Critical dynamics: complexity, universality and "hydro++"

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Outline

- Cumulants evolution: universality lost.

S. Mukherjee, R. Venugopalan and YY, 1506.00645, PRC; 1512.08022, QM proceedings.

- Kibble-Zurek dynamics: universality regained!

S. Mukherjee, R. Venugopalan and YY, 1605.09341.

- *New development*: Hydro + +.

M. Stephanov and YY, in preparation.

Cumulants evolution

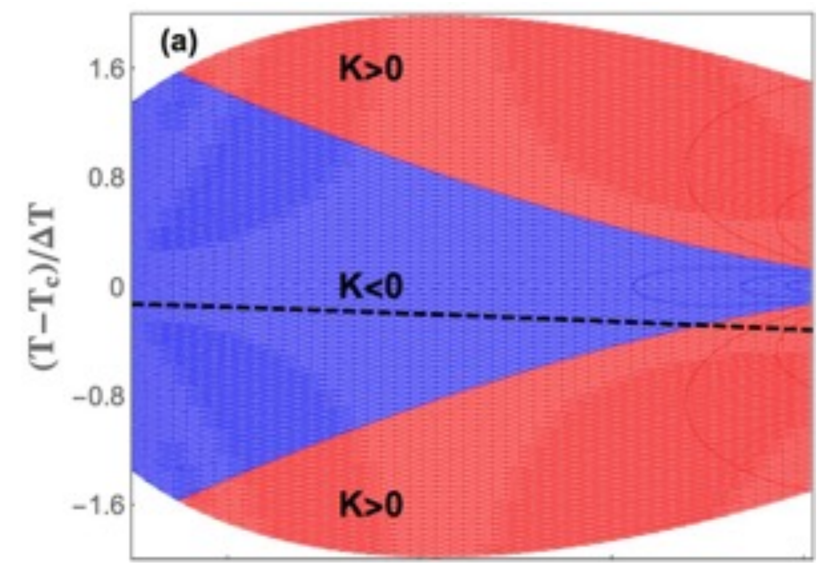
Observables: $\langle (\delta N)^n \rangle \sim \kappa_n \sim \langle (\delta\sigma)^n \rangle$

Universality: $\kappa_n^{\rm eq} \sim \xi_{\rm eq}^{\#} \times f_n^{\rm eq}(\theta)$

- Enhanced non-Gaussian fluctuations.

$$\kappa_4^{\rm eq} \sim \xi_{\rm eq}^7$$

- Universal pattern in sign.



(equilibrium kurtosis)

Critical dynamics matters

- Critical cumulants are unavoidably out of equilibrium :

$$\tau_\sigma \sim \xi_{eq}^z, \qquad z \approx 3$$

- Non-equilibrium evolution of correlation length (Gaussian cumulants): Berdinkov-Rajagopal, 1999.

- Open question (as of 2014): evolution of non-Gaussian cumulants along trajectories passing the QCD critical regime .

Evolution equations for cumulants

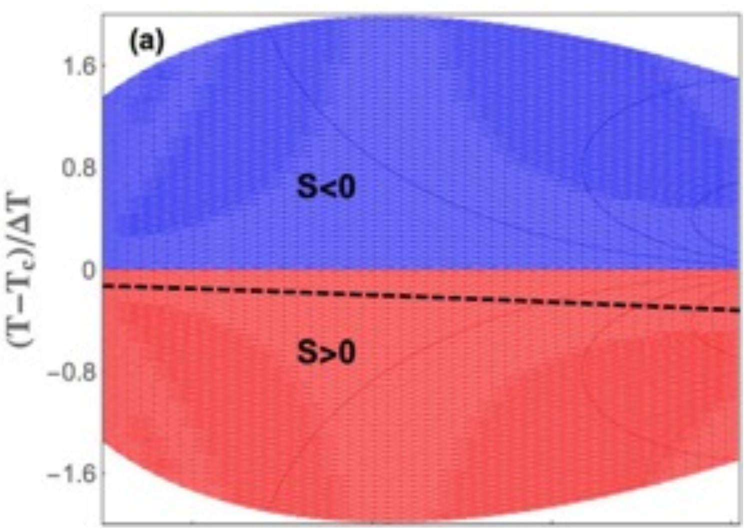
- Critical mode receives "random kicks" from *thermalized* hard modes => Generalized Langevin equation.

- Averaging out the noise => A set of novel evolution equations for cumulants (S. Mukherjee, R. Venugopalan and YY, 1506.00645, PRC).

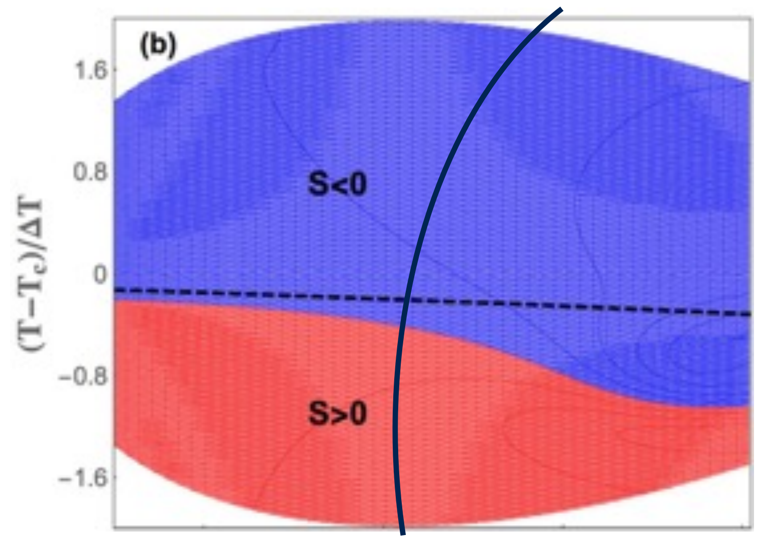
$$\frac{d\kappa_n(\tau)}{d\tau} = \frac{1}{\tau_\sigma(\tau)} L_n\left[\kappa_1, \kappa_2, \ldots, \kappa_n; \mu_B(\tau), T(\tau), \text{mapping}\right]$$

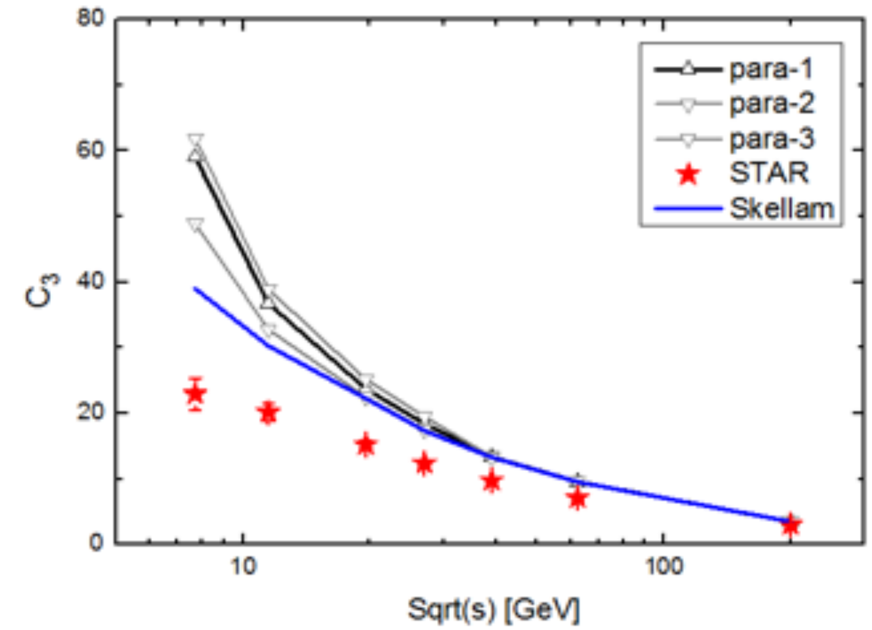
$$\tau_\sigma \sim \xi_{\rm eq}^z, \qquad z \approx 3$$

Equilibrium

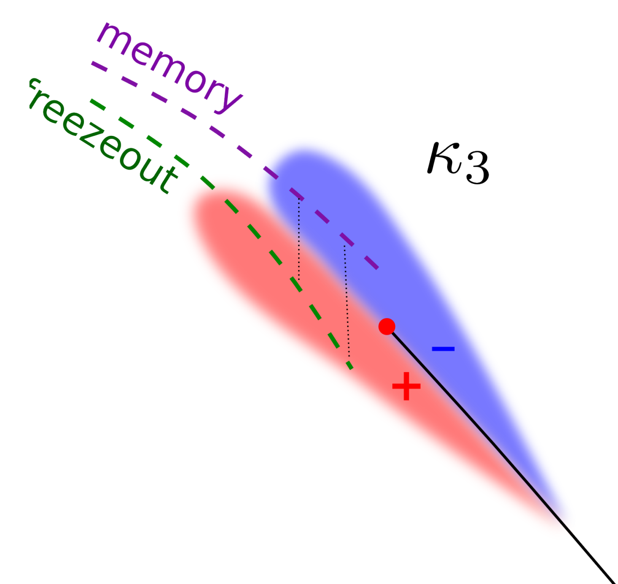


non-equilibrium



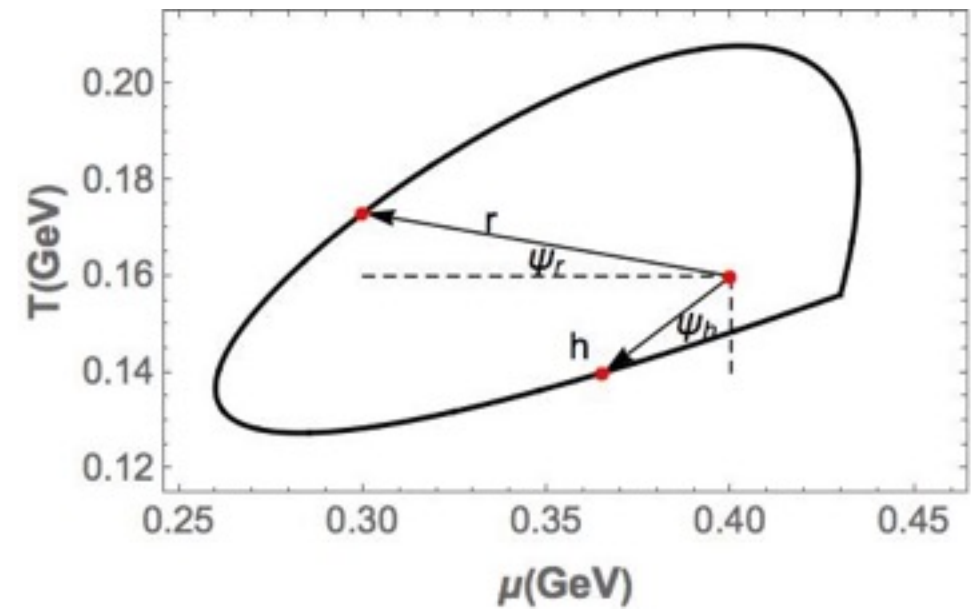


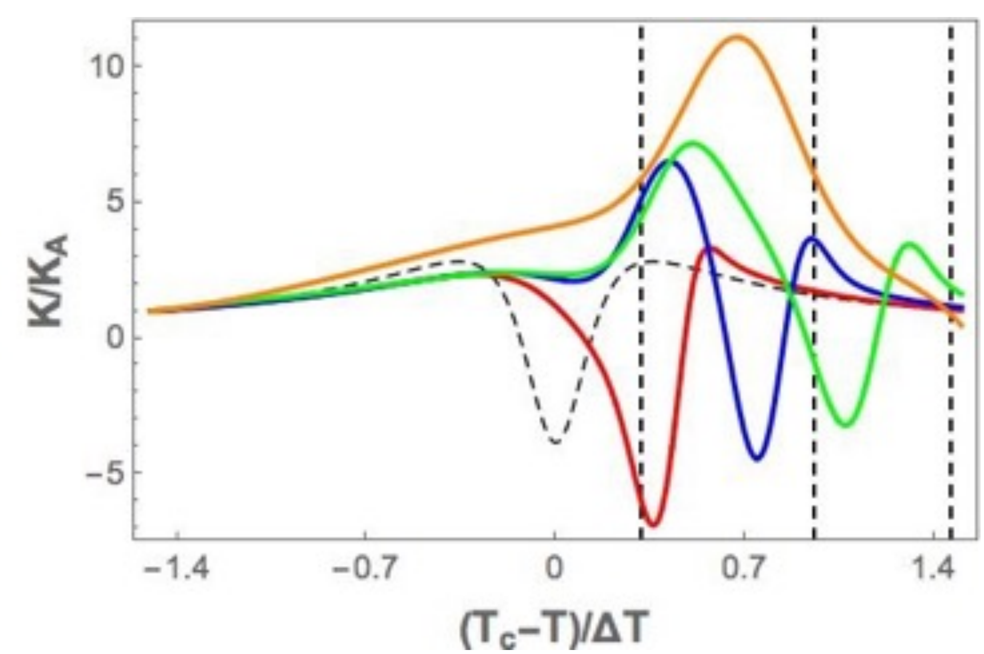
- "Sign puzzle" of skewness: "remembrance of things past".



Complexity

- Evolution depends on many non-universal inputs:
  - mapping, location of critical point, width of critical regime.
  - relaxation time.
  - Trajectories in phase diagram.
- The non-equilibrium cumulants look complicated.





- Is universality lost in complexity?
  - Which non-universal inputs (collectively denoted by Γ) dominate the dynamics?
  - What has been "memorized"?

Answers to those questions are connected by:

Kibble-Zurek dynamics

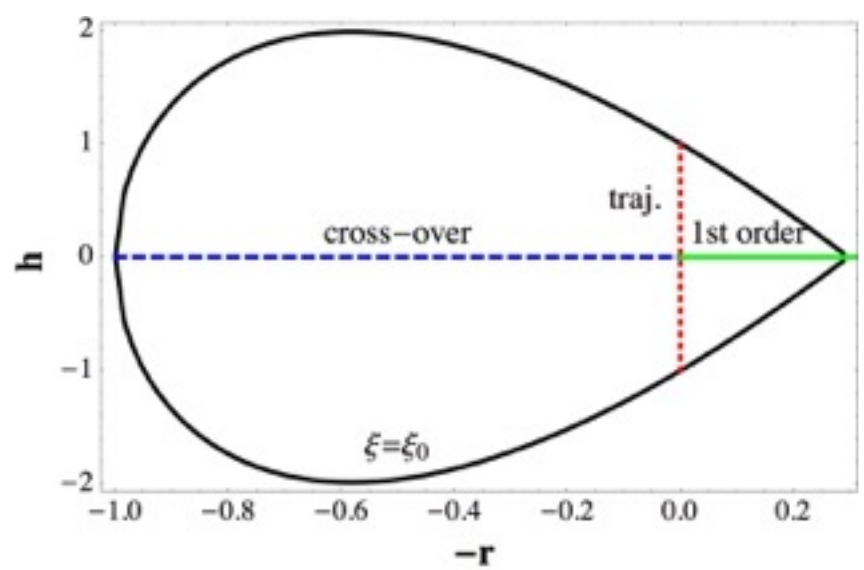
Kibble-Zurek dynamics

The formation and evolution of topological defects in cosmological phase transitions. (*T.W. Kibble, Physics Reports 67, 183 (1980))*

Generalized to vortex generation in superfluids. (*W. H. Zurek, "Cosmological experiments in superfluid helium?", Nature 317, 505 (1985))*

Kibble-Zurek dynamics in a little bang?

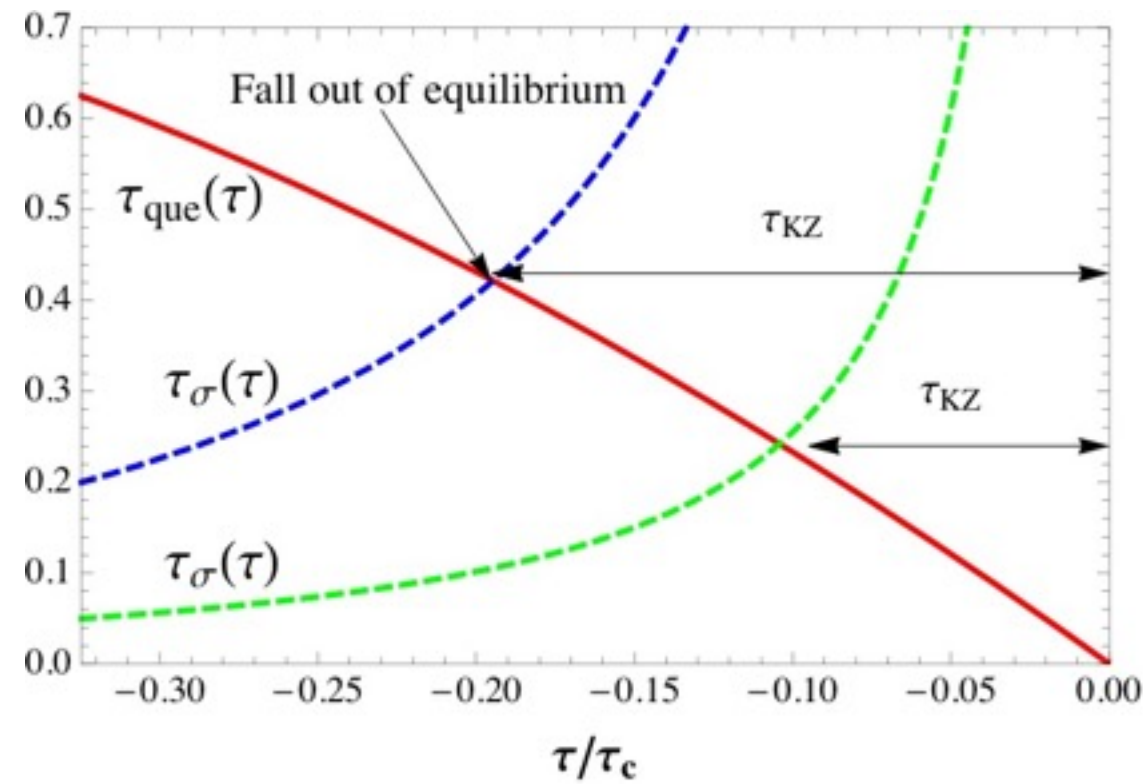
Consider a trajectory passing the critical point.



(μ_B, T) ⟷ 3d Ising model (r,h)

Comparing relaxation time and "quench" time.

$$\tau_\sigma(\tau) > \tau^{\xi}_{\rm que} \equiv |\frac{\xi_{\rm eq}}{\partial_\tau \xi_{\rm eq}}|$$

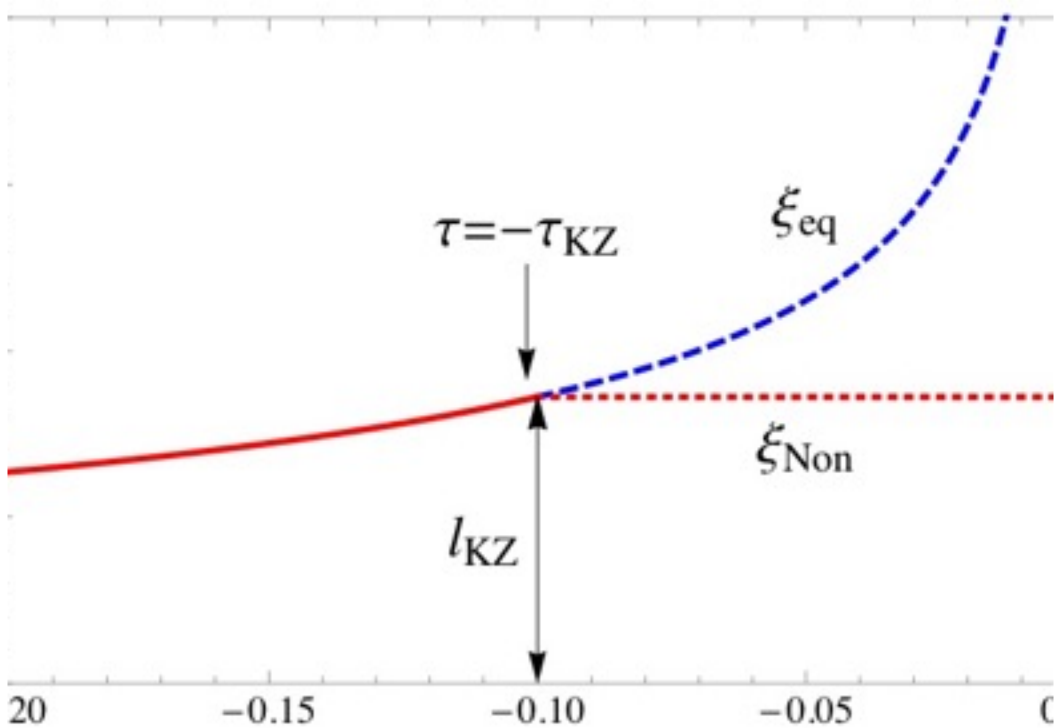


- An emergent time scale for non-equilibrium dynamics.

$$\tau_\sigma(\tau_{\rm KZ}) = \tau_{\rm que}(\tau_{\rm KZ})$$

The evolution is frozen .

$$l_{\rm KZ} = \xi_{\rm eq}(\tau_{\rm KZ})$$



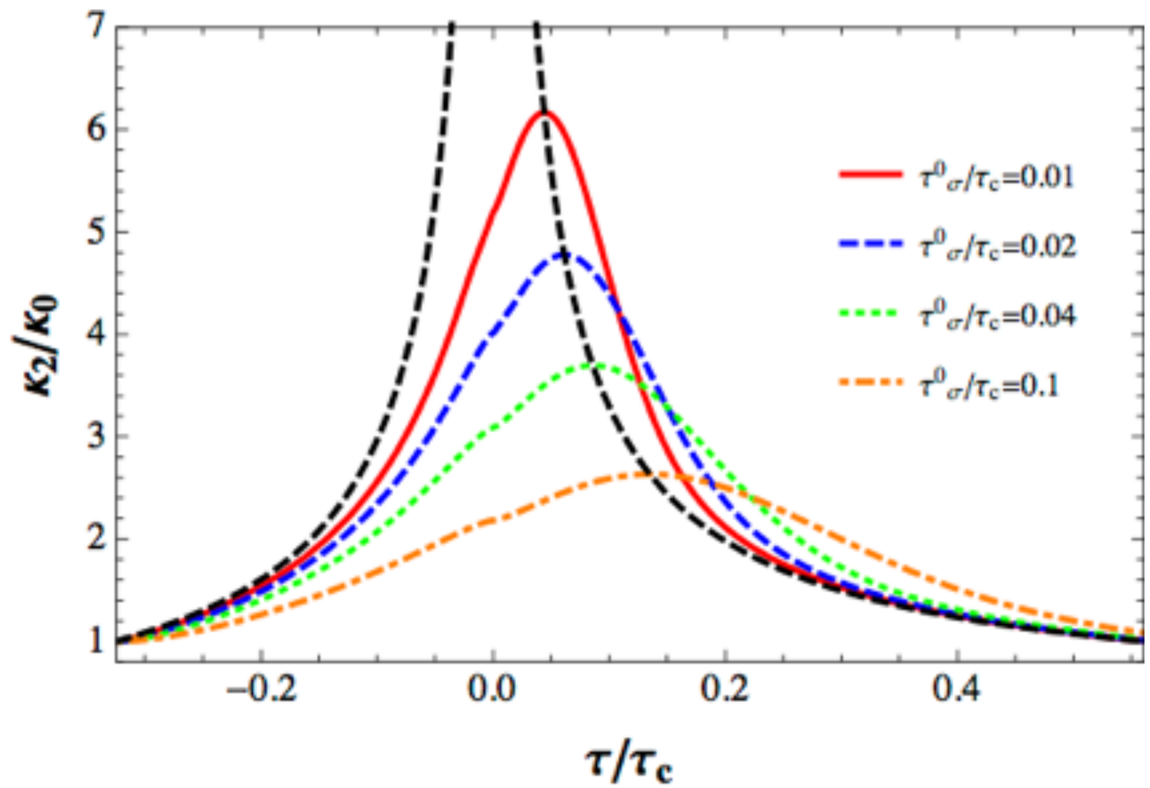
- Kibble-Zurek dynamics: non-equilibrium critical evolutions are characterized by $l_{\rm KZ}, \tau_{\rm KZ}$.

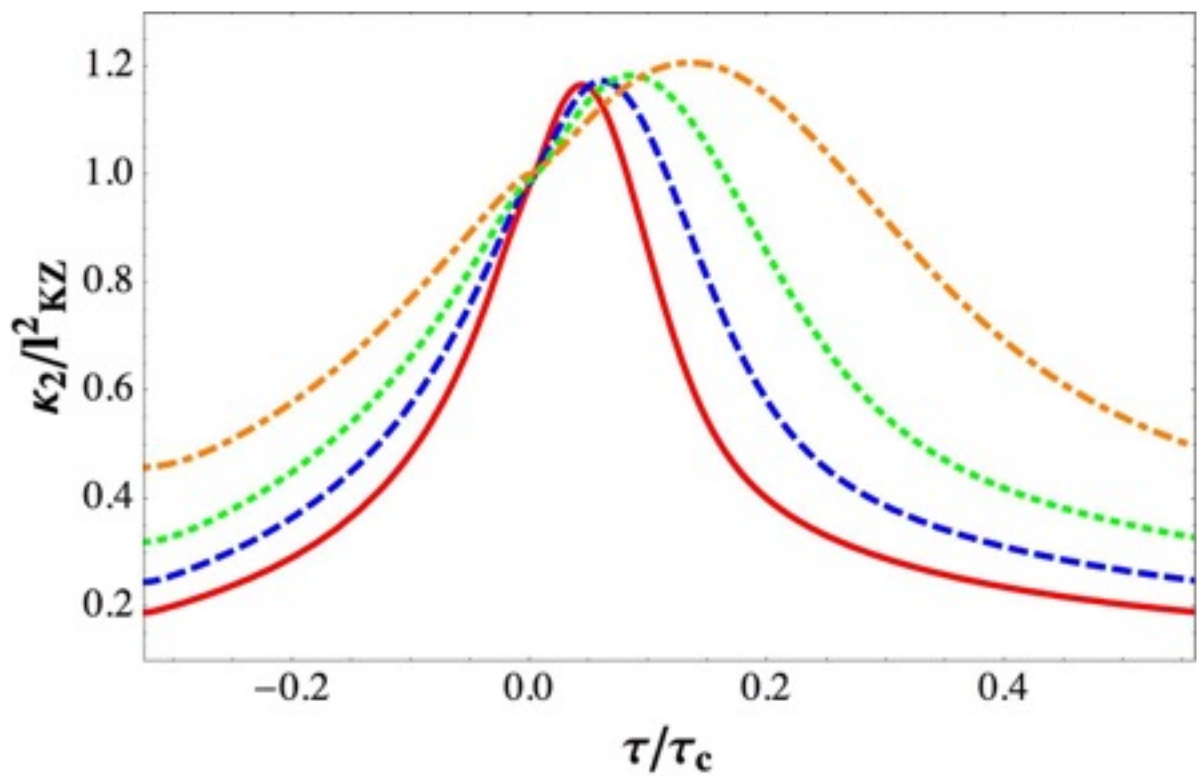
- Given bulk evolution and critical equation of state, $l_{\rm KZ}, \tau_{\rm KZ}$ can be determined from the definition:

$$\tau_\sigma(\tau_{\rm KZ}) = \tau_{\rm que}(\tau_{\rm KZ}) \qquad\qquad l_{\rm KZ} = \xi_{\rm eq}(\tau_{\rm KZ})$$

- An illustrative example: Berdnikov-Rajagopal model revisited.

Scaling with length is not enough

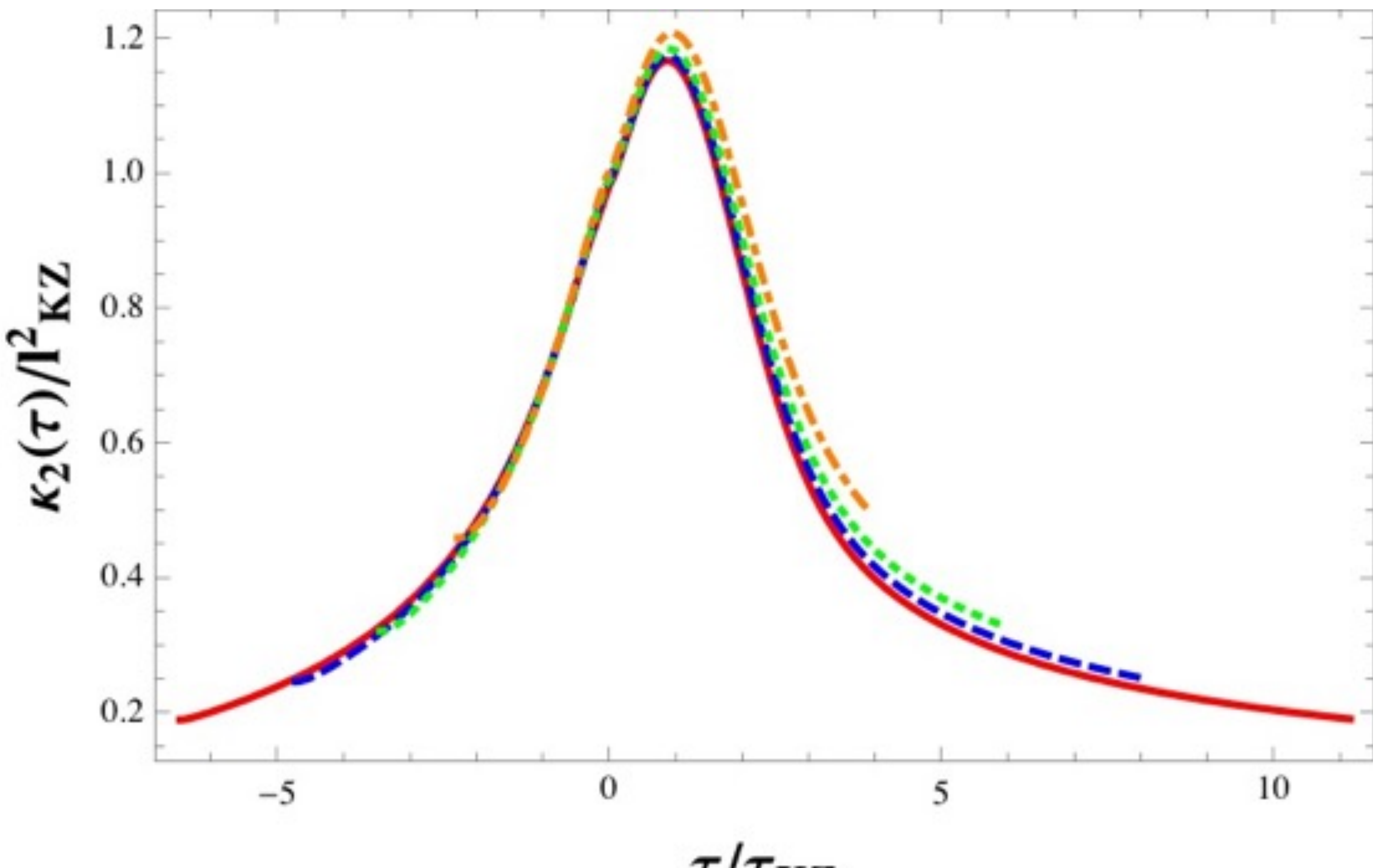




- Rescale Gaussian cumulants determined from Berdnikov-Rajagopal model by l^2_{KZ} .

- The peak value looks universal, but time-dependence does not.

- A step forward: let us rescale time by τ_{KZ} !



- We illustrated the existence of a scaling function:

$$\kappa_2(\tau;\Gamma) \sim l^2_{\rm KZ}(\Gamma)\ \underbrace{f_2\left(\tau/\tau_{\rm KZ}(\Gamma)\right)}_{\rm Universal}$$

(Γ: non-universal inputs)

- NB: the study of non-equilibrium scaling function is a new frontier in condensed matter cummunity: new physics in an old paper!

The Kibble-Zurek Problem: Universality and the Scaling Limit

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PUPT-2405

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(Dated: September 20, 2012)

PRL **109,** 015701 (2012) PHYSICAL REVIEW LETTERS week ending 6 JULY 2012

Nonequilibrium Dynamic Critical Scaling of the Quantum Ising Chain

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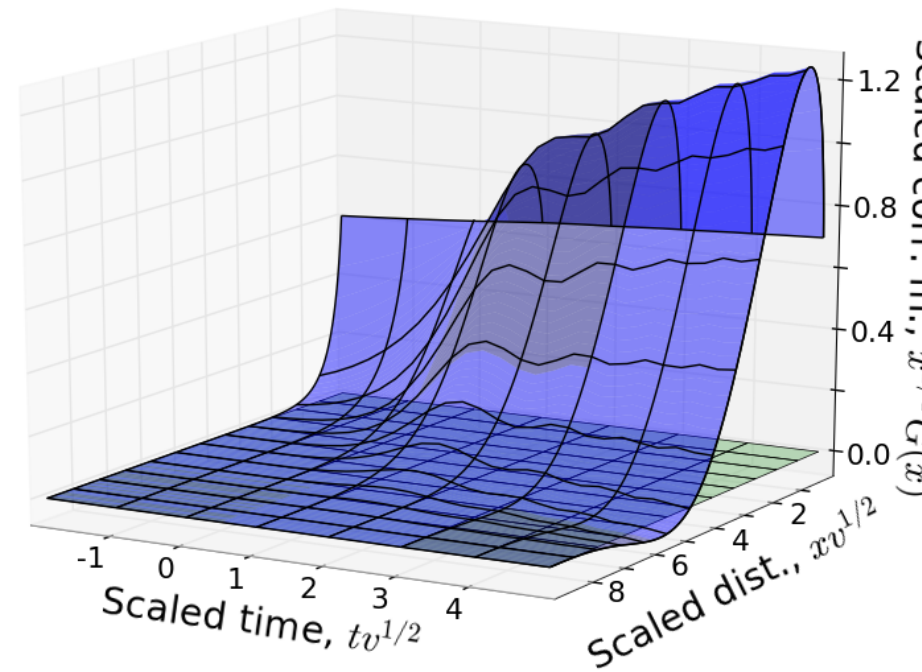
(Received 2 February 2012; published 2 July 2012)

We solve for the time-dependent finite-size scaling functions of the one-dimensional transverse-field Ising chain during a linear-in-time ramp of the field through the quantum critical point. We then simulate Mott-insulating bosons in a tilted potential, an experimentally studied system in the same equilibrium universality class, and demonstrate that universality holds for the dynamics as well. We find qualitatively



PRL **116,** 080601 (2016) PHYSICAL REVIEW LETTERS week ending 26 FEBRUARY 2016

Universality in the Dynamics of Second-Order Phase Transitions

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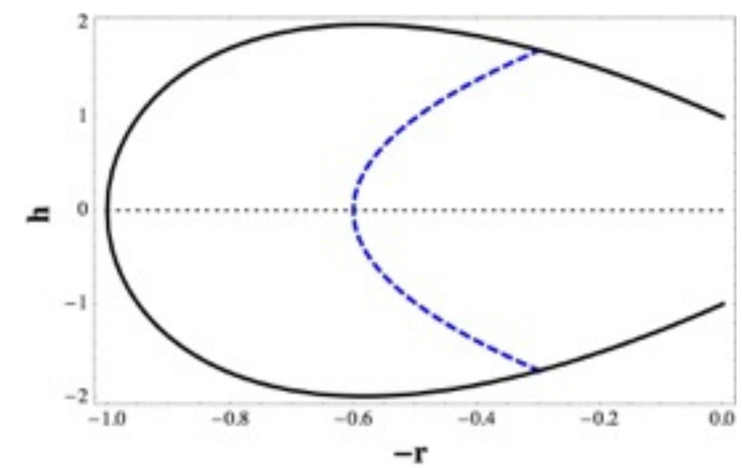
(Received 18 November 2013; revised manuscript received 10 February 2015; published 26 February 2016)

New ingredients when applied to heavy-ion collisions:

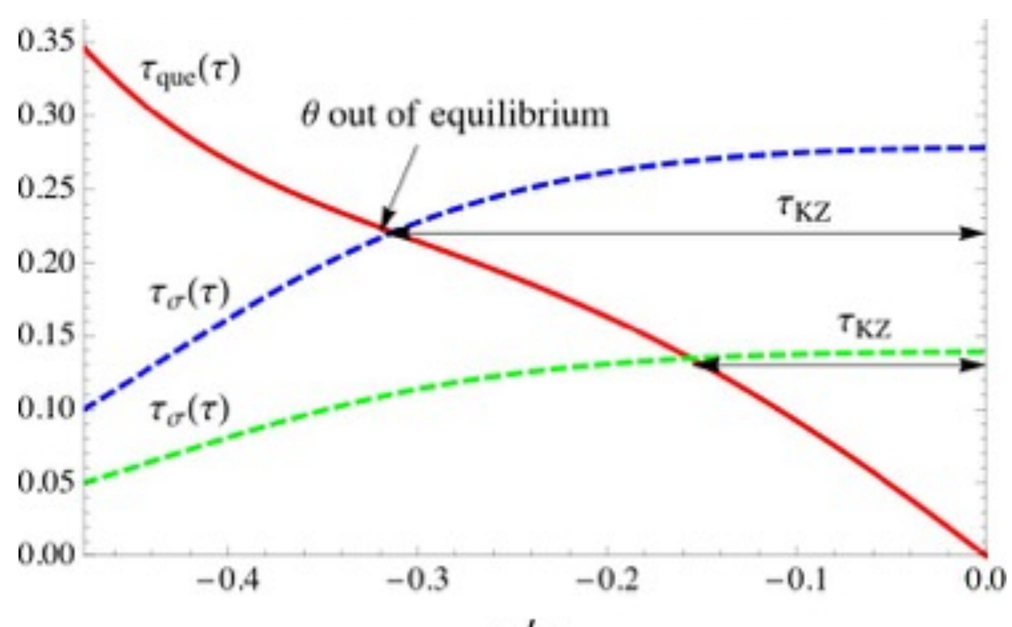
- Extending scaling hypothesis for non-Gaussian cumulants.
- Applying non-equilibrium scaling for trajectories away from the critical point:

A new realization of KZ dynamics

- For a generic trajectory near the cross-over line, $\theta(\tau)$ flips sign.



- Relaxation time remains finite, but θ changes too fast!



- A new non-equilibrium scaling variable: ("memory of sign"). $\theta_{\rm KZ} = \theta(-\tau_{\rm KZ})$

- Non-equilibrium skewness: $\theta_{\rm F} > 0, \theta_{\rm KZ} < 0$

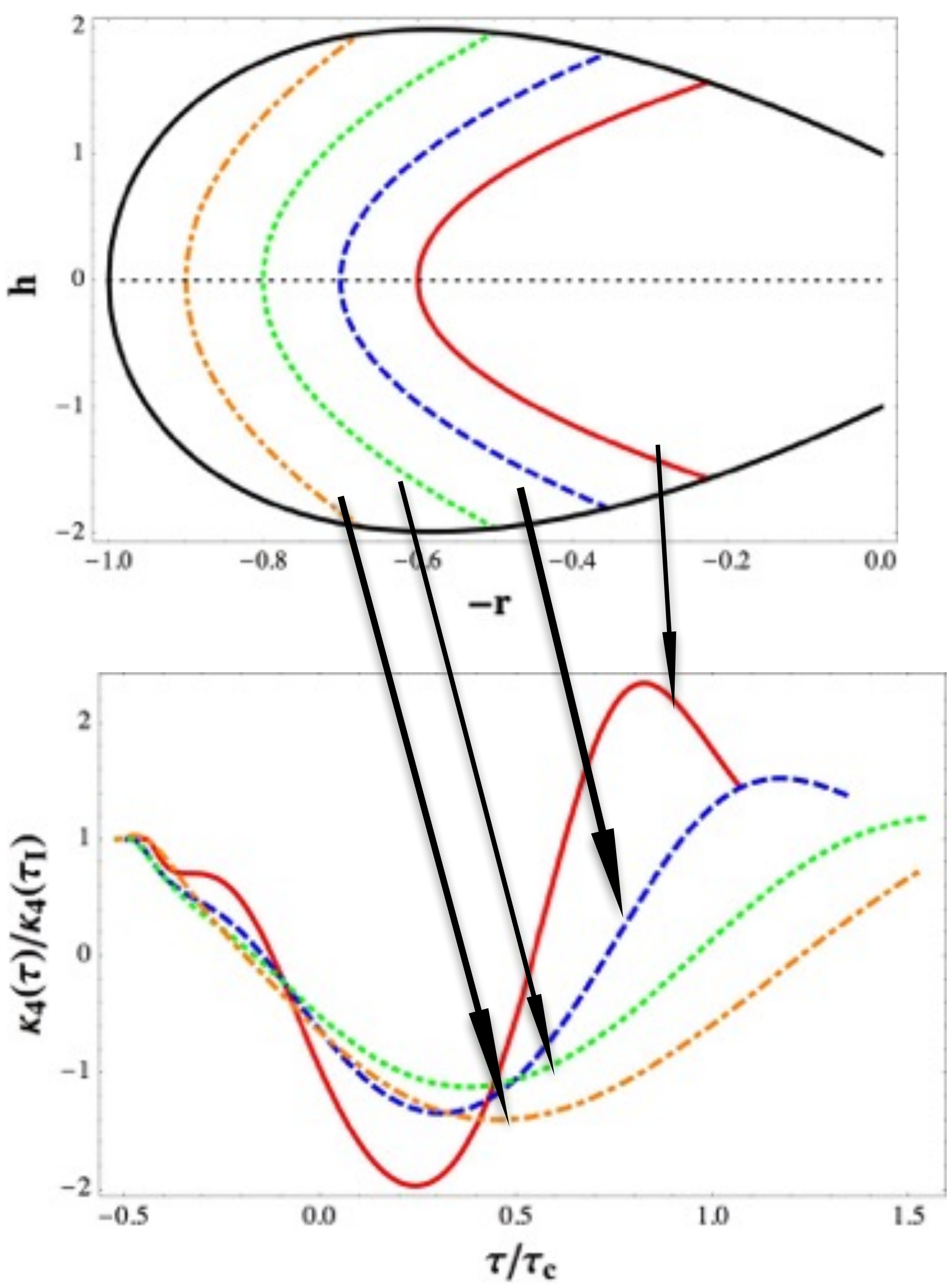
- Generalized scaling hypothesis (S. Mukherjee, R. Venugopalan and YY, 1605.09341):

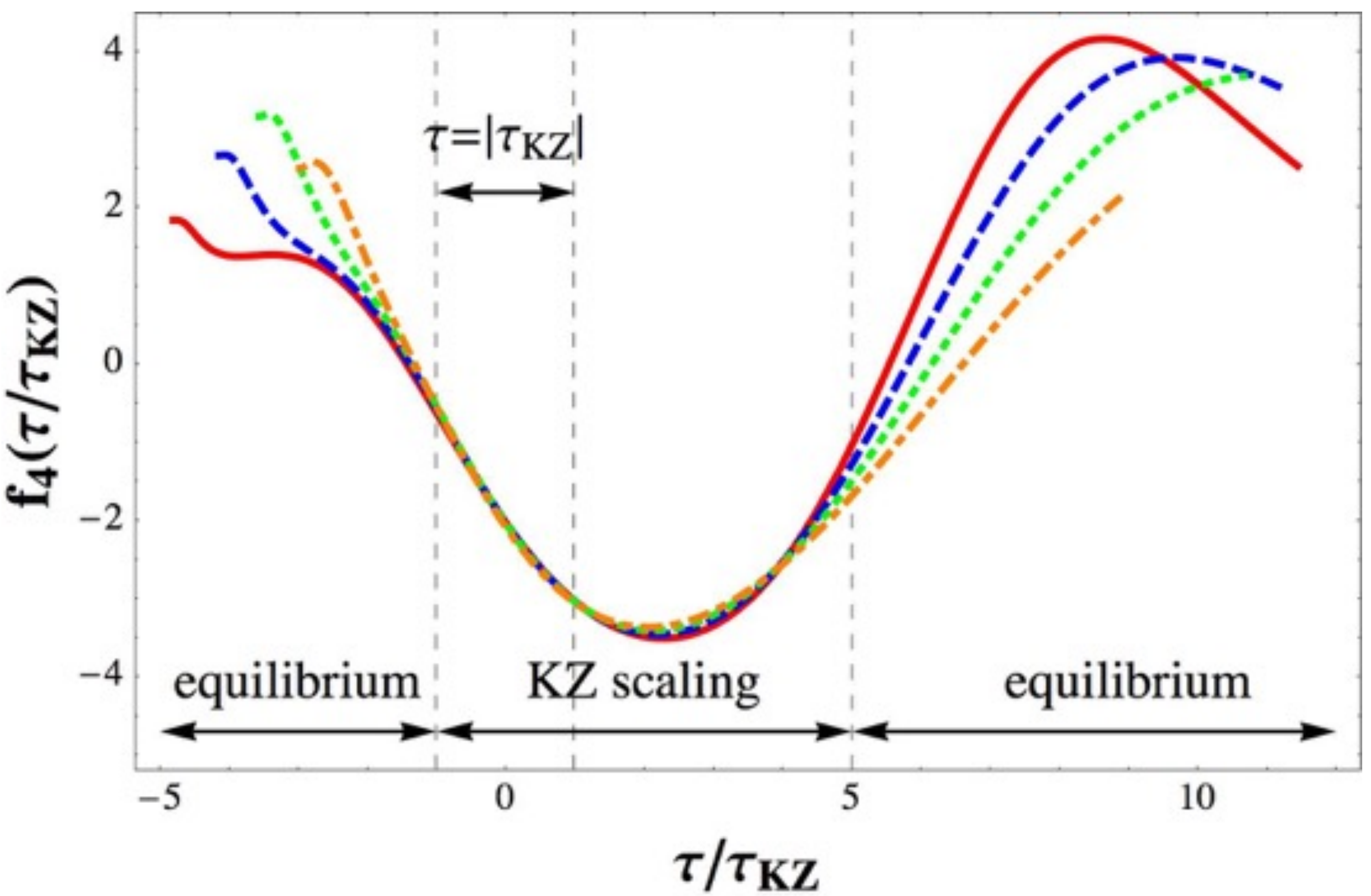
$$\kappa_n(\tau;\Gamma) \sim l_{\rm KZ}^{\#}(\Gamma)\, f_n(\tilde{t}, \theta_{\rm KZ})$$

- Testing scaling hypothesis: different trajectories, same θ_{KZ} .

- Equilibrium cumulants, $\tau_{\mathrm{KZ}}, l_{\mathrm{KZ}}$ are different for those trajectories.

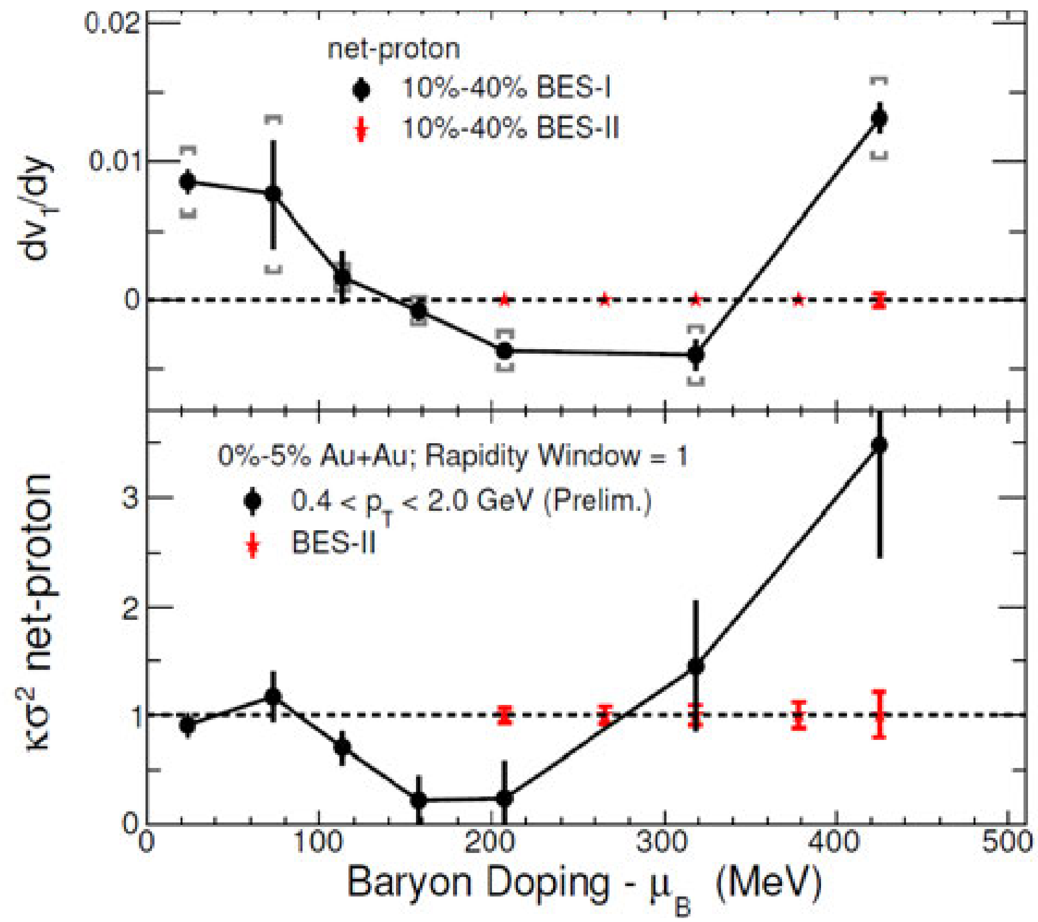
- Expectation from the scaling hypothesis: scaling functions are independent of trajectories.



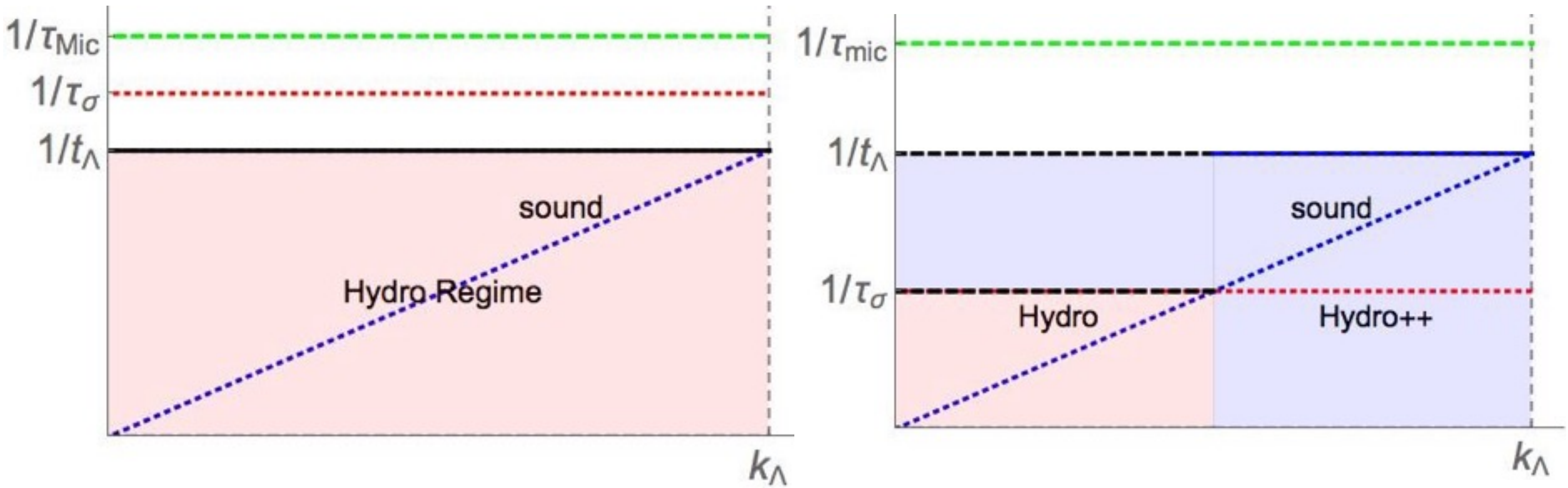


"Trilogy of Universality": universality lost, universality regained, *universality discovered* (to be composed)

Hydro++



A consistent description of flow and fluctuation observables is required.



- *Hydro + +* : a general framework for hydro with a slow mode (e.g. hydro + axial charge density, M. Stephanov, Ho-Ung Yee and YY 1501.00222).

- Applications to critical dynamics: the additional slow mode is the critical mode.

- Generalized entropy can be constructed as:

$$s(\epsilon, n; \sigma) = s_0(\epsilon, n) + \frac{1}{2!} a_2(\epsilon, n) \left[\sigma - \sigma_0(\epsilon, n)\right]^2 + \frac{1}{3!} a_3(\epsilon, n) \left[\sigma - \sigma_0(\epsilon, n)\right]^3 + \dots$$

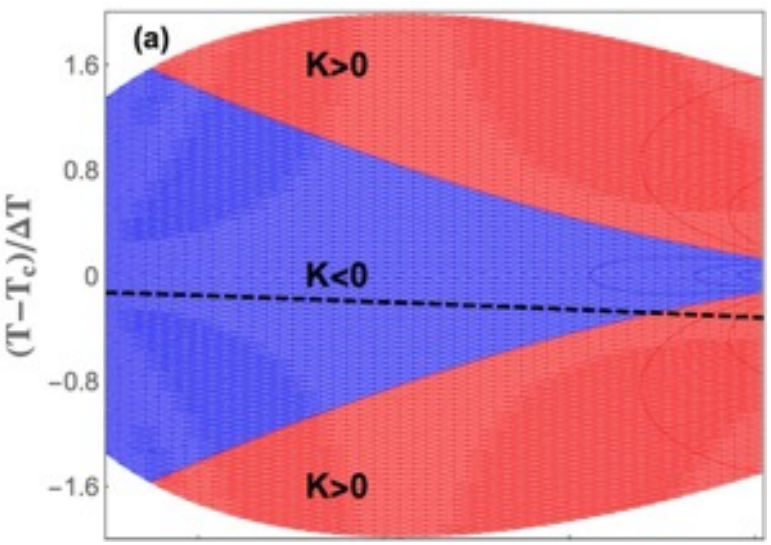
(completely fixed by Ising model and mapping relation)

- The 6th equation (can be matched to evolution equation for the first cumulant):

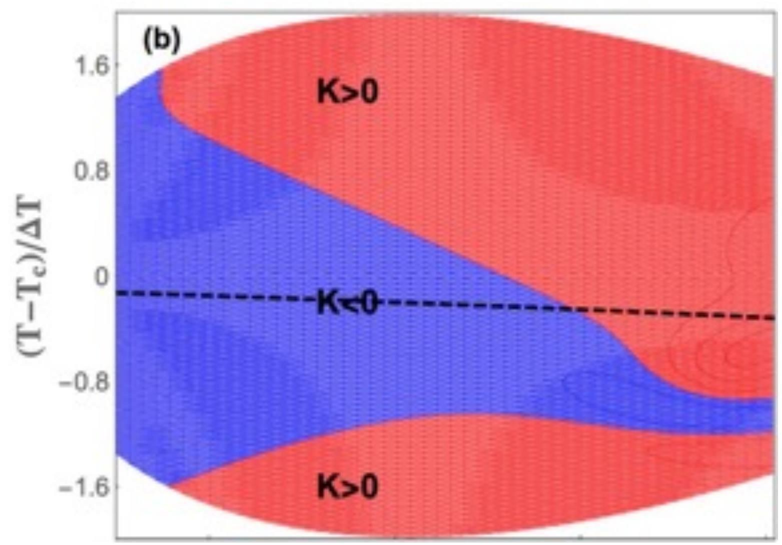
$$u^\mu \partial_\mu \sigma = -\gamma \, \partial_\sigma \, s(\epsilon, n; \sigma)$$

- Illustrative example: Bjorken expansion (M.Stephanov and YY in preparation).

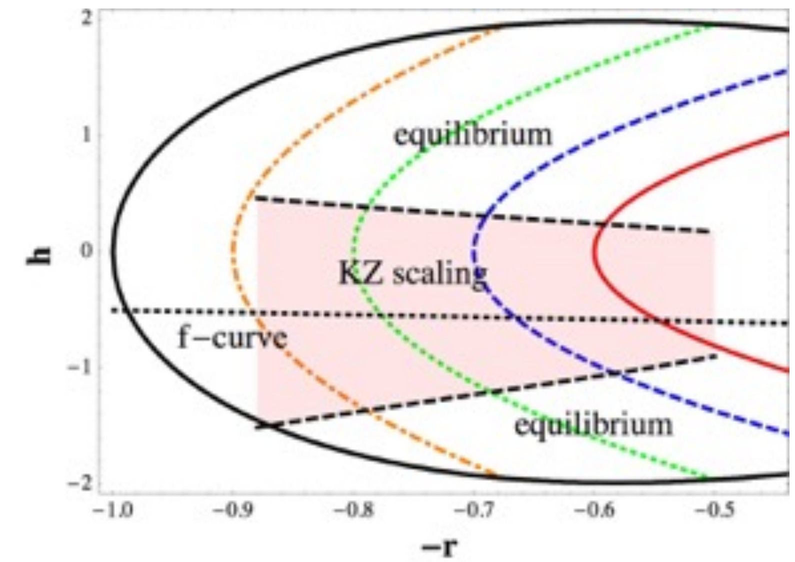
From better cartoon towards quantitive studies



Equilibrium, 11'



non-equilibrium, 15'



Scaling, 16'

Hydro ++: next stage, new physics, let us do it together!

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