(The good, bad and ugly of) Optical potentials and nucleon scattering from ab-initio Green function

Andrea Idini

C. Barbieri

Toward Predictive Theories of Nuclear Reactions Across the Isotopic Chart
INT - Seattle, 21 Mar

Optical Potentials

Objective: an effective, consistent description of structure and reactions with a single formalism.
(Hopefully) Predictive power of nuclear reactions measurements over a range of exotic isotopes.

Method: Optical potential derived from Self Consistent Green Function and χEFT interactions.

1. reproduce nuclear bulk properties, i.e. binding energy and radii; ***NNLO**$_{sat}$*
2. use the same description to consistently generate an optical potential reproducing elastic scattering data.

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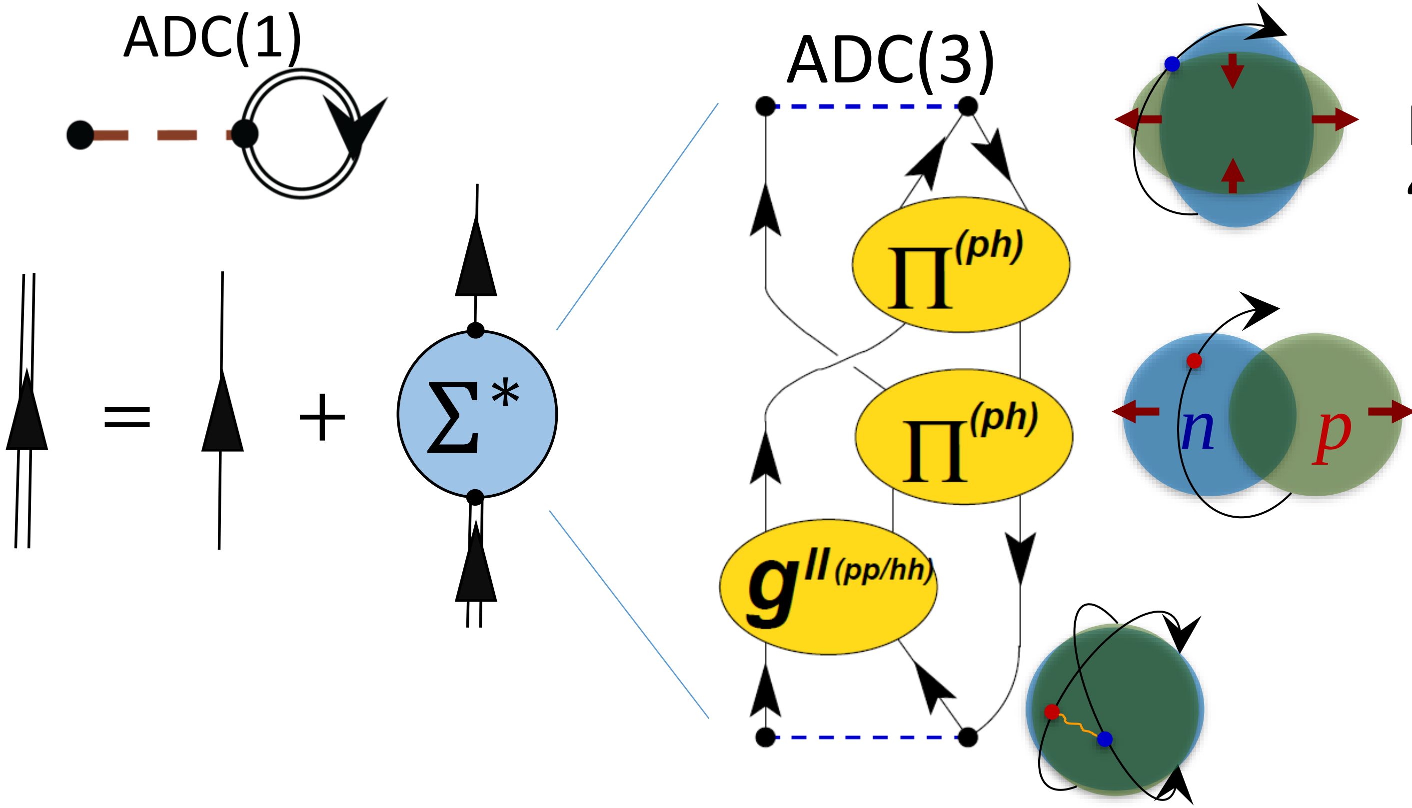
Green Functions *(Dyson Equation)*

$$g_{\alpha\beta}(\omega) = g^0_{\alpha\beta}(\omega) + \sum_{\gamma\delta} g^0_{\alpha\gamma}(\omega)\Sigma^{\star}_{\gamma\delta}(\omega) g_{\delta\beta}(\omega)$$

HF
ADC(1)

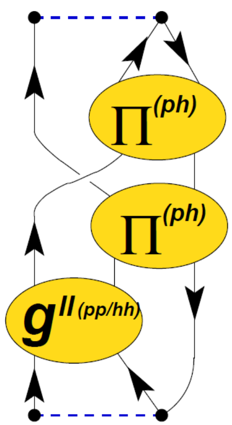
Faddeev RPA
ADC(3)



Particle hole
'polarization'
propagator
(ph-RPA)

Particle-particle (pp-RPA)
two-body correlation 'ladder' propagator

$$\Sigma^{\star}_{\alpha\beta}(\omega) = \Sigma^{(\infty)}_{\alpha\beta} + \sum_{i,j} \mathbf{M}^{\dagger}_{\alpha,i} \left[\frac{1}{\omega - (\mathbf{K}^{>} + \mathbf{C}) + i\eta} \right]_{i,j} \mathbf{M}_{j,\beta} + \sum_{r,s} \mathbf{N}_{\alpha,r} \left[\frac{1}{\omega - (\mathbf{K}^{<} + \mathbf{D}) - i\eta} \right]_{r,s} \mathbf{N}^{\dagger}_{s,\beta}$$

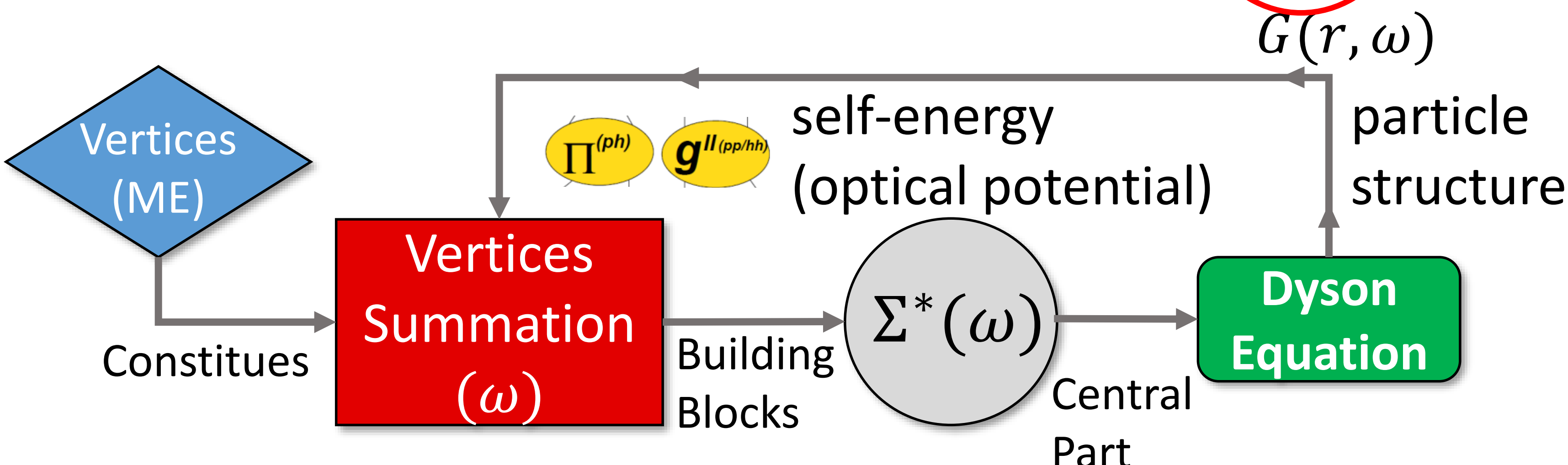


$$\begin{pmatrix} \widehat{T} + \Sigma^{(\infty)} & M^{\dagger} & N \\ M & E^{>} + C & \\ N^{\dagger} & & E^{<} + D \end{pmatrix} \begin{pmatrix} \mathcal{Z}^i \\ \mathcal{W}^i \\ \mathcal{V}^i \end{pmatrix} = \begin{pmatrix} \mathcal{Z}^i \\ \mathcal{W}^i \\ \mathcal{V}^i \end{pmatrix} \varepsilon_i$$

2p1h ($M^{\dagger}$)

2p2h (C)

Unknown



More details in

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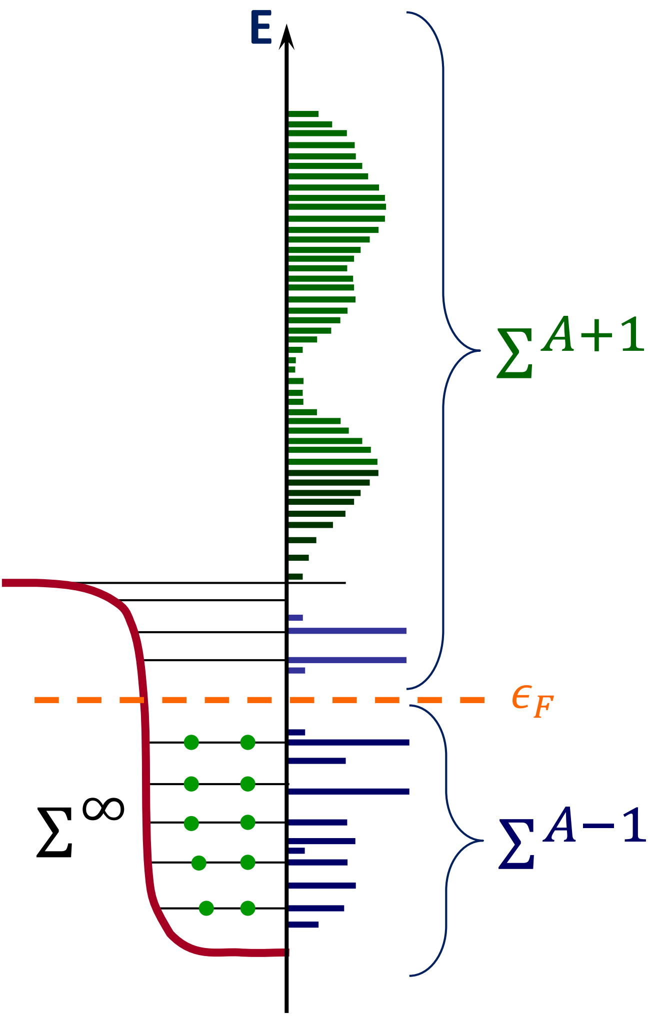
C. Barbieri, A. Carbone, Lect. Notes Phys. arXiv:1611.03923 [nucl-th]

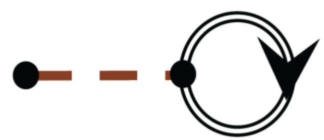
Nucleon elastic scattering



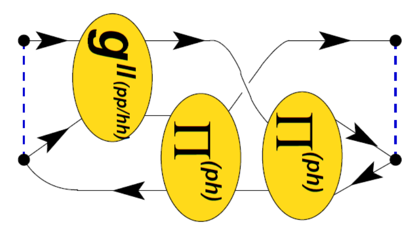
The irreducible self-energy is a nucleon-nucleus optical potential*

$$\Sigma^{*}_{\alpha\beta}(r, r'; \omega) = \Sigma^{\infty}_{\alpha\beta} + \sum_i \frac{m^i_\alpha \, m^{i*}_\beta}{\omega - \epsilon_i \pm i\eta}$$

correlated mean-field



resonances beyond mean-field



➔ This provides *consistent* many-body and scattering wave functions

*Mahaux & Sartor, Adv. Nucl. Phys. 20 (1991), Escher & Jennings PRC66:034313 (2002)

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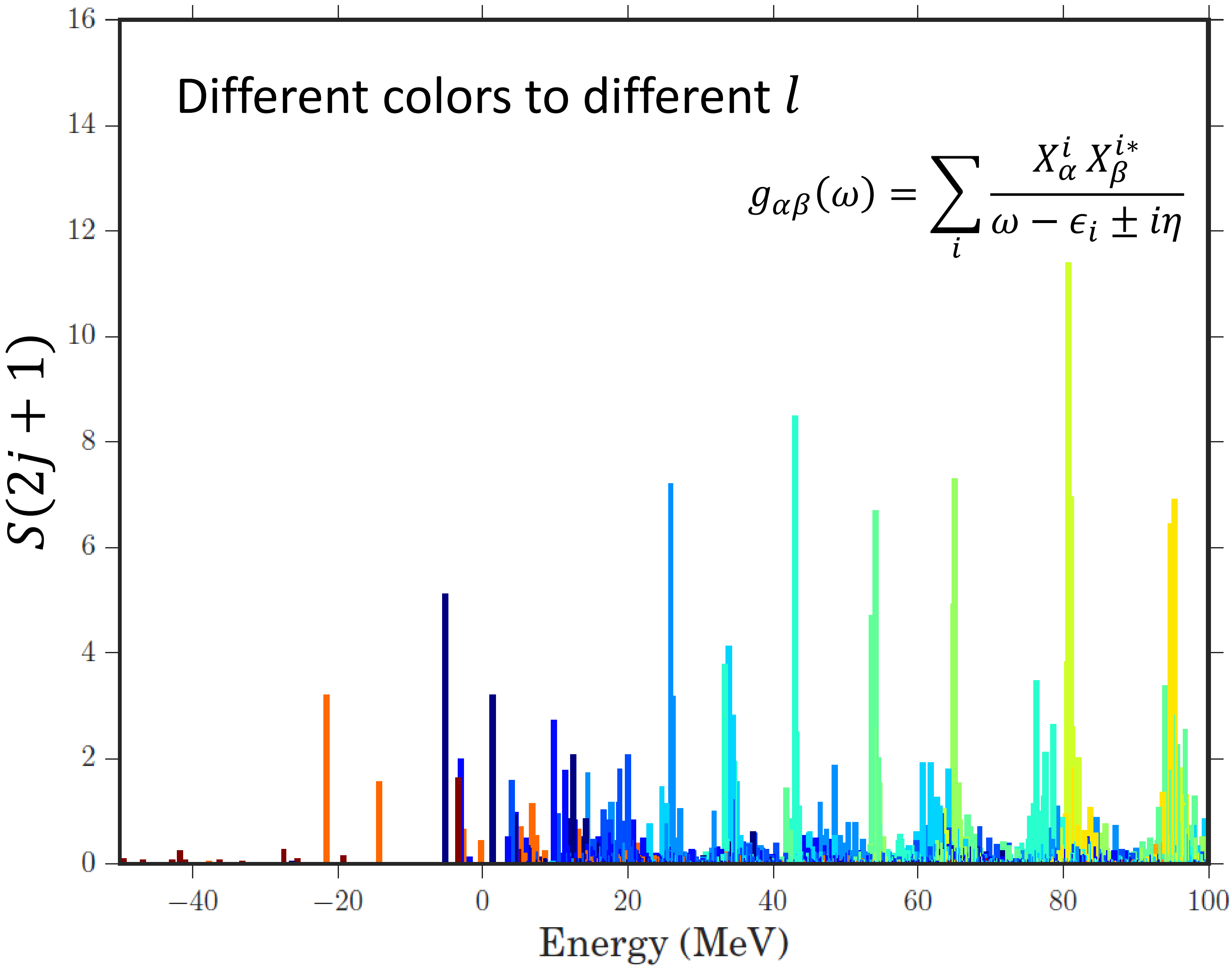
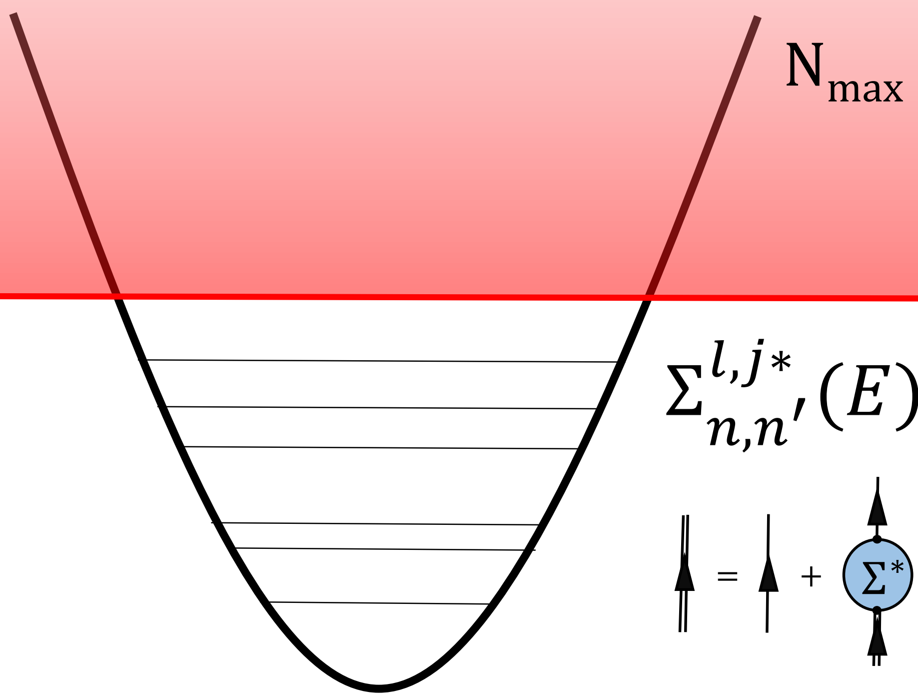
^{16}O neutron propagator



- Solve Dyson equation in HO Space, find $\Sigma^{l,j*}_{n,n'}(E)$

- diagonalize in full continuum momentum space $\Sigma^{l,j*}(k,k',E)$

$$\frac{k^2}{2m}\psi_{l,j}(k) + \int dk'k'^2 \left(\Sigma^{l,j*}(k,k',E)\right)\psi_{l,j}(k') = \mathrm{E}\,\psi_{l,j}(k)$$



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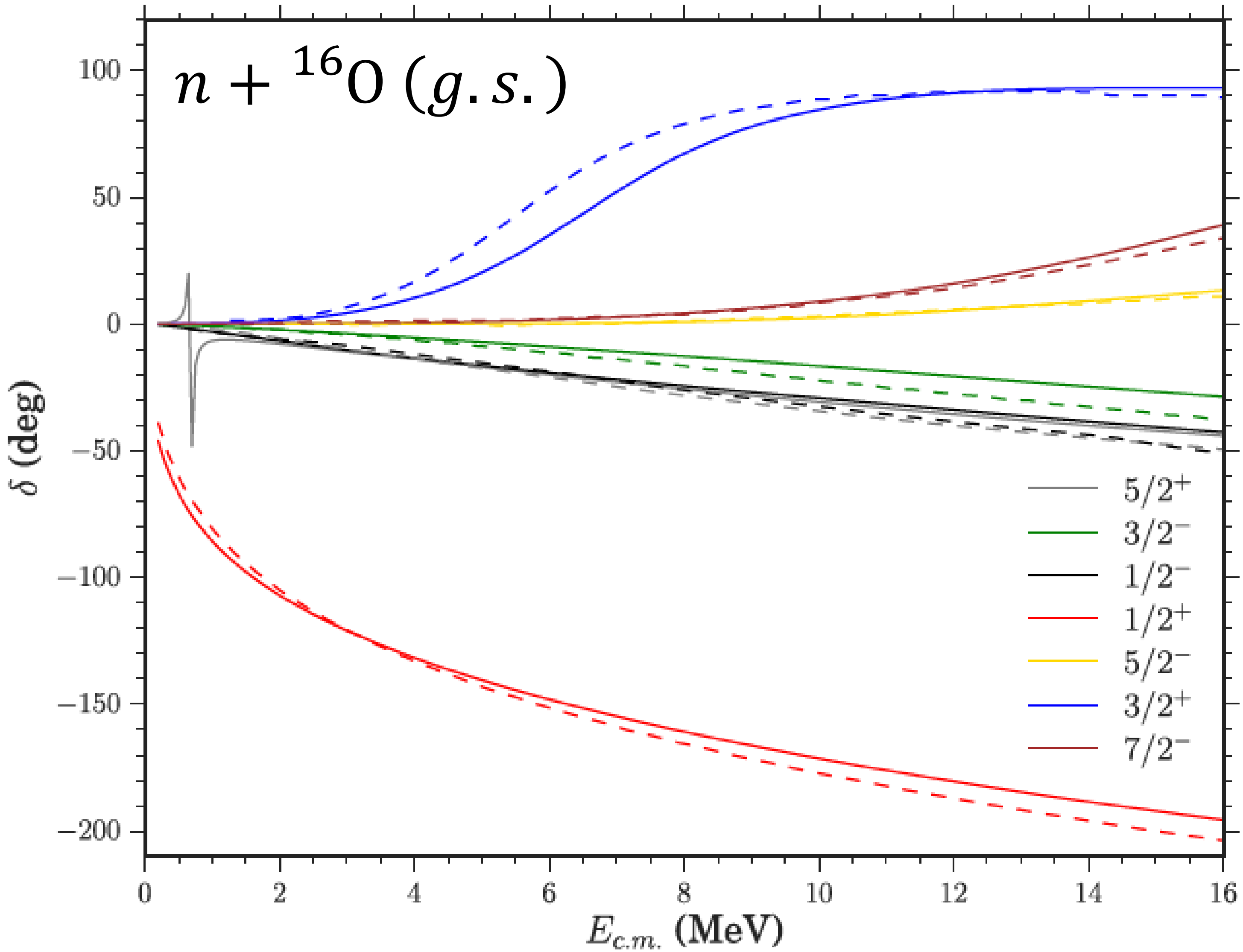
RESULTS

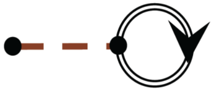
arXiv:1612.01478 [nucl-th]

SRG-N^{3}LO, $\Lambda = 2.66\ \mathrm{fm}^{-1}$



Navràtil, Roth, Quaglioni, PRC82, 034609 (2010)

Σ^{∞}



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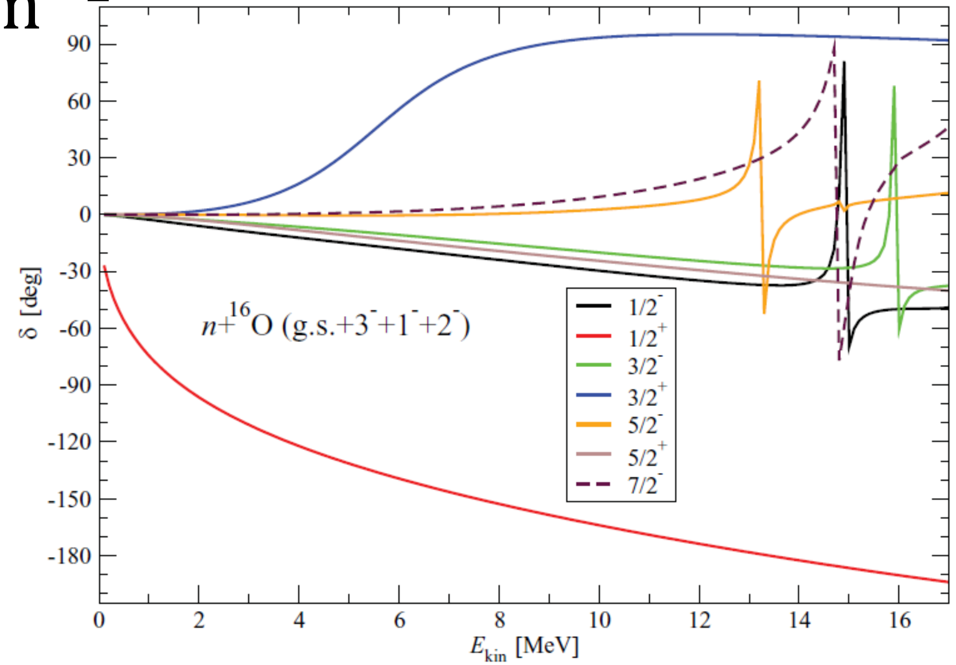
SRG-N^{3}LO, $\Lambda = 2.66\ \mathrm{fm}^{-1}$

Navràtil, Roth, Quaglioni,
PRC82, 034609 (2010)

$n + {}^{16}\mathrm{O}$



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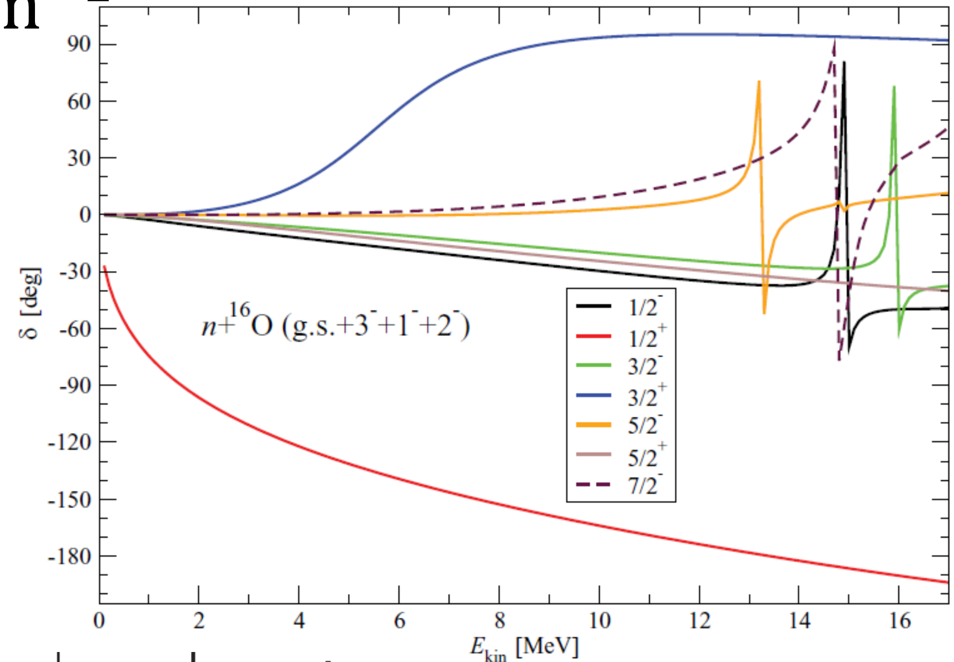

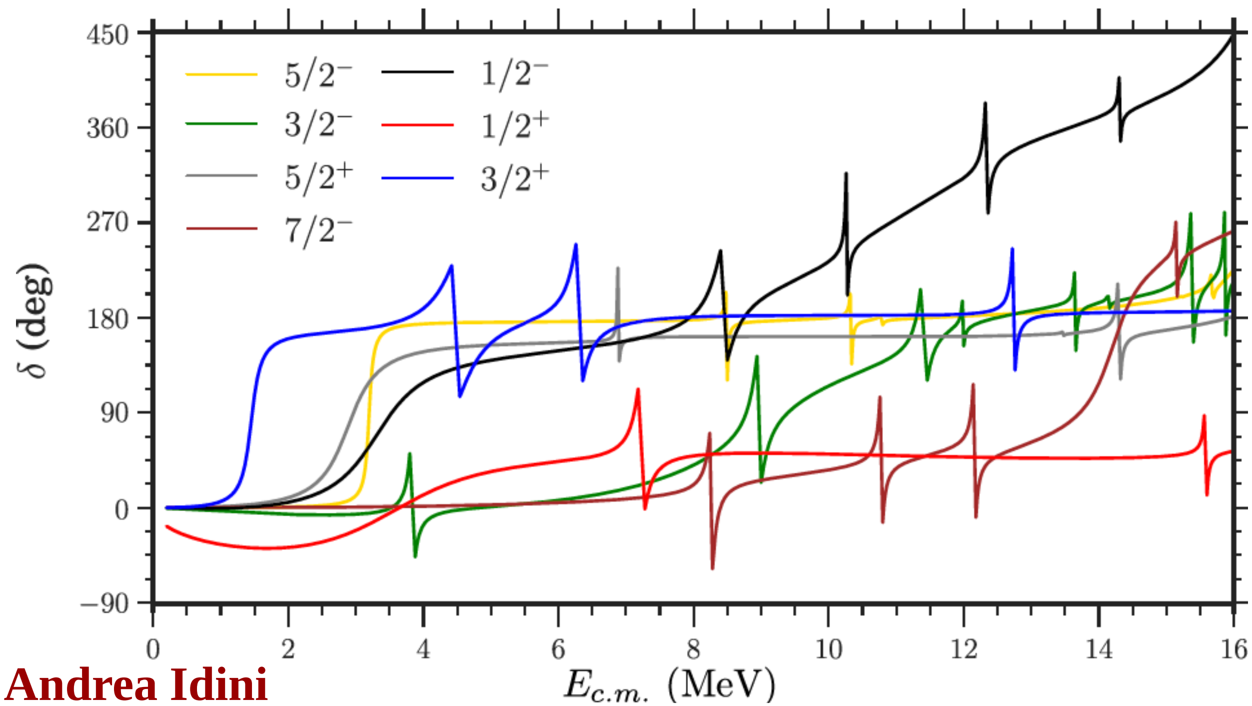

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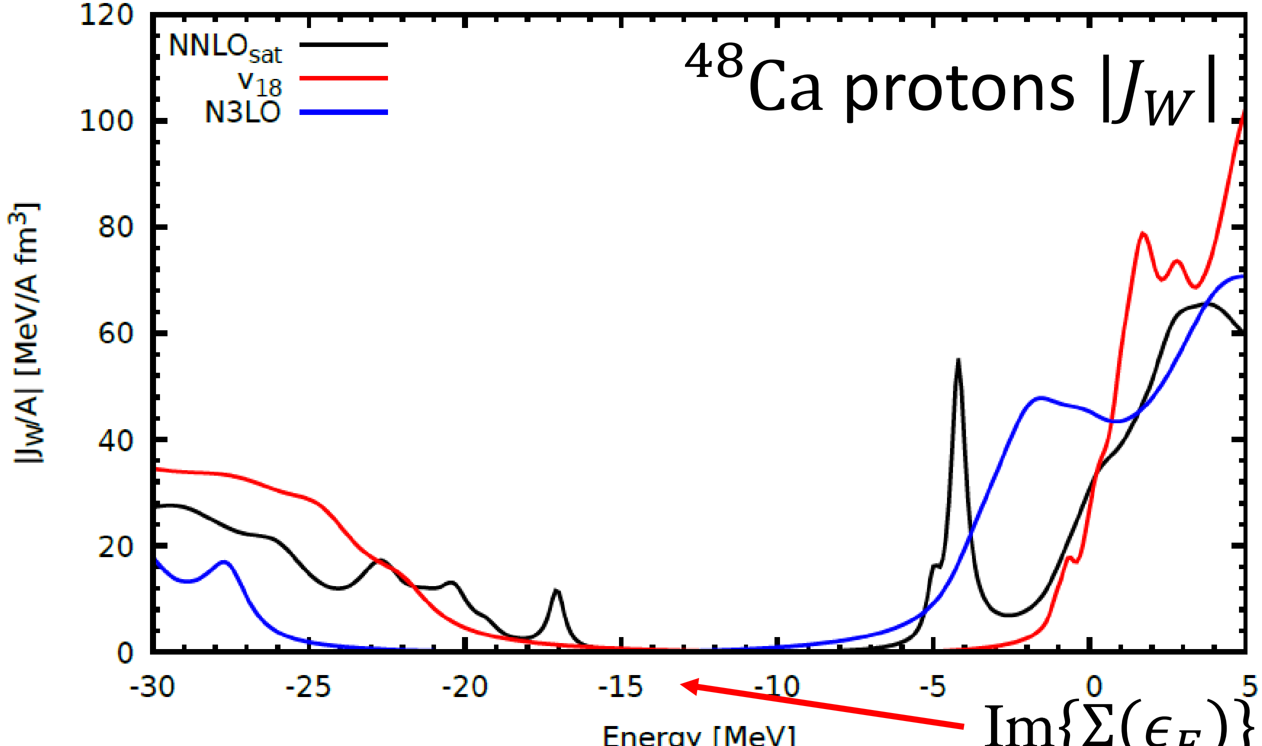
Volume integrals

$$J_W^\ell(E) = 4\pi \int drr^2 \int dr' r'^2 \mathrm{Im}\ \Sigma_0^\ell(r, r', E)$$

Non local potential

$$J_V^\ell(E) = 4\pi \int drr^2 \int dr' r'^2 \mathrm{Re}\ \Sigma_0^\ell(r, r'; E).$$

$$\tilde{\Sigma}^{\ell j}_{n_a, n_b}(E) = \sum_r \frac{m^r_{n_a} m^r_{n_b}}{E - \varepsilon_r \pm i\eta}$$



different Fermi energies and particle-hole gap for different interactions

$\mathrm{Im}\{\Sigma(\epsilon_F)\} = 0$

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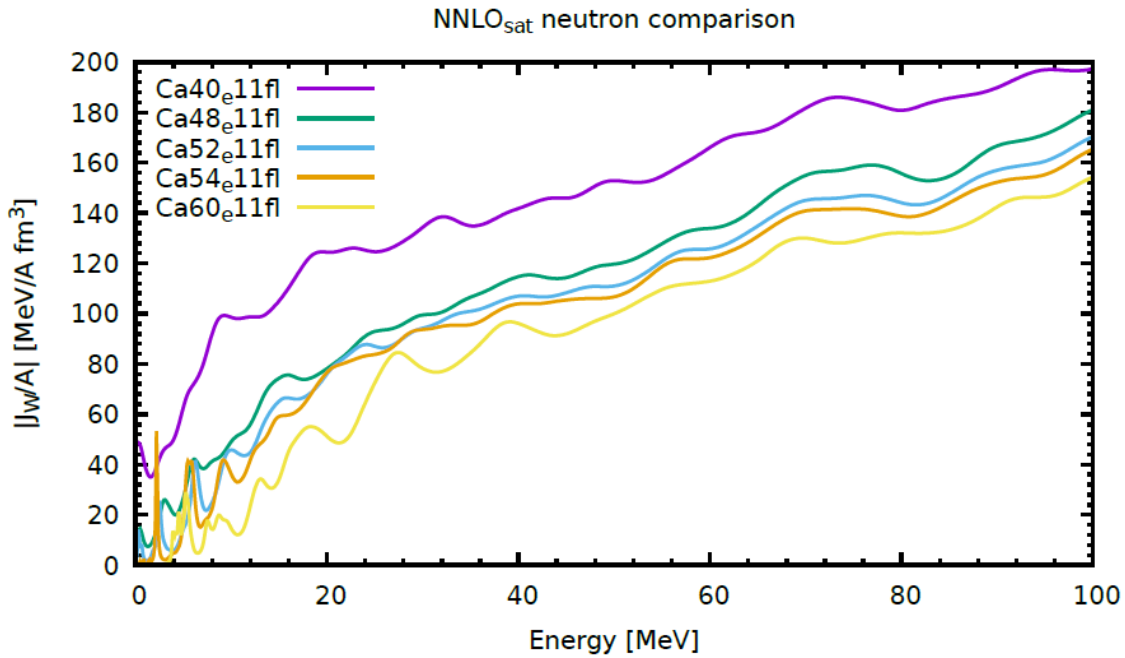
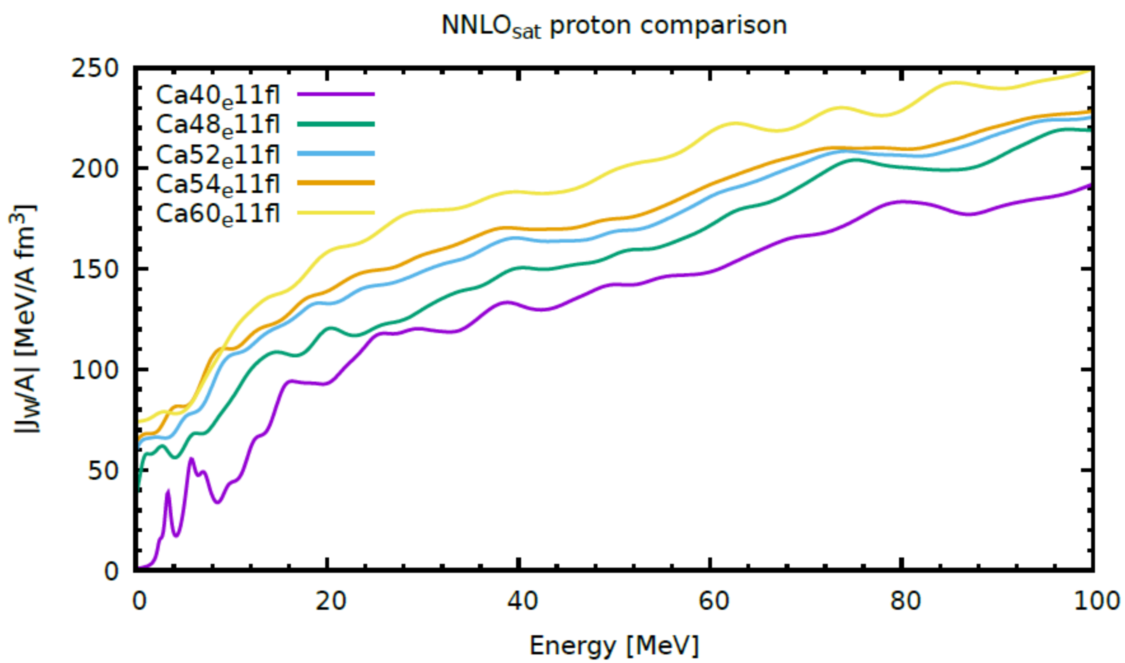
S. Waldecker et al. PRC**84**, 034616(2011)

Ca isotopes

neutron and proton volume integrals of self energies.





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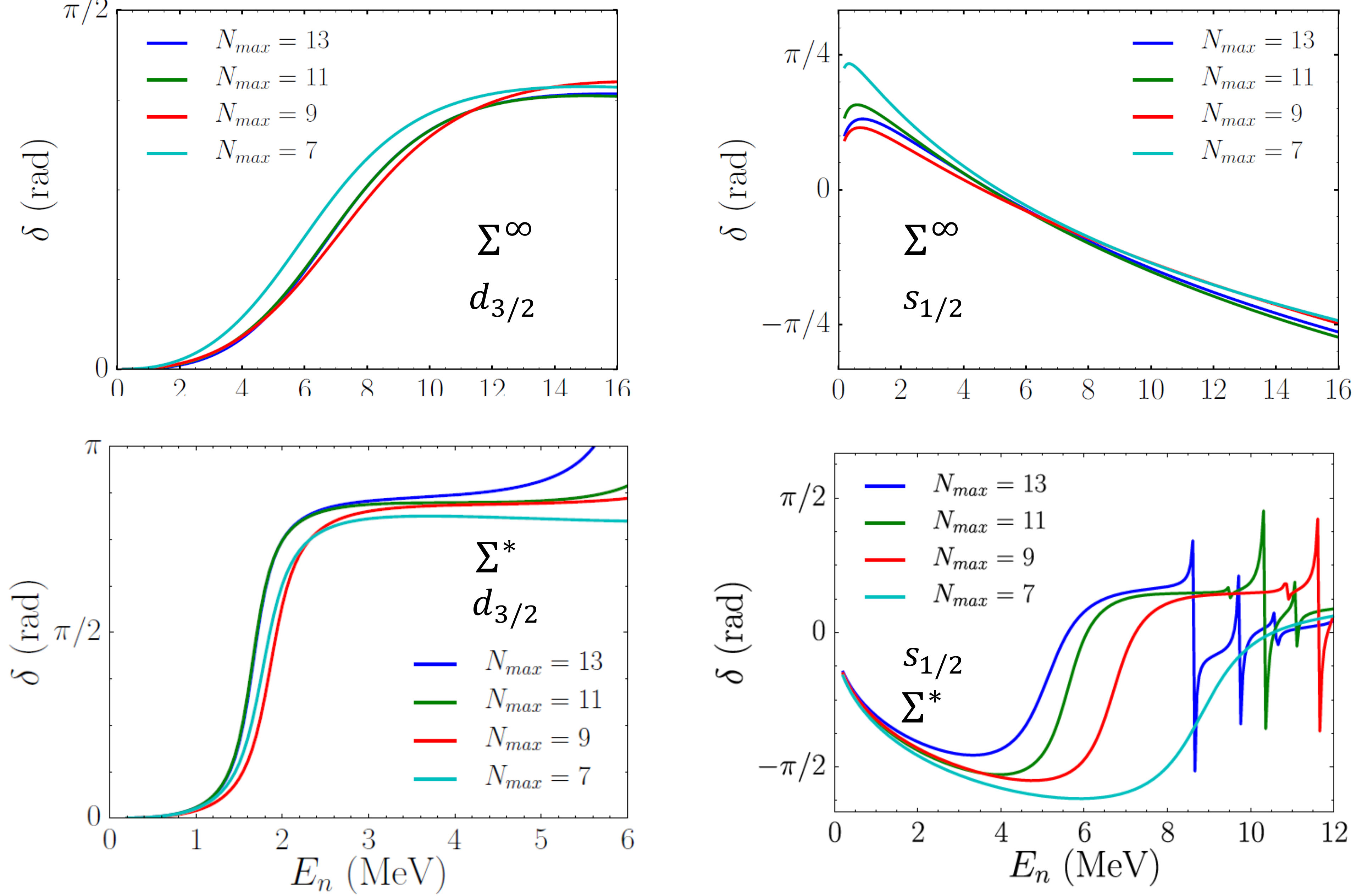
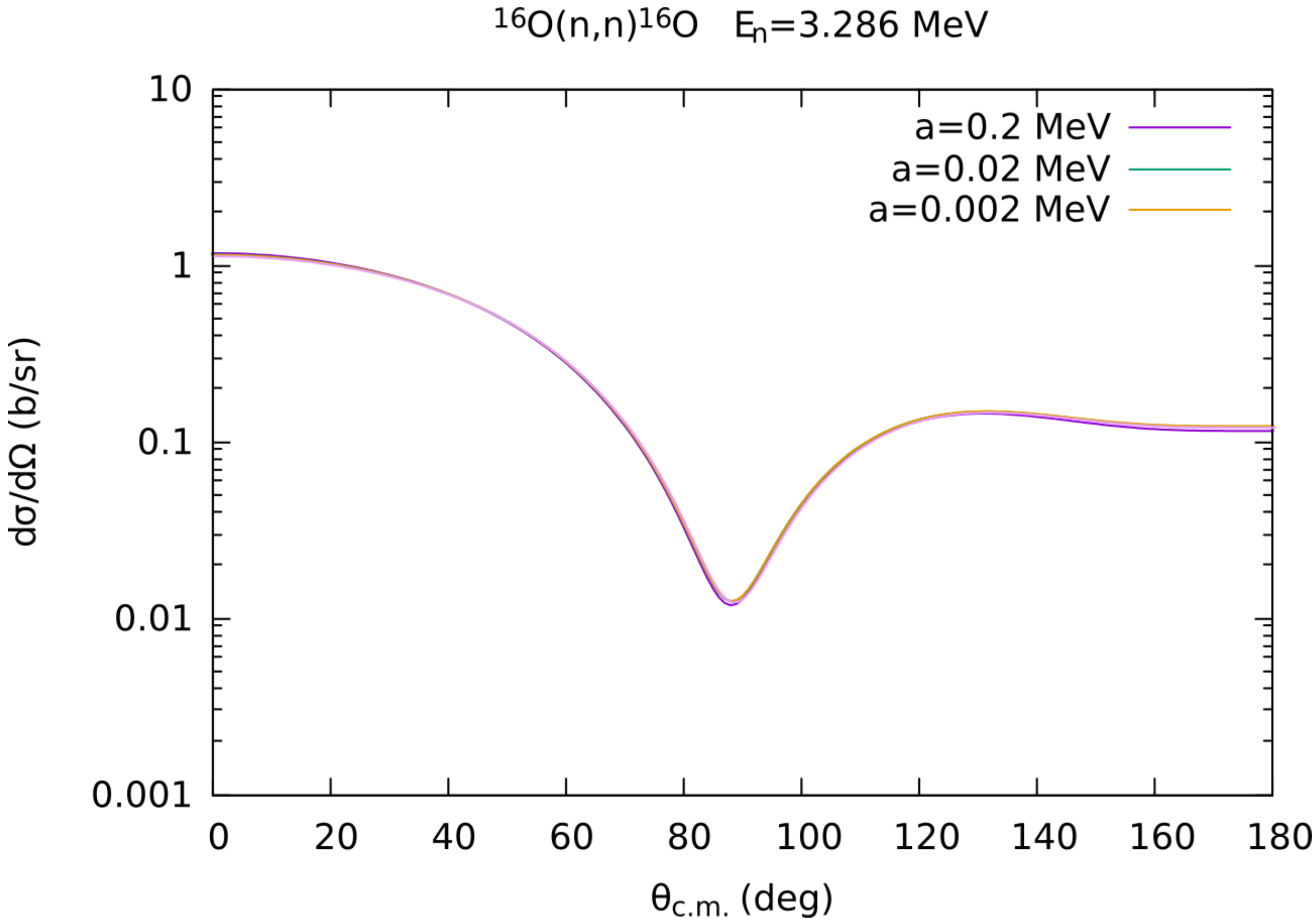
N_{max} convergence



«Imaginary» Parameter

$$\Gamma(E) = \frac{1}{\pi} \frac{a\,(E - E_F)^2}{(E - E_F)^2 - b^2}$$

$b = 22.36$ MeV



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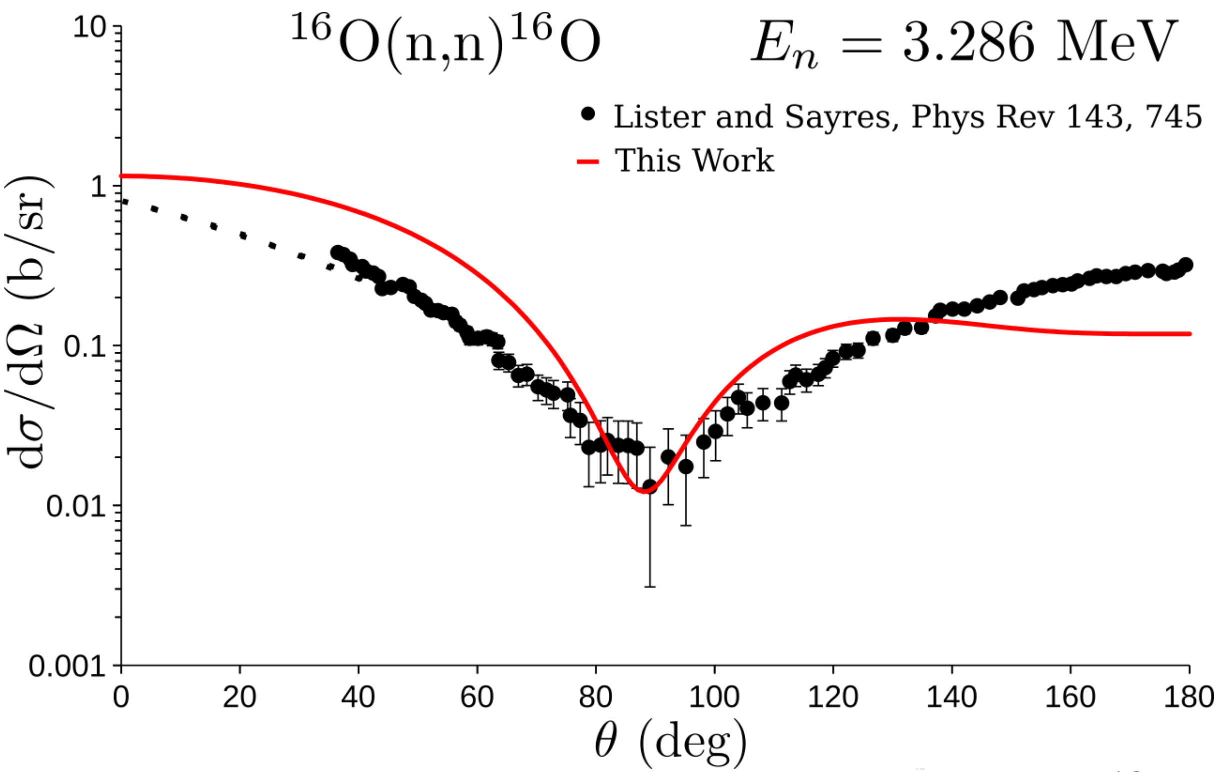

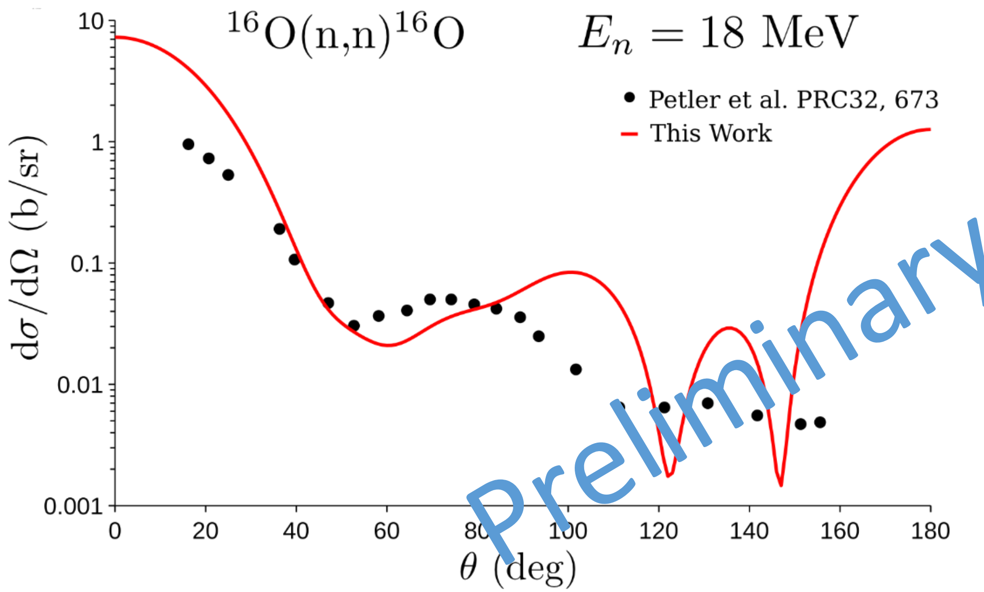

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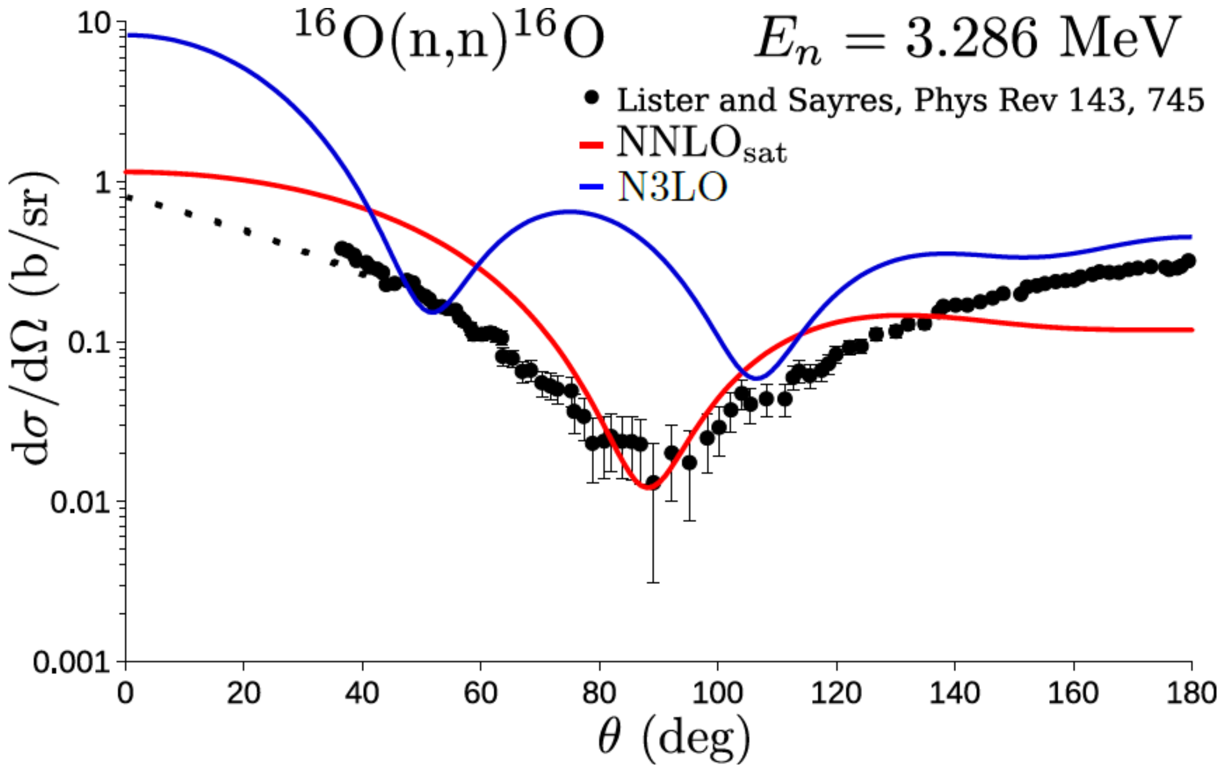

$^{16}O \, \langle r_p \rangle$

experiment	2.699±0.005 fm
NNLO$_{sat}$	2.734 fm
N3LO NN	2.354 fm

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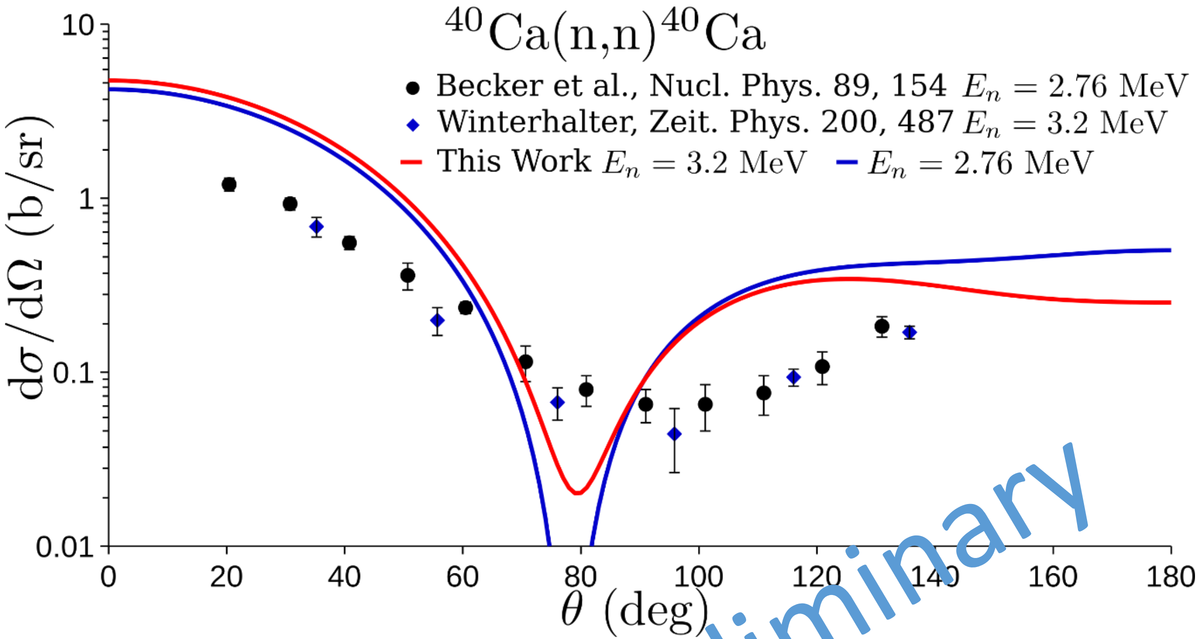
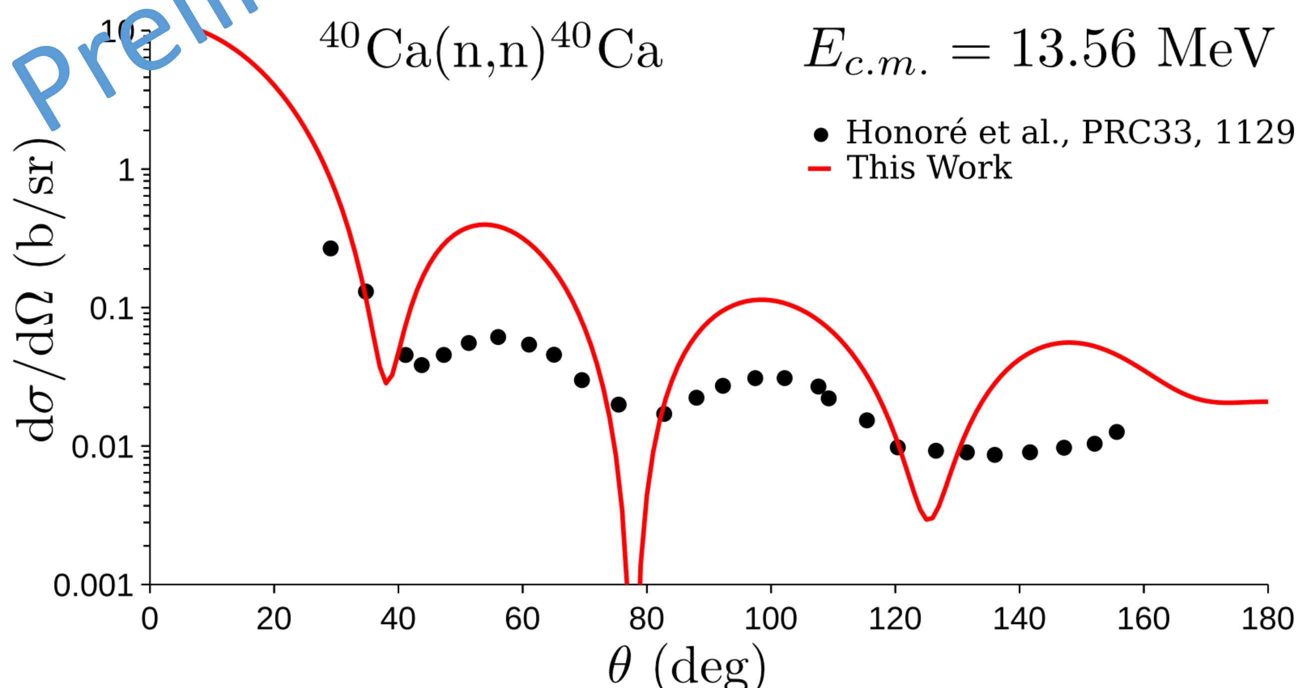
N_{max}=11



Preliminary



Dipole Response and Polarizability

With
Francesco Raimondi

$$\sigma_\gamma(E) = 4\pi^2 \alpha E R(E)$$

$$\alpha_D = 2\alpha \int dE\, R_Z(E)/E$$

$$R(E) = \sum_i \left|\langle \psi_i^A | \hat{D} | \psi_0^A \rangle\right|^2 \delta(E_i - E)$$

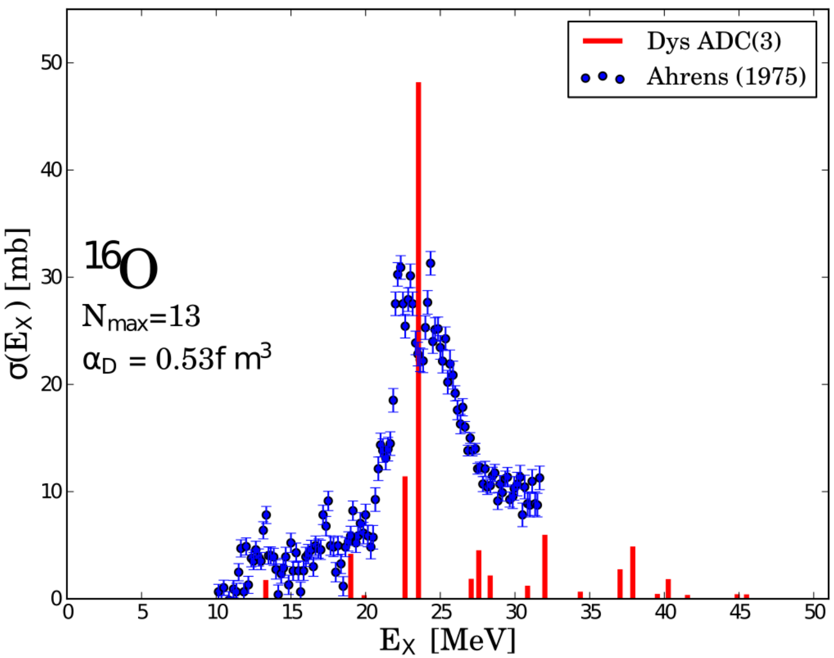
$$\sum_{\alpha\beta} \langle \alpha | \hat{D} | \beta \rangle \langle \psi_i^A | c_\alpha^+ c_\beta | \psi_0^A \rangle$$

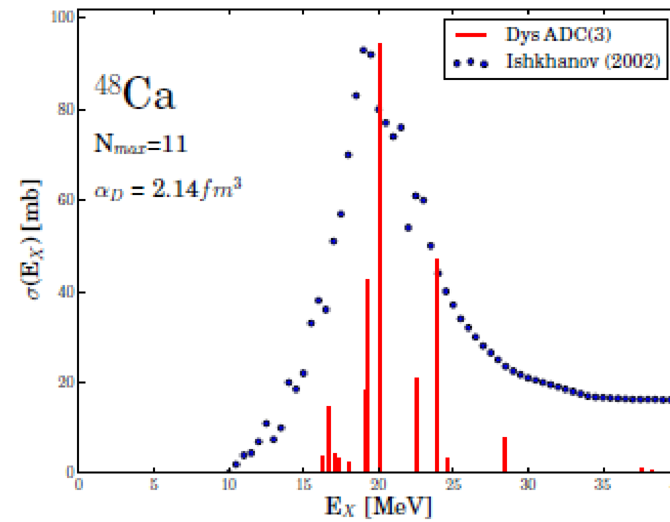
HO ME

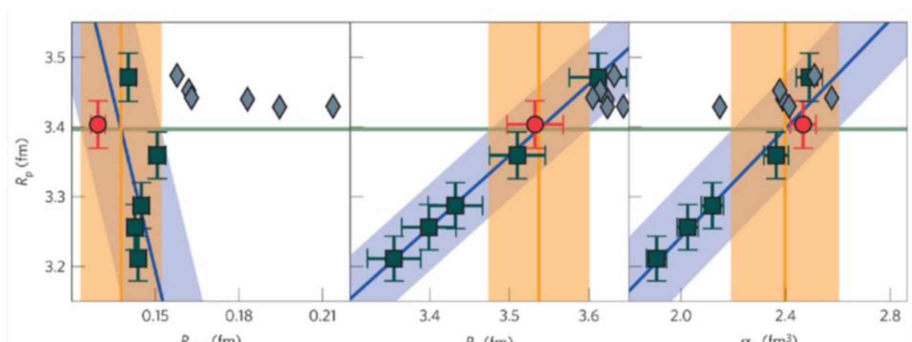
RPA

Dipole polarizability α_D (fm^3)

Nucleus	SCGF	CC/LIT	Exp
^{40}Ca	1.89	1.47 $(1.87)_{thresh}$	1.87(3)
^{48}Ca	2.14	2.45	2.07(22)







Hagen *et al.*, *Nature Physics* **12**, 186

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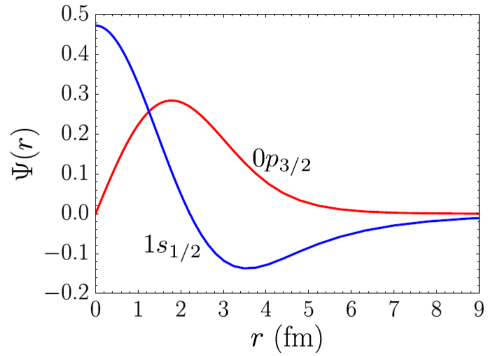
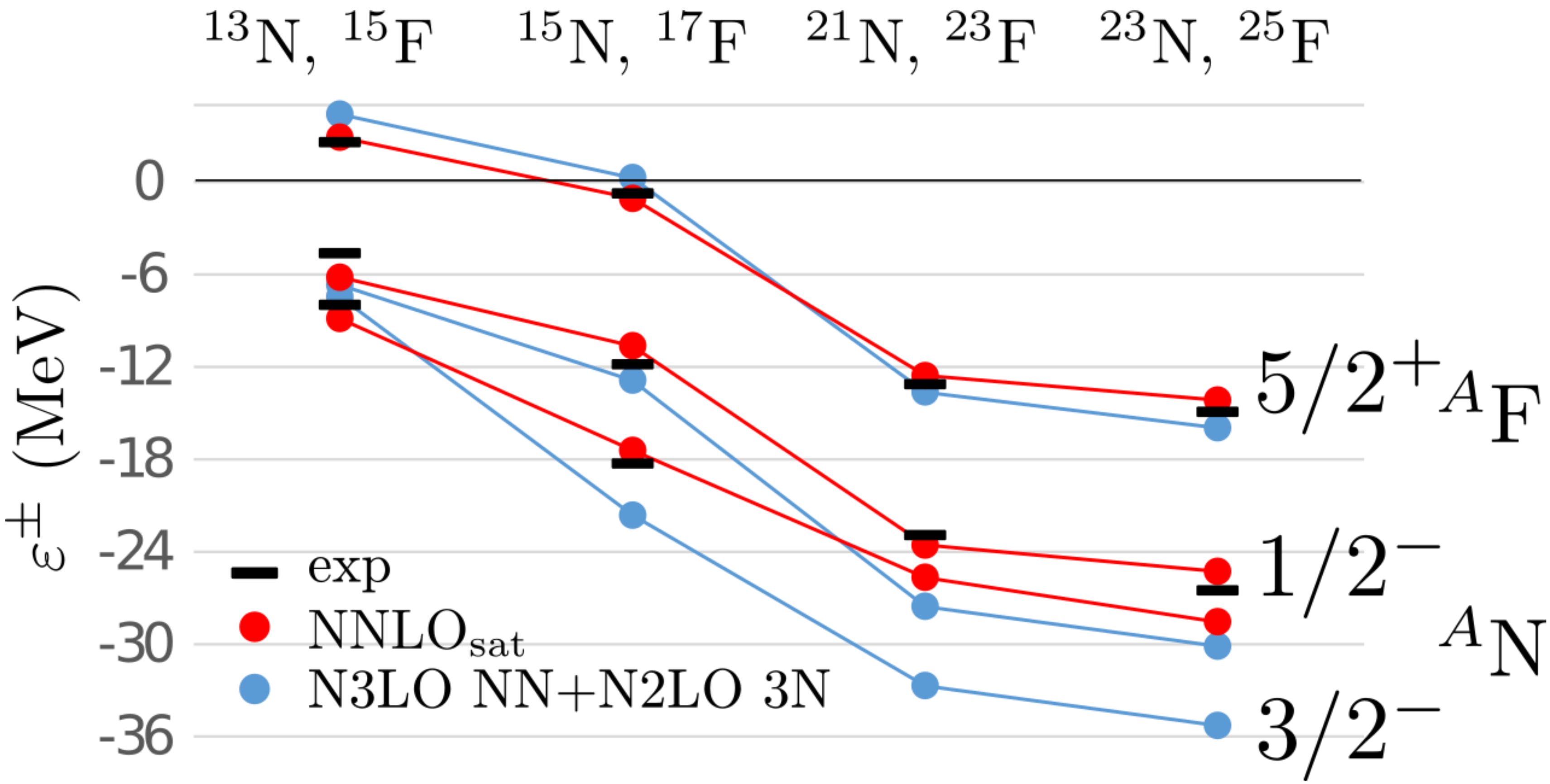
Overlap function

$$\Psi_i(r) = \sqrt{A}\int dr_1 \ldots dr_A \Phi^+_{(A-1)}(r_1, \ldots, r_{A-1})\Phi^+_{(A)}(r_1, \ldots, r_A)$$





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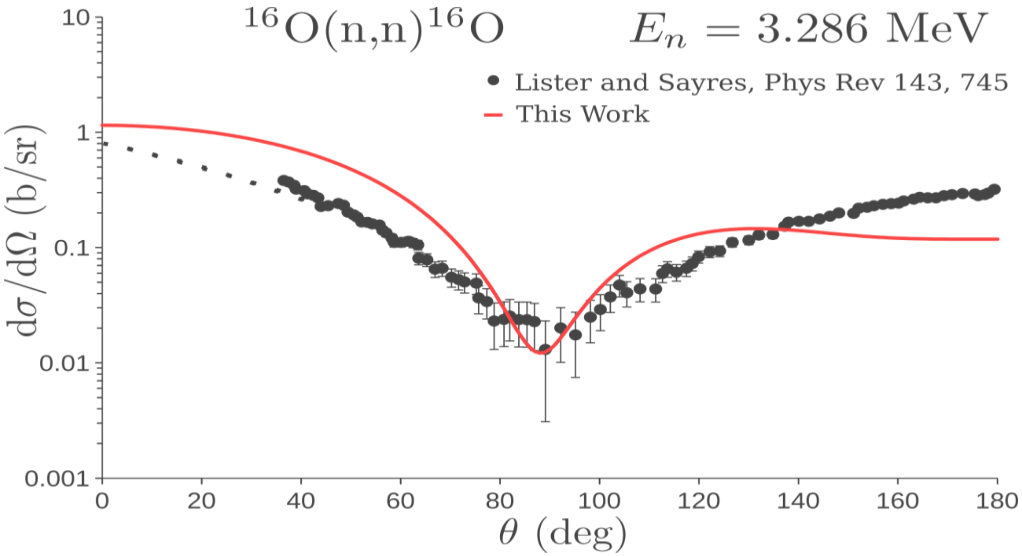
EM results from A. Cipollone PRC**92**, 014306 (2015)

Conclusions and Perspectives

- We are developing an interesting tool to study nuclear reactions effectively.
  We have defined a non-local generalized optical potential corresponding to nuclear self energy.
- This tool is useful to probe properties of nuclear interactions.
- *Radii, saturation and bulk properties are fundamental!*
  (but not enough? Where do we want to go?)



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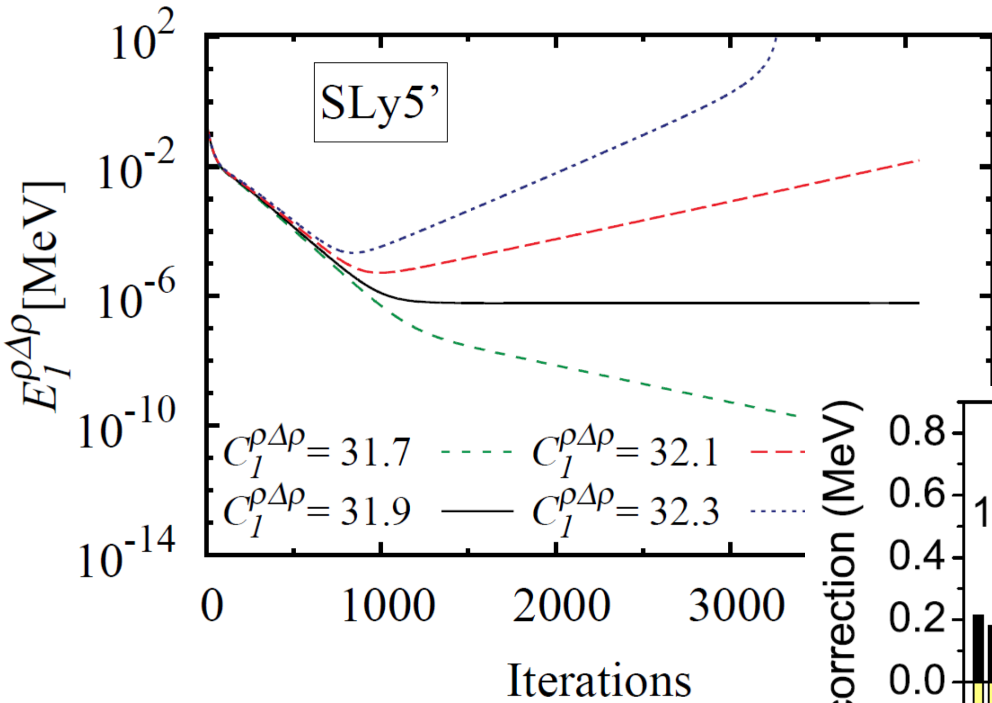
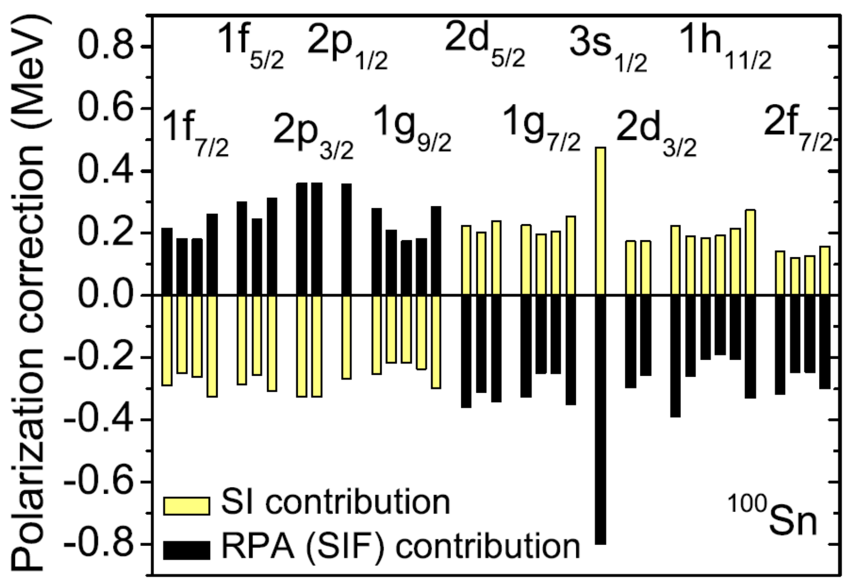
Density dependence introduces instabilities in the mean field

Hellemans et al. PRC 88, 064323 (2013)



$$\rho^{\alpha}\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right)$$

$$\alpha = 1/6$$



Tarpanov et al. PRC89, 014307 (2014)

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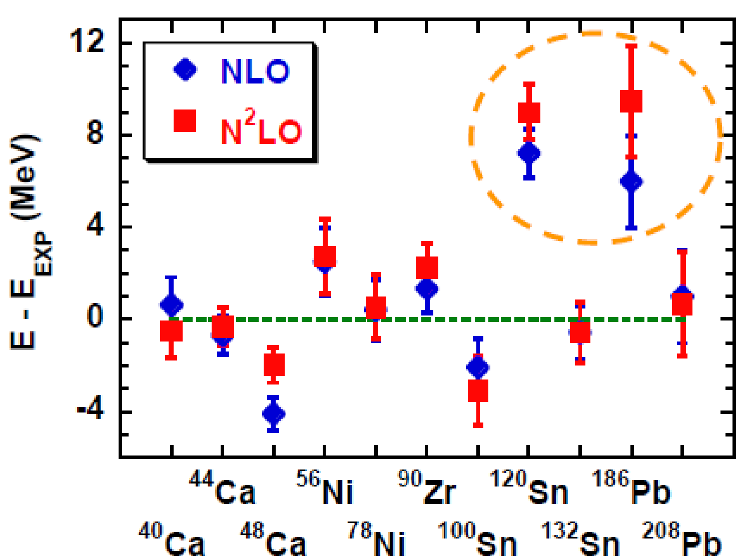
Finite Range pseudopotential

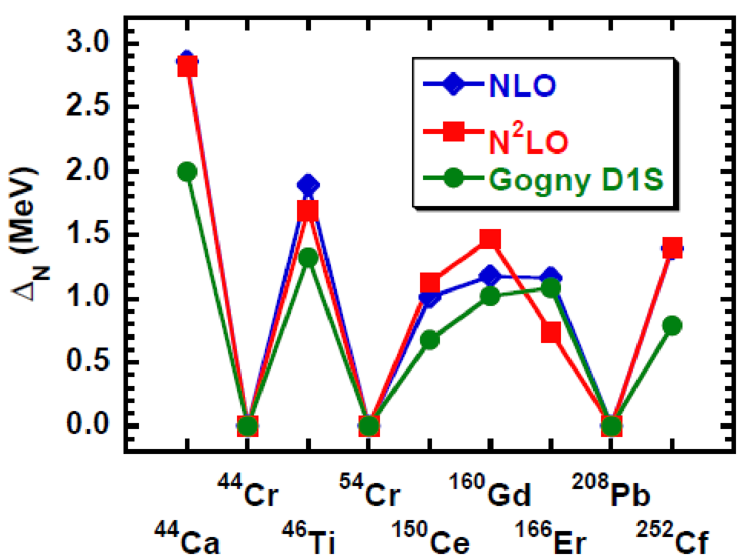
$$\mathcal{V}_k(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4) = \left(W_k\hat{1}_\sigma\hat{1}_\tau + B_k\hat{1}_\tau\hat{P}^\sigma - H_k\hat{1}_\sigma\hat{P}^\tau - M_k\hat{P}^\sigma\hat{P}^\tau\right) \times \hat{O}_k(\mathbf{k}_{12},\mathbf{k}_{34})\delta(\mathbf{r}_{13})\delta(\mathbf{r}_{24})g_a(\mathbf{r}_{12})\,,$$

K. Bennaceur
J. Dobaczewski
M. Korteleinen

with $k = 0,\ 1,$ or 2.

$$\mathcal{V}_\delta(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4) = t_0\left(1 + x_0\hat{P}^\sigma\right)\delta(\mathbf{r}_{13})\delta(\mathbf{r}_{24})\delta(\mathbf{r}_{12})$$





J.Phys. G44 (2017) no.4, 045106

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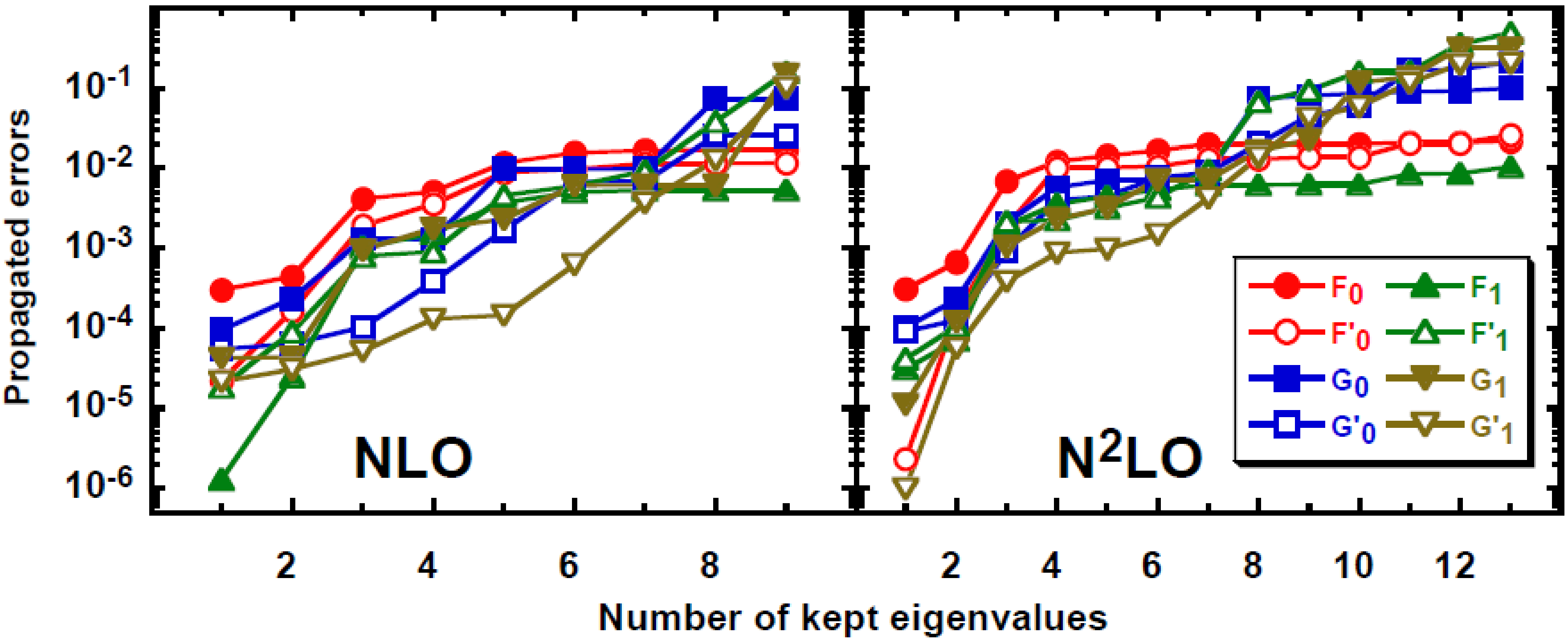
Landau parameters and error analysis

Error Analysis and Landau parameters guide us to know which channel of the interaction are less constrained

$F \Rightarrow S = 0, T = 0 \qquad F' \Rightarrow S = 0, T = 1$

$G \Rightarrow S = 1, T = 0 \qquad G' \Rightarrow S = 1, T = 1$



A.Idini, K. Bennaceur, J. Dobaczewski, arXiv:1612.00378

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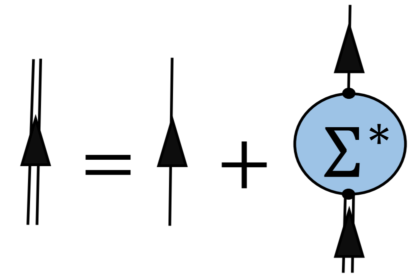
Why Green's Functions?

Dyson Equation

$$g_{\alpha\beta}(\omega) = g^0_{\alpha\beta}(\omega) + \sum_{\gamma\delta} g^0_{\alpha\gamma}(\omega)\Sigma^{\star}_{\gamma\delta}(\omega)g_{\delta\beta}(\omega)$$



Equation of motion

$$\left(E + \frac{\hbar^2}{2m}\nabla^2_r\right)G(\mathbf{r},\mathbf{r}';E) - \int d\mathbf{r}''\Sigma(\mathbf{r},\mathbf{r}'';E)G(\mathbf{r}'',\mathbf{r}';E) = \delta(\mathbf{r}-\mathbf{r}')$$

Corresponding Hamiltonian

$$\mathcal{H}_{\mathcal{M}}(\mathbf{r},\mathbf{r}') = -\frac{\hbar^2}{2m}\nabla^2_r\delta(\mathbf{r}-\mathbf{r}') + \Sigma(\mathbf{r},\mathbf{r}';E+i\epsilon)$$

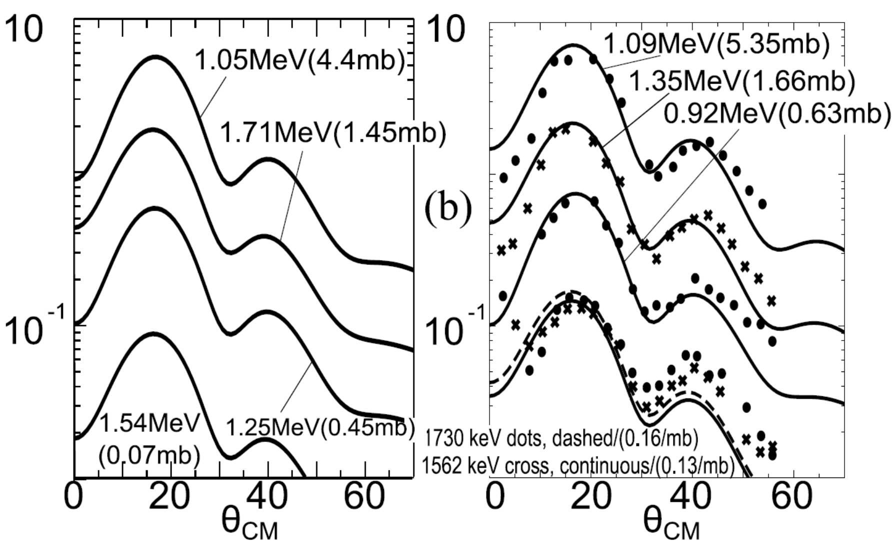
Σ corresponds to the Feshbach's generalized optical potential

Escher & Jennings PRC66 034313 (2002)

Why optical potentials?

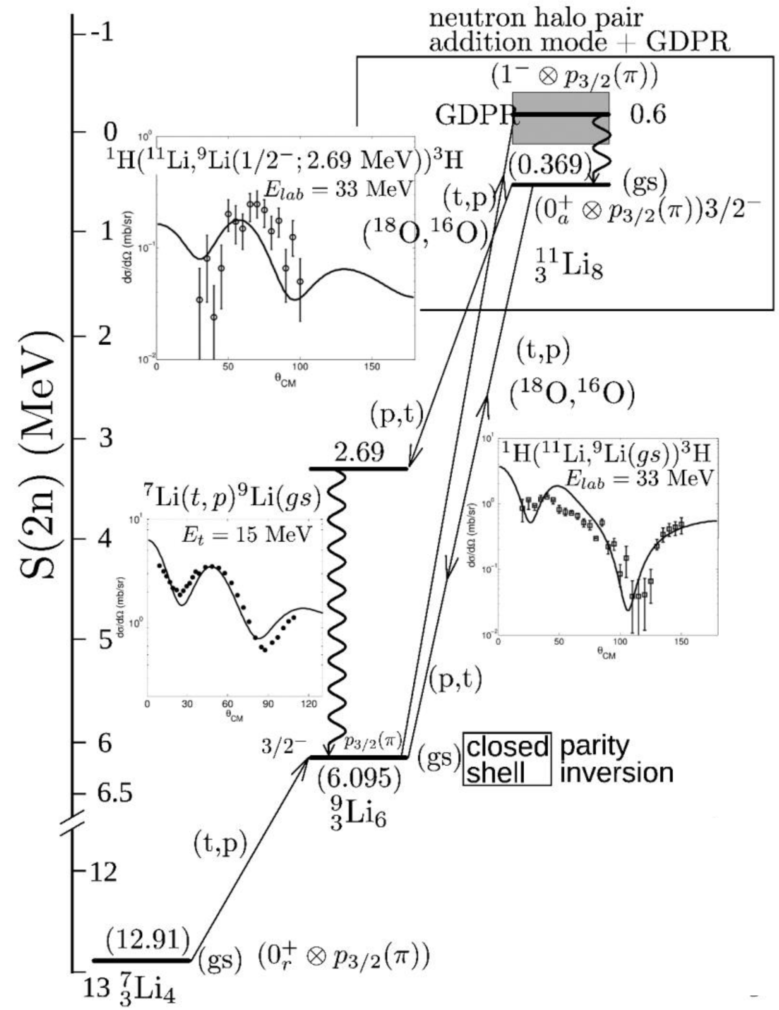
- Optical potentials **reduce many-body complexity** decoupling structure contribution and reactions dynamics.

1 particle transfer



A.I. et al. Phys. Rev. C **92**, 031304 (2015)

2 particle transfer



Broglia et al. *Phys. Scr.* **91** 06301* (2016)

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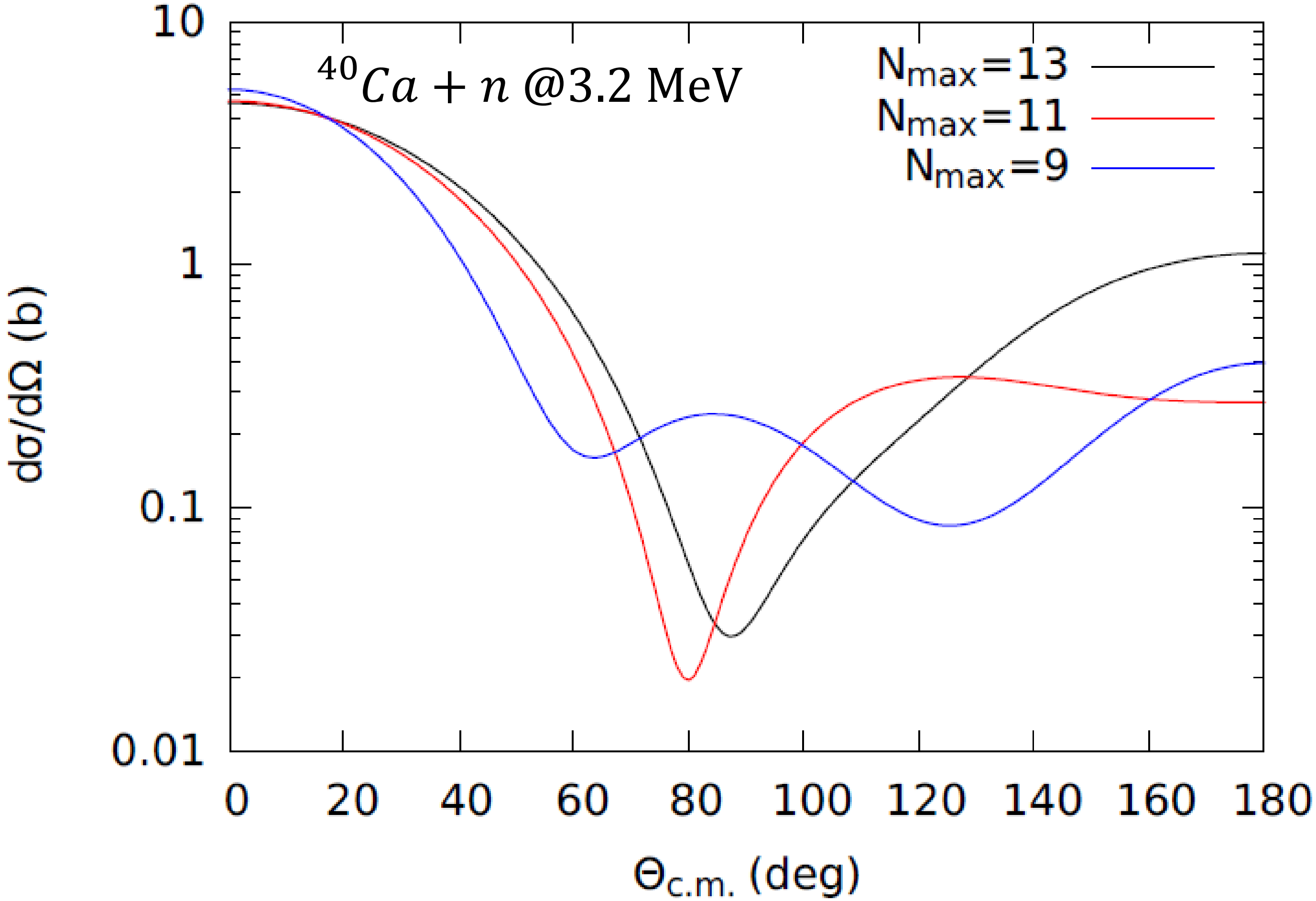
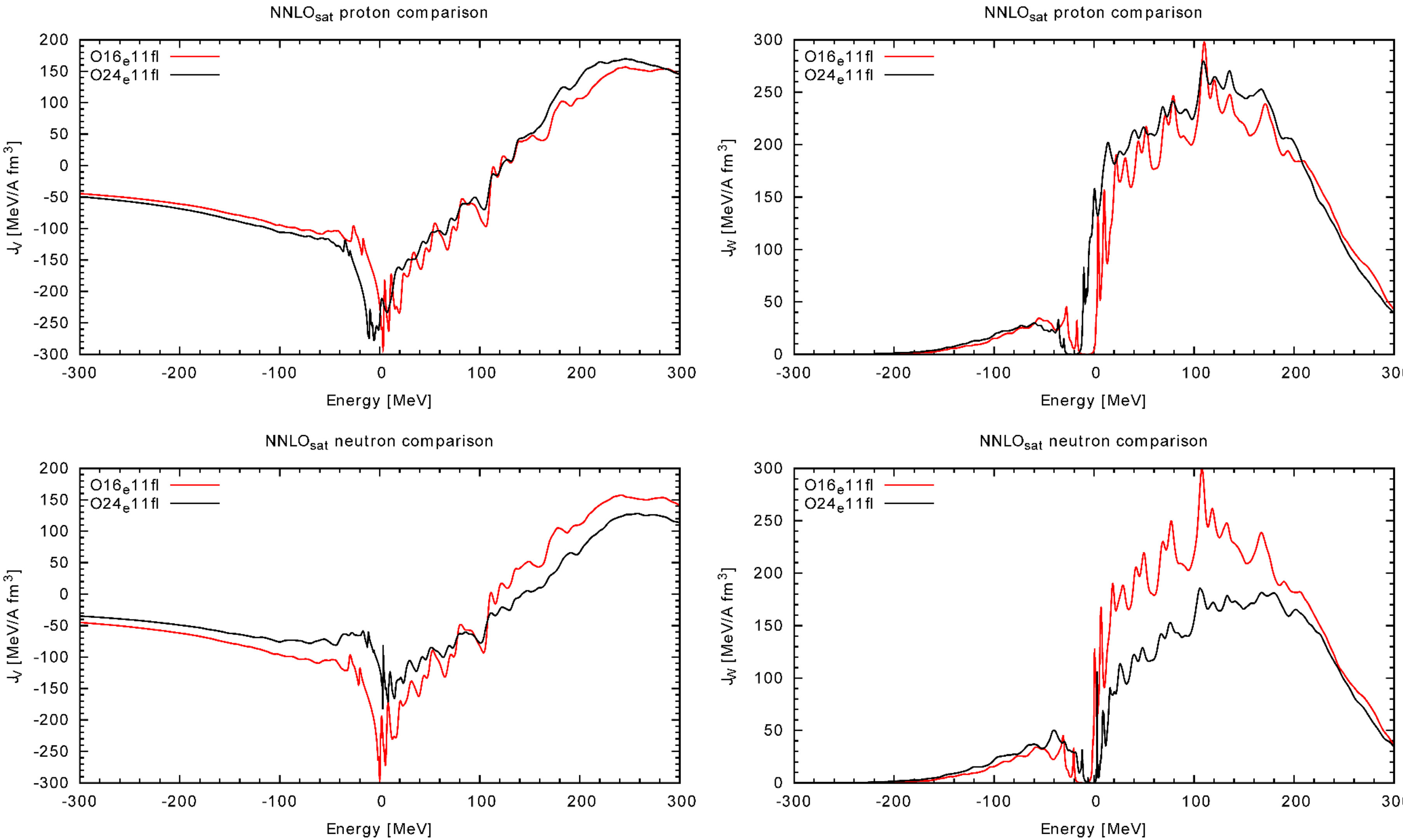


$^{40}Ca + n$ @3.2 MeV
Nmax=13
Nmax=11
Nmax=9
10
1
0.1
0.01
0
20
40
60
80
100
120
140
160
180
dσ/dΩ (b)
Θc.m. (deg)

^{16}O and ^{24}O


NNLOsat proton comparison
O16e11fl
O24e11fl
JV [MeV/A fm3]
Energy [MeV]
NNLOsat proton comparison
O16e11fl
O24e11fl
JW [MeV/A fm3]
Energy [MeV]
NNLOsat neutron comparison
O16e11fl
O24e11fl
JV [MeV/A fm3]
Energy [MeV]
NNLOsat neutron comparison
O16e11fl
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Energy [MeV]