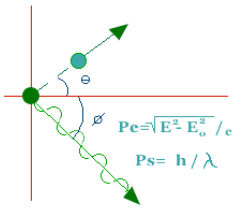


Latest developments and future applications of the dispersive optical model

INT 17-1a
3/16/2017



Wim Dickhoff
Bob Charity
Lee Sobotka

Hossein Mahzoon (Ph.D. 2015)
Mack Atkinson

Cole Pruitt
Shouwan Chen (Hefei U)
Natalya Calleya
Michael Keim

- Motivation
- Green's functions/propagator method
 - vehicle for ab initio calculations
 - as a framework to link data at positive and negative energy (and to generate predictions for exotic nuclei)
- > dispersive optical model (DOM <- Claude Mahaux)
- Recent DOM extension to non-local potentials
- Revisit the $(e,e'p)$ data from NIKHEF
- Neutron skin in ^{48}Ca (importance of total xsections)
- Ongoing and future applications
- Conclusions

Recent review:

WD, Bob Charity, Hossein Mahzoon
J. Phys. G: Nucl. Part. Phys. 44 (2017) 033001

Motivation

- Rare isotope physics requires a **much** stronger link between nuclear reactions and nuclear structure descriptions
- We need an ab initio approach for optical potential → optical potentials must therefore become **nonlocal** and **dispersive**
- Current status to extract structure information from nuclear reactions involving strongly interacting probes **unsatisfactory**
- Intermediate step: dispersive optical model as originally proposed by Claude Mahaux → some **extensions** discussed here

Optical potential \leftrightarrow nucleon self-energy

- e.g. Bell and Squires \rightarrow elastic T-matrix = reducible self-energy
- e.g. Mahaux and Sartor *Adv. Nucl. Phys.* **20**, 1 (1991)
 - relate dynamic (energy-dependent) real part to imaginary part
 - employ subtracted dispersion relation

General dispersion relation for self-energy:

$$\text{Re } \Sigma(E) = \Sigma^{HF} - \frac{1}{\pi} \mathcal{P} \int_{E_T^+}^{\infty} dE' \frac{\text{Im } \Sigma(E')}{E - E'} + \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{E_T^-} dE' \frac{\text{Im } \Sigma(E')}{E - E'}$$

Calculated at the Fermi energy $\varepsilon_F = \frac{1}{2} \{ (E_0^{A+1} - E_0^A) + (E_0^A - E_0^{A-1}) \}$

$$\text{Re } \Sigma(\varepsilon_F) = \Sigma^{HF} - \frac{1}{\pi} \mathcal{P} \int_{E_T^+}^{\infty} dE' \frac{\text{Im } \Sigma(E')}{\varepsilon_F - E'} + \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{E_T^-} dE' \frac{\text{Im } \Sigma(E')}{\varepsilon_F - E'}$$

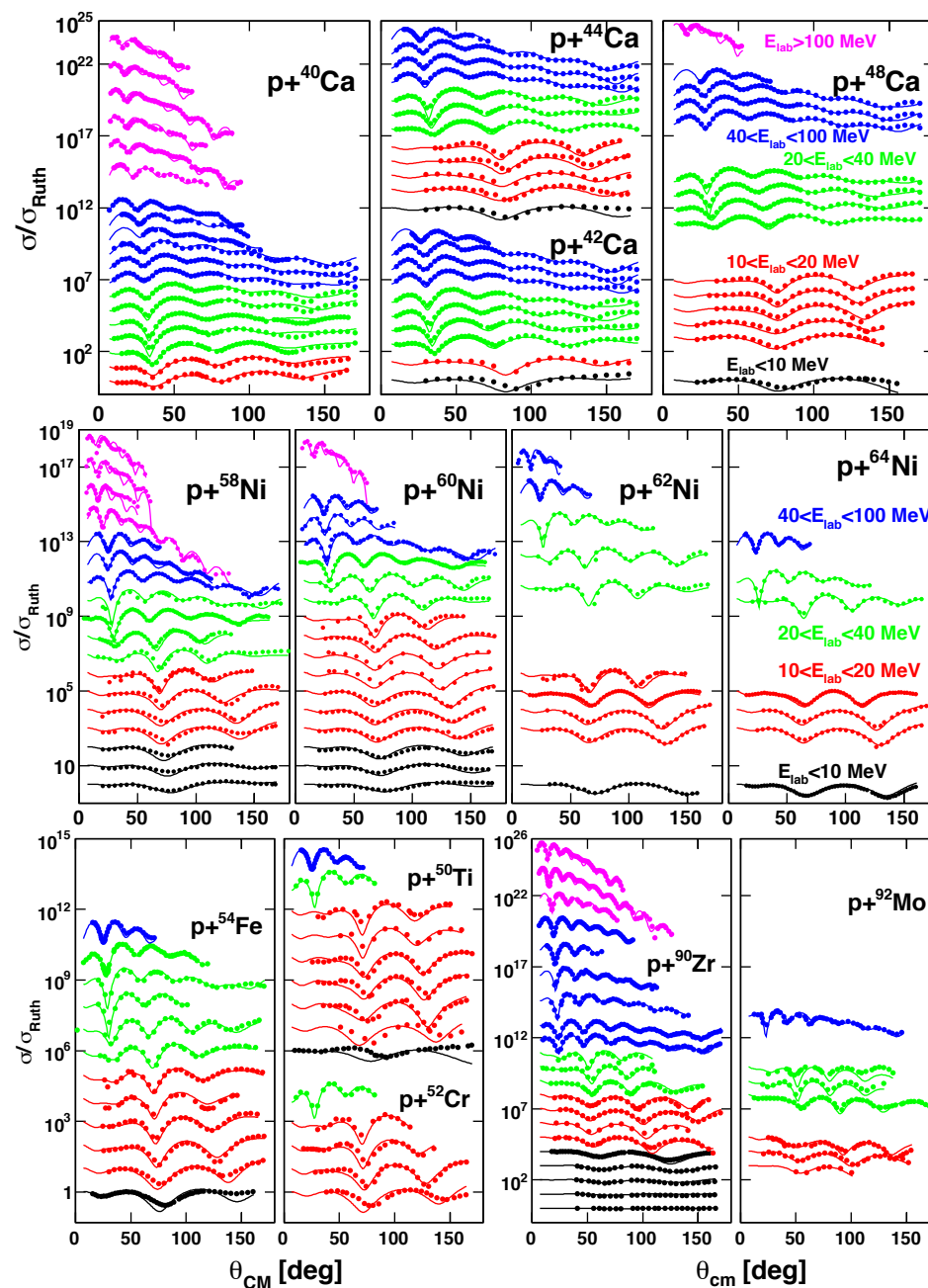
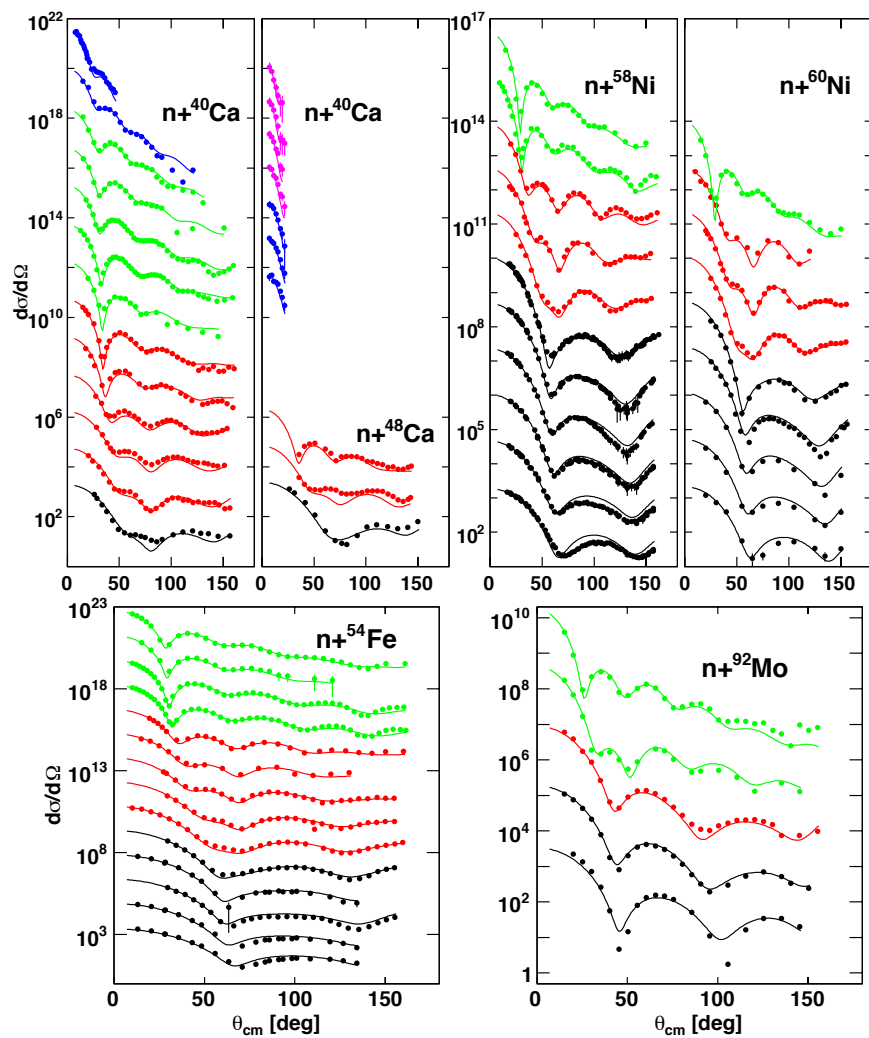
Subtract

$$\text{Re } \Sigma(E) = \text{Re } \widetilde{\Sigma}^{HF}(\varepsilon_F)$$

$$- \frac{1}{\pi} (\varepsilon_F - E) \mathcal{P} \int_{E_T^+}^{\infty} dE' \frac{\text{Im } \Sigma(E')}{(E - E')(\varepsilon_F - E')} + \frac{1}{\pi} (\varepsilon_F - E) \mathcal{P} \int_{-\infty}^{E_T^-} dE' \frac{\text{Im } \Sigma(E')}{(E - E')(\varepsilon_F - E')}$$

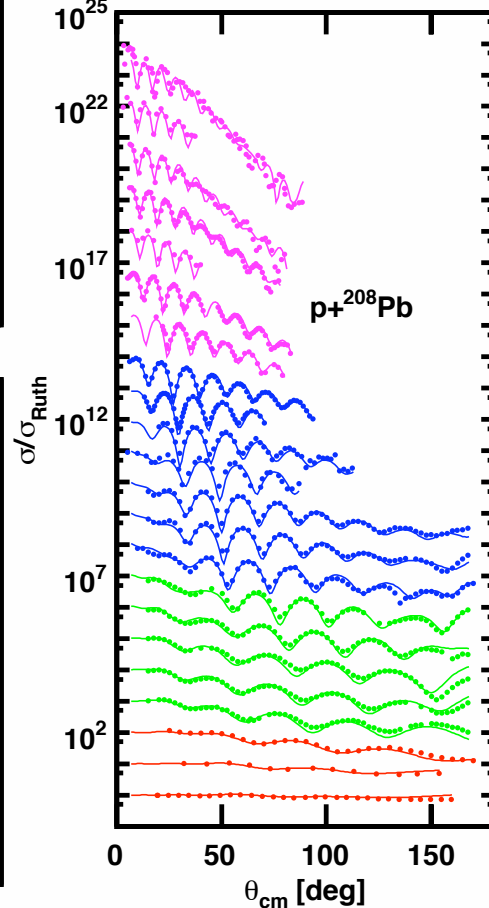
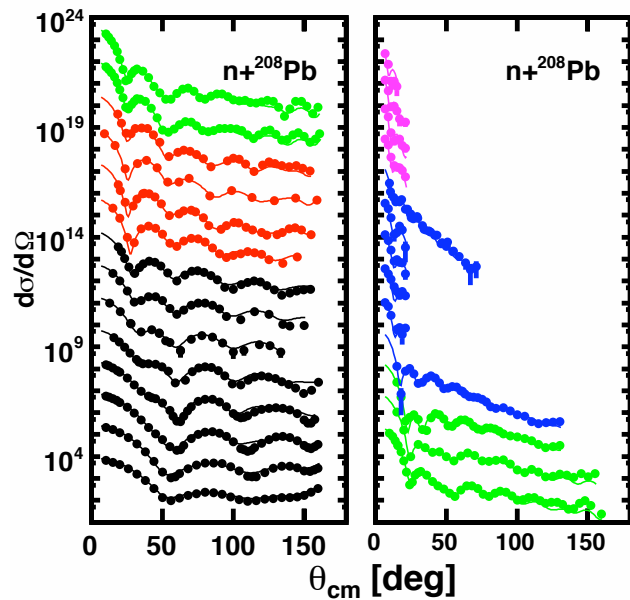
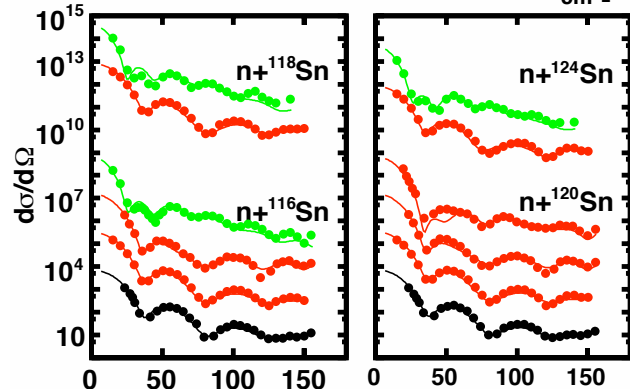
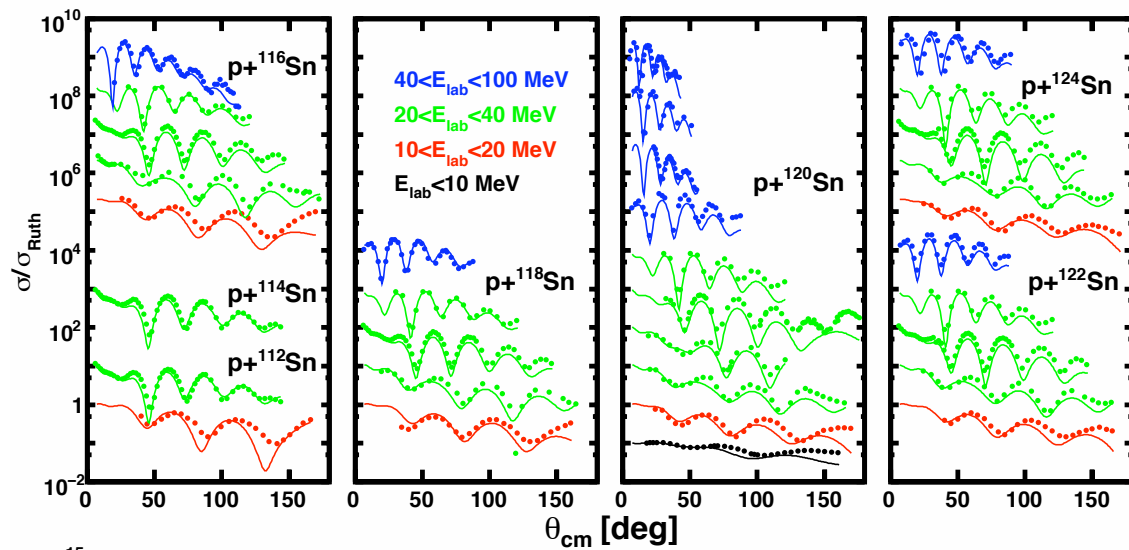
Elastic scattering data for protons and neutrons

- Local DOM implementation



J. Mueller et al.

PRC83,064605 (2011), 1-32



Local DOM analysis

J. Mueller et al.
 PRC83,064605 (2011), 1-32

$^{132}\text{Sn}(d,p)$

How does it work when the potentials are extrapolated?

- Ingredients from local DOM
 - Overlap function
 - p and n optical potential
- Reaction model ADWA (Ron Johnson)
- **MSU-WashU**:--> N. B. Nguyen, S. J. Waldecker, F. M. Nuñez, R. J. Charity, and W. H. Dickhoff
- $^{40,48}\text{Ca}, ^{132}\text{Sn}, ^{208}\text{Pb}(d,p)$

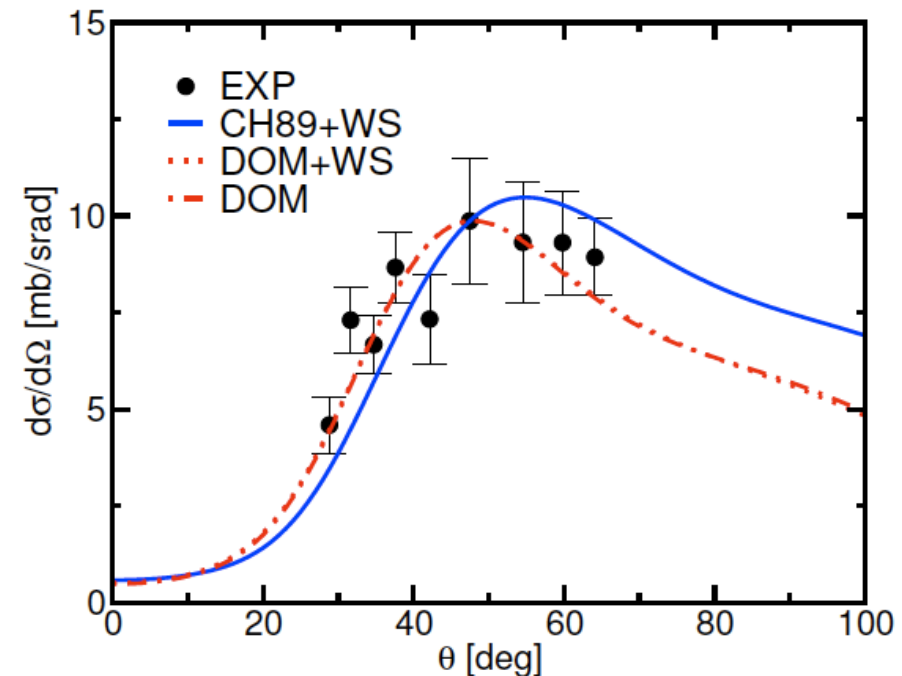
[Phys. Rev. C84, 044611 \(2011\), 1-9](#)

• Data: K.L. Jones et al., Nature 465, 454 (2010)

• $E_d = 9.46 \text{ MeV}$ $^{132}\text{Sn}(d,p)^{133}\text{Sn}$

- CH89+ws --> $S_{1f7/2} = 1.1$

- DOM --> $S_{1f7/2} = 0.72$



Nonlocal DOM implementation PRL112,162503(2014)

- Particle number --> **nonlocal** imaginary part
- Ab initio FRPA & SRC --> different nonlocal properties above and below the Fermi energy Phys. Rev. C84, 034616 (2011) & Phys. Rev.C84, 044319 (2011)
- **Include** charge density in fit
- Describe high-momentum nucleons <--> (e,e'p) data from JLab

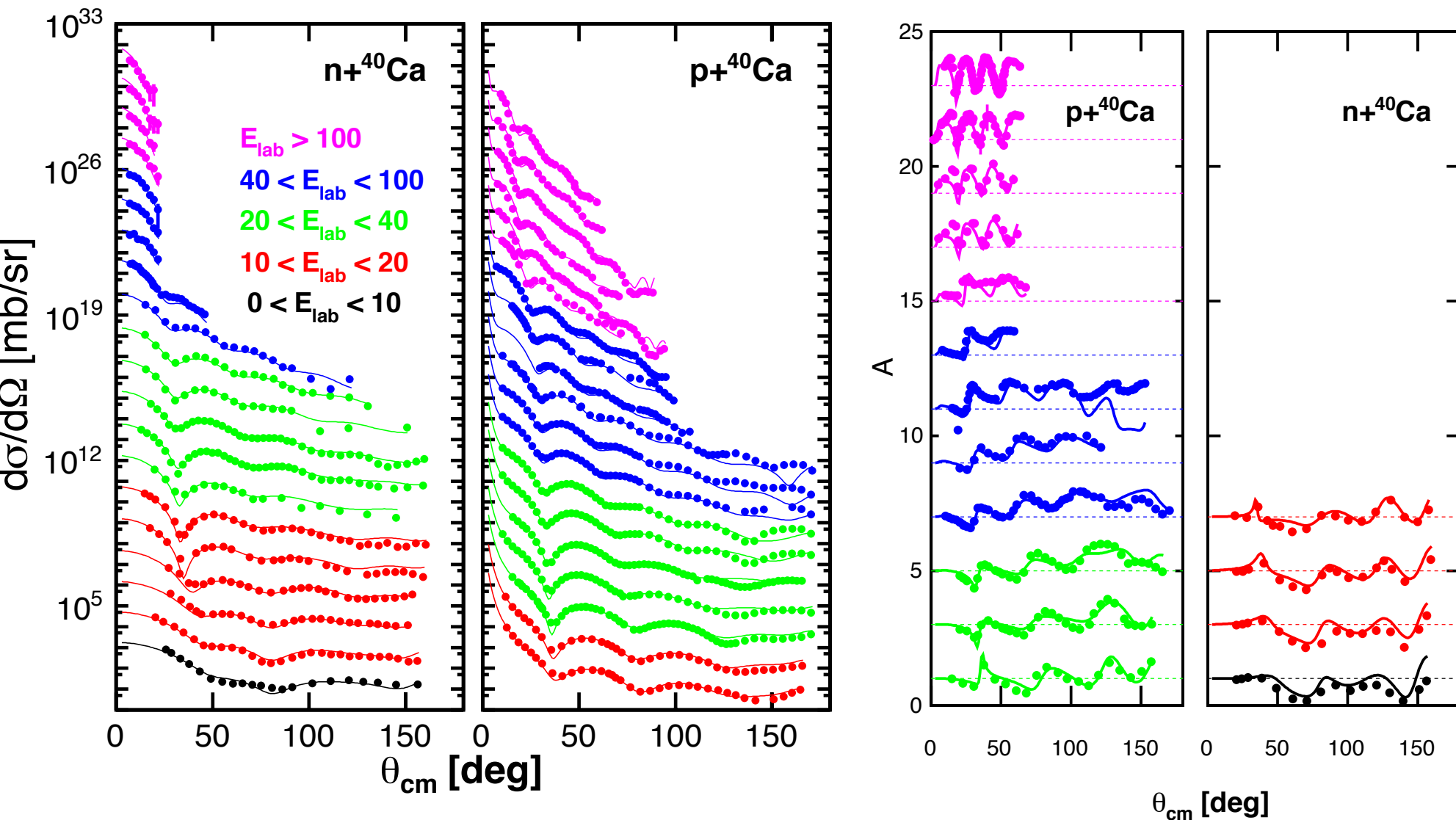
Implications

- Changes the description of hadronic reactions because interior nucleon wave functions depend on non-locality
- Consistency test of interpretation (e,e'p) reaction (**see later**)
- Independent "experimental" statement on size of three-body contribution to the energy of the ground state--> two-body only:

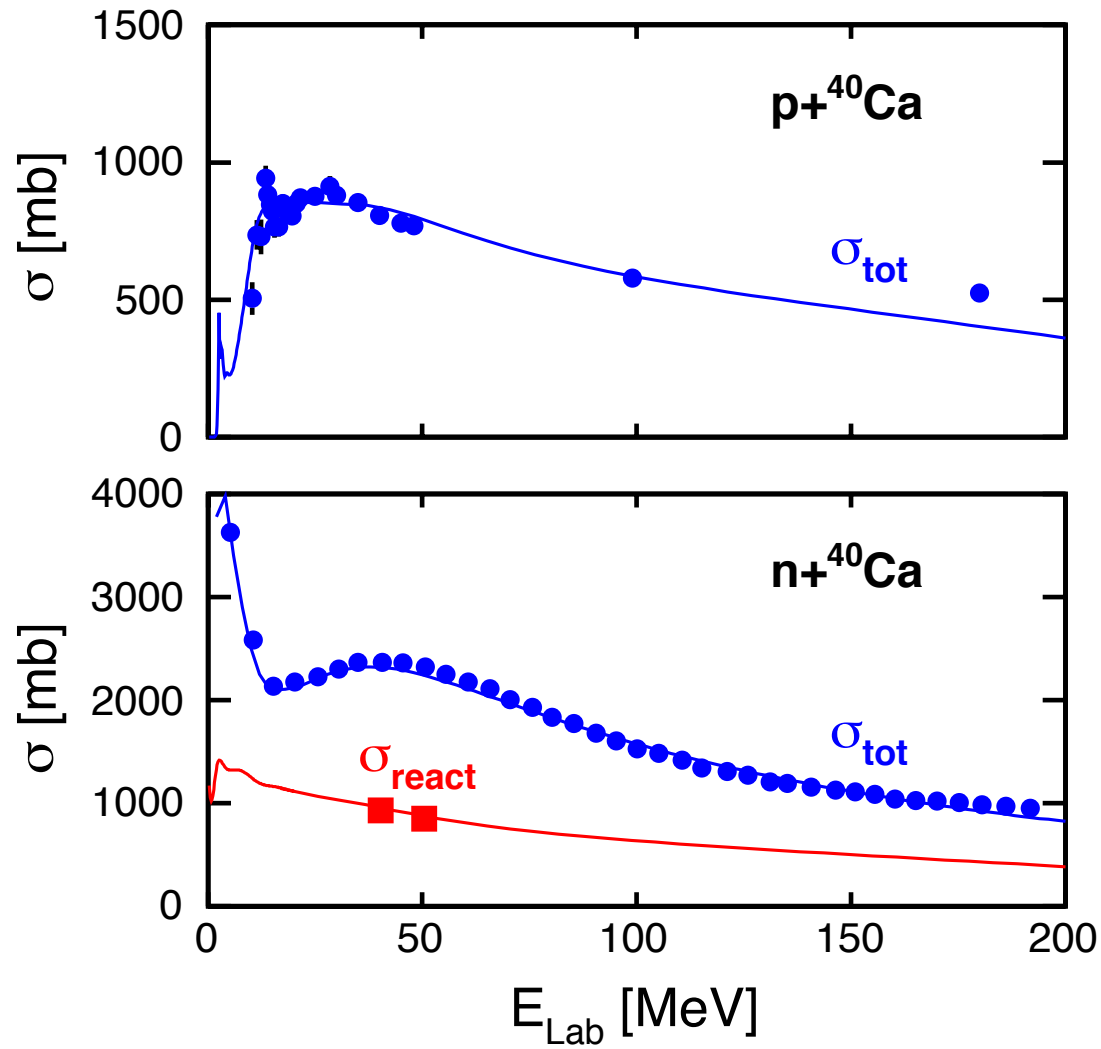
$$E/A = \frac{1}{2A} \sum_{\ell j} (2j+1) \int_0^\infty dk k^2 \frac{k^2}{2m} n_{\ell j}(k) + \frac{1}{2A} \sum_{\ell j} (2j+1) \int_0^\infty dk k^2 \int_{-\infty}^{\epsilon_F} dE E S_{\ell j}(k; E)$$

reactions and structure

Differential cross sections and analyzing powers



Reaction (p&n) and total (n) cross sections

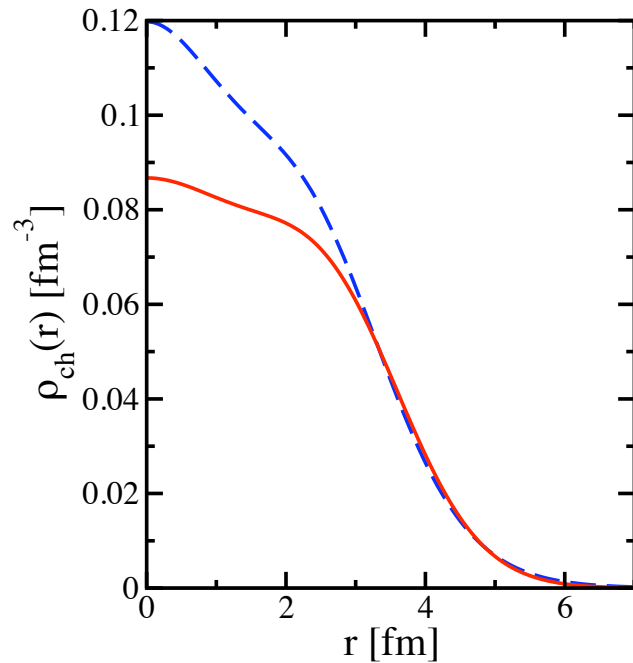


Critical experimental data → charge density

Local version

radius correct...

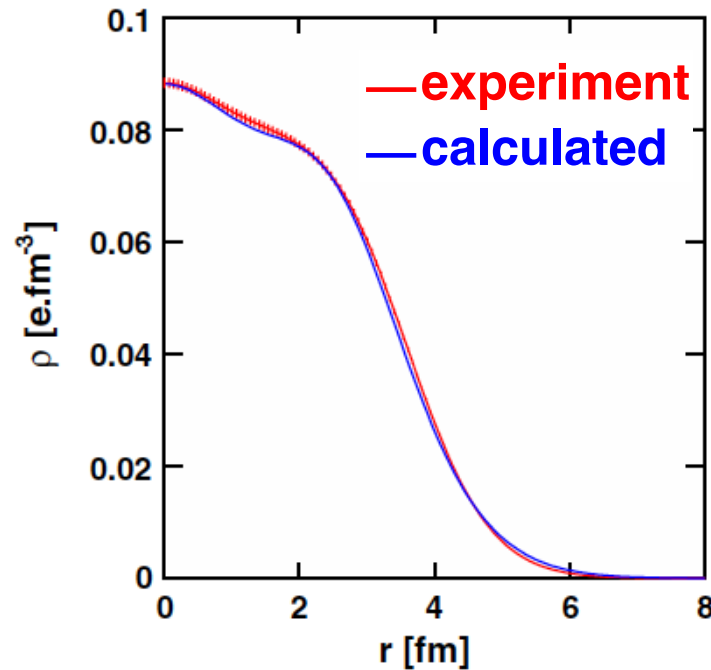
PRC82, 054306 (2010)



Charge density ^{40}Ca

Non-locality essential

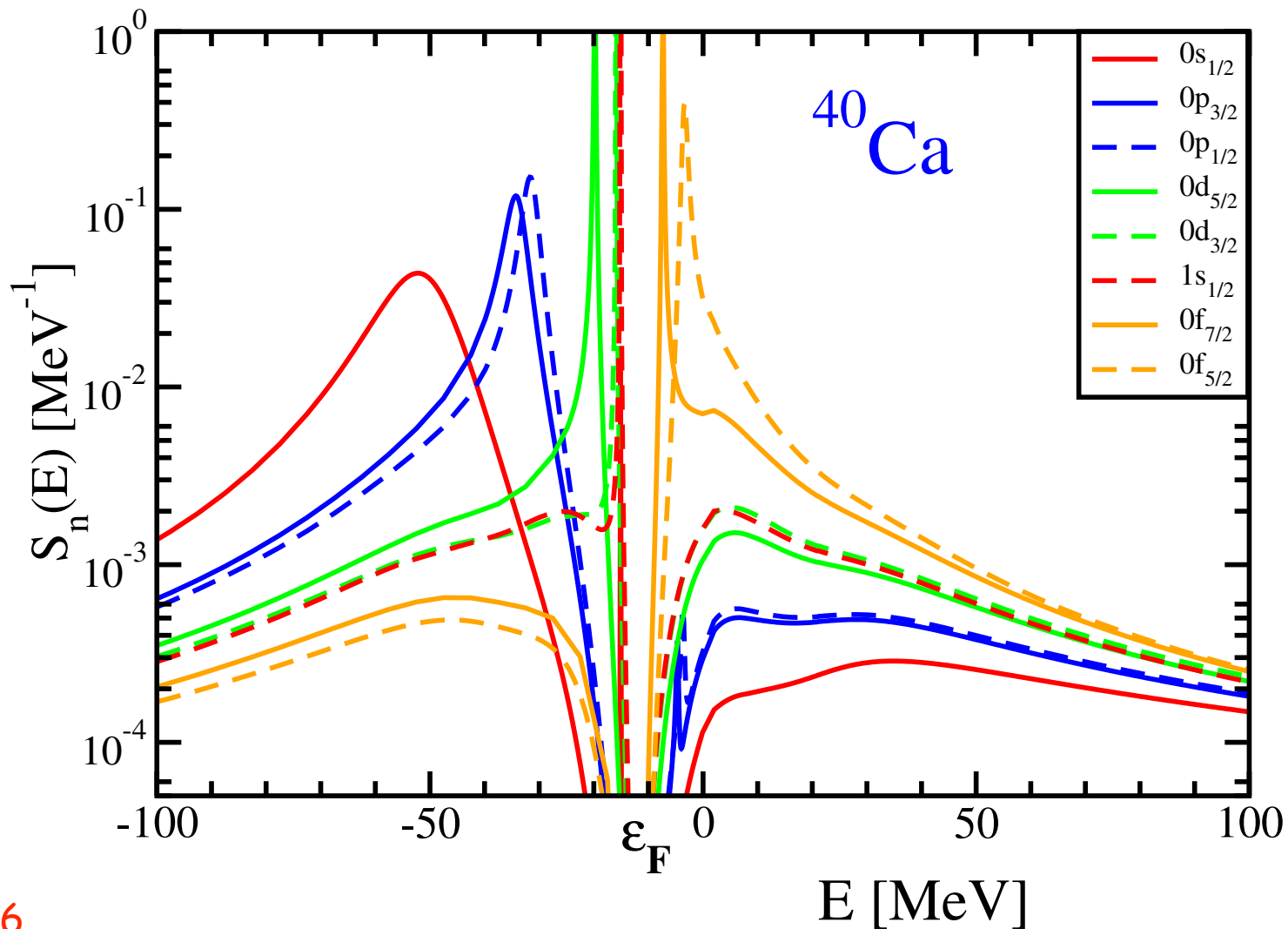
PRL 112,162503(2014)



High-momentum nucleons → JLab can also be described → E/A

Spectral function for bound states

- [0,200] MeV → constrained by elastic scattering data



$$S_{0d_{3/2}} = 0.76$$

$$S_{1s_{1/2}} = 0.78$$

0.15-0.25 larger than NIKHEF analysis!

PRC90, 061603(R) (2014)

reactions and structure

Quantitatively

- Orbit closer to the continuum \rightarrow more strength in the continuum
- Note “particle” orbits
- Drip-line nuclei have valence orbits very near the continuum

Table 1: Occupation and depletion numbers for bound orbits in ^{40}Ca . $d_{nlj}[0, 200]$ depletion numbers have been integrated from 0 to 200 MeV. The fraction of the sum rule that is exhausted, is illustrated by $n_{nlj} + d_{nlj}[\varepsilon_F, 200]$. Last column $d_{nlj}[0, 200]$ depletion numbers for the CDBonn calculation.

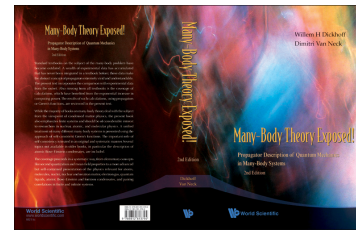
orbit	n_{nlj} DOM	$d_{nlj}[0, 200]$ DOM	$n_{nlj} + d_{nlj}[\varepsilon_F, 200]$ DOM	$d_{nlj}[0, 200]$ CDBonn
$0s_{1/2}$	0.926	0.032	0.958	0.035
$0p_{3/2}$	0.914	0.047	0.961	0.036
$1p_{1/2}$	0.906	0.051	0.957	0.038
$0d_{5/2}$	0.883	0.081	0.964	0.040
$1s_{1/2}$	0.871	0.091	0.962	0.038
$0d_{3/2}$	0.859	0.097	0.966	0.041
$0f_{7/2}$	0.046	0.202	0.970	0.034
$0f_{5/2}$	0.036	0.320	0.947	0.036

Another look at (e,e'p) data

- collaboration with Louk Lapikás and Henk Blok
- Data published at $E_p = 100$ MeV Kramer thesis NIKHEF for $^{40}\text{Ca}(e,e'p)^{39}\text{K}$
Phys.Lett.B227(1989)199
Results: $S(d_{3/2})=0.65$ and $S(s_{1/2})=0.51\dots?$
- More data at 70 and 135 MeV (only in a conference paper)
- What do these spectroscopic factor numbers really represent?
 - Assume DWIA for the reaction description
 - Use kinematics (momentum transfer parallel to initial proton momentum) favoring simplest part of the excitation operator (no two-body current)
 - Overlap function:
 - WS with radius adjusted to shape of cross section
 - Depth adjusted to separation energy
 - Distorted proton wave from standard "global optical potential"
 - Fit normalization of overlap function to data -> spectroscopic factor

Why go back there?

FSI and $(e, e' p) \Leftrightarrow$ analysis

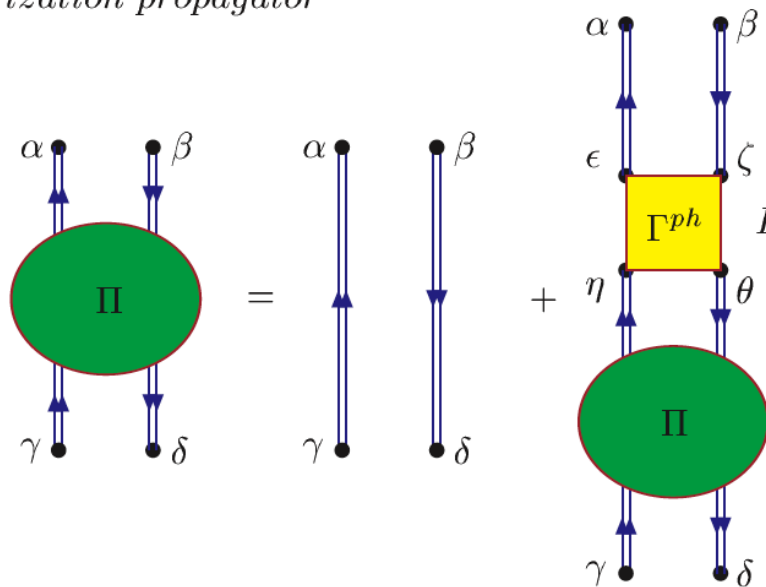


$$\hat{O} = \sum_{\alpha, \beta} \langle \alpha | O | \beta \rangle a_{\alpha}^{\dagger} a_{\beta} \quad \text{Electron Scattering} \Rightarrow \text{one-body operator}$$

$$\left| \langle \Psi_n^A | \hat{O} | \Psi_0^A \rangle \right|^2 = \sum \langle \alpha | O | \beta \rangle^* \langle \gamma | O | \delta \rangle \langle \Psi_0^A | a_{\alpha}^{\dagger} a_{\beta} | \Psi_n^A \rangle \langle \Psi_n^A | a_{\gamma}^{\dagger} a_{\delta} | \Psi_0^A \rangle$$

Requires (imaginary part of) exact polarization propagator

Polarization propagator



Choose kinematics: \Rightarrow only first term

$$\langle \Psi_m^{A+1} | a_{\alpha}^{\dagger} | \Psi_0^A \rangle$$

\Rightarrow Elastic scattering (phenomenology)

$$\langle \Psi_n^{A-1} | a_{\beta} | \Psi_0^A \rangle$$

\Rightarrow Quasihole wave function

“Absolute” spectroscopic factors?

reactions and structure

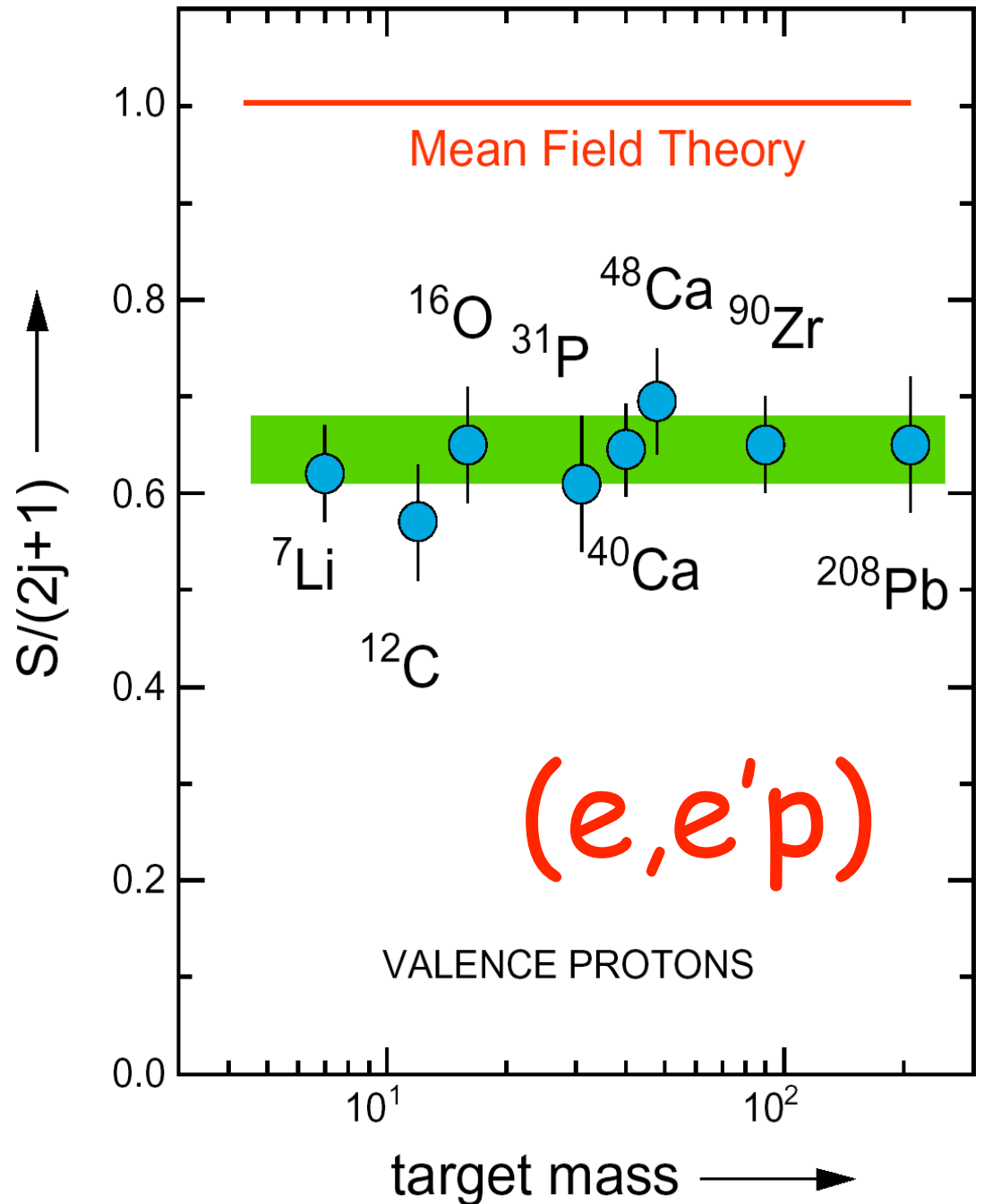
Removal probability for valence protons from NIKHEF data

L. Lapikás, Nucl. Phys. A553,297c (1993)

$S \approx 0.65$ for valence protons
Reduction \Rightarrow both SRC and LRC

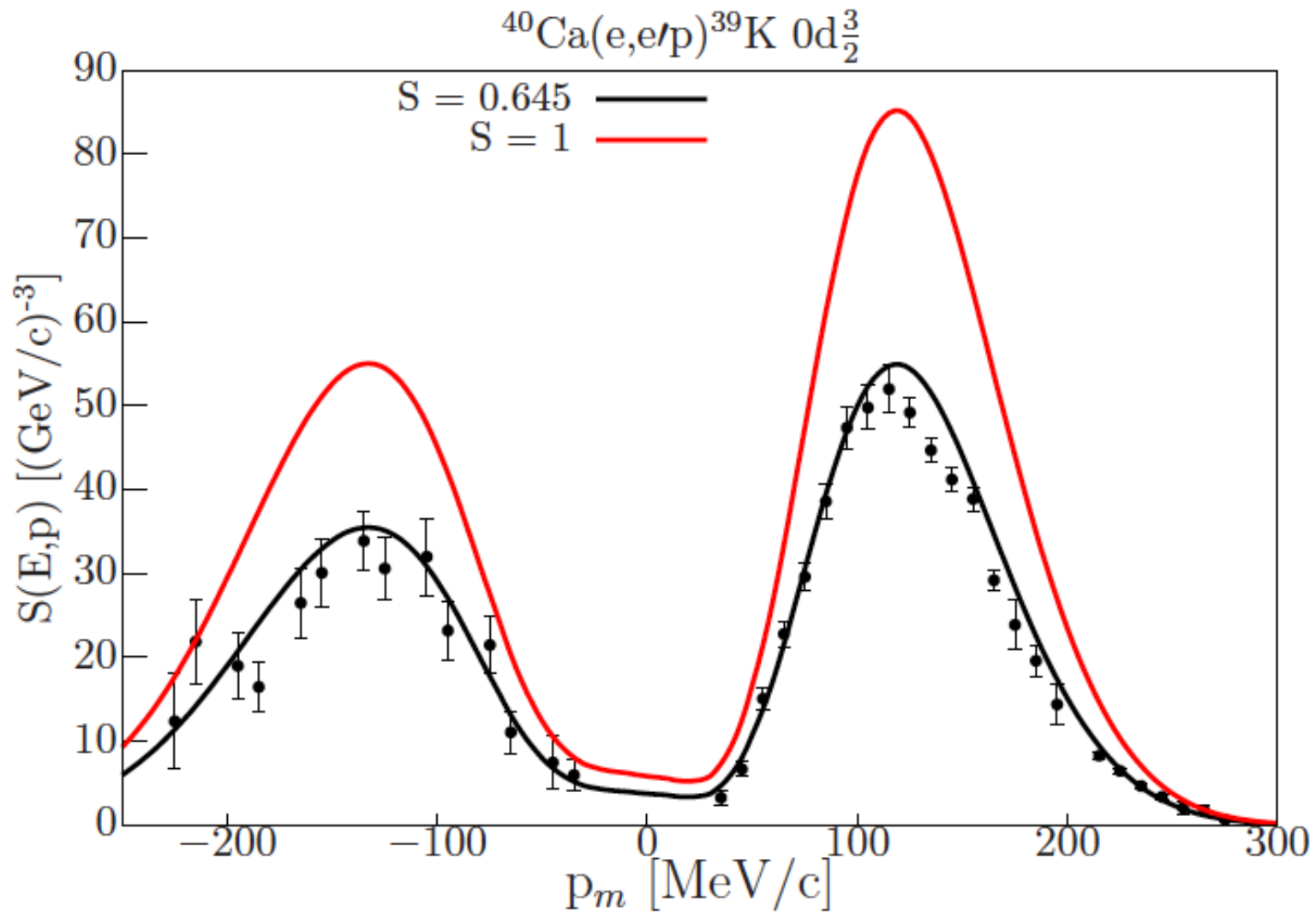
Weak probe but propagation in the nucleus of removed proton using standard optical potentials to generate distorted wave \rightarrow associated uncertainty $\sim 5-15\%$

Why: details of the interior scattering wave function uncertain since non-locality is not constrained (so far.....) but now available for ^{40}Ca !



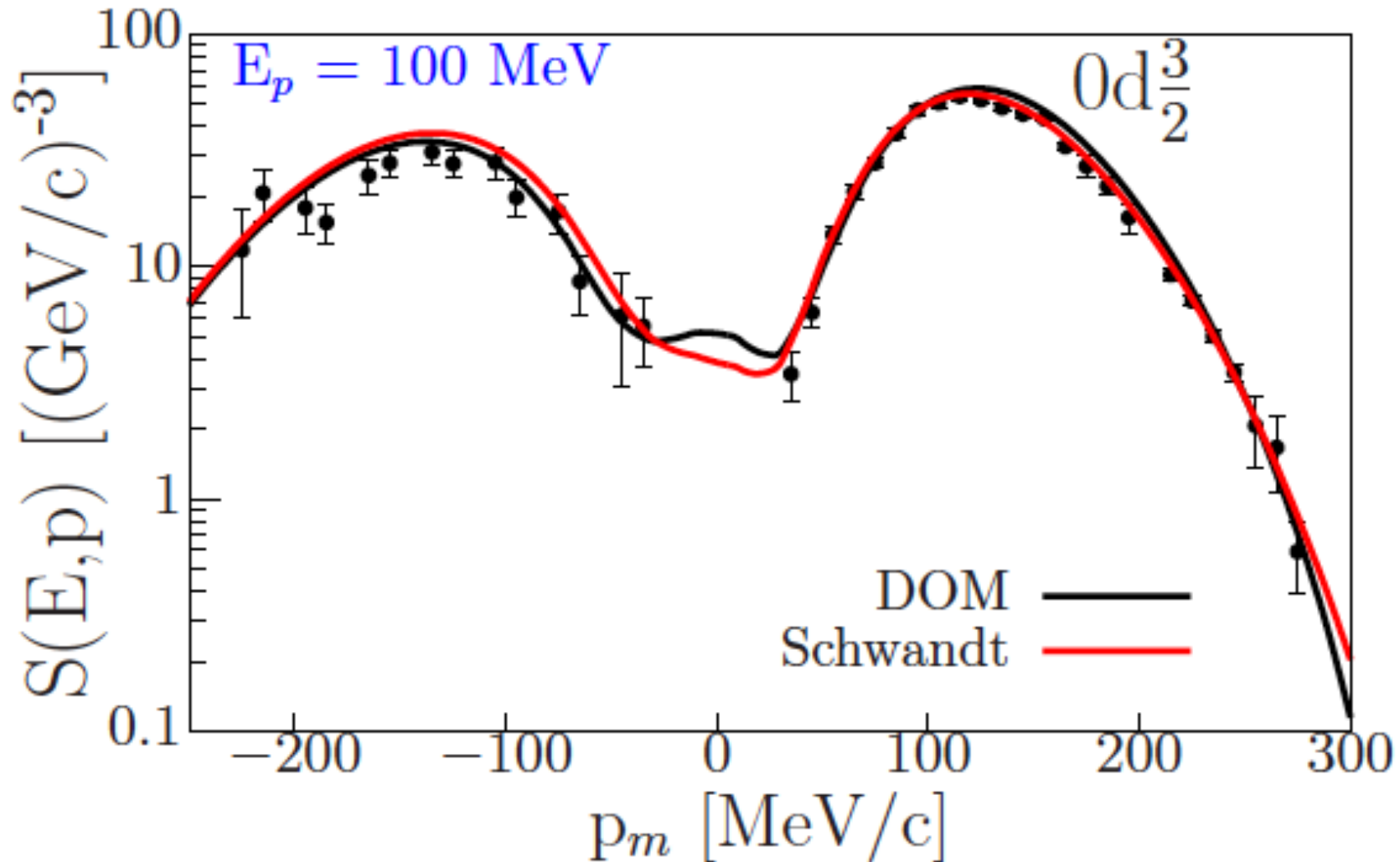
NIKHEF analysis

- Schwandt et al. (1981) optical potential



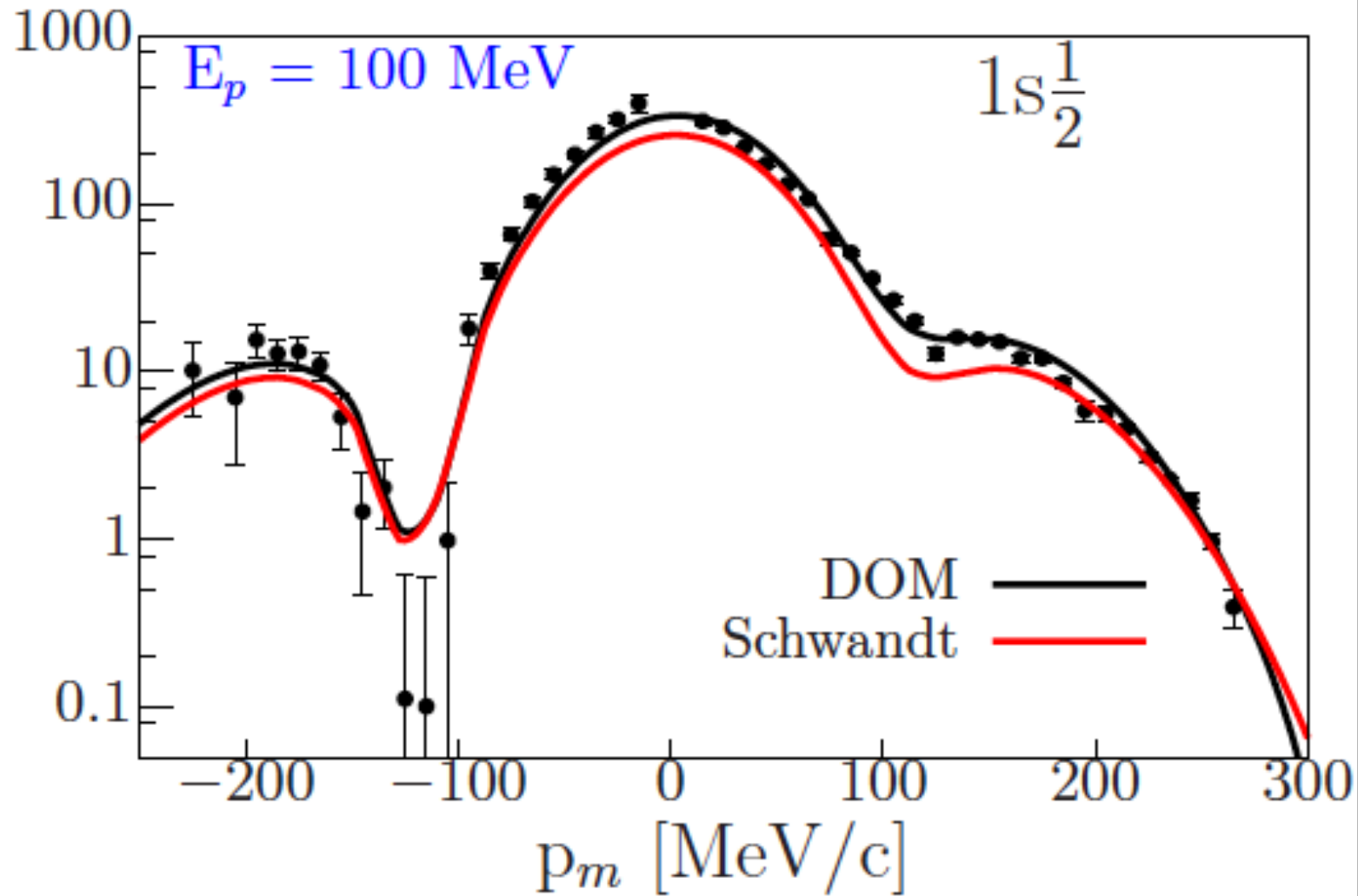
DOM non-local ingredients $E_p = 100$ MeV

- $S(d_{3/2})=0.75$ indirectly constrained by other data so **not** adjusted
- NIKHEF: $S(d_{3/2})=0.65\pm 0.06$



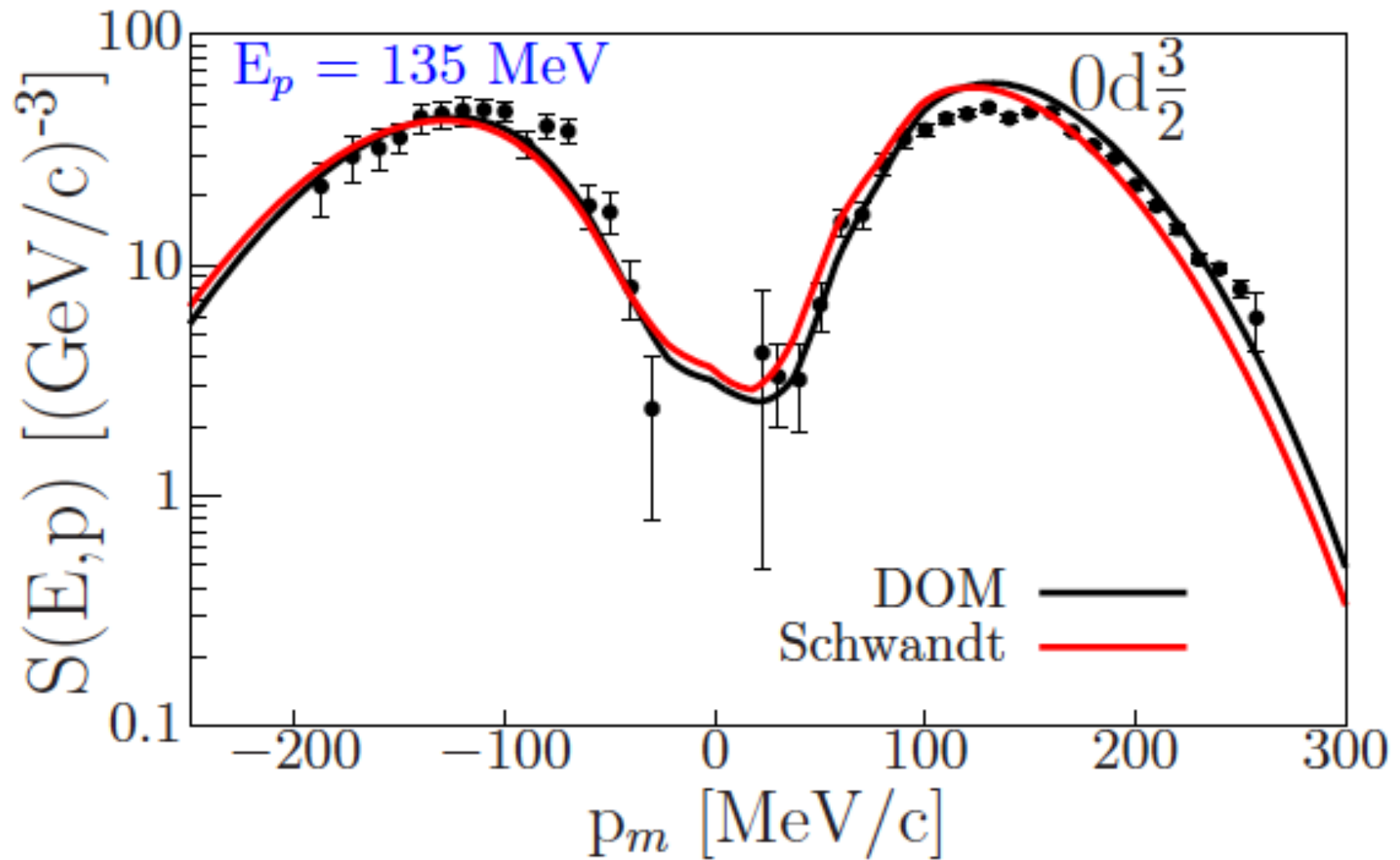
$$E_p = 100 \text{ MeV}$$

- $S(s_{1/2})=0.78$ indirectly constrained by other data so **not** adjusted
- NIKHEF: $S(s_{1/2})=0.51\pm 0.05$



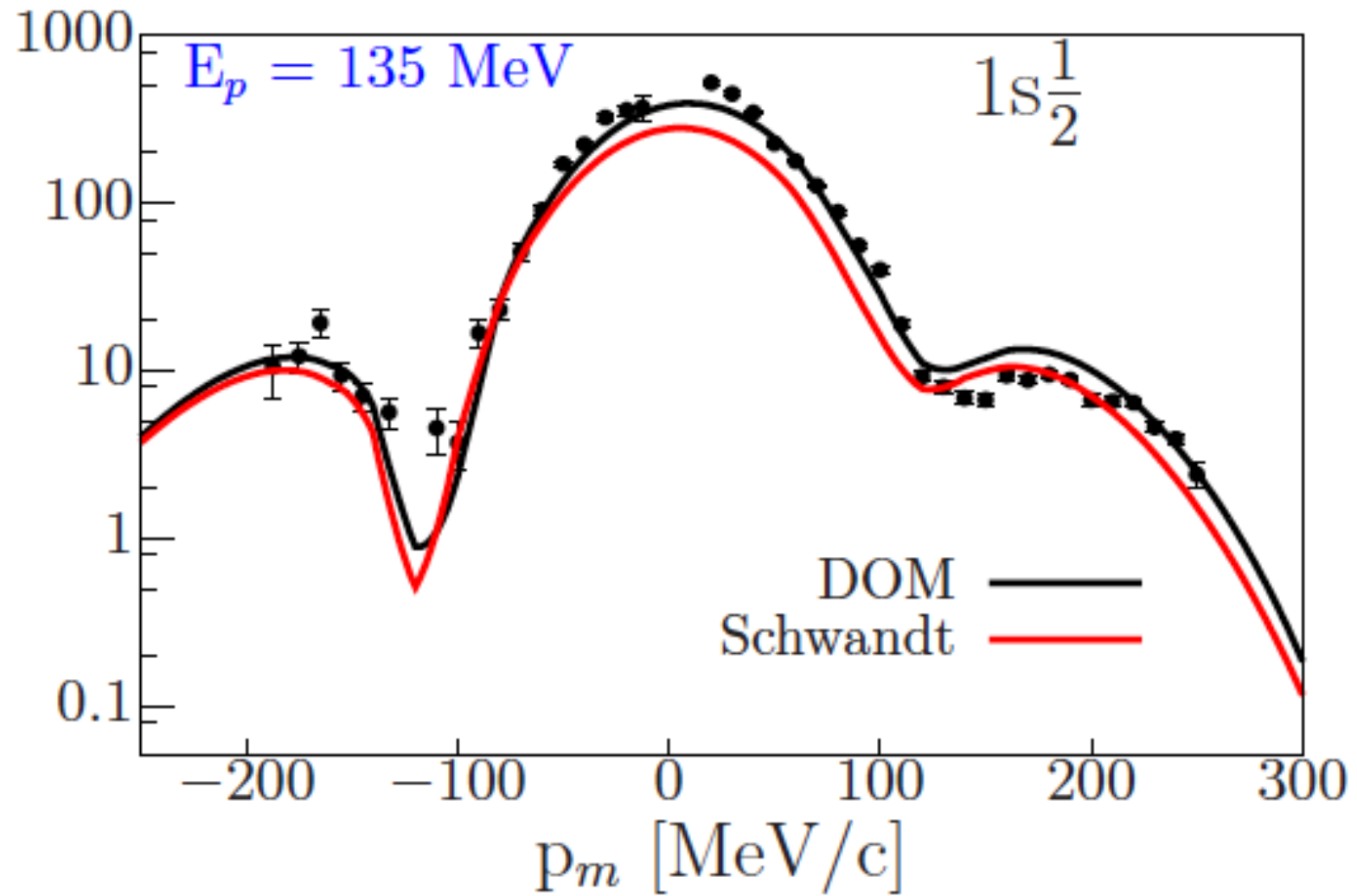
$$E_p = 135 \text{ MeV}$$

- Still reasonable? Perhaps not...



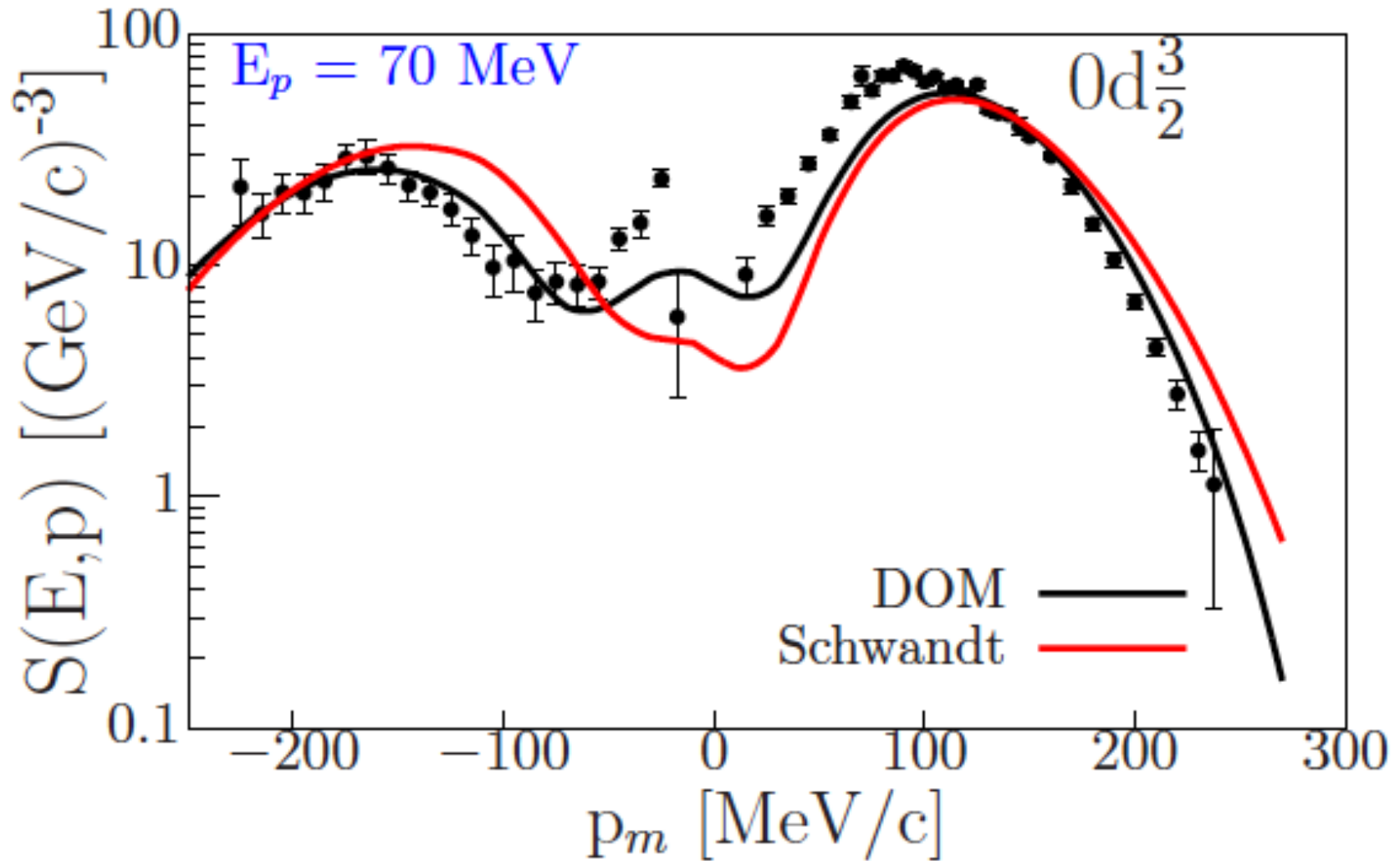
$$E_p = 135 \text{ MeV}$$

- Too high excitation energy?



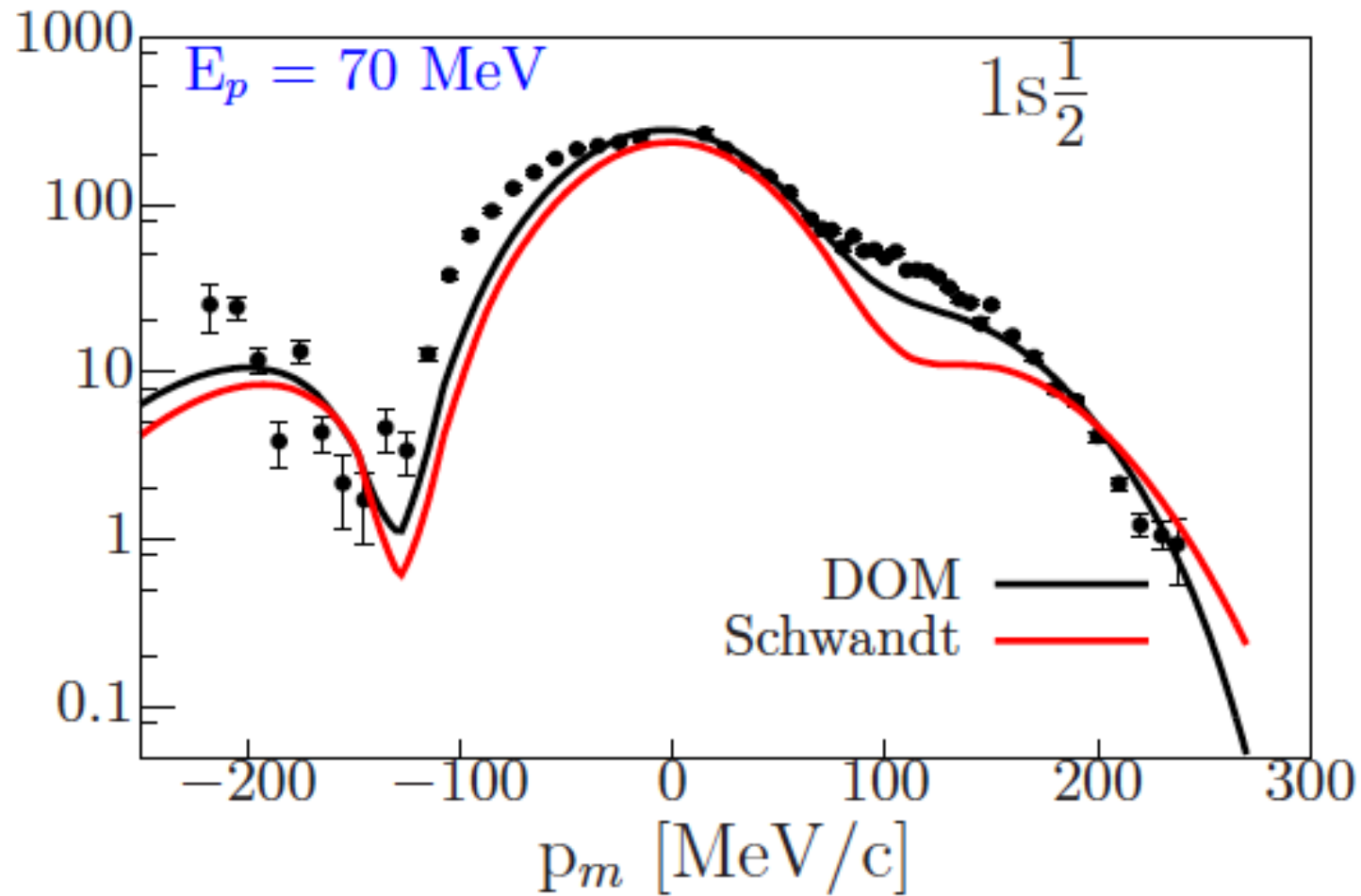
$$E_p = 70 \text{ MeV}$$

- Reaction model no longer good enough and there is more transverse excitation



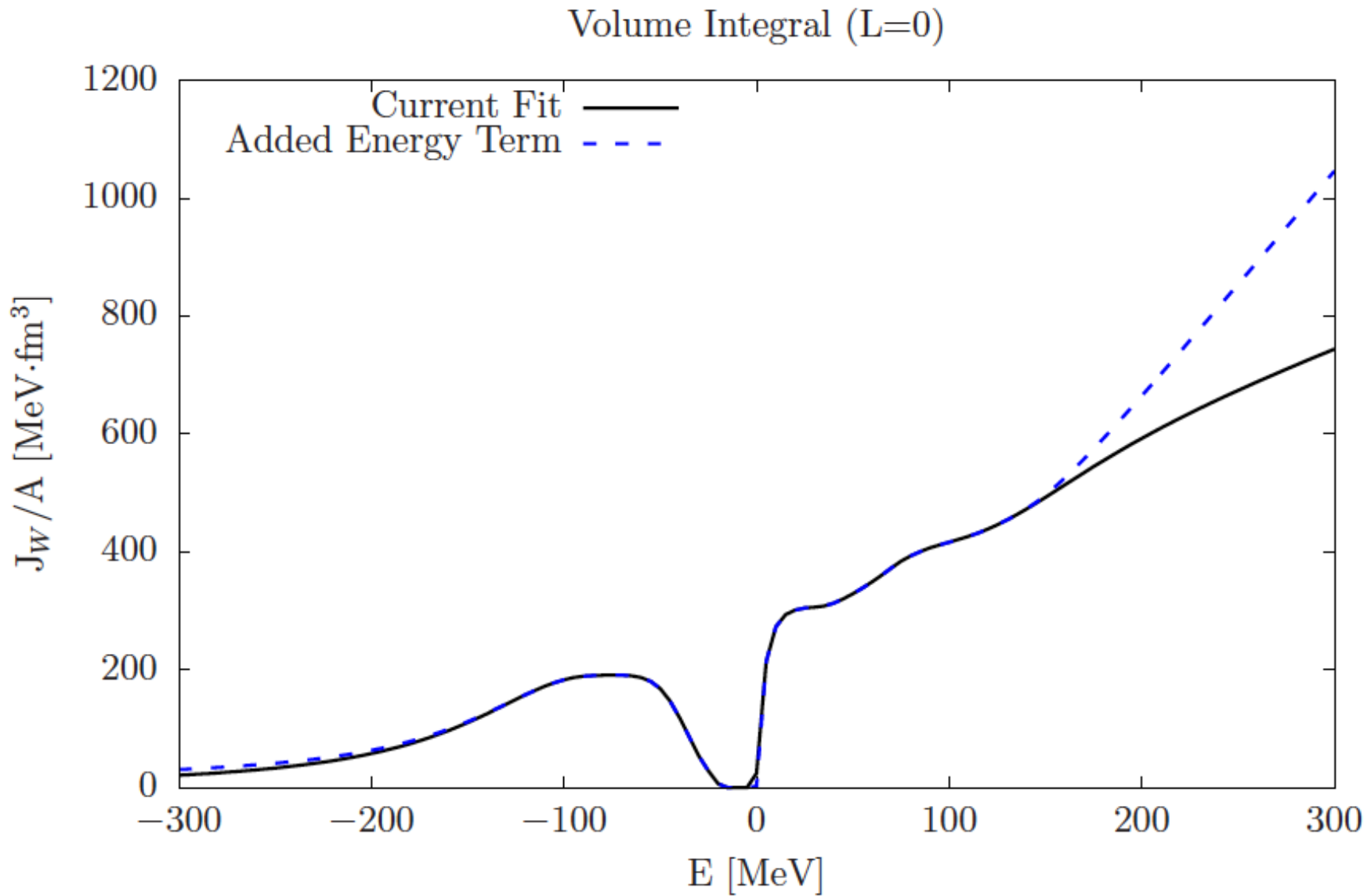
$$E_p = 70 \text{ MeV}$$

- Limitation of $(e,e'p)$?



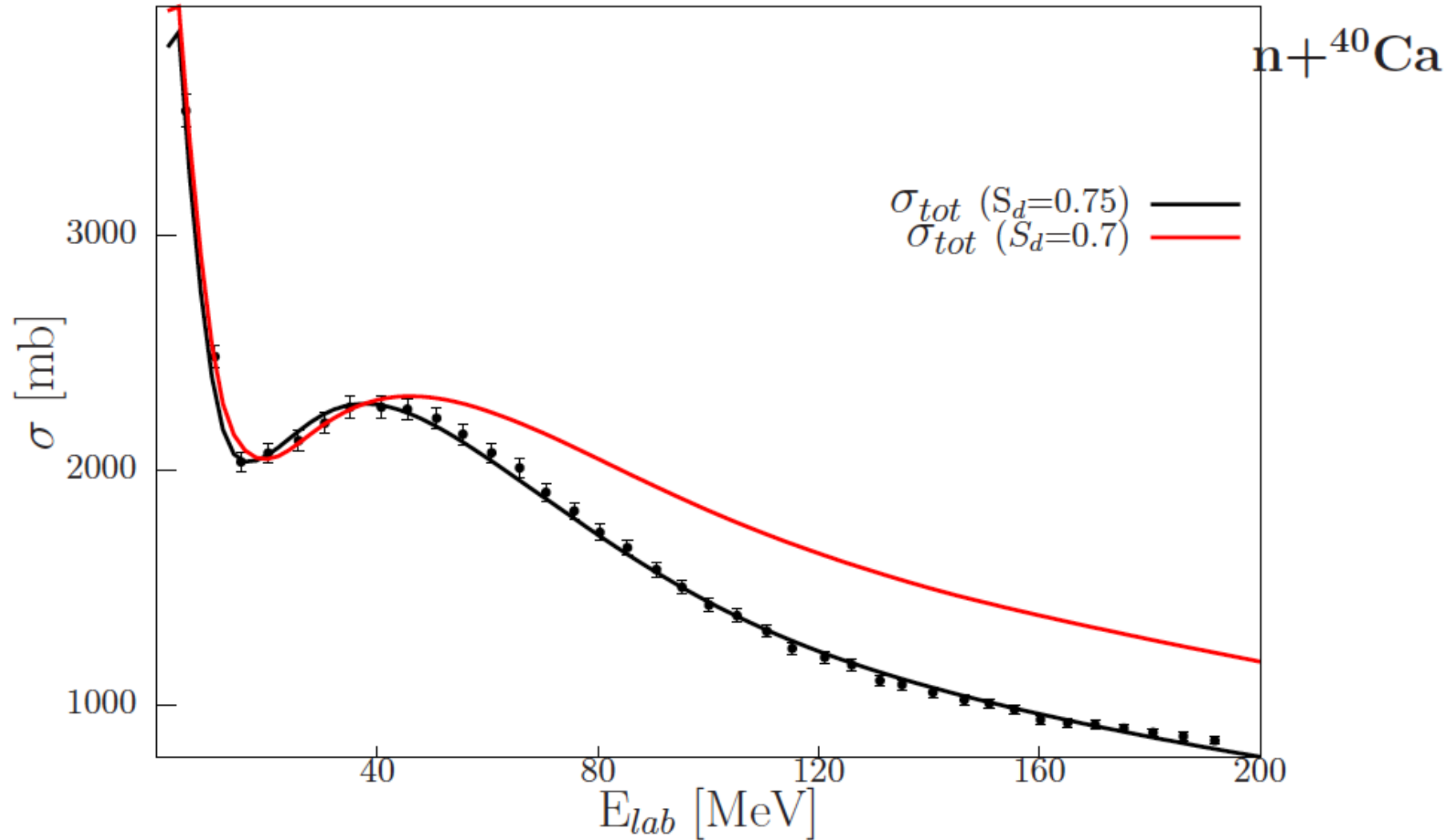
- What about further reducing the spectroscopic factor?
- What happens with other data?

Remove strength to higher energy

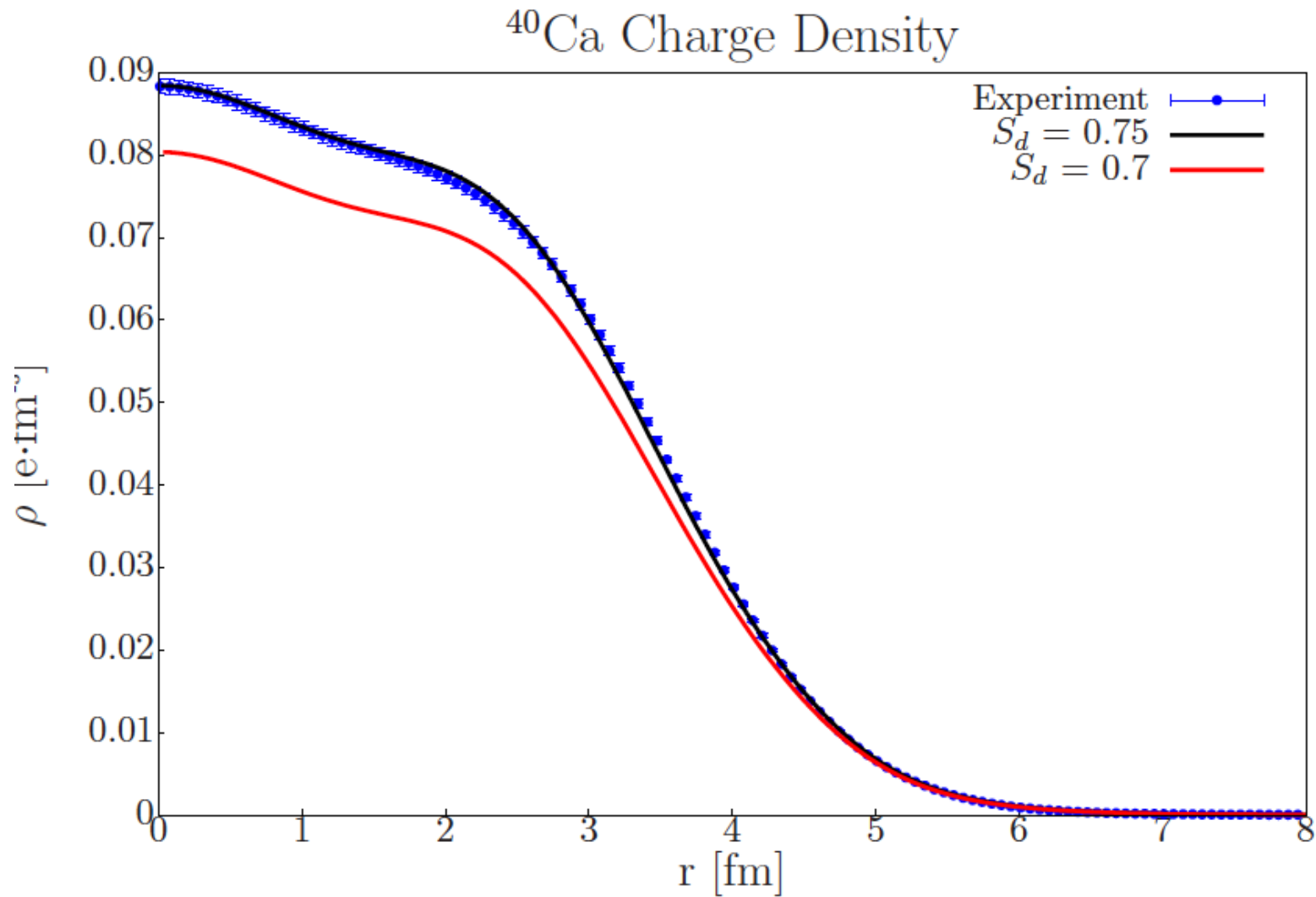


Problems

- Total neutron cross section

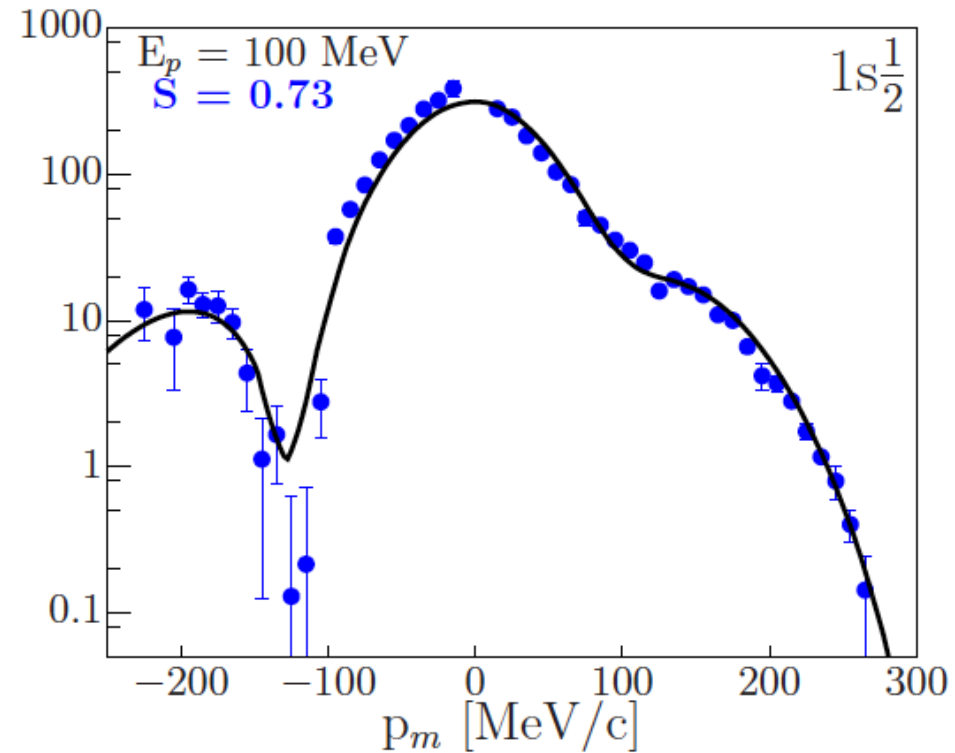
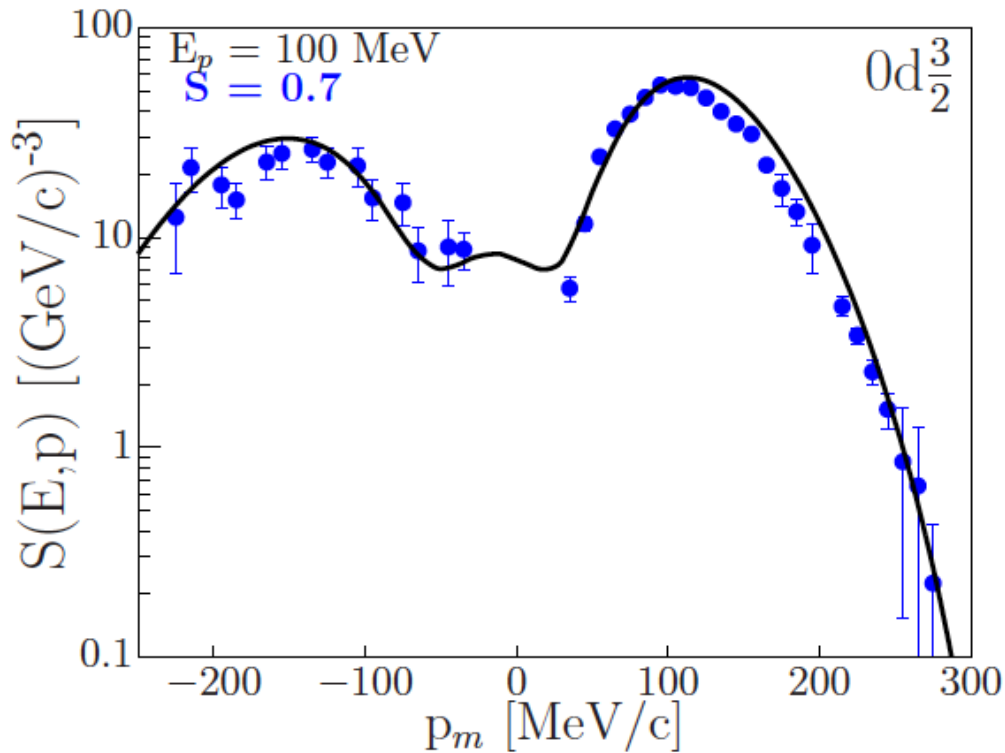


More problems



Only looking at (e,e'p) data

- Visual slightly better with smaller normalization
- But larger values seem ruled out



Message

- Nonlocal dispersive potentials yield consistent input
- Constraints from other data generate spectroscopic factors ~ 0.75 in ^{40}Ca
- Implications for transfer reactions significant
- (p,2p) reaction for stable targets can be constrained
- Consistent with inelastic electron scattering data

Lessons from the past probably forgotten?

PHYSICAL REVIEW C

VOLUME 45, NUMBER 6

JUNE 1992

High resolution electron scattering from high spin states in ^{208}Pb

J. P. Connelly,* D. J. DeAngelis, J. H. Heisenberg, F. W. Hersman, W. Kim, M. Leuschner,
T. E. Milliman, and J. Wise†

Department of Physics, University of New Hampshire, Durham, New Hampshire 03824

C. N. Papanicolas

Department of Physics and Nuclear Physics Laboratory, University of Illinois, 1110 West Green Street, Urbana, Illinois 61801

(Received 2 March 1992)

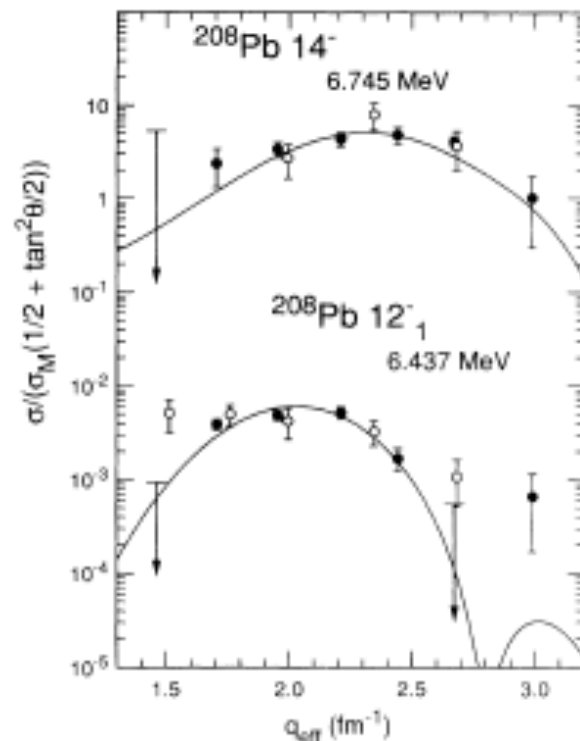


TABLE I. High spin transitions seen in this experiment with the dominant 1p-1h configuration and the normalization factor (N_q) of the Woods-Saxon DWBA fits to the data. An asterisk indicates the assignment of J^π is from this experiment.

Energy (MeV)	J^π	1p-1h configuration	N_q
5.010	9^+	$\nu(2g_{9/2}, 1i_{13/2}^{-1})$	0.54 ± 0.01
5.260	9^{+*}	$\pi(1h_{9/2}, 1h_{11/2}^{-1})$	0.53 ± 0.04
5.291	11^{+*}	$\nu(2g_{9/2}, 1i_{13/2}^{-1})$	0.38 ± 0.03
5.860	11^{+*}	$\nu(1i_{11/2}, 1i_{13/2}^{-1})$	0.61 ± 0.05
5.954	9^{+*}	$\nu(1i_{11/2}, 1i_{13/2}^{-1})$	0.50 ± 0.05
6.110	12^+	$\pi(2f_{7/2}, 1h_{11/2}^{-1})$	0.19 ± 0.03
6.283	10^{-*}	$\nu(1j_{15/2}, 1i_{13/2}^{-1})$	0.39 ± 0.06
6.437	12^-	$\nu(1j_{15/2}, 1i_{13/2}^{-1})$	0.64 ± 0.07
6.437	12^-	$\nu(1j_{15/2}, 1i_{13/2}^{-1})$	0.46 ± 0.07
6.745	14^-	$\nu(1j_{15/2}, 1i_{13/2}^{-1})$	0.53 ± 0.04
6.833	$(8^-)^*$	$\pi(1i_{13/2}, 1h_{11/2}^{-1})$	0.58 ± 0.05
6.859	9^{-*}	$\pi(1i_{13/2}, 1h_{11/2}^{-1})$	0.55 ± 0.02
6.879	7^{-*}	$\pi(1i_{13/2}, 1h_{11/2}^{-1})$	0.39 ± 0.01
6.884	10^{-*}	$\pi(1i_{13/2}, 1h_{11/2}^{-1})$	0.32 ± 0.09
7.064	12^-	$\pi(1i_{13/2}, 1h_{11/2}^{-1})$	0.32 ± 0.05
7.086	12^{-*}	$\pi(1i_{13/2}, 1h_{11/2}^{-1})$	0.18 ± 0.02

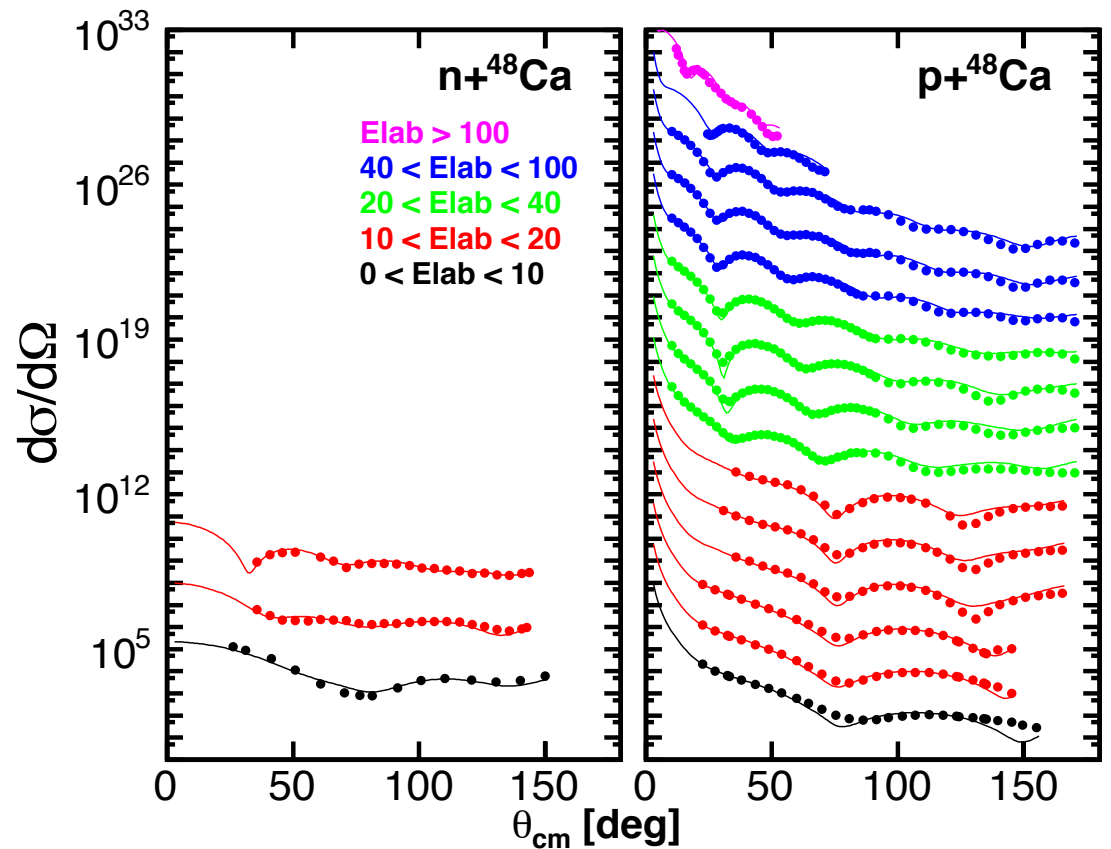
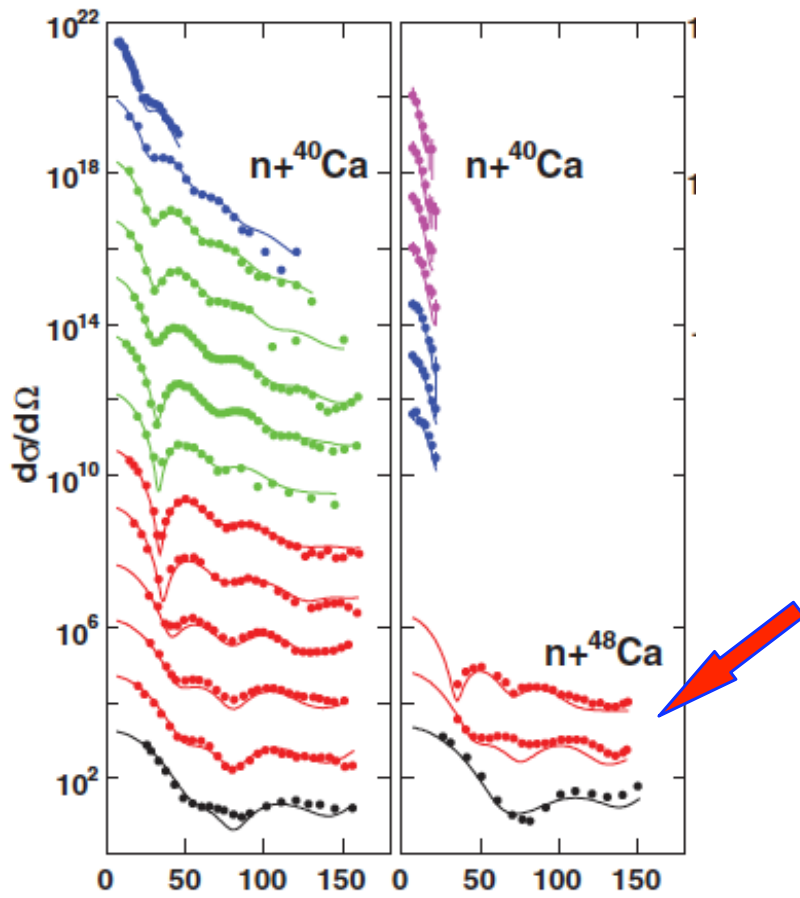
FIG. 4. M_{14} (6.745 MeV) and M_{12} (6.347 MeV) form factors with DWBA Woods-Saxon fits. The 6.745 MeV form factor is scaled by 1000. Forward angle data are represented by the open data points; the 155° data are presented by the solid data points.

New DOM results for ^{48}Ca

- Change of proton properties when 8 neutrons are added to ^{40}Ca ?
- Change of neutron properties?
- Can hard to measure quantities be indirectly constrained?

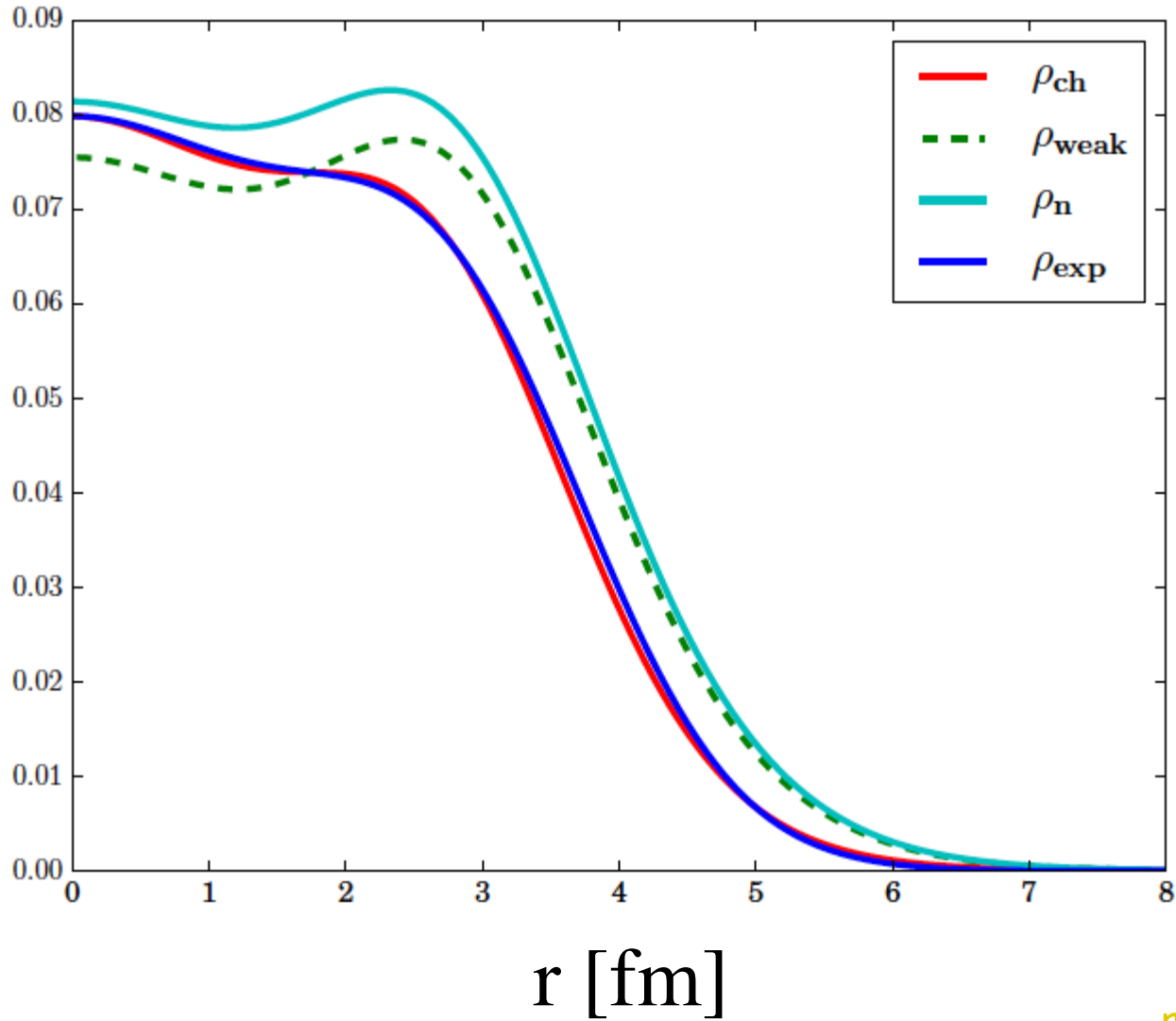
What about neutrons?

- ^{48}Ca \rightarrow charge density has been measured
- Recent neutron elastic scattering **data** \rightarrow PRC83,064605(2011)
- Local DOM **OLD** Nonlocal DOM **NEW**



Results ^{48}Ca

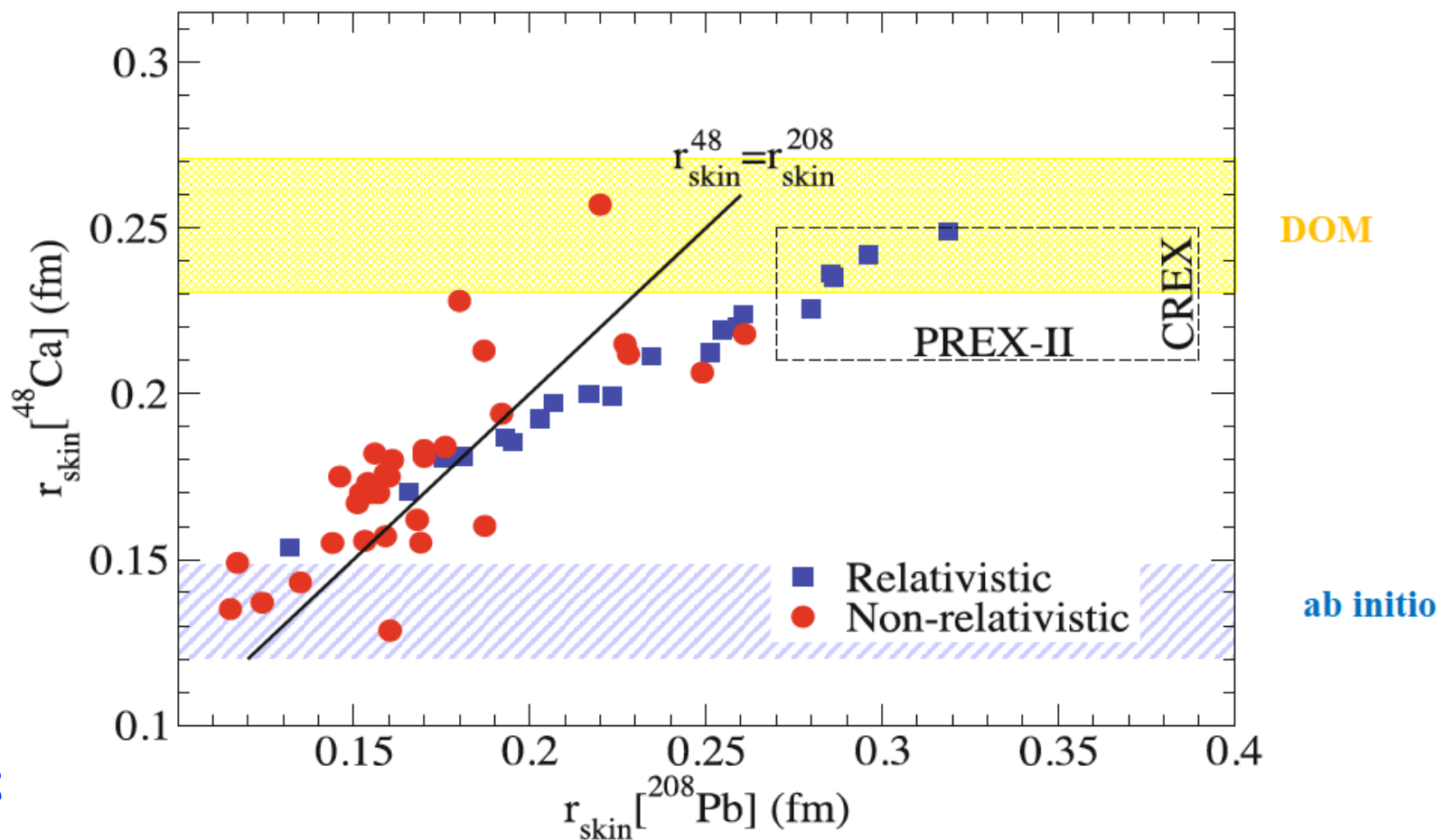
- Density distributions
- DOM \rightarrow neutron distribution $\rightarrow R_n - R_p$



Comparison of neutron skin with other calculations and future experiments...

- Figure adapted from

C.J. Horowitz, K.S. Kumar, and R. Michaels, Eur. Phys. J. A (2014)



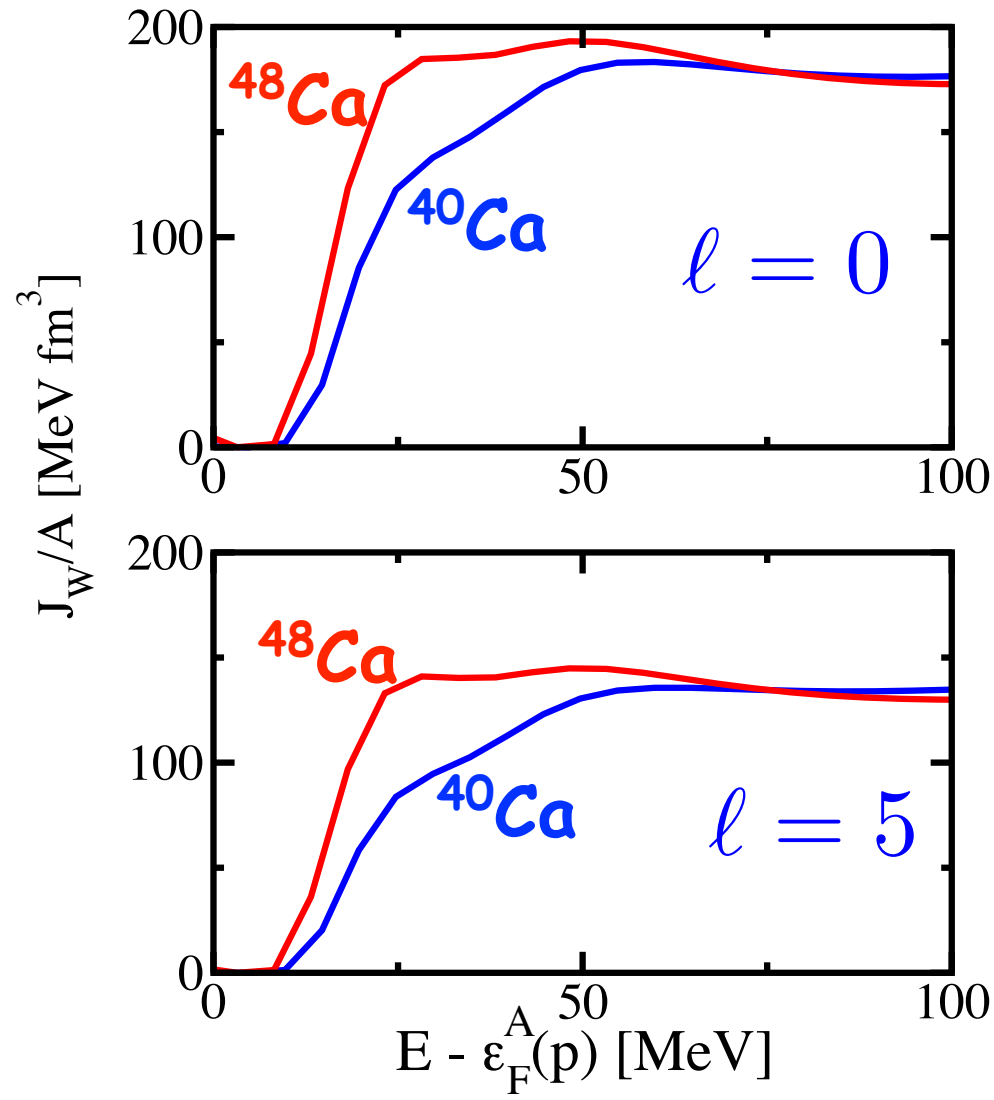
- "Ab initio":

G. Hagen et al., Nature Phys. 12, 186 (2016)

--> drip line

Volume integrals for $^{40-48}\text{Ca}$

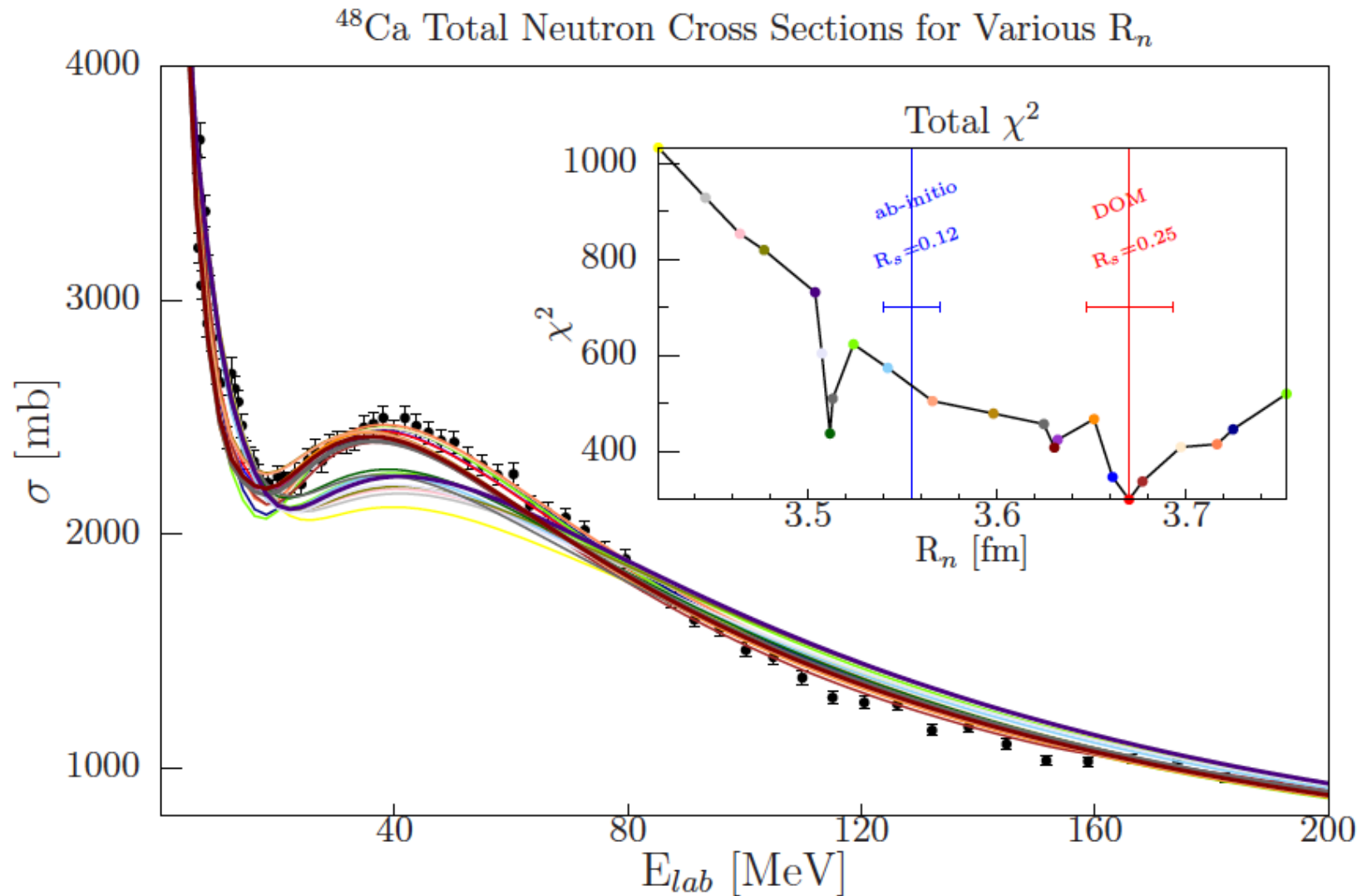
Protons see the same interior but a different surface!



--> drip line

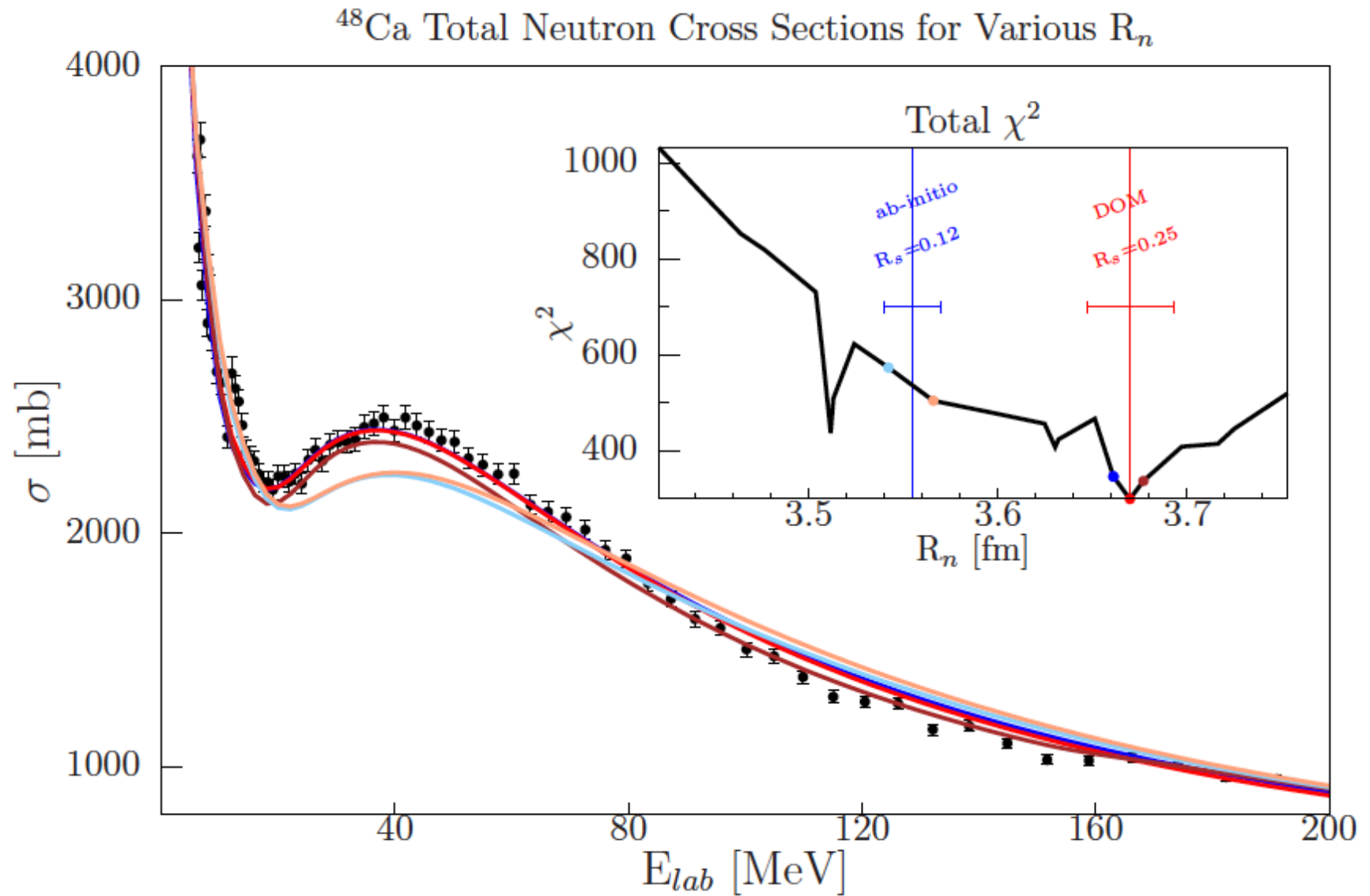
Constraining the neutron radius

- How robust is this result



--> drip line

Less clutter



- CREX will decide!

--> drip line

Quantitative comparison of ^{40}Ca and ^{48}Ca

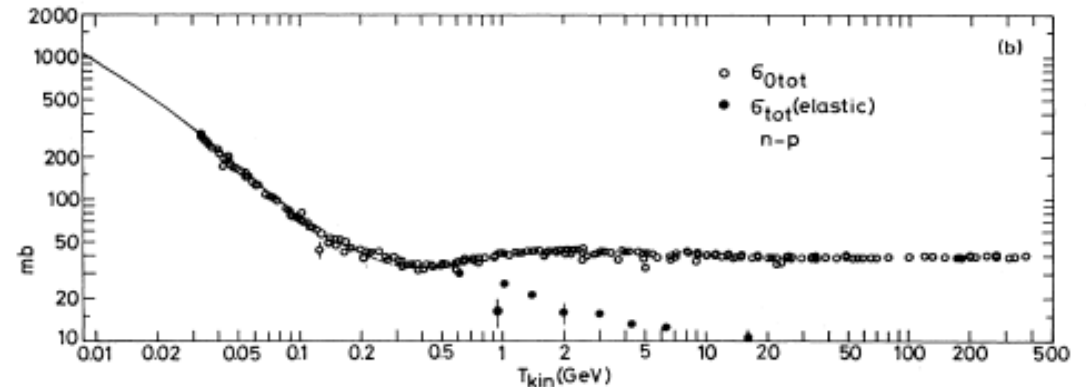
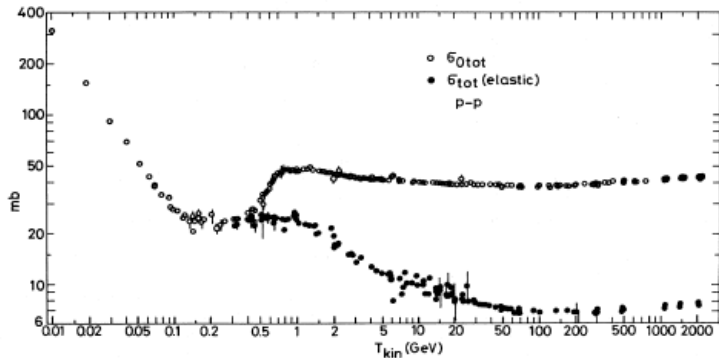
Spectroscopic factors	^{40}Ca	p ^{48}Ca	n ^{48}Ca
$0d_{3/2}$	0.76	0.65 ↓	0.80 ↑
$1s_{1/2}$	0.78	0.71 ↓	0.83 ↑
$0f_{7/2}$	0.73	0.59 ↓	0.84 ↑

Ongoing work

- ^{208}Pb fit \rightarrow neutron skin prediction
- $^{48}\text{Ca}(e,e'p)$
- ^{112}Sn and ^{124}Sn total neutron cross sections being analyzed
- future ^{64}Ni measurement of total neutron cross section
- $^{14,20}\text{O}$ elastic proton scattering
- Local then nonlocal fit to Sn, Ni, O isotopes
- Integrate DOM ingredients with (d,p) - (n, γ) surrogate- and (p,d) codes
- Insert correlated Hartree-Fock contribution from realistic NN interactions in DOM self-energy \rightarrow tensor force included in mean field
- Extrapolations to the respective drip lines available
- Analyze energy density as a function of density and nucleon asymmetry
- **Ab initio optical potential calculations initiated CC and Green's function method**

Future plans

- Include higher energy data (proton elastic scattering) using a Dirac formulation



- $(p,2p)$ and (p,pn) reactions
- extend DOM to deuteron
- Construct functional derivative of DOM self-energy \rightarrow excited states
- Improve functional form of self-energy (computationally expensive)

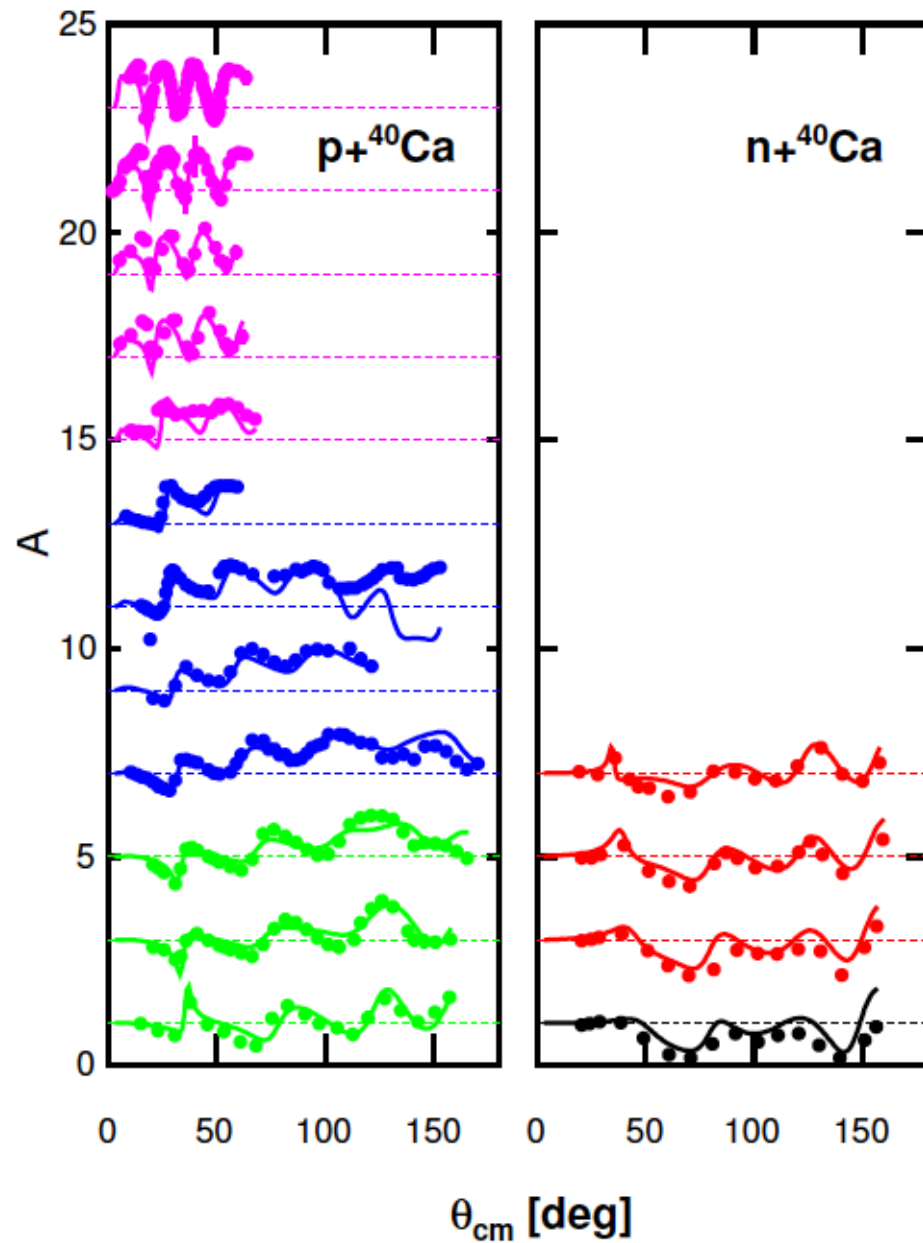
Conclusions

- It **is** possible to link nuclear reactions and nuclear structure
- Vehicle: **nonlocal** version of **Dispersive Optical Model** (Green's function method) as developed by Mahaux → **DSM**
- Can be used as input for analyzing nuclear reactions
- Can predict properties of exotic nuclei
- "Benchmark" for ab initio calculations: e.g. V_{NNN} → binding
- Can describe ground-state properties
 - charge density & momentum distribution
 - spectral properties including high-momentum Jefferson Lab data
- **Elastic scattering** determines depletion of bound orbitals
- **Outlook:** reanalyze many reactions with nonlocal potentials...
- For $N \geq Z$ sensitive to properties of neutrons → weak charge prediction, **large neutron skin**, perhaps more... reactions and structure

Polarization data in ^{40}Ca

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supplement



reactions and structure