The Super-Radiant Mechanism in Nuclear Physics

NSCL

- N.Auerbach
- TAU and MSU

Coherence in Spontaneous Radiation Process

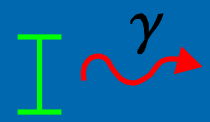
- R.H.Dicke, Phys.Rev. **93**, 99 (1954)
- *“In the usual treatment of spontaneous radiation by a gas, the radiation process is calculated as though the separate molecules radiate independently of each other…..*
- *It is clear that this model is incapable of describing a coherent spontaneous radiation process…This simplified picture overlooks the fact that all the molecules are interacting with a common radiation field and hence cannot be treated as independent.”...*
- *”A gas that radiates strongly because of coherence will be called “super-radiant””.*

Superradiance, collectivization by decay

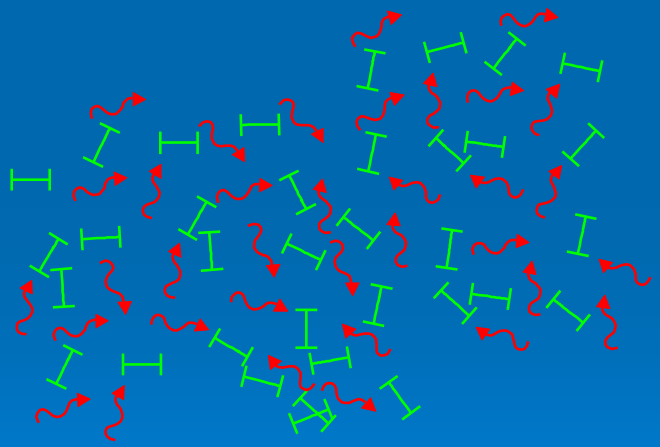
Dicke coherent state

N identical two-level atoms coupled via common radiation

Single atom γ



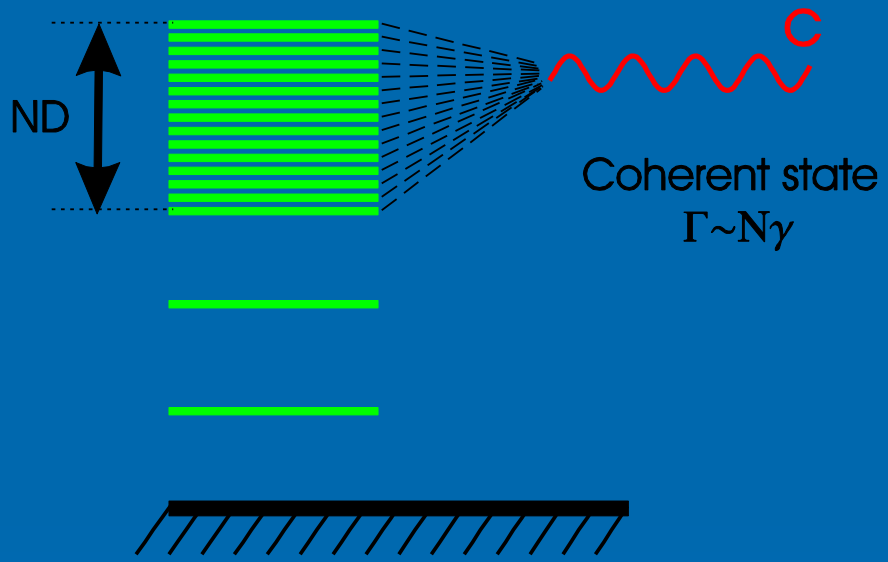
Coherent state $\Gamma \sim N\gamma$



Analog in nuclei

Interaction via continuum

(Trapped states) self-organization



"Super-radiant" state

- The special unstable state is often referred to as the *"super-radiant" (SR),* in analogy to the Dicke coherent state of a set of two-level atoms coupled through a common radiation field. Here, the coherence is generated by the common decay channel. The stable states are *trapped* and decoupled from the continuum.
- *R.H.Dicke, Phys.Rev. 93, 99 (1954)*

The Effective Hamiltonian

The total wave function of the system,

$$|\Psi\rangle = Q|\Psi\rangle + P|\Psi\rangle$$

satisfies the Schrodinger equation

$$H|\Psi\rangle = E|\Psi\rangle$$

that can be decomposed into a set of coupled equations:

$$(E - H_{QQ})Q|\Psi\rangle = H_{QP}P|\Psi\rangle$$

and

$$(E - H_{PP})P|\Psi\rangle = H_{PQ}Q|\Psi\rangle$$

where the notation is $H_{AB} = AHB$

Effective Hamiltonian (cont'd)

Eliminating $P|\Psi\rangle$ we obtain:

$$(E - H_{QQ}^{ef})Q|\Psi\rangle = 0$$

with the effective Hamiltonian:

$$\mathrm{H}_{\mathrm{QQ}}^{ef} = H_{QQ} + H_{QP}\frac{1}{E^{(+)} - H_{PP}}H_{PQ}$$

Here $E^{(+)} = E + i0$

The second term of the effective Hamiltonian contains a real and imaginary part of the propagator

$$\mathrm{G}^{(+)}(E) = \frac{1}{E^{(+)} - H_{PP}}$$

The Effective Hamiltonian (cont'd)

These originate from the principal value and delta function $\delta\left(E - H_{PP}\right)$.

The imaginary part, $-\left(i/2\right)W$ is given by:

$$W = 2\pi \sum_{c} H_{QP} \left|c\right\rangle \left\langle c\right| H_{PQ}$$

where c are the open channels.

The effective Hamiltonian in Q-space is non-Hermitian

$$\mathrm{H}^{\mathrm{eff}} = \bar{H} - \frac{i}{2} W$$

where $\bar{H} \equiv \bar{H}_{QQ}$ is a symmetric real matrix that includes, apart from the original Hamiltonian of the Q-space , H_{QQ} ,

the principal value contribution of the QP-coupling.

The cross section for a reaction $a \to b$ is determined by the square of the scattering amplitude:

$$T^{ba}\left(E\right) = \sum_{q,q'} \left\langle a\right| H_{QP} \left|q\right\rangle \left(\frac{1}{E^{(+)} - H^{eff}} \right)_{qq'} \left\langle q'\right| H_{QP} \left|b\right\rangle \ ,$$

$$S = 1 - iT$$

The eigenvalues of H^{eff} , $\varepsilon = \mathrm{E}\text{-}\left(i/2\right)\Gamma$, are complex poles of the scattering matrix, corresponding to the resonances in the cross section.

Single Channel

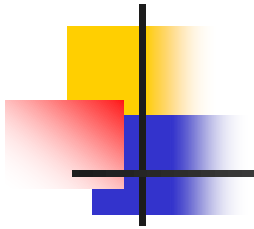
- To demonstrate in a simple way the effect of the anti-Hermitian term we look at the case of a single channel. Then the matrix W has a completely separable form:

$$\langle q|W|q'\rangle = 2\pi A_q^c A_{q'}^{c*} \text{ , where}$$

$$\mathrm{A}_{\mathrm{q}}^{\mathrm{c}} = \langle q|H_{QP}|c\rangle$$

Separable interaction

$$H = \varepsilon I - \frac{i}{2}\begin{bmatrix} A_1^2 & A_1A_2 & A_1A_3 \ldots\ldots \\ A_2\,A_1 & A_2^2 & A_2A_3 \ldots\ldots \\ A_3A_1 & A_3A_2 & A_3^2 \ldots\ldots \\ . & & \\ . & & \end{bmatrix}$$



- The W matrix is in general non-diagonal and the matrix elements are highly correlated.

“Super-radiant” state

- The rank of the factorized matrix is 1, so that all eigenvalues of W are zero, except one that has the value equal to the trace of this matrix:

$$\Gamma_0 = \sum_q \langle q|W|q\rangle = 2\pi \sum_q \left|A_q^c\right|^2 \equiv \sum_q \Gamma_q^{\uparrow}$$

$$(\Gamma_q^{\uparrow} = 2\pi \left|A_q^c\right|^2)$$

General case

- The phenomenon of super-radiance survives in a general situation of N intrinsic states and N_c open channels provided $N_c << N$, if the mean level spacing D of internal states and their characteristic decay widths satisfy the conditions

$$\kappa^c = \frac{\Gamma_q^{\uparrow}(c)}{D} > 1$$

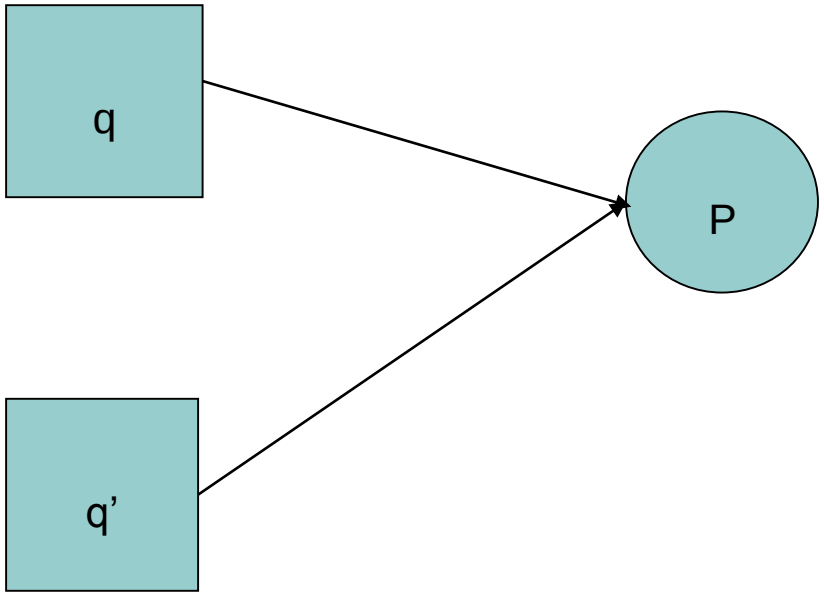
General case

- In this regime of overlapping resonances their interaction through the common continuum channels leads to restructuring of the complex energy spectrum, similarly to the formation of Dicke's coherent state. Since the rank of the factorized W matrix is N_c , it has only N_c non-zero positive eigenvalues.
- The intrinsic space Q is now divided into the SR subspace of dimension N_c and the subspace of trapped states It> of dimension $N-N_c$.

- For example for two channels the matrix elements of W will have the form:

$$W_{ij} = A_i A_j + B_i B_j$$

Effective interaction in the Q-space



Doorways

- Frequently only a subset of intrinsic states {Q} connects directly to the {P} space of channels. The rest of states in {Q} will connect to {P} states due to the admixtures of these selected states . The special states coupled directly to the continuum are the *doorways Id>.* They form the doorway subspace {D}. The corresponding projection operator will be denoted as *D.*
- The remaining states will be denoted as $\left|\tilde{q}\right\rangle$

Doorways

The full Hamiltonian can be decomposed in the following way :

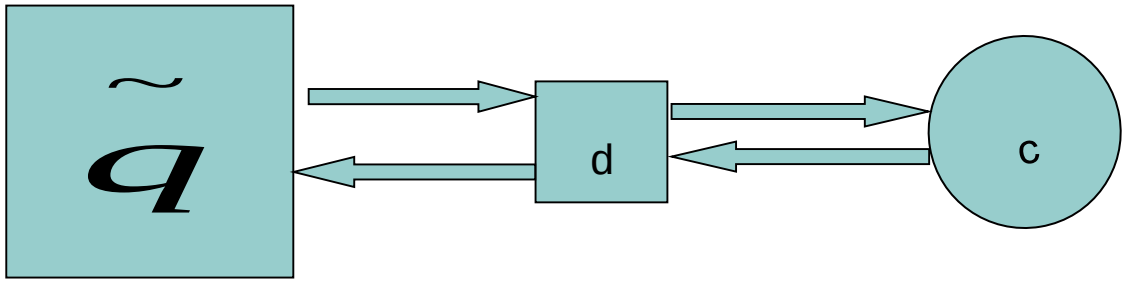
$$\mathrm{H} = \left(H_{\tilde{Q}\tilde{Q}} + H_{DD} + H_{\tilde{Q}D} + H_{D\tilde{Q}} \right) + \left(H_{PP} + H_{DP} + H_{PD} \right)$$

Diagonalizing the first part in this expression will give back the states $|\mathrm{q}\rangle$ with the components $|\mathrm{d}\rangle$ mixed with states $|\tilde{\mathrm{q}}\rangle$

(Note that there is no $H_{P\tilde{Q}}$ in the above expression)

Doorways (Corridors)

Doorways



Single doorway

Assume one important doorway $|d\rangle$. The matrix elements of the effective operator W in the Q-space are now given by:

$$\langle q|W|q'\rangle = 2\pi \sum_{c=1}^{N_c} \langle q|H_{DP}|c\rangle\langle c|H_{PD}|q'\rangle$$

Under the doorway assumption,

$\langle q|H_{DP}|c\rangle = \langle q|d\rangle\langle d|H_{DP}|c\rangle$, where $\langle q|d\rangle$ is the admixture of the doorway to the state $|q\rangle$

Then,

$$\langle q|W|q'\rangle = 2\pi\langle q|d\rangle\langle d|q'\rangle\sum_c |\langle d|H_{DP}|c\rangle|^2$$

This matrix element is again separable, irrespective of the number of channels, and again one finds a single broad state with a widths:

$$\Gamma_s = 2\pi\sum_q |\langle q|d\rangle|^2 \sum_c |\langle d|H_{DP}|c\rangle|^2$$

naturally this width is simply the decay width $\Gamma_d^{\uparrow}$ of the doorway.

When is this picture valid?

The criterion of validity is that the average spacing between levels in {Q} -space is smaller than the decay width of such a state "before" the SR is set at work. Consider the spreading width , $\Gamma_{\mathrm{d}}^{\downarrow}$, of the doorway state for the fragmentation into compound states $|\tilde{\mathrm{q}}\rangle$. If $\mathrm{N_q}$ is the number of compound states in the interval covered by the spreading width, their average energy spacing is :

$$\bar{D} \approx \frac{\Gamma_d^{\downarrow}}{N_q}$$

Before the SR mechanism is turned on, the average decay width of a $|\mathrm{q}\rangle$ is

$\Gamma_{\mathrm{q}}^{\uparrow} = 2\pi \sum_c \left|\langle q | H_{QP} | c \rangle\right|^2$ that can be estimated:

$$\Gamma_q^{\uparrow} = \frac{\Gamma_s}{N_q} \quad \text{, therefore :} \quad \frac{\Gamma_q^{\uparrow}}{D_q} \approx \frac{\Gamma_s}{\Gamma_d^{\downarrow}} \approx \frac{\Gamma_d^{\uparrow}}{\Gamma_d^{\downarrow}} \quad \text{thus,} \quad \frac{\Gamma_{\mathrm{d}}^{\uparrow}}{\Gamma_d^{\downarrow}} > 1$$

Examples

- *Isobaric analog state (IAS).*

The IAS, $|\mathrm{A}\rangle$ is the result of action of the isospin lowering operator T_- on the parent state $|\pi\rangle$,

$|\mathrm{A}\rangle = const \cdot T_- |\pi\rangle$

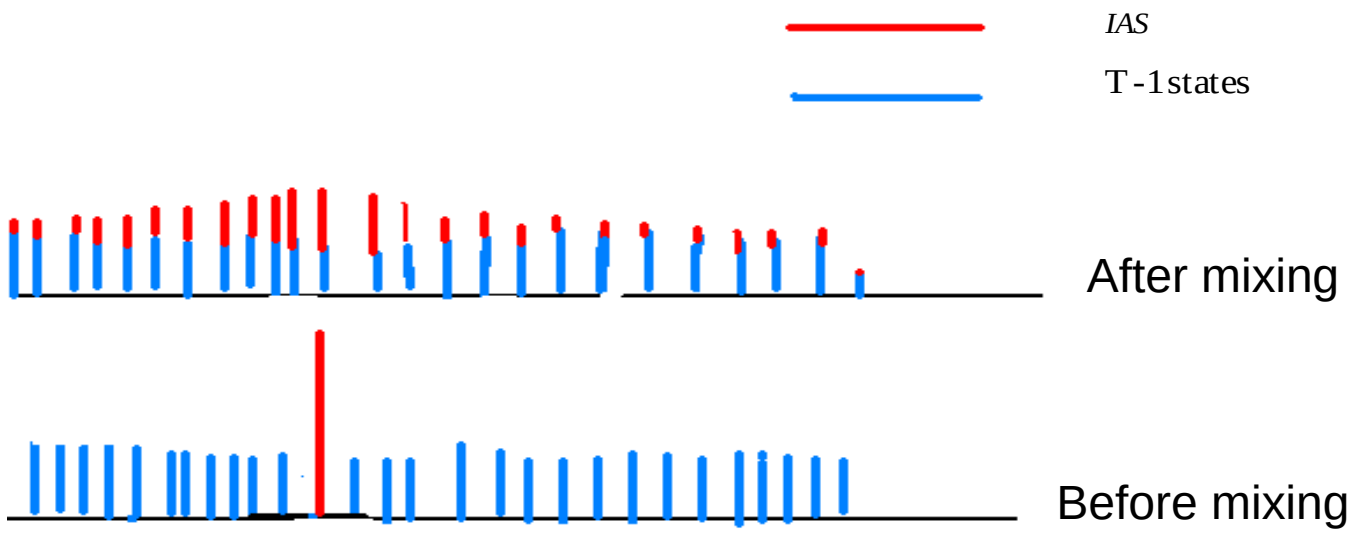
In the compound nucleus, the IAS is surrounded by many compound states $|q\rangle$ of lower isospin $\mathrm{T}_< = T - 1$. The Coulomb interaction does not conserve isospin fragmenting the strength of the IAS over many states $|q\rangle$ that results in the spreading width $\Gamma_A^{\downarrow}$ of the IAS.

If located above the threshold, the IAS can also decay into several continuum channels that gives rise to the decay width $\Gamma_A^{\uparrow}$. In heavy nuclei $\Gamma_A^{\uparrow} > \Gamma_A^{\downarrow}$.

The SR mechanism is relevant to this case, providing an explanation why the IAS appears as a single resonance with a decay width given by that of $|A\rangle$,

$\Gamma_{IAS}^{\uparrow} = 2\pi \left| \langle A | H_{QP} | P \rangle \right|^2$

Mixing of the IAS with T-1 states



Example ^{208}Pb

In ^{208}Bi the total width of the IAR is about $\Gamma_A = 250keV$, of which $\Gamma_A^{\uparrow} = 170keV$ and $\Gamma_A^{\downarrow} = 80keV$.

Certainly $\frac{\Gamma_A^{\uparrow}}{\Gamma_A^{\downarrow}} > 1$

Example; *Giant Resonances (GR)*

- One describes the giant resonances in nuclei in terms of 1p-1h configurations. The residual interaction forms collective states out of these configurations. However, usually the GRs are located in the particle continuum. The 1p-1h are surrounded by a vast spectrum of 2p-2h excitations which will mix with the GR.

Denote these states as $|b\rangle$, and the giant resonance by $|G\rangle$. The 2p - 2h states will decay into the continuum via the admixture $\langle G|b\rangle$ (of the GR into the 2p - 2h states).

The GR serves as a doorway. The W matrix elements are given by:

$$\langle b|W|b'\rangle = \langle b|G\rangle\langle G|b'\rangle\sum_c \langle G|V|c\rangle\langle c|V|G\rangle$$

Again the W matrix is of rank one and the SR state will have the decay width:

$$\Gamma_G^{\uparrow} = 2\pi\sum_c \left|\langle G|V|c\rangle\right|^2 \text{, under the condition that } \frac{\Gamma_G^{\uparrow}}{\Gamma_G^{\downarrow}} > 1$$

The super-radiant mechanism applied to intermediate energy nuclear physics; examples.

The SR mechanism is applied to several intermediate (and high) energy phenomena.

N-Nbar excitations

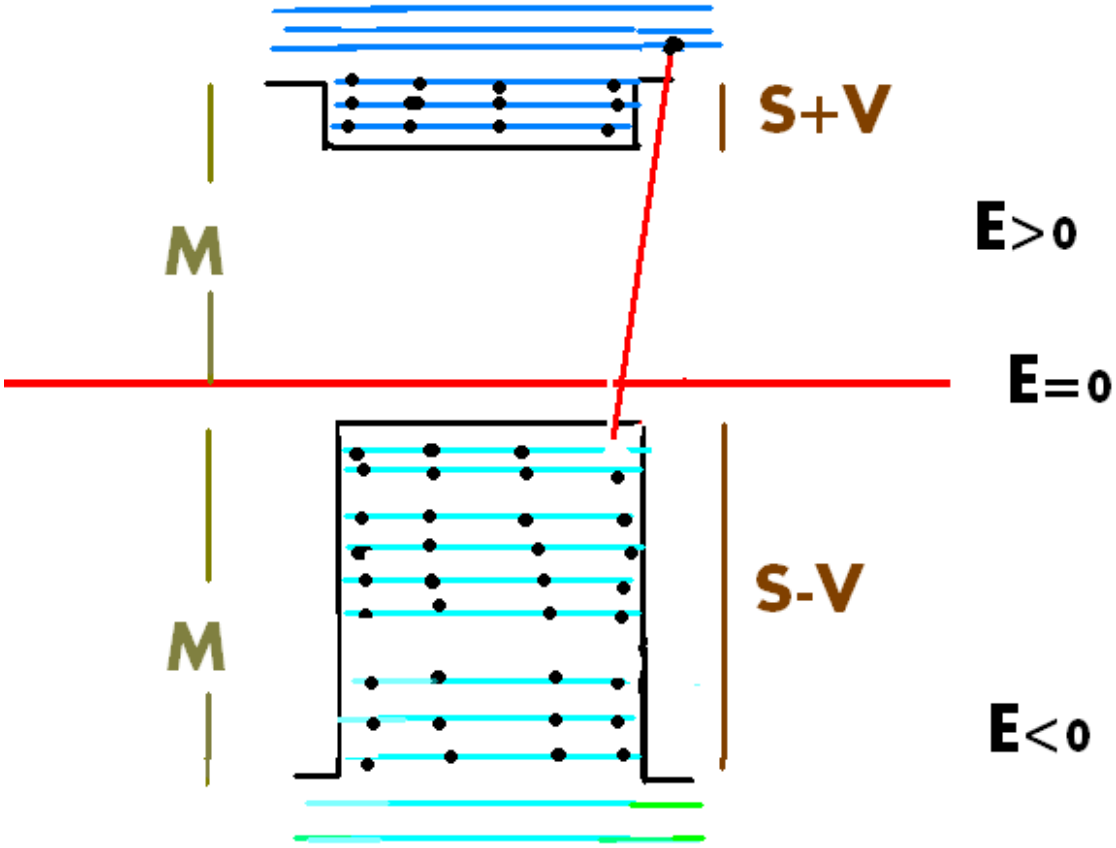


TABLE I. The real and imaginary parts of the eigenvalues for $N\overline{N}$ excitations in the ^{16}O nucleus (see text).

$N\overline{N}$ state	ϵ_i (MeV)	$\Gamma_i/2$ (MeV)
1	1115	0.03
2	1119	0.02
3	1121	0.19
4	1124	0.22
5	1125	78.53
6	1129	0.36
7	1133	0.88
8	1141	0.23
9	1146	1.46

M. L'Huillier, N. V. Giai and N. A.
Nucl. Phys. A519, 83 (1990).

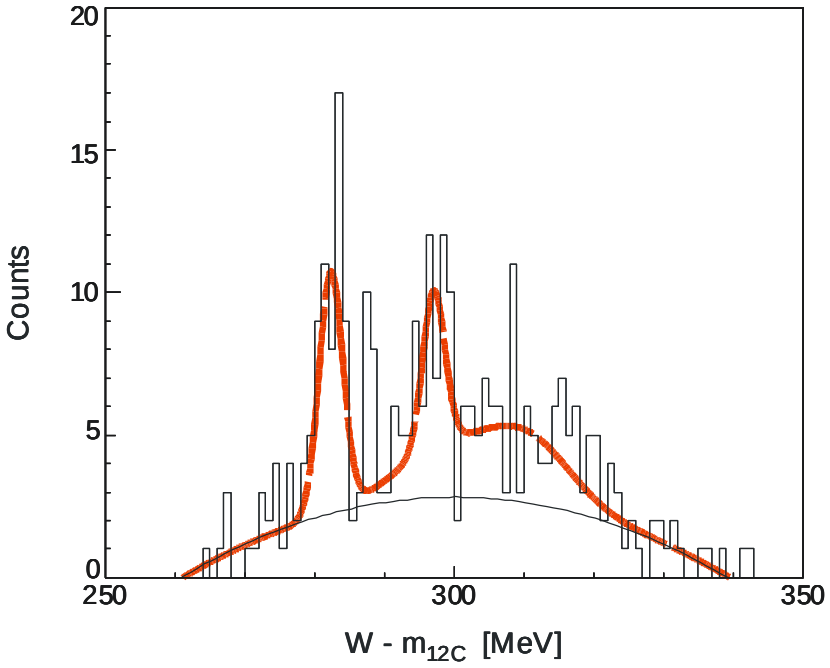
N. Auerbach, "Super-radiant States in Intermediate Energy Nuclear Physics" Phys. Rev. C50, 1606 (1994).

We could envisage such a mechanism taking placing in deeply bound hadronic atoms (π-mesic atoms, antiproton atoms) where due to the coupling to the absorption channel narrow resonances will appear in addition to a wide super-radiant state. The system of dibaryons is possibly another example. The coupling of several dibaryon channels might result in some quasistationary dibaryon states due to the effects of the mechanism we discussed here. The list of possibilities can be made longer. We should emphasize, however, that the physical scenario described here is often not fulfilled. The residual nuclear interaction has of course a real and an imaginary part. The real part of the interaction will cause the redistribution of the unperturbed excitation strength and some of the narrow resonances that emerge will have small excitation strength and will not be observable. Thus the situation described in this work is special and only in selected physical circumstances will it arise. Only future studies in this direction may hold the answer to whether and where in fact such a mechanism is taking place in nature.

Multi-quark states

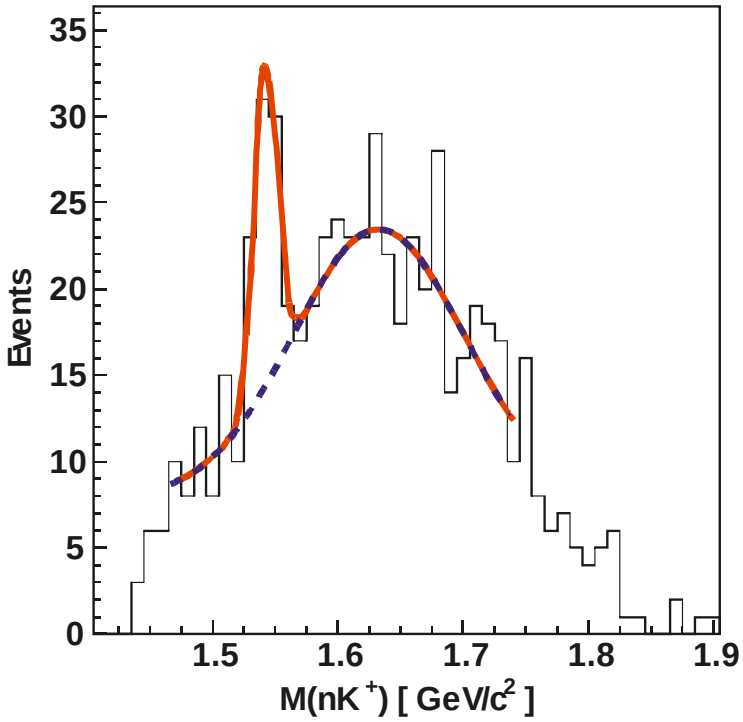
- The SR mechanism is universal and can take place in very distinctively different systems.
  It is possible that in the sector of quark physics there are situations in which preconditions exist for the appearance SR states followed by very narrow trapped resonances. Some examples are taken from the multi-quark systems. In some cases it is claimed that uncharacteristically narrow resonances are observed. For example the 1545 MeV pentaquark, the X(3872) tetraquark and other tetraquarks.

Superradiance in resonant spectra



Narrow resonances and broad superradiant state in ^{12}C 👎

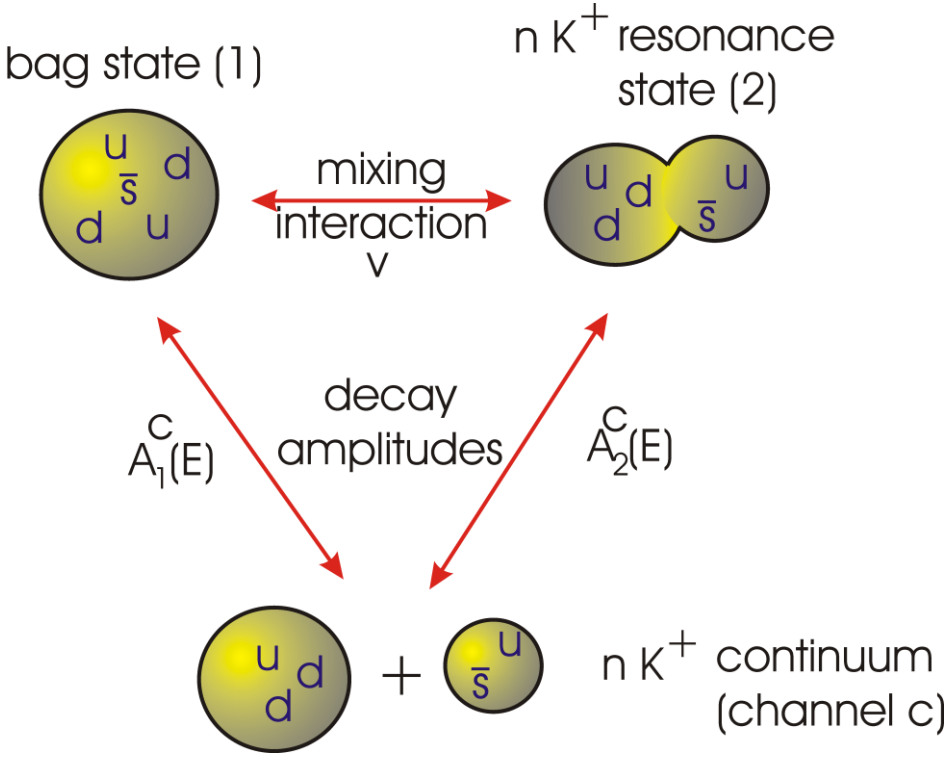
Bartsch *et.al.* Eur. Phys. J. A **4**, 209 (1999)



Pentaquark as a possible candidate for superradiance

Stepanyan *et.al.* hep-ex/0307018

Θ^+ pentaquark as a two-state interference

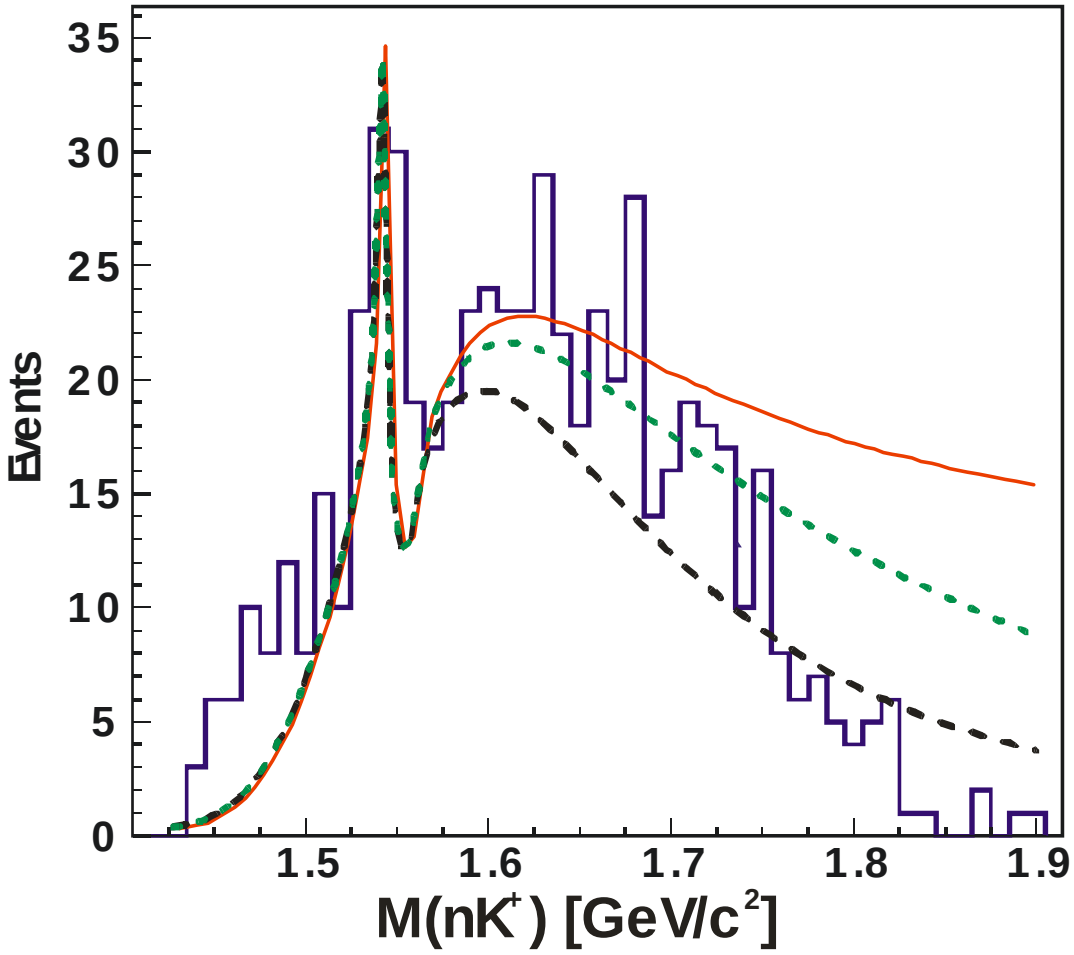


$$\gamma_i = \mathrm{A}_i^2 \,, i = 1,2$$

Effective Hamiltonian

$$\mathcal{H}(E) = \begin{pmatrix} \epsilon_1 - \frac{i}{2}\gamma_1 & v - \frac{i}{2}A_1 A_2 \\ v - \frac{i}{2}A_1 A_2 & \epsilon_2 - \frac{i}{2}\gamma_2 \end{pmatrix}$$

Kn scattering crossection



Sensible parameters
under requirement
•Resonant energy E_r=1540 MeV
•Kn threshold energy
•Width of broad peak

□1535 MeV
□1(Er) =120 MeV
□2 =1560 MeV
□2(Er) =60 MeV
v=1 MeV
□□□□□ (green), 500 (red) MeV

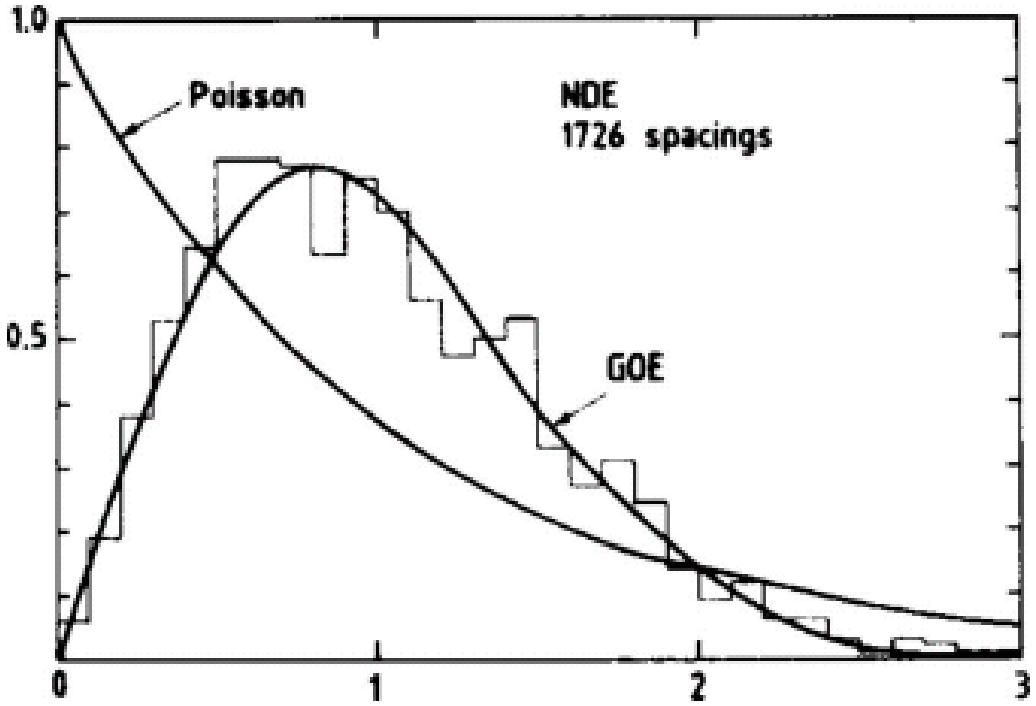
Other examples

One could envisage other situations in the field of intermediate energy when the SR mechanism might produce narrow states in addition to a very broad state. Narrow resonances in deeply bound hadronic atoms (pionic, anti-nucleonic), in deeply bound anti-kaons, in sigma hypernuclei, etc

The PTD

- One of the best and historically advanced examples of the manifestation of the interplay between intrinsic dynamics and decay channels is given by low-energy neutron resonances in complex nuclei. The series of these well-pronounced separated resonances were studied long ago and later gave rise to the ‘‘Nuclear Data Ensemble’’. Interpreting these resonances as quasi-stationary levels of the compound nucleus formed after the neutron capture, agreement was found with predictions of the Gaussian orthogonal ensemble (GOE) of random matrices.
- With exceedingly complicated wave functions of compound states, the statistical distribution of their components is close to Gaussian. The neutron decay implements the analysis of a specific component related to the channel ‘‘neutron in continuum plus a target nucleus in its ground state.’’ The neutron width is proportional to the squared amplitude of this component, and the width distribution appropriate for one channel [Porter-Thomas distribution (PTD)].

Random Matrix Theory (RMT)



Porter-Thomas Distribution (PTD)

- In the case of chaos the distribution of widths is given by the χ^2_ν with $\nu = 1$

$$P(\Gamma) = (2\pi\Gamma/\langle\Gamma\rangle)^{-1/2} e^{-\frac{\Gamma}{2\langle\Gamma\rangle}}$$

Recent Experiment

- Recent experiments with improved accuracy give
- evidence of significant deviations from the PTD so that the attempts to still use the χ^2 distribution for the fit invariably
- require ν<1.

The new result was interpreted as a consequence of an unknown non-statistical mechanism or just a breakdown of nuclear theory as was claimed in the related article.

Our Work. (G. L. Celardo, N. Auerbach,F. M. Izrailev,and V. G. Zelevinsky PRL 106, 042501 (2011))

- The goal of our work was to point out that a correct
- description of unstable quantum states in a complicated
- many-body system naturally leads to deviations from the
- PTD, of the same type as observed . The
- random matrix theory was formulated for local statistics in
- a closed quantum system with a discrete spectrum governed
- by a very complicated Hermitian Hamiltonian.
- However, the presence of open decay channels and
- therefore the finite lifetime of intrinsic states unavoidably
- lead to new phenomena outside of the GOE framework.
- We have seen that the coupling to the open channels is described by a very special W-matrix that has correlated matrix elements even when the Hermitian part has no correlations.

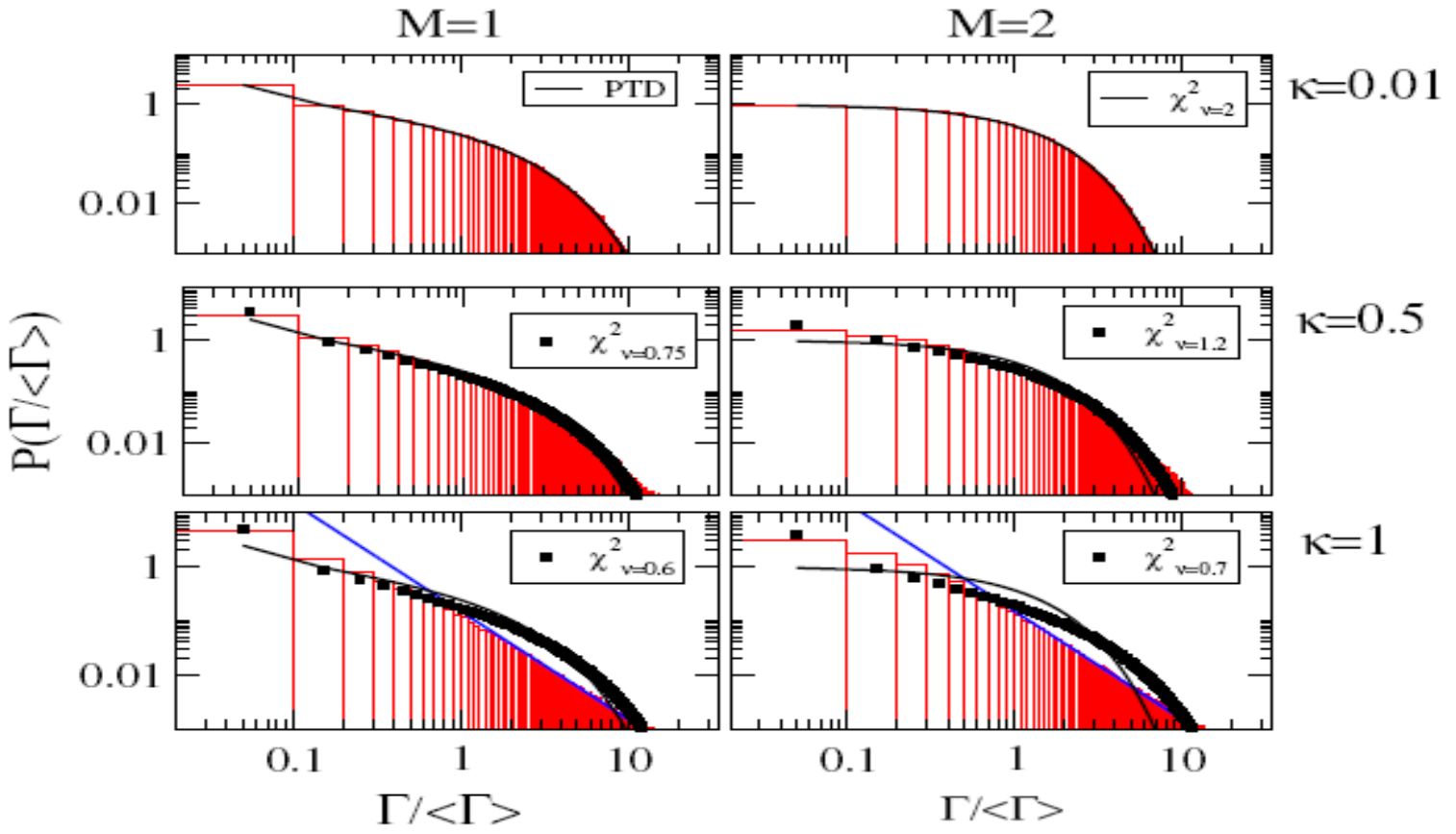
Compound Nucleus. Random Matrix Theory (RMT).

- The matrix H has dimension N that in the nuclear case should include a large number of shell-model many-body states important for the dynamics in the energy range under consideration; in the region of neutron resonances, N ~ $10^{5-6} >> M$. With the trace of W equal to η , the parameter defining the dynamics is the ratio of typical "bare" widths η/N of individual states to the energy spacing D. At small values of this parameter, the resonance widths obey the PTD. With widths increasing, the system moves to the regime of overlapping resonances.

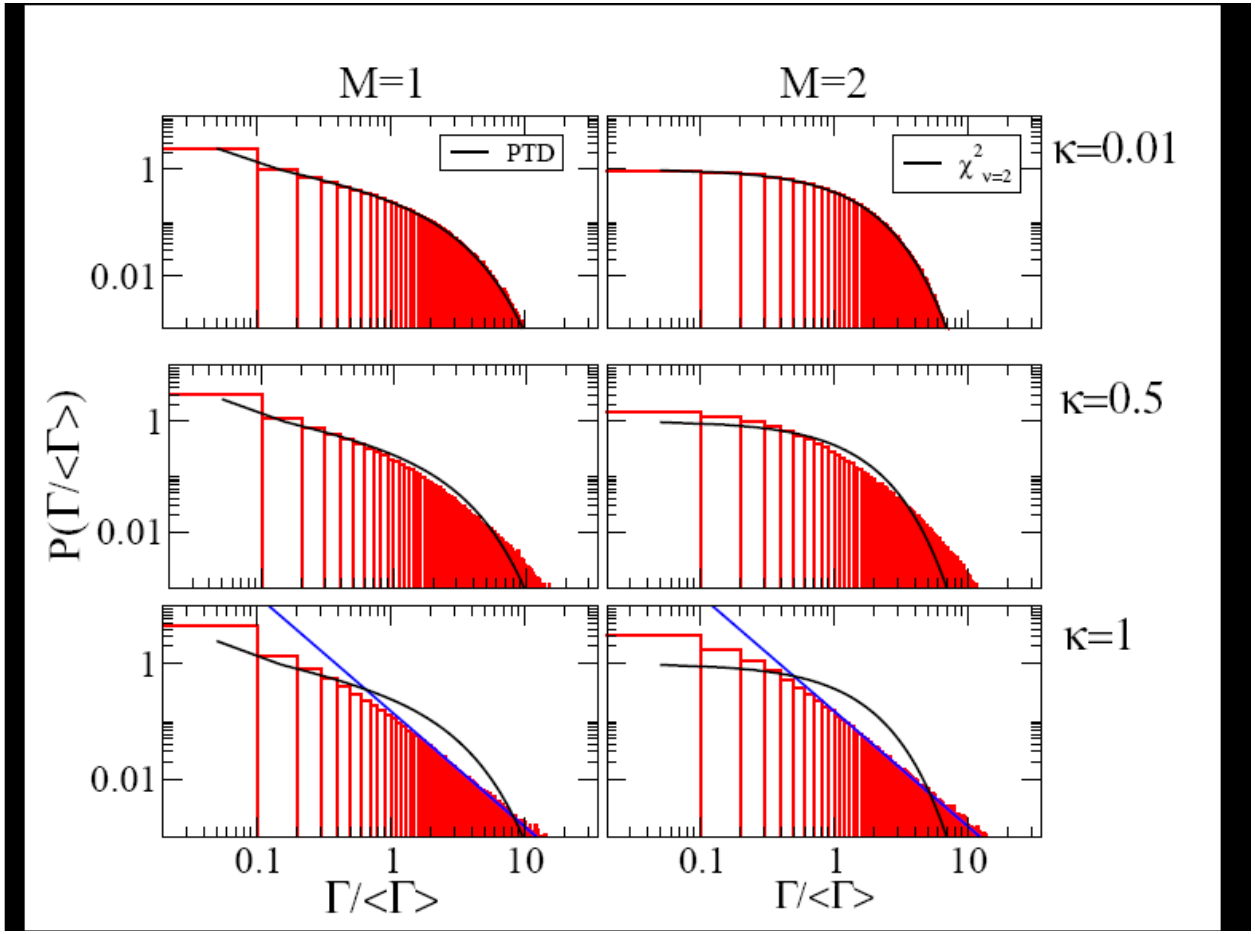
Coupling to the continuum

- The individual components of a typical intrinsic state are Gaussian distributed uncorrelated quantities, and the neutron widths, being proportional to the squares of those components, display the PTD. However, the correct description of the dynamics with the continuum coupling shows the limited character of this prediction. When the coupling is
- weak, $\kappa = \eta / ND << 1$, we indeed expect to see well isolated resonances with the PTD of the widths. With growing continuum coupling (increase of energy from the threshold), the deviations become more and more pronounced. At $\kappa =$ 1, a kind of a phase transition occurs with the sharp redistribution of widths and the segregation of a superradiant state accumulating the lion's share of the whole summed width, an analog of a giant resonance along the imaginary energy axis.

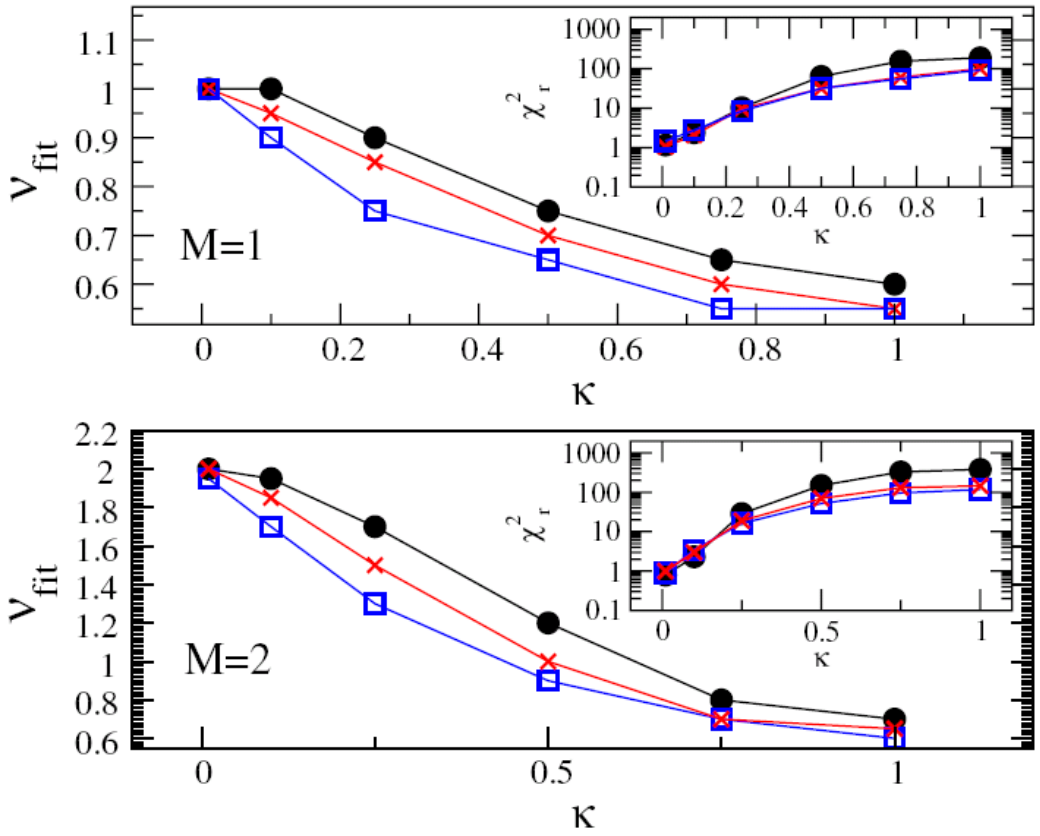
Results


M=1
M=2
PTD
$\chi^2_{\nu=2}$
$\chi^2_{\nu=0.75}$
$\chi^2_{\nu=1.2}$
$\chi^2_{\nu=0.6}$
$\chi^2_{\nu=0.7}$
κ=0.01
κ=0.5
κ=1
P(Γ/<Γ>)
1
0.01
0.1
1
10
Γ/<Γ>

Results



Results (for GOE and TBRE)



Bottom Line

- We analyze the statistics of resonance widths in a many-body Fermi system with open decay channels.
- Depending on the strength of continuum coupling, such a system reveals growing deviations from the standard chi-square (Porter-Thomas) width distribution. The deviations emerge from the process of increasing interaction of intrinsic states through common decay channels; in the limit of perfect coupling this process leads to the superradiance phase transition.The results presented here are important for the understanding of recent experimental data concerning the width distribution for neutron resonances in nuclei.