters in nuclei vol.3", (2014) 1.

R. Lazauskas

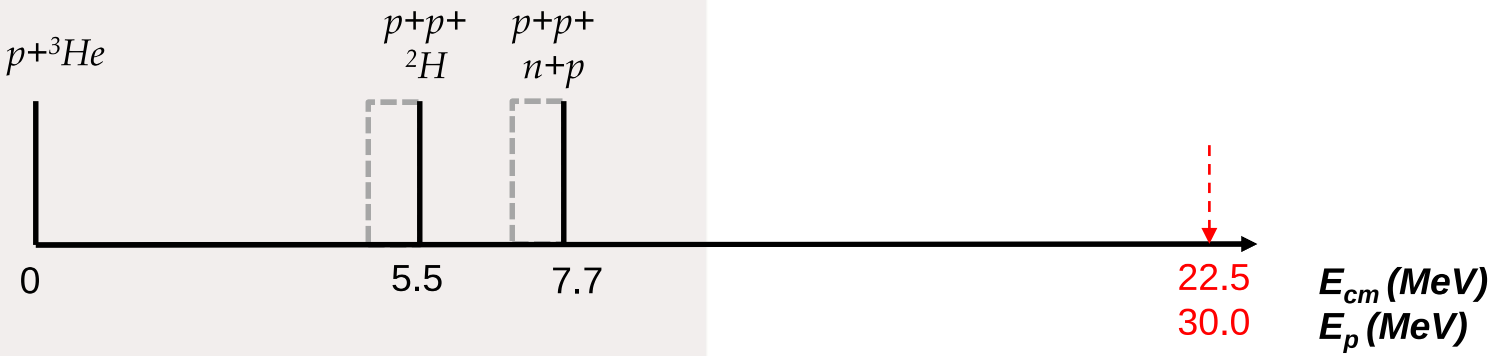
p-^{3}He scattering/realistic NN interactions





*A. Deltuva and A.C. Fonseca, Phys. Rev. C87 (2013) 054002

**Exp: B.T. Murdoch et al., Phys. Rev. C21(1984) 2001; J. Birchall, Phys. Rev. C29(1984) 2009.



R. Lazauskas

Price to pay

- C.S. outgoing waves asymtoticaly converges as $e^{-|k_{cp}|r\sin(\theta)}$
  - Slow convergence for small k_{cp}, difficulties in close-threshold region
  - Convergence is faster for large θ
- One should work with c.s. potential or c.s. basis functions
  - Imposes upper limit for the q to be used, since

$V(r)\sim\exp(-\mu r^n) \rightarrow \exp(-\mu r^n e^{in\theta})$ → Starts to diverge for θ>π/(2n)

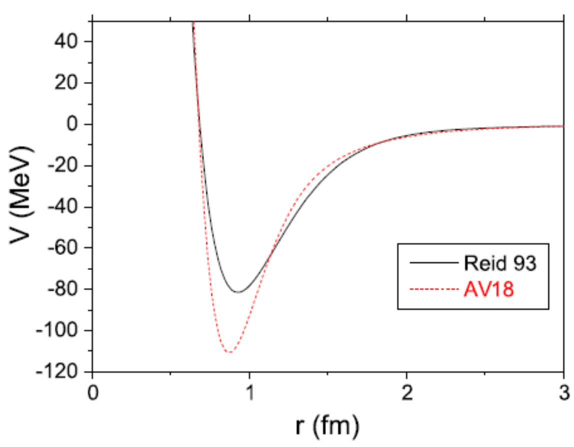
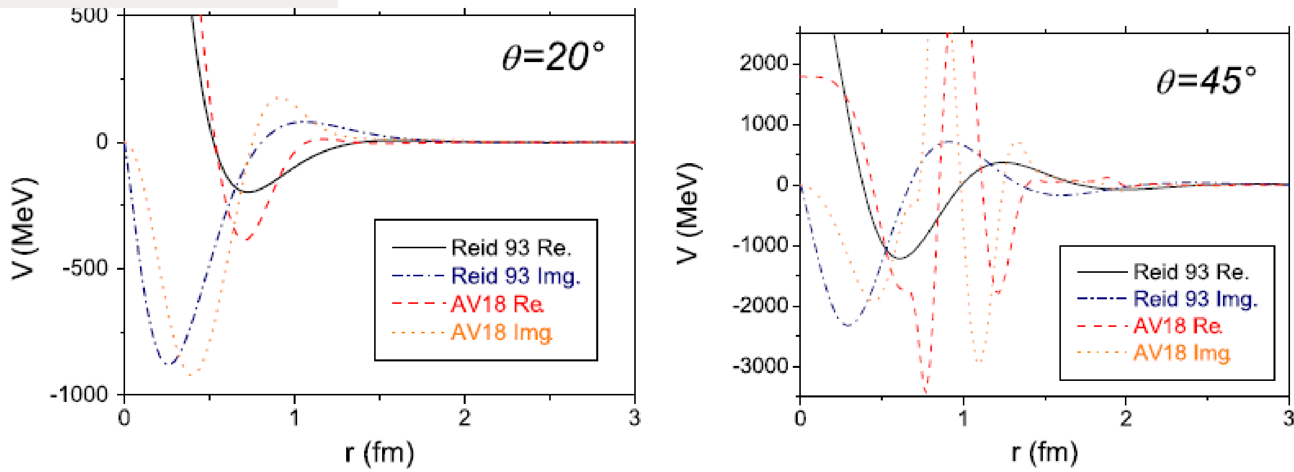


FIG. 1. (Color online) Reid 93 and AV18 $^1S_0$ nn potentials.



Conclusion

• FY eq. formalism remains reference in few-body scattering calculations. Four-nucleon scattering problem approaches fixed status. The first solution of 5-body FY equations is presented.

• At the same time several promising methods emerge to solve scattering problems based on trivial boundary conditions.

• In particular, complex-scaling method has been revived in nuclear physics. This method enables to solve few-body scattering problem employing standard bound state techniques (feasible by almost any config. space bound state technique and requires very limited effort to be implemented)

• Simple extension of the formalism to many-body scattering case

• Very accurate results are already obtained for 3-body and 4-body elastic and breakup scattering

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