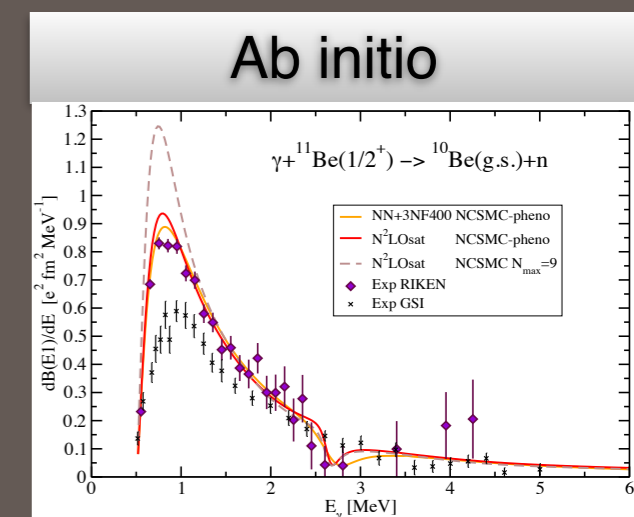
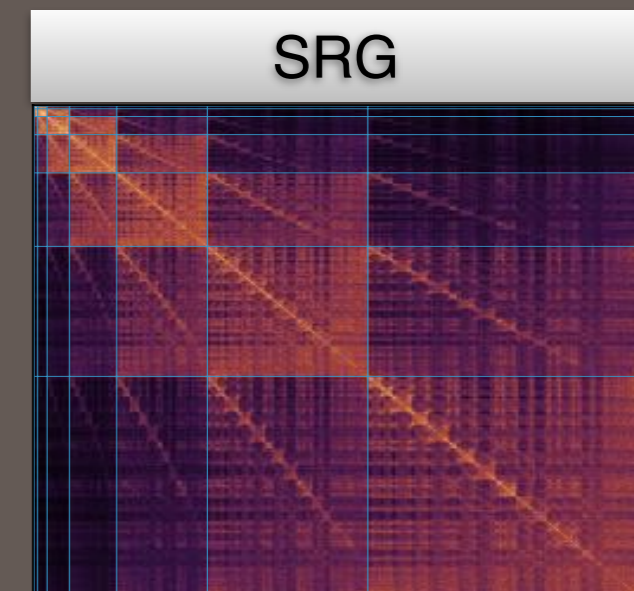
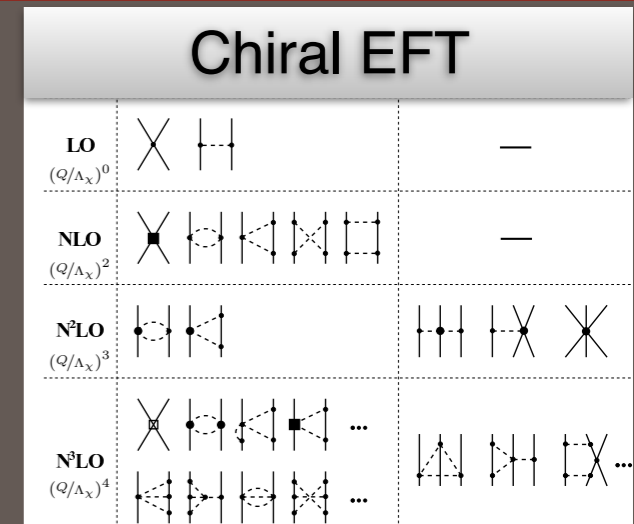


Predictive Power of Chiral Interactions for Nuclear Structure and Reaction Calculations in the p-Shell

INT Program INT-17-1a
 March 27 2017, Seattle

Angelo Calci | TRIUMF

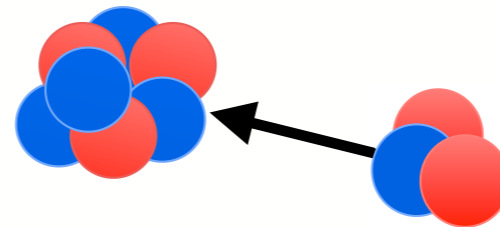


ab initio description of nuclei

QCD-based interaction
realistic NN+3N interactions

bound states &
spectroscopy

resonances &
scattering states



**(Importance Truncated)
NCSM**

ab initio description of
nuclear clusters

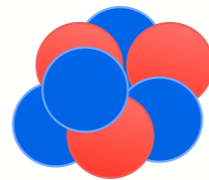
**Resonating Group
Method (RGM)**

describing relative
motion of clusters

NCSM with Continuum
continuum effects
in spectroscopy

ab initio description of nuclei

QCD-based interaction
realistic NN+3N interactions



Chiral NN+3N Interactions

Weinberg, van Kolck, Machleidt, Entem, Meissner, Epelbaum, Krebs, Bernard,...

● standard interaction:

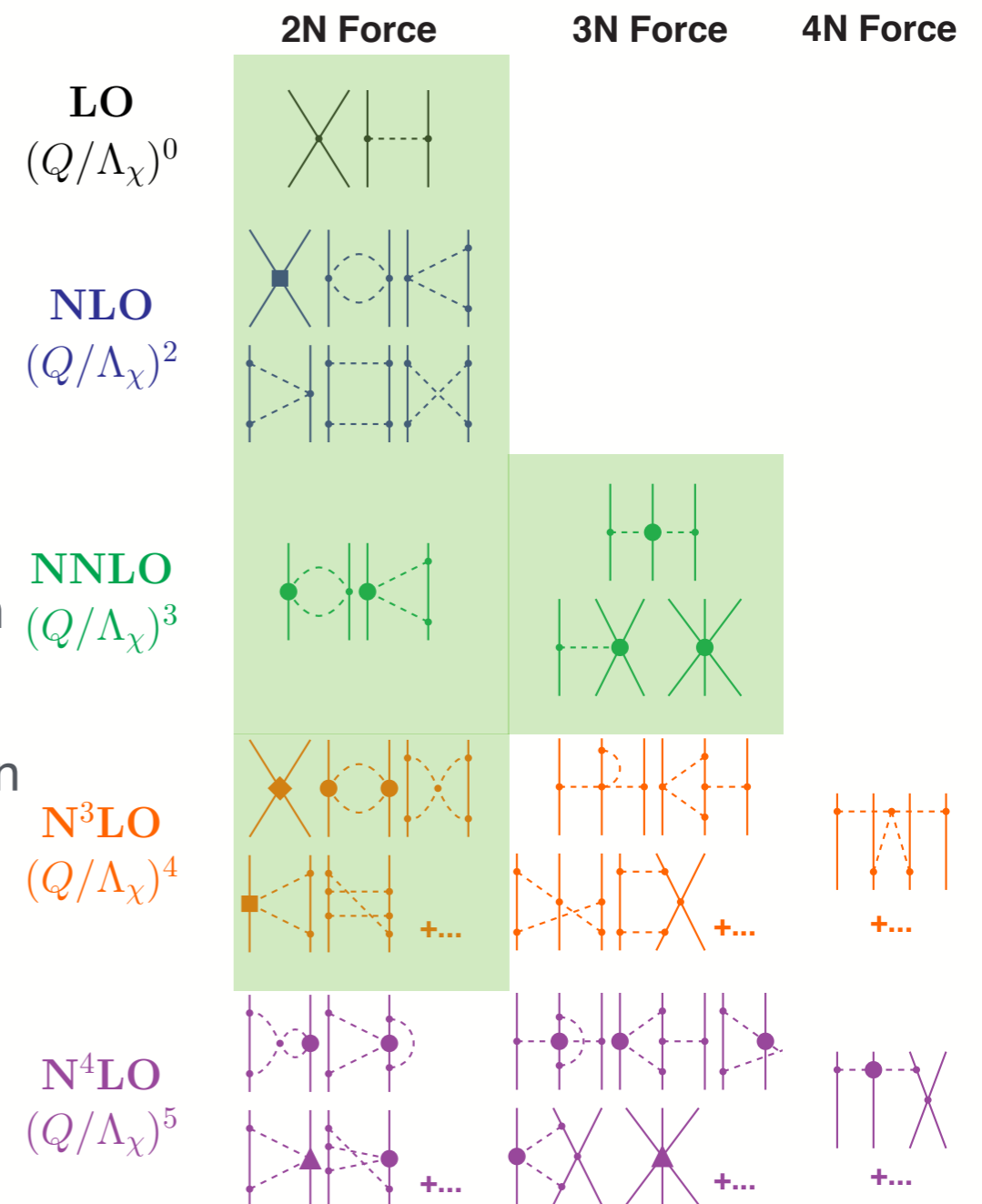
- NN @ N³LO: Entem & Machleidt, 500MeV cutoff
- 3N @ N²LO: Navrátil, local, 500MeV cutoffs & modifications of the 3N force

● optimized N²LO interaction:

- NN: Ekström et al., 500MeV cutoff, LECs fitted with POUNDerS
- 3N: Navrátil, local, 500MeV cutoff, fit to ⁴He & Triton

● EGM N²LO interaction:

- NN: Epelbaum et al., 450, . . . , 600 MeV cutoff
- 3N: Epelbaum et al., 450, . . . , 600 MeV cutoff, nonlocal



Chiral NN+3N Interactions

Weinberg, van Kolck, Machleidt, Entem, Meissner, Epelbaum, Krebs, Bernard,...

● standard interaction:

- NN @ N³LO: Entem & Machleidt, 500MeV cutoff
- 3N @ N²LO: Navrátil, local, 500MeV cutoffs & modifications of the 3N force

● chiral interactions are not unique:

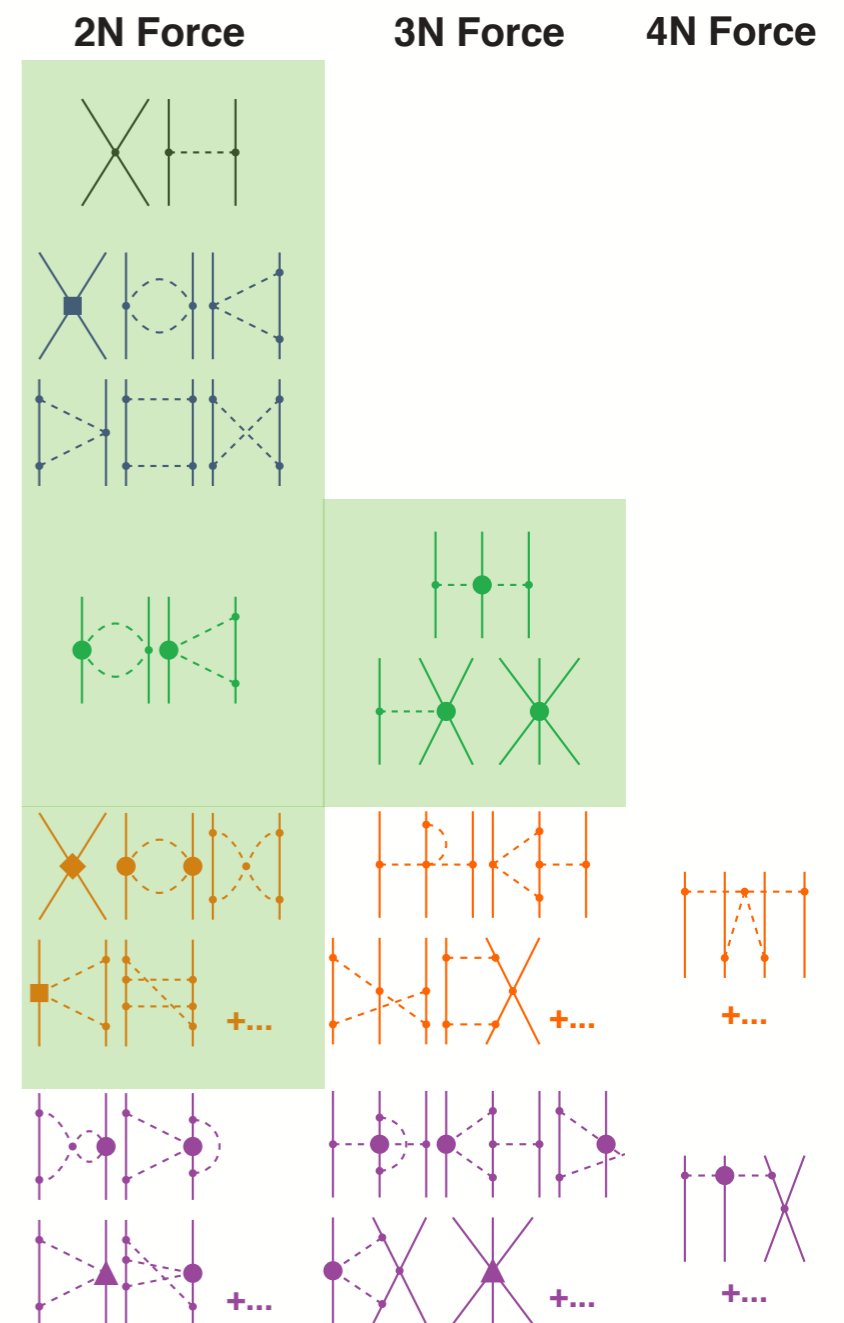
- chiral order
- regularization
- fit of low-energy constants (LECs)
- (power counting)

- NN: Epelbaum et al., 450, . . . , 600 MeV cutoff
- 3N: Epelbaum et al., 450, . . . , 600 MeV cutoff, nonlocal

LO
(Q/Λ_χ)⁰

NLO
²

N⁴LO
(Q/Λ_χ)⁵



Next Generation Interactions

Weinberg, van Kolck, Machleidt, Entem, Meissner, Epelbaum, Krebs, Bernard,...

- **standard interaction:**

- NN @ N³LO: Entem & Machleidt, 500MeV cutoff
- 3N @ N²LO: Navrátil, local cutoffs

- **N²LO_{SAT} interaction:**

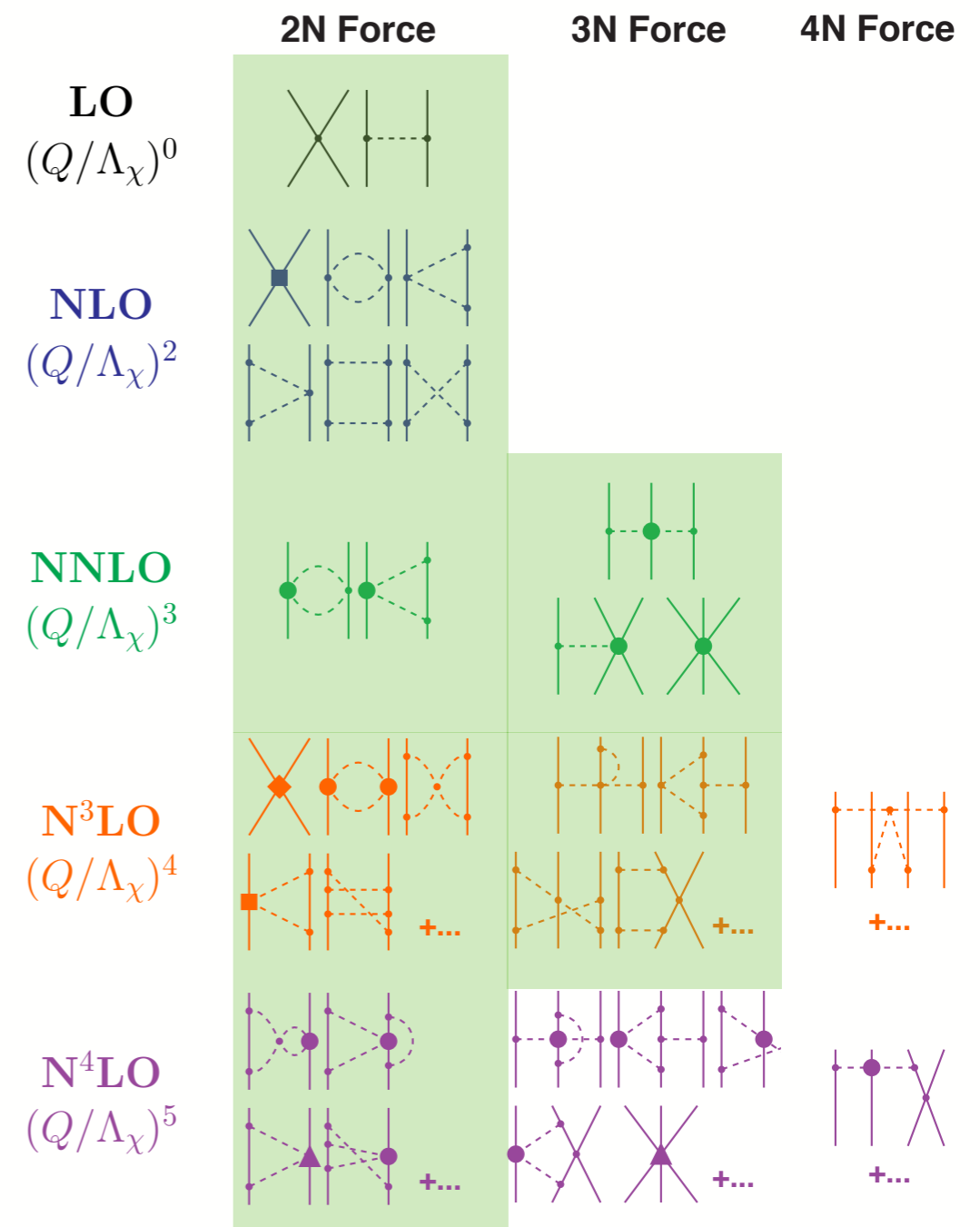
- NN+3N: Ekström et al., nonlocal 450MeV cutoff, simultaneous fit to NN data and selected many-body observables

- **LENPIC interaction:**

- NN up to N⁴LO: Epelbaum et al., semi-local cutoff
- 3N up to N³LO: under construction

- **N⁴LO(500):**

- NN @ N⁴LO: Machleidt et al., 500MeV cutoff



Similarity Renormalization Group (SRG)

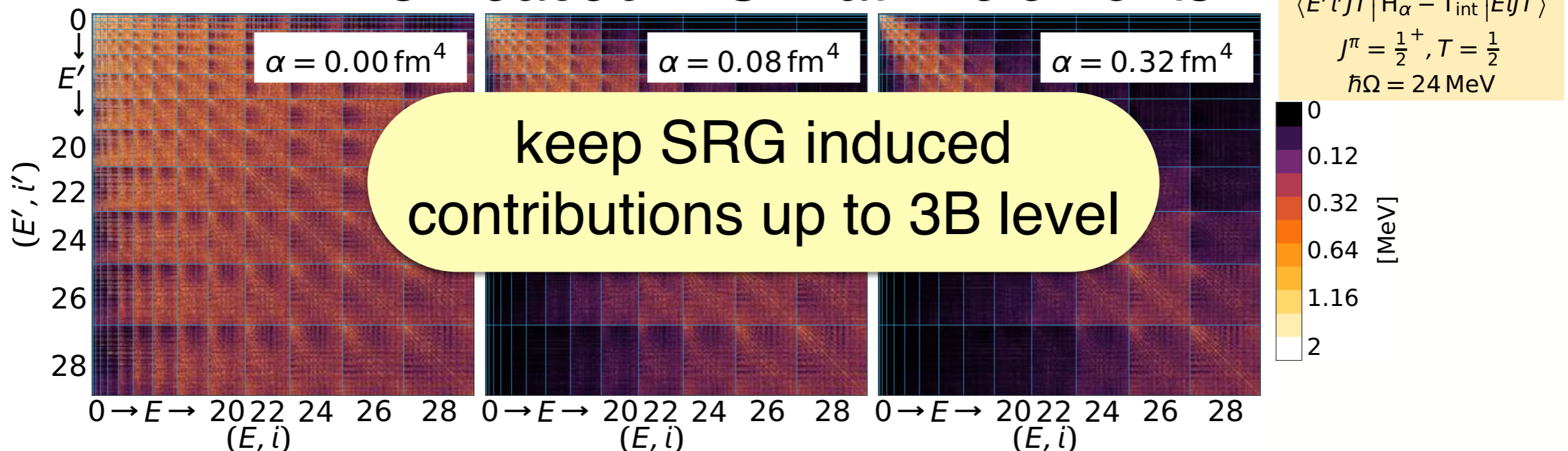
accelerate convergence by **pre-diagonalizing** the Hamiltonian with respect to the many-body basis

- unitary transformation leads to evolution equation**

$$\frac{d}{d\alpha} \tilde{H}_\alpha = [\eta_\alpha, \tilde{H}_\alpha] \quad \text{with} \quad \eta_\alpha = (2\mu)^2 [T_{\text{int}}, \tilde{H}_\alpha] = -\eta_\alpha^\dagger$$

advantages of SRG: **flexibility** and **simplicity**

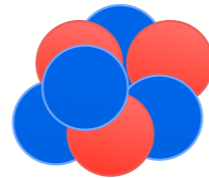
3B-Jacobi HO matrix elements



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**(Importance Truncated)
NCSM**

ab initio description of
nuclear clusters

No-Core Shell Model (NCSM)

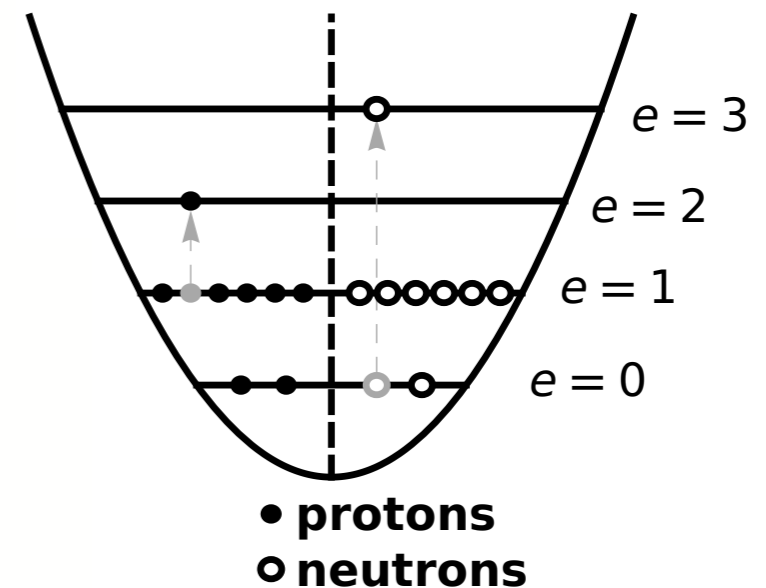
- solving the eigenvalue problem:

$$H |\Psi_n\rangle = E_n |\Psi_n\rangle$$

- **model space:**

spanned by Slater determinants with unperturbed excitation energy

up to $N_{max}\hbar\Omega$



No-Core Shell Model (NCSM)

- solving the eigenvalue problem:

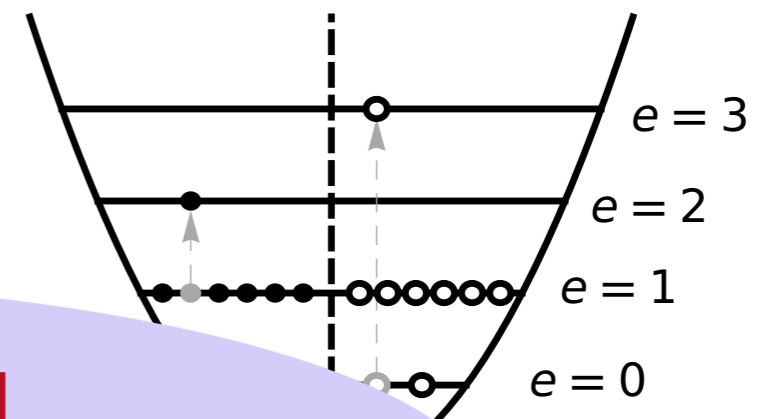
$$H |\Psi_n\rangle = E_n |\Psi_n\rangle$$

- **model space:**

spanned by
 unperturbed
 up to $N_{max} \hbar\Omega$

problem of NCSM

enormous increase of model space with
 particle number A



No-Core Shell Model (NCSM)

- solving the eigenvalue problem:

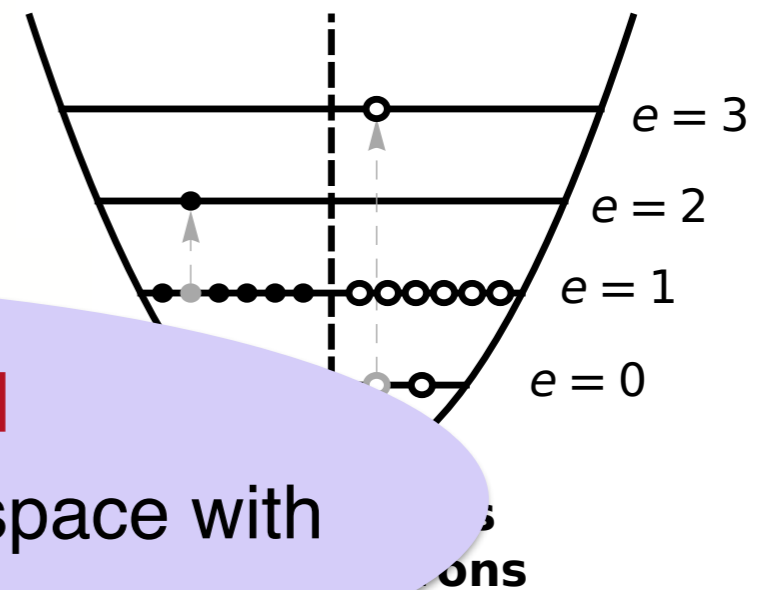
$$H |\Psi_n\rangle = E_n |\Psi_n\rangle$$

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Importance Truncated NCSM

- a priori determination of relevant basis states via first-order perturbation theory

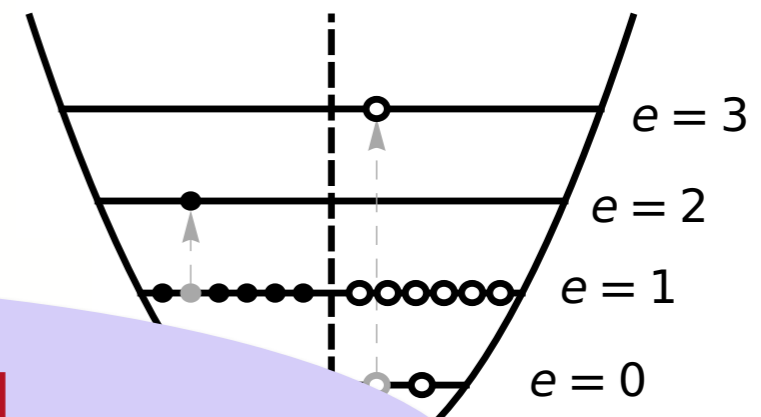
$$K_\nu = - \frac{\langle \Phi_\nu | H_{\text{int}} | \Psi_{\text{ref}} \rangle}{\epsilon_\nu - \epsilon_{\text{ref}}}$$

- **importance truncated space** spanned by basis states with $|K_\nu| \geq K_{\text{min}}$

No-Core Shell Model (NCSM)

- solving the eigenvalue problem:

$$H |\Psi_n\rangle = E_n |\Psi_n\rangle$$



- **model space:**

spanned by
unperturbed
up to $N_{max} \hbar\Omega$

problem of NCSM

enormous increase of model space with
particle number A

Importance Truncation

- **extrapolation** of $\kappa_{min} \rightarrow 0$ recovers effect of omitted contributions
- IT-NCSM provides **same results** as full NCSM
- **expands** application **range** to larger A
- **importance truncated space** spanned by basis states with $|K_\nu| \geq \kappa_{min}$

Sensitivity on chiral 3N interactions

- analyze the sensitivity of spectra on **low-energy constants** (C_i, C_D, C_E) and **cutoff** (Λ) of the chiral 3N interaction at N²LO



- why this is interesting:

- impact of N³LO contributions:** some N³LO diagrams can be absorbed into the N²LO structure by shifting the c_i constants

$$\bar{c}_1 = c_1 - \frac{g_A^2 M_\pi}{64\pi F_\pi^2}, \quad \bar{c}_3 = c_3 + \frac{g_A^4 M_\pi}{16\pi F_\pi^2}, \quad \bar{c}_4 = c_4 - \frac{g_A^4 M_\pi}{16\pi F_\pi^2} \quad \text{(Bernard et al., Ishikawa, Robilotta)}$$

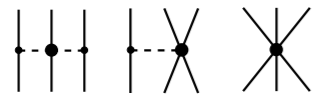
- uncertainty propagation:** sizable variations of the c_i from different extractions (also affects NN)

$$c_1 = -1.23\dots - 0.76, \quad c_3 = -5.94\dots - 3.20, \quad c_4 = 3.40\dots 5.40 \text{ [GeV}^{-1}\text{]}$$

- cutoff dependence:** does the cutoff choice in the 3N interaction affect nuclear structure observables?

Sensitivity on chiral 3N interactions

- analyze the sensitivity of spectra on **low-energy constants** (C_i, C_D, C_E) and **cutoff** (Λ) of the chiral 3N interaction at N²LO



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(Bernard et al.,
Ishikawa, Robilotta)

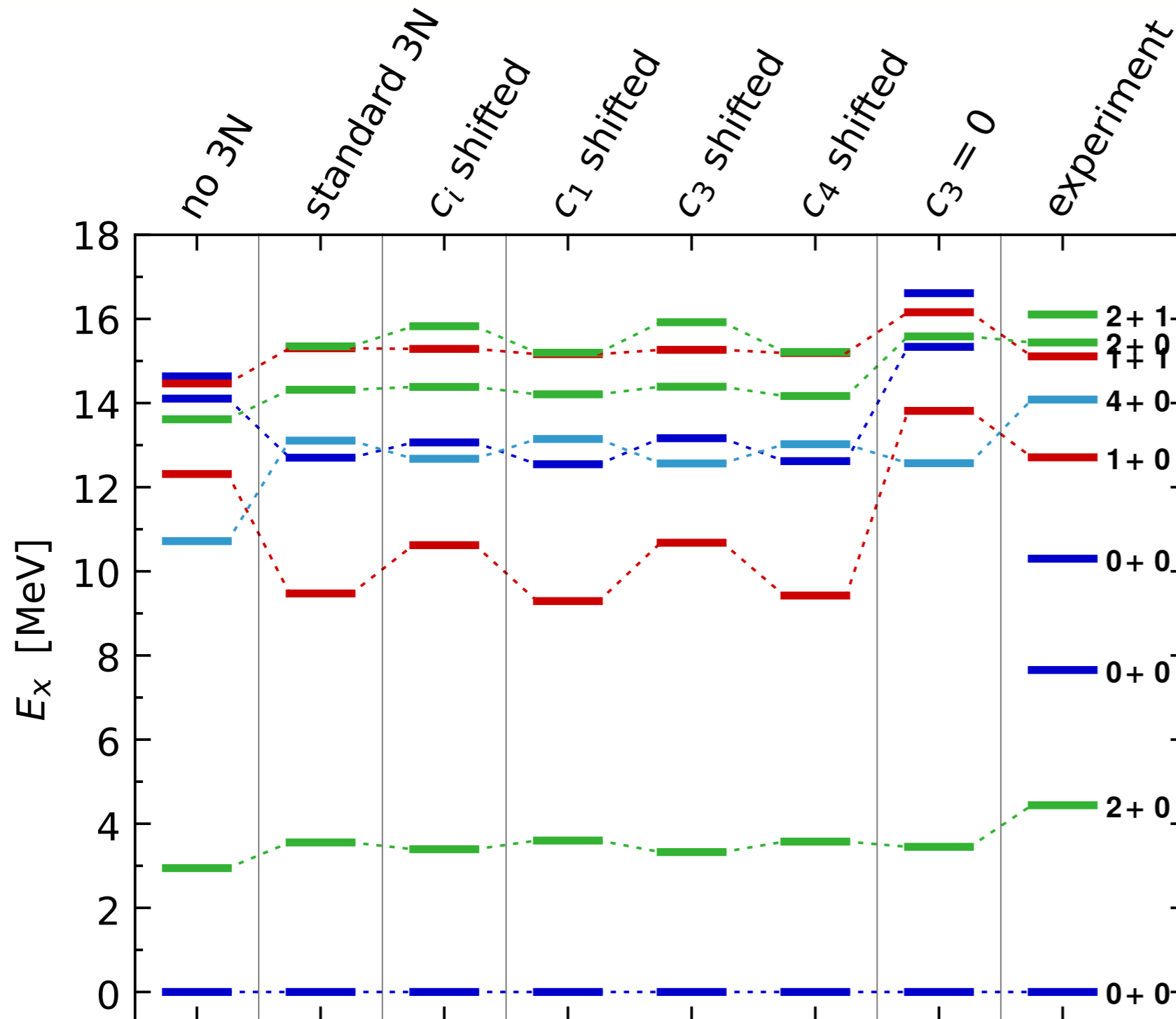
- uncertainty propagation:** sizable variations in c_i provide different extractions (also affects NN) **constraints** for chiral

$$c_1 = -1.23... - 0.76, \quad c_3 = -5.94... - 3$$

provide different constraints for chiral Hamiltonians and quantify uncertainties

- cutoff dependence:** does the cutoff of the 3N interaction affect nuclear structure observables?

^{12}C : Sensitivity to c_i



- many states are rather c_i independent
- first $1+$ state shows strong c_3 dependence

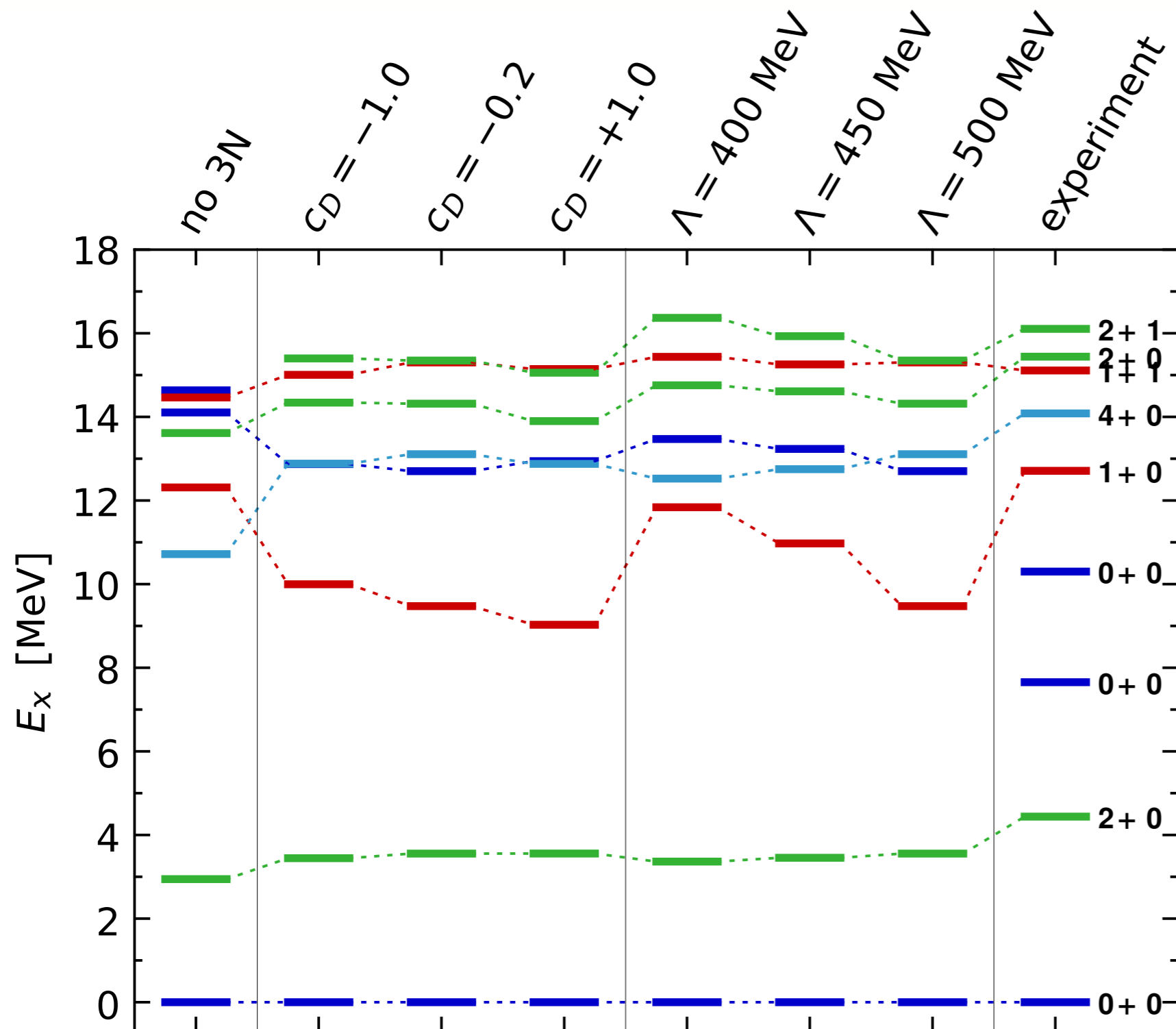
IT-NCSM

$$\hbar\Omega = 16 \text{ MeV}$$

$$N_{\text{max}} = 8$$

$$\alpha = 0.08 \text{ fm}^4$$

^{12}C : Sensitivity to c_D and cutoff

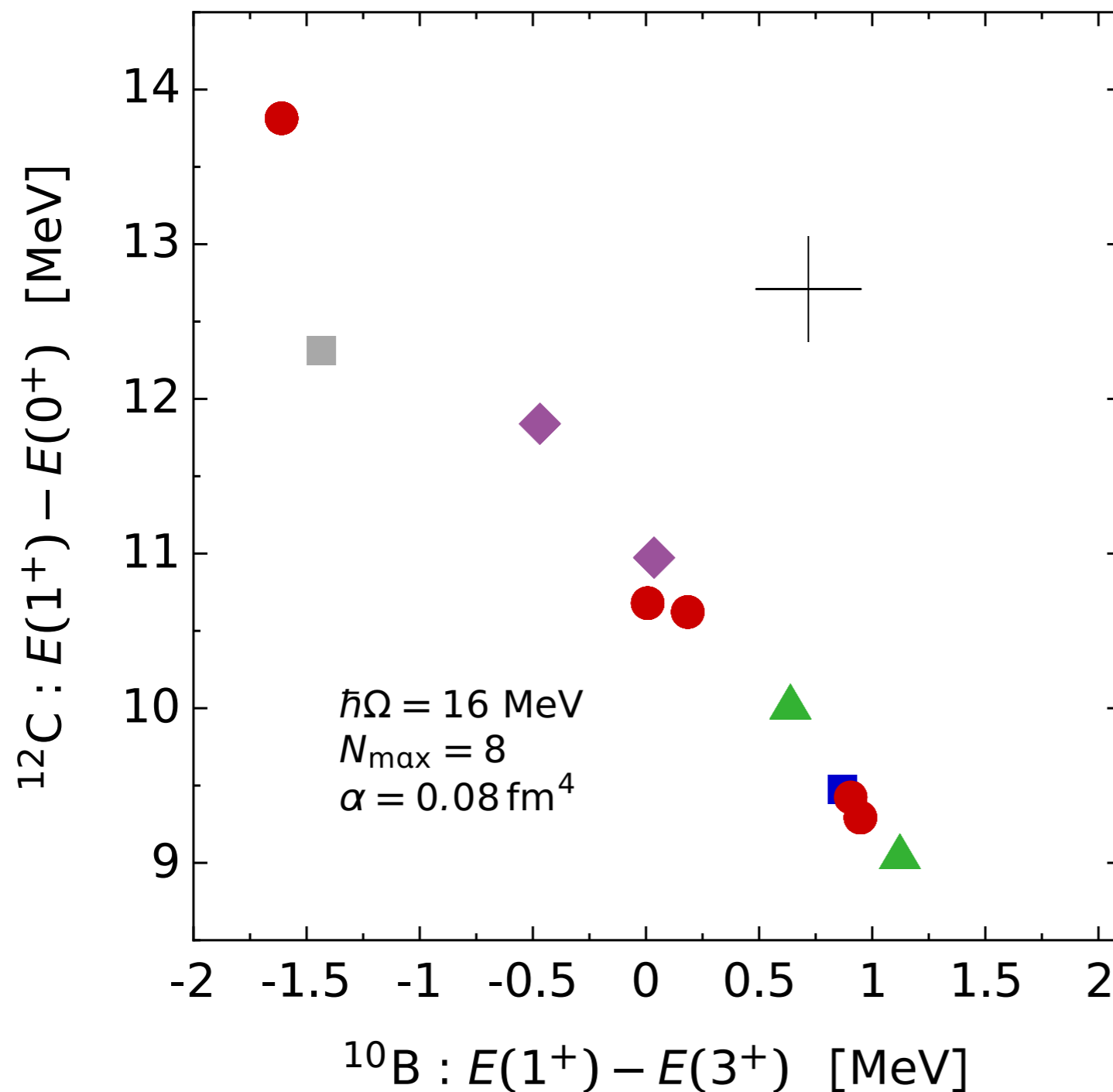


- moderate dependence on c_D , stronger dependence on Λ
- again first 1^+ state is most sensitive

IT-NCSM

$\hbar\Omega = 16$ MeV
 $N_{\text{max}} = 8$
 $\alpha = 0.08 \text{ fm}^4$

Correlation Analysis: $^{12}\text{C}(1^+)$ vs. $^{10}\text{B}(1^+)$

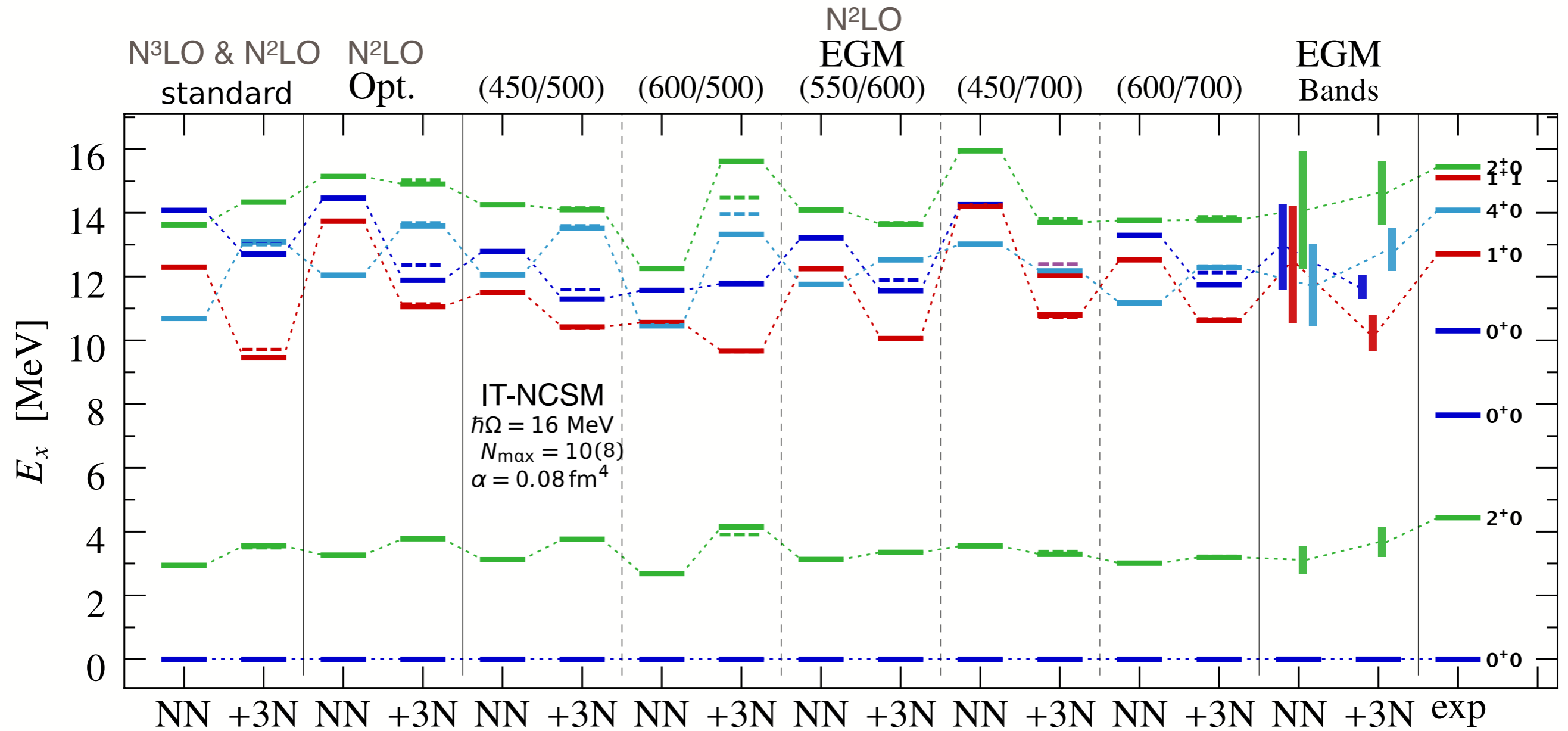


- correlation does not agree with experiment
- hints at problems with E&M NN interaction

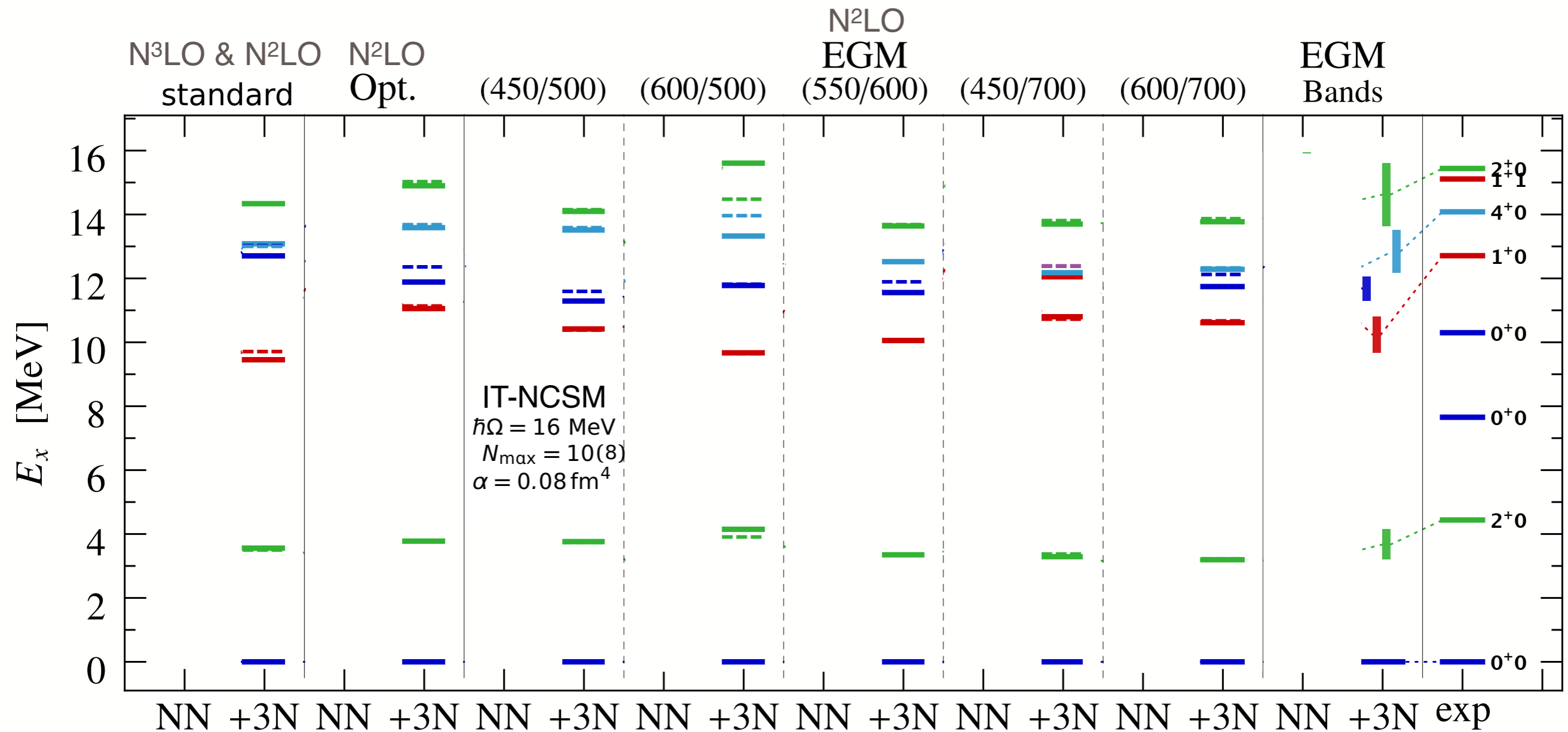
standard

- no 3N
- std 3N
- c_i var
- ▲ c_D var
- ◆ Λ var
- + exp

^{12}C : Cutoff Dependence

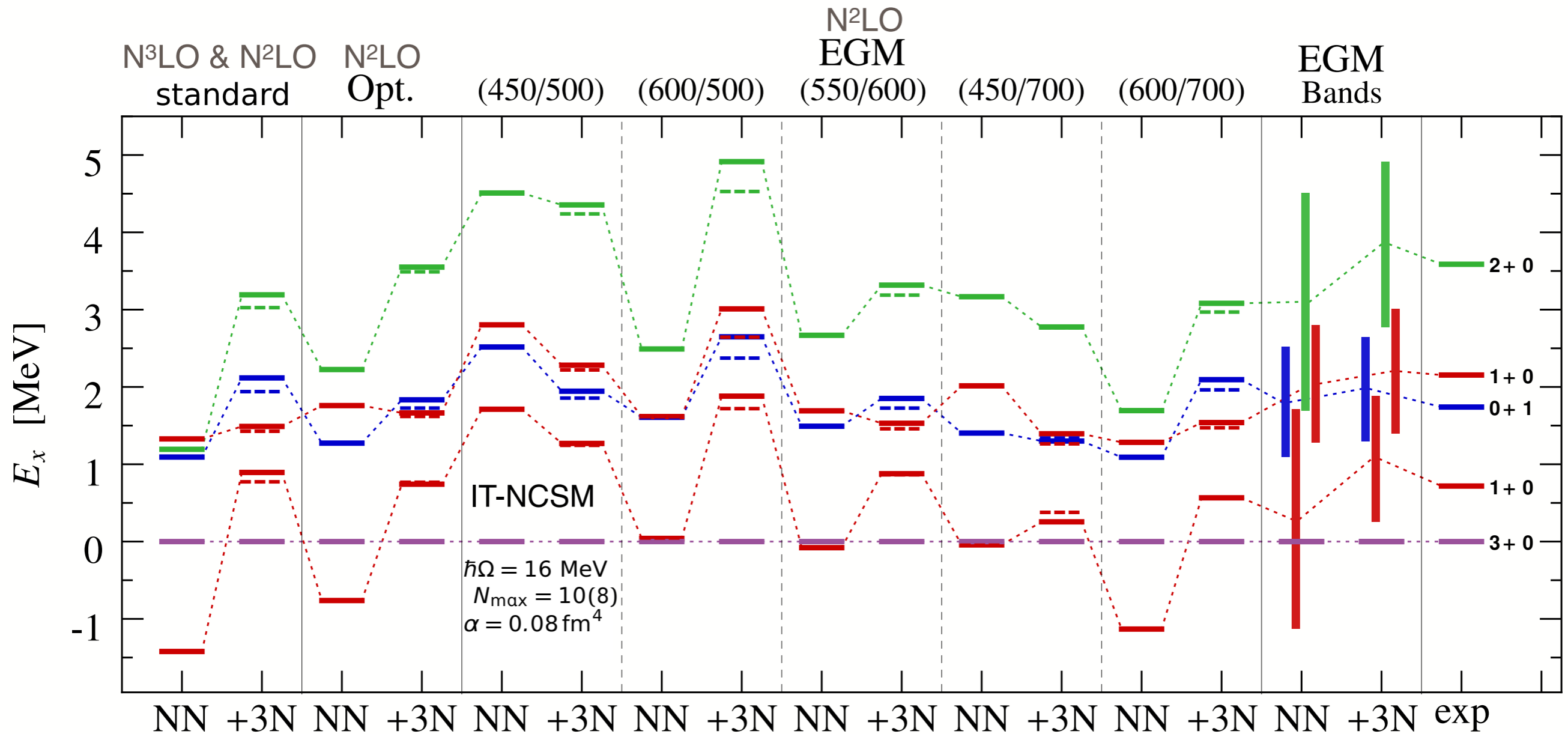


^{12}C : Cutoff Dependence



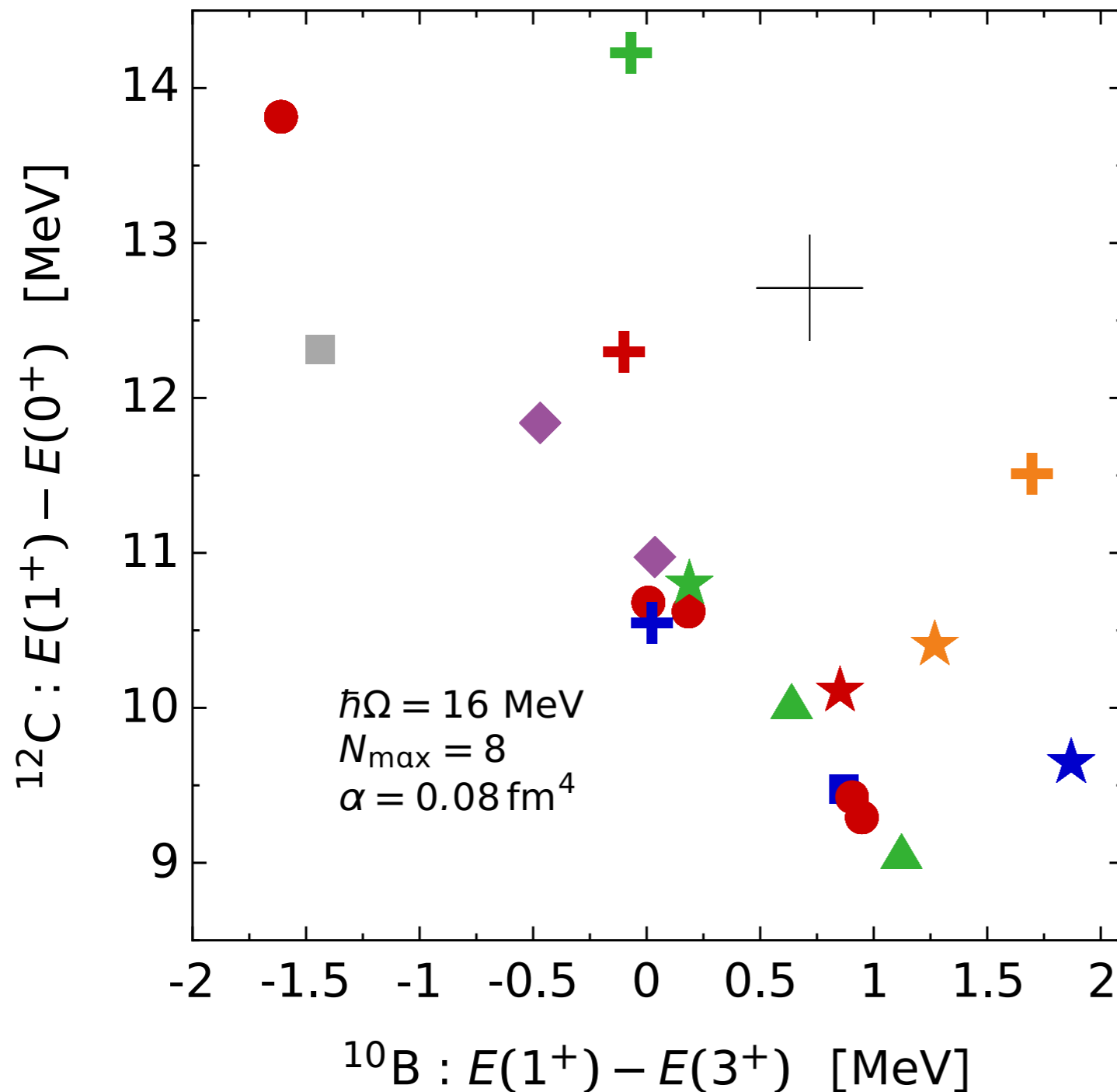
- small cutoff dependence for NN+3N

^{10}B : Cutoff Dependence



- complex system with compressed spectrum
- accurate predictions within large uncertainties

Correlation Analysis: $^{12}\text{C}(1^+)$ vs. $^{10}\text{B}(1^+)$



- no obvious correlation for NN modifications

standard

- no 3N
- std 3N
- c_i var
- ▲ c_D var
- ◆ Λ var

+ exp

EGM

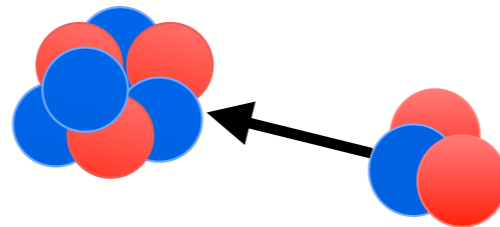
- ✦ no 3N (450,500)
- ✦ no 3N (600,500)
- ✦ no 3N (550,600)
- ✦ no 3N (450,700)
- ★ 3N (450,500)
- ★ 3N (600,500)
- ★ 3N (550,600)
- ★ 3N (450,700)

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in spectroscopy

Can *Ab Initio* Theory Explain the Phenomenon of Parity Inversion in ^{11}Be ?

Angelo Calci,^{1,*} Petr Navrátil,^{1,†} Robert Roth,² Jérémy Dohet-Eraly,^{1,‡} Sofia Quaglioni,³ and Guillaume Hupin^{4,5}

¹TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

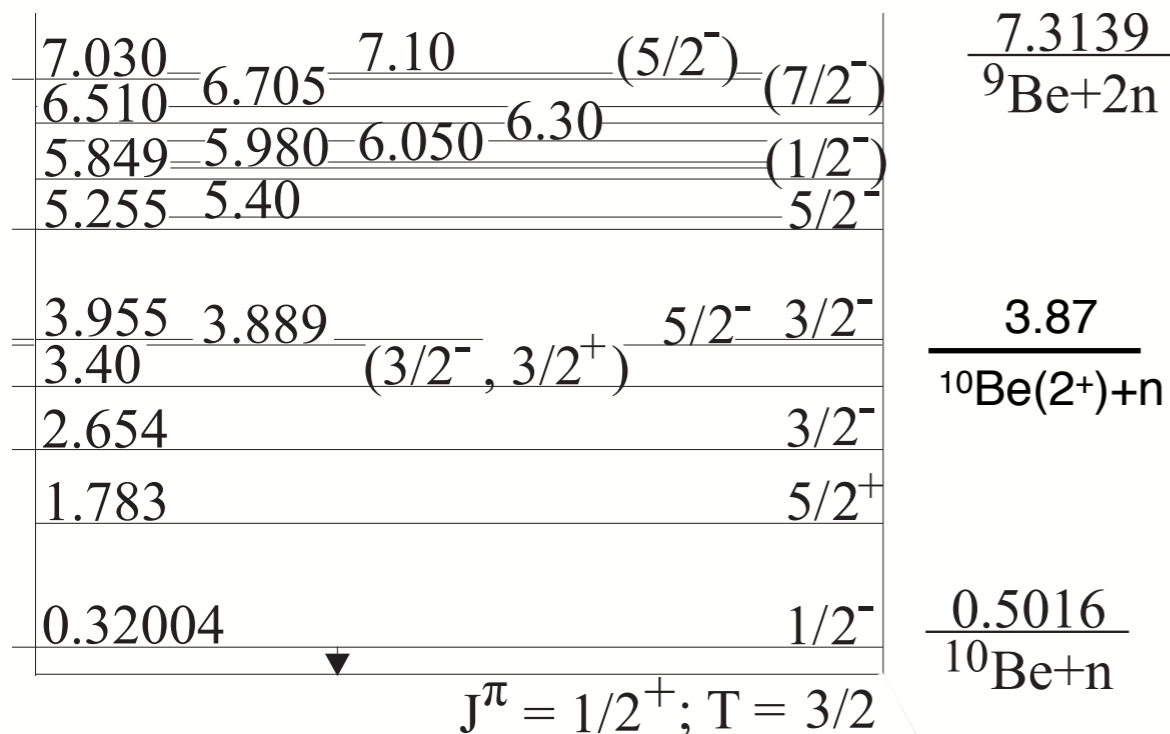
²Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany

³Lawrence Livermore National Laboratory, P.O. Box 808, L-414, Livermore, California 94551, USA

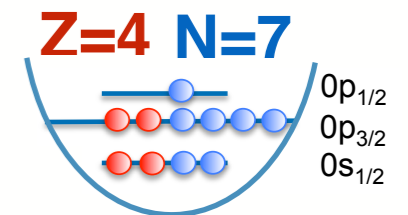
⁴Institut de Physique Nucléaire, Université Paris-Sud, IN2P3/CNRS, F-91406 Orsay Cedex, France

⁵CEA, DAM, DIF, F-91297 Arpajon, France

Spectrum



- **parity inversion**
shell model predicts
g.s. to be $J^\pi=1/2^-$
- **Halo structure**
weakly bound $J=1/2$ states
spectrum dominated by $n-^{10}\text{Be}$



Neutron-rich halo Nucleus ^{11}Be

PRL 117, 242501 (2016)

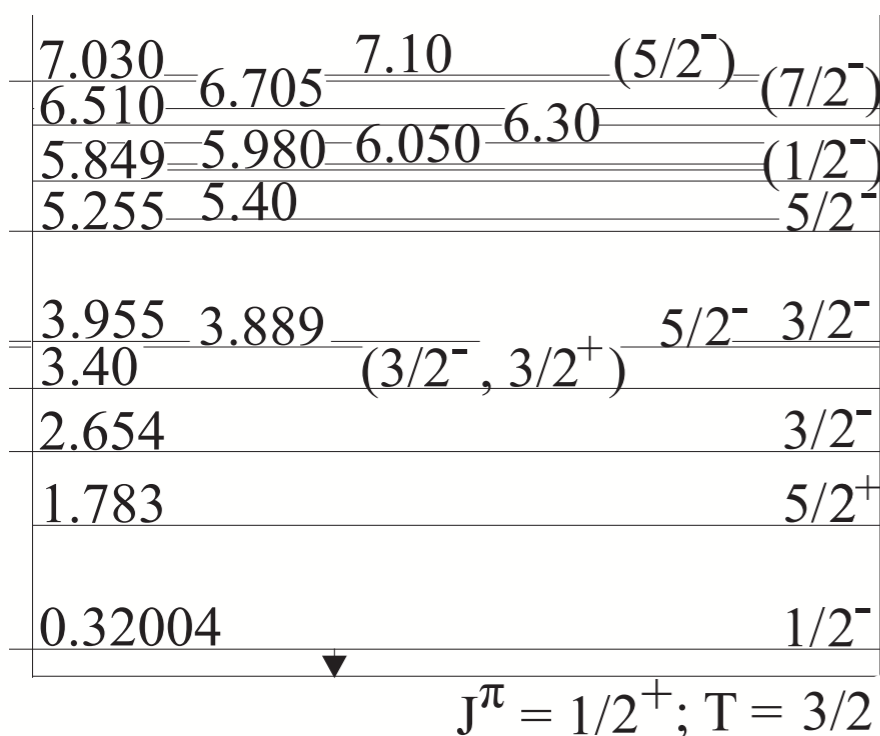
PHYSICAL REVIEW LETTERS

week ending
9 DECEMBER 2016

Can *Ab Initio* Theory Explain the Phenomenon of Parity Inversion in ^{11}Be ?

Angelo Calci,^{1,*} Petr Navrátil,² ...
¹TRIUMF, Vancouver, Canada
²Institut für Experimentelle Kernphysik, Universität Wien, Austria
³Lawrence Livermore National Laboratory, Livermore, California 94551, USA
⁴Institut de Physique Nucléaire, Université de Strasbourg, Strasbourg, France

Can **ab initio** theory describe this complicated system?

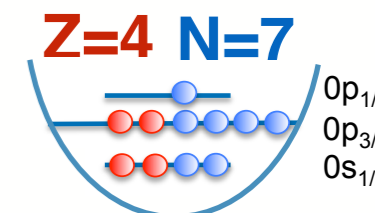


$$\frac{7.3139}{^9\text{Be}+2n}$$

$$\frac{3.87}{^{10}\text{Be}(2^+)+n}$$

$$\frac{0.5016}{^{10}\text{Be}+n}$$

- **parity inversion**
shell model predicts g.s. to be $J^\pi=1/2^-$
- **Halo structure**
weakly bound $J=1/2$ states
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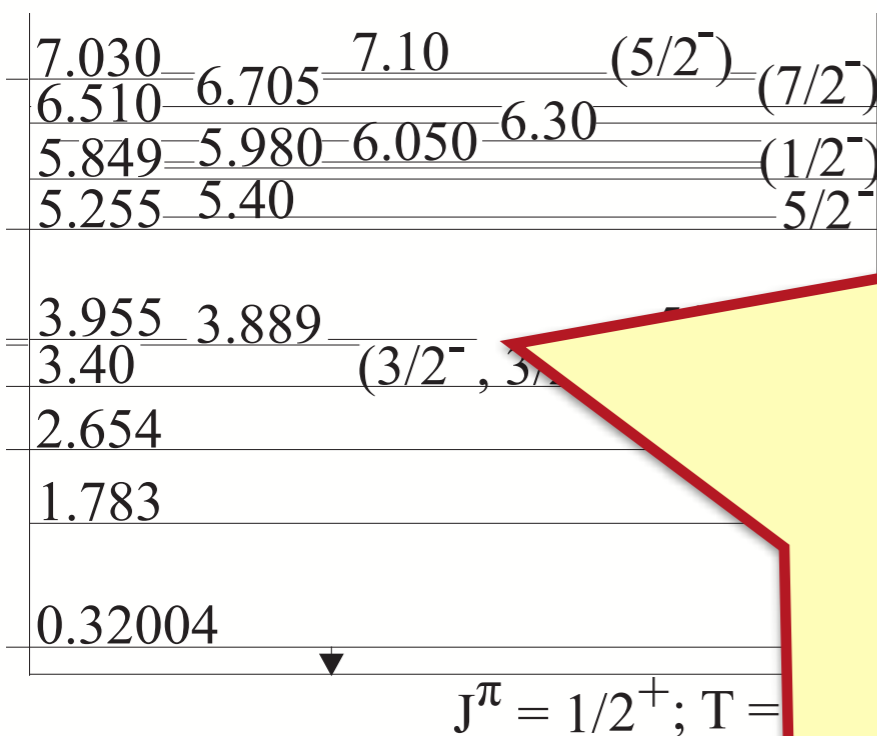
⁵TRIUMF, Vancouver, BC V6P 6S1, Canada

⁶Physikalisches Institut der Universität Bonn, Germany

⁷Lawrence Livermore National Laboratory, Livermore, California 94551, USA

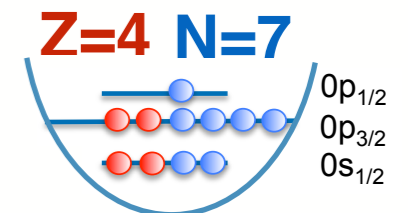
⁸CEA Saclay, 91191 Gif-sur-Yvette Cedex, France

Can **ab initio** theory describe this complicated system?



$\frac{7.3139}{^9\text{Be}+2n}$

- **parity inversion** shell model predicts states to be $J^\pi=1/2^-$



YES

and $J=1/2$ states

dominated by $n-^{10}\text{Be}$

Neutron-rich halo Nucleus ^{11}Be

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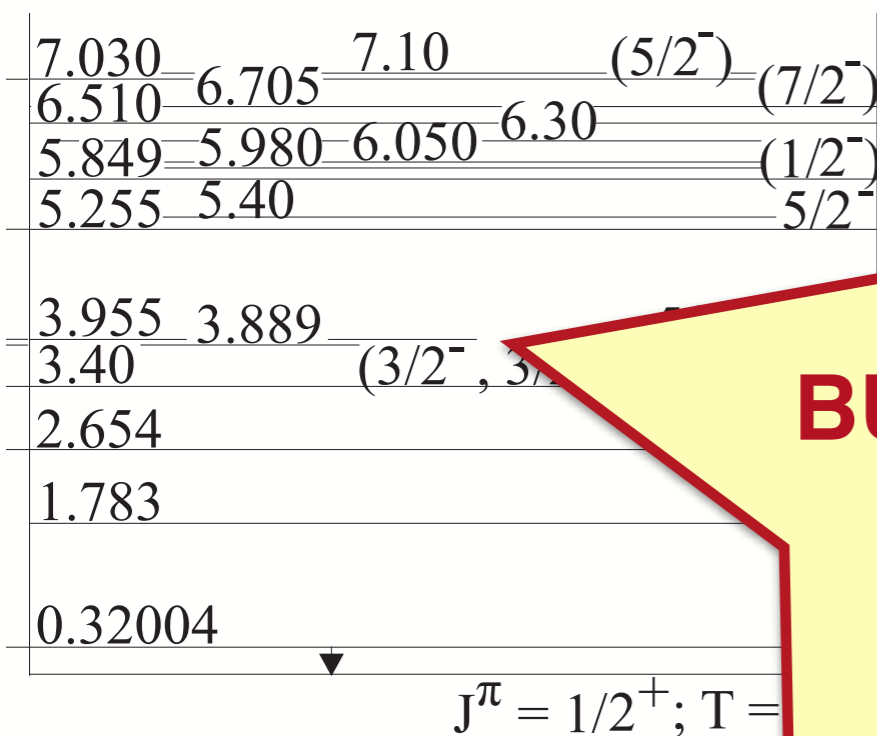
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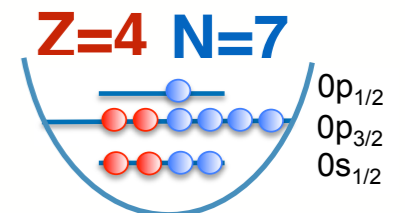
⁸IN2P3-CNRS, IJL, Université de Strasbourg

Can **ab initio** theory describe this complicated system?



$\frac{7.3139}{^9\text{Be}+2n}$

- **parity inversion** shell model predicts states to be $J^\pi = 1/2^-$



YES

BUT... huge challenge for interaction and many-body method


and $J=1/2$ states dominated by $n-^{10}\text{Be}$

NCSM with Continuum (NCSMC)

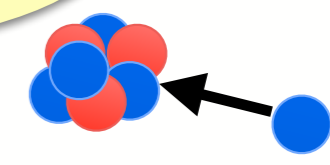
Baroni, Navrátil, Quaglioni *Phys. Rev. Lett.* 110, 022505 (2013)

- representing $H |\psi^{J\pi T}\rangle = E |\psi^{J\pi T}\rangle$ using the **over-complete basis**

$$|\psi^{J\pi T}\rangle = \sum_{\lambda} c_{\lambda} |\Psi_{A E_{\lambda} J^{\pi T}}\rangle + \sum_{\nu} \int dr r^2 \frac{\chi_{\nu}(r)}{r} |\xi_{\nu r}^{J\pi T}\rangle$$



expansion in A-body
NCSM eigenstates



relative motion of clusters
NCSM/RGM expansion

- leads to NCSMC equation

$$\begin{pmatrix} H_{NCSM} & h \\ h & \mathcal{H} \end{pmatrix} \begin{pmatrix} c \\ \chi(r)/r \end{pmatrix} = E \begin{pmatrix} \mathbb{1} & g \\ g & \mathbb{1} \end{pmatrix} \begin{pmatrix} c \\ \chi(r)/r \end{pmatrix}$$

- with 3N contributions in

H_{NCSM}

covered by
NCSM

h

given by
 $\langle \Psi_{A E_{\lambda} J^{\pi T}} | H | \xi_{\nu r}^{J\pi T} \rangle$

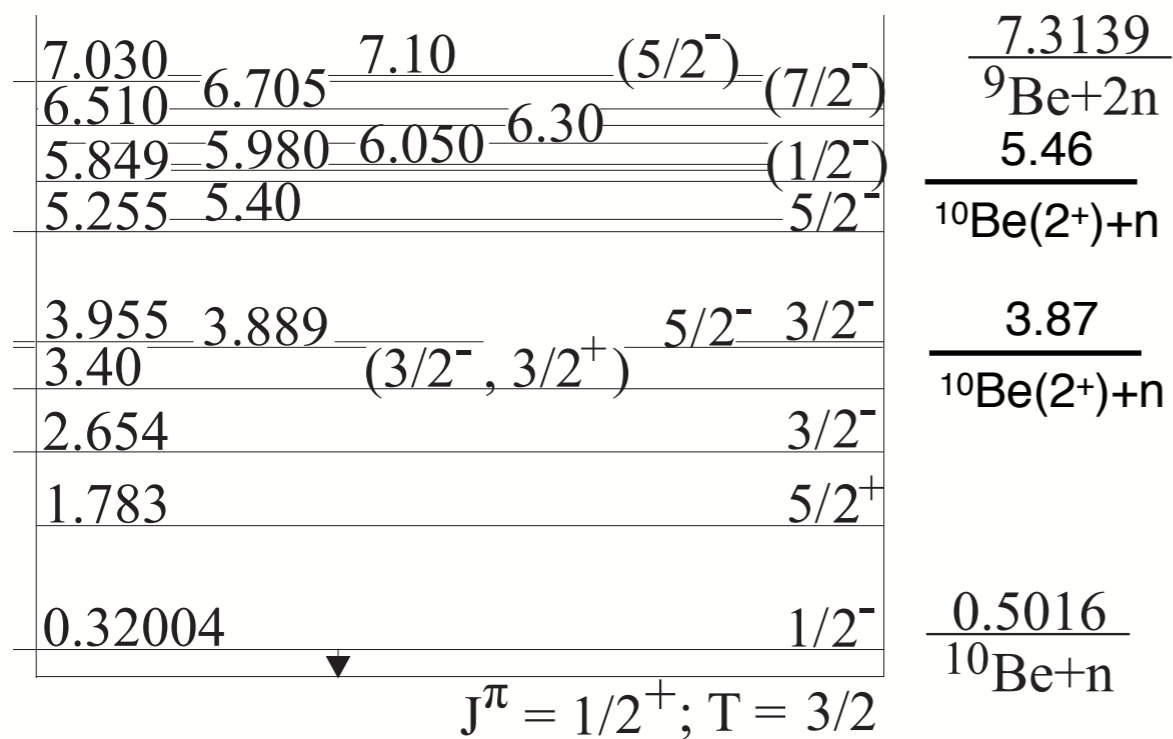
\mathcal{H}

contains NCSM/RGM
Hamiltonian kernel

^{11}Be : Ab initio NCSMC calculations

- Halo structure**

spectrum dominated by n- ^{10}Be halo structure



^{11}Be

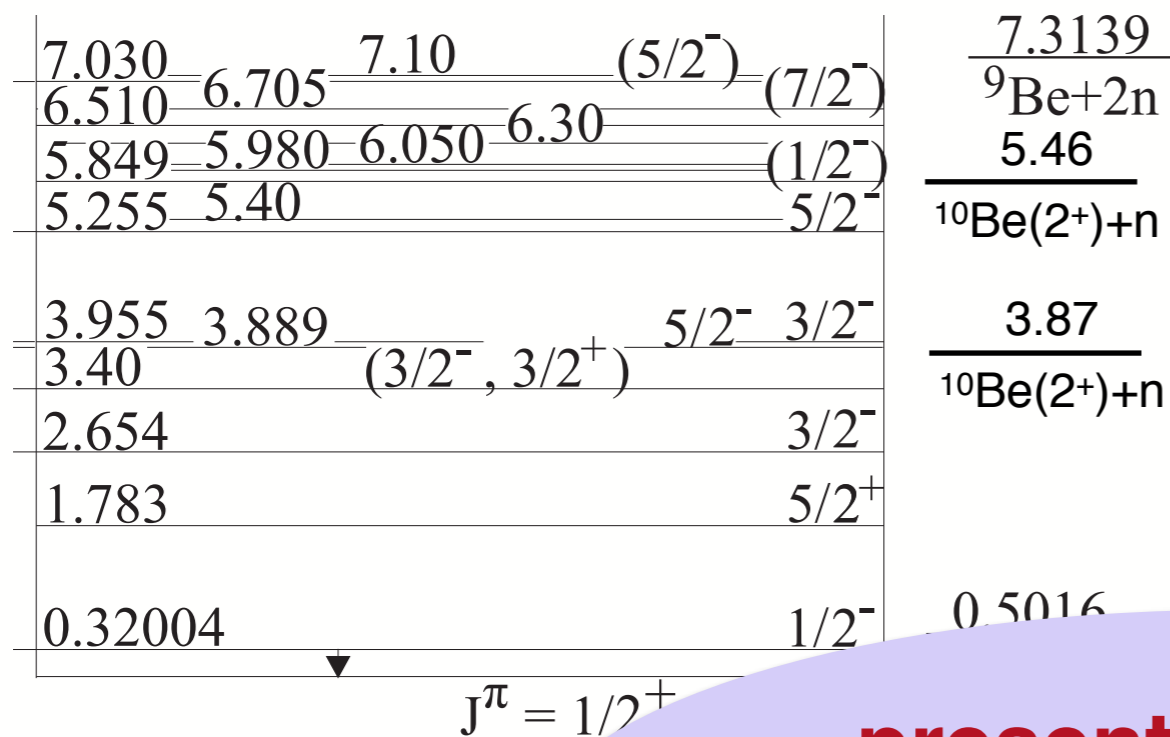
NCSM input

- calculations use NCSM vectors and energies as input
- include n- ^{10}Be continuum (0⁺, 2⁺, 2⁺ states of ^{10}Be)
- include ^{11}Be short-range correlations:
 - 4 negative parity (at least)
 - 3 positive parity states of ^{11}Be

^{11}Be : Ab initio NCSMC calculations

- Halo structure**

spectrum dominated by n- ^{10}Be halo structure



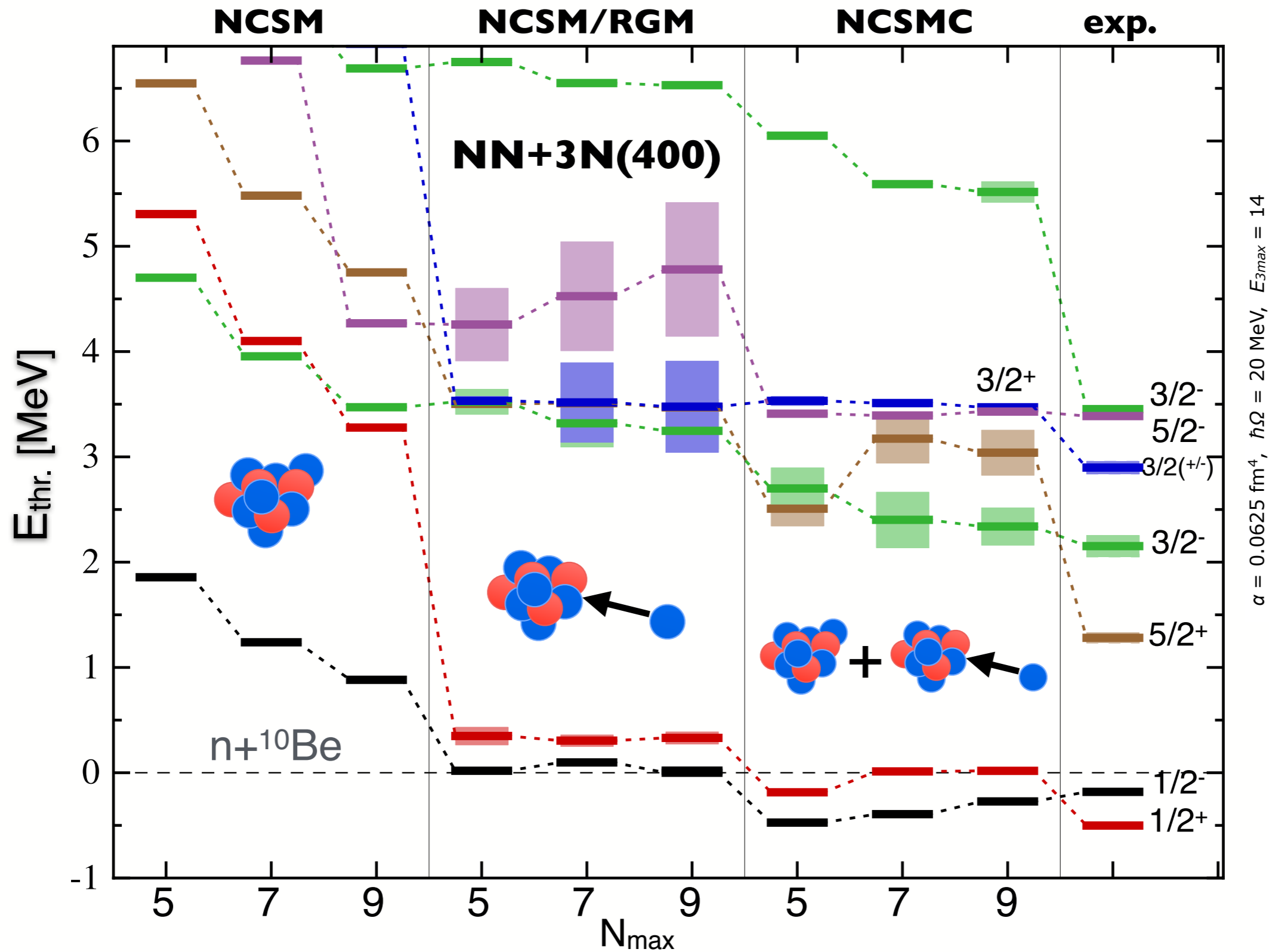
^{11}Be

presented NCSMC results are converged w.r.t. to NCSM input

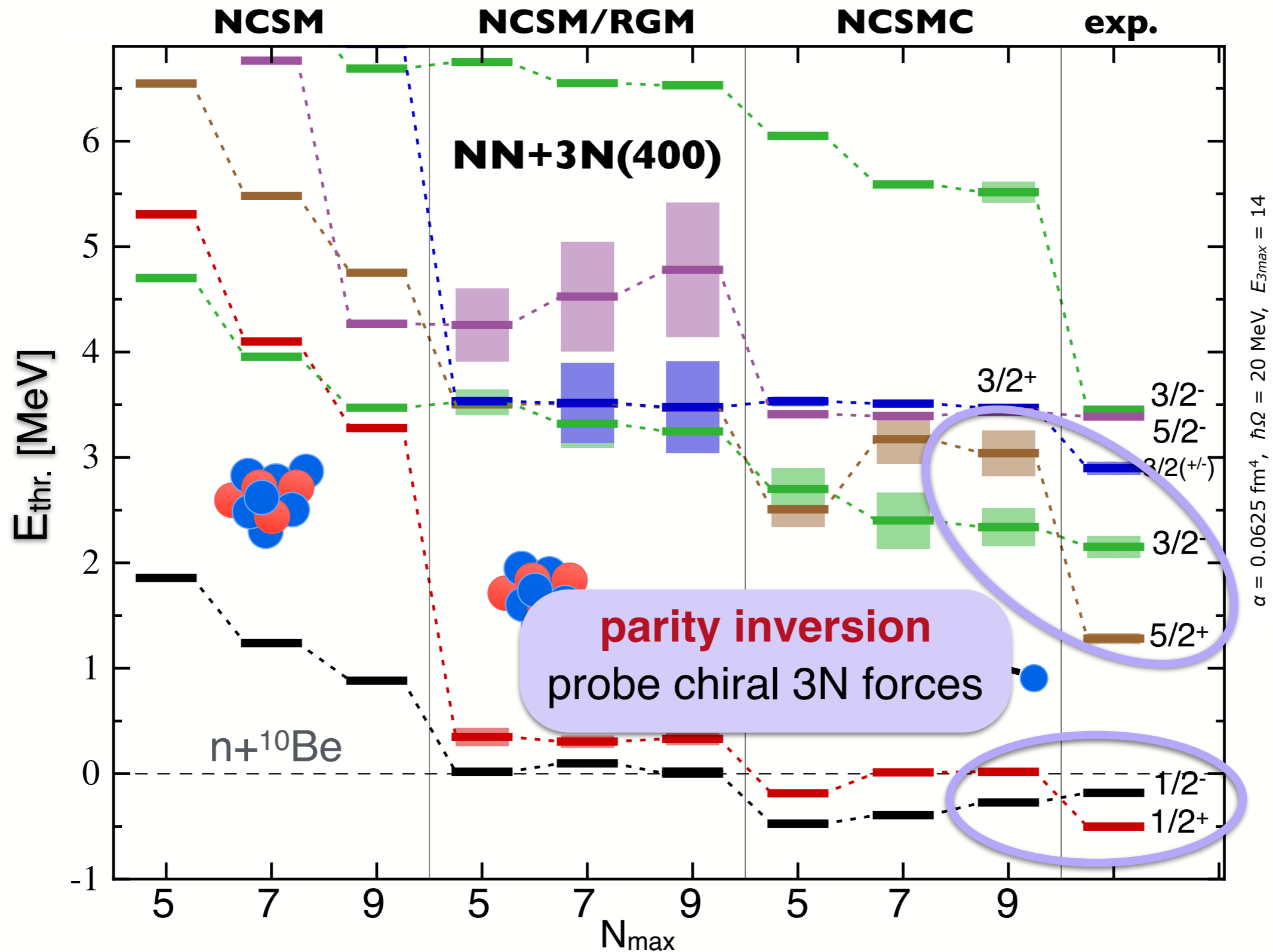
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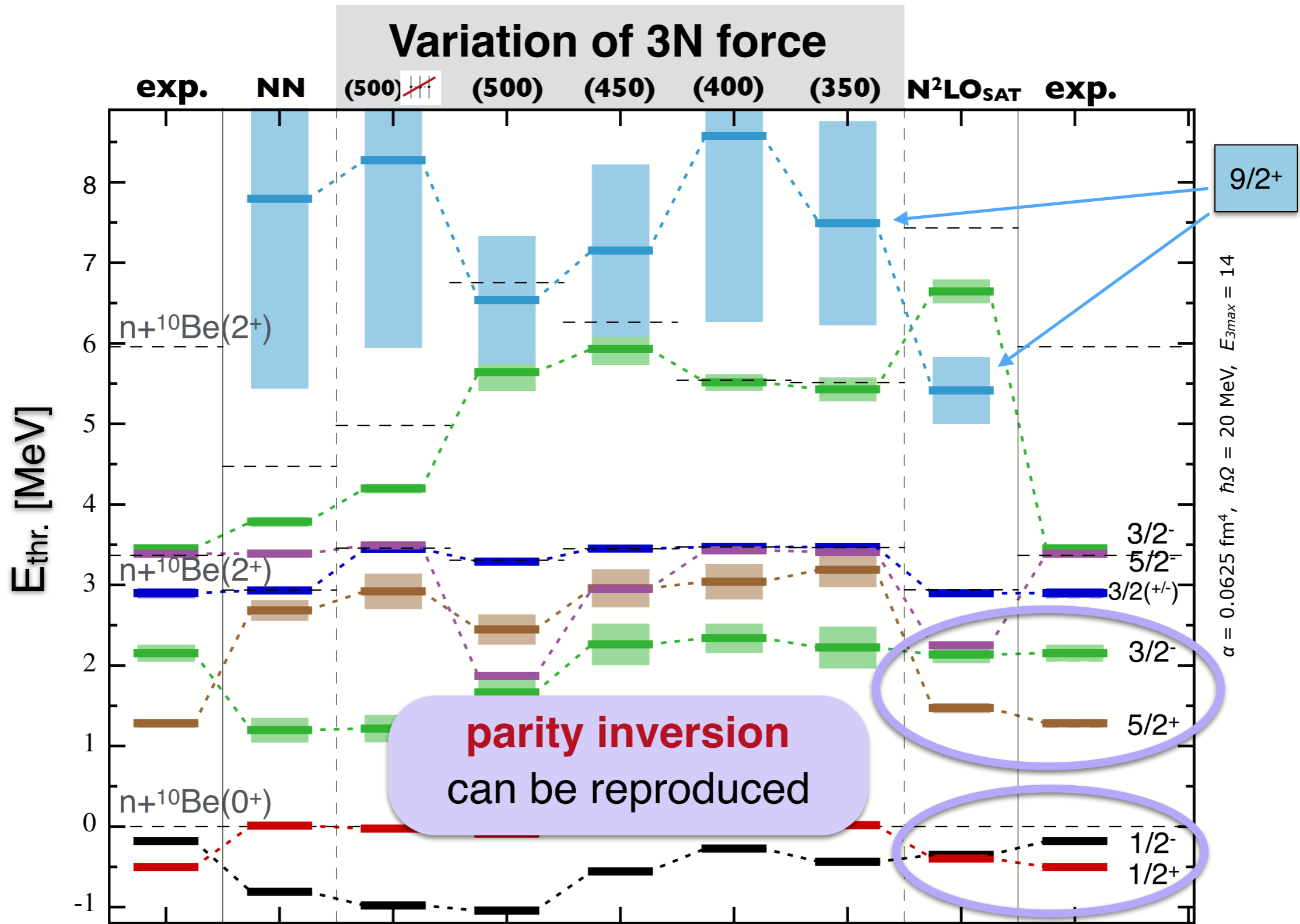
^{11}Be excitation spectrum



^{11}Be excitation spectrum



^{11}Be excitation spectrum



^{11}Be : Photodisintegration process & E1 transition

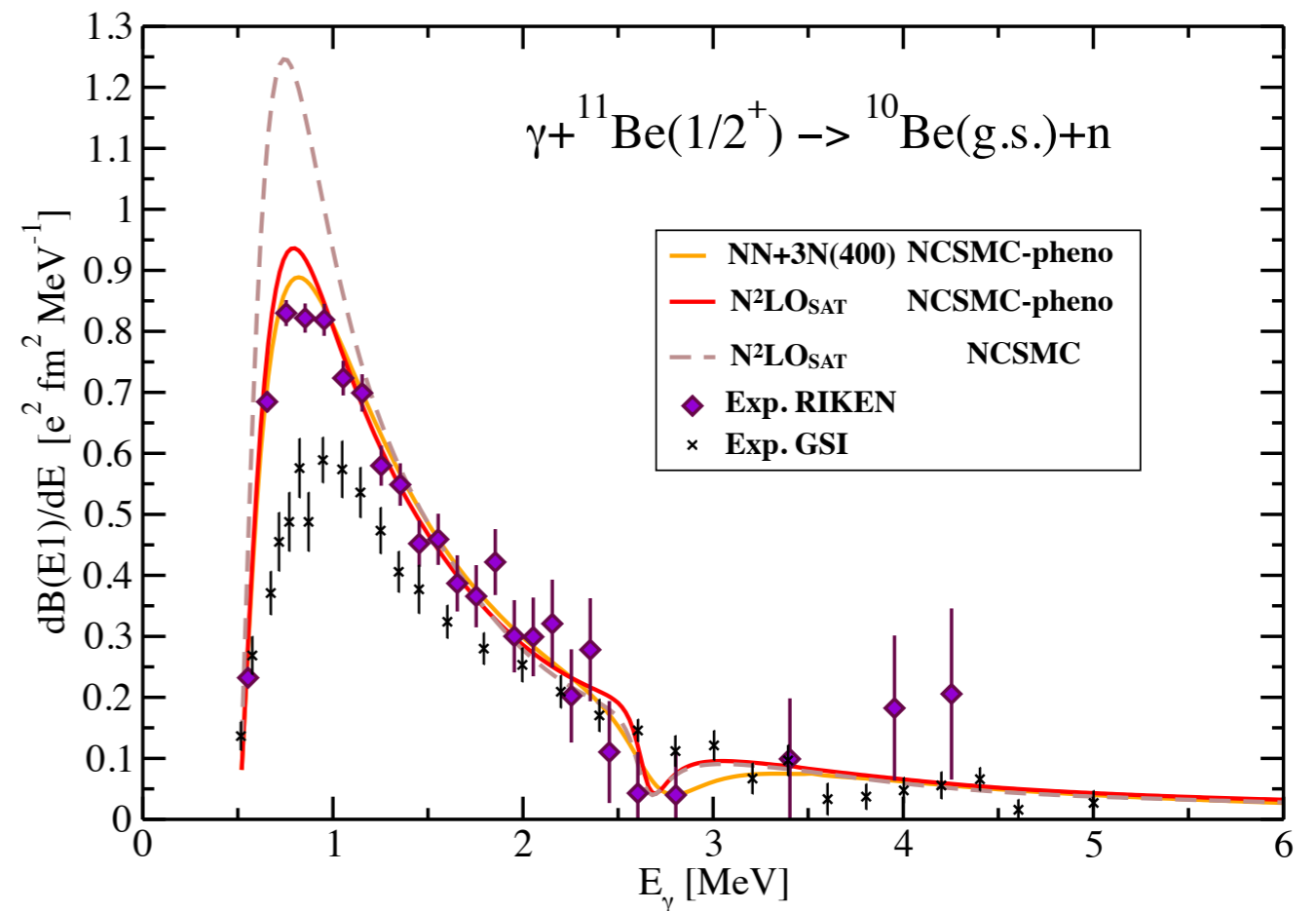
$B(E1:1/2^- \rightarrow 1/2^+)$ [e^2fm^2]

	NCSM	NCSMC	NCSMC-pheno	exp.
NN+3N(400)	0.0005	-	0.146	0.102(2)*
$\text{N}^2\text{LO}_{\text{SAT}}$	0.0005	0.127	0.117	

*Kwan et al. Phys. Lett. B 732, 210 (2014)

- **strongest known E1** transition between low-lying states (attributed to halo structure)

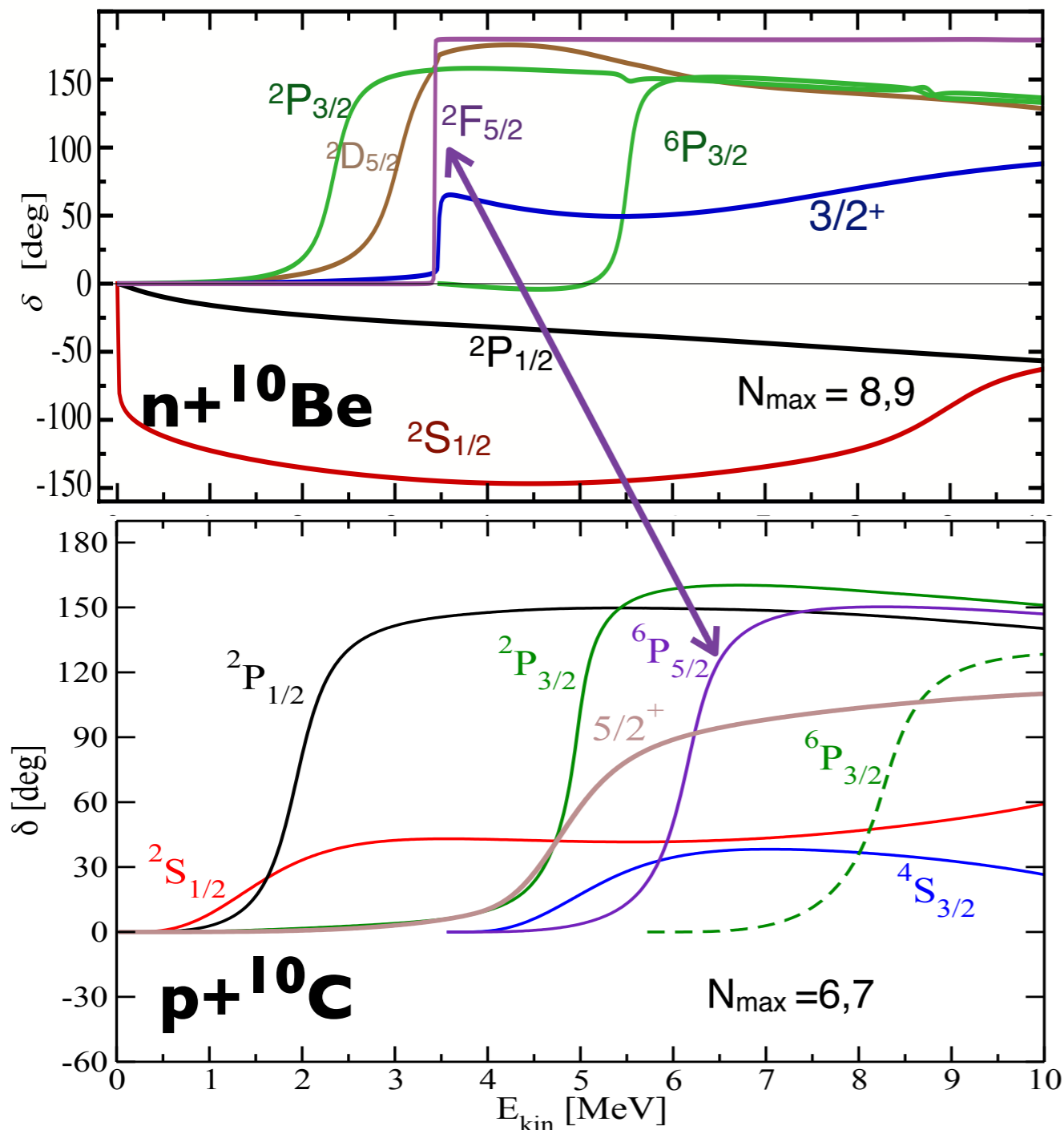
- reproduced **only** with **continuum effects**



- **conflicting** experimental **measurements**
- ab initio results:
 - **discriminate** between measurements
 - **predict dip** at $3/2^-$ resonance energy

Mirror nuclei: ^{11}Be and ^{11}N

standard NN+3N(400)



$\alpha = 0.0625 \text{ fm}^4, \hbar\Omega = 20 \text{ MeV}, E_{3\text{max}} = 14$

7.030	6.705	7.10	(5/2 ⁻)	(7/2 ⁻)	$\frac{7.3139}{^9\text{Be}+2\text{n}}$
6.510	5.980	6.050	6.30	(1/2 ⁻)	
5.849	5.40			5/2 ⁻	
3.955	3.889			5/2 ⁻ 3/2 ⁻	$\frac{3.87}{^{10}\text{Be}(2^+)+\text{n}}$
3.40			(3/2 ⁻ , 3/2 ⁺)		
2.654				3/2 ⁻	
1.783				5/2 ⁺	
0.32004				1/2 ⁻	$\frac{0.5016}{^{10}\text{Be}+\text{n}}$

$J^\pi = 1/2^+; T = 3/2$

^{11}Be

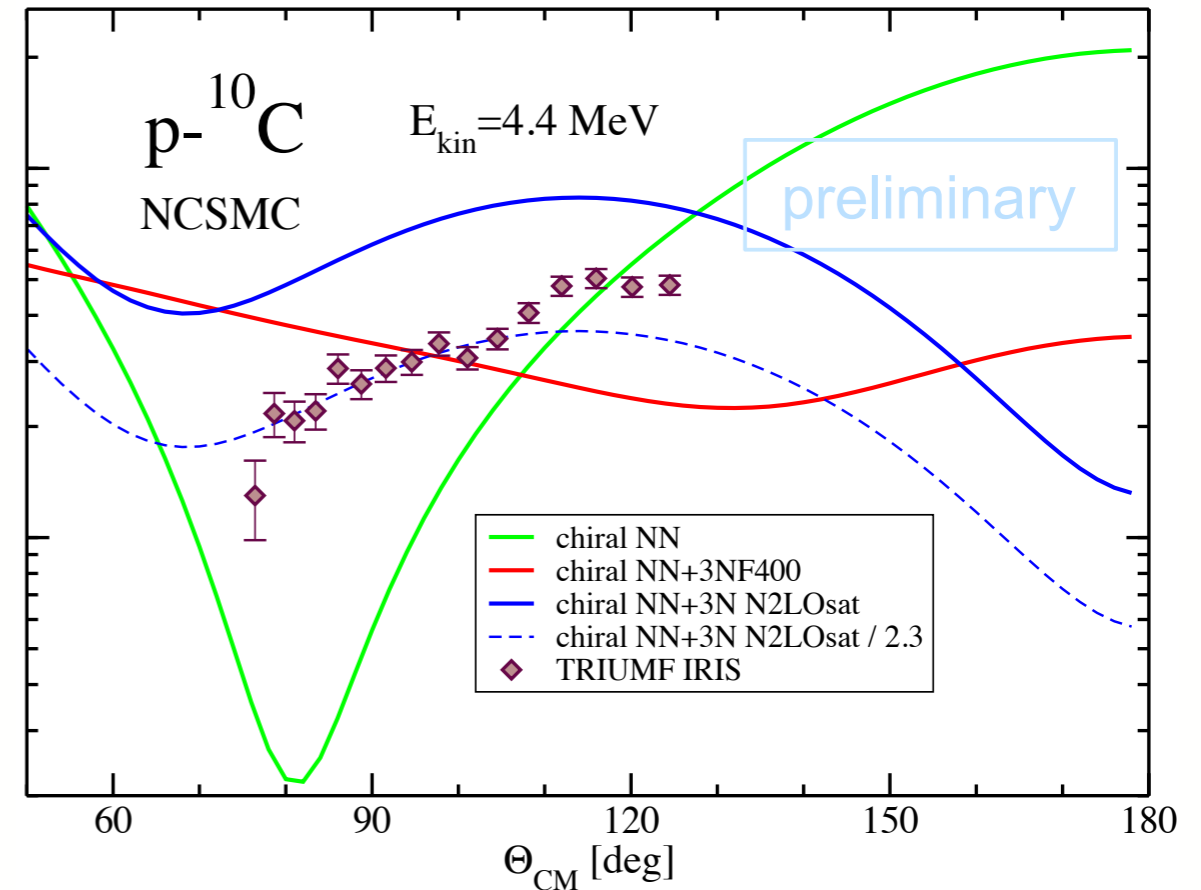
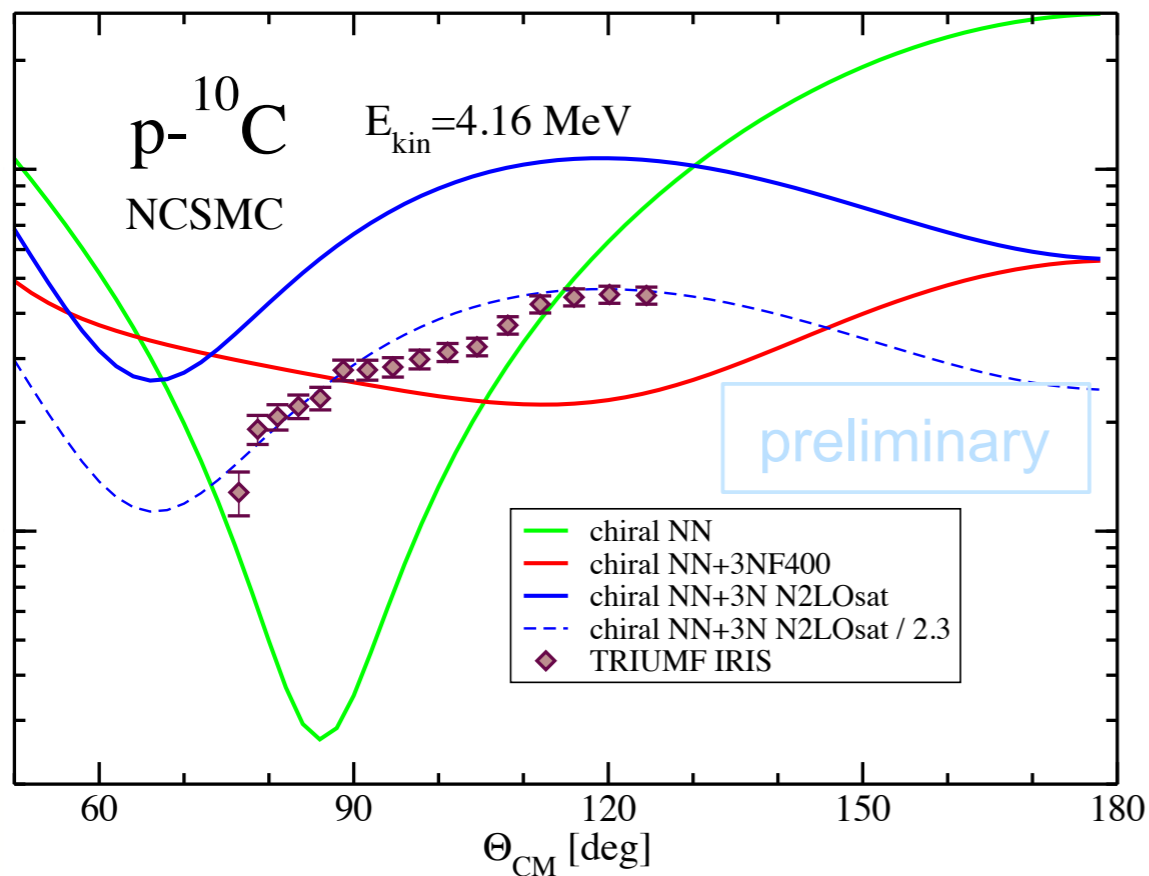
3.63	(5/2 ⁻)	
2.86	3/2 ⁻	
2.20	5/2 ⁺	$\frac{2.5174}{^9\text{B}+2\text{p}}$
1.57		
0.73	1/2 ⁻	

$J^\pi = 1/2^+; T = 3/2$

$p+^{10}\text{C}$ Scattering: Structure of ^{11}N resonances

Mirror System

elastic scattering allows discrimination among chiral nuclear forces

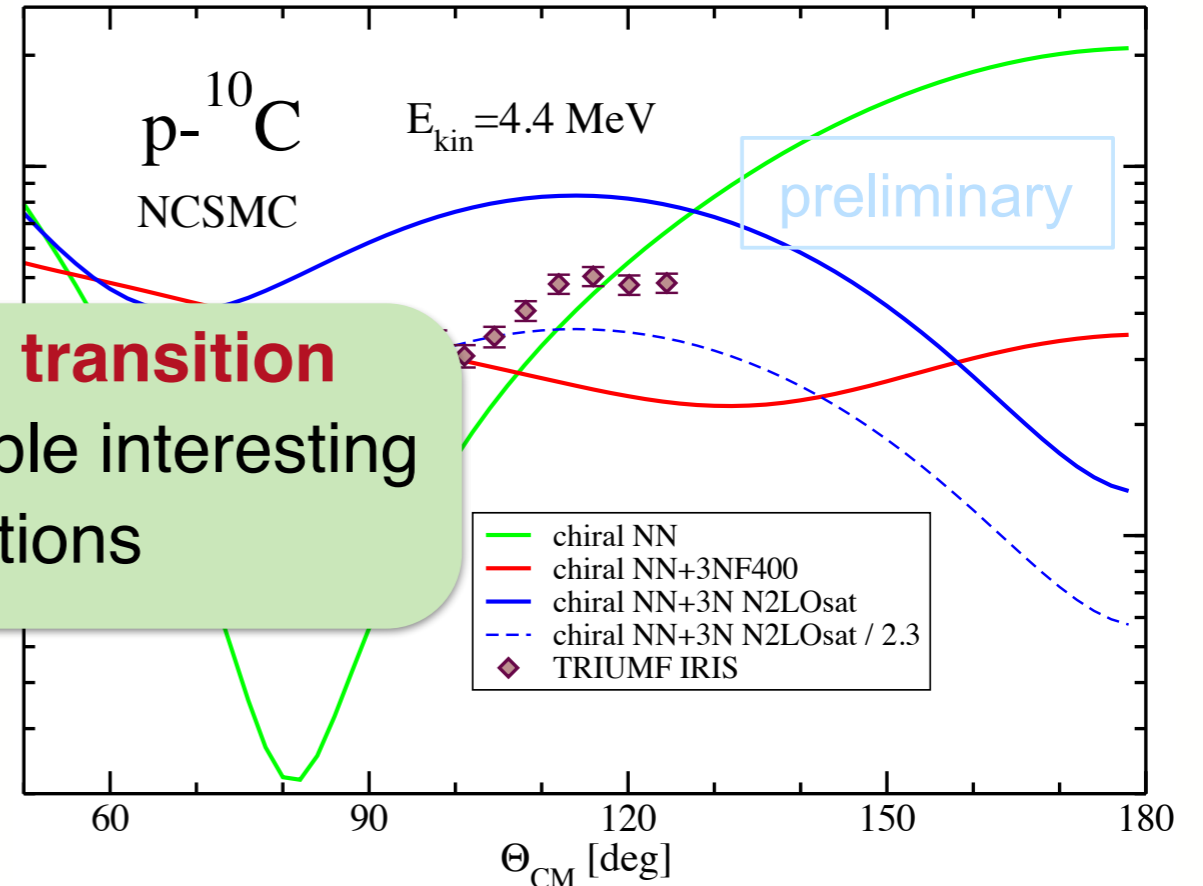
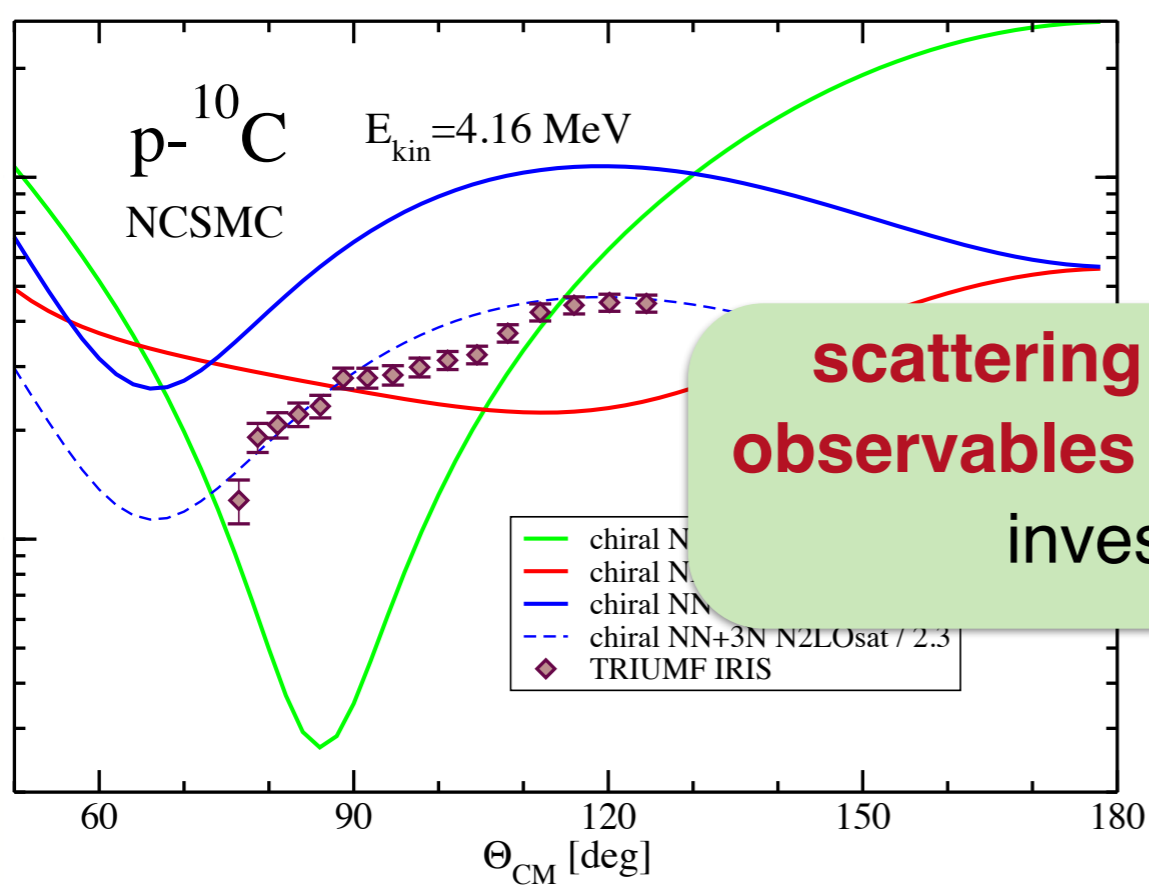


IRIS collaboration:
A. Kumar, R. Kanungo, A. Sanetullaev *et al.*

A. Calci, P. Navratil, G. Hupin, S. Quaglioni, R. Roth *et al.* with IRIS collaboration, in preparation

p+¹⁰C Scattering: Structure of ¹¹N resonances

Mirror System
 elastic scattering allows discrimination
 among chiral nuclear forces



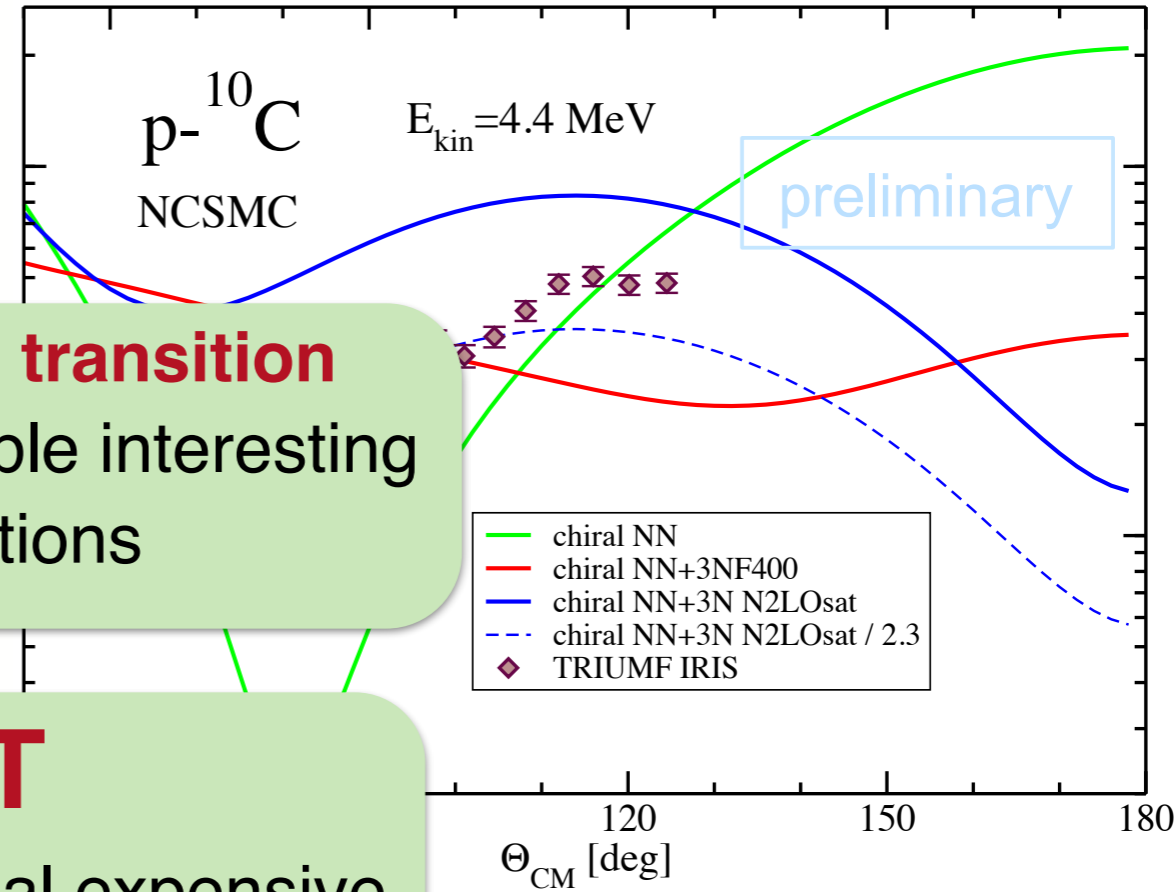
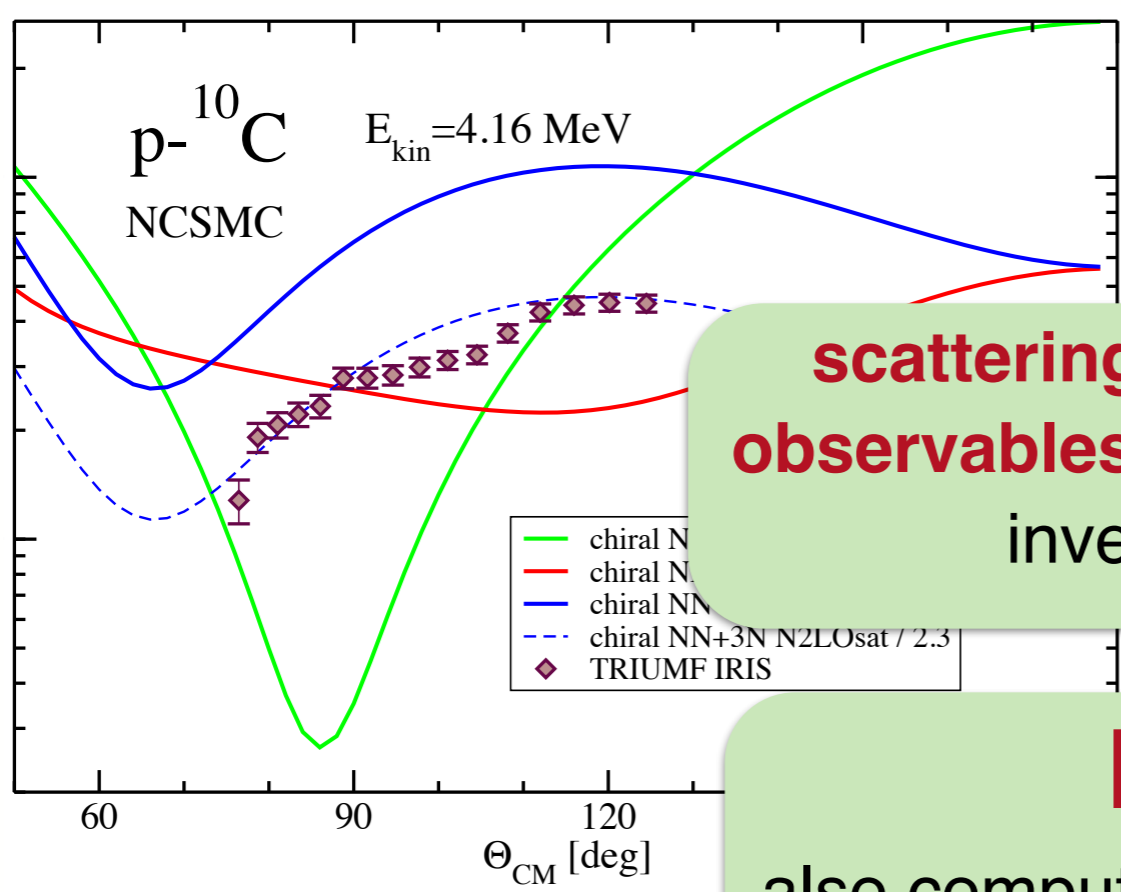
scattering and transition observables enable interesting investigations

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p+¹⁰C Scattering: Structure of ¹¹N resonances

Mirror System
 elastic scattering allows discrimination
 among chiral nuclear forces



scattering and transition observables enable interesting investigations

BUT
 also computational expensive

IRIS collaboration:
 A. Kumar, R. Kanungo, A. Sanetullaev *et al.*

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NCSMC with approximated 3N forces

with
P. Navrátil, R. Roth, E. Gebrerufael

NCSM with Continuum (NCSMC)

- representing $H|\psi^{J\pi T}\rangle = E|\psi^{J\pi T}\rangle$ using the **over-complete basis**

$$|\psi^{J\pi T}\rangle = \sum_{\lambda} c_{\lambda} |\Psi_{A E_{\lambda} J^{\pi T}}\rangle + \sum_{\nu} \int dr r^2 \frac{\chi_{\nu}(r)}{r} |\xi_{\nu r}^{J\pi T}\rangle$$

expansion in A-body
NCSM eigenstates

relative
NCSM

- leads to NCSMC equation

$$\begin{pmatrix} H_{NCSM} & h \\ h & \mathcal{H} \end{pmatrix} \begin{pmatrix} c \\ \chi(r)/r \end{pmatrix} = E \begin{pmatrix} 1 \\ g \end{pmatrix}$$

- with 3N contributions in

H_{NCSM}

covered by
NCSM

h

given by
 $\langle \Psi_{A E_{\lambda} J^{\pi T}} | H | \xi_{\nu r}^{J\pi T} \rangle$

\mathcal{H}

contains NCSM/RGM
Hamiltonian Kernel



Normal-ordering (NO) approximation

- standard tool to **reduce particle rank**
- generally NO can be considered as basis transformation

$$V_{3N} \approx \underbrace{\tilde{V}_{0N} + \tilde{V}_{1N} + \tilde{V}_{2N}}_{\text{contain information of reference state and initial 3N force}} + \cancel{\tilde{V}_{3N}}$$

contain information of reference state and initial 3N force

- interested in direct description of **open-shell systems**
 - multi-reference normal ordering (MR-NO)
 - generalization of wicks theorem [Kutzelnigg, Mukherjee]

NCSM/RGM kernels with MR-NO contributions

- reduces computational costs tremendously
- impressively accurate approximation

Derive NCSM/RGM Kernels

0B kernel

dominant 0B kernel contribution included in target eigenstates
 \Rightarrow only MR-NO 1B and 2B kernels contribute

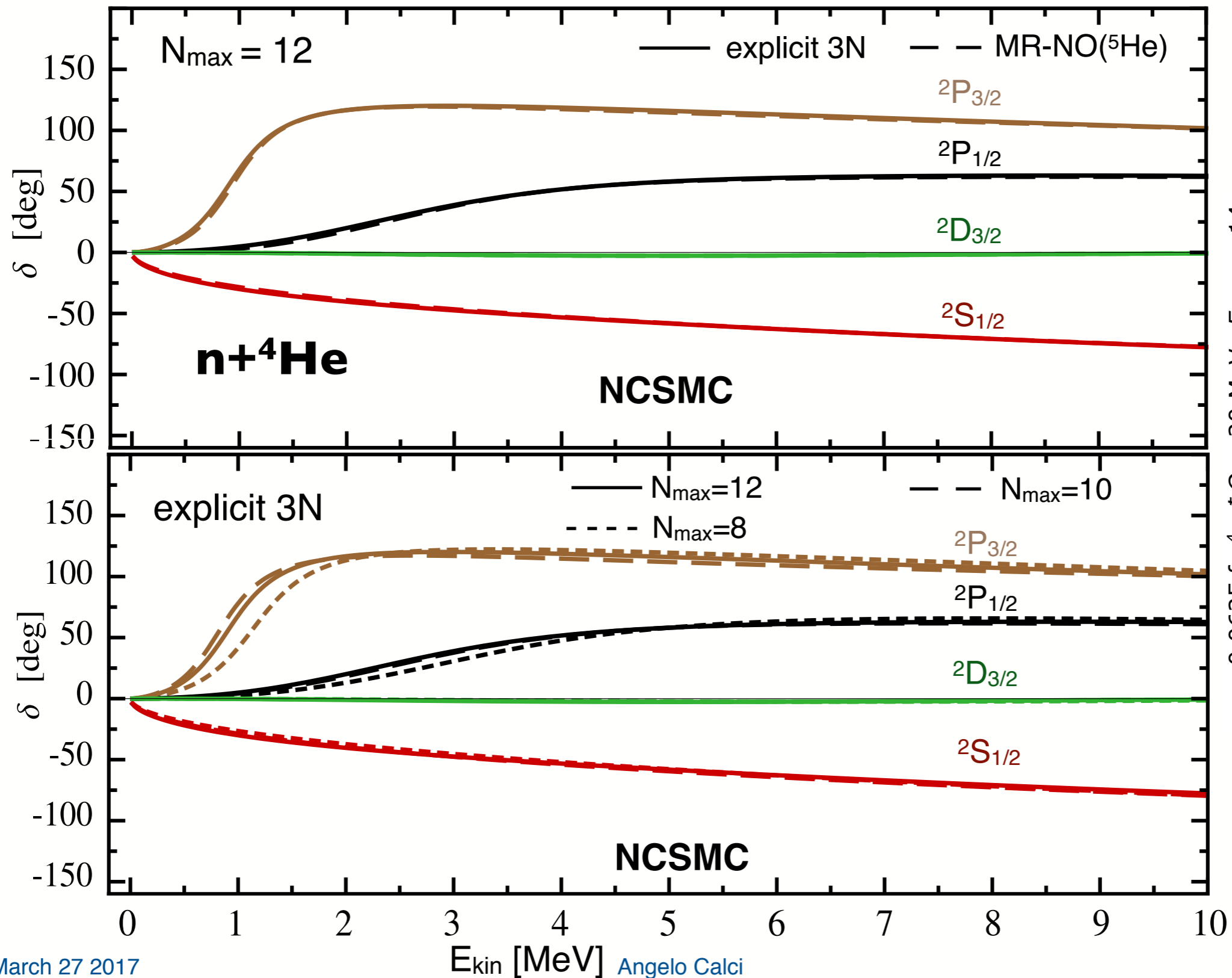
1B kernel

$$\begin{aligned}
 & {}_{SD} \langle \epsilon_{\nu'n'}^{\mathcal{J}\pi T} | \mathbf{V}_A | \epsilon_{\nu n}^{\mathcal{J}\pi T} \rangle_{SD} \\
 = & \sum_{M_1 m_j} \sum_{M_{T_1} m_t} \sum_{M'_1 m'_j} \sum_{M'_{T_1} m'_t} \begin{pmatrix} I_1 & j & | & \mathcal{J} \\ M_1 & m_j & | & \mathcal{M} \end{pmatrix} \begin{pmatrix} T_1 & \frac{1}{2} & | & T \\ M_{T_1} & m_t & | & M_T \end{pmatrix} \begin{pmatrix} I'_1 & j' & | & \mathcal{J} \\ M'_1 & m'_j & | & \mathcal{M} \end{pmatrix} \begin{pmatrix} T'_1 & \frac{1}{2} & | & T \\ M'_{T_1} & m'_t & | & M_T \end{pmatrix} \\
 \times & {}_{SD} \langle \psi'_{A-1} E'_1 I'_1 \pi'_1 M'_1 T'_1 M'_{T_1} | \psi_{A-1} E_1 I_1 \pi_1 M_1 T_1 M_{T_1} \rangle_{SD} \\
 \times & \langle n'l'j'm'_j \frac{1}{2} m'_t | \mathbf{V}_A | nljm_j \frac{1}{2} m_t \rangle \\
 & - {}_{SD} \langle \epsilon_{\nu'n'}^{\mathcal{J}\pi T} | \mathbf{V}_A \mathbf{T}_{A-1,A} | \epsilon_{\nu n}^{\mathcal{J}\pi T} \rangle_{SD} \\
 = & -\frac{1}{A-1} \sum_{M_1 m_j} \sum_{M_{T_1} m_t} \sum_{M'_1 m'_j} \sum_{M'_{T_1} m'_t} \begin{pmatrix} I_1 & j & | & \mathcal{J} \\ M_1 & m_j & | & \mathcal{M} \end{pmatrix} \begin{pmatrix} T_1 & \frac{1}{2} & | & T \\ M_{T_1} & m_t & | & M_T \end{pmatrix} \begin{pmatrix} I'_1 & j' & | & \mathcal{J} \\ M'_1 & m'_j & | & \mathcal{M} \end{pmatrix} \begin{pmatrix} T'_1 & \frac{1}{2} & | & T \\ M'_{T_1} & m'_t & | & M_T \end{pmatrix} \\
 \times & \sum_{\alpha_{A-1}} {}_{SD} \langle \psi'_{A-1} E'_1 I'_1 \pi'_1 M'_1 T'_1 M'_{T_1} | \mathbf{a}_{nljm_j m_t}^\dagger \mathbf{a}_{\alpha_{A-1}} | \psi_{A-1} E_1 I_1 \pi_1 M_1 T_1 M_{T_1} \rangle_{SD} \\
 \times & \langle n'l'j'm'_j \frac{1}{2} m'_t | \mathbf{V}_A | \alpha_{A-1} \rangle
 \end{aligned}$$

2B kernel

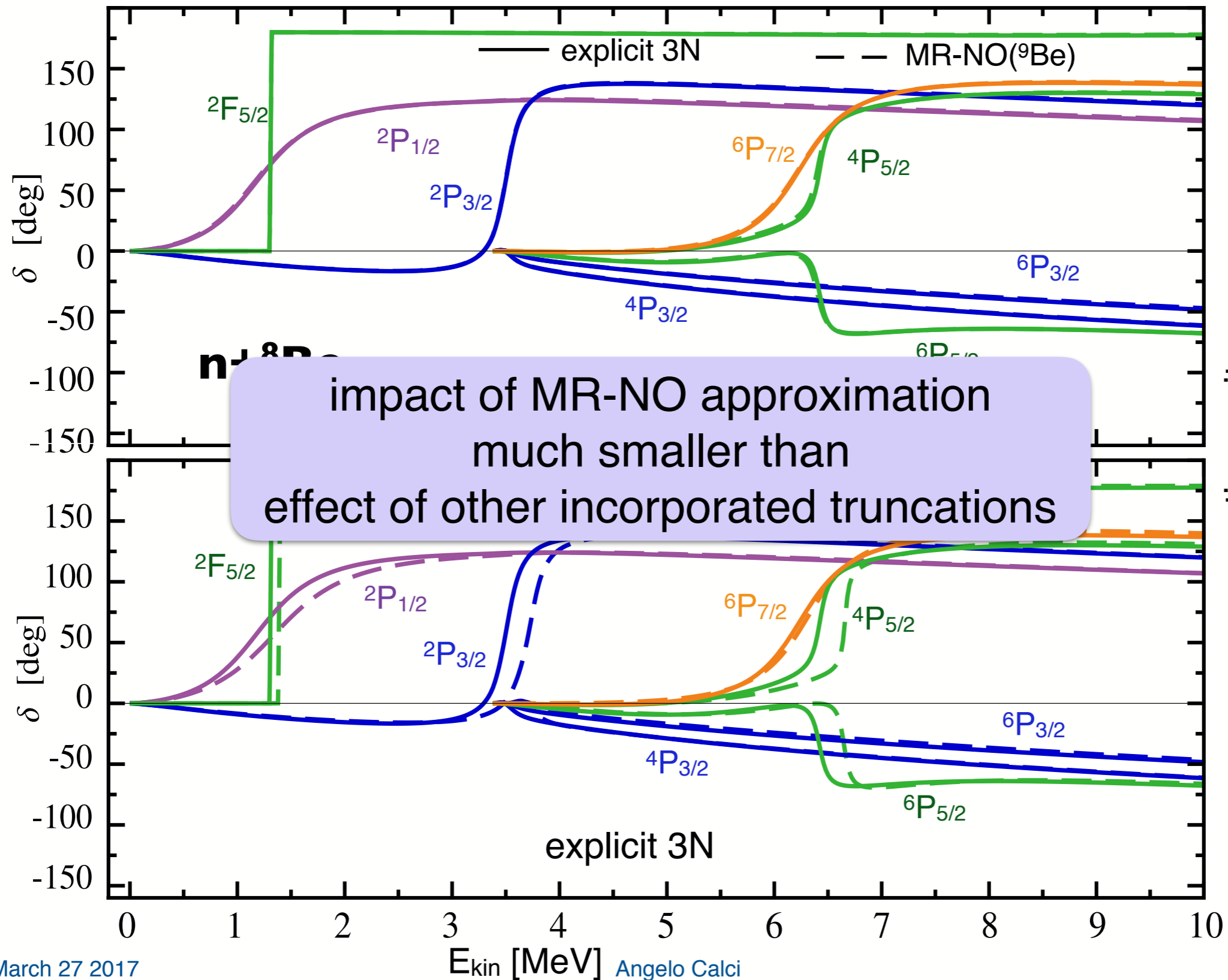
...

NCSMC: Impact of 3N in Kernels



NN+3N(500)

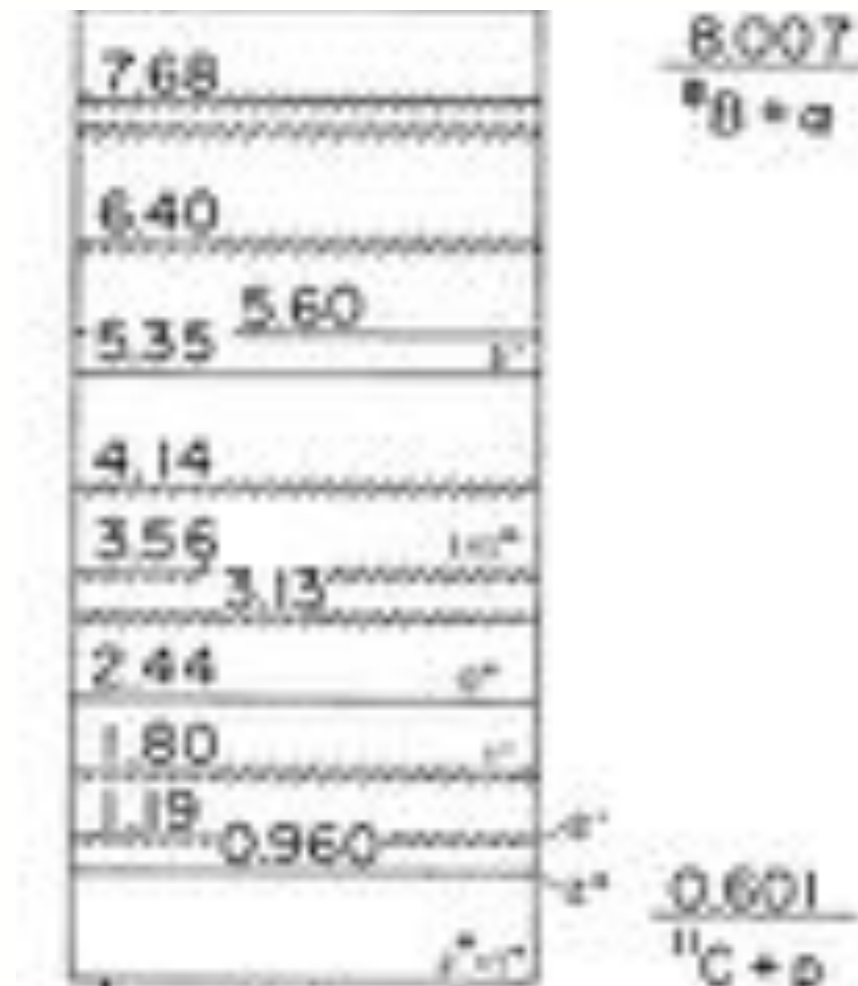
NCSMC: Impact of 3N in Kernels



First application: ^{12}N

- ideal candidate**

weakly bound $J=1^+$ state
dominated by $p\text{-}^{11}\text{C}$



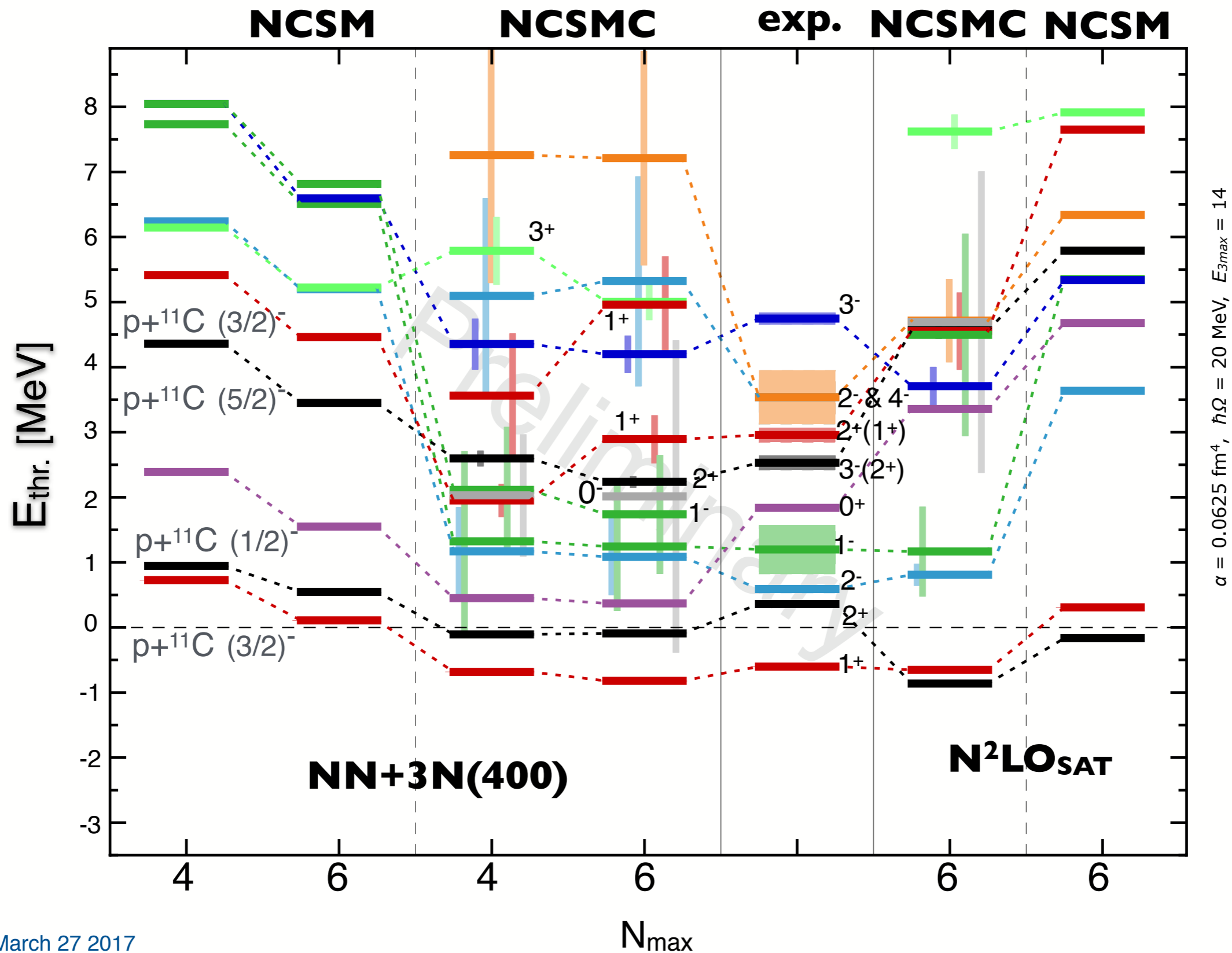
^{12}N

- some low lying resonances not measured precisely
- $^{11}\text{C}(p,\gamma)^{12}\text{N}$ can bypass triple-alpha process
- planned experiment at TUDA facility at TRIUMF

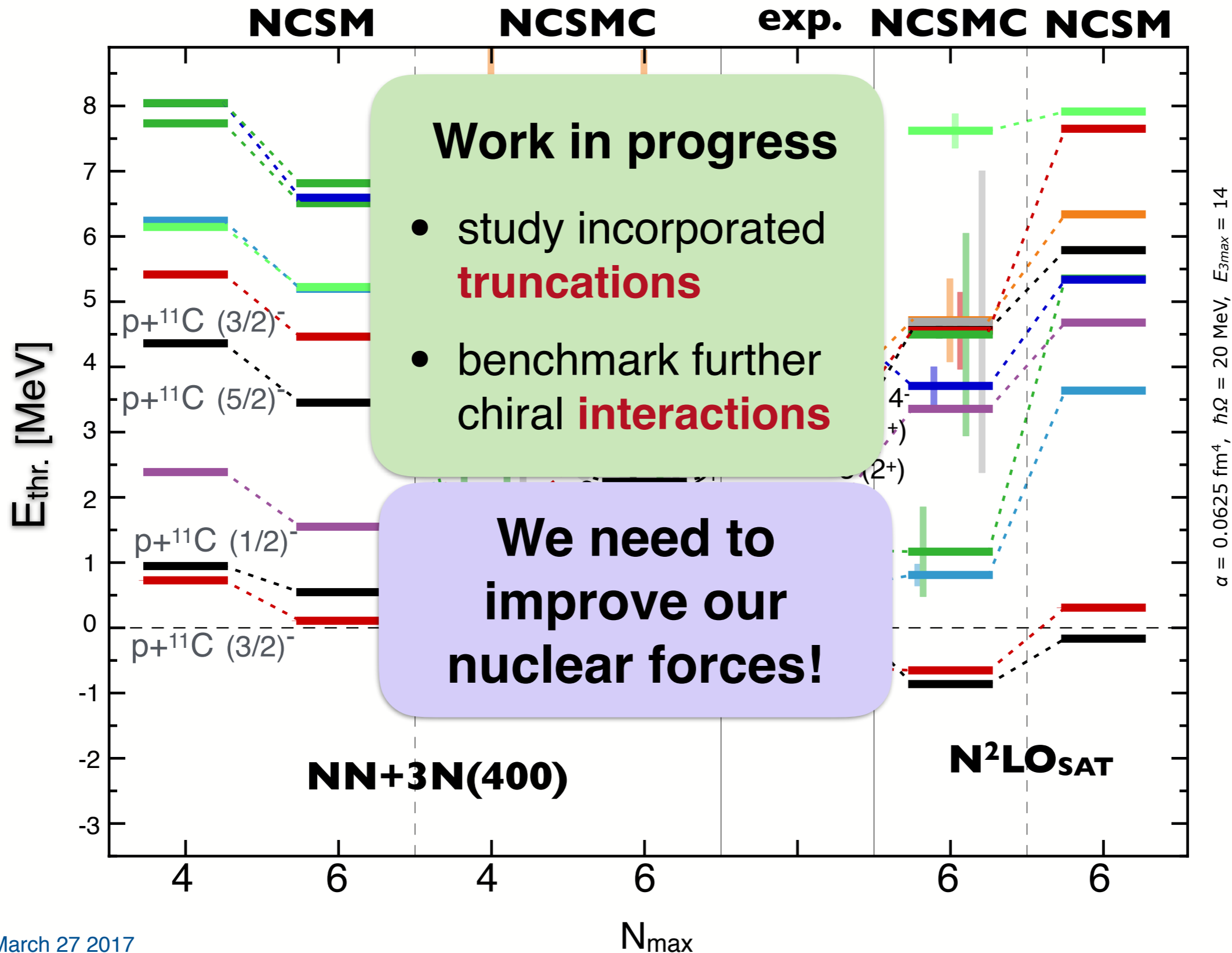
ab initio NCSMC

- include $p\text{-}^{11}\text{C}$ continuum ($3/2^-, 1/2^-, 5/2^-, 3/2^-$ states of ^{11}C)
- include 4 negative and 6 positive parity states of ^{12}N
- MR-NO with respect to $N_{\text{max}}=0$ eigenstate of ^{12}N

^{12}N spectrum with continuum effects

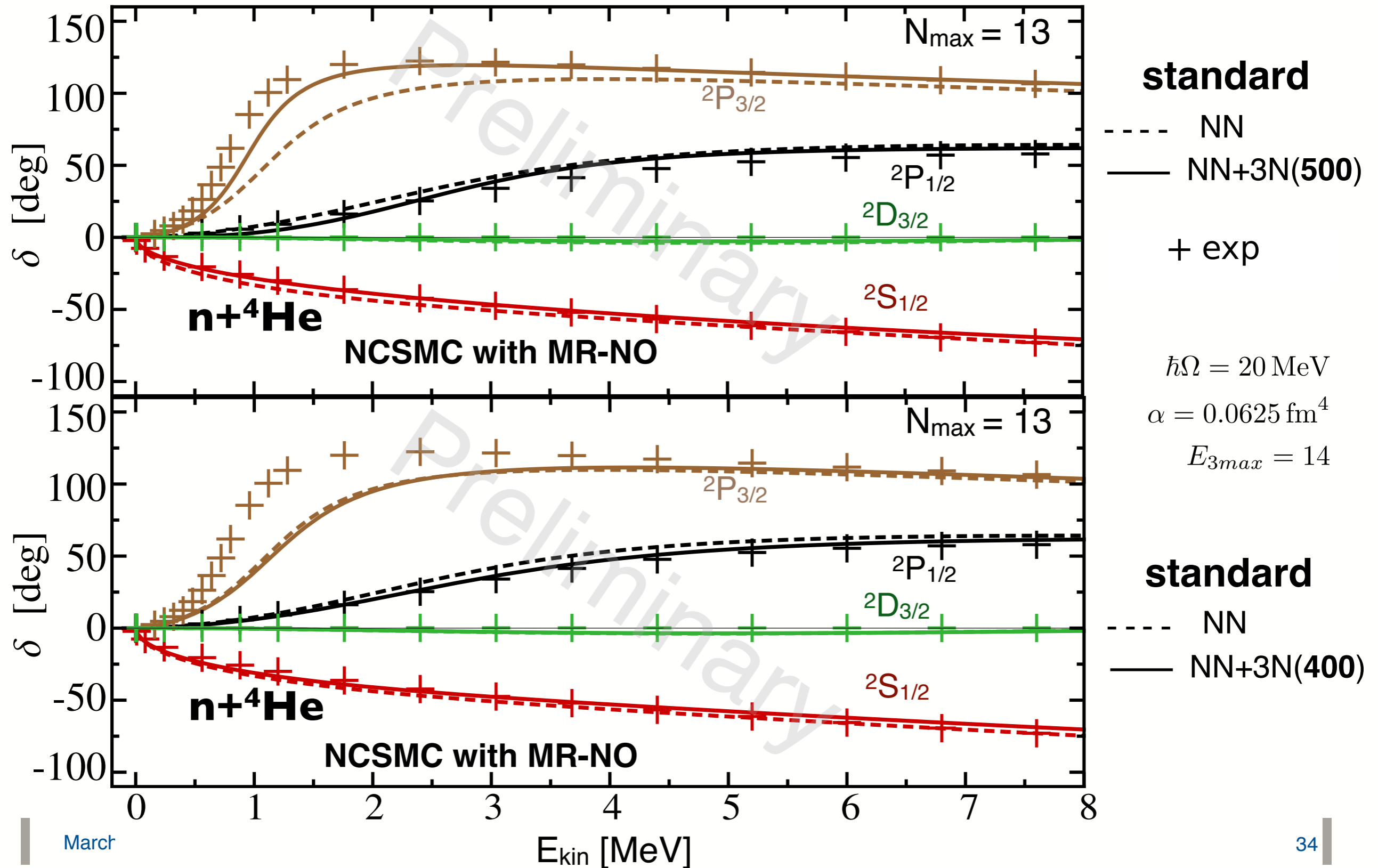


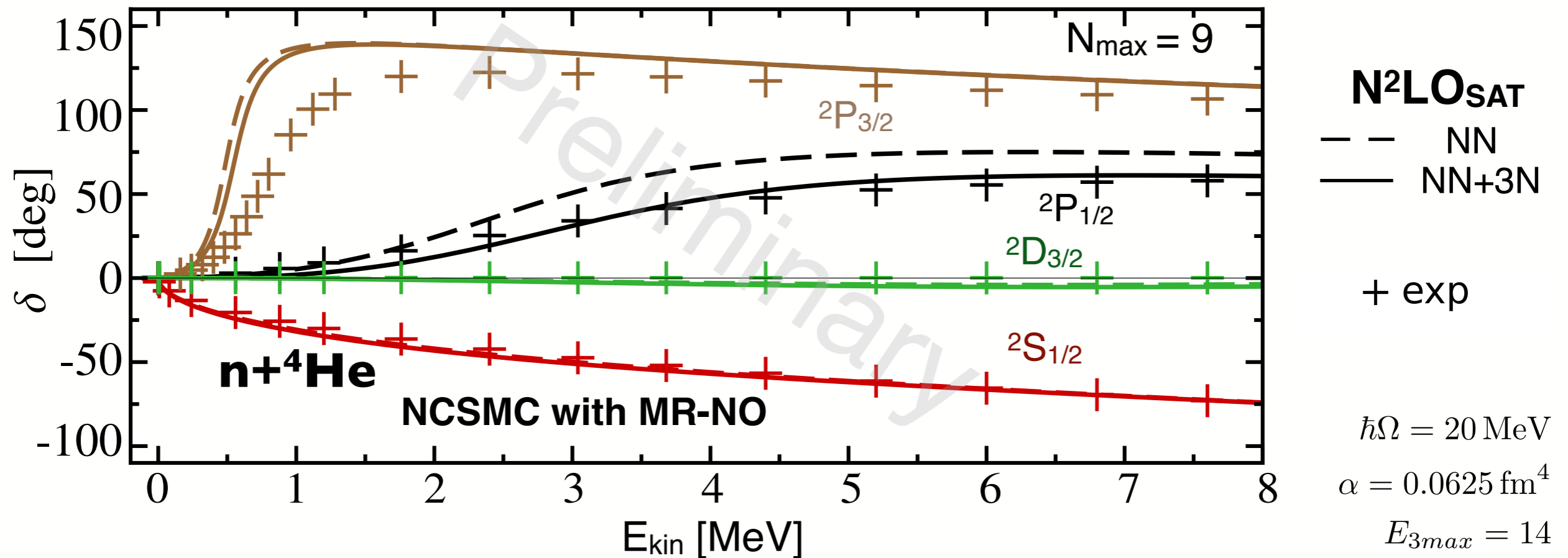
^{12}N spectrum with continuum effects



Probe chiral interaction in light nuclear scattering

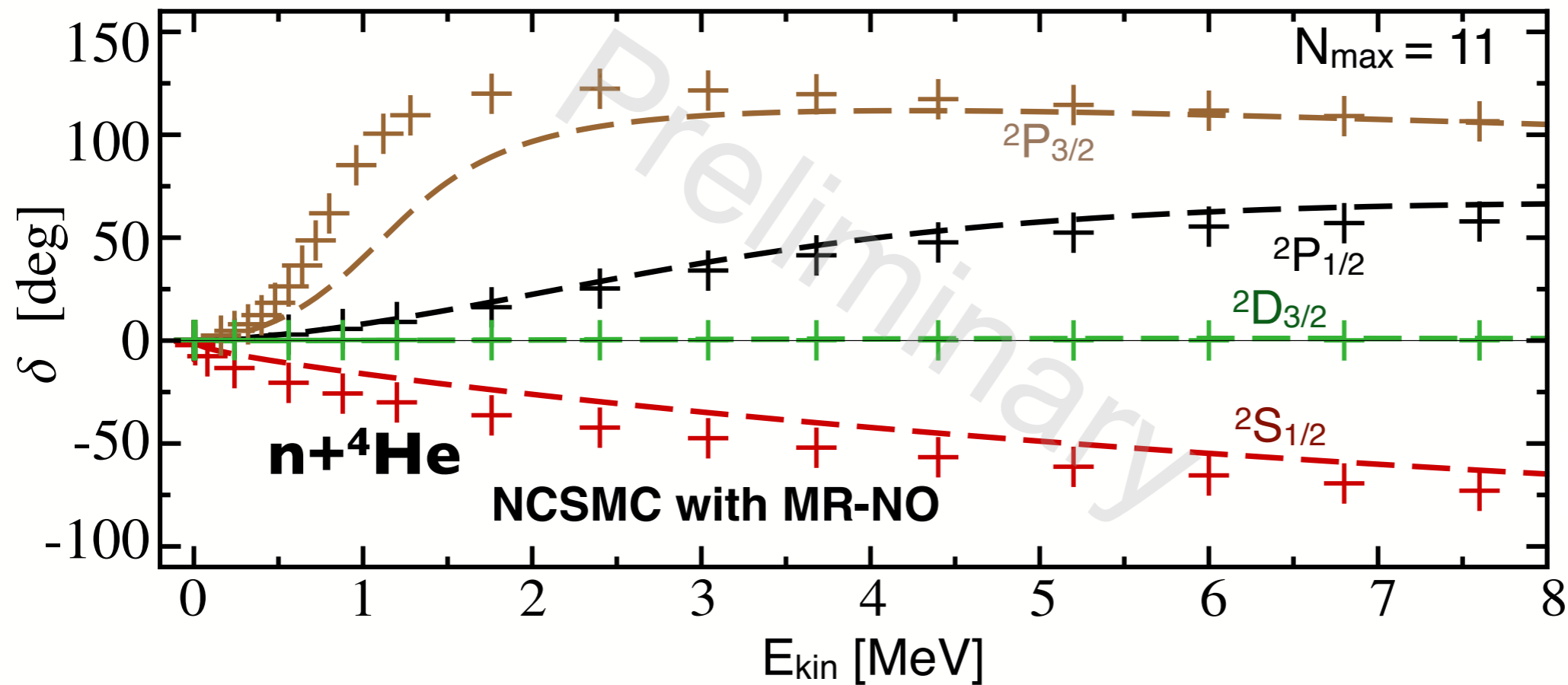
n-⁴He: Standard interaction





- $P_{3/2}$ - $P_{1/2}$ splitting sensitive to details of nuclear force
- under- or overestimated by NN+3N(400) or N²LO_{SAT} interaction

n-⁴He with LENPIC interaction



LENPIC
interaction
N²LO
 $R = 1.0$ fm

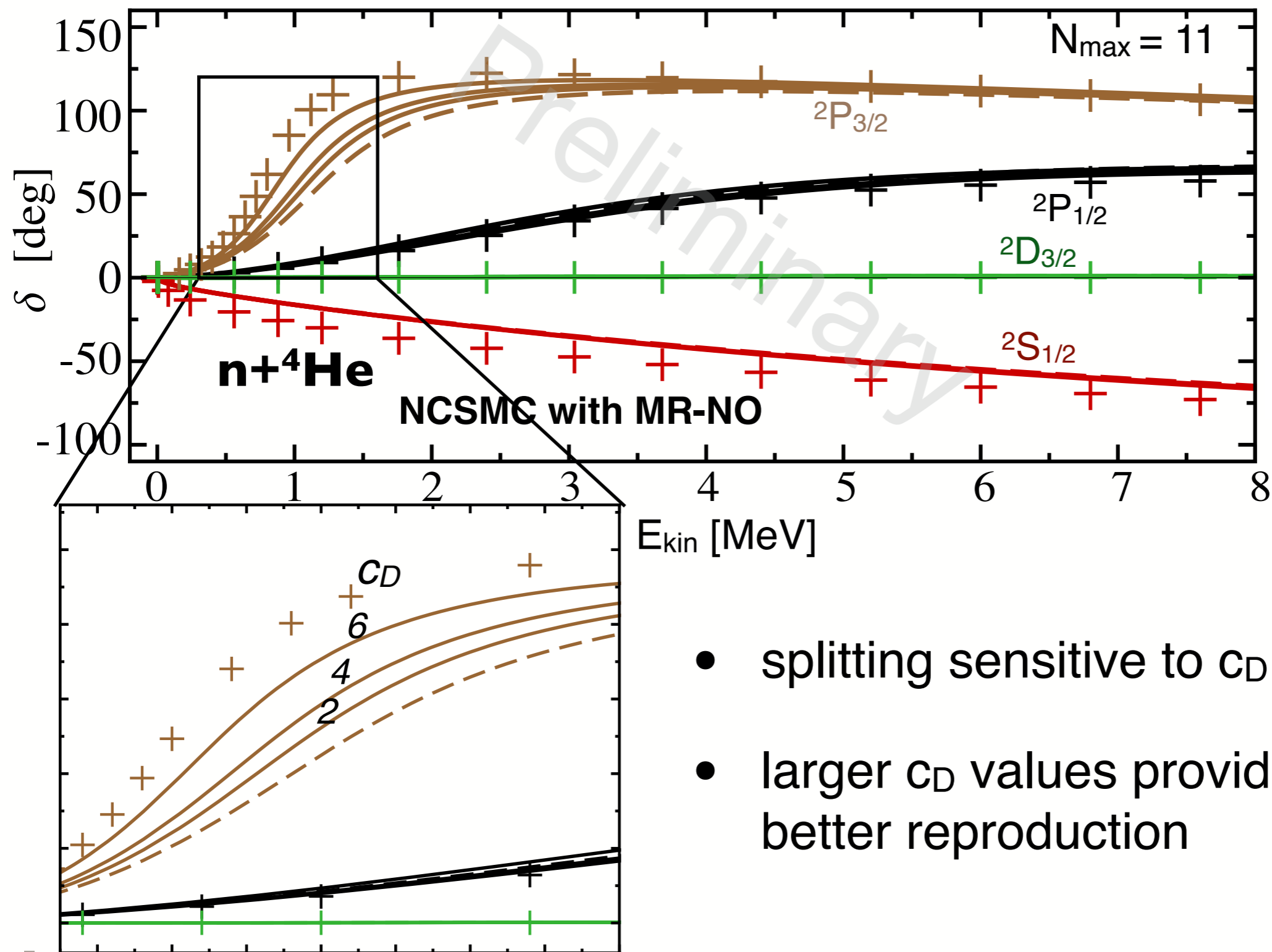
$$\hbar\Omega = 24 \text{ MeV}$$

$$\alpha = 0.08 \text{ fm}^4$$

$$E_{3max} = 14$$

- splitting underestimated without 3N interaction

n-⁴He with LENPIC interaction



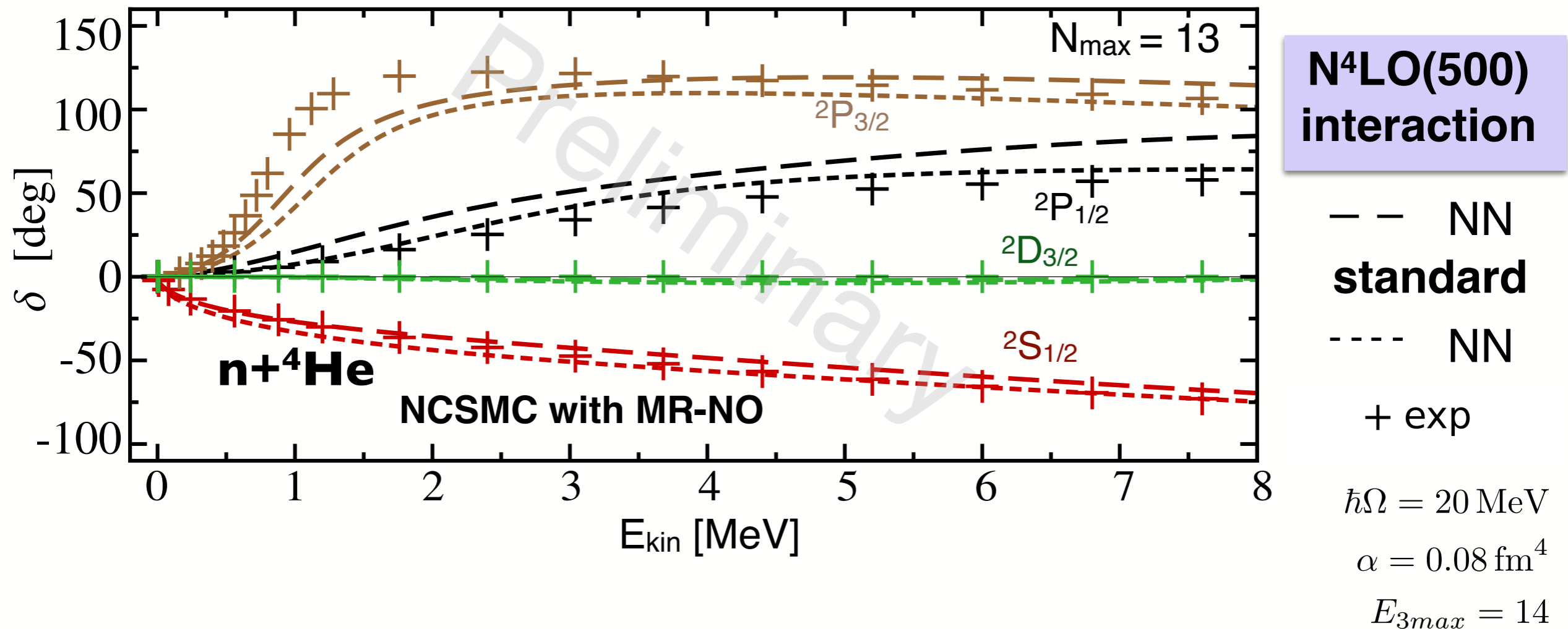
LENPIC interaction
 N²LO
 R = 1.0 fm

--- NN
 — NN+3N
 + exp
 c_D = 2, 4, 6
 c_E fitted to Triton g.s.

- splitting sensitive to c_D
- larger c_D values provide better reproduction

$\hbar\Omega = 24 \text{ MeV}$
 $\alpha = 0.08 \text{ fm}^4$
 $E_{3max} = 14$

n-⁴He with N⁴LO(500) interaction



- promising splitting properties of N⁴LO(500) NN interaction

- **insufficient knowledge of nuclear force** provides largest uncertainties in ab initio calculations
- **p-shell spectra** provide powerful testbed for chiral potential
- **combination of NCSMC with MR-NO** allows to include continuum effects at strongly reduced cost
 - enables heavier targets and **complex projectiles**
 - **probe future interactions** in weakly-bound system
 - splitting of $P_{3/2}$ - $P_{1/2}$ phase shifts in n - ^4He can be used to **constrain 3N** interaction

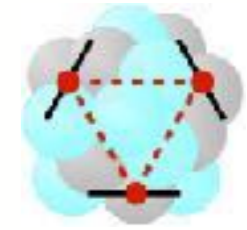
Thank you! Merci!

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LENPIC



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COMPUTING TIME