From Cluster Model, Ab-inito Theory To Halo Effective Field Theory

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Toward Predictive Theories of Nuclear Reactions Across the Isotopic Chart
Institute for Nuclear Theory, 03/24/2017

Outline

- Introduction to effective field theory

  → see also van Kolck's talk

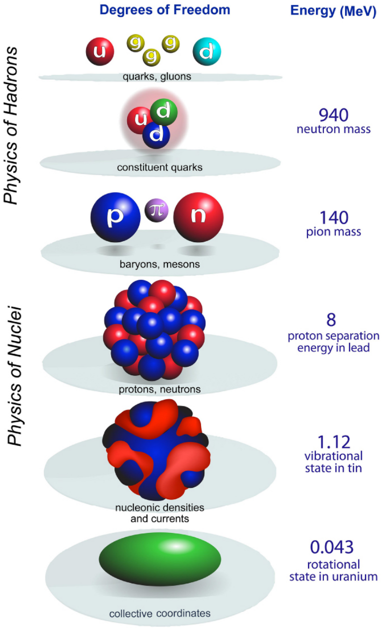
- Cluster models
- EFT studies of halo structures:
  - $1n$-halo (s-wave, p-wave)
  - $1p$-halo and α-clusters)
  - $2n$-halo
- ~~Electromagnetic reactions in halo nuclei~~

  → see Rupak's talk & Phillips's talk

- Discussions:
  - EFT construction and power counting
  - Universality in halo nuclei
  - Connect EFT with cluster models, ab initio theories, and experiments

Hammer, CJ, Phillips, arXiv:1702.08605

Energy-scale hierarchy in nuclear physics



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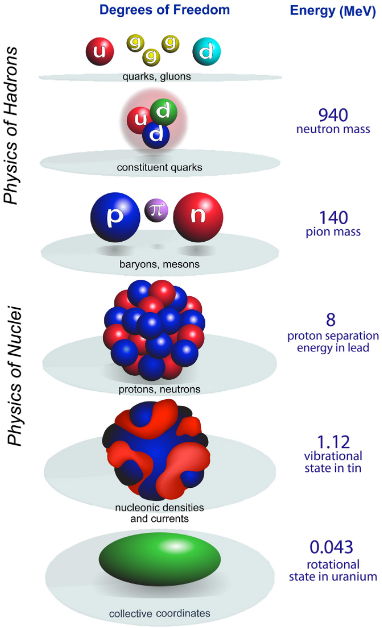
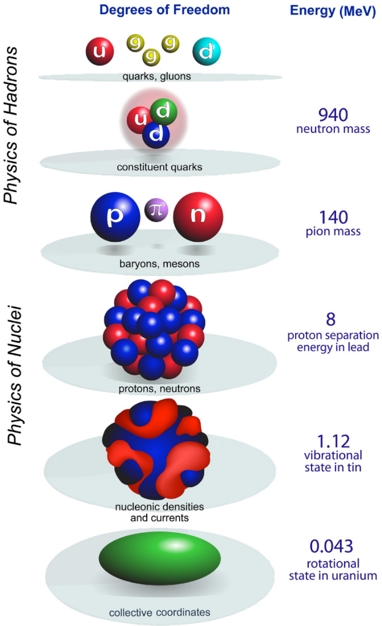


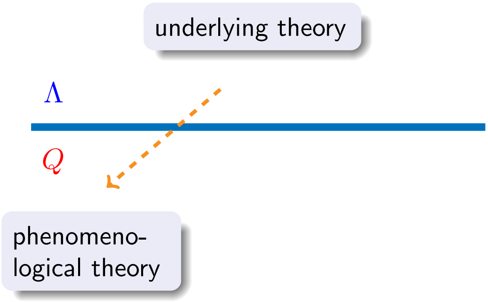
underlying theory

Λ

Q

Energy-scale hierarchy in nuclear physics





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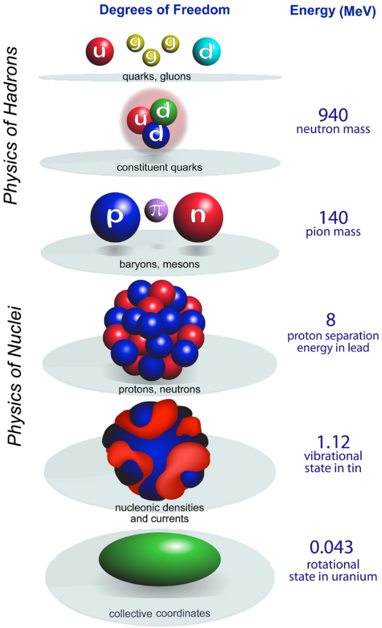

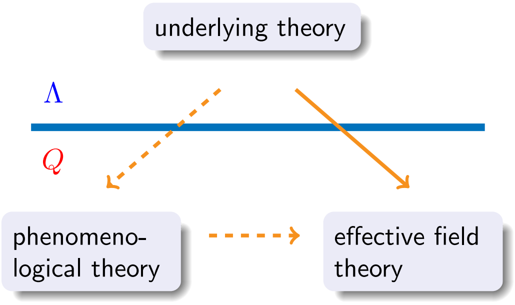

- In RG evolution:
  - effective d.o.f. appears from symmetry breaking
  - EFT emerges systematically from underlying theory
  - EFT “inherits” good aspects from phenomenology

Key elements of an EFT

- separation of scales:
  - observables at typical momentum scale Q
  - short-range physics at scale Λ, where $\Lambda \gg Q$
  - Q-physics is affected by short-range effects at Λ through quantum tunning
  - Q-physics is insensitive to details in Λ-physics

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  - effective Lagrangian:

$$\mathcal{L} = \sum_{\nu,i} c_\nu (Q/\Lambda)^\nu \hat{O}_{\nu,i}$$

  - a limited number of LECs enter at a given EFT order
  - predict observables at Q-scale with controlled uncertainties at each order

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  - a limited number of LECs enter at a given EFT order
  - predict observables at Q-scale with controlled uncertainties at each order
- universality at a given EFT order:
  - observables are correlated through a limited number of LECs

Atomic nuclei

lattice QCD

Λ ~GeV

$Q \sim m_\pi \quad \leftarrow (2M_N E/A)^{1/2}$

Atomic nuclei

lattice QCD

$\Lambda \sim$GeV

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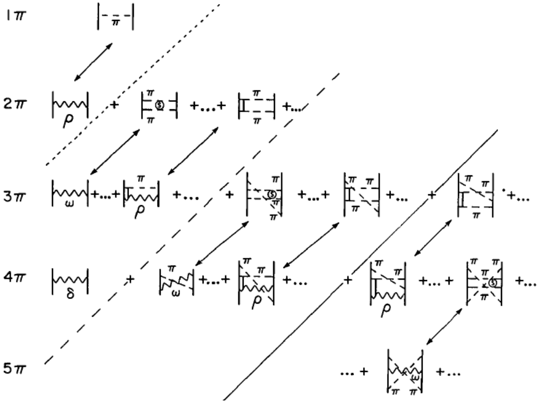
Bonn full model

short-range physics is added phenomenologically



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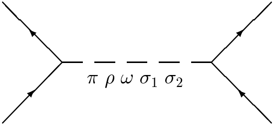
CD-Bonn potential



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Atomic nuclei

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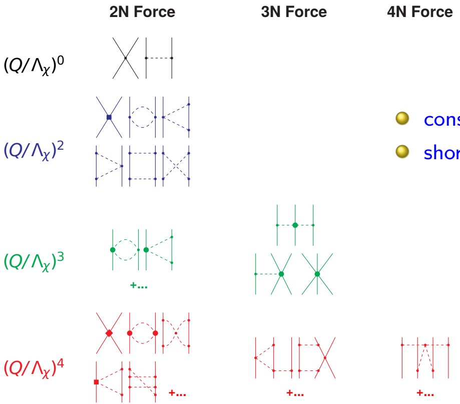
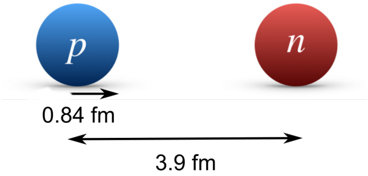
chiral EFT potential



- constructed in systematic expansions of Q/Λ_χ
- short-range physics is embedded in LECs

Halo nuclei and nuclear clustering

- ^{2}H
  - simplest neutron halo



Halo nuclei and nuclear clustering

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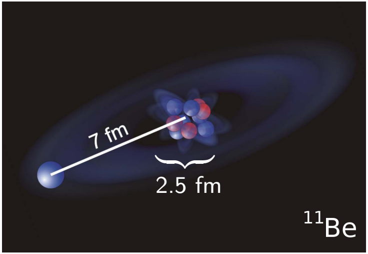
- neutron halos with a compound nuclear core
  - ^{6}He, ^{11}Be, ...



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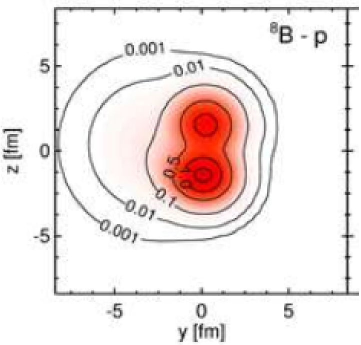
- proton halos
  - ^{17}F* (s-wave halo)
  - ^{8}B (p-wave halo):



FMD calculation (T. Neff, GSI)

Halo nuclei and nuclear clustering

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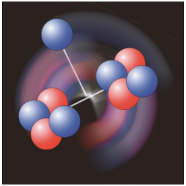
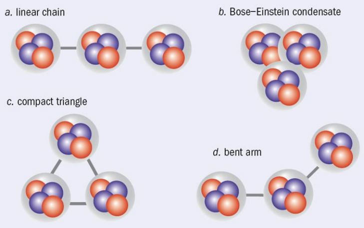
- α-clustering
  - ^{9}Be: $\alpha + \alpha + n$
  - ^{8}Be, ^{12}C*, ^{16}O*





Halo physics near clustering threshold

ab initio theory

$\Lambda \sim \sqrt{m_n E_c^*}$

$Q \sim \sqrt{m_n S_n}$

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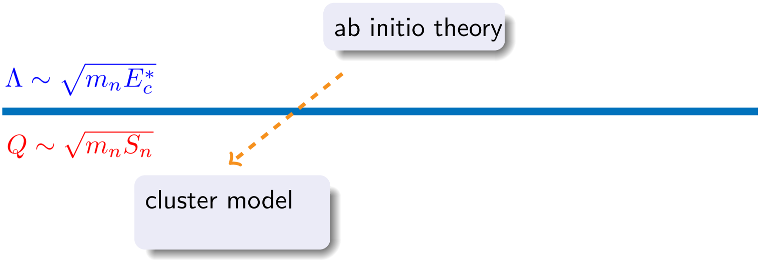
It is in general difficult to tackle threshold physics in ab initio theories

- difficulties in studying continuum problem in many-body calculations
  improved by NCSMC, GSM-Bergren, Lattice-EFT scattering, LIT, $\cdots$
- systematic uncertainty controls in chiral potentials
  issues in power counting, fitting parameter correlations, $\cdots$
- threshold observables may converge slower in Q/Λ than binding energy does
  $Q_{\text{halo}} \ll Q_{\chi\text{EFT}} \approx (2M_N E/A)^{1/2}$

Halo physics near clustering threshold



^{23}N in a cluster model

	^{21}N	^{22}N	^{23}N
S_{1n} [MeV]	$4.59(11)$	$1.28(21)$	$1.79(36)$
S_{2n} [MeV]	$6.75(10)$	$5.87(20)$	$3.07(31)$

AME2012

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- **We study ^{23}N in $n + n +^{21}$N cluster model**

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- **Faddeev equation in hyperspherical harmonics expansion**
  numerical tool: FaCE [Thompson, Nunes, Danilin, Comp. Phys. Comm. '04]

Phenomenological Interactions

- realistic nn: Gogny-Pires-De Tourreil (GPT)
- phenomenological n-^{21}N: Wood Saxon

$$V_{n\text{-core}}(r) = -\frac{V_0}{1+\exp(\frac{r-r_0}{a})} - \frac{V_{\text{so}}}{ra}\frac{\exp(\frac{r-r_0}{a})}{(1+\exp(\frac{r-r_0}{a}))^2}\mathbf{L}\cdot\mathbf{S}$$

- core-neutrons occupy $(0s_{1/2})^2$ $(0p_{3/2})^4$ $(0p_{1/2})^2$ $(0d_{5/2})^6$ shells
  $\epsilon(0d_{5/2}) = S_{1n}[^{21}\text{N}]$
- valence neutrons occupy either $(1s_{1/2})^2$ or $(0d_{3/2})^2$
  $\epsilon(1s_{1/2}) = S_{1n}[^{22}\text{N}]$

^{23}N G.S. & Excited Halo States

- We tune $V_{n\text{-core}}$ to reproduce
  ^{21}N $S_{1n} = 4.59(11)$ MeV
  ^{22}N $S_{1n} = 1.28^{+21}_{-21}$ MeV
- We predict S_{2n} and r_m

S_{2n} MeV	r_m fm	S^*_{2n} MeV	r^*_m fm
4.13	2.969	0.315	4.272
3.64	2.985	0.185	4.358
3.13	3.004	0.069	4.476

Experiment: $S_{2n} = 3.07(31)$ MeV

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Experiment: $S_{2n} = 3.07(31)$ MeV

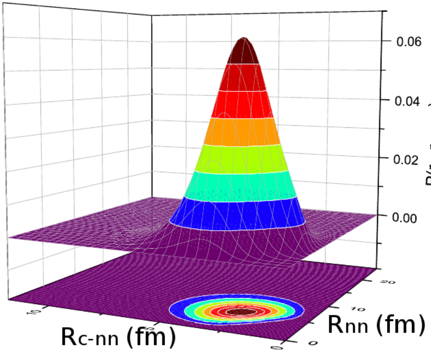
- add 3BF $V_3(\rho) = W_0 e^{-\rho^2/\rho_0^2}$ to reproduce
  ^{23}N $S_{2n} = 3.07$ MeV
- Predictions in S_{2n} and r_m

S_{2n} MeV	r_m fm	S^*_{2n} MeV	r^*_m fm
3.07	3.022	0.195	4.629
3.07	3.019	0.128	4.790
3.07	3.011	0.064	5.011

3BF can sometimes be small when short-range physics is already included

^{23}N Probability Density Distributions

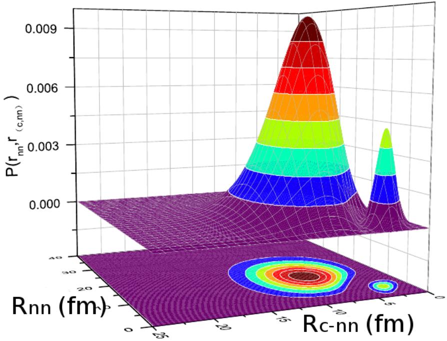
ground state



$(1s_{1/2})^2$ 95%

$(0d_{3/2})^2$ 5%

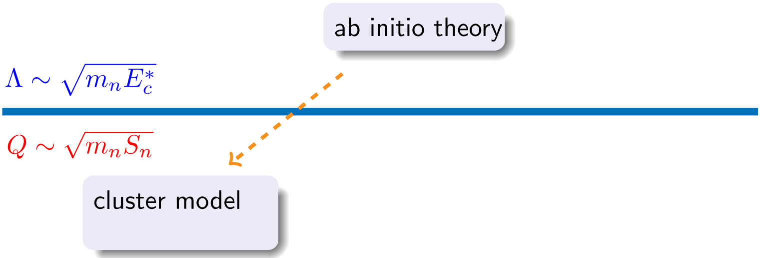
excited state



$(1s_{1/2})^2$ 77%

$(0d_{3/2})^2$ 23%

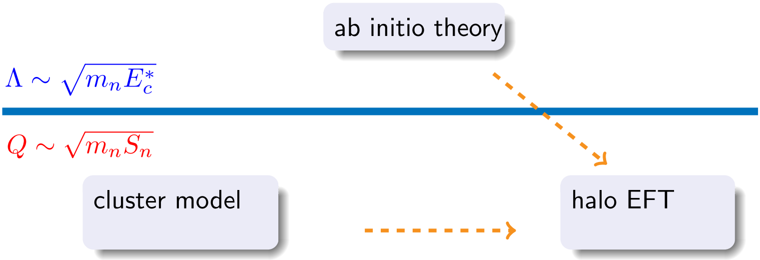
Halo physics near clustering threshold



difficulties in cluster models:

- how to reduce model dependence?
- how to quantify theory uncertainties?
- how to systematically improve accuracy?

Halo physics near clustering threshold



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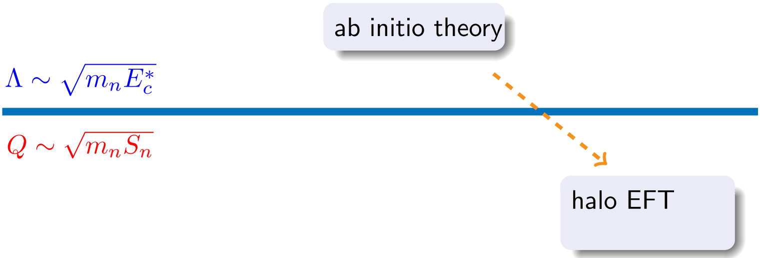


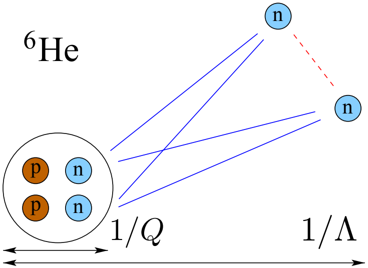
- cluster configuration in halo EFT:

  core + valence nucleons d.o.f.

- separation of scales:

  $Q \ll \Lambda \rightarrow$ systematic expansion in observables

- short-range effects from underlying theory:
  - anti-symmetrization of core nucleons is not explicitly included in halo EFT
  - those short-range effects are embedded in LECs



Halo Effective Field Theory

- We adopt EFT with contact interactions to describe clustering in halo nuclei
- introduce auxiliary dimer fields for bound/resonance states

$$
\begin{aligned}
\mathscr{L} &= \mathscr{L}_1 + \mathscr{L}_2 + \mathscr{L}_3 \\
\mathscr{L}_1 &= n^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_n} \right) n + c^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_c} \right) c \\
\mathscr{L}_2 &= s^\dagger \left[\eta_0 \left(i\partial_0 + \frac{\nabla^2}{4m_n} \right) + \Delta_0 \right] s + \sigma^\dagger \left[\eta_1 \left(i\partial_0 + \frac{\nabla^2}{2(m_n + m_c)} \right) + \Delta_1 \right] \sigma \\
&\quad + g_0 \left[s^\dagger (nn) + \text{h.c.} \right] + g_1 \left[\sigma^\dagger (nc) + \text{h.c.} \right], \\
\mathscr{L}_3 &= h\, (\sigma n)^\dagger (\sigma n)
\end{aligned}
$$

- 2-body contact (LO)

$= -i\sqrt{2}g$

$g \leftarrow$ 2-body observable

- 3-body contact (LO)

$= ih$

$h \leftarrow$ 3-body observable

One-neutron s-wave halos

- **scattering amplitude:** $t_0(k) = \frac{2\pi}{\mu}\left(\frac{1}{a_0} - \frac{r_0}{2}k^2 + ik\right)^{-1}$
  - in low-energy bound/virtual state: $a_0 \sim 1/Q$; $r_0 \sim 1/\Lambda$
  - expand t-matrix in r_0/a_0

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- **tune coupling**
  - LO: $a_0 = \left(\frac{2\pi\Delta}{\mu g^2} + \Lambda\right)^{-1}$
  - NLO: $r_0 = -\eta\frac{2\pi}{\mu^2 g^2}$

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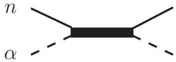
- **tune coupling**
  - LO: $a_0 = \left(\frac{2\pi\Delta}{\mu g^2} + \Lambda\right)^{-1}$
  - NLO: $r_0 = -\eta\frac{2\pi}{\mu^2 g^2}$
- **pole expansion**: $t_0(k) = \frac{2\pi}{\mu}\frac{C_\sigma^2/C_{\sigma,LO}^2}{\gamma_0 + ik} + \mathrm{regular}$
  - ANC: $\psi_0(\boldsymbol{r}) = C_\sigma Y_{00}(\hat{r})\exp(-\gamma_{0,\sigma}r)/r$
  - LO: $C_{\sigma,LO} = \sqrt{2\gamma_0}$
  - NLO: $C_\sigma/C_{\sigma,LO} = 1/\sqrt{1-\gamma_0 r_0}$

One-neutron s-wave halos

	^{2}H	^{11}Be	^{15}C	^{19}C
EXP				
S_{1n} [MeV]	2.224573(2)	0.50164(25)	1.2181(8)	0.58(9)
E_c^* [MeV]	140	3.36803(3)	6.0938(2)	1.62(2)
$\langle r_{nc}^2 \rangle^{1/2}$ [fm]	3.936(12)	6.05(23)	4.15(50)	6.6(5)
	3.95014(156)	5.7(4)	7.2$\pm$4.0	6.8(7)
		5.77(16)	4.5(5)	5.8(3)
EFT				
Q/Λ	0.33	0.39	0.45	0.6
r_0/a_0	0.32	0.32	0.43	0.33
$C_\sigma/C_{\sigma,LO}$	1.295	1.3	1.63	1.3
$\langle r_{nc}^2 \rangle^{1/2}$ [fm]	3.954	6.85	4.93	5.72

P-wave neutron halos

- nc interaction in a p-wave bound/resonance state



$$= \frac{2\pi}{\mu} \frac{\vec{p} \cdot \vec{q}}{-1/a_1 + r_1 k^2/2 - ik^3}$$

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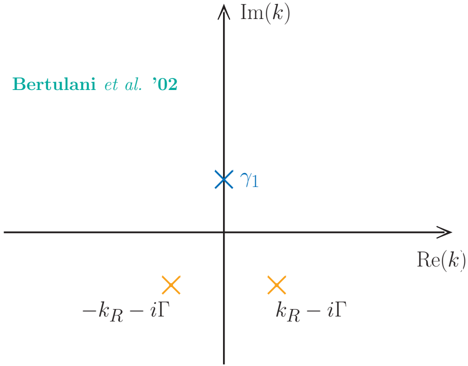
$$= \frac{2\pi}{\mu} \frac{\vec{p} \cdot \vec{q}}{-1/a_1 + r_1 k^2/2 - ik^3}$$

- causality $r_1 \not\to 0$ Nishida '12
- both a_1 and r_1 enter at leading order
- p-wave power counting
  - resum ik^3: $1/a_1 \sim Q^3$, $r_1 \sim Q$ [Bertulani, Hammer, van Kolck NPA '02]
  - perturbative ik^3: $1/a_1 \sim Q^2\Lambda$, $r_1 \sim \Lambda$ [Bedaque, Hammer, van Kolck PLB '03]
    - $a_1 < 0$: shallow resonance: ^{5}He $(3/2^-)$
    - $a_1 > 0$: shallow bound state: ^{11}Be $(1/2^-)$, ^{8}Li (2^+), ^{8}Li* (1^+)

p-wave power counting: ik^3 resummation

- ik^3 resuming: Bertulani, Hammer, van Kolck NPA '02
  - $1/a_1 \sim Q^3 \quad r_1 \sim Q$
  - two fine tunings at LO
- for $a_1 < 0$: ^{5}He $(3/2^-)$
  - a broad shallow resonance
  - a shallow bound state (spurious)

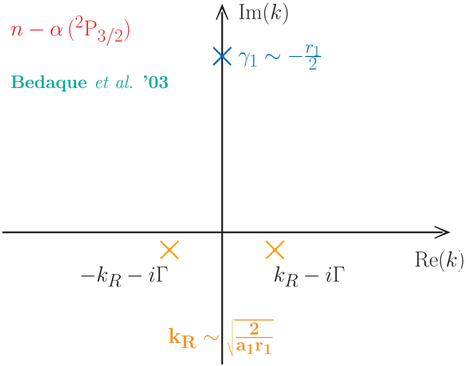


p-wave power counting: perturbative ik^3

- perturbative ik^3:
  - $1/a_1 \sim Q^2\Lambda \quad r_1 \sim \Lambda$
  - one fine tuning at LO

Bedaque, Hammer, van Kolck PLB '03

- for $a_1 < 0$: ^{5}He $(3/2^-)$
  - a narrow shallow resonance
  - a deep bound state (unphysical)



p-wave power counting: perturbative ik^3

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- for $a_1 < 0$: ^{5}He $(3/2^-)$
  - a narrow shallow resonance
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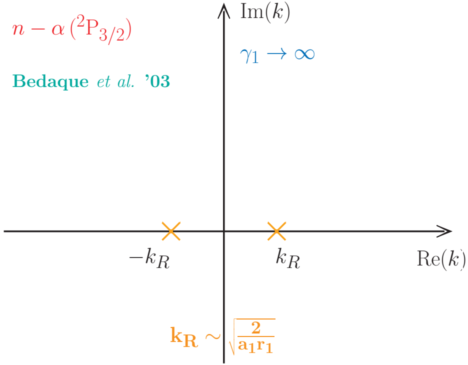
  - drop ik^3 term at LO:
    $\Gamma \to 0$, $\gamma_1 \to \infty$



p-wave power counting: perturbative ik^3

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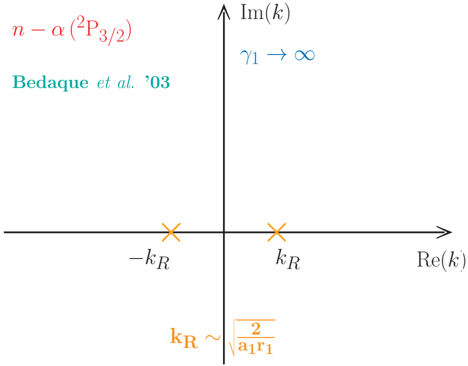
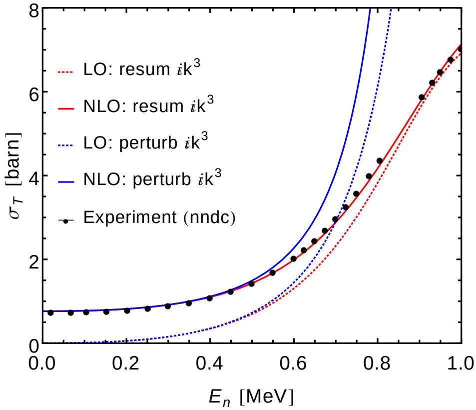
  - a narrow shallow resonance
  - a deep bound state (unphysical)
  - drop ik^3 term at LO:
    $\Gamma \to 0$, $\gamma_1 \to \infty$
- perturbative ik^3 is only rigorous for $|k - k_R| \gg Q^2/\Lambda$

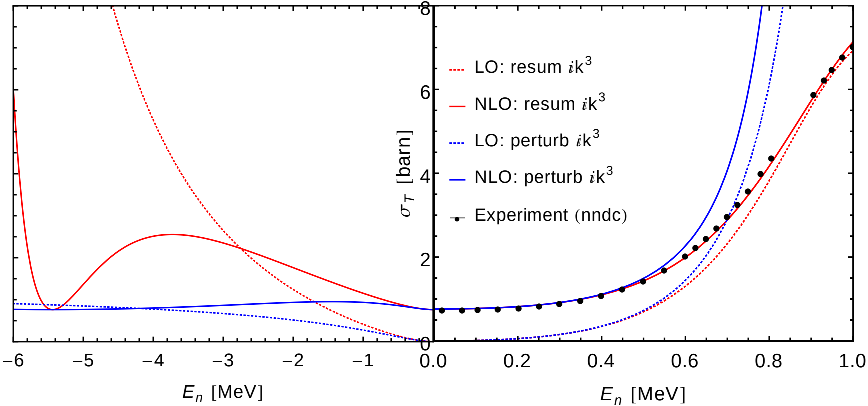


$n - \alpha$ scattering cross sections



- ik^3 resummation works better when E is close to E_R

$n - \alpha$ scattering cross sections



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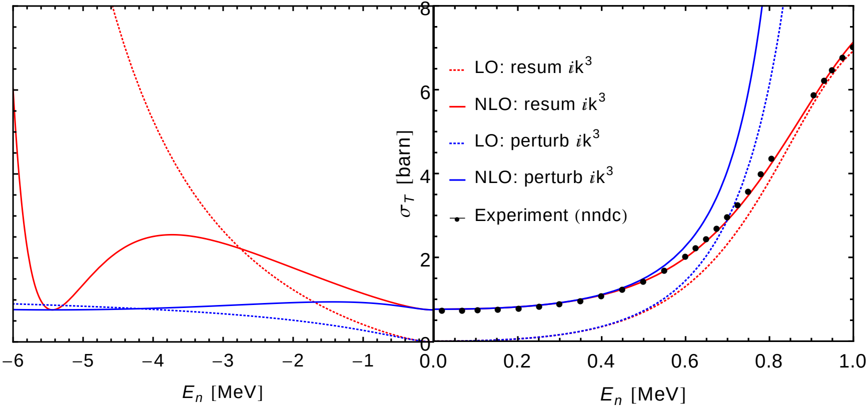
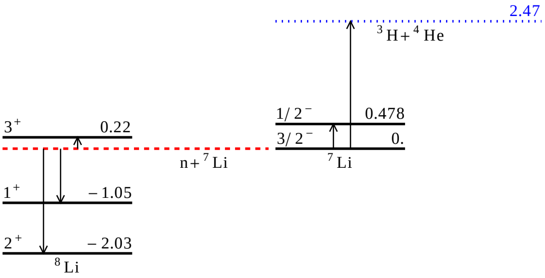


- ik^3 resummation works better when E is close to E_R
- ik^3 resummation generates spurious bound-state pole; perturbative ik^3 does not
- In ^{6}He three-body calculation, $E \to -\Lambda$ in loop integral
  - Rotureau, van Kolck, FBS '13: ik^3 resummation + spurious pole subtraction
  - C.J., Elster, Phillips, PRC '14: perturbative ik^3 (free from spurious pole)

Bound p-wave halos in ^{8}Li



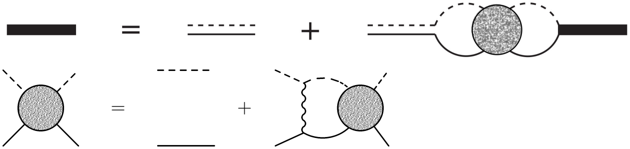
p-wave ANCs in each n-^{7}Li channel

	$C_{(^3P_2)}$	$C_{(^5P_2)}$	$C_{(^3P_2^*)}$	$\tilde{C}_{(^3P_1)}$	$\tilde{C}_{(^5P_1)}$	$\tilde{C}_{(^1P_1^*)}$	$\tilde{C}_{(^3P_1^*)}$
VMC	$-0.283(12)$	$-0.591(12)$	$-0.384(6)$	$0.220(6)$	$0.197(5)$	$-0.195(3)$	$-0.214(3)$
EXP	$-0.284(23)$	$-0.593(23)$		$0.187(16)$	$0.217(13)$		

ANCs: input in EFT calculations of neutron capture reactions
Zhang, Nollett, Phillips, PRC '14

Halo EFT with Coulomb

- In halo/clustering systems with Coulomb interactions, a new scale $k_c = Q_c \alpha_{em} \mu$ enters
  - $k_c \gtrsim Q$: Coulomb interaction is nonperturbative

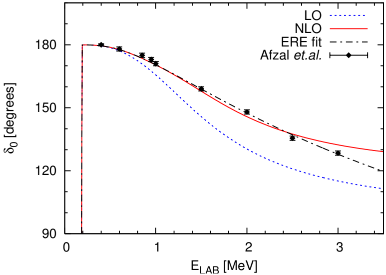


p-p scattering [Kong, Ravndal, PLB '99; NPA '10]
p-α and α-α scattering [Higa, Hammer, van Kolck, NPA '08; Higa, FBS '11]
^{17}F* [Ryberg, Forssén, Hammer, Platter, PRC '14; AnnPhys '16]

  - $k_c \ll Q$: Coulomb interaction is perturbative
^{3}H and ^{3}He [König, Grießhammer, Hammer, van Kolck, JPG '16]

Fine tuning in α clustering and proton-halos

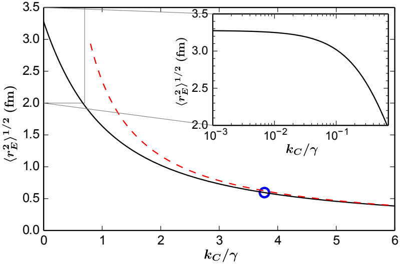
α-α narrow resonance



Higa, Hammer, van Kolck, NPA '08

- $k_c \gg k_R$
- LO $k_R^2 \approx 2/(a_0 r_0)$
- NLO a_0, r_0, P_0

universality in proton halos



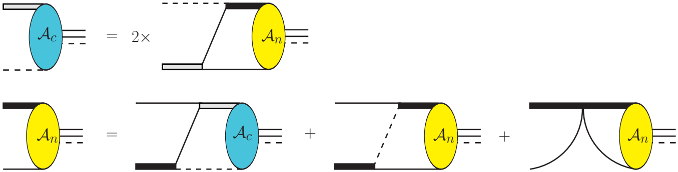
Ryberg, Forssén, Hammer, Platter, AnnPhy '16

- for $k_c \gg \gamma$:

  large cancellation btw Coulomb repulsion and strong interaction

$2n$ halos in Faddeev formalism

- **solving transition amplitudes $\mathcal{A}_c$ and $\mathcal{A}_n$**



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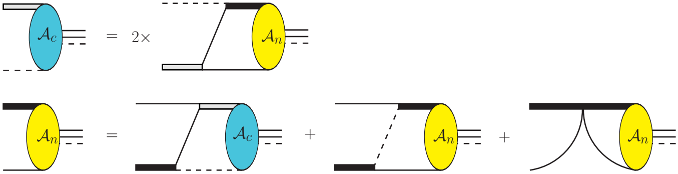
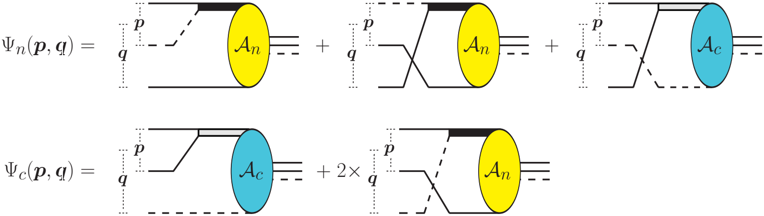


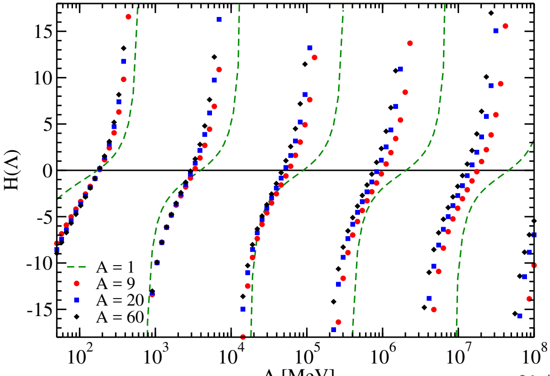
- **three-body wave functions**



Three-body renormalization

- **running of three-body coupling**
  - tune $H(\Lambda) = \Lambda^2 h / 2mg^2$:
    reproduce one observable in a $2n$-halo

Hammer, CJ, Phillips, arXiv:1702.08605



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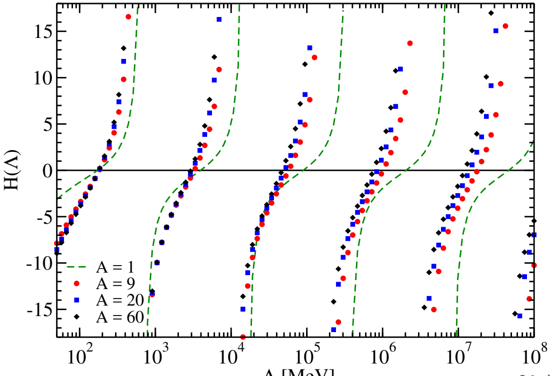
  - $H(\Lambda)$ periodic for $\Lambda \to \lambda\Lambda$ [$A = 1$ Bedaque *et al.* '00]

Hammer, CJ, Phillips, arXiv:1702.08605



Three-body renormalization

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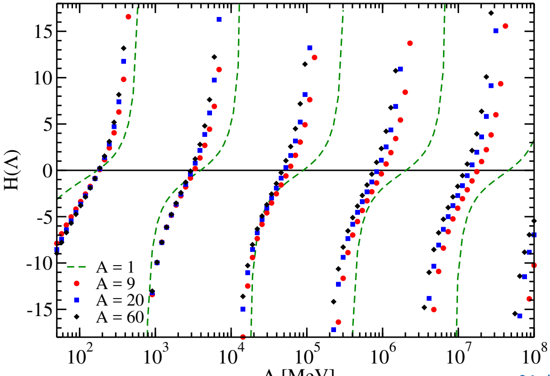
  - $H(\Lambda)$ periodic for $\Lambda \to \lambda\Lambda$ [$A = 1$ Bedaque *et al.* '00]
  - $H(\Lambda)$ appears as RG limit cycle [Mohr et al., AnnPhys '06]
  - discrete scale invariance $\to$ Efimov physics

Hammer, CJ, Phillips,
arXiv:1702.08605



EFT For $2n$ s-wave Halos

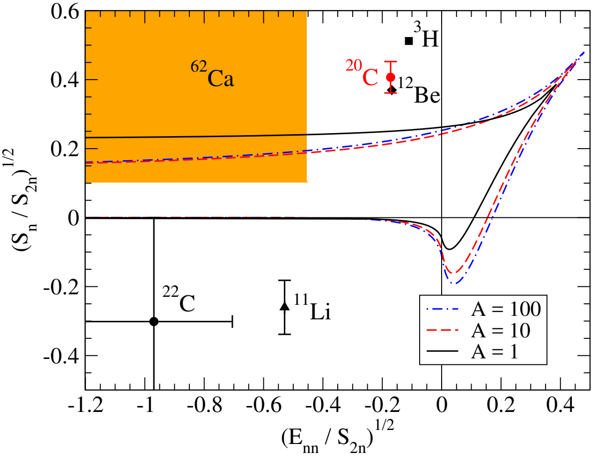
- n-core in s-wave virtual/real bound state:
  ^{11}Li, ^{12}Be, ^{20}C [Canham, Hammer, EPJA '08, NPA '10]
  ^{22}C Acharya, C.J., Phillips, PLB '13

- charge radius of 2n s-wave halos
  [Hagen, Hammer, Platter, EPJA '13]
  [Vanasse, PRC '17]

- heaviest 2n s-wave halo:
  ^{62}Ca [Hagen, Hagen, Hammer, Platter, PRL '13]
  fit n-^{60}Ca scattering length from coupled-cluster calculations

Universality in $2n$ s-wave halo

- contour constraints on ground-state energy S_{2n} if the excited-state energy $B_3^* = \max\{0, E_{nn}, S_{1n}\}$



Canham, Hammer, EPJA '08; Frederico *et al.* PPNP '12;

Hammer, CJ, Phillips, arXiv:1702.08605

^{22}C: $2n$ Halo

	^{20}C	^{21}C	^{22}C
bound/unbound	bound		
ground state	0^+		
binding/virtual energy [MeV]	S_{2n}: 3.50(24) AME2012		
matter radius r_m [fm]	2.97(5) Ozawa et al. '01 $2.97^{+0.03}_{-0.05}$ Togano et al. '16		

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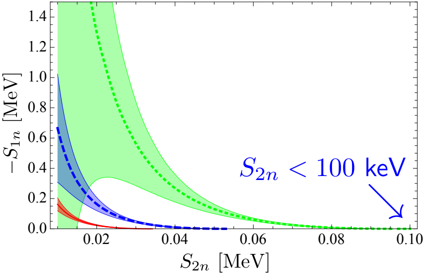
- Halo EFT [Acharya, CJ, Phillips, PLB '13]
  we fit to ^{22}C matter radius to constrain:
  - S_{1n} in ^{21}C ($a < 0$)
  - S_{2n} in ^{22}C

Correlations in ^{22}C

$$\langle r_m^2 \rangle_{2n-\mathrm{halo}} = \frac{1}{m_n S_{2n}} f\left(\frac{E_{nn}}{S_{2n}}, \frac{S_{1n}}{S_{2n}}; A\right)$$

Acharya, C.J., Phillips, PLB '13

experimental input:
$\langle r_m^2 \rangle_{2n-\mathrm{halo}} - \frac{10}{11}\langle r_m^2 \rangle_{\mathrm{core}} = 21^{+10}_{-10}$ fm^2
Ozawa *et al.* PRL '10



Acharya, CJ, Phillips, PLB '13

bands: uncertainty from NLO EFT
$\sim \max\left\{\frac{\sqrt{mE_{nn}}}{\Lambda}, \frac{\sqrt{mS_{1n}}}{\Lambda}, \frac{\sqrt{mS_{2n}}}{\Lambda}\right\}$

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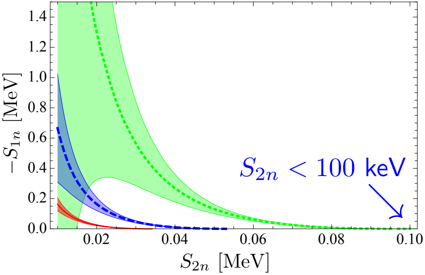

Other experimental bound:

- AME2012
  $S_{2n} < 170$ keV
- Gaudefroy et al., PRL '12
  $S_{1n} < -2.9$ MeV

Acharya, CJ, Phillips, PLB '13

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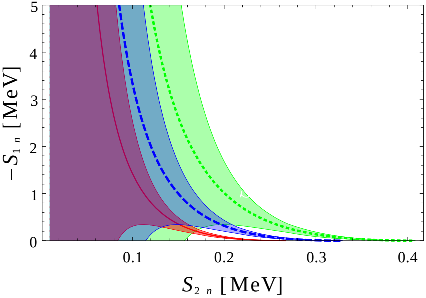
Acharya, C.J., Phillips, PLB '13

new experimental input:
$\langle r_m^2 \rangle_{2n-\text{halo}} - \frac{10}{11} \langle r_m^2 \rangle_{\text{core}} = 3.81^{+0.82}_{-0.71}$ fm^2
Togano *et al.* PLB '16



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Hammer, CJ, Phillips, arXiv:1702.08605

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Acharya, C.J., Phillips, PLB '13

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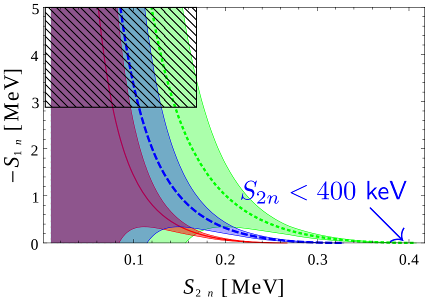

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- Gaudefroy et al., PRL '12

  $S_{1n} < -2.9$ MeV

Hammer, CJ, Phillips, arXiv:1702.08605

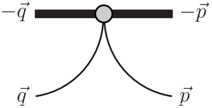
^{6}He: $2n$ Halo with p-wave nc interactions

- *ab intio* **calculation**
  - no-core shell model Navrátil *et al.* '01; Sääf, Forssén '14
  - NCSM-RGM/Continuum Romero *et al.* '14 '16
  - Green's function Monte Carlo Pieper *et al.* '01; '08
  - hyperspherical harmonics (EIHH) Bacca *et al.* '12
- **Halo EFT in ^{6}He ground state**
  - EFT+Gamow shell model Rotureau, van Kolck Few Body Syst. '13
  - EFT+Faddeev equation C.J., Elster, Phillips, PRC '14
  - diff power-counting Ryberg, Forssén, Platter, arXiv:1701.08576

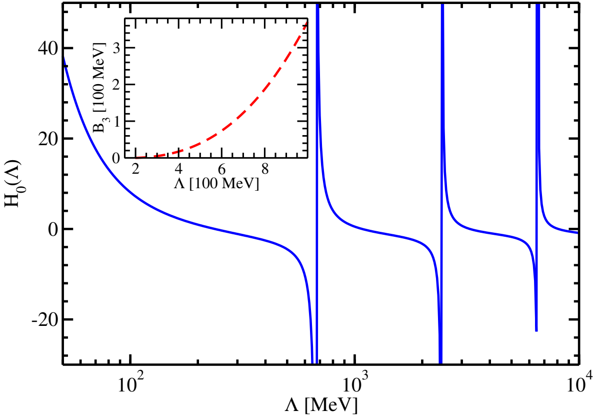
Running of 3BF Coupling

- p-wave 3BF:

  reproduce $S_{2n} = 0.973$ MeV



$= M_n\, qp\, \frac{H(\Lambda)}{\Lambda^2}$



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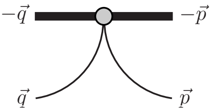
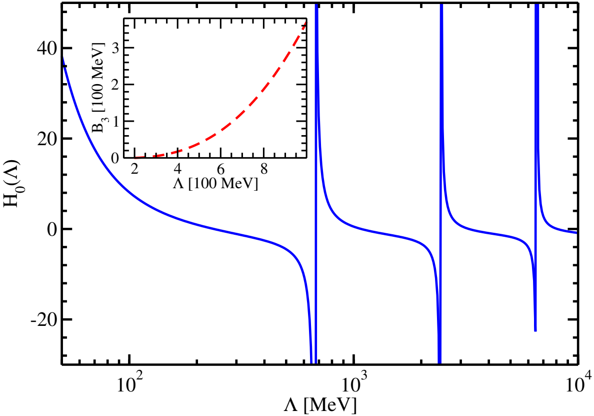


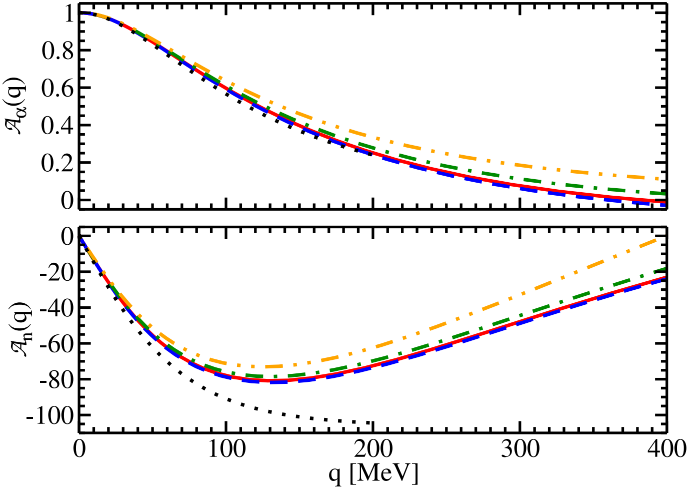
$$= M_n \, qp \, \frac{H(\Lambda)}{\Lambda^2}$$

- discrete scaling symmetry is broken due to p-wave interactions



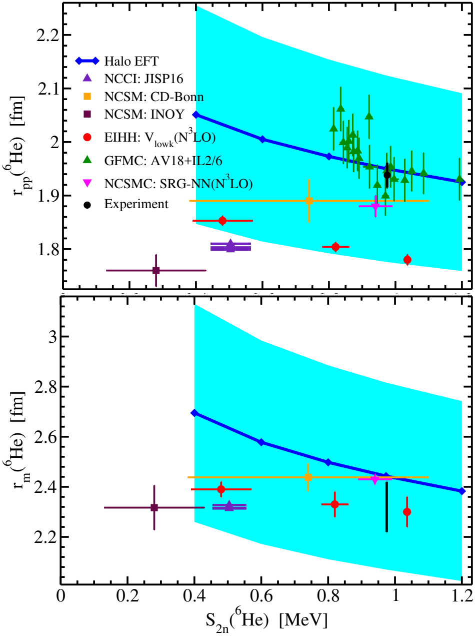
Renormalized Faddeev Components

$\mathcal{A}_\alpha$ and $\mathcal{A}_n$ are cutoff independent



C.J., Elster, Phillips, PRC '14

Universal correlations btw ^{6}He radii & S_{2n}



[Preliminary]

- **He-6 point-proton radius**
- **He-6 point-nucleon radius**

compare with

NCCI: Caprio, Maris, Vary, PRC '14

NCSM: Caurier, Navratil, PRC '06

EIHH: Bacca, Barnea, Schwenk, PRC '12

GFMC: Pieper, RNC '08

NCSMC: Romero et al., PRL '16

Halo EFT: preliminary (uncertainty)

Summary

- Effective field theory comes with limited powers determined by Q and Λ, different EFTs may be efficient at different energy regimes

- Halo EFT describes near-threshold physics in halo nuclei in a controlled expansion in Q/Λ

- Halo EFT rejuvenate cluster models with a systematic uncertainty estimates

- Halo EFT can be connected with *ab initio* calculations
    - adopt inputs from *ab initio* results
    - benchmark with *ab initio* calculations
    - explain correlations among observables in *ab initio* work