Scale and scheme dependence of nuclear processes

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Refs: *"The longitudinal response function of the deuteron in chiral EFT,"*
C.-J. Yang and D.R. Phillips, EJP A49 (2013)

"Deuteron electrodisintegration with unitarily evolved potentials,"
S. More, S. König, rjf, and K. Hebeler, PRC (2015)

"Scale dependence of deuteron electrodisintegration,"
S. More, S. Bogner, rjf (in preparation)

Some particular motivations for thinking about this here

- Nature of optical potentials for *N*–*A*
  - Filomena Nunes (and other recent talks): “bipolar thinking” of effective interaction vs. ab initio self-energy
  - Surrey group: sensitivity to high-*np* momenta and D-state component in (d, p) reactions [e.g., PRL **117** (2016)]
- Short-range correlations (SRC) in nuclear structure and reactions
  - JLab SRC/EMC correlation experiments [e.g., Hen et al., RMP]
  - Chen et al. analysis using EFT and OPE [arXiv:1607.03065]
  - Nuclear contacts (cf. cold atoms), $\beta\beta 0\nu$, ...
- And, as usual, what about spectroscopic factors and the like?

Outline

Overview: scale and scheme dependence

Test case: deuteron electrodisintegration

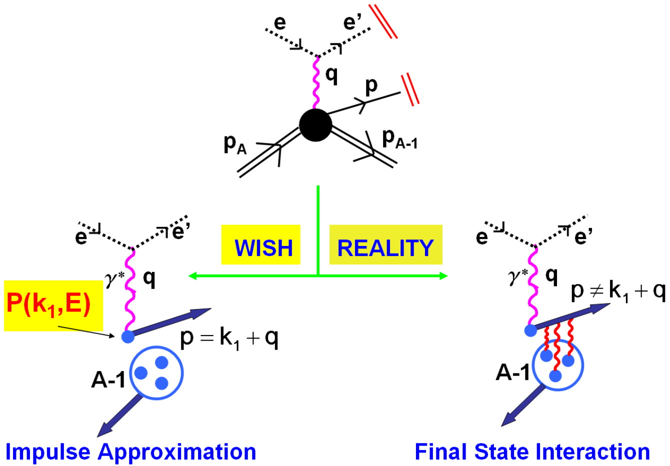
Summary and take-aways

Many back-up slides

Context for scale/scheme dependence: knock-out reactions

- E.g., $(p, 2p)$ or high-momentum electron scattering on nuclei
- Goal is *process independent* determination of properties
- If impulse approximation (IA) in some form is really valid, then direct extraction of nuclear properties is possible
- More generally, process independence requires a controlled factorization of structure and reaction mechanism
- But dividing point is not unique, so scale/scheme dependent
- Understanding this dependence is important for:
  - robust extractions from experiments
  - to correctly use the structure information in other processes
  - to understand the impact of approximations for both

Standard story for $(e, e'p)$ [from C. Ciofi degli Atti]



- In IA: "missing" momentum $p_m = k_1$ and energy $E_m = E$
- Choose E_m to select a discrete final state for range of p_m
- FSI and meson exchange currents treated as *add-on* theoretical corrections to IA? But mixing with structure is scale dependent!

Let's check a textbook for conventional wisdom . . .

S. Wong, "Introductory Nuclear Physics", pg. 358-9:

> *"Let us recapture what is happening when an intermediate energy nucleon is scattered off a nucleus. ... The three parts of a calculation — optical potential, nucleon-nucleon interactions, and nuclear wave functions — are three distinct parts of the problem and may be treated quite independently of each other."*

Even if not so explicit, this viewpoint is often implicit.

Note: there's no problem with an ab initio calculation that treats all elements consistently. (Still need factorization to extract properties.)

Parton distributions as paradigm [Marco Stratman]

Deep-inelastic scattering (DIS) according to pQCD

the physical structure fct. is independent of μ_f
(this will lead to the concept of renormalization group eqs.)

both, pdf's and the short-dist. coefficient depend on μ_f
(choice of μ_f: shifting terms between long- and short-distance parts)

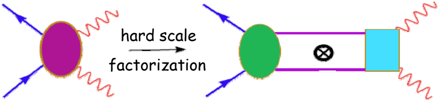
$$F_2(x,Q^2) = x \sum_{a=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_a(\xi,\mu_f^2) \left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_s(\mu_r)}{2\pi}\left[P_{qq}\left(\frac{x}{\xi}\right)\ln\frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})\left(\frac{x}{\xi}\right)\right]\right]$$

yet another scale: μ_r
due to the renormalization
of ultraviolet divergencies

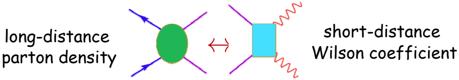
short-distance "Wilson coefficient"

choice of the factorization scheme

Factorization: high-E QCD vs. low-E nuclear



$$F_2(x, Q^2) \sim \sum_a f_a(x, \mu_f) \otimes \widehat{F}_2^a(x, Q/\mu_f)$$



- Separation between long- and short-distance physics is not unique $\Longrightarrow$ introduce μ_f
- Choice of μ_f defines border between long/short distance
- Form factor F_2 is independent of μ_f, but pieces are not
- Q^2 running of $f_a(x, Q^2)$ comes from choosing μ_f to optimize extraction from experiment

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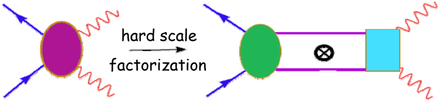

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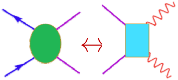

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- Also has factorization assumptions (e.g., from D. Bazin ECT* talk, 5/2011)

Observable: cross section

Structure model: spectroscopic factor

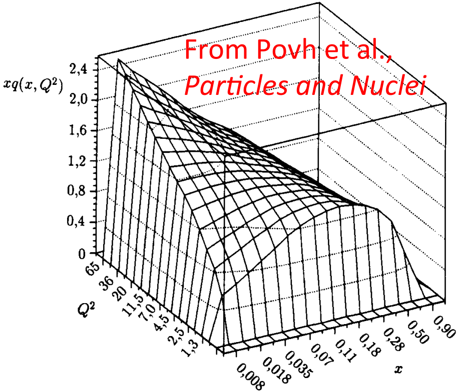
Reaction model: single-particle cross section

$$\sigma^{if} = \sum_{|J_f - J_i| \leq j \leq J_f + J_i} S_j^{if} \sigma_{sp}$$

- Is the factorization general/robust? (Process dependence?)
- What is the scale/scheme dependence of extracted properties (and the reaction model)?
- What are the trade-offs? (Does simpler structure always mean much more complicated reaction?)

Use RG as tool to address questions

Parton vs. nuclear momentum distributions



- The quark distribution $q(x, Q^2)$ is scale *and* scheme dependent
- $x\ q(x, Q^2)$ measures the share of momentum carried by the quarks in a particular x-interval
- $q(x, Q^2)$ and $q(x, Q_0^2)$ are related by RG evolution equations

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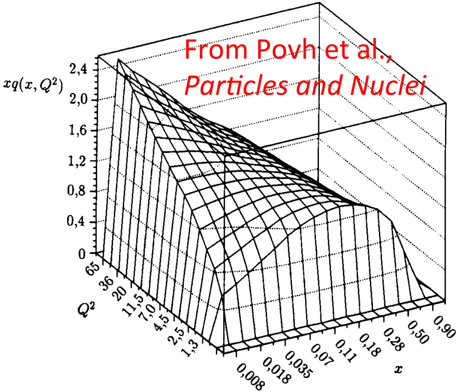

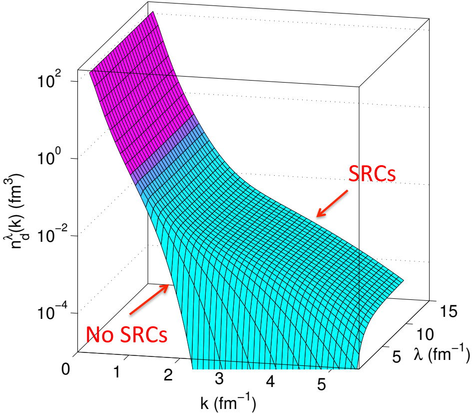

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- Deuteron momentum distribution is scale *and* scheme dependent
- Initial AV18 potential evolved with SRG from $\lambda = \infty$ to $\lambda = 1.5\,\text{fm}^{-1}$
- High momentum tail shrinks as λ decreases (lower resolution)

Scheming for parton distributions

Need schemes for both renormalization and factorization

From the “Handbook of perturbative QCD” by G. Sterman et al.

“Short-distance finite parts at higher orders may be apportioned arbitrarily between the C’s and ϕ’s. A prescription that eliminates this ambiguity is what we mean by a factorization scheme. . . . The two most commonly used schemes, called DIS and $\overline{MS}$, reflect two different uses to which the freedom in factorization may be put.”

“The choice of scheme is a matter of taste and convenience, but it is absolutely crucial to use schemes consistently, and to know in which scheme any given calculation, or comparison to data, is carried out.”

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Specifying a scheme in low-energy nuclear physics includes specifying a potential and *consistent* currents, including regulators, and how a reaction is analyzed. (EFT is a good framework for this!)

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- To make calculations easier or more convergent
  - QCD running coupling and scale: improved perturbation theory; choosing a gauge: e.g., Coulomb or Lorenz
  - Low-k potential: improve many-body convergence, or to make microscopic connection to shell model or dft
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- Scale and scheme for nuclear reactions?
  - Plan: use renormalization group (RG) to consistently relate scales and quantitatively probe ambiguities

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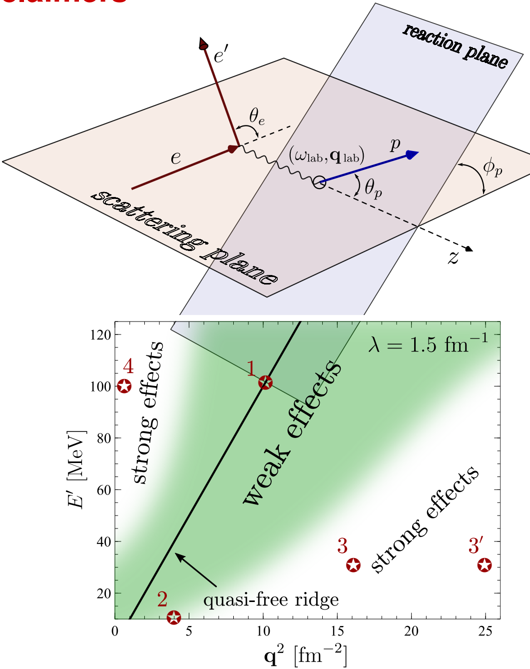
Set up for ^{2}H($e, e'p$) and disclaimers

- Simplest knock-out process: no complications of three-body forces; neglect relativity, etc.
- f_L only: $\frac{d\sigma}{d\Omega} \propto v_L\, f_L + v_T\, f_T + \cdots$
  $\Longrightarrow f_L \sim \sum_{m_s, m_J} \left|\langle\psi_f|J_0(q)|\psi_i\rangle\right|^2$
- $|\psi_i\rangle$ is always deuteron
- FSI: $|\psi_f\rangle = |\phi^{p'}\rangle + G_0(E')t(E')|\phi^{p'}\rangle$



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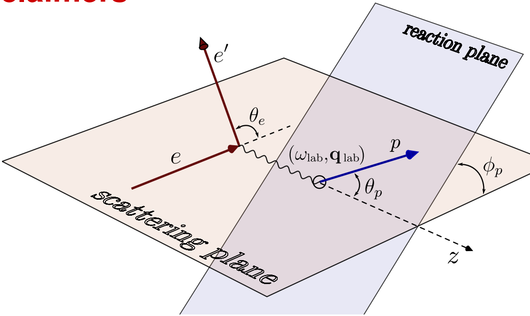
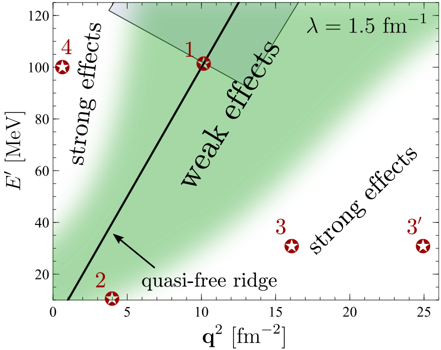
- Use AV18 as "high-resolution" initial potential ($\lambda = \infty$)
- Initially only one-body current
  $\langle \mathbf{k}_1 T_1 | J_0(\mathbf{q}) | \mathbf{k}_2 T = 0\rangle =$
  $\frac{1}{2}\left(G_E^p + (-1)^{T_1} G_E^n\right)\delta(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{q}/2) +$
  $\frac{1}{2}\left((-1)^{T_1} G_E^p + G_E^n\right)\delta(\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{q}/2)$





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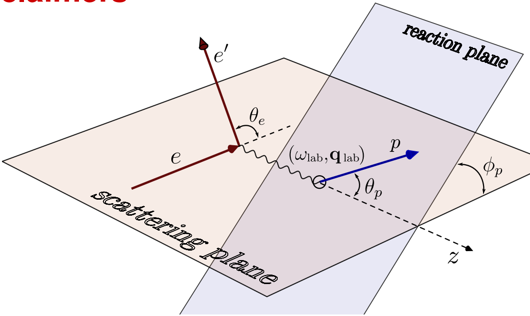
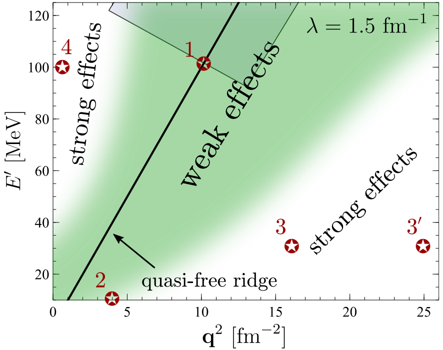
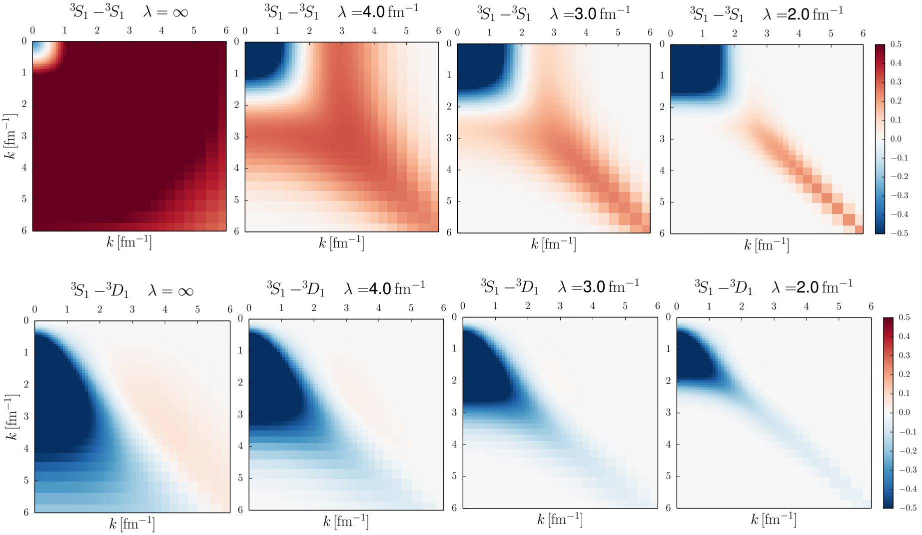
- Use similarity RG evolution to probe scale and scheme dependence vs. kinematics





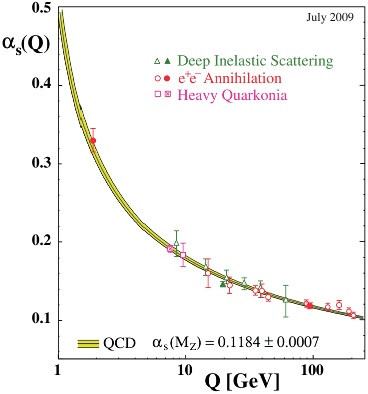
SRG evolution of AV18 potential $dH_s/ds = [[G_s, H_s], H_s]$, $G_s = T$



Notes: unitary transformation $\widehat{O}_\lambda = \widehat{U}_\lambda \widehat{O}_{\lambda=\infty} \widehat{U}_\lambda^\dagger$; λ sets decoupling scale:

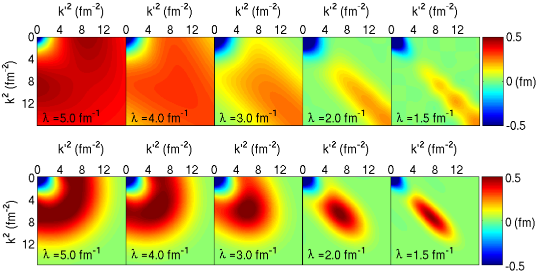
$V_\lambda(k,k') \approx V_{\lambda=\infty}(k,k') e^{-\left(\frac{k^2-k'^2}{\lambda^2}\right)^2}$ (nonlocality!); scheme dependence from G_s

Running QCD $\alpha_s(Q^2)$ vs. running nuclear V_λ



- The QCD coupling is *scale* dependent (cf. low-E QED): $\alpha_s(Q^2) \approx [\beta_0 \ln(Q^2/\Lambda^2_{\text{QCD}})]^{-1}$
- The QCD coupling strength α_s is *scheme* dependent (e.g., "V" scheme used on lattice, or $\overline{\text{MS}}$)

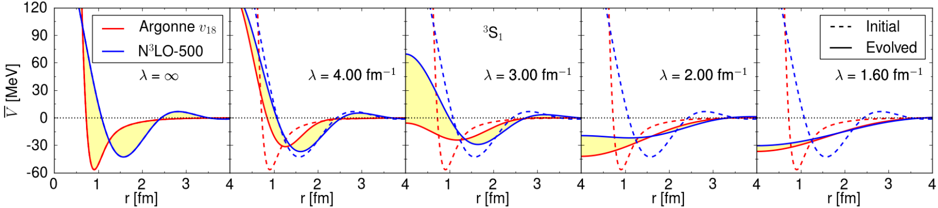
- Vary scale ("resolution") with RG
- Scale dependence: SRG (or $V_{\text{low }k}$) running of initial potential with λ (decoupling or separation scale)



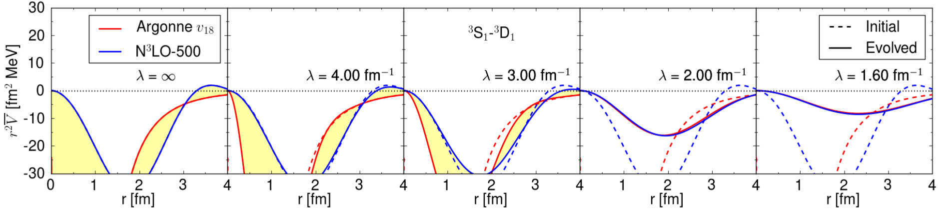
- Scheme dependence: SRG generator and AV18 vs. N^{3}LO (plus associated 3NFs) but note flow to universality at low k
- All λ are (NN) phase equivalent!
- Shift contributions between interaction and sums over intermediate states

Visualizing the softening of NN interactions

- Project non-local NN potential: $\overline{V}_\lambda(r) = \int d^3r'\, V_\lambda(r, r')$
  - Roughly gives action of potential on long-wavelength nucleons
- Central part (S-wave) [Note: The V_λ's are all phase equivalent!]



- Tensor part (S-D mixing) [graphs from K. Wendt et al., PRC (2012)]



$\Longrightarrow$ Note the flow to universal potentials!

Source of scale-dependence for low-E processes

- Measured cross section as convolution: reaction $\otimes$ structure
  - but separate parts are not unique, *only* the combination
- Short-range unitary transformation U leaves m.e.'s invariant:

$$O_{mn} \equiv \langle \Psi_m | \widehat{O} | \Psi_n \rangle = \left(\langle \Psi_m | U^\dagger \right) U \widehat{O} U^\dagger \left(U | \Psi_n \rangle \right) = \langle \widetilde{\Psi}_m | \widetilde{O} | \widetilde{\Psi}_n \rangle \equiv \widetilde{O}_{\widetilde{m}\widetilde{n}}$$

Note: matrix elements of operator $\widehat{O}$ itself between the transformed states are in general modified:

$$O_{\widetilde{m}\widetilde{n}} \equiv \langle \widetilde{\Psi}_m | O | \widetilde{\Psi}_n \rangle \neq O_{mn} \quad \Longrightarrow \quad \text{e.g., } \langle \Psi_n^{A-1} | a_\alpha | \Psi_0^A \rangle \text{ changes}$$

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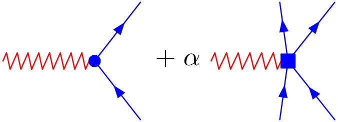
- In a low-energy effective theory, transformations that modify *short-range* unresolved physics $\Longrightarrow$ equally valid states. So $\widetilde{O}_{mn} \neq O_{mn} \Longrightarrow$ scale/scheme dependent observables.
- RG unitary transformations change the decoupling scale $\Longrightarrow$ change the factorization scale. Use to characterize and explore scale and scheme and process dependence!

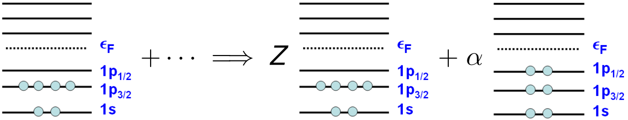
All pieces mix with unitary transformation

- A one-body current becomes many-body (cf. EFT current):

$\widehat{U}\widehat{\rho}(\mathbf{q})\widehat{U}^\dagger = $  $+ \cdots$

- New wf correlations have appeared (or disappeared):

$\widehat{U}|\Psi_0^A\rangle = \widehat{U}$ [levels] $+ \cdots \Longrightarrow Z$ [levels] $+ \alpha$ [levels] $+ \cdots$



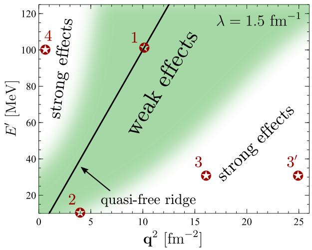
  - Similarly with $|\Psi_f\rangle = a_{\mathbf{p}}^\dagger|\Psi_n^{A-1}\rangle$
  - E.g., *spectroscopic factors* are generally scale dependent

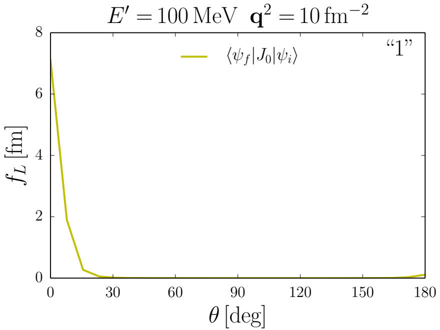
- Final state interactions (FSI) are also modified by $\widehat{U}$

- Bottom line: the cross section is unchanged *only* if all pieces are included, *with the same U*: $H(\lambda)$, current operator, FSI, ...

Results at the quasi-free ridge (QFR) [More et al. (2015)]

- Recall: $f_L \sim \sum_{m_s, m_J} \left| \langle \psi_f | J_0 | \psi_i \rangle \right|^2$
- At the quasi-free ridge, IA works because proton and neutron already on shell $E'(\text{in MeV}) \approx 10\,\mathbf{q}^2(\text{in fm}^{-2})$
- Long-range part of the wave function probed at QFR $\rightarrow$ invariant under SRG evolution





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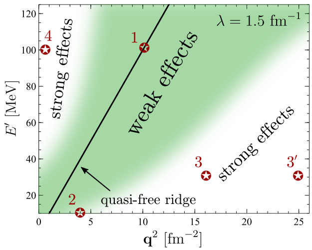

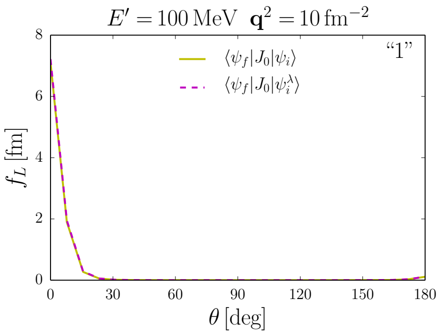

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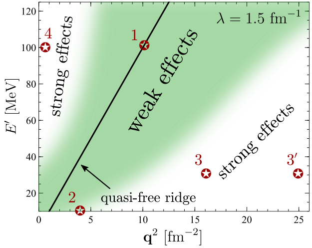

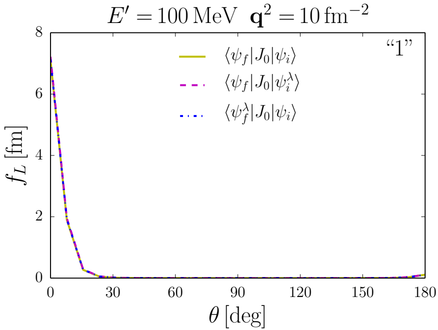

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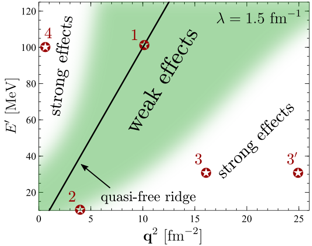
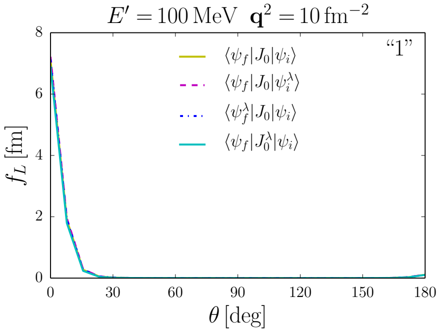
- Long-range part of the wave function probed at QFR $\rightarrow$ invariant under SRG evolution





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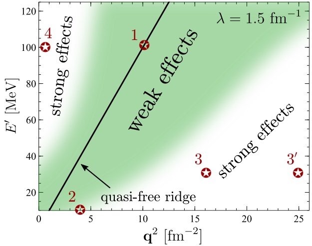

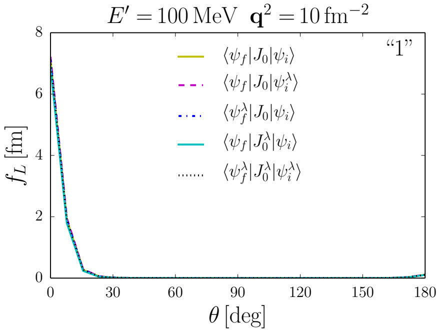

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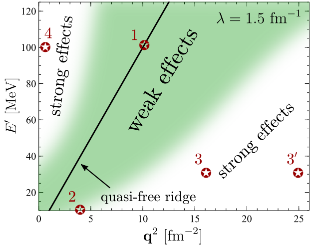

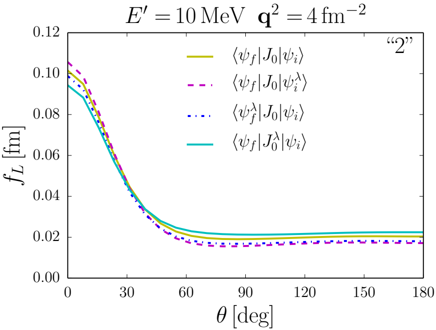

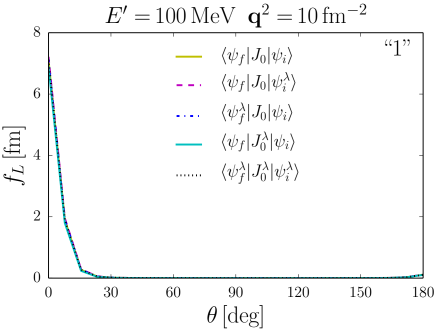

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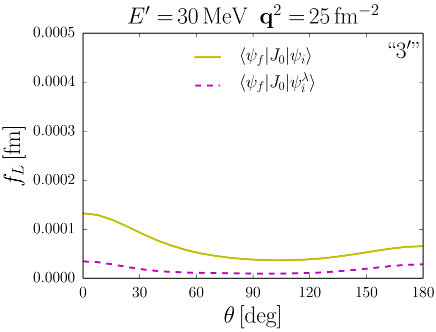

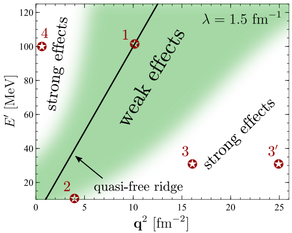

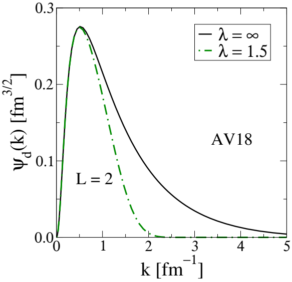

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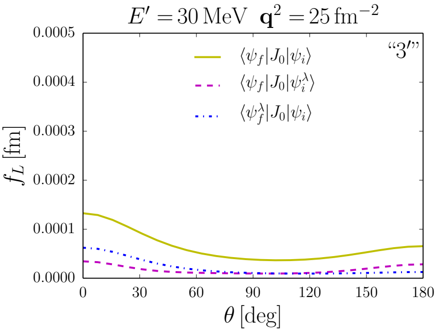

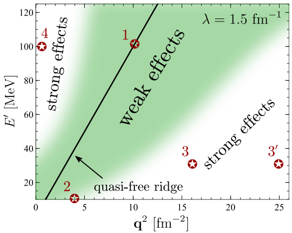

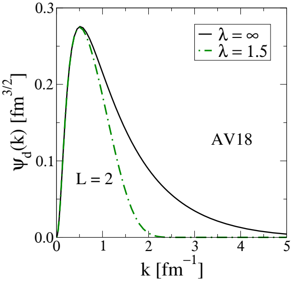

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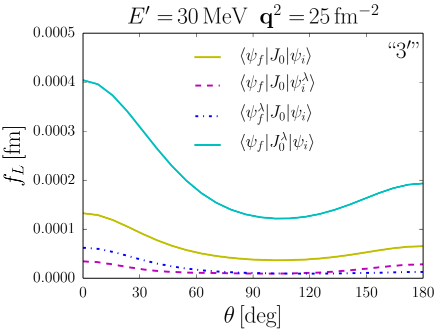

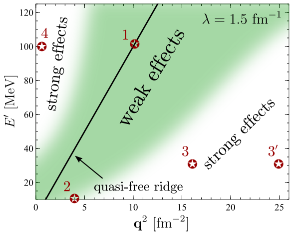

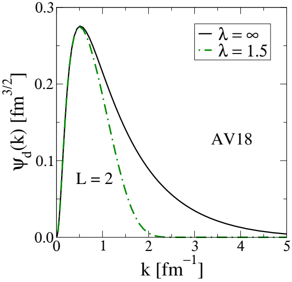

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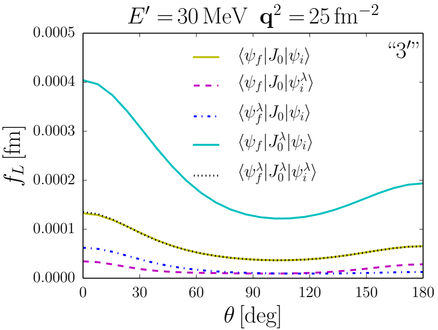

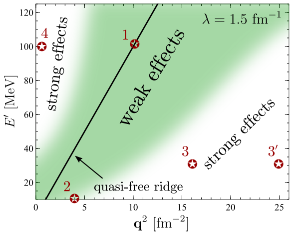

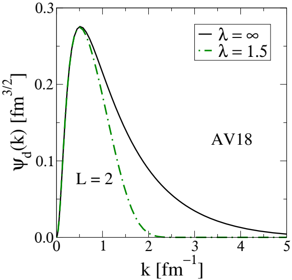

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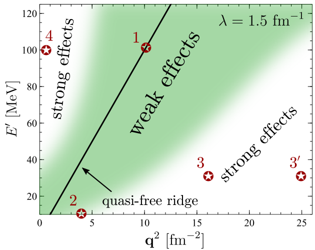

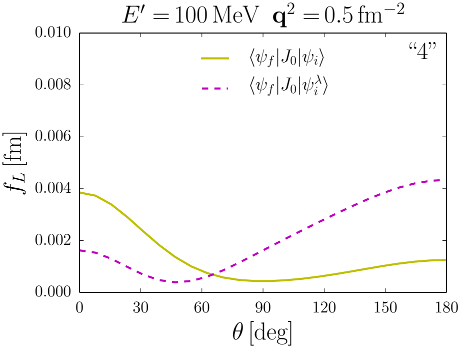

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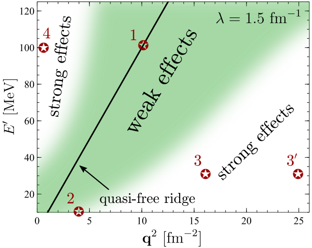

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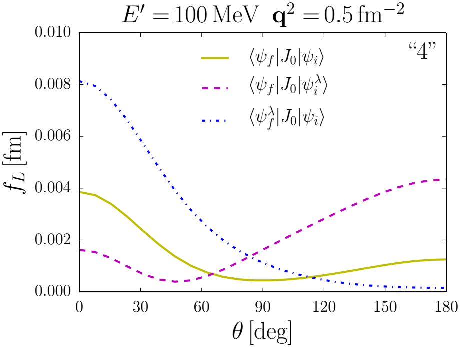

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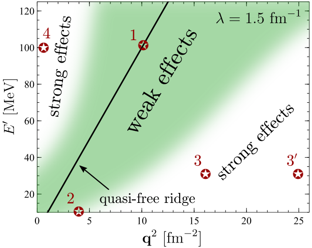

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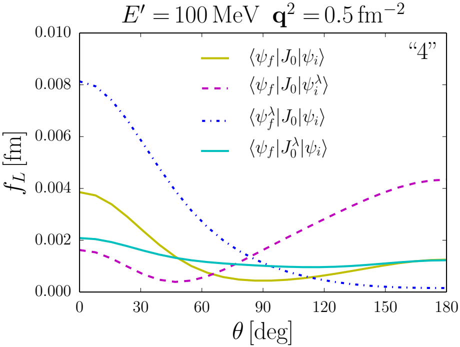

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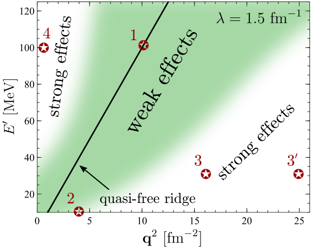

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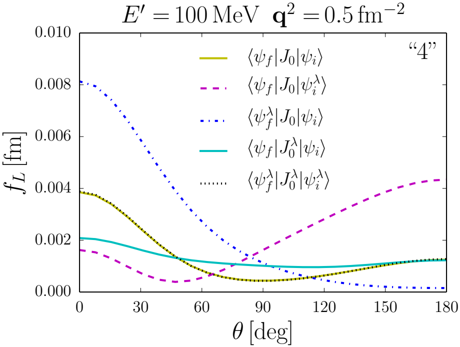
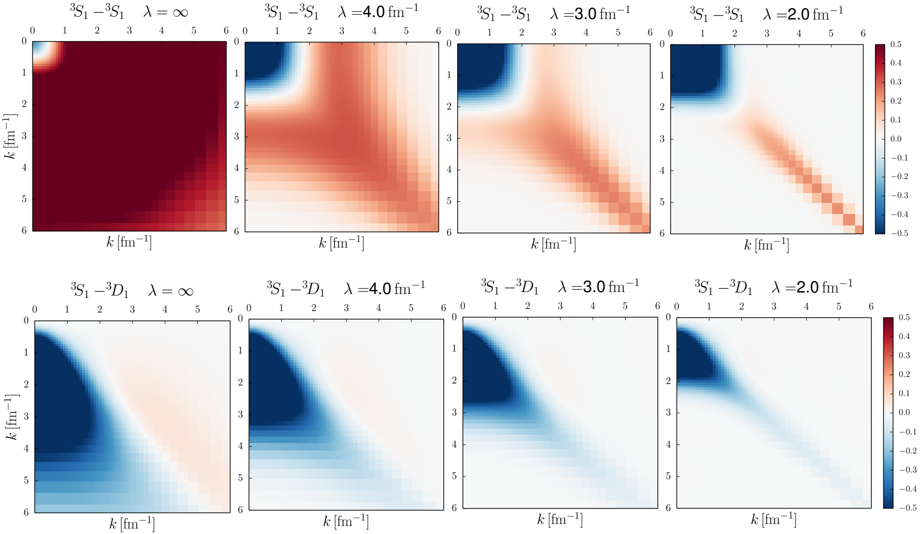


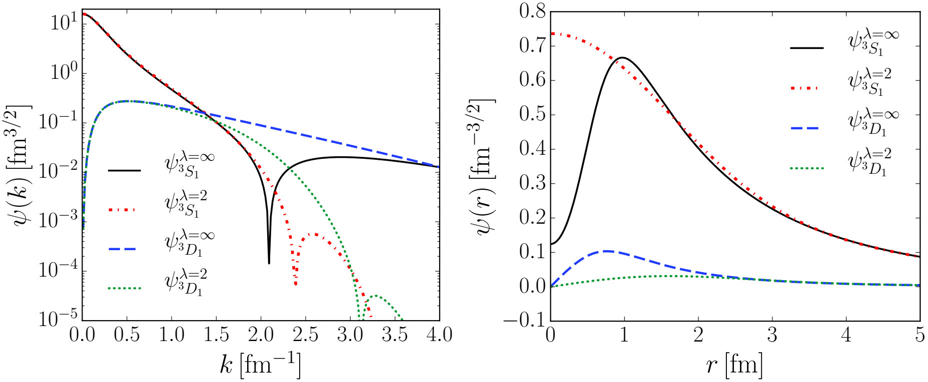
SRG evolution of AV18 potential $dH_s/ds = [[G_s, H_s], H_s]$, $G_s = T$



Notes: unitary transformation $\widehat{O}_\lambda = \widehat{U}_\lambda \widehat{O}_{\lambda=\infty} \widehat{U}_\lambda^\dagger$; λ sets decoupling scale:

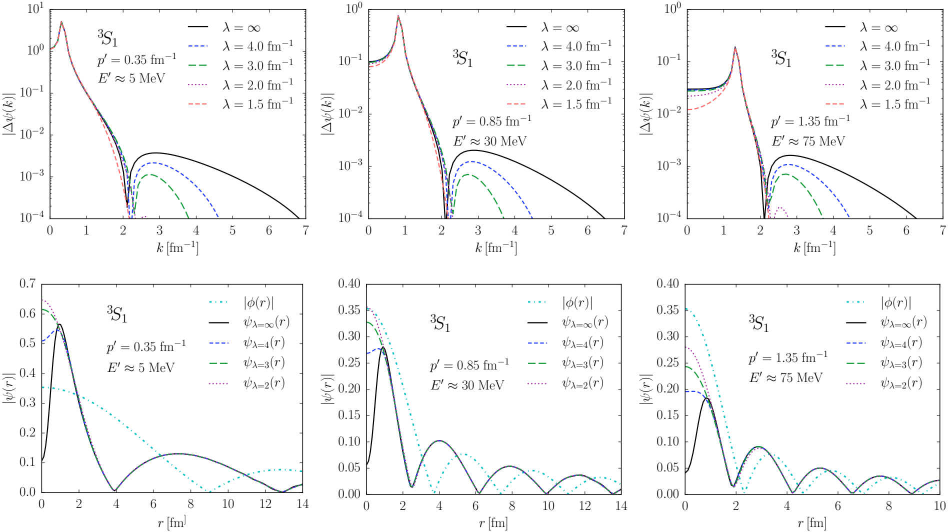
$V_\lambda(k,k') \approx V_{\lambda=\infty}(k,k') e^{-\left(\frac{k^2-k'^2}{\lambda^2}\right)^2}$ (nonlocality!); scheme dependence from G_s

Deuteron wave functions at two resolution scales



- S-wave part: high-momentum tail from coupling of low- and high-momentum by AV18 ($\lambda = \infty$) evolved away as λ reduced
- Consequent filling of wound at small *r* (SRCs disappear!)
- D-wave part: reduced S-D tensor coupling lowers D-state probability
- Note that *r*-space tails (i.e., ANCs) are RG invariant

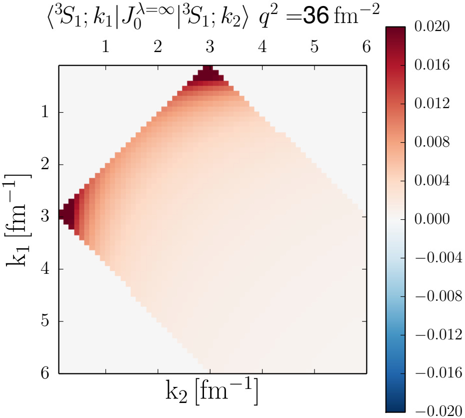
S-wave scattering wave functions at different scales

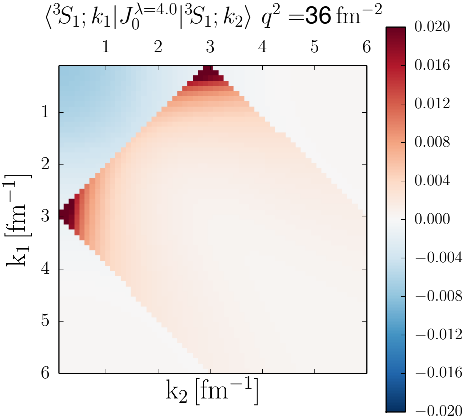


- High-k tail and small r wound evolved away as λ reduced (but same δ)
- Local decoupling shows up as p' increases: suppressed low k

Evolution of current with decreasing resolution λ

$^3S_1$ component at fixed $q = 6\ \text{fm}^{-1}$; one-body peaked at $q/2$





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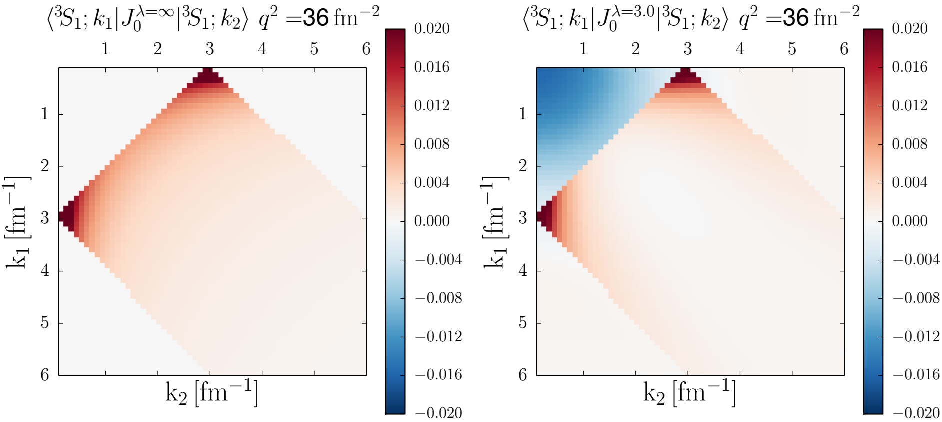

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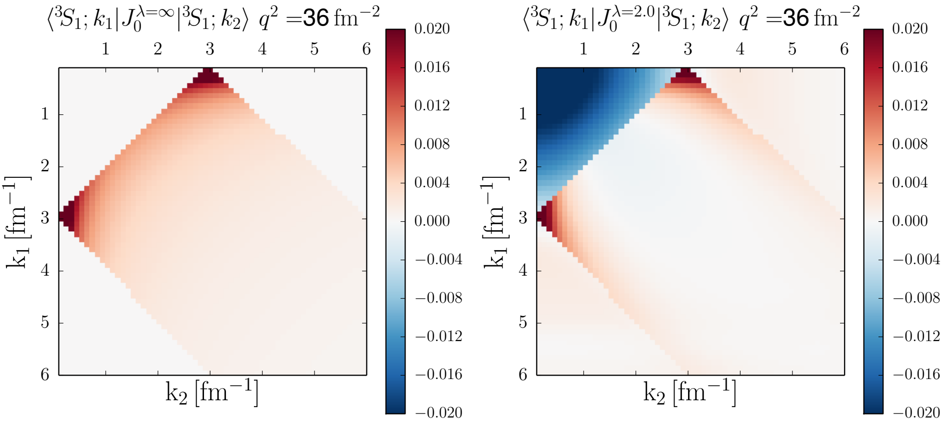

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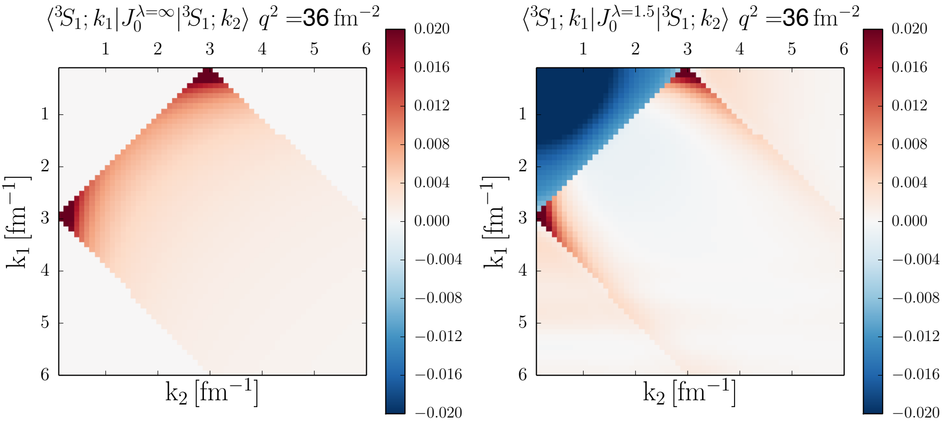

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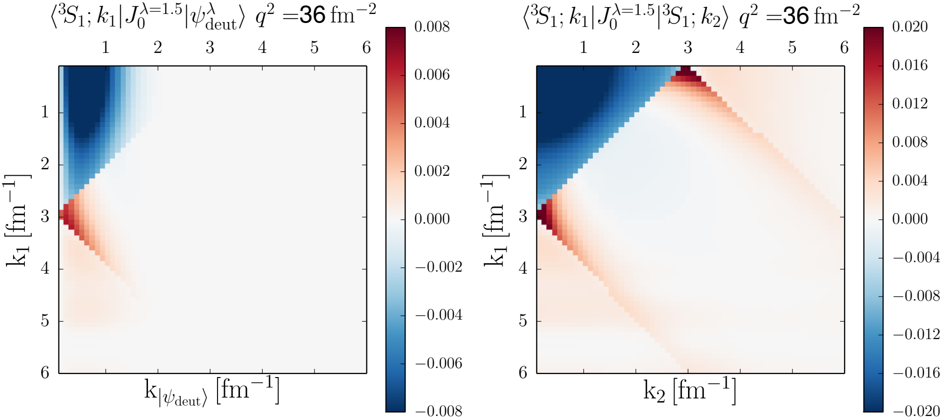

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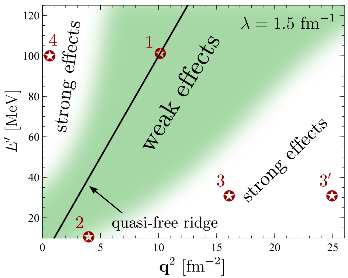
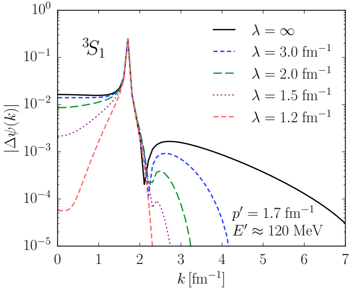
- Evolved deuteron wf filters current (and then filtered by $|\psi_f\rangle$)

FSI at large final $p' = 1.7\ \text{fm}^{-1} \Longrightarrow$ scan in $\mathbf{q}^2$ with fixed $|\psi_f^\lambda\rangle$





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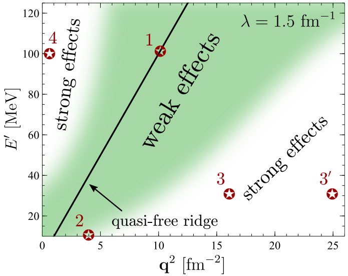

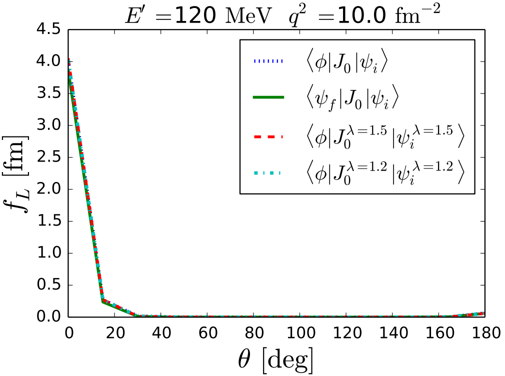

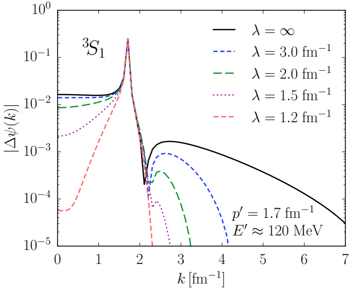

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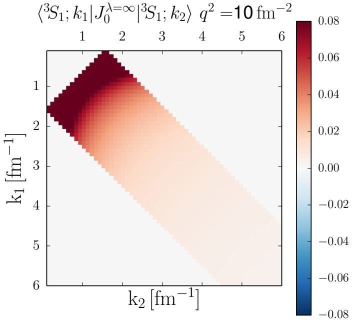

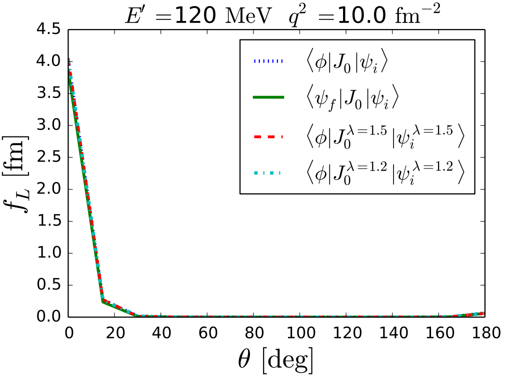

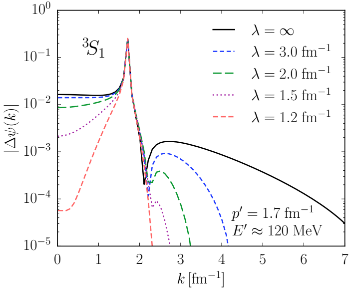

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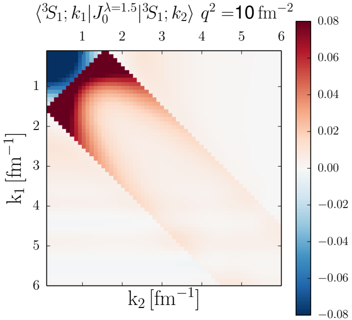

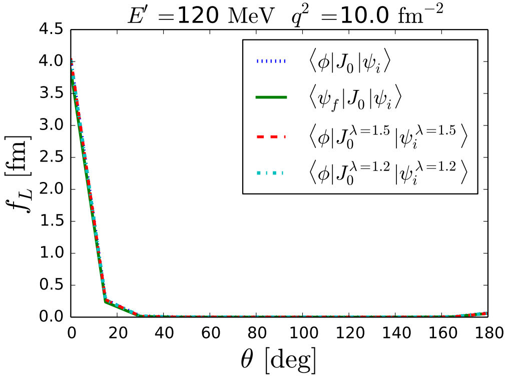

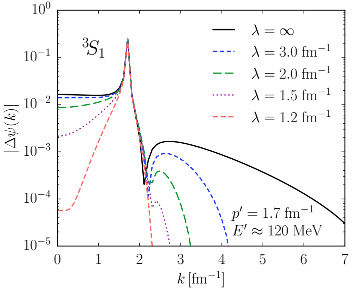

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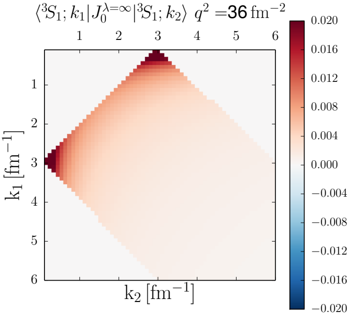

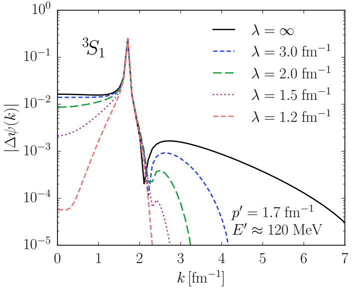

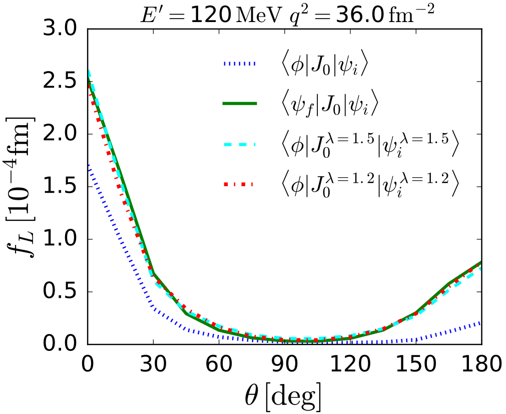

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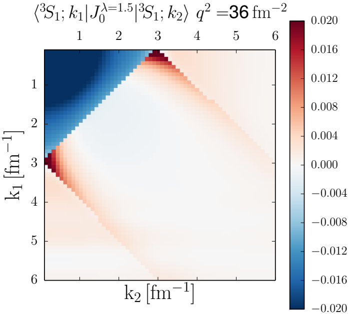

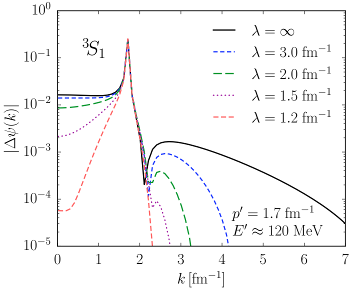

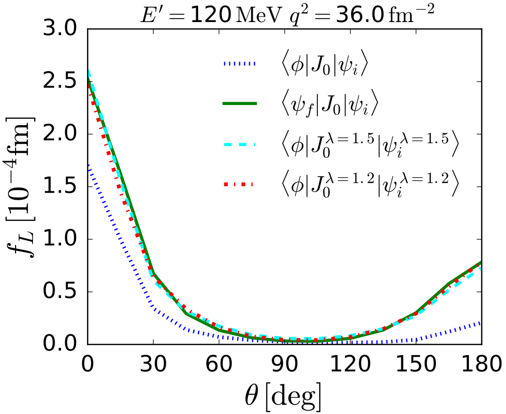

- At larger q^2, unevolved high-k tail in $|\psi_f\rangle$ gives significant FSI strength
- But when $q/2 > p', \lambda$, evolved $|\psi_f^\lambda\rangle$ picks up $k \approx p'$ while $|\psi_i\rangle$ restricts $k' < \lambda$ $\Longrightarrow$ small FSI

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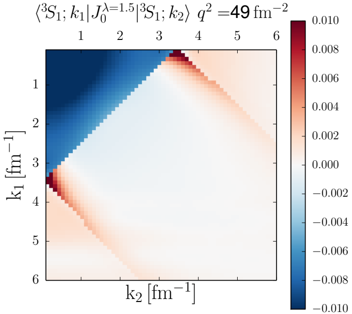
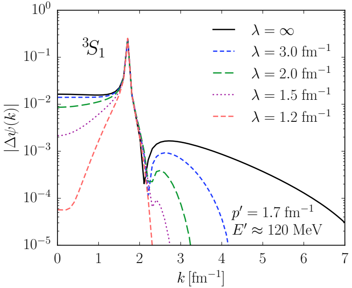
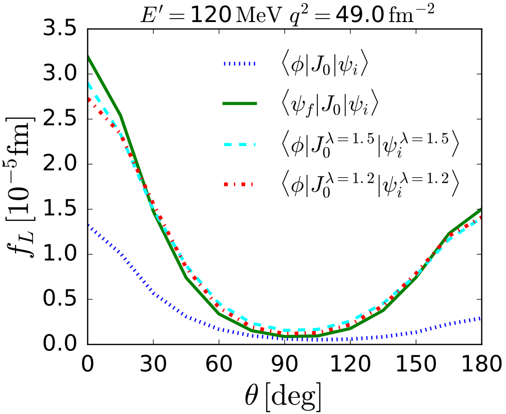
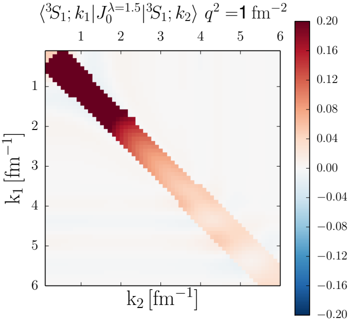


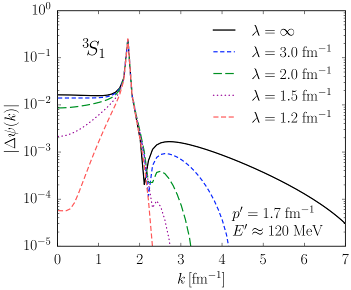


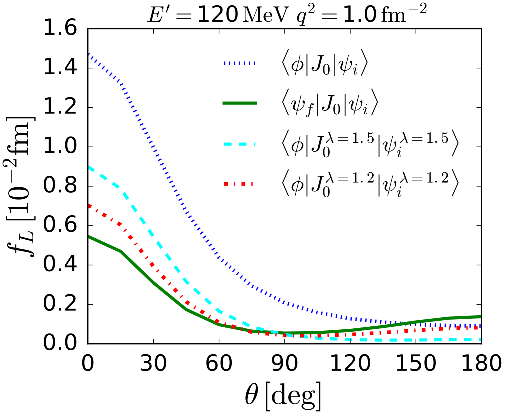


- At still larger q^2, unevolved high-k tail in $|\psi_f\rangle$ gives even more FSI strength
- But when $q/2 > p', \lambda$, evolved $|\psi_f^\lambda\rangle$ picks up $k \approx p'$ while $|\psi_i\rangle$ restricts $k' < \lambda$
  $\Longrightarrow$ small FSI by same argument

FSI at large final $p' = 1.7\ \mathrm{fm}^{-1} \Longrightarrow$ scan in $\mathbf{q^2}$ with fixed $|\psi_f^\lambda\rangle$



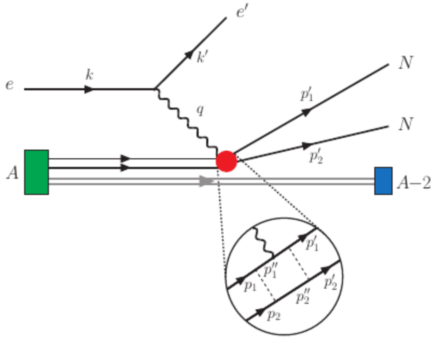




- At low q^2, $|\psi_f\rangle$ probes current for $k > q/2$ $\Longrightarrow$ large FSI again
- Reduced but significant FSI dependent on λ

Current evolution and SRC story

- So FSI can be simpler at low resolution. What about short-distance physics in deuteron?
- Varying λ shuffles physics between current and structure parts
- What happens to SRCs?
- λ decreases $\rightarrow$ blob size increases. One-body current operator develops two-body (and higher-body) components

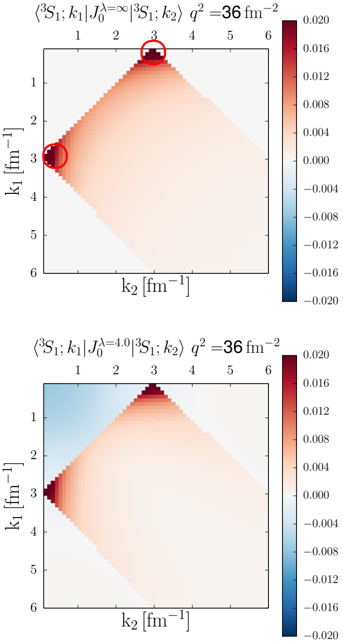


at high resolution

- $\langle \mathbf{k}_1 T_1 | J_0(\mathbf{q}) | \mathbf{k}_2 T = 0 \rangle =$
$$\frac{1}{2}\left(G_E^p + (-1)_1^T G_E^n\right)\delta(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{q}/2) + \frac{1}{2}\left((-1)_1^T G_E^p + G_E^n\right)\delta(\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{q}/2)$$
- Naive expectation: RG changes to $J_0(q)$ complicate calculations

Derivative expansion for the current

- Consider region $p' < q/2$ and calculate f_L; recall $\langle\psi_f|J_0(q)|\psi_i\rangle = \langle\psi_f^\lambda|J_0^\lambda(q)|\psi_i^\lambda\rangle$ $\Longrightarrow |\psi_f^\lambda\rangle$ and $|\psi_i^\lambda\rangle$ filter current
- $\therefore$ Low-momentum part of $J_0^\lambda(q)$ will be selected



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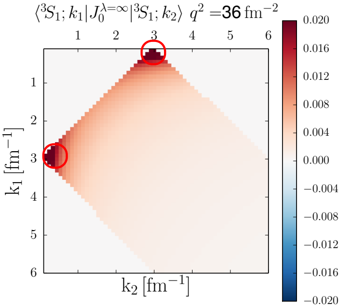

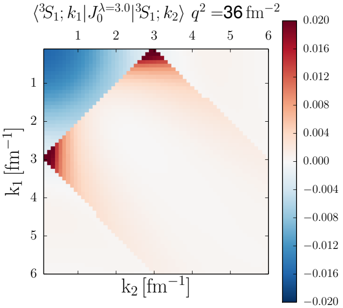

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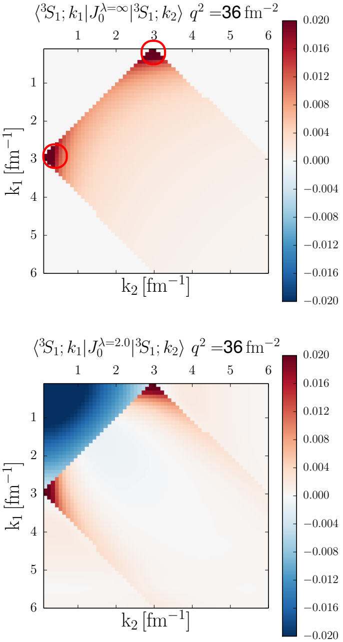

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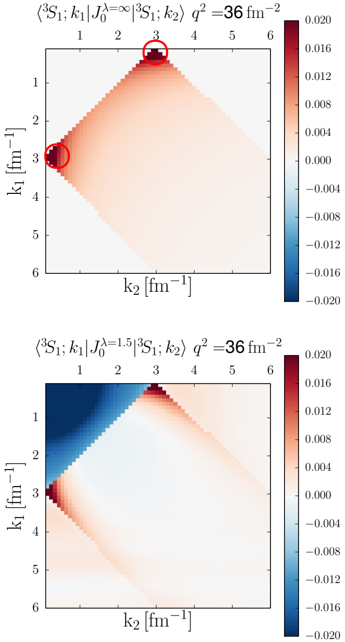

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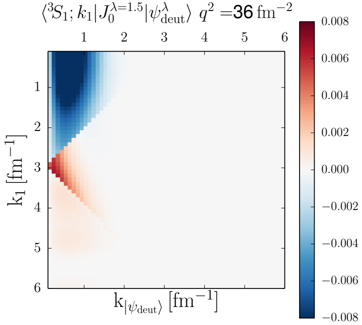

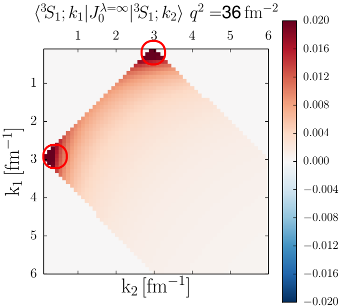

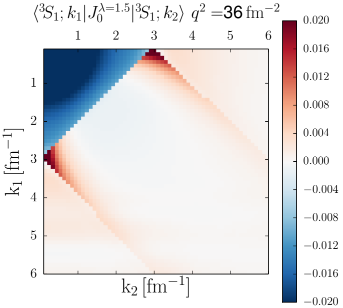

- Simple: $\langle {}^3S_1; k_1|J_0^\lambda(q)|{}^3S_1; k_2\rangle$
  $= g_0^q + g_2^q(k_1^2 + k_2^2)/\lambda^2 + \cdots$

Simple treatment of matrix elements

- Add expansions in other waves: $\langle {}^3P_1; k_1 | J_0^\lambda(q) | {}^3S_1; k_2 \rangle = \frac{k_1}{\lambda}(g_1^q + g_3^q k_2^2/\lambda^2) + \cdots$
- Only S-wave part of deuteron wf needed: $\langle \psi_f^\lambda | J_0^\lambda(q) | \psi_i^\lambda \rangle \approx \langle \psi_f^\lambda | J_0^\lambda(q) | \psi_{i\,{}^3S_1}^\lambda \rangle$
- Thus: $\langle \psi_f^\lambda | J_0^\lambda | \psi_{i\,{}^3S_1}^\lambda \rangle$

$$= \langle \psi_f^\lambda | {}^3S_1 \rangle \underbrace{\langle {}^3S_1 | J_0^\lambda | {}^3S_1 \rangle}_{\text{use deriv. exp.}} \langle {}^3S_1 | \psi_{i\,{}^3S_1}^\lambda \rangle + \langle \psi_f^\lambda | {}^3P_1 \rangle \underbrace{\langle {}^3P_1 | J_0^\lambda | {}^3S_1 \rangle}_{\text{use deriv. exp.}} \langle {}^3S_1 | \psi_{i\,{}^3S_1}^\lambda \rangle + \cdots$$

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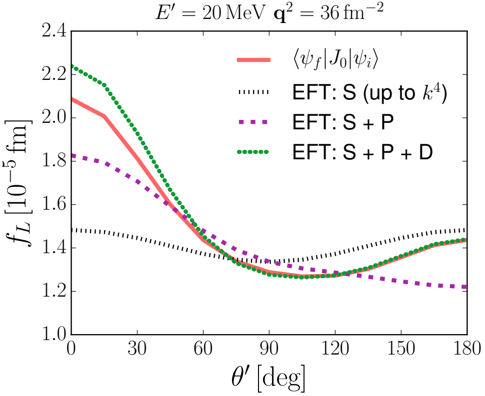
- End result is very simple:

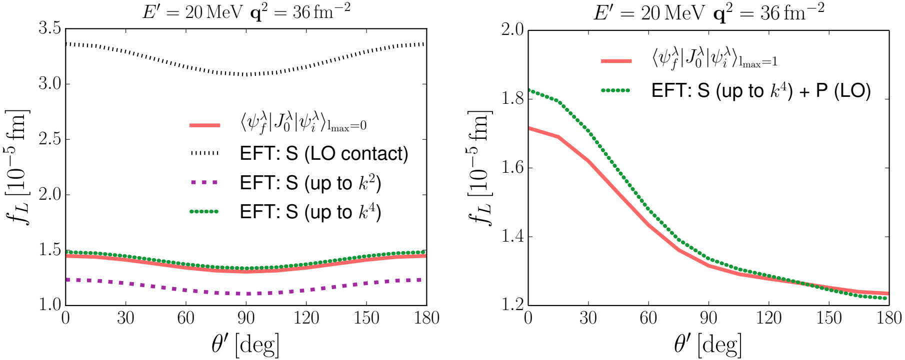
$$\langle \psi_f^\lambda | J_0^\lambda(q) | \psi_{\mathrm{deut}}^\lambda \rangle = g_0^q \; \psi_f^{\lambda\,*}(r)\, \psi_{\mathrm{deut}}^\lambda(r) \Big|_{r=0} + \cdots$$

- And it works! $f_L^{\mathrm{approx}} \approx f_L^{\mathrm{exact}}$
- Agreement improves with higher-order terms in expansion



Convergence in partial wave channels

[Note: still not fully understood yet!]



- $\langle\psi_f^\lambda|J_0^\lambda|\psi_i^\lambda\rangle_{\mathrm{l_{max}=0}} \equiv \langle\psi_f^\lambda;{}^3S_1|J_0^\lambda{}_{\mathrm{exact}}|\psi_i^\lambda;{}^3S_1\rangle$
- $\langle\psi_f^\lambda|J_0^\lambda|\psi_i^\lambda\rangle_{\mathrm{l_{max}=1}} \equiv \langle\psi_f^\lambda;{}^3S_1|J_0^\lambda{}_{\mathrm{exact}}|\psi_i^\lambda;{}^3S_1\rangle + \sum_{i=0,1,2}\langle\psi_f^\lambda;{}^3P_i|J_0^\lambda{}_{\mathrm{exact}}|\psi_i^\lambda;{}^3S_1\rangle$
- $\langle{}^3P_i;k_1|J_0^\lambda{}_{\mathrm{EFT}}|{}^3S_1;k_2\rangle_{\mathrm{LO}} \equiv g_{P_i}^q\,k_1$

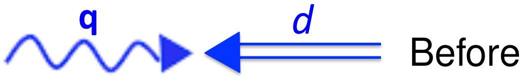
Simple pictures at high and low resolution

Can we account for the cross section at *both* high and low resolution with simple pictures?

Work in final neutron-proton rest frame at $\theta = 0^\circ$

Assume photon momentum absorbed entirely by proton

Scattering on the quasi-free ridge:



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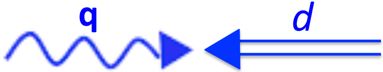

Before

After $p_f' \approx q/2$

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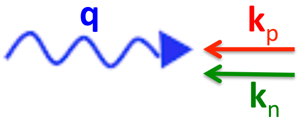
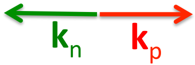


Before $p'_i \approx 0$



After $p'_f \approx q/2$

Deuteron wave function probed at low momentum

Simple pictures at high and low resolution

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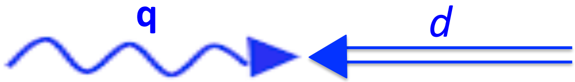
Assume photon momentum absorbed entirely by proton

Scattering near threshold with SRC kinematics:



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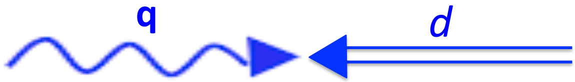

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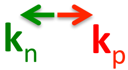

After $p_f' \approx$ small

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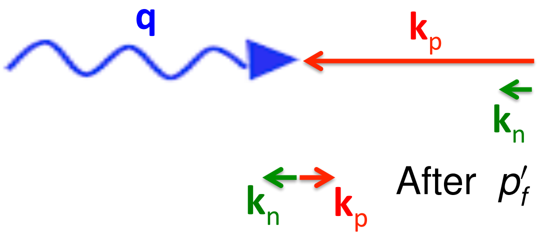

Before $p_i' \approx$ large

After $p_f' \approx$ small

Cross section from short-range correlation

Simple pictures at high and low resolution

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Work in final neutron-proton rest frame at $\theta = 0^\circ$

Assume photon momentum absorbed entirely by proton

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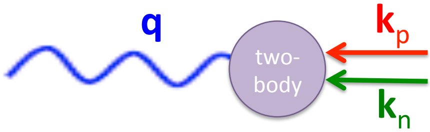
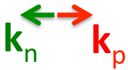


Before $p_i' \approx$ low

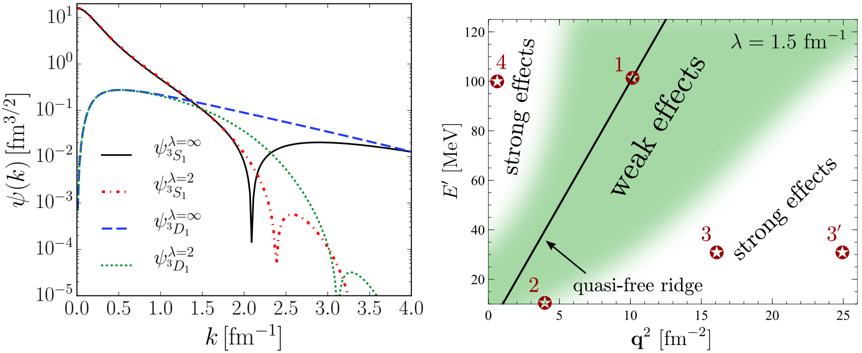


After $p_f' \approx$ small

Cross section from low momentum!

Scale dependence of D-state contribution

- Is *sensitivity* to the deuteron *D*-state probability scale-independent?



Scale dependence of D-state contribution

- Is *sensitivity* to the deuteron *D*-state probability scale-independent?
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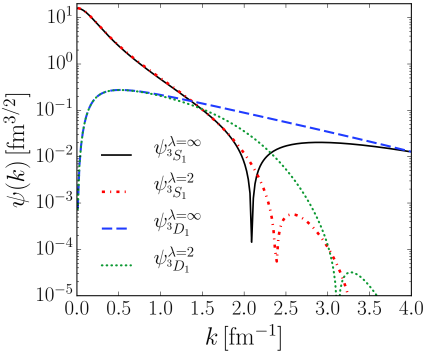

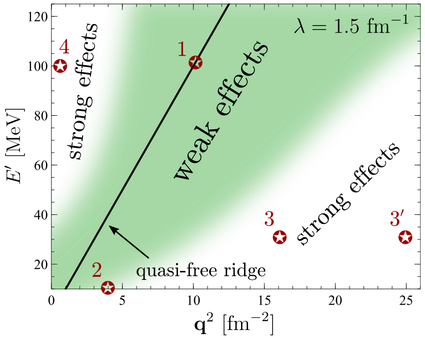

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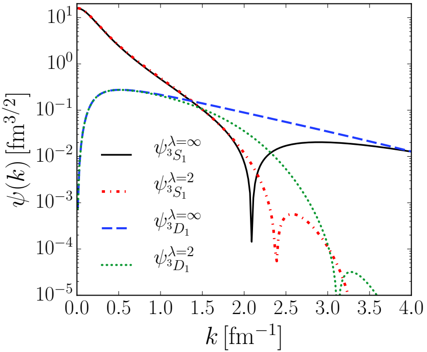

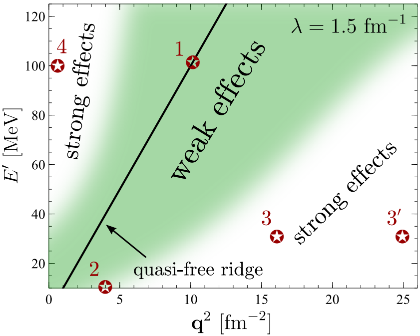

- Consider $E' = 20\,\text{MeV}$ with $q^2 = 36\,\text{fm}^{-2}$

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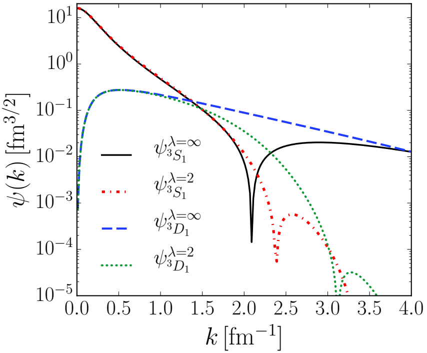

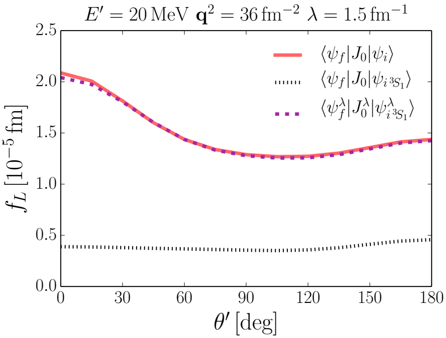

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- Unevolved contribution to f_L mostly *D*-state but all *S*-state for evolved

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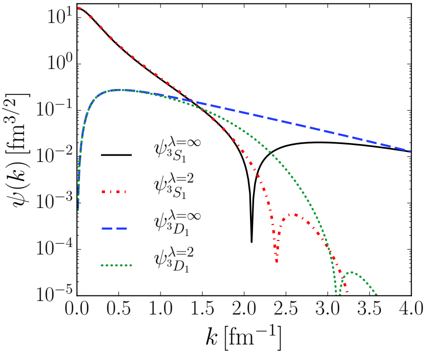
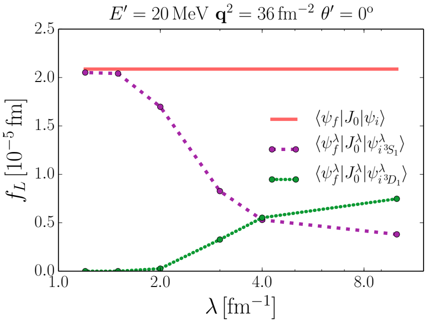




- Consider $E' = 20\,\mathrm{MeV}$ with $q^2 = 36\,\mathrm{fm}^{-2}$
- Unevolved contribution to f_L mostly *D*-state but all *S*-state for evolved
- λ evolution shows switch from *D*-channel to *S*-channel

Outline

Overview: scale and scheme dependence

Test case: deuteron electrodisintegration

Summary and take-aways

Many back-up slides

Take-away points from "toy" model study

- Scale dependence appears in many places, but systematic
- Case studies show:
  - Decoupling of the final state by RG evolution leads to decreased contribution from FSI $\Longrightarrow$ increased validity of IA
  - RG generated scale separation makes low-resolution potentials well suited for (high-q) reaction calculations (OPE!)
  - Intuitive picture of reaction can change qualitatively
  - Sensitivity to specific parts of nuclear wave function can be highly scale dependent
  - Explanation of factorization straightforward in low-momentum picture [in back-up slides]
- While extreme kinematics here, non-negligible effects expected for more ordinary kinematics
- Next steps: initial 2-body current, f_T, extend to $A > 2$

How should one choose a scale and/or scheme?

- To make calculations easier or more convergent
  - QCD running coupling and scale: improved perturbation theory; choosing a gauge: e.g., Coulomb or Lorenz
  - Low-*k* potential: improve many-body convergence, or to make microscopic connection to shell model or dft
  - (Near-) local potential: quantum Monte Carlo methods work
- Better interpretation or intuition $\Longrightarrow$ predictability
  - SRC phenomenology for high-*q* electron scattering?
- Allowing for cleanest extraction from experiment
  - Can one "optimize" validity of impulse approximation?
  - In inclusive high-*E* QCD, use Q^2 of experiment
  - Ideally extract at one scale, evolve to others using RG
- Scale and scheme for nuclear reactions?

Outline

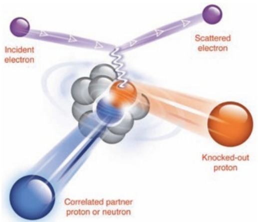
Overview: scale and scheme dependence

Test case: deuteron electrodisintegration

Summary and take-aways

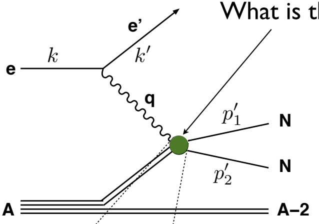
Many back-up slides

Large Q^2 scattering at different RG decoupling scales



Subedi et al., Science 320, 1476 (2008)

What is this vertex?



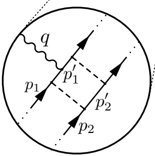
$$q = k - k'$$

$$\nu = E_k - E_{k'}$$

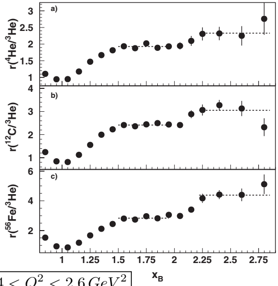
$$Q^2 = -q^2$$

$$x_B = \frac{Q^2}{2m_N\nu}$$

Higinbotham, arXiv:1010.4433



SRC interpretation:
NN interaction can scatter states with $p_1, p_2 \lesssim k_F$ to intermediate states with $p_1', p_2' \gg k_F$ which are knocked out by the photon

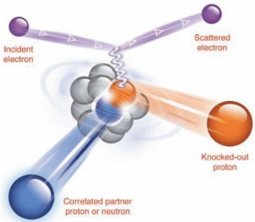


$1.4 < Q^2 < 2.6\, GeV^2$

Egiyan et al. PRL 96, 1082501 (2006)

SRC explanation relies on high-momentum nucleons in structure

Large Q^2 scattering at different RG decoupling scales



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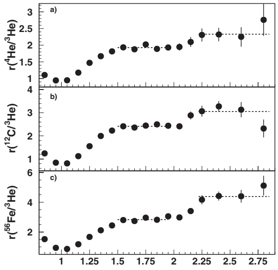

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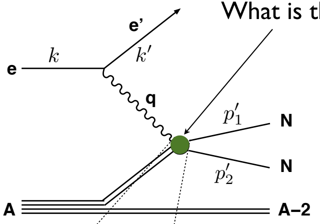

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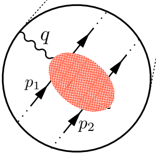

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How to explain cross sections in terms of low-momentum interactions?

Vertex depends on the resolution!

RG evolution changes physics *interpretation* but not cross section!

U-factorization with SRG [Anderson et al., arXiv:1008.1569]

- Factorization: $U_\lambda(k,q) \to K_\lambda(k) Q_\lambda(q)$ when $k < \lambda$ and $q \gg \lambda$
- Operator product expansion for nonrelativistic wf's (see Lepage)

$$\Psi_\alpha^\infty(q) \approx \gamma^\lambda(q) \int_0^\lambda p^2 dp\, Z(\lambda) \Psi_\alpha^\lambda(p) + \eta^\lambda(q) \int_0^\lambda p^2 dp\, p^2\, Z(\lambda)\, \Psi_\alpha^\lambda(p) + \cdots$$

- Construct unitary transformation to get $U_\lambda(k,q) \approx K_\lambda(k) Q_\lambda(q)$

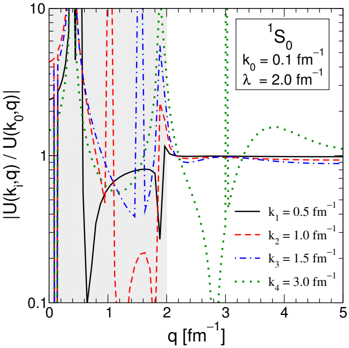
$$U_\lambda(k,q) = \sum_\alpha \langle k|\psi_\alpha^\lambda\rangle \langle \psi_\alpha^\infty|q\rangle \to \Big[\sum_\alpha^{\alpha_{low}} \langle k|\psi_\alpha^\lambda\rangle \int_0^\lambda p^2 dp\, Z(\lambda) \Psi_\alpha^\lambda(p)\Big] \gamma^\lambda(q) + \cdots$$

- Test of factorization of U:

$$\frac{U_\lambda(k_i,q)}{U_\lambda(k_0,q)} \to \frac{K_\lambda(k_i) Q_\lambda(q)}{K_\lambda(k_0) Q_\lambda(q)},$$

so for $q \gg \lambda \Rightarrow \dfrac{K_\lambda(k_i)}{K_\lambda(k_0)} \xrightarrow{\text{LO}} 1$

- Look for plateaus: $k_i \lesssim 2\,\text{fm}^{-1} \lesssim q$ $\Longrightarrow$ it works!
- Leading order $\Longrightarrow$ contact term!



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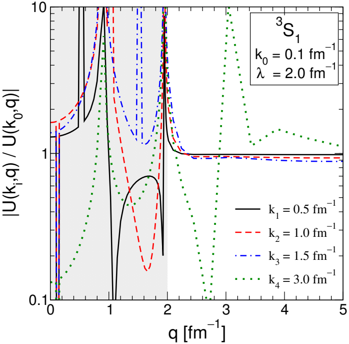

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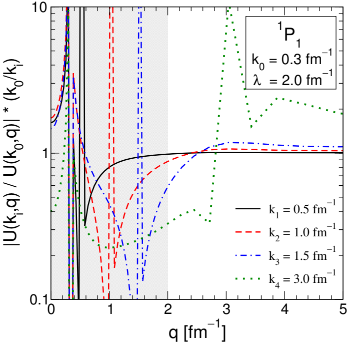
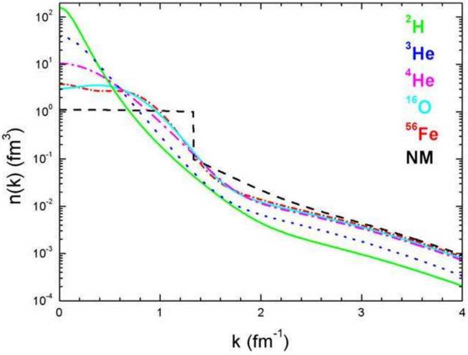


Nuclear scaling from RG factorization (schematic!)

- RG unitary transformation with scale separation: $\widehat{U} \to U_\lambda(k, q)$
- Factorization: when $k < \lambda$ and $q \gg \lambda$, $U_\lambda(k,q) \to K_\lambda(k) Q_\lambda(q)$

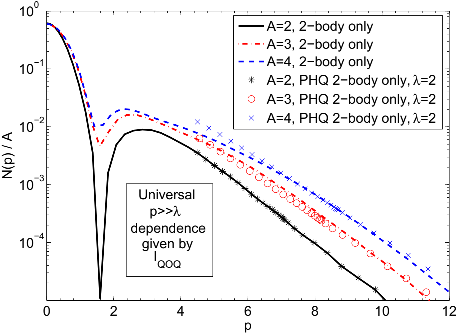
$$\frac{n_A(q)}{n_d(q)} = \frac{\langle A | a^\dagger_{\mathbf{q}} a_{\mathbf{q}} | A \rangle}{\langle d | a^\dagger_{\mathbf{q}} a_{\mathbf{q}} | d \rangle} \underset{\widehat{U}^\dagger \widehat{U} = 1}{\overset{\text{RG}}{\Longrightarrow}} \quad \widehat{U}|d\rangle \to |\widetilde{d}\rangle \,, \; \widehat{U}|A\rangle \to |\widetilde{A}\rangle \,, \; \widehat{U} a^\dagger_{\mathbf{q}} a_{\mathbf{q}} \widehat{U}^\dagger$$

$\Longrightarrow n_A(q) \approx C_A n_D(q)$ at large q



[From C. Ciofi degli Atti and S. Simula]

Test case: A bosons in toy 1D model



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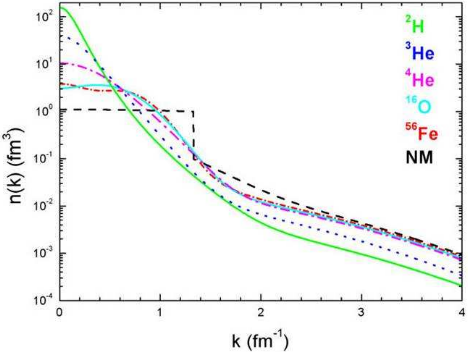

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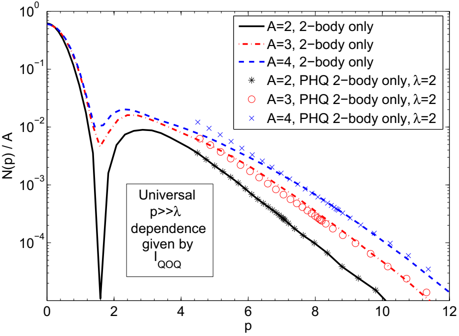

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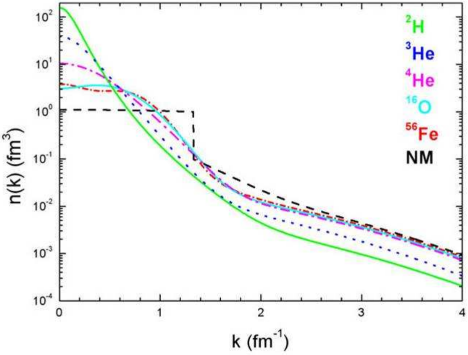

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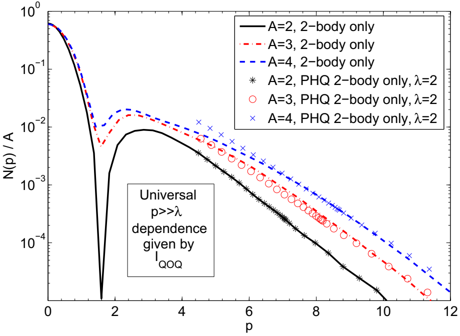

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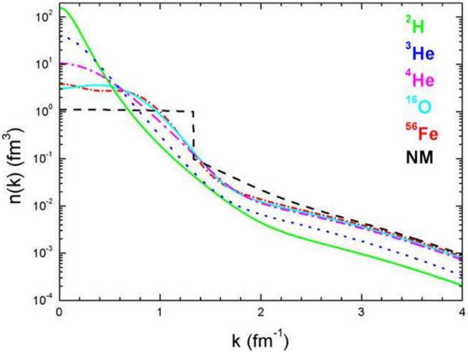

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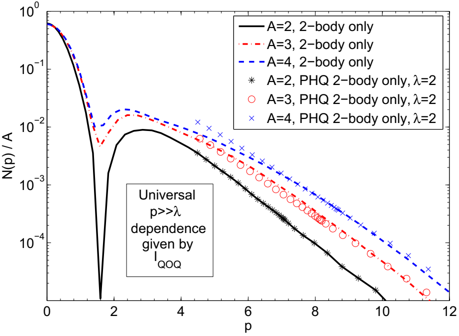

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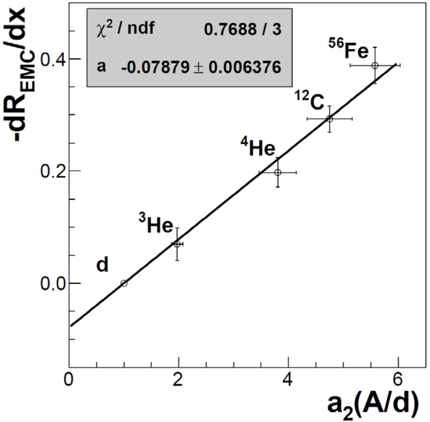
Scaling and EMC correlation via low resolution

- SRG factorization, e.g., $U_\lambda(k,q) \to K_\lambda(k) Q_\lambda(q)$ when $k < \lambda$ and $q \gg \lambda$
  - Dependence on high-q independent of A $\Longrightarrow$ universal [cf. Neff et al.]
  - A dependence from low-momentum matrix elements $\Longrightarrow$ calculate!
- EMC from EFT using OPE:
  - Isolate A dependence, which factorizes from x
  - EMC A dependence from long-distance matrix elements



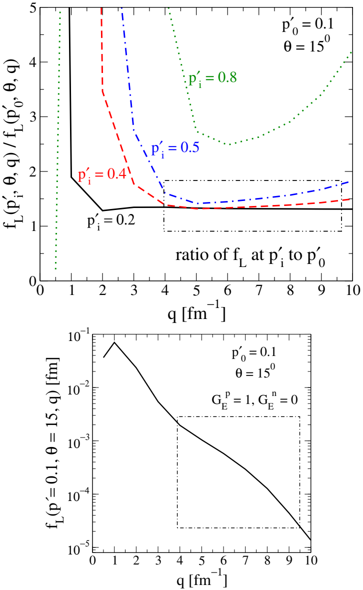
L.B. Weinstein, et al., Phys. Rev. Lett. 106, 052301 (2011)

If the same leading operators dominate, then does linear A dependence of ratios follow immediately?

Need to do quantitative calculations to explore!

q-factorization of f_L

- $f_L \equiv f_L(p', \theta; q)$
  p' and θ: outgoing nucleon
  q: momentum transfer
- For $p' \ll q$, f_L scales with q
  $f_L(p', \theta; q) \to g(p', \theta) B(q)$
- Note that f_L is a strong function of q



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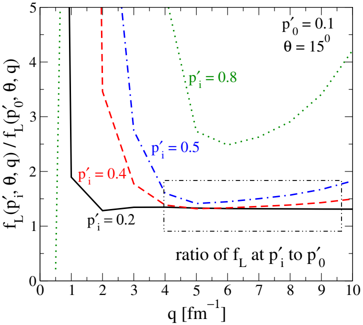
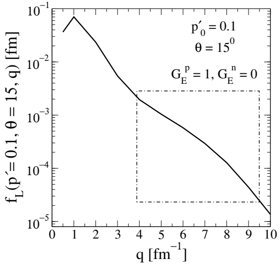
- Note that f_L is a strong function of q
- Follows from the LO term in EFT expansion:
  $\langle \psi_f^\lambda | J_0^\lambda(q) | \psi_{\rm deut}^\lambda \rangle \approx$
  $g_0^q \ \psi_f^{\lambda *}(p'; r)\, \psi_{\rm deut}^\lambda(r) \Big|_{r=0}$





Uses of the renormalization group (RG) [cf. S. Weinberg (1981)]

- Improving perturbation theory; e.g., in QCD calculations
  - Mismatch of energy scales can generate large logarithms
  - Shift between couplings and loop integrals to reduce logs
- Identifying universality in critical phenomena
  - Filter out short-distance degrees of freedom
- Simplifying calculations of nuclear structure/reactions
  - Make nuclear physics look more like quantum chemistry!

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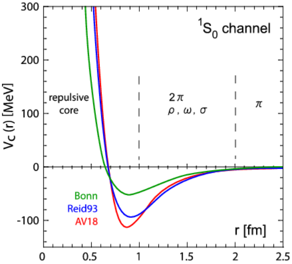
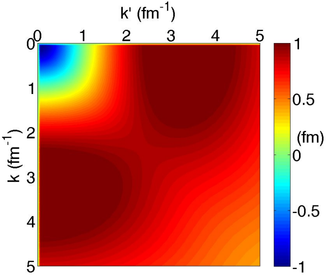


AV18, Bonn, Reid93



$\langle k | V_{\mathrm{AV18}} | k' \rangle$

Coupling of low-k/high-k modes: non-perturbative, strong correlations, . . .

Remedy: Use RG to decouple modes $\Longrightarrow$ low resolution

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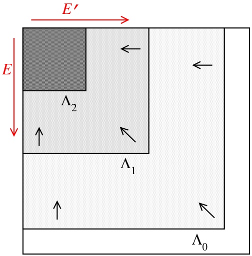
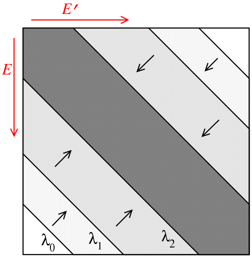


"$V_{\text{low }k}$"



Similarity RG

- $V_{\text{low }k}$: lower cutoff Λ_i in k, k' via $dT(k, k'; k^2)/d\Lambda = 0$
- SRG: drive H toward diagonal with flow equation

$$dH_s/ds = [[G_s, H_s], H_s]$$

Continuous unitary transforms (cf. running couplings)

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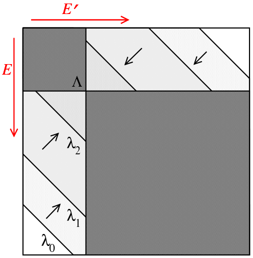
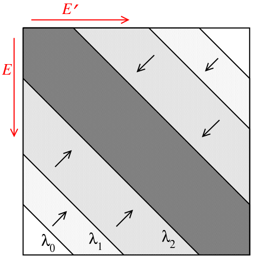


Block diagonal SRG



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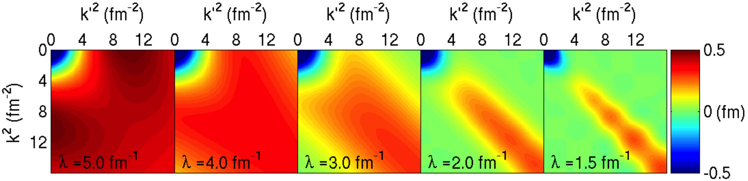
AV18:



- Decoupling naturally visualized in momentum space for $G_s = T$
  - Phase-shift equivalent! Width of diagonal given by $\lambda^2 = 1/\sqrt{s}$
  - What does this look like in coordinate space?

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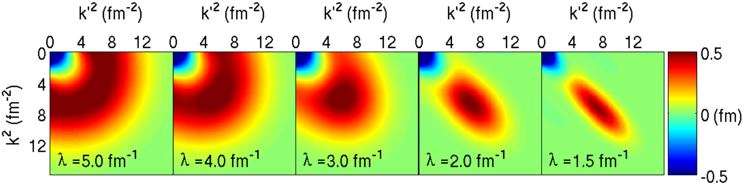
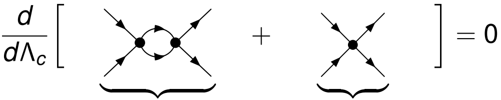
N^3LO:
(500 MeV)



- Decoupling naturally visualized in momentum space for $G_s = T$
  - Phase-shift equivalent! Width of diagonal given by $\lambda^2 = 1/\sqrt{s}$
  - What does this look like in coordinate space?

Compare changing a cutoff in an EFT to RG decoupling

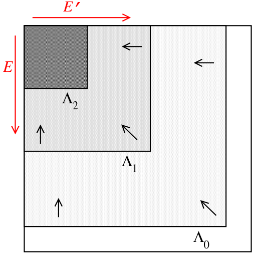
- (Local) field theory version in perturbation theory (diagrams)
  - Loops (sums over intermediate states) $\overset{\Delta\Lambda_c}{\Longleftrightarrow}$ LECs



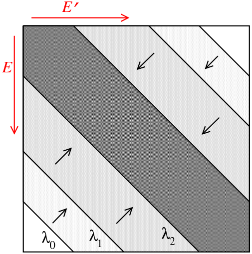
$$\int^{\Lambda_c} \frac{d^3q}{(2\pi)^3} \frac{C_0 M C_0}{k^2 - q^2 + i\epsilon} \qquad C_0(\Lambda_c) \propto \frac{\Lambda_c}{2\pi^2} + \cdots$$

  - Momentum-dependent vertices $\Longrightarrow$ Taylor expansion in k^2
  - This implements an operator product expansion!
- Claim: $V_{\text{low }k}$ RG and SRG decoupling work analogously

“$V_{\text{low }k}$”

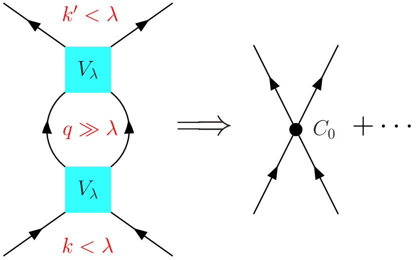


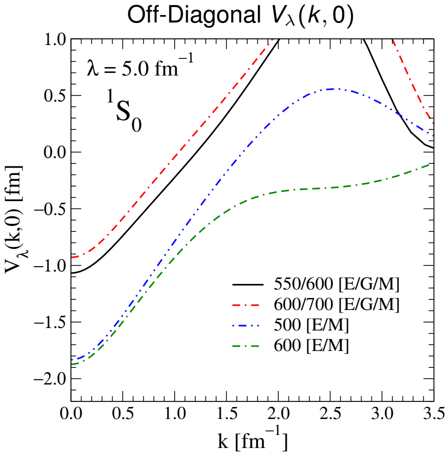
SRG (“T” generator)



Approach to universality (fate of high-q physics!)

Run NN to lower λ via SRG $\Longrightarrow$ $\approx$Universal low-k V_{NN}





$q \gg \lambda$ (or Λ) intermediate states
$\Longrightarrow$ change is $\approx$ contact terms:

$$C_0 \delta^3(\mathbf{x} - \mathbf{x}') + \cdots$$

[cf. $\mathcal{L}_{\rm eft} = \cdots + \frac{1}{2} C_0 (\psi^\dagger \psi)^2 + \cdots$]

- Similar pattern with phenomenological potentials (e.g., AV18)

Factorization: $\Delta V_\lambda(k, k') = \int U_\lambda(k, q) V_\lambda(q, q') U^\dagger_\lambda(q', k')$ for $k, k' < \lambda$, $q, q' \gg \lambda$

$$\stackrel{U_\lambda \to K \cdot Q}{\longrightarrow} K(k)[\int Q(q) V_\lambda(q, q') Q(q')] K(k')$$ with $K(k) \approx 1$!

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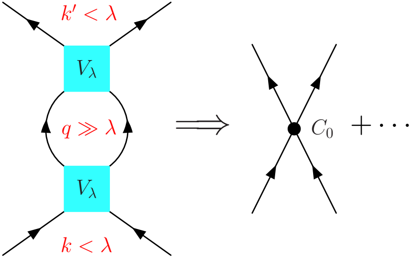

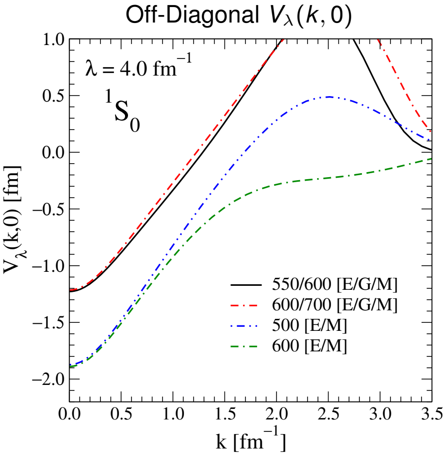

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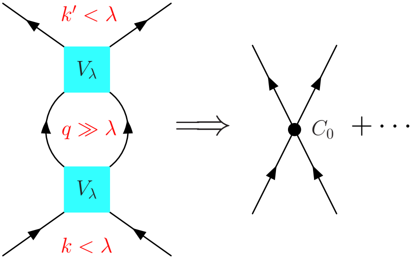

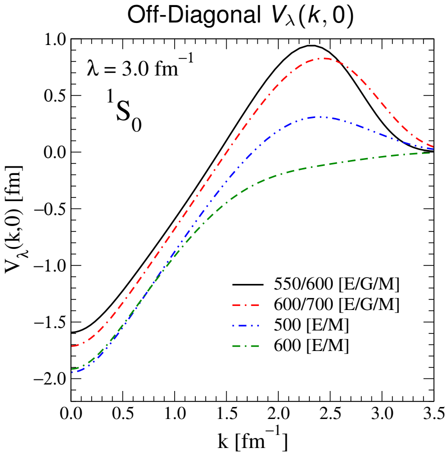

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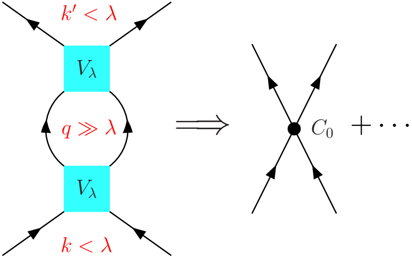

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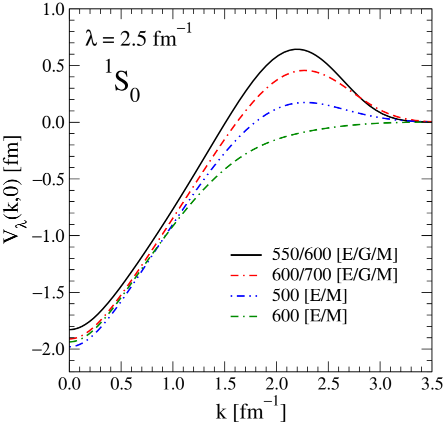
Off-Diagonal $V_\lambda(k,0)$



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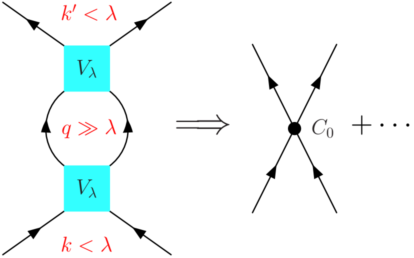

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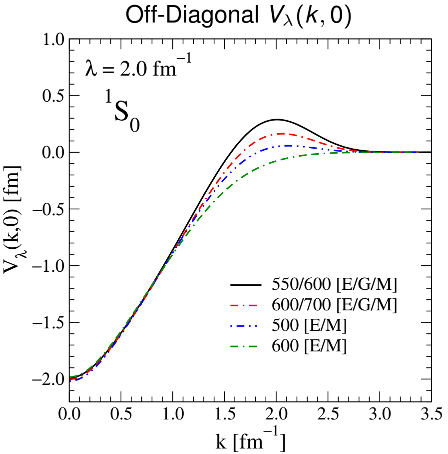

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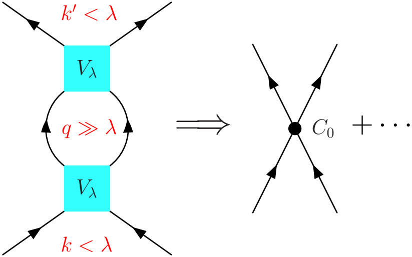

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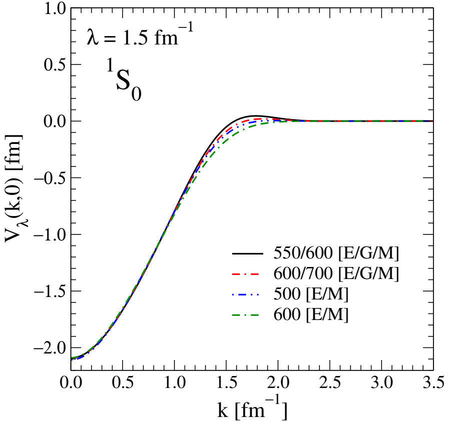

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EMC effect from the EFT perspective

- Exploit scale separation between short- and long-distance physics
  - *Match* complete set of operator matrix elements (power count!)
  - Cf. needing a *model* of short-distance nucleon dynamics
  - Distinguish long-distance nuclear from nucleon physics
- EMC and effective field theory (examples)
  - "DVCS-dissociation of the deuteron and the EMC effect"
    [S.R. Beane and M.J. Savage, Nucl. Phys. A 761, 259 (2005)]

    "By constructing all the operators required to reproduce the matrix elements of the twist-2 operators in multi-nucleon systems, one sees that operators involving more than one nucleon are not forbidden by the symmetries of the strong interaction, and therefore must be present. While observation of the EMC effect twenty years ago may have been surprising to some, in fact, its absence would have been far more surprising."
  - "Universality of the EMC Effect"
    [J.-W. Chen and W. Detmold, Phys. Lett. B 625, 165 (2005)]
  - "SRCs and the EMC Effect in EFT" [Chen et al., arXiv:1607.03065]

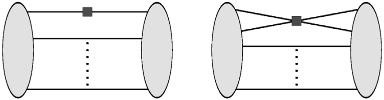
A dependence of the EMC effect is long-distance physics!

- EFT treatment by Chen and Detmold [Phys. Lett. B 625, 165 (2005)]

$$F_2^A(x) = \sum_i Q_i^2 \, x q_i^A(x) \qquad \Longrightarrow \qquad R_A(x) = F_2^A(x)/AF_2^N(x)$$

"The x dependence of $R_A(x)$ is governed by short-distance physics, while the overall magnitude (the A dependence) of the EMC effect is governed by long distance matrix elements calculable using traditional nuclear physics."

- Match matrix elements: leading-order nucleon operators to isoscalar twist-two quark operators



$$\Longrightarrow \langle x^2 \rangle_q v^{\mu_0} \cdots v^{\mu_n} N^\dagger N [1 + \alpha_n N^\dagger N] + \cdots$$

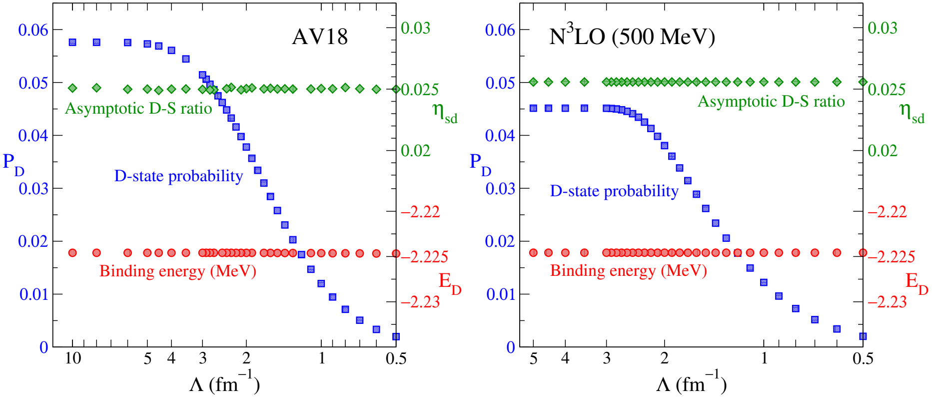
$$R_A(x) = \frac{F_2^A(x)}{AF_2^N(x)} = 1 + g_{F_2}(x)\mathcal{G}(A) \quad \text{where} \quad \mathcal{G}(A) = \langle A|(N^\dagger N)^2|A\rangle / A\Lambda_0$$

$\Longrightarrow$ the slope $\frac{dR_A}{dx}$ scales with $\mathcal{G}(A)$ [Why is this not cited more?]

Partial list of ‘non-observables’ references

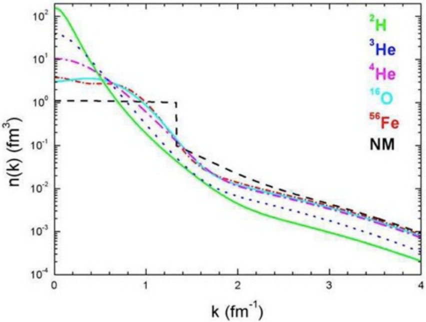
- *Equivalent Hamiltonians in scattering theory*, H. Ekstein, (1960)
- *Measurability of the deuteron D state probability*, J.L. Friar, (1979)
- *Problems in determining nuclear bound state wave functions*, R.D. Amado, (1979)
- *Nucleon nucleon bremsstrahlung: An example of the impossibility of measuring off-shell amplitudes*, H.W. Fearing, (1998)
- *Are occupation numbers observable?*, rjf and H.-W. Hammer, (2002)
- *Unitary correlation in nuclear reaction theory: Separation of nuclear reactions and spectroscopic factors*, A.M. Mukhamedzhanov and A.S. Kadyrov, (2010)
- *Non-observability of spectroscopic factors*, B.K. Jennings, (2011)
- *How should one formulate, extract, and interpret ‘non-observables’ for nuclei?*, rjf and A. Schwenk, (2010) [in J. Phys. G focus issue on Open Problems in Nuclear Structure Theory, edited by J. Dobaczewski]

Deuteron true and scheme-dependent observables

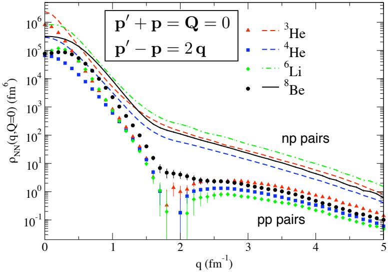


- Unitary transformations labeled by Λ ($V_{\mathrm{low}\,k}$ here)
  $\Longrightarrow$ soften interactions by lowering resolution (how far?)
  $\Longrightarrow$ reduced short-range and tensor correlations
- D-state probability changes (cf. spectroscopic factors)
- Asymptotic D-S ratio is unchanged (cf. ANC's)

Momentum distributions in nuclei



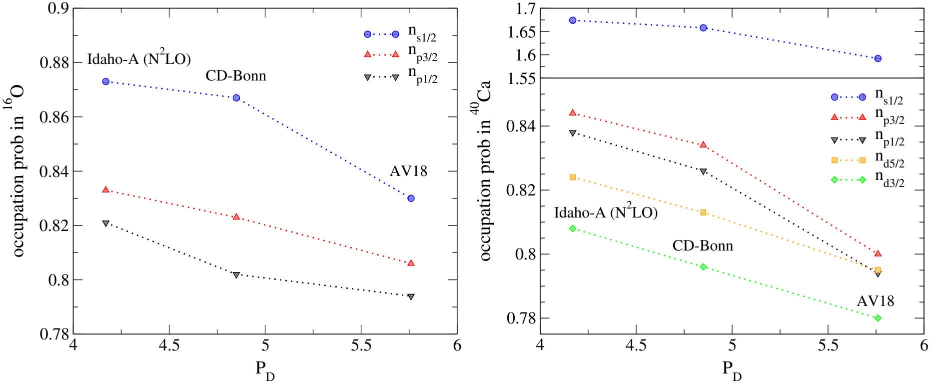
taken from Ciofi degli Atti, Simula PRC 53, 1689 (1996)



Schiavilla et al. PRL 98, 132501 (2007)

Correlation of P_D with spectroscopic factors

Calculations from Gad and Muether, Phys. Rev. C **66**, 044361 (2002)



- Increased occupation probability with increased non-locality and correlated reduction in short-range tensor strength
- Is the correlation quantitatively predictable?

Cutoff dependence in coupled cluster calculations

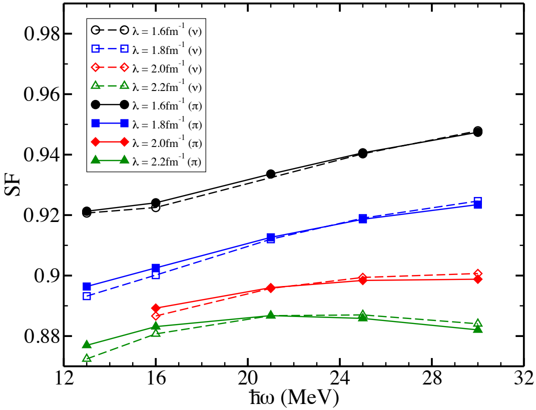


FIG. 4: (Color online) Spectroscopic factor SF$(1/2^-)$ for neutron and proton removal as a function of the oscillator spacing $\hbar\omega$ for nucleon-nucleon interactions with different cutoffs in a model space with $N = 6$.

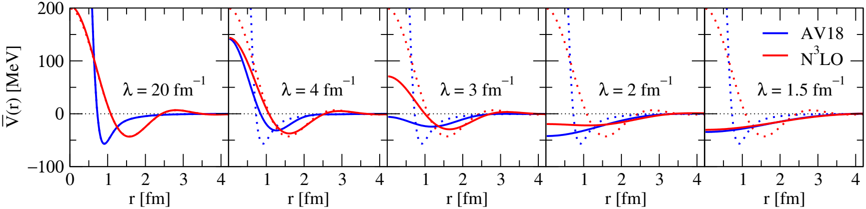
[From Ø. Jensen et al., PRC **82**, 014310 (2010)]

- SF increases as SRG resolution λ decreases from 2.2 to 1.6 fm^{-1}

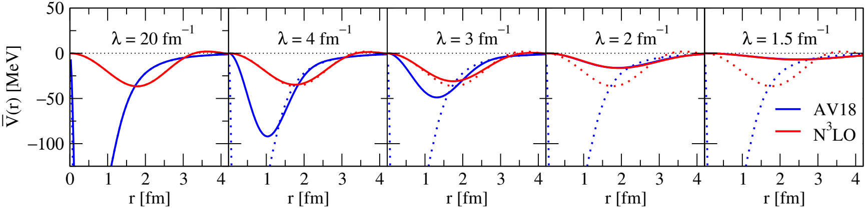
Wave functions are more single-particle-like as Λ/λ decreases, but do reaction operators become significantly less one-body?

Changing the scheme: (short-range) NN potential

- $V_{\text{low }k}$ or SRG unitary transformations to soften interactions
- Project non-local NN potential: $\overline{V}_\lambda(r) = \int d^3r'\, V_\lambda(r,r')$
  - Roughly gives action of potential on long-wavelength nucleons
- Central part (S-wave) [Note: The V_λ's are all phase equivalent!]

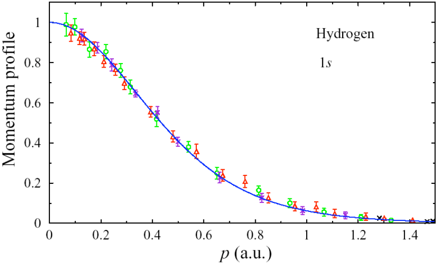


- Tensor part (S-D mixing) [graphs from K. Wendt]



Are wave functions measurable? [from W. Dickhoff]

Atoms studied with the (e,2e) reaction

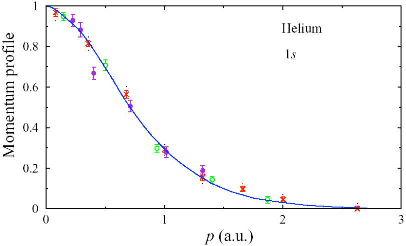


$$\varphi_{1s}(p) = 2^{3/2}\pi \frac{1}{(1+p^2)^2}$$

Hydrogen 1s wave function
"seen" experimentally
Phys. Lett. 86A, 139 (1981)

And so on for other atoms …

Helium
in Phys. Rev. A8, 2494 (1973)



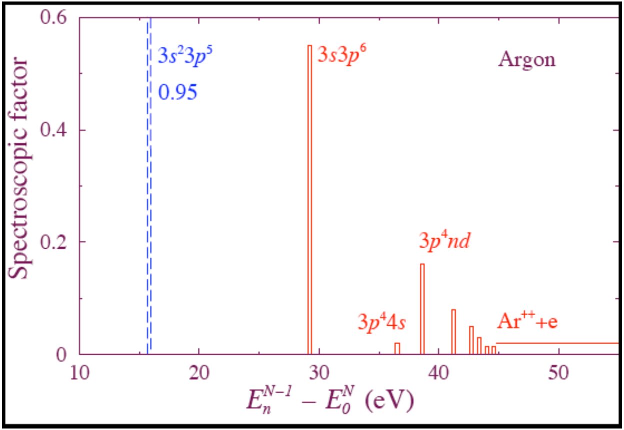
- But compare approximations for $(e, 2e)$ on atoms to those for $(e, e'p)$ on nuclei! (Impulse approx., FSI, vertex, …)

Spectroscopic factors in atoms

For a bound final *N*-1 state the spectroscopic factor is given by $S = \int d\vec{p} \left| \left\langle \Psi_n^{N-1} \middle| a_{\vec{p}} \middle| \Psi_0^N \right\rangle \right|^2$

For H and He the 1*s* electron spectroscopic factor is 1

For Ne the valence 2*p* electron has *S*=0.92 with two additional fragments, each carrying 0.04, at higher energy.



Argon
3*p* and 3*s* strength

Closed-shell atoms
$n(\alpha) = 0$ or 1

One-body scattering, small scheme dependence $\Longrightarrow$ robust SF

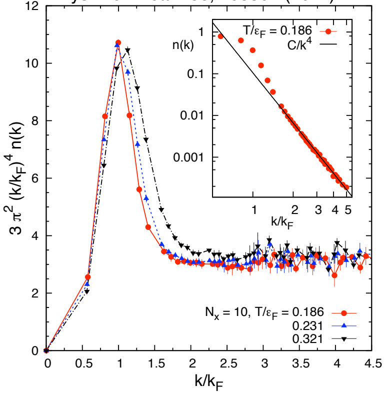
Unitary cold atoms: Is $n(k)$ observable?

- Tail of momentum distribution + contact [Tan; Braaten/Platter]

$$n(k) \stackrel{k\to\infty}{\longrightarrow} \frac{C}{k^4}$$

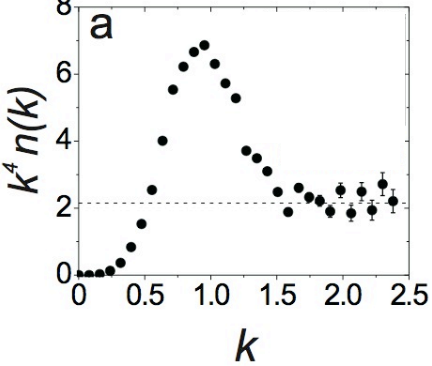
Theory (lattice)

J. E. Drut, T. A. Lähde, T. Ten
Phys. Rev. Lett. **106**, 205302 (2011)



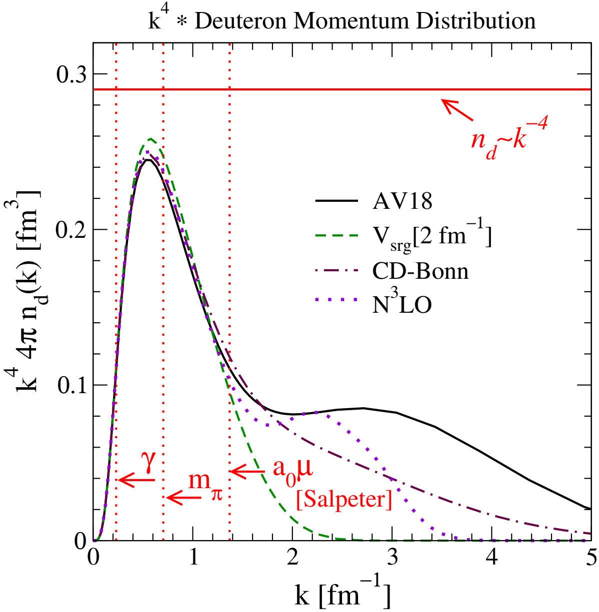
Experiment

J. T. Stewart et al
PRL **104**, 235301 (2010)



- When $R/a_s \ll kR \ll 1 \Longrightarrow$ tiny scheme dependence

Is the tail of $n(k)$ for nuclei measurable? (cf. SRC's)



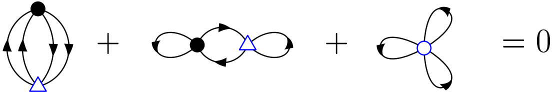
- E.g., extract from electron scattering?
- No region where $1/a_s \ll k \ll 1/R$
- Scheme dependent high-momentum tail!
- $n(k)$ from V_{SRG} has *no* high-momentum components!
- But $n(k)$ from $U a_k^\dagger a_k U^\dagger$ is unchanged $\Longrightarrow$ two-body operator!

Using EFT and field redefinitions as tool

- EFT: $\mathcal{L}_{\rm eft} = \psi^\dagger \left[i\frac{\partial}{\partial t} + \frac{\nabla^2}{2M} \right] \psi - \frac{C_0}{2}(\psi^\dagger\psi)^2 - \frac{D_0}{6}(\psi^\dagger\psi)^3 + \ldots$
  - general short-range interactions, but not unique!
- Try simple field redefinition to check scheme dependence:

$$\psi \longrightarrow \psi + \alpha \frac{4\pi}{\Lambda^3}(\psi^\dagger\psi)\psi \qquad \alpha \sim \mathcal{O}(1) \Longrightarrow \text{“natural”} \Longrightarrow \text{estimate!}$$

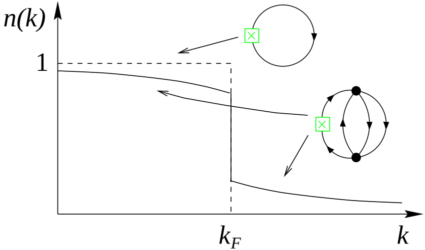
  - “new” vertices: 2–body off-shell $\triangle$, 3–body $\circ \propto \frac{8\pi\alpha}{\Lambda^3} C_0 (\psi^\dagger\psi)^3$
  - asymptotic “on-shell” quantities (S-matrix elements) must be unchanged by redefinition
- Energy density is model (α) independent *if* all terms kept
  - sum of new terms is zero, so energy is unchanged



- What about momentum occupation number?

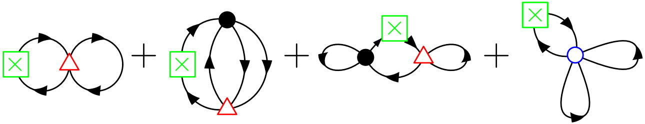
Occupation No. $\Longrightarrow$ Momentum Distribution

- Insert $a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \Longrightarrow$ ⊠



- But nonzero contribution $\Delta n(k)$ from induced vertices:

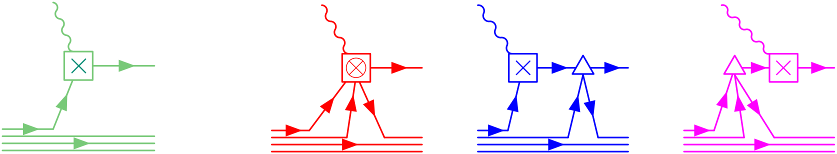
$$\Delta n(k) =$$



- There is no *preferred* definition for transformed operator
  - $\Longrightarrow$ only defined for specific convention
  - $\Longrightarrow$ momentum distributions for different schemes differ

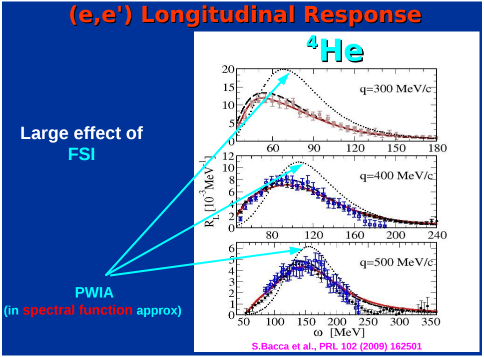
Analysis of (e,e'p) Experiments? [cf. (e,2e) on atoms]

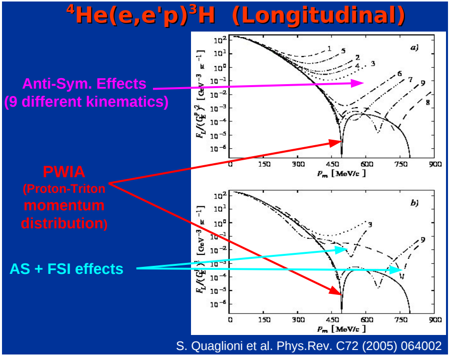
- Suppose external source $J(x)$ coupled to fermions
  - EFT: need most general current coupled to $J(x)$ for all α
- Consider lowest order with simplest ($\alpha = 0$) current
  - if $\alpha = 0$, just impulse approximation $J\psi^\dagger\psi$



  - if $\alpha \neq 0$ [recall $\psi \longrightarrow \psi + \alpha\frac{4\pi}{\Lambda^3}(\psi^\dagger\psi)\psi$], then same cross section *only* if vertex contribution from modified operator *and* modified final (and initial) state interactions are included
- There are *always* contributions from all three at each order
  - sub-leading pieces are mixed by field redefinitions
    $\Longrightarrow$ isolating $J\psi^\dagger\psi$ is model dependent
  - How large is ambiguity? Set by natural size $\alpha \sim \mathcal{O}(1)$

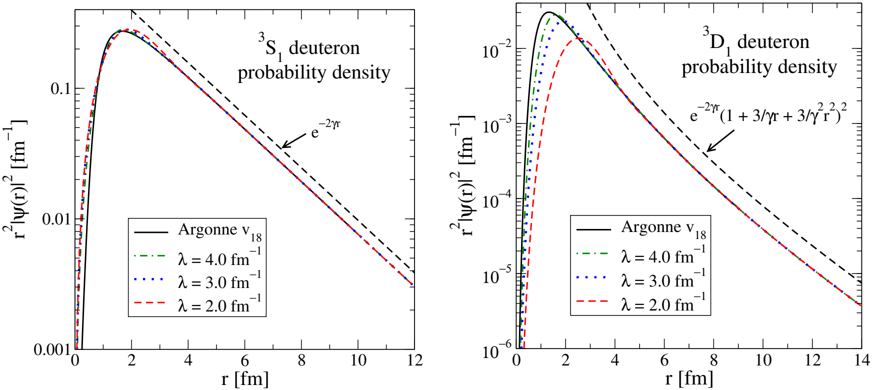
Ab initio electron scattering with LIT [from G. Orlandini]





- Ab initio calculations of longitudinal (e, e') response functions show importance of FSI for quasi-elastic regime
  - PWIA fails for quasi-elastic peak and at low ω
  - FSI effects decrease with q in peak but not at low ω
- Direct proton knockout and neglect of FSI tested for $(e, e'p)$
  - Both antisymmetrization effects and FSI play important roles
  - Approximate estimates of FSI effects can be poor

Why are ANC's different? Coordinate space



- ANC's, like phase shifts, are asymptotic properties
  $\Longrightarrow$ short-range unitary transformations do not alter them
  [e.g., see Mukhamedzhanov/Kadyrov, PRC **82** (2010)]
- In contrast, SF's rely on *interior* wave function overlap
- (Note difference in S-wave and D-wave ambiguities)

Why are ANC's different? Momentum space

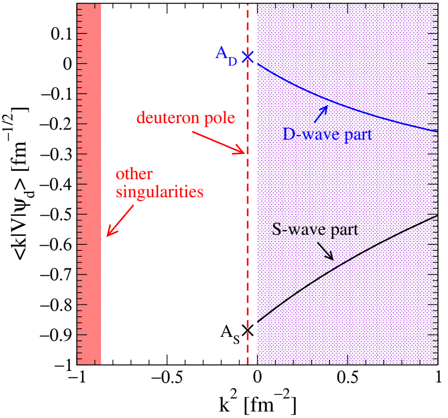
[based on R.D. Amado, PRC **19** (1979)]

1. $$\frac{k^2}{2\mu}\langle \mathbf{k}|\psi_n\rangle + \langle \mathbf{k}|V|\psi_n\rangle = -\frac{\gamma_n^2}{2\mu}\langle \mathbf{k}|\psi_n\rangle$$
$$\Longrightarrow \langle \mathbf{k}|\psi_n\rangle = -\frac{2\mu\langle \mathbf{k}|V|\psi_n\rangle}{k^2+\gamma_n^2}$$

2. $$\langle \mathbf{r}|\psi_n\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}}\langle \mathbf{k}|\psi_n\rangle \xrightarrow{|\mathbf{r}|\to\infty} A_n e^{-\gamma_n r}/r$$

3. integral dominated by pole from 1.

4. extrapolate $\langle \mathbf{k}|V|\psi_n\rangle$ to $k^2 = -\gamma_n^2$



- Or, residue from extrapolating on-shell T-matrix to deuteron pole $\Longrightarrow$ invariant under unitary transformations
- Inverse scattering puzzle: A_n uniquely determined because assumed longest-range part of V from one-pion exchange
- Next vertex singularity at $-(\gamma + m_\pi)^2 \Longrightarrow$ same for FSI

What about long-range correlations?

- SF calculations with FRPA
- N^3LO Hamiltonian
  - Soft $\Longrightarrow$ small SRC
  - SRC contribution changes dramatically with lower resolution
- Compare short-range correlations (SRC) to long-range correlations from particle-vibration coupling
- LRC $\gg$ SRC!!
- Are long-range correlations scheme dependent?

C. Barbieri, PRL 103 (2009)

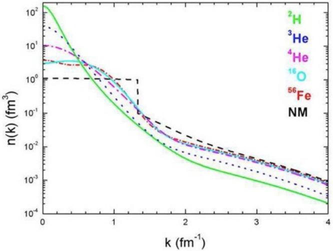
TABLE I. Spectroscopic factors (given as a fraction of the IPM) for valence orbits around ^{56}Ni. For the SC FRPA calculation in the large harmonic oscillator space, the values shown are obtained by including only SRC, SRC and LRC from particle-vibration couplings (full FRPA), and by SRC, particle-vibration couplings and extra correlations due to configuration mixing (FRPA + ΔZ_α). The last three columns give the results of SC FRPA and SM in the restricted $1p0f$ model space. The ΔZ_αs are the differences between the last two results and are taken as corrections for the SM correlations that are not already included in the FRPA formalism.

	10 osc. shells			Exp. [29]	$1p0f$ space		
	FRPA (SRC)	Full FRPA	FRPA $+\Delta Z_\alpha$		FRPA	SM	ΔZ_α
^{57}Ni:							
$\nu 1p_{1/2}$	0.96	0.63	0.61		0.79	0.77	-0.02
$\nu 0f_{5/2}$	0.95	0.59	0.55		0.79	0.75	-0.04
$\nu 1p_{3/2}$	0.95	0.65	0.62	0.58(11)	0.82	0.79	-0.03
^{55}Ni:							
$\nu 0f_{7/2}$	0.95	0.72	0.69		0.89	0.86	-0.03
^{57}Cu:							
$\pi 1p_{1/2}$	0.96	0.66	0.62		0.80	0.76	-0.04
$\pi 0f_{5/2}$	0.96	0.60	0.58		0.80	0.78	-0.02
$\pi 1p_{3/2}$	0.96	0.67	0.65		0.81	0.79	-0.02
^{55}Co:							
$\pi 0f_{7/2}$	0.95	0.73	0.71		0.89	0.87	-0.02

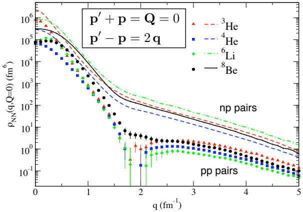
Parton distributions as paradigm: Factorization

- PDF analysis: part of convolution for cross section can be calculated reliably for given experimental conditions so that the remaining part can be extracted as a universal quantity, to be related to other processes and kinematic conditions
- For hard-scattering processes with large momentum transfer scale Q, *factorization* allows separation of momentum and distance scales in reaction
  - The time scale for binding interactions in the rest frame is time dilated in the center-of-mass frame, so the interaction of an electron with a hadron in deep-inelastic scattering is with single non-interacting partons
  - Short-distance part calculated systematically in low-order perturbative QCD; long-distance part identified in PDF's (momentum distribution for partons in hadrons)
- PDF's relate deep inelastic scattering of leptons, Drell-Yan, jet production, and more
  - Measure in limited set of reactions and then perturbative calculations of hard scattering and PDF evolution enable first principles predictions of cross sections for other processes

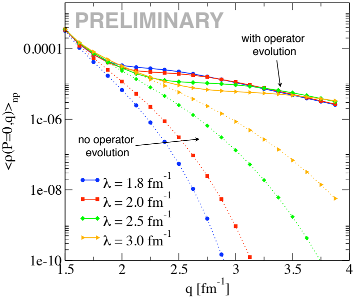
Simpler calculations of pair densities

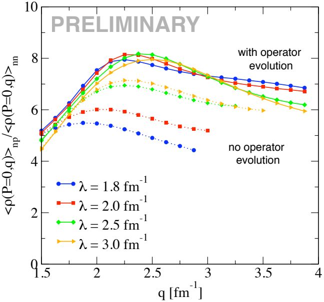


taken from Ciofi degli Atti, Simula PRC 53, 1689 (1996)



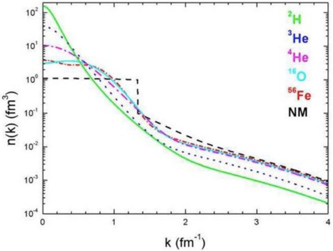
Schiavilla et al. PRL 98, 132501 (2007)





Many-body perturbation theory may be sufficient at low resolution!

Simpler calculations of pair densities



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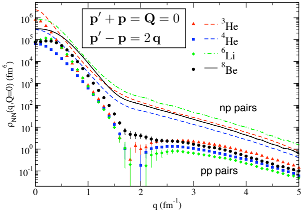

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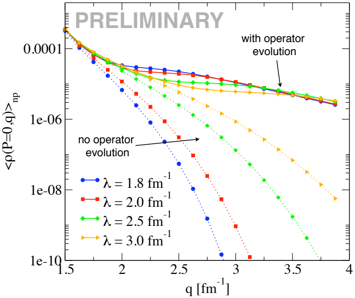

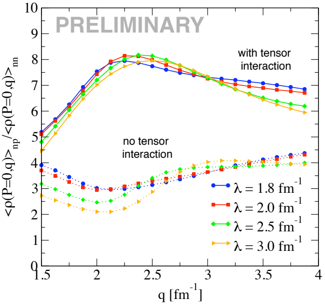

Many-body perturbation theory may be sufficient at low resolution!