

Canada's national laboratory for particle and nuclear physics Laboratoire national canadien pour la recherche en physique nucléaire et en physique des particules

# **Predictive Power of Chiral Interactions for Nuclear Structure and Reaction Calculations in the p-Shell**

**INT Program INT-17-1a**  March 27 2017, Seattle



Owned and operated as a joint venture by a consortium of Canadian universities via a contribution through the National Research Council Canada Propriété d'un consortium d'universités canadiennes, géré en co-entreprise à partir d'une contribution administrée par le Conseil national de recherches Canada

**Angelo Calci | TRIUMF**



# **Outline**





# **Outline**

## **many-body eigenvalue problem ab initio description of nuclei**

#### D. |
|} **based int** |
|-**QCD-based interaction**

realistic NN+3N interactions





*Weinberg, van Kolck, Machleidt, Entem, Meissner, Epelbaum, Krebs, Bernard,... Chiral Eff based nuclear forces in the chiral effection* forces in the case of the chiral experiment of the chiral experi *Chiral EFT based nuclear forces* 12 *Chiral Eff based nuclear forces in the chiral eff based forces in the chiral city in the Chiral Eff based nuclear forces in the chiral spectrum* forces in the content force of the content force of the content forces in the content of t

## **• standard interaction:**

- NN @ N<sup>3</sup>LO: Entem & Machleidt, 500MeV cutoff (*Q/*⇤)<sup>0</sup>
- 3N @ N2LO: Navrátil, local, 500MeV cutoffs & modifications of the 3N force

## **• optimized N2LO interaction:**

- NN: Ekström et al., 500MeV cutoff, LECs fitted with **POUNDerS** NNLO NNLO NNLO vith  $(Q/\Lambda_\chi)^3$
- 3N: Navrátil, local, 500MeV cutoff, fit to <sup>4</sup>He & Triton

## **• EGM N2LO interaction:**

- NN: Epelbaum et al., 450, ..., 600 MeV cutoff N<sup>4</sup>LO
- 3N: Epelbaum et al., 450, . . . , 600 MeV cutoff, nonlocal (*Q/*⇤)<sup>5</sup>



March 27 2017

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*Weinberg, van Kolck, Machleidt, Entem, Meissner, Epelbaum, Krebs, Bernard,...*



**RIUMF** 



# Next Generation Interactions

*Weinberg, van Kolck, Machleidt, Entem, Meissner, Epelbaum, Krebs, Bernard,...*

## **• standard interaction:**

- NN @ N<sup>3</sup>LO: Entem & Machleidt, 500MeV cutoff (*Q/*⇤)<sup>0</sup>
- 3N @ N<sup>2</sup>LO: Navrátil, local cutoffs

## **• N2LOSAT interaction:**

• NN+3N: Ekström et al., nonlocal 450MeV cutoff, simultaneous fit to NN data and selected many-body observables

## **• LENPIC interaction:**

- NN up to N<sup>4</sup>LO: Epelbaum et al., semi-local cutoff
- 3N up to N<sup>3</sup>LO: under construction
- **• N4LO(500):**
	- NN @ N4LO: Machleidt et al., 500MeV cutoff



Similarity Renormalization Group (SRG) **accelerate** convergence by **pre-diagonalizing** the Hamiltonian with respect to the many-body basis Similarity Renormalization Group (SRG) **accelerate** convergence by **pre-diagonalizing** the Hamiltonian with respect to the many-body basis **accelerate** convergence by **pre-diagonalizing** the Hamiltonian ■ **unitary transformation** leads to **evolution equation** d **accelerate** convergence by **pre-diagonalizing** the Hamiltonian with respect to the many-body basis

**• unitary transformation leads to evolution equation** ■ **unitary transformation** leads to **evolution equation** lld<br> **ry transformation leads to evolution equation** α ■ **unitary transformation** leads to **evolution equation**

$$
\frac{d}{d\alpha}\widetilde{H}_{\alpha} = [\eta_{\alpha}, \widetilde{H}_{\alpha}] \quad \text{with} \quad \eta_{\alpha} = (2\mu)^2 [\text{T}_{\text{int}}, \widetilde{H}_{\alpha}] = -\eta_{\alpha}^{\dagger}
$$

#### advantages of SRG: **flexibility** and **simplicity** advantages of SRG: **flexibility** and **simplicity** and **Binding Corporation** and **Sinding** ages of SRG: **flexibility** and **simplicity** and





# **Outline**



• solving the eigenvalue problem: the eigenvalue problem: ■ **solving the eigenvalue problem** H**|**Ψn**〉 =** En**|**Ψn**〉**

H !  $|\Psi_n\rangle = E_n$  $|\Psi_{n}\rangle$ 

**• model space:** ■ **many-body basis**: Slater determinants **|**ν**〉** composed of  $\mathbf s$  are single-particle-par

spanned by Slater determinants with unperturbed excitation energy up to  $N_{max}$ ħΩ ad by Clater d .<br>מח<sup>i</sup>חי ν  $\overline{\phantom{a}}$ ν run  $\frac{1}{2}$  with unperturbed excitation energy of up the up that  $\frac{1}{2}$  with  $\frac{1}{2}$  and  $\frac$ 



■ **solving the eigenvalue problem** • solving the eigenvalue problem: the eigenvalue problem: H**|**Ψn**〉 =** En**|**Ψn**〉**  $e = 3$ !  $|\Psi_n\rangle = E_n$  $|\Psi_{n}\rangle$ H  $e = 2$ ■ **many-body basis**: Slater determinants **|**ν**〉 • model space:**  $e = 1$ 000000 composed of harmonic oscillator single-particle  $e = 0$ spanned h spanned **by Super determinants with Spanned Association problem of NCSM** -<br>חסוו*ב* .<br>ዝና<del>በ</del>  $r_{\mathbf{Q}}$ enormous increase of model space with unpertui enormous increase ν ν **ons** . particle number *A* up to *N<sub>max</sub>h*Ω **N<sub>mov</sub>ft**  $\frac{1}{2}$  with unperturbed excitation energy of up the upper second excitation energy of up the upper second energy of up to  $\frac{1}{2}$ 



particle number A

## **Importance Truncated NCSM Importance Truncated NCSM**

• a priori determination of relevant basis states via first-order ■ a priori determination of relevant basis states via first-order perturbation theory produced in a set  $\alpha$  $\langle \Phi_{\nu}$  $\overline{1}$  $\mid$  H $_{\sf int}$  $|\Psi_{\text{ref}}\rangle$ 

κν **= −**

εν **−** εref 4

**• importance truncated space** spanned by basis states with lk **|**κν**| ≥** κmin **|**κν**| ≥** κmin

MI

Angelo Calci 8

**Importance Truncated NCSM**

■ a priori determination of relevant basis states via first-order





particle number A

 $\frac{1}{2}$  with unperturbed excitation energy of up the upper second excitation energy of up the upper second energy of up to  $\frac{1}{2}$ 

MI

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- 16O: Origin of Induced 4N 16O: Origin of Induced 4N 16O: Origin of Induced 4N ■ analyze the sensitivity of spectra on low-energy constants ( $c_i$ ,  $c_D$ ,  $c_E$ ) and **cutoff** ( $\Lambda$ ) of the chiral 3N interaction at N<sup>2</sup>LO  $++$   $+$   $+$   $+$   $+$ 
	- why this is interesting:
		- **impact of N<sup>3</sup>LO contributions**: some N<sup>3</sup>LO diagrams can be absorbed into the N<sup>2</sup>LO structure by shifting the  $c_i$  constants<br> $q^2 A^{\pi}$   $q^4 A^{\pi}$   $q^4 A^{\pi}$   $q^4 A^{\pi}$  (Bernard et al.,

$$
\bar{c}_1 = c_1 - \frac{g_A^2 M_\pi}{64\pi F_\pi^2}, \quad \bar{c}_3 = c_3 + \frac{g_A^4 M_\pi}{16\pi F_\pi^2}, \quad \bar{c}_4 = c_4 - \frac{g_A^4 M_\pi}{16\pi F_\pi^2}
$$
 (Bernard et al.,  
Ishikawa, Robilotta)

• **uncertainty propagation**: sizable variations of the  $c_i$  from different extractions (also affects NN) Nm<sup>x</sup> Nm<sup>x</sup> Nm<sup>x</sup> ● **! "** ncertainty p Nm<sup>x</sup> Nm<sup>x</sup> Nm<sup>x</sup> **Opagation**. Sizable valiations of ● **!" UNCERTANILY PROPAGALION.** SIZADIt Nm<sup>x</sup> Nm<sup>x</sup> Nm<sup>x</sup>  $\frac{1}{2}$ 

 $c_1 = -1.23... - 0.76$ ,  $c_3 = -5.94... - 3.20$ ,  $c_4 = 3.40...5.40$   $[GeV^{-1}]$ Λ **=** 2.24fm**−**<sup>1</sup> Λ **=** 1.88fm**−**<sup>1</sup> Λ **=** 1.58fm**−**<sup>1</sup> ℏΩ **=** 20 MeV  $= 0.4$ Λ **=** 2.24fm**−**<sup>1</sup> Λ **=** 1.88fm**−**<sup>1</sup> Λ **=** 1.58fm**−**<sup>1</sup>  $\overline{1}$ ℏΩ **=** 20 MeV  $\overline{1}$  0.76 Λ **=** 2.24fm**−**<sup>1</sup> Λ **=** 1.88fm**−**<sup>1</sup> Λ **=** 1.58fm**−**<sup>1</sup>

**• cutoff dependence**: does the cutoff choice in the 3N interaction affect nuclear structure observables?

- 16O: Origin of Induced 4N 16O: Origin of Induced 4N 16O: Origin of Induced 4N ■ analyze the sensitivity of spectra on low-energy constants ( $c_i$ ,  $c_D$ ,  $c_E$ ) and **cutoff** ( $\Lambda$ ) of the chiral 3N interaction at N<sup>2</sup>LO  $+$   $+$   $+$   $+$   $+$   $+$ 
	- why this is interesting:
		- **impact of N<sup>3</sup>LO contributions**: some N<sup>3</sup>LO diagrams can be absorbed into the N<sup>2</sup>LO structure by shifting the  $c_i$  constants<br> $q^2 A^{\pi}$   $q^4 A^{\pi}$   $q^4 A^{\pi}$   $q^4 A^{\pi}$  (Bernard et al.,

$$
\bar{c}_1 = c_1 - \frac{g_A^2 M_\pi}{64\pi F_\pi^2}, \quad \bar{c}_3 = c_3 + \frac{g_A^4 M_\pi}{16\pi F_\pi^2}, \quad \bar{c}_4 = c_4 - \frac{g_A^4 M_\pi}{16\pi F_\pi^2}
$$
 (Bernard et al.,  
Ishikawa, Robilotta)

- **uncertainty propagation**: sizable variation and the contrainty propagation: sizable variante contrainty different extractions (also affects NN)  $c_1 = -1.23... -0.76$ ,  $c_3 = -5.94... -3$  **Hamiltonians and 2 6 and provide constraints** for chiral Hamiltonians and **quantify**  Nm<sup>x</sup> Nm<sup>x</sup> Nm<sup>x</sup> Nmx (1999) and the state of ● **! "** Λ **=** 2.24fm**−**<sup>1</sup> Λ **=** 1.88fm**−**<sup>1</sup> Λ **=** 1.58fm**−**<sup>1</sup> ℏΩ **=** 20 MeV Nm<sup>x</sup> Nm<sup>x</sup> Nm<sup>x</sup> **C directed inty propagation**. Sizable **2** ● **!" d**  $\overline{P}$  0.16  $\overline{P}$  $\mathcal{L}_1$   $\mathcal{L}_2$  = −5.94… − 1.  $\overline{1}$ ℏΩ **=** 20 MeV Nm<sup>x</sup> Nm<sup>x</sup> Nm<sup>x</sup>  $\frac{1}{2}$  $\overline{1}$  0.76 Λ **=** 2.24fm**−**<sup>1</sup> Λ **=** 1.88fm**−**<sup>1</sup> Λ **=** 1.58fm**−**<sup>1</sup>
	- **cutoff dependence**: does the cutoff **change interactional dependence**: does the cutoff concertainties on affect nuclear structure observables?



- many states many states are rather independent are rather *ci*
	- $\ddotsc$  fixet  $\ddotsc$ • first  $1+$  state shows strong *c3* dependence

ℏΩ **=** 16 MeV ℏΩ **=** 16 MeV  $N_{\text{max}} = 8$  $\alpha = 0.08$  fm  $^4$ **IT-NCSM**

П

### and the contract of the Cutoff of the Cu 12C: Sensitivity to *c<sub>D</sub> and cutoff*



- on *c<sub>D</sub>*,  $\frac{1}{2}$  ctronger stronger dependependence moderate dependence stronger on Λ
- again first 1**<sup>+</sup>** • again first  $1^+$ state is most sensitive

 $\hbar\Omega = 16$  MeV **IT-NCSM**

 $N_{\text{max}} = 8$  $\alpha = 0.08$  fm<sup>4</sup> α *b* 0.00 m

## **Correlation Analysis: 12C(1+) vs. 10B(1+)** Correlation Analysis: 12C**(**1**+)** vs. 10B**(**1**+)** Correlation Analysis: 12C**(**1**+)** vs. 10B**(**1**+)**



- correlation does not agree with experiment
- **+** exp  $\overline{\phantom{a}}$ with E&M NN interaction





![](_page_18_Figure_0.jpeg)

• small cutoff dependence for NN+3N

### **POTRIUMF**

10B: Cutoff Dependence (10B: Cutoff Dependence de la Cutoff Dependence de la Cutoff Dependence de la Cutoff<br>10B: Cutoff Dependence de la Cutoff Dependence de la Cutoff Dependence de la Cutoff Dependence de la Cutoff D 12C: Cutoff Dependence (12C: Cutoff Dependence de la Constantino de la Constantino de la Constantino de la Con<br>12C: Cutoff Dependence (12C: Cutoff Dependence de la Constantino de la Constantino de la Constantino de la Co

#### n Chiral Order der N3LO+N2LO  $\int$ hiral $\int$ rdar $\int$ 12C: Cutoff Dependence (1982)<br>12C: Cutoff Dependence (1982)<br>12C: Cutoff Dependence (1982) ndence on Chiral Order N2LO+N2LO  $450000$   $\alpha$  MeV  $\alpha$

standard

Epelbaum

![](_page_19_Figure_3.jpeg)

Li: Alternative Interactions

![](_page_20_Picture_0.jpeg)

## **Correlation Analysis: 12C(1+) vs. 10B(1+)** Correlation Analysis: 12C**(**1**+)** vs. 10B**(**1**+)** Correlation Analysis: 12C**(**1**+)** vs. 10B**(**1**+)**

![](_page_20_Figure_2.jpeg)

![](_page_21_Picture_0.jpeg)

# **Outline**

![](_page_21_Figure_2.jpeg)

![](_page_22_Picture_0.jpeg)

**RIUMF** 

PRL 117, 242501 (2016) PHYSICAL REVIEW LETTERS week ending week ending

9 DECEMBER 2016

#### Can Ab Initio Theory Explain the Phenomenon of Parity Inversion in <sup>11</sup>Be?

Angelo Calci,<sup>1,\*</sup> Petr Navrátil,<sup>1,†</sup> Robert Roth,<sup>2</sup> Jérémy Dohet-Eraly,<sup>1,‡</sup> Sofia Quaglioni,<sup>3</sup> and Guillaume Hupin<sup>4,5</sup>

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#### **Neutron-rich halo nucleus 11Be**  Spectrum <sup>n</sup> <sup>þ</sup> <sup>10</sup>Be halo structure, is investigated from first principles using chiral two- and three-nucleon forces.

![](_page_22_Figure_11.jpeg)

• **parity inversion** g.s. to be  $J<sup>π=1/2</sup>$  $\eta$  e to be  $\overline{\Pi}$ =1/2-

![](_page_22_Picture_13.jpeg)

### <sup>2+)+n</sup> • Halo structure – In reality, 11Be g.s. is **J<sup>π</sup>=1/2+** - parity inversion

weakly bound J=1/2 states spectrum dominated by n-<sup>10</sup>Be state also bound  $\sim$  180 keV  $\sim$ re way the way the material to the sense of the interactions are constrained to the interactions are constrained to  $\mu$ de data can domina

![](_page_23_Picture_0.jpeg)

# Neutron-rich halo Nucleus 11Be

PRL 117, 242501 (2016) PHYSICAL REVIEW LETTERS week ending week ending

9 DECEMBER 2016

![](_page_23_Figure_5.jpeg)

### Can Ab Initio Theory Explain the Phenomenon of Parity Inversion in <sup>11</sup>Be?

![](_page_24_Picture_0.jpeg)

# Neutron-rich halo Nucleus 11Be

PRL 117, 242501 (2016) PHYSICAL REVIEW LETTERS week ending week ending

9 DECEMBER 2016

![](_page_24_Figure_5.jpeg)

#### Can Ab Initio Theory Explain the Phenomenon of Parity Inversion in <sup>11</sup>Be?

![](_page_25_Picture_0.jpeg)

# Neutron-rich halo Nucleus 11Be

PRL 117, 242501 (2016) PHYSICAL REVIEW LETTERS week ending week ending

9 DECEMBER 2016

![](_page_25_Figure_5.jpeg)

#### Can Ab Initio Theory Explain the Phenomenon of Parity Inversion in  $^{11}$ Be?

**RIUMF** 

# NCSM with Continuum (NCSMC)

• representing  $H|\Psi^{J\pi\tau}\rangle = E|\Psi^{J\pi\tau}\rangle$  using the over-complete basis H  $|\Psi^{J\pi T}\rangle = E$  $|\Psi^{J\pi T}\rangle$ *Baroni, Navrátil, Quaglioni Phys. Rev. Lett. 110, 022505 (2013)*

$$
|\Psi^{J\pi T}\rangle = \sum_{\lambda} c_{\lambda} |\Psi_A E_{\lambda} J^{\pi} T\rangle + \sum_{\nu} \int dr r^2 \frac{\chi_{\nu}(r)}{r} |\xi_{\nu r}^{J\pi T}
$$

expansion in A-body NCSM eigenstates

relative motion of clusters NCSM/RGM expansion

 $\setminus$ 

• leads to NCSMC equation

 $\overline{\phantom{a}}$ 

$$
\left(\begin{array}{cc}H_{NCSM} & h \\ h & \mathcal{H}\end{array}\right)\left(\begin{array}{c} c \\ \chi(r)/r\end{array}\right)=E\left(\begin{array}{cc} \mathbb{I} & g \\ g & \mathbb{I}\end{array}\right)\left(\begin{array}{c} c \\ \chi(r)/r\end{array}\right)
$$

\n- with 3N contributions in 
$$
H_{NCSM}
$$
  $h$   $H_{NCSM}$   $h$   $H_{NCSM}$   $qiven by$  contains NCSM/RAM NCSM  $(\Psi_A E_{\lambda' J}^{T} T | H | \xi_{vr}^{J \pi T})$  Hamiltonian kernel  $qgen$   $Qatri$   $Angelo$   $Calci$
\n

![](_page_27_Picture_0.jpeg)

### • **Halo structure**

spectrum dominated by  $n^{-10}$ Be halo structure

![](_page_27_Figure_3.jpeg)

## **NCSM input**

- calculations use NCSM vectors and energies as input
- include n-10Be continuum  $(0^+,2^+,2^+$  states of  $^{10}Be)$
- include <sup>11</sup>Be short-range correlations:
	- 4 negative parity (at least) 3 positive parity states of 11Be

![](_page_28_Picture_0.jpeg)

**NCSM input**

and energies as input

• calculations use NCSM vectors

### • **Halo structure**

spectrum dominated by  $n^{-10}$ Be halo structure

![](_page_28_Figure_3.jpeg)

![](_page_29_Picture_0.jpeg)

### 11Be excitation spectrum 2 3 4 *E*thr. [MeV]

![](_page_29_Figure_2.jpeg)

1<br>11<br>11

![](_page_30_Picture_0.jpeg)

### 11Be excitation spectrum 2 3 4 *E*thr. [MeV]

![](_page_30_Figure_2.jpeg)

1<br>11<br>11

March 27 2017 **2017** 20 November 20 November 20 Angelo Calci

![](_page_31_Picture_0.jpeg)

## 11Be excitation spectrum 4 5 *E*thr. [MeV]

![](_page_31_Figure_2.jpeg)

March 27 2017 **21** 21 21 22

5

6

Angelo Calci ! drastic difference for the 1/2+ state right at threshold

## **TRIUMF** 11Be: Photodisintegration process & E1 transition

![](_page_32_Picture_246.jpeg)

![](_page_32_Picture_247.jpeg)

\*Kwan et al. Phys. Lett. B 732, 210 (2014)

- **strongest known E1** transition between low-lying states (attributed to halo structure)
- reproduced **only** with **continuum effects**

![](_page_32_Figure_6.jpeg)

- **conflicting** experimental **measurements**
- ab initio results:
	- **discriminate** between measurements
	- **predict dip** at 3/2- resonance energy

![](_page_33_Picture_0.jpeg)

.

**negative parity positive parity**

### Mirror nuclei:<sup>11</sup>Be and <sup>11</sup>N *Langhammer, Navrátil, Quaglioni, Hupin, Calci, Roth; Phys. Rev. C 91, 021301(R) (2015)*

![](_page_33_Figure_2.jpeg)

March 27 2017 **Angelo Calci Angelo Calci** 23

#### p+10C Scattering: Structure of 11N resonances **p+10C scattering: structure of 11N resonances**  60  $\overline{\mathsf{J}}$ ,<br>C  $\frac{1}{2}$ <u>1</u> nanc  $\overline{a}$ te  $\overline{\phantom{a}}$  $\overline{\phantom{0}}$ 5/2+ 60 ring: Structure of TIN resonanc 21L  $\overline{\phantom{a}}$

30

#### $\blacktriangleright$ ia chii 150 180  $\overline{\text{P}}$ ering allows discrim 5/2+ 90 among chiral nuclear forces  $\overline{\phantom{0}}$ elastic scattering allows discrimination Mirror System 120 **180**  $\mathbf{H}$ among chiral nuclear torce  $\frac{1}{2}$ 90 elastic scattering allows discrimination 5/2+ and chiral nuclear ford  $\Delta$ elastic scallering allows discrimination Discrimination among chiral nuclear forces

90

**p+10C scattering: structure of 11N resonances** 

3/2+

3/2+

![](_page_34_Figure_2.jpeg)

niaboration.<br>Igo, A. Sanetullaev *et al.* Nacceles A. Calci, P. Navratil, G. Hupin, S. Quaglioni, R. Roth *et al* with IRIS collaboration, in preparation

3/2-

P5/2

30

 $\mathbb{R}^n$ 

A. Kumar, R. Kanungo, A. Sanetullaev *et al.*  A. Kumar, R. Kanungo, A. Sanetullaev *et al.*  A. Kumar, R. Kanungo, A. Sanetullaev *et al.* 

90

Ekin [MeV]

Ekin [MeV]

S1/2

S1/2

#### p+10C Scattering: Structure of 11N resonances **p+10C scattering: structure of 11N resonances**  60  $\overline{\mathsf{J}}$ ,<br>C  $\frac{1}{2}$ <u>1</u> nanc  $\overline{a}$ te  $\overline{\phantom{a}}$  $\overline{\phantom{0}}$ 5/2+ 60 ring: Structure of TIN resonanc **RIUMF**  $\overline{\phantom{a}}$

30

#### $\blacktriangleright$ ia chii 150 180  $\overline{\text{P}}$ ering allows discrim 90 among chiral nuclear forces  $\overline{a}$ elastic scattering allows discrimination Mirror System 120 **180**  $\mathbf{H}$ among chiral nuclear torce  $\frac{1}{2}$ elastic scattering allows discrimination 5/2+ and chiral nuclear ford  $\Delta$ elastic scallering allows discrimination Discrimination among chiral nuclear forces

90

5/2+

**p+10C scattering: structure of 11N resonances** 

3/2+

3/2+

![](_page_35_Figure_2.jpeg)

IRIS collaboration: IRIS collaboration: IRIS collaboration: A. Kumar, R. Kanungo, A. Sanetullaev *et al.*  A. Kumar, R. Kanungo, A. Sanetullaev *et al.*  A. Kumar, R. Kanungo, A. Sanetullaev *et al.*  IRIS collaboration:

90

Ekin [MeV]

Ekin [MeV]

S1/2

S1/2

30

niaboration.<br>Igo, A. Sanetullaev *et al.* Nacceles A. Calci, P. Navratil, G. Hupin, S. Quaglioni, R. Roth *et al* with IRIS collaboration, in preparation

3/2-

90

P5/2

30

 $\mathbb{R}^n$ 

#### p+10C Scattering: Structure of 11N resonances **p+10C scattering: structure of 11N resonances**  60  $\overline{\mathsf{J}}$ ,<br>C  $\frac{1}{2}$ <u>1</u> nanc  $\overline{a}$ te  $\overline{\phantom{a}}$  $\overline{\phantom{0}}$ 5/2+ 60 ring: Structure of TIN resonanc **RIUMF**  $\overline{\phantom{a}}$

30

#### $\blacktriangleright$ ia chii 150 180  $\overline{\text{P}}$ ering allows discrim 90 among chiral nuclear forces  $\overline{a}$ elastic scattering allows discrimination Mirror System 120 **180**  $\mathbf{H}$ among chiral nuclear torce  $\frac{1}{2}$ elastic scattering allows discrimination 5/2+ and chiral nuclear ford  $\Delta$ elastic scallering allows discrimination

90

5/2+

**p+10C scattering: structure of 11N resonances** 

3/2+

3/2+

![](_page_36_Figure_2.jpeg)

90

Ekin [MeV]

Ekin [MeV]

S1/2

S1/2

30

3/2-

90

P5/2

30

 $\mathbb{R}^n$ 

![](_page_37_Picture_0.jpeg)

# NCSMC with approximated 3N forces

with P. Navrátil, R. Roth, E. Gebrerufael NCSM with Continuum (NCSMC)

• representing  $H|\Psi^{J\pi T}\rangle = E|\Psi^{J\pi T}\rangle$  using the **over-complete basis** H  $|\Psi^{J\pi T}\rangle = E$  $|\Psi^{J\pi T}\rangle$ 

**RETRIUMF** 

![](_page_38_Figure_2.jpeg)

# red the mode of the set **@TRIUMF**<br>Normal-orde **order** ring (NO) approximation ˜ Normal-ordering (NO) approximation

contain information in the reference standard tool to reduce particle rank • standard tool to **reduce particle rank**

 $\approx$ 

• generally NO can be considered as basis transformation

 $\overline{\phantom{a}}$ contain information of reference state and initial 3N force

- interested in direct description of **open-shell systems**
	- multi-reference normal ordering (MR-NO)

 $V_{3N} \approx \tilde{V}_{0N} + \tilde{V}_{1N} + \tilde{V}_{2N} + \tilde{V}_{3N}$ 

• generalization of wicks theorem [Kutzelnigg, Mukherjee]

## **NCSM/RGM kernels with MR-NO contributions**

- reduces computational costs tremendously
- impressively accurate approximation

<sup>2</sup>*<sup>N</sup>* + *V*

![](_page_40_Picture_0.jpeg)

#### **Derive NCSM/RGM Kernels Derive NCSM/RGM Kernels**  $\mathcal{L}$ *MT*1*m<sup>t</sup> M*0 1*m*<sup>0</sup> *j M*0 *<sup>T</sup>*1*m*<sup>0</sup> *t* ✓ *<sup>I</sup>*<sup>1</sup> *<sup>j</sup> <sup>J</sup> M*<sup>1</sup> *m<sup>j</sup> M* ◆ ✓ *T*<sup>1</sup> <sup>2</sup> *T M<sup>T</sup>*<sup>1</sup> *m<sup>t</sup> M<sup>T</sup>* ◆ ✓ *I*<sup>0</sup> <sup>1</sup> *j*<sup>0</sup> *J M*<sup>0</sup> <sup>1</sup> *m*<sup>0</sup> *<sup>j</sup> M* ◆ ✓ *T*<sup>0</sup> 1 <sup>2</sup> *T P***1** *M* ◆ ⇡0 <sup>1</sup>*M*<sup>0</sup> *l* 0 *j*0 *m*<sup>0</sup> *m*<sup>0</sup> ⇡1*M*1*T*1*M<sup>T</sup>*<sup>1</sup> *<sup>&</sup>gt;SD <sup>|</sup>nljm<sup>j</sup>* 1 *<sup>t</sup>|O| <sup>A</sup>*<sup>1</sup>*E*1*I*<sup>1</sup> *m<sup>t</sup> >* Derive NCSM/RGM Kerne **BALLARD BLOOM AFRANAI** *<sup>A</sup>*<sup>1</sup>*E*<sup>0</sup> 1*I*<sup>0</sup> 1 ⇡0 <sup>1</sup>*M*<sup>0</sup> 1*T*<sup>0</sup> 1*M*<sup>0</sup> *<sup>T</sup>*<sup>1</sup> *| < n*<sup>0</sup> *l* 0 *j*0 *m*<sup>0</sup> *j m*<sup>0</sup> *<sup>t</sup>|O| <sup>A</sup>*<sup>1</sup>*E*1*I*<sup>1</sup> ⇡1*M*1*T*1*M<sup>T</sup>*<sup>1</sup> *<sup>&</sup>gt;SD <sup>|</sup>nljm<sup>j</sup>* 1 2 *m<sup>t</sup> >* <u>2</u> 2 2 2 2 2 2 2 2 2 2 2 0.2 0N exchange of the second sec

#### $\bigcap$   $\bigcap$  karn <u>0.2 0N external product</u> *SD <* ✏ ⌫0*n*<sup>0</sup> *|V* <sup>0</sup>*<sup>N</sup> T <sup>A</sup>*1*,A|*✏ 0B kernel *A* 1

dom nant 0B kernel contribution inclu *M I*<sup>*M*</sup> *M<sup>T</sup>*<sup>1</sup> *m<sup>t</sup> M<sup>T</sup>* **led in tar** *M*<sup>0</sup> <sup>1</sup> *m*<sup>0</sup> let e ens *M*<sup>0</sup> *m*<sup>0</sup> *<sup>t</sup> M<sup>T</sup>* dominant 0B kernel contribution included in target eigensta *<sup>A</sup>*<sup>1</sup>*E*<sup>0</sup> 1*I*<sup>0</sup> ⇡0 <sup>1</sup>*M*<sup>0</sup> 1*T*<sup>0</sup> 1*M*<sup>0</sup> *<sup>T</sup>*<sup>1</sup> *|a† a<sup>n</sup>*0*l*0*j*0*m*<sup>0</sup> *| <sup>A</sup>*<sup>1</sup>*E*1*I*<sup>1</sup> ⇡1*M*1*T*1*M<sup>T</sup>*<sup>1</sup> *>SD*  $\overline{\mathsf{U}}$  11 *inant M*1*m<sup>j</sup>*  $\sqrt{ }$ *MT*1*m<sup>t</sup>*  $\overline{1}$ *j T*1 *t*  $\overline{\phantom{a}}$ ✓ *<sup>I</sup>*<sup>1</sup> *<sup>j</sup> <sup>J</sup> M*<sup>1</sup> *m<sup>j</sup> M* ◆ ✓ *T*<sup>1</sup> <sup>2</sup> *T M I M I M T*  $\overline{a}$  *jo tora in targe* + **aigenets** *M*<sup>0</sup> *T*<sup>1</sup> *m*<sup>0</sup> *<sup>t</sup> M<sup>T</sup>* ⇥ *SD <* <sup>0</sup>  $\Rightarrow$  only MR-NO 1B and 2B kernels contribute **dominant** dominant 0B kernel contribution included in target eigenstates 0.3 1N direct

### 0.3 1N direct 0.3 1N direct *SD <* ✏ *J* ⇡*T* ⌫0*n*<sup>0</sup> *|V <sup>A</sup>|*✏ *<sup>J</sup>* ⇡*<sup>T</sup>* ⌫*<sup>n</sup> >SD* 1B kernel

 $\delta_{SD} < \epsilon_{\nu' n'}^{\mathcal{J} \pi T} |{\boldsymbol{V}}_A | \epsilon_{\nu n}^{\mathcal{J} \pi T} >_{SD} \delta_{NL}^{\mathcal{J} \pi T}$ 

 $\sum$  $m_j$  *M*  $\overline{\phantom{a}}$  $m_t$ *m*<sup>*t*</sup>  $M$  $\overline{\phantom{a}}$  $_1^\prime$  $m^\prime_j$   $\Lambda$  $\overline{\phantom{a}}$  $T_1$ <sup>*m* $'_t$ </sup>  $\left\{\begin{array}{c|c}I_1 & j & J\end{array}\right\}$  $M_1$  *m<sub>j</sub>* |  $\mathcal{M}$  $\left( \begin{array}{cc} T_1 & \frac{1}{2} \end{array} \right)$  $M_{T_1}$  *m<sub>t</sub>* |  $M_1$  $\left( \begin{array}{cc} I'_1 & j'_1 \ I'_2 & j'_2 \end{array} \right)$  $M'_1$   $m'_j$  | *J* ◆ ✓ *T*<sup>0</sup> 1  $\frac{1}{2}$ ,  $\left\langle M'_{T_1} \mid m'_t \mid \right.$  $\begin{array}{cc} \begin{array}{c} \mathcal{L} \end{array} & \sum \end{array} \begin{array}{c} \sum \end{array} \begin{array}{c} \begin{array}{c} \mathcal{L} \\ \mathcal{M} \end{array} \begin{array}{c} \begin{array}{c} \mathcal{L} \\ \mathcal{M} \end{array} \end{array} \begin{array}{c} \begin{array}{c} \mathcal{L} \\ \mathcal{M} \end{array} \end{array} \begin{array}{c} \begin{array}{c} \mathcal{L} \\ \mathcal{M} \end{array} \end{array} \begin{array}{c} \begin{array}{c} \mathcal{L} \\ \mathcal{M} \end{$  $M_1m_j$  $\sum$  $M_{T_1}$  $m_t$  $\sum$  $M_1'm_j'$  $\sum$  $M'_{T_1}m'_t$  $\left( \begin{array}{ccc} I_1 & j \end{array} \right| \mathcal{J}$  $M_1$  *m<sub>j</sub>* |  $\mathcal{M}$  $\left( \begin{array}{cc} T_1 & \frac{1}{2} \end{array} \right)$  *T*  $M_{T_1}$  *m*<sub>t</sub> |  $M_T$  $\left( \begin{array}{cc} I'_1 & j'_2 \end{array} \right)$  $M_1'$  *m'<sub>j</sub>* |  $\mathcal{M}$  $\bigwedge$   $T_1'$  $\frac{1}{2}$  | T  $M'_{T_1}$   $m'_t \mid M_T$  $\sum_{i=1}^{n} (I_i - j \mid \mathcal{J}) (T_1 - \frac{1}{2} \mid T) (I'_1 - j' \mid \mathcal{J}) (T'_1 - \frac{1}{2} \mid T)$ ⇥ *SD <* <sup>0</sup> *<sup>A</sup>*<sup>1</sup>*E*<sup>0</sup> 1*I*<sup>0</sup> 1  $\sum_{i=1}^{\infty} \left( \begin{array}{cc} 1 & j \\ M_1 & m_i \end{array} \right) \left( \begin{array}{cc} J & 1 & \frac{1}{2} \\ M_T & m_i \end{array} \right)$ 1 1

*<sup>J</sup>* ⇡*<sup>T</sup>* ⌫*<sup>n</sup> <sup>&</sup>gt;SD*<sup>=</sup> *<sup>V</sup>* <sup>0</sup>*<sup>N</sup> · NormKernel*

- $SD \leq \psi'_{A-1} E'_1 I'^{\pi_1}_1$  $\times$  *sp*  $\lt \psi'_{A-1} E'_1 I'_1^{\pi'_1} M'_1 T'_1 M'_{T_1} |\psi_{A-1} E_1 I_1^{\pi_1} M_1 T_1 M_{T_1} >_{SL}$  $\int_0^{\pi_1} M_1' T_1' M_{T_1}' \left| \psi_{A-1} E_1 I_1^{\pi_1} M_1 T_1 M_{T_1} \right| >_{SD}$  $\frac{1}{50}$   $\frac{1}{2}$   $\int_1^{\prime} \pi_1^{\prime} M_1^{\prime} T_1^{\prime} M_m^{\prime}$
- $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$  $\overline{a}$  $m_j$ <sup>*t*</sup><sub>*f*</sub><sup>*m*</sup> $\frac{1}{6}$  $\overline{ }$  $\times$   $\langle n'l'j'm'_{j}\frac{1}{2}m'_{t}|V_{A}|nljm_{j}\frac{1}{2}m_{t}\rangle$ 1 2  $m_t^{\prime}$ <sup>*|*</sup>*V<sub>A</sub>|* $nljm_j$ 1 2  $\times$   $\langle n'l'j'm'_{i} \frac{1}{2} m'_{t} | V_{A} | n l j m_{j} \frac{1}{2} m_{t} \rangle$

 $-$ s $_D<\epsilon^{\mathcal{J}\pi T}_{\nu'n'}|{\boldsymbol{V}}_A{\boldsymbol{T}}_{A-1,A}|\epsilon^{\mathcal{J}\pi T}_{\nu'n} >_{SD}$  $\frac{1}{4-1}$   $\sum_{i=1}^{n}$   $\sum_{i=1}^{n}$  $\sqrt{ }$  $\ddot{x}$  $\ddot{\phantom{0}}$  $\frac{1}{\sqrt{2}}$  $T_1$  $A-1 \sum_{M_1 m_j} \sum_{M_{T_1} m_t} \sum_{M'_1 m'_j} \sum_{M'_{T_1} m'_t} \left( M_1 \ m_j \mid \mathcal{M} \ \right) \left( M_{T_1} \ m_t \mid M_T \ \right) \left( M'_1 \ m'_j \mid \mathcal{M} \ \right) \left( M'_{T_1} \ m'_t \mid \mathcal{M} \right)$  $J'$   $\vert a^{\dagger} \vert$  **q**  $\vert \psi \rangle$  *F<sub>1</sub>, T<sub>1</sub>, T<sub>1</sub>, N<sub>1</sub>, <i>T*<sub>1</sub>  $M_{\tau} > \epsilon$  $= -\frac{1}{4}$  $A-1$  $\sum$  $M_1m_j$  $\sum$  $M_{T_1}$  $m_t$  $\sum$  $M_1'm_j'$  $\sum$  $M'_{T_1}m'_t$  $\left( \begin{array}{ccc} I_1 & j \end{array} \right| \mathcal{J}$  $M_1$  *m<sub>j</sub>* |  $\mathcal{M}$  $\left( \begin{array}{cc} T_1 & \frac{1}{2} \end{array} \right)$  *T*  $M_{T_1}$  *m<sub>t</sub>* |  $M_T$  $\left( \begin{array}{cc} I'_1 & j'_1 \end{array} \right)$  $M'_1$  *m'<sub>j</sub>* |  $\mathcal{M}$  $\bigwedge$   $T_1'$  $\frac{1}{2}$   $T$  $M'_{T_1}$   $m'_t \mid M_T$ ◆

- $\sum_{i}$   $SD$  $\sim \psi_f$  $-1^{L_1}$  $1$ <sup>*M*</sup>  $M_{T_1}'T_1'M_{T_1}'|\bm{a}_{nljm_jm_t}^\dagger\bm{a}_{\alpha_A}$  $\times$   $\sum_{SD}$   $\langle \psi'_{A-1} E'_1 I'_1 \pi'_1 M'_1 T'_1 M'_{T_1} | \bm{a}_{nljm,m_t}^{\dagger} \bm{a}_{\alpha_{A-1}} | \psi_{A-1} E_1 I_1 \pi_1 M_1 T_1 M_{T_1} \rangle_{SD}$  $\frac{a_{A-1}}{a_{A-1}}$  $\frac{2}{\alpha}$   $\frac{5}{\alpha}$  $\overline{\phantom{a}}$  $\mathbf{r}$ / $\pi'$  $\overline{U}$  $\bm{a}_{nljm_jm_t}\bm{a}_{\alpha_A}$  $-1$ *L*<sub>1</sub>*I*<sub>1</sub><sup>*m*</sup><sub>1</sub><sup>*I*</sup>  $\times \sum_{i} S_{i} D \langle \psi'_{A-1} E'_{1} I'_{1} \pi'_{1} M'_{1} T'_{1} M'_{T_{1}} | \mathbf{a}_{nljm_jm_t}^{\dagger} \mathbf{a}_{\alpha_{A-1}} | \psi_{A-1} E_{1} I_{1} \pi_{1} M_{1} T_{1} M_{T_{1}} \rangle_{SD}$  $\alpha_{A-1}$  $SD \leq \psi'_{A-1} E'_1 I'_1$  $\frac{\pi_1^{\prime} M_1^{\prime} T_1^{\prime} M_{T_1}^{\prime}}{|a_{n l j m_j m_t}^{\dagger} a_{\alpha_{A-1}}|} \psi_{A-1} E_1 {I_1}^{\pi_1} {M_1} T_1 {M_{T_1}} >_{SD}$
- $\leq n/\ell$  $\chi \quad < n'l'j'm'_j\frac{1}{2}m'_t|\bm{V}_A|\alpha_{A-1}>$  $\overline{\phantom{0}}$  $\times$   $\langle n'l'j'm'_j\frac{1}{2}m'_t|\boldsymbol{V}_A|\alpha_{A-1}\rangle$ 1 2  $m_t'|\boldsymbol{V}_A|\alpha_{A-1} >$

#### JUNE KAMALER DI SERVICE SERVIC  $\overline{2}$  No  $\overline{1}$  $\overline{\phantom{a}}$   $\overline{\$ ⇥ *< n*<sup>0</sup> *l* 0 *j*0 *m*<sup>0</sup> *j* 2 *m*<sup>0</sup> *<sup>t</sup>|V <sup>A</sup>|*↵*<sup>A</sup>*<sup>1</sup> *>* 0.5 NN direct 2B kernel

<sup>=</sup> <sup>1</sup>

*SD <* ✏

…

![](_page_41_Picture_0.jpeg)

## NCSMC: Impact of 3N in Kernels  $\overline{\phantom{a}}$ 3  $\overline{\mathsf{N}}$

![](_page_41_Figure_2.jpeg)

![](_page_42_Picture_0.jpeg)

## NCSMC: Impact of 3N in Kernels 2 3  $\overline{\mathbf{C}}$

![](_page_42_Figure_2.jpeg)

![](_page_43_Picture_0.jpeg)

# First application: 12N

• **ideal candidate** 

weakly bound J=1+ state dominated by p-11C

![](_page_43_Figure_4.jpeg)

- some low lying resonances not measured precisely
- $11C(p,y)$ <sup>12</sup>N can bypass triple-alpha process
- planed experiment at TUDA facility at TRIUMF

## **ab initio NCSMC**

- include p-<sup>11</sup>C continuum (3/2- ,1/2- ,5/2- ,3/2- states of 11C)
- include 4 negative and 6 positive parity states of 12N
- MR-NO with respect to  $N_{max}=0$ eigenstate of 12N

### **@TRIUMF**

### 12N spectrum with continuum effects 9Be: NO. NCC WEST VEHICLE WITH SPECIFIC UP 2  $\bm{\nabla}$ *E*thr. [MeV]

![](_page_44_Figure_2.jpeg)

### **RETRIUMF**

### 12N spectrum with continuum effects 9Be: NO. NCC WEST VEHICLE WITH SPECIFIC UP 2  $\bm{\nabla}$ *E*thr. [MeV]

![](_page_45_Figure_2.jpeg)

![](_page_46_Picture_0.jpeg)

# Probe chiral interaction in light nuclear scattering

![](_page_47_Picture_0.jpeg)

# n-4He: Standard interaction

![](_page_47_Figure_2.jpeg)

![](_page_48_Picture_0.jpeg)

# n-<sup>4</sup>He with N<sup>2</sup>LO<sub>SAT</sub>

![](_page_48_Figure_2.jpeg)

- 9 P3/2 P1/2 splitting sensitive to details of nuclear force
- under- or overestimated by NN+3N(400) or N<sup>2</sup>LO<sub>SAT</sub> interaction

![](_page_49_Picture_0.jpeg)

# Correlation Analysis: 12C**(**1**+)** vs. 10B**(**1**+)** n-4He with LENPIC interaction

![](_page_49_Figure_2.jpeg)

• splitting underestimated without 3N interaction  $\mathsf{\Pi}$  $n$ 

 $\hbar\Omega = 24 \,\mathrm{MeV}$  $\alpha = 0.08 \, \mathrm{fm}^4$  $E_{3max} = 14$  $\overline{D}$  c<sub>D</sub>  $\overline{O}$  c<sub>D</sub>  $\overline{O}$  ( $\overline{O}$  cD<sub>0</sub>)  $\mu L = 24 \text{N}$ 

![](_page_50_Picture_0.jpeg)

# n-<sup>4</sup>He with LENPIC interaction

![](_page_50_Figure_2.jpeg)

March 27 2017 **Angelo Calci** 37 **Angelo Calci** 37

![](_page_51_Picture_0.jpeg)

## n-<sup>4</sup>He with N<sup>4</sup>LO(500) interaction n-4He with N4LO(500) interaction

![](_page_51_Figure_2.jpeg)

• promising splitting properties of N4LO(500) NN interaction sir<br>. a properties of N<sup>4</sup>LO(500) NN interaction

![](_page_52_Picture_0.jpeg)

# **Outlook**

- **insufficient knowledge of nuclear force** provides largest uncertainties in ab initio calculations
- **p-shell spectra** provide powerful testbed for chiral potential
- **combination of NCSMC with MR-NO** allows to include continuum effects at strongly reduced cost
	- enables heavier targets and **complex projectiles**
	- **probe future interactions** in weakly-bound system
	- splitting of  $P_{3/2}$   $P_{1/2}$  phase shifts in n-4He can be used to **constrain 3N** interaction

## **RIUMF**

# Epilogue Thank you! Merci!

**LENPIC**

## ■ **thanks to my collaborators •** thanks to my collaborators

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- Michigan State University, USA **Istituto Nazionale di Fisica Nucleare, Pisa, Italy** • J. Dohet-Eraly<br>Patitute Nationals di Fisice N

![](_page_53_Picture_7.jpeg)

![](_page_53_Picture_8.jpeg)

![](_page_53_Picture_9.jpeg)

U. TIUTTETU-TI<del>C</del>UUI<br>LLNL Livermore, USA C. Romero-Redondo

**NRC-CNR** 

- $\mathbb{R}$  Low-Energy  $\mathbb{R}$ **Nuclear Property Property** vici. Hupin Université Paris-Sud, France • G. Hupin
	- International **Collaboration**<br>Collaboration • H. Hergert, S. Bogner MSU, USA

![](_page_53_Picture_13.jpeg)

![](_page_53_Picture_14.jpeg)

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