

Phase diagram in the vector meson extended PQM model

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*Exploring the QCD Phase Diagram through Energy Scans
INT, Seattle*

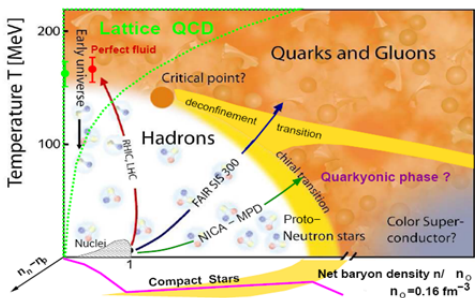
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Overview

- 1 Introduction
 - Motivation
- 2 The model
 - Axial(vector) meson extended linear σ -model
 - Parametrization at $T = 0$
 - Polyakov loop
- 3 eLSM at finite T/μ_B
 - Extremum equations for $\phi_{N/S}$ and $\Phi, \bar{\Phi}$
- 4 Results
 - T dependence of the order parameters
 - Critical endpoint
 - Phase diagram
- 5 Summary

QCD phase diagram

Phase diagram in the $T - \mu_B - \mu_I$ space



- At $\mu_B = 0$ $T_c = 151$ MeV
Y. Aoki, *et al.*, PLB **643**, 46 (2006)
- Is there a CP?
($T_{CP}=162$ MeV, $\mu_{CP}=360$ MeV, Fodor-Katz)
- At $T = 0$ in μ_B where is the phase boundary?

Details of the phase diagram are heavily studied theoretically (Lattice, EFT), and experimentally (RHIC, LHC, FAIR, NICA)

Motivation

- At $\mu = 0$ we know the properties of strong interactions from the lattice in theoretical side and from STAR/PHENIX and from ALICE in the experimental side. On the other hand, for $\mu \gg 0$ at the moment no theory and no experiment provide reasonable information.
- What is the order of phase transition on the $T=0$ line? Is there a CEP?
- Equation of state for neutron stars.
- How the masses change in medium?

Idea

- Build an effective model having the right global symmetry pattern.
- Compare the thermodynamics of the model with lattice at $\mu = 0$
- Extrapolate to high μ .

Effective models

Since QCD is very hard to solve \longrightarrow low energy effective models were set up \longrightarrow reflecting the global symmetries of QCD

- Nambu-Jona-Lasinio model (+Kobayashi-Maskawa-t'Hooft)
- Chiral perturbation theory
- Linear and nonlinear (it does not contain degrees of freedom relevant at high T) sigma model
- To study the phase diagram, we introduced the constituent quarks
- For mimicing confinement, we add the Polyakov loop variables.

extended Polyakov-Quark-Meson model

Chiral symmetry

If the quark masses are zero (chiral limit) \implies QCD invariant under the following global transformation (**chiral symmetry**):

$$q_L = (1 - \gamma_5)/2q, \quad q_R = (1 + \gamma_5)/2q \quad \text{only the mass term mixes}$$
$$U(3)_V q = \exp(-i\alpha t)q \quad U(3)_A q = \exp(-i\beta\gamma_5 t)q$$

$$U(3)_L \times U(3)_R \simeq U(3)_V \times U(3)_A =$$
$$SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$$

$U(1)_V$ term \longrightarrow baryon number conservation

$U(1)_A$ term \longrightarrow broken through axial anomaly

$SU(3)_A$ term \longrightarrow broken down by any quark mass

$SU(3)_V$ term \longrightarrow broken down to $SU(2)_V$ if $m_u = m_d \neq m_s$
 \longrightarrow totally broken if $m_u \neq m_d \neq m_s$ (**in nature**)

Meson fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^8 \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & K^0 & \eta_S \end{pmatrix} (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_S = \sum_{i=0}^8 \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & K_S^0 & \sigma_S \end{pmatrix} (\sim \bar{q}_i q_j)$$

Particle content:

Pseudoscalars: $\pi(138)$, $K(495)$, $\eta(548)$, $\eta'(958)$

Scalars: $a_0(980 \text{ or } 1450)$, $K_0^*(800 \text{ or } 1430)$,

(σ_N, σ_S) : 2 of $f_0(500, 980, 1370, 1500, 1710)$

Structure of scalar mesons

	Mass (MeV)	width (MeV)	decays
$A_0(980)$	980 ± 20	$50 - 100$	$\pi\pi$ dominant
$A_0(1450)$	1474 ± 19	265 ± 13	$\pi\eta, \pi\eta', K\bar{K}$
$K_s(800) = \kappa$	682 ± 29	547 ± 24	$K\pi$
$K_s(1430)$	1425 ± 50	270 ± 80	$K\pi$ dominant
$f_0(500) = \sigma$	400–550	400 – 700	$\pi\pi$ dominant
$f_0(980)$	980 ± 20	40 – 100	$\pi\pi$ dominant
$f_0(1370)$	1200–1500	200 – 500	$\pi\pi \approx 250, K\bar{K} \approx 150$
$f_0(1500)$	1505 ± 6	109 ± 7	$\pi\pi \approx 38, K\bar{K} \approx 9.4$
$f_0(1710)$	1722 ± 6	135 ± 7	$\pi\pi \approx 30, K\bar{K} \approx 71$

Possible scalar states: $\bar{q}q, \bar{q}q\bar{q}q$, meson-meson molecules, glueballs

pseudoscalar nonet: π, K, η, η' , scalar nonet: $A_0, K_0, 2 f_0$

multiquark states: $f_0(980), A_0(980) f_0(600), K_0(800) ???$

meson-meson bound state ($K\bar{K}$): $f_0(980) ???$

glueballs: $f_0(1500)$ (weak coupling to $\gamma\gamma$), $f_0(1710) ???$

Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^8 \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega_N - \rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & K^{*0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^8 b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N} + a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N} - a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & K_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle content:

Vector mesons: $\rho(770)$, $K^*(894)$, $\omega_N = \omega(782)$, $\omega_S = \phi(1020)$

Axial vectors: $a_1(1230)$, $K_1(1270)$, $f_{1N}(1280)$, $f_{1S}(1426)$

Lagrangian (2/1)

$$\begin{aligned}
\mathcal{L}_{\text{Tot}} = & \text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\
& - \frac{1}{4} \text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \text{Tr} \left[\left(\frac{m_1^2}{2} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + \text{Tr}[H(\Phi + \Phi^\dagger)] \\
& + c_1 (\det \Phi + \det \Phi^\dagger) + i \frac{g_2}{2} (\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
& + \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(L_\mu^2 + R_\mu^2) + h_2 \text{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \text{Tr}(L_\mu \Phi R^\mu \Phi^\dagger) \\
& + g_3 [\text{Tr}(L_\mu L_\nu L^\mu L^\nu) + \text{Tr}(R_\mu R_\nu R^\mu R^\nu)] + g_4 [\text{Tr}(L_\mu L^\mu L_\nu L^\nu) \\
& + \text{Tr}(R_\mu R^\mu R_\nu R^\nu)] + g_5 \text{Tr}(L_\mu L^\mu) \text{Tr}(R_\nu R^\nu) + g_6 [\text{Tr}(L_\mu L^\mu) \text{Tr}(L_\nu L^\nu) \\
& + \text{Tr}(R_\mu R^\mu) \text{Tr}(R_\nu R^\nu)] \\
& + \bar{\Psi} i \not{\partial} \Psi - g_F \bar{\Psi} (\Phi_S + i\gamma_5 \Phi_{PS}) \Psi + g_V \bar{\Psi} \gamma^\mu \left(V_\mu + \frac{g_A}{g_V} \gamma_5 A_\mu \right) \Psi
\end{aligned}$$

+ Polyakov loops

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, D.H. Rischke,
 Phys. Rev. D87 (2013) 014011

Lagrangian (2/2)

where

$$D^\mu \Phi = \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu) - ieA_e^\mu [T_3, \Phi]$$

$$\Phi = \sum_{i=0}^8 (\sigma_i + i\pi_i) T_i, \quad H = \sum_{i=0}^8 h_i T_i \quad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^8 (\rho_i^\mu - b_i^\mu) T_i, \quad L^\mu = \sum_{i=0}^8 (\rho_i^\mu + b_i^\mu) T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA_e^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu [T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA_e^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu [T_3, R^\mu]\}$$

$$\bar{\Psi} = (\bar{u}, \bar{d}, \bar{s})$$

non strange – strange base:

$$\varphi_N = \sqrt{2/3}\varphi_0 + \sqrt{1/3}\varphi_8,$$

$$\varphi_S = \sqrt{1/3}\varphi_0 - \sqrt{2/3}\varphi_8, \quad \varphi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)$$

broken symmetry: non-zero condensates $\langle \sigma_N \rangle, \langle \sigma_S \rangle \longleftrightarrow \phi_N, \phi_S$

Symmetry properties of the model

Global $U(3)_L \times U(3)_R$ transformation:

$$\begin{aligned}\Phi &\rightarrow U_L \Phi U_R^\dagger \\ L^\mu &\rightarrow U_L L^\mu U_L^\dagger \\ R^\mu &\rightarrow U_R R^\mu U_R^\dagger\end{aligned}$$

Consequences (using the unitarity of U's):

$$\begin{aligned}D^\mu \Phi &\rightarrow U_L D^\mu \Phi U_R^\dagger \\ L^{\mu\nu} &\rightarrow U_L L^{\mu\nu} U_L^\dagger \\ R^{\mu\nu} &\rightarrow U_R R^{\mu\nu} U_R^\dagger\end{aligned}$$

$$(\text{Tr}(\Phi^\dagger \Phi))' = \text{Tr}(U_R \Phi^\dagger U_L^\dagger U_L \Phi U_R^\dagger) = \text{Tr}(U_R \Phi^\dagger \Phi U_R^\dagger) = \text{Tr}(\Phi^\dagger \Phi U_R^\dagger U_R) = \text{Tr}(\Phi^\dagger \Phi)$$

All terms are invariant except the determinant and the explicit symmetry breaking term.

Determinant term

$$U_L = e^{-i\omega_L^a T^a} \quad U_R = e^{-i\omega_R^a T^a}$$

$$\omega_V^a = 0.5(\omega_L^a + \omega_R^a) \quad \omega_A^a = 0.5(\omega_L^a - \omega_R^a)$$

By $SU(3)_L \times SU(3)_R$ transformation (if $\omega_L^0 = \omega_R^0 = 0 = \omega_V^0 = \omega_A^0$)

$$(\det \Phi)' = \det(U_L \Phi U_R^\dagger) = \det U_L \det \Phi \det U_R^\dagger = \det \Phi$$

Similarly $\det \Phi^\dagger$ is also invariant.

If $\omega_V^0 \neq 0$ and all the other ω 's are 0 ($[T^a, T^0] = 0$)

$$(\det \Phi)' = \det(e^{-i\omega_V^0 T^0} \Phi e^{i\omega_V^0 T^0}) = \det(e^{-i\omega_V^0 T^0} e^{i\omega_V^0 T^0} \Phi) = \det \Phi$$

On the other hand, if $\omega_A^0 \neq 0$ and all the other ω 's are 0

$$(\det \Phi)' = \det(e^{-i\omega_A^0 T^0} \Phi e^{-i\omega_A^0 T^0}) = \det(e^{-i\omega_A^0 T^0} e^{-i\omega_A^0 T^0} \Phi) = e^{-i2\omega_A^0} \det \Phi \text{Tr} T^0$$

So the determinant term is invariant under $U(3)_V \times SU(3)_A$ transformation and breaks explicitly the $U(1)_A$ symmetry.

Axial(vector) meson extended linear σ -modelExplicit breaking term: $\text{Tr}[\hat{\epsilon}(\Phi + \Phi^\dagger)]$

$$\hat{\epsilon} = \sum_{i=0}^8 \epsilon_i T_i = \begin{pmatrix} \frac{\epsilon_N}{2} & 0 & 0 \\ 0 & \frac{\epsilon_N}{2} & 0 \\ 0 & 0 & \frac{\epsilon_S}{\sqrt{2}} \end{pmatrix} \quad \text{only } \epsilon^0, \epsilon^8 \neq 0$$

- axial transformation: if at least $\epsilon^0 \neq 0$ $U(3)_A$ is broken:

$$(\text{Tr}[\hat{\epsilon}(\Phi)])' = \text{Tr}(e^{-i2\omega_A^a T^a} \hat{\epsilon} \Phi)$$

- vector transformation

$$(\text{Tr}[\hat{\epsilon}(\Phi)])' = \text{Tr}(e^{-i\omega_V^a T^a} \hat{\epsilon} e^{i\omega_V^a T^a} \Phi)$$

Since $[\hat{\epsilon}, T^0] = 0$, $U(1)_V$ symmetry is preserved.

If all $\epsilon^a = 0$ except ϵ^0 , $U(3)_V$ is preserved.

If ϵ^8 also non zero, then since $[T^K, T^8] = 0$ if $k = 1, 2, 3$, $U(1)_V \times SU(2)_V$ survives (isospin symmetry)

(If $\epsilon^3 \neq 0$ too, then the isospin symmetry is broken, only $U(1)_V$.)

Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \rightarrow \sigma_{N/S} + \phi_{N/S} \quad \phi_{N/S} \equiv \langle \sigma_{N/S} \rangle$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like $\text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)]$:

$$\pi_N - a_{1N}^\mu : -g_1 \phi_N a_{1N}^\mu \partial_\mu \pi_N,$$

$$\pi - a_1^\mu : -g_1 \phi_N (a_1^{\mu+} \partial_\mu \pi^- + a_1^{\mu 0} \partial_\mu \pi^0) + \text{h.c.},$$

$$\pi_S - a_{1S}^\mu : -\sqrt{2} g_1 \phi_S a_{1S}^\mu \partial_\mu \pi_S,$$

$$K_S - K_\mu^* : \frac{ig_1}{2} (\sqrt{2} \phi_S - \phi_N) (\bar{K}_\mu^{*0} \partial^\mu K_S^0 + K_\mu^{*-} \partial^\mu K_S^+) + \text{h.c.},$$

$$K - K_1^\mu : -\frac{g_1}{2} (\phi_N + \sqrt{2} \phi_S) (K_1^{\mu 0} \partial_\mu \bar{K}^0 + K_1^{\mu+} \partial_\mu K^-) + \text{h.c.}.$$

Diagonalization \rightarrow Wave function renormalization

Determination of the parameters of the Lagrangian

16 unknown parameters ($m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, \mathbf{g}_F, \mathbf{g}_V, \mathbf{g}_A$) \longrightarrow Determined by the **min. of χ^2** :

$$\chi^2(x_1, \dots, x_N) = \sum_{i=1}^M \left[\frac{Q_i(x_1, \dots, x_N) - Q_i^{\text{exp}}}{\delta Q_i} \right]^2,$$

where $(x_1, \dots, x_N) = (m_0, \lambda_1, \lambda_2, \dots)$, $Q_i(x_1, \dots, x_N)$ calculated from the model, while Q_i^{exp} taken from the PDG

multiparametric minimalization \longrightarrow **MINUIT**

- PCAC \rightarrow 2 physical quantities: f_π, f_K
- Tree-level masses \rightarrow 15 physical quantities:
 $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^*}, m_{a_1}, m_{f_1^H}, m_{a_0}, m_{K_S}, m_{f_0^L}, m_{f_0^H}$
- Decay widths \rightarrow 12 physical quantities:
 $\Gamma_{\rho \rightarrow \pi\pi}, \Gamma_{\Phi \rightarrow KK}, \Gamma_{K^* \rightarrow K\pi}, \Gamma_{a_1 \rightarrow \pi\gamma}, \Gamma_{a_1 \rightarrow \rho\pi}, \Gamma_{f_1 \rightarrow KK^*}, \Gamma_{a_0}, \Gamma_{K_S \rightarrow K\pi},$
 $\Gamma_{f_0^L \rightarrow \pi\pi}, \Gamma_{f_0^L \rightarrow KK}, \Gamma_{f_0^H \rightarrow \pi\pi}, \Gamma_{f_0^H \rightarrow KK}$
- $T_c = 155$ MeV from lattice

Parametrization at $T = 0$

Results

	Cal(GeV)	Mass		Cal(GeV)	Width
m_π	0.1405	0.1380	f_π	0.0955	0.0922
m_K	0.4995	0.4956	f_K	0.1094	0.1100
m_η	0.5421	0.5479	$\Gamma_{f_0L \rightarrow KK}$	0.0	0.0
$m_{\eta'}$	0.9643	0.9578	$\Gamma_{f_0H \rightarrow KK}$	0.0	0.0
m_ρ	0.8064	0.7755	Γ_ρ	0.1515	0.149
m_ϕ	0.9901	1.0195	Γ_ϕ	0.003534	0.003545
m_{K^*}	0.9152	0.8938	Γ_{K^*}	0.04777	0.048
m_{f_1H}	1.4160	1.4264	$\Gamma_{f_1 \rightarrow KK}$	0.04451	0.0445
m_{a_1}	1.0766	1.2300	$\Gamma_{A_1 \rightarrow \rho\pi}$	0.1994	0.425
m_{K_1}	1.2999	1.2720	$\Gamma_{A_1 \rightarrow \gamma\pi}$	0.0003670	0.000640
m_{a_0}	0.7208	0.980	Γ_{A_0}	0.06834	0.075
$m_{K_0^*}$	0.7529	0.682	$\Gamma_{K_0^*}$	0.006001	0.00547
m_{f_0L}	0.2823	0.475	$\Gamma_{f_0L \rightarrow \pi\pi}$	0.5542	0.550
m_{f_0H}	0.7376	0.990	$\Gamma_{f_0H \rightarrow \pi\pi}$	0.08166	0.07
m_{ud}	0.3224	0.308	m_s	0.4577	0.483

Parametrization at $T = 0$

Parameters

Parameter	Value
ϕ_N [GeV]	0.1411
ϕ_S [GeV]	0.1416
m_0^2 [GeV ²]	$2.3925E - 4$
m_1^2 [GeV ²]	$6.3298E - 8$
λ_1	-1.6738
λ_2	23.5078
δ_S [GeV ²]	0.1133
c_1 [GeV]	1.3086
g_1	5.6156
g_2	3.0467
h_1	27.4617
h_2	4.2281
h_3	5.9839
g_F	4.5708
M_0 [GeV]	0.3511

- with this set
 $f_0^I = 0.2837$ GeV

Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\text{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\text{Tr}_c \bar{L}(\vec{x})}{N_c}$ with

$$L(x) = \mathcal{P} \exp \left[i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right]$$

→ signals center symmetry (\mathbb{Z}_3) breaking at the deconfinement transition

low T : confined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle = 0$

high T : deconfined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle \neq 0$

Polyakov gauge: the temporal component of the gauge field is time independent and can be gauge rotated to a diagonal form in the color space

$$G_{4,d}(\vec{x}) = \phi_3(\vec{x})\lambda_3 + \phi_8(\vec{x})\lambda_8; \quad \lambda_3, \lambda_8 : \text{Gell-Mann matrices.}$$

In this gauge the Polyakov loop operator is

$$L(\vec{x}) = \text{diag}(e^{i\beta\phi_+(\vec{x})}, e^{i\beta\phi_-(\vec{x})}, e^{-i\beta(\phi_+(\vec{x})+\phi_-(\vec{x}))})$$

where $\phi_\pm(\vec{x}) = \pm\phi_3(\vec{x}) + \phi_8(\vec{x})/\sqrt{3}$

Form of the potential

I.) Simple **polynomial potential** invariant under \mathbb{Z}_3 and charge conjugation: R.D.Pisarski, PRD 62, 111501

$$\frac{\mathcal{U}_{\text{poly}}(\Phi, \bar{\Phi})}{T^4} = -\frac{b_2(T)}{2} \bar{\Phi} \Phi - \frac{b_3}{6} (\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4} (\bar{\Phi} \Phi)^2$$

with
$$b_2(T) = a_0 + a_1 \frac{T_0}{T} + a_2 \frac{T_0^2}{T^2} + a_3 \frac{T_0^3}{T^3}$$

II.) **Logarithmic potential** coming from the $SU(3)$ Haar measure of group integration
K. Fukushima, Phys. Lett. **B591**, 277 (2004)

$$\frac{\mathcal{U}_{\text{log}}(\Phi, \bar{\Phi})}{T^4} = -\frac{1}{2} a(T) \Phi \bar{\Phi} + b(T) \ln \left[1 - 6 \Phi \bar{\Phi} + 4 (\Phi^3 + \bar{\Phi}^3) - 3 (\Phi \bar{\Phi})^2 \right]$$

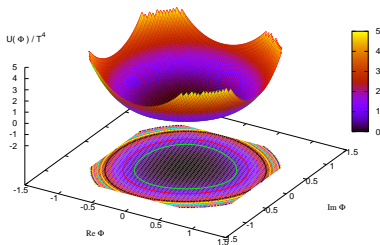
with
$$a(T) = a_0 + a_1 \frac{T_0}{T} + a_2 \frac{T_0^2}{T^2}, \quad b(T) = b_3 \frac{T_0^3}{T^3}$$

$\mathcal{U}(\Phi, \bar{\Phi})$ models the free energy of a pure gauge theory
→ the parameters are fitted to the pure gauge lattice data

Polyakov loop potential

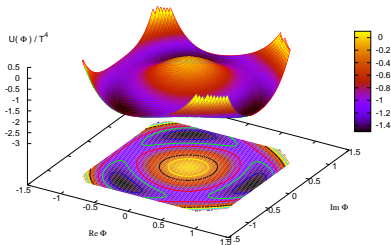
“Color confinement”

$\langle \Phi \rangle = 0 \rightarrow$ no breaking of \mathbb{Z}_3
one minimum



“Color deconfinement”

$\langle \Phi \rangle \neq 0 \rightarrow$ spontaneous breaking of \mathbb{Z}_3
minima at $0, 2\pi/3, -2\pi/3$
one of them spontaneously selected



from H. Hansen et al., PRD75, 065004 (2007)

Effects of quarks: Improved Polyakov loops potential

- N_f -dependence of T_0 estimated: $T_0(N_f = 2 + 1) = 182$ MeV for $m_s = 95$ MeV

L.M. Haas et al., PRD 87, 076004 see also B.-J. Schaefer et al., PRD 76, 074023

- Within FRG, the glue potential $U^{glue}(\Phi, \bar{\Phi})$ coming from the gauge dof propagating in the presence of dynamical quarks can be matched to the potential $U^{YM}(\Phi, \bar{\Phi})$ of the SU(3) YM theory by relating the reduced temperatures:

$$\frac{U^{glue}}{T^4}(\Phi, \bar{\Phi}, t_{glue}) = \frac{U^{YM}}{(T^{YM})^4}(\Phi, \bar{\Phi}, t_{YM}(t_{glue})), \quad t_{YM}(t_{glue}) \approx 0.57 t_{glue}$$

$$t_{glue} \equiv \frac{T - T_c^{glue}}{T_c^{glue}}, \quad t_{YM} \equiv \frac{T^{YM} - T_0^{YM}}{T_0^{YM}}, \quad T_c^{glue} \in (180, 270) \text{ MeV}$$

L.M.Haas et al., PRD 87, 076004 (2013)

Effects of Polyakov loops on FD statistics

Inclusion of the Polyakov loop modifies the Fermi-Dirac distribution function

$$f(E_p - \mu_q) \longrightarrow f_{\bar{\Phi}}^+(E_p) = \frac{(\bar{\Phi} + 2\Phi e^{-\beta(E_p - \mu_q)}) e^{-\beta(E_p - \mu_q)} + e^{-3\beta(E_p - \mu_q)}}{1 + 3(\bar{\Phi} + \Phi e^{-\beta(E_p - \mu_q)}) e^{-\beta(E_p - \mu_q)} + e^{-3\beta(E_p - \mu_q)}}$$

$$f(E_p + \mu_q) \longrightarrow f_{\Phi}^-(E_p) = \frac{(\Phi + 2\bar{\Phi} e^{-\beta(E_p + \mu_q)}) e^{-\beta(E_p + \mu_q)} + e^{-3\beta(E_p + \mu_q)}}{1 + 3(\Phi + \bar{\Phi} e^{-\beta(E_p + \mu_q)}) e^{-\beta(E_p + \mu_q)} + e^{-3\beta(E_p + \mu_q)}}$$

$$\Phi, \bar{\Phi} \rightarrow 0 \implies f_{\Phi}^{\pm}(E_p) \rightarrow f(3(E_p \pm \mu_q)) \quad \Phi, \bar{\Phi} \rightarrow 1 \implies f_{\Phi}^{\pm}(E_p) \rightarrow f(E_p \pm \mu_q)$$

three-particle state appears: mimics confinement of quarks within baryons

at $T = 0$ there is no difference between models with and without Polyakov loop

Extremum equations for $\phi_{N/S}$ and $\Phi, \bar{\Phi}$ T/μ_B dependence of the Polyakov-loops

$$\frac{\partial \Omega}{\partial \Phi} = \frac{\partial \Omega}{\partial \bar{\Phi}} \Big|_{\varphi_N = \phi_N, \varphi_S = \phi_S} = 0, \quad \Omega : \text{grand canonical potential}$$

$$- \frac{d}{d\Phi} \left(\frac{U(\Phi, \bar{\Phi})}{T^4} \right) + \frac{2N_c}{T^3} \sum_{q=u,d,s} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left(\frac{e^{-\beta E_q^-(p)}}{g_q^-(p)} + \frac{e^{-2\beta E_q^+(p)}}{g_q^+(p)} \right) = 0$$

$$- \frac{d}{d\bar{\Phi}} \left(\frac{U(\Phi, \bar{\Phi})}{T^4} \right) + \frac{2N_c}{T^3} \sum_{q=u,d,s} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left(\frac{e^{-\beta E_q^+(p)}}{g_q^+(p)} + \frac{e^{-2\beta E_q^-(p)}}{g_q^-(p)} \right) = 0$$

$$g_q^+(p) = 1 + 3 \left(\bar{\Phi} + \Phi e^{-\beta E_q^+(p)} \right) e^{-\beta E_q^+(p)} + e^{-3\beta E_q^+(p)}$$

$$g_q^-(p) = 1 + 3 \left(\Phi + \bar{\Phi} e^{-\beta E_q^-(p)} \right) e^{-\beta E_q^-(p)} + e^{-3\beta E_q^-(p)}$$

$$E_q^\pm(p) = E_q(p) \mp \mu_B/3, \quad E_{u/d}(p) = \sqrt{p^2 + m_{u/d}^2}, \quad E_s(p) = \sqrt{p^2 + m_s^2}$$

Extremum equations for $\phi_{N/S}$ and $\Phi, \bar{\Phi}$

T/μ_B dependence of the condensates ($\phi_{N/S}$)

$$\frac{\partial \Omega}{\partial \phi_N} = \frac{\partial \Omega}{\partial \phi_S} \Big|_{\Phi, \bar{\Phi}} = 0, \quad (\text{after the SSB})$$

Hybrid approach: fermions at one-loop, mesons at tree-level (their effects are much smaller)

$$m_0^2 \phi_N + \left(\lambda_1 + \frac{1}{2} \lambda_2 \right) \phi_N^3 + \lambda_1 \phi_N \phi_S^2 - h_N + \frac{g_F}{2} N_c (\langle u\bar{u} \rangle_T + \langle d\bar{d} \rangle_T) = 0$$

$$m_0^2 \phi_S + (\lambda_1 + \lambda_2) \phi_S^3 + \lambda_1 \phi_N^2 \phi_S - h_S + \frac{g_F}{\sqrt{2}} N_c \langle s\bar{s} \rangle_T = 0$$

$$\langle q\bar{q} \rangle_T = -4m_q \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_q(p)} (1 - f_\Phi^-(E_q(p)) - f_\Phi^+(E_q(p)))$$

Extremum equations for $\phi_{N/S}$ and $\Phi, \bar{\Phi}$

Masses

$$M_{i,ab}^2 = \left. \frac{\partial^2 \Omega(T, \mu_f)}{\partial \varphi_{i,a} \partial \varphi_{i,b}} \right|_{\min} = m_{i,ab}^2 + \Delta_0 m_{i,ab}^2 + \Delta_T m_{i,ab}^2,$$

$m_{i,ab}^2 \rightarrow$ tree-level mass matrix,

$\Delta_0 / \Delta_T m_{i,ab}^2 \rightarrow$ fermion vacuum/thermal fluctuation,

$$\Delta_0 m_{i,ab}^2 = \left. \frac{\partial^2 \Omega_{q\bar{q}}^{\text{vac}}}{\partial \varphi_{i,a} \partial \varphi_{i,b}} \right|_{\min} = -\frac{3}{8\pi^2} \sum_{f=u,d,s} \left[\left(\frac{3}{2} + \log \frac{m_f^2}{M^2} \right) m_{f,a}^{2(i)} m_{f,b}^{2(i)} + m_f^2 \left(\frac{1}{2} + \log \frac{m_f^2}{M^2} \right) m_{f,ab}^{2(i)} \right],$$

$$\begin{aligned} \Delta_T m_{i,ab}^2 = \left. \frac{\partial^2 \Omega_{q\bar{q}}^{\text{th}}}{\partial \varphi_{i,a} \partial \varphi_{i,b}} \right|_{\min} &= 6 \sum_{f=u,d,s} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_f(p)} \left[(f_f^+(p) + f_f^-(p)) \left(m_{f,ab}^{2(i)} - \frac{m_{f,a}^{2(i)} m_{f,b}^{2(i)}}{2E_f^2(p)} \right) \right. \\ &\quad \left. + (B_f^+(p) + B_f^-(p)) \frac{m_{f,a}^{2(i)} m_{f,b}^{2(i)}}{2TE_f(p)} \right], \end{aligned}$$

where $m_{f,a}^{2(i)} \equiv \partial m_f^2 / \partial \varphi_{i,a}$, $m_{f,ab}^{2(i)} \equiv \partial^2 m_f^2 / \partial \varphi_{i,a} \partial \varphi_{i,b}$

Features of our approach

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables, $\Phi, \bar{\Phi}$ with U^{YM} or U^{glue}
- u,d,s constituent quarks, ($m_u = m_d$)
- no mesonic fluctuations included in the grand canonical potential:

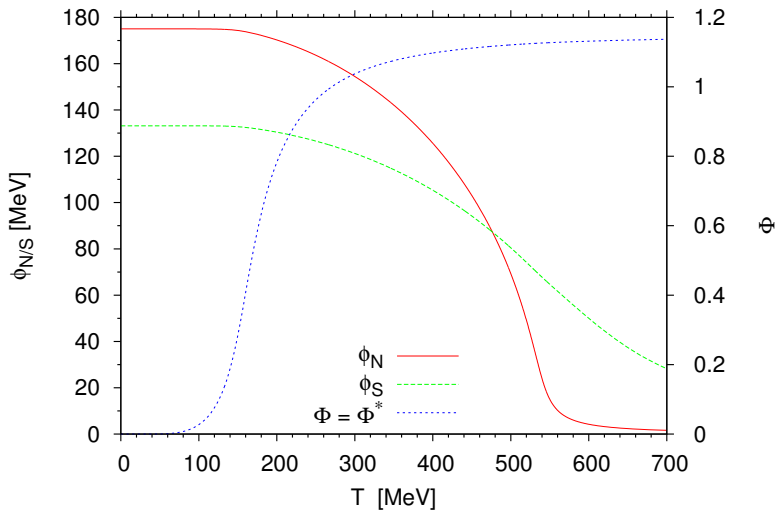
$$\Omega(T, \mu_q) = -\frac{1}{\beta V} \ln(Z)$$

- Fermion **vacuum** and **thermal** fluctuations
- quarks do not couple to (axial) vector meson yet
- Four order parameters ($\phi_N, \phi_S, \Phi, \bar{\Phi}$) \rightarrow four T/μ -dependent equations
- thermal contribution of π, K, f_0^L included in the pressure

T dependence of the order parameters

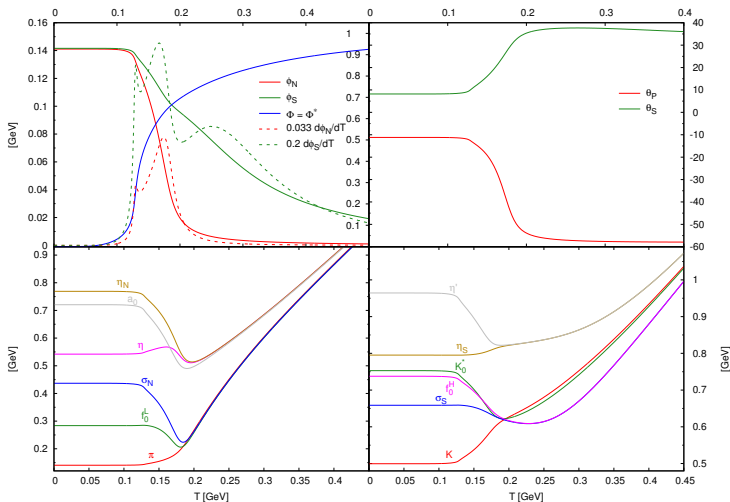
$\phi_{N/S}(T), \Phi/\bar{\Phi}(T)$ *with* Polyakov loop $m_{f_0^L} = 1326$ MeV

Condensates and Polyakov loop variables with vacuum fluctuations



T dependence of the order parameters

With low mass scalars, $m_{f^L} = 300$ MeV



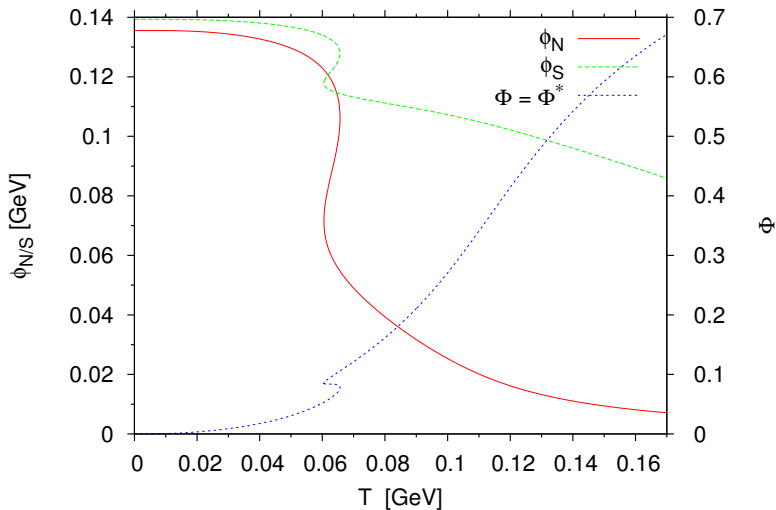
Our pattern: $m_\eta \leq m_{\eta_N} < m_{\eta_S} \leq m_{\eta'}$ in contrast to others

Schaefer, PRD79 014018, Tiwari PRD88, 074017

T dependence of the order parameters

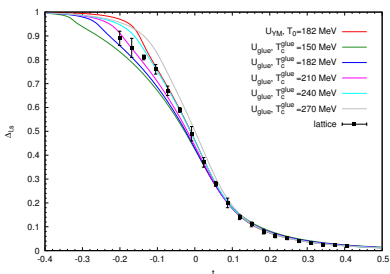
With low mass scalars 1st order phase transition

Order params. in T , A0LKsLf013, $\mu_B=0.849$ GeV, $m_{f_0^L}=0.25$ GeV



T dependence of the order parameters

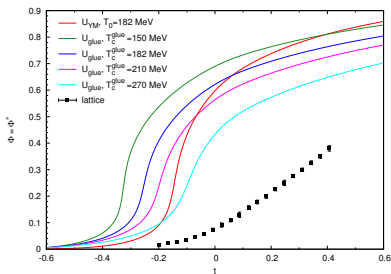
T -dependence of condensates compared to lattice results



subtracted chiral condensate

$$\Delta = \frac{(\Phi_N - h_N/h_S \Phi_S)_T}{(\Phi_N - h_N/h_S \Phi_S)_{T=0}}$$

U^{glue} with $T_C^{glue} \in (210 - 240)$ MeV gives good agreement with the lattice result of [Borsanyi et al., JHEP 1009, 073 \(2010\)](#)



- lattice shows smooth transition
- our result is completely off
- renormalization of the Polyakov loop may explain part of the discrepancy [Andersen et al., PRD92, 114504](#)

Thermodynamical Observables

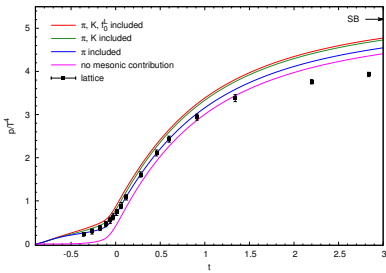
We include mesonic thermal contribution to p for (π, K, f_0^I)

$$\Delta p(T) = -nT \int \frac{d^3q}{(2\pi)^3} \ln(1 - e^{-\beta E(q)}), \quad E(q) = \sqrt{q^2 + m^2}$$

- pressure: $p(T, \mu_q) = \Omega_H(T = 0, \mu_q) - \Omega_H(T, \mu_q)$
- entropy density: $s = \frac{\partial p}{\partial T}$
- quark number density: $\rho_q = \frac{\partial p}{\partial \mu_q}$
- energy density: $\epsilon = -p + Ts + \mu_q \rho_q$
- scaled interaction measure: $\frac{\Delta}{T^4} = \frac{\epsilon - 3p}{T^4}$
- speed of sound at $\mu_q = 0$: $c_s^2 = \frac{\partial p}{\partial \epsilon}$

T dependence of the order parameters

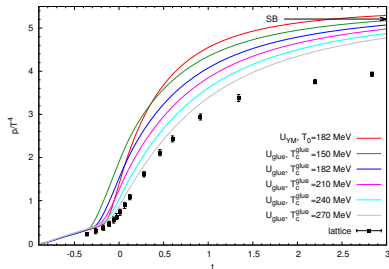
Normalized pressure



we use U^{glue} with $T_c^{glue} = 270$ MeV

pion dominates at low T

at high T pressure overshoots the lattice data



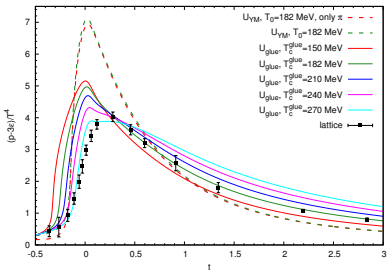
overshooting increases with decreasing T_c^{glue}

lattice: [Borsányi et al., JHEP 1011, 077 \(2010\)](#)

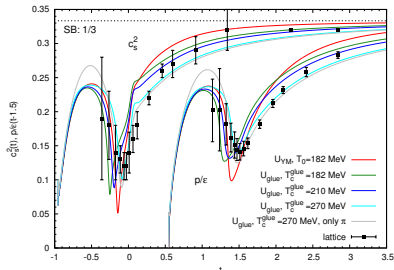
T dependence of the order parameters

Observables

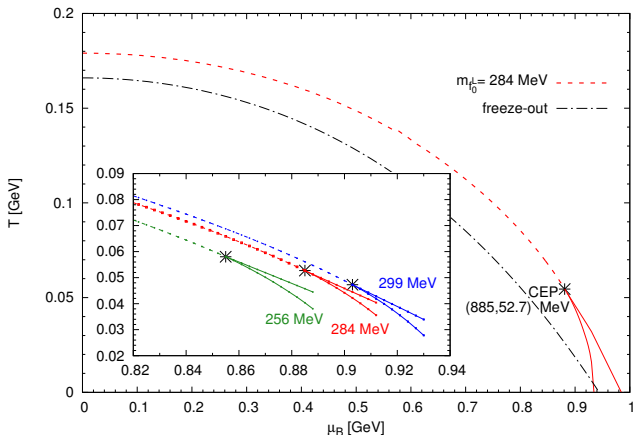
interaction measure



speed of sound

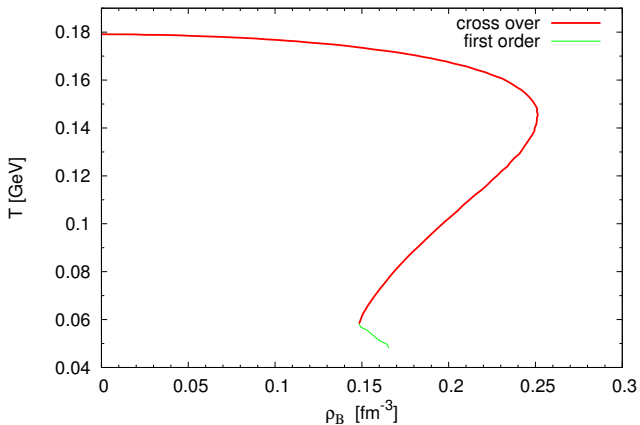


$T - \mu_B$ phase diagram



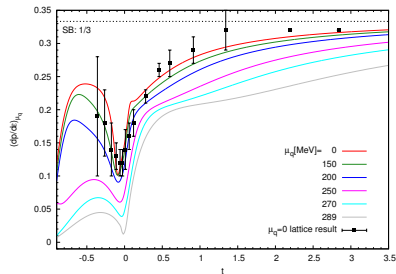
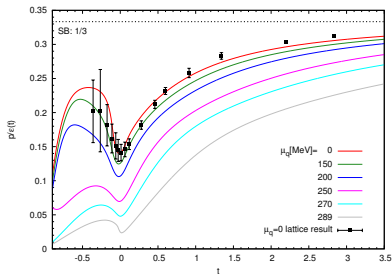
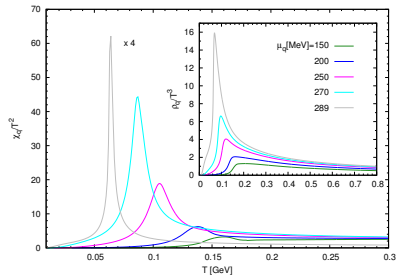
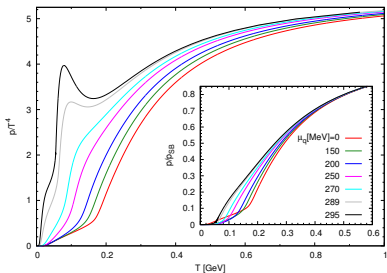
- we use U^{glue} with $T_c^{glue} = 210$ MeV
- freeze-out curve from Cleymans et al., J.Phys.G32, S165

Phase diagram

 $T - \rho_B$ phase diagram

general feature of PQM (or similar models): at $T=0$ the phase transition is at very low density, pushing it to higher values the phase transition becomes cross over.

Phase diagram

Observables at $\mu_B \neq 0$ 

Summary and Conclusions

- The thermodynamics of the ePQM was studied after parametrizing of the model with a modification of the method used in [Parganlija et al., PRD87, 014011](#)
- 40 possible assignments of the scalars to the nonet states were investigated. Lowest χ^2 for $a_0^{\bar{q}q} \rightarrow a_0(980)$, $K_0^{*,\bar{q}q} \rightarrow K_0^*(980)$, $f_0^{l,\bar{q}q} \rightarrow f_0(500)$, $f_0^{h,\bar{q}q} \rightarrow f_0(980)$
- The phase transition temperature requires low mass (≤ 400 MeV) f_0
- For the best set of parameters CEP was found in the $T - \mu_B$ plane
- The T -dependence of various thermodynamical observables measured on the lattice is reasonable well reproduced with an improved Polyakov loop potential. [L.M. Hass et al., PRD87, 076004](#)
- The model's predictions are unrealistic at large μ_B

- To do...
- Improve the vacuum phenomenology by tetraquarks (and glueballs)
- coupling the quarks to the (axial)vectors
- including mesonic fluctuations
- find a way to improve the high density behaviour

Thank you for your attention!