

Integrating ab initio structure models into accurate reaction calculations using EFT

Pierre Capel, Daniel Phillips and Hans-Werner Hammer



ECOLE
POLYTECHNIQUE
DE BRUXELLES



TECHNISCHE
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Halo nuclei

Exotic nuclear structures are found far from stability

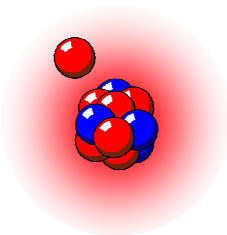
In particular halo nuclei with peculiar quantal structure :

- Light, **n-rich** nuclei
- Low S_n or S_{2n}

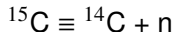
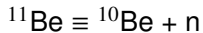
Exhibit **large matter radius**

due to strongly clusterised structure :

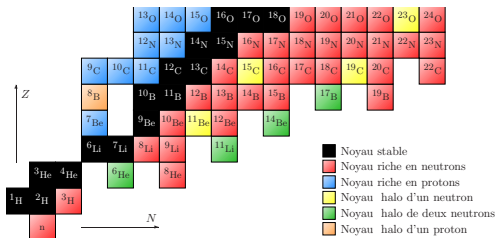
neutrons tunnel far from the **core** and form a **halo**



One-neutron halo



Two-neutron halo



Proton haloes are possible but less probable : ${}^8\text{B}$, ${}^{17}\text{F}$

Reactions with halo nuclei

Halo nuclei are **fascinating** objects
but difficult to study [$\tau_{1/2}(^{11}\text{Be})= 13 \text{ s}$]
 \Rightarrow require **indirect** techniques, like reactions

Elastic scattering

Breakup \equiv dissociation of **halo** from **core**
by interaction with target

Need good understanding of the reaction mechanism
i.e. an accurate **theoretical description** of reaction
coupled to a realistic model of projectile

Framework

Projectile (P) modelled as a two-body system :
 core (c) + loosely bound nucleon (f) described by

$$H_0 = T_r + V_{cf}(\mathbf{r})$$

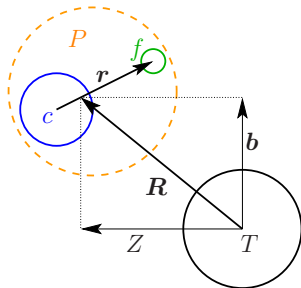
V_{cf} adjusted to reproduce
 bound state Φ_0
 and resonances

Target T seen as
 structureless particle

P - T interaction simulated by optical potentials
 \Rightarrow breakup reduces to **three-body** scattering problem :

$$\left[T_R + H_0 + V_{cT} + V_{fT} \right] \Psi(\mathbf{r}, \mathbf{R}) = E_T \Psi(\mathbf{r}, \mathbf{R})$$

with initial condition $\Psi(\mathbf{r}, \mathbf{R}) \xrightarrow{Z \rightarrow -\infty} e^{iKZ + \dots} \Phi_0(\mathbf{r})$



Dynamical eikonal approximation (DEA)

Three-body scattering problem :

$$\left[T_R + H_0 + V_{cT} + V_{fT} \right] \Psi(\mathbf{r}, \mathbf{R}) = E_T \Psi(\mathbf{r}, \mathbf{R})$$

with condition $\Psi \xrightarrow{Z \rightarrow -\infty} e^{iKZ} \Phi_0$

Eikonal approximation : factorise $\Psi = e^{iKZ} \widehat{\Psi}$

$$T_R \Psi = e^{iKZ} \left[T_R + v P_Z + \frac{\mu_{PT}}{2} v^2 \right] \widehat{\Psi}$$

Neglecting T_R vs P_Z and using $E_T = \frac{1}{2} \mu_{PT} v^2 + \epsilon_0$

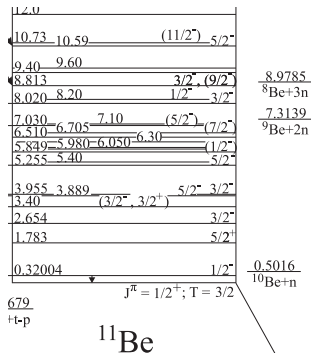
$$i\hbar v \frac{\partial}{\partial Z} \widehat{\Psi}(\mathbf{r}, \mathbf{b}, Z) = [H_0 - \epsilon_0 + V_{cT} + V_{fT}] \widehat{\Psi}(\mathbf{r}, \mathbf{b}, Z)$$

solved for each \mathbf{b} with condition $\widehat{\Psi} \xrightarrow{Z \rightarrow -\infty} \Phi_0(\mathbf{r})$

This is the dynamical eikonal approximation (DEA)

[Baye, P. C., Goldstein, PRL 95, 082502 (2005)]

(Usual) **eikonal** includes the **adiabatic approximation** : $(H_0 - \epsilon_0) \approx 0$



- $\frac{1}{2}^+$ ground state :
 $\epsilon_{\frac{1}{2}^+} = -0.504$ MeV
 In our model, seen as $1s_{\frac{1}{2}}$ neutron
 bound to ${}^{10}\text{Be}(0^+)$
- $\frac{1}{2}^-$ bound excited state :
 $\epsilon_{\frac{1}{2}^-} = -0.184$ MeV
 In our model, seen as $0p_{\frac{1}{2}}$ neutron
 bound to ${}^{10}\text{Be}(0^+)$
- $\frac{5}{2}^+$ bound excited state :
 $\epsilon_{\frac{5}{2}^+} = 1.274$ MeV
 In our model, seen as a $d_{\frac{5}{2}}$ resonance

Usual phenomenological description

In reaction models, projectile \equiv **two-body** system :

$$H_0 = T_r + V_{cn}(\mathbf{r}),$$

where V_{cn} is a phenomenological Woods-Saxon that reproduces the basic nuclear properties of the projectile (binding energy, J^π, \dots)

Nowadays **ab initio** calculations of such exotic nuclei are available
Can we use them within a reaction code ?

But do we need to go that far ?

Breakup reactions are mostly peripheral, i.e., probe :

- ANC of the ground state [P.C. & Nunes, PRC 75, 054609 (2007)]
- phaseshifts in the continuum [P.C. & Nunes, PRC 73, 014615 (2006)]

\Rightarrow **constrain** two-body description by **ab initio** prediction

Ab initio description of ^{11}Be

A recent **ab initio** calculation of ^{11}Be has been performed

[A. Calci *et al.* PRL 117, 242501 (2016)]

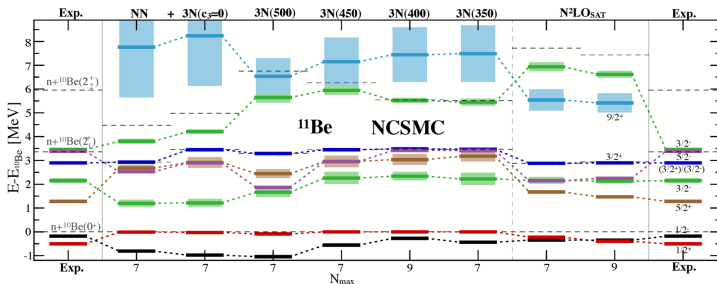
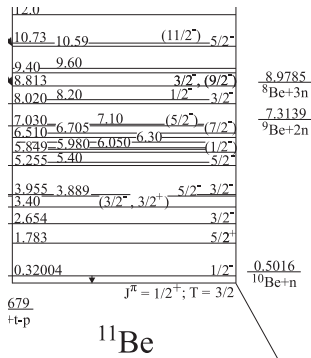


FIG. 2. NCSMC spectrum of ^{11}Be with respect to the $n + ^{10}\text{Be}$ threshold. Dashed black lines indicate the energies of the ^{10}Be states. Light boxes indicate resonance widths. Experimental energies are taken from Refs. [1,51].

Difficult to reproduce the shell inversion

⇒ include phenomenology to obtain the correct ordering

Ab initio description of ^{11}Be bound states



- $\frac{1}{2}^+$ ground state :
 $\epsilon_{\frac{1}{2}^+} = -0.500 \text{ MeV}$
 $C_{\frac{1}{2}^+} = 0.786 \text{ fm}^{-1/2}$
 $S_{1s\frac{1}{2}} = 0.90$
- $\frac{1}{2}^-$ bound excited state :
 $\epsilon_{\frac{1}{2}^-} = -0.184 \text{ MeV}$
 $C_{\frac{1}{2}^-} = 0.129 \text{ fm}^{-1/2}$
 $S_{0p\frac{1}{2}} = 0.85$

Ab initio description of ^{10}Be -n continuum

Provides the most accurate calculation for the ^{10}Be -n continuum

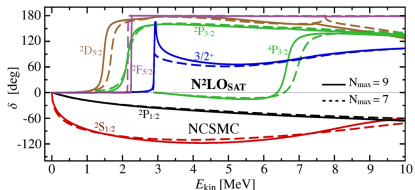


FIG. 3. The $n + ^{10}\text{Be}$ phase shifts as a function of the kinetic energy in the center-of-mass frame. NCSMC phase shifts for the $N^2\text{LO}_{\text{SAT}}$ interaction are compared for two model spaces indicated by N_{max} .

Idea : constrain the ^{10}Be -n potential in the reaction code to reproduce **ab initio** bound states ANC and δ_{lj} .

^{10}Be -n potential

Replace the ^{10}Be -n interaction by **effective** potentials in each partial wave

Use the spirit of **halo EFT** : separation of scales (in energy or in distance)

Use narrow Gaussian potentials

$$V_{lj}(r) = V_0 e^{-\frac{r^2}{2\sigma^2}} + V_2 r^2 e^{-\frac{r^2}{2\sigma^2}}$$

Fit V_0 and V_2 to reproduce ϵ_{lj} , and C_{lj} (bound states)

or Γ_{lj} for resonances

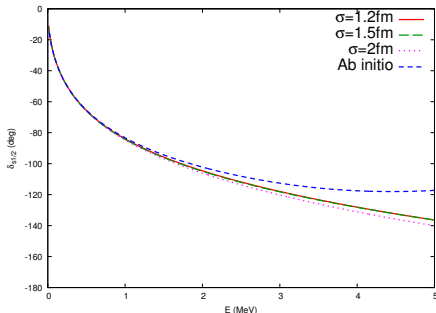
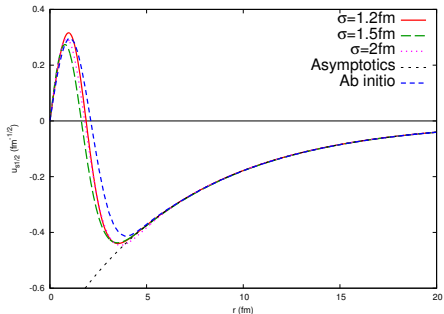
$\sigma = 1.2, 1.5$ or 2 fm is a parameter used to evaluate the sensitivity of the calculations to this effective model

$s_{\frac{1}{2}}$: potentials fitted to $\epsilon_{\frac{1}{2}^+}$ and $C_{\frac{1}{2}^+}$

Potentials fitted to $\epsilon_{1s_{\frac{1}{2}}} = -0.504 \text{ MeV}$ and $C_{1s_{\frac{1}{2}}} = 0.786 \text{ fm}^{-1/2}$

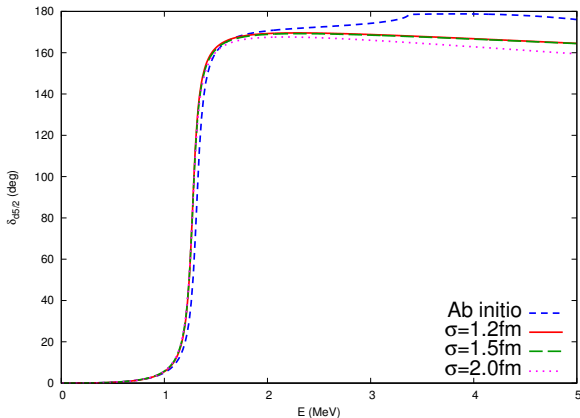
Ground-state wave function

$s_{\frac{1}{2}}$ phaseshifts



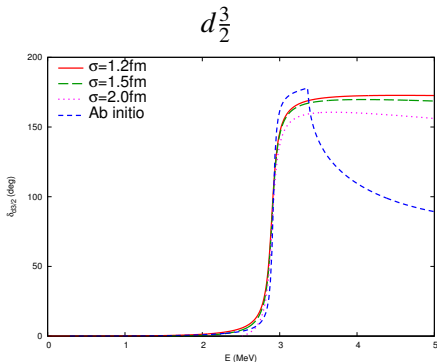
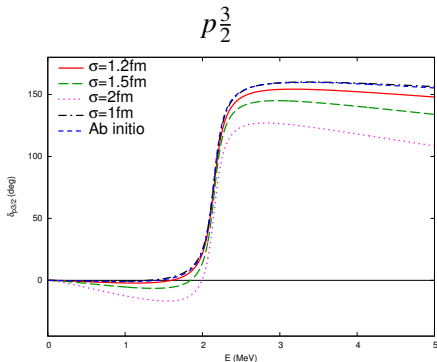
- Wave functions : **same** asymptotics but **different** interior
- $\delta_{s_{\frac{1}{2}}}$: all effective potentials are in **good agreement** with **ab initio** up to 1.5 MeV (same effective-range expansion)
- Similar results obtained for $p_{\frac{1}{2}}$ (excited bound state)

$d_{\frac{5}{2}}^5$: potentials fitted to $\epsilon_{\frac{5}{2}^+}^{\text{res}}$ and $\Gamma_{\frac{5}{2}^+}$



- **Identical** $\delta_{d_{\frac{5}{2}}^5}$ up to 1.5 MeV
up to 5 MeV for the narrow potentials ($\sigma = 1.2$ or 1.5 fm)
- **Excellent agreement** with **ab initio** results up to 2 MeV

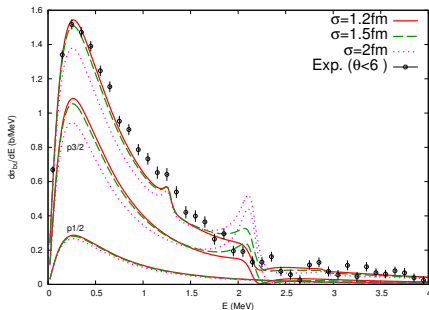
$p_{\frac{3}{2}}$ and $d_{\frac{3}{2}}$: potentials fitted to ϵ^{res} and Γ



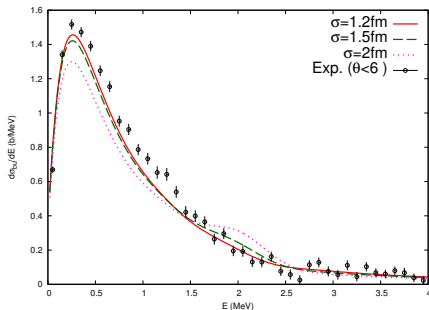
- Large variation in δ obtained by effective potentials
Broad potential ($\sigma = 2\text{ fm}$) cannot reproduce correct behaviour
- Fair agreement with **ab initio** results up to 2.5 MeV
- ^{10}Be core excitation @ 3.4 MeV not described in effective model

$^{11}\text{Be} + \text{Pb} \rightarrow ^{10}\text{Be} + n + \text{Pb}$ @ 69 A MeV

Total breakup cross section
and p contributions



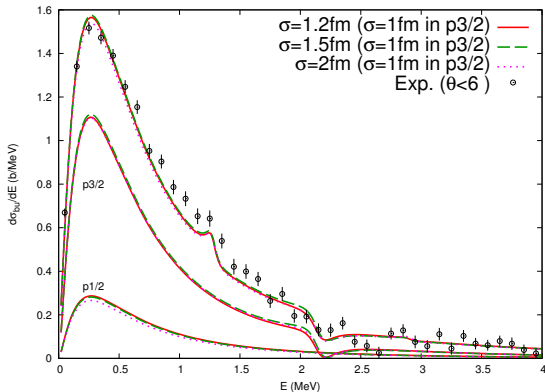
Folded with experimental resolution



- Major differences in $p_{3/2}$ partial wave ; due to differences in $\delta_{p_{3/2}}$
- Broad potential ($\sigma = 2 \text{ fm}$) produces unrealistic $p_{3/2}$ contribution
- Excellent agreement with data [Fukuda *et al.* PRC 70, 054606 (2004)]

Role of $\delta_{p3/2}$

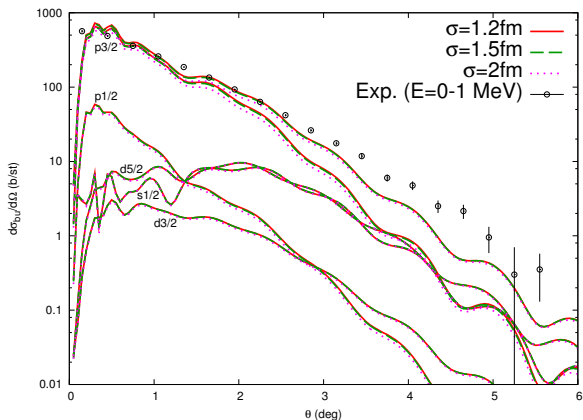
Calculations repeated with different potentials ($\sigma = 1.2, 1.5$ or 2 fm) but in $p_{3/2}$, where $\sigma = 1$ fm (perfect agreement with *ab initio*)



All potentials provide **the same** $p_{3/2}$ contribution

- confirms the **peripherality** of reaction (no influence of the internal part)
- shows the significant role of **phaseshifts**

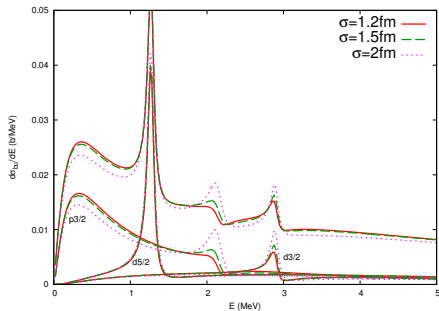
$^{11}\text{Be} + \text{Pb} \rightarrow ^{10}\text{Be} + n + \text{Pb}$ @ 69 A MeV



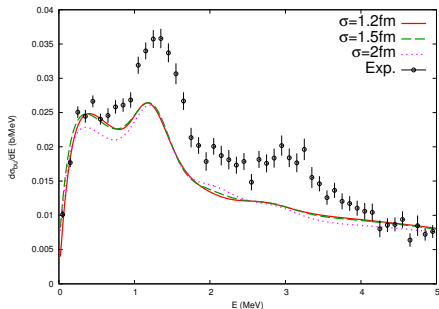
- Good agreement with experiment [Fukuda *et al.* PRC 70, 054606 (2004)]
- All potentials provide similar cross sections ($\sigma = 2$ fm slightly lower)

$^{11}\text{Be} + \text{C} \rightarrow ^{10}\text{Be} + \text{n} + \text{C}$ @ 67A MeV

Total breakup cross section and dominant contributions



Folded with experimental resolution
[Fukuda *et al.* PRC 70, 054606 (2004)]



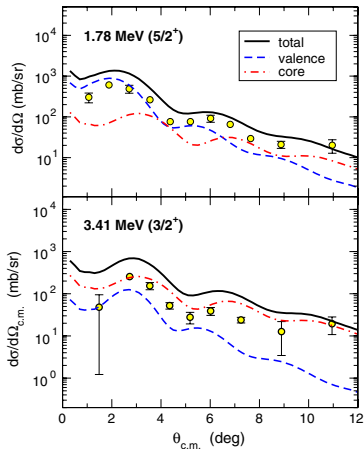
- All potentials produce similar breakup cross sections (but $\sigma = 2\text{fm}$)
- In nuclear breakup, **resonances** play significant role
- Order of magnitude of experiment well reproduced
- But **resonant breakup** not correctly described due to **short-range** details missing in the effective model (?)

Effect of core-excitation in resonant breakup

$^{11}\text{Be} + \text{C} \rightarrow ^{10}\text{Be} + \text{n} + \text{C}$ @ 67 A MeV

computed in an extended DWBA model including **core excitation**

[A. Moro & J.A. Lay, PRL 109, 232502 (2012)]

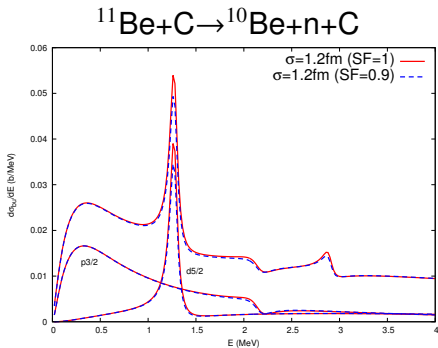
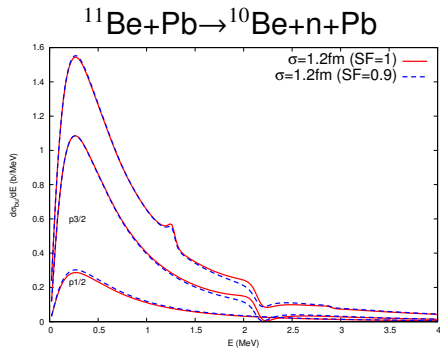


- Breakup due to the excitation of the **valence** neutron and of the **core** are considered
- **Both** are needed to reproduce the oscillatory pattern of experiment
- **Core excitation** dominates the $\frac{3}{2}^+$ resonant breakup
- Confirms the missing short-range details in our effective model

SF vs ANC

Calci *et al.* predict $\mathcal{S}_{1s\frac{1}{2}} = 0.90$, but we use $\mathcal{S}_{1s\frac{1}{2}} = 1 \dots$

\Rightarrow repeat calculations with $\mathcal{S}_{1s\frac{1}{2}} = 0.90$ (keeping $C_{\frac{1}{2}^+} = 0.786 \text{ fm}^{-1/2}$)



No difference \Rightarrow SF cannot be extracted from these measurements

One exception : **resonant** breakup, where SF plays a role

\Rightarrow influence of the short-range details (?)

Summary and prospect

- Exotic nuclei studied mostly through **reactions**
- Mechanism of reactions with halo nuclei understood
How to improve the projectile description in reaction models ?
- **Ab initio** models **too expensive** to be used in reaction codes
⇒ include the predictions that matter in **effective model**
- Using **Gaussian** potentials, we reproduce the **ANC** and **phase shifts** predicted by **ab initio** calculations
- Our study confirms
 - **peripherality** of breakup reactions
 - influence of the continuum through **phase shifts**
- Using **ab initio** predictions gives **excellent agreement** with data
 - efficient way to include the significant degrees of freedom
 - provides an estimate the influence of omitted mechanisms
e.g., resonances include short-range details

Thanks to my collaborators

Daniel Baye
Gerald Goldstein



Achim Schwenk
Hans-Werner Hammer



Daniel Phillips



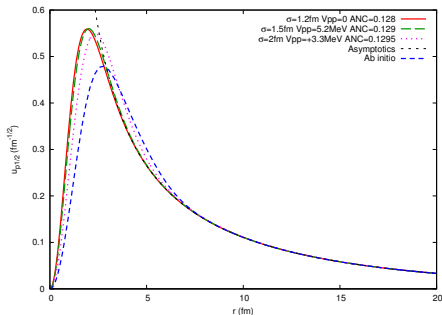
Filomena Nunes



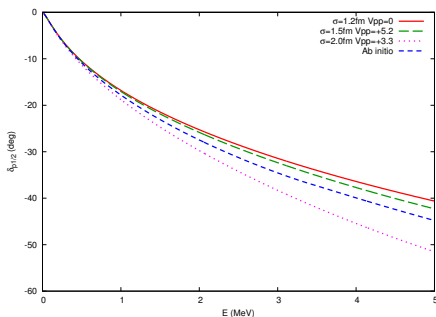
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Potentials fitted to $\epsilon_{0p_{\frac{1}{2}}} = -0.184$ MeV and $C_{0p_{\frac{1}{2}}} = 0.129$ fm $^{-1/2}$

Excited-state wave function



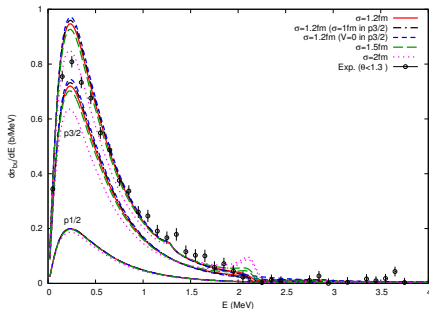
$p_{1/2}$ phaseshifts



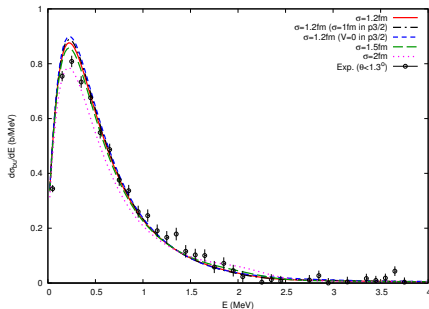
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Fair agreement with **ab initio** results up to 1 MeV

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Total breakup cross section
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[Fukuda *et al.* PRC 70, 054606 (2004)]



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