Functional renormalization group studies of QCD - a review

Exploring the QCD Phase Diagram
through Energy Scans, INT-16-3
INT, Seattle, WA

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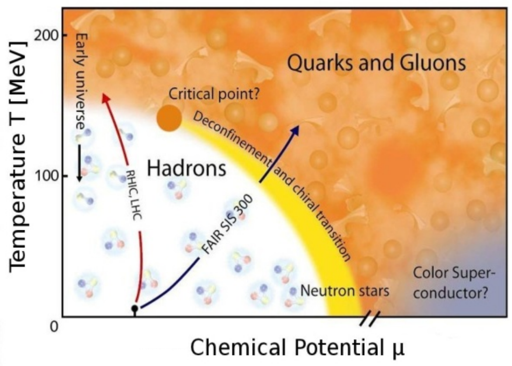
DFG Deutsche Forschungsgemeinschaft

BERKELEY LAB

Fundamental challenges

1. Understanding the **phase structure of QCD** from first principles

Phase structure at large chemical potentials largely unknown due to **sign problem** in lattice QCD…

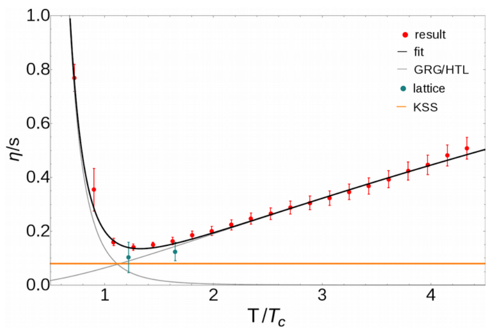


- adapted from GSI

2. Understanding the **fundamental properties** of strongly interacting matter from its **microscopic description**

Hadron spectrum
pole masses, decay constants, form factors, scattering amplitudes,...

Realtime observables
elementary spectral functions, transport coefficients...



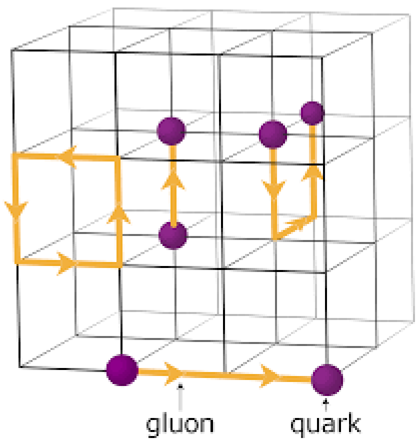
- Christiansen, Haas, Pawlowski, NSt PRL **115** (2015) 11, 112002

Difficult to obtain in Euclidean approaches due to analytic continuation

Nonperturbative approaches

Both challenges require **first-principle approaches**:

Lattice QCD



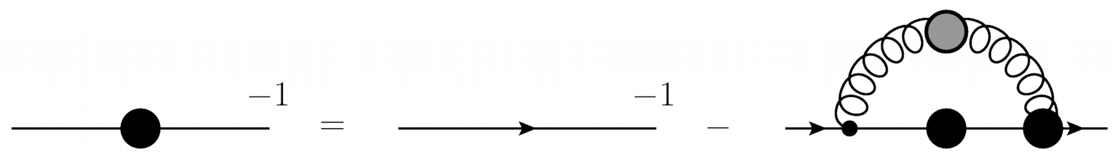
jicfus.jp

Functional approaches

- Dyson-Schwinger equations (DSE)
- n-particle irreducible methods (nPI)
- **Functional Renormalization Group (FRG)**

use relations between off-shell Green's functions

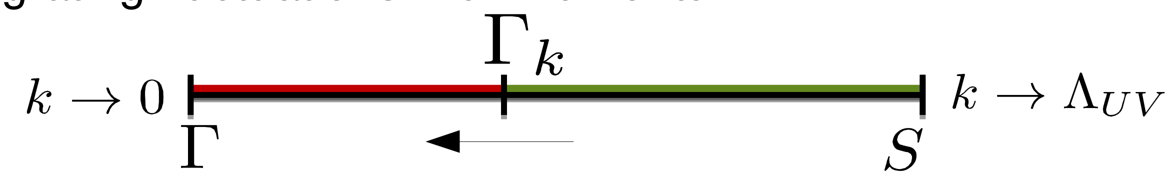
e.g. quark propagator DSE



✓ Complementary to the lattice
✓ No sign problem
✓ Calculation of realtime observables

Functional RG for QCD

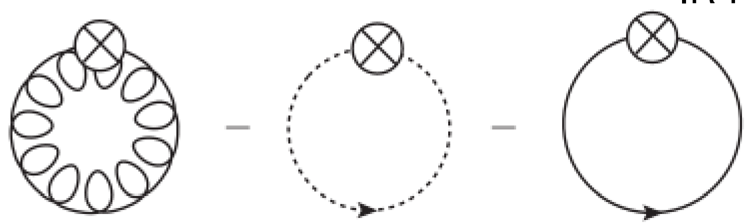
Spirit of **Wilson RG**: Calculate full quantum effective action by integrating fluctuations with momentum k



Functional Renormalization Group (FRG)

Master equation:

IR-Regulator

$k\partial_k\Gamma_k = \frac{1}{2}$ 

full field- and momentum-dependent propagators

Free energy/
Grand potential

Gluon fluctuations

Ghost fluctuations

Quark fluctuations

+flow equations for n-point functions via functional differentiation

Yang-Mills
& Confinement

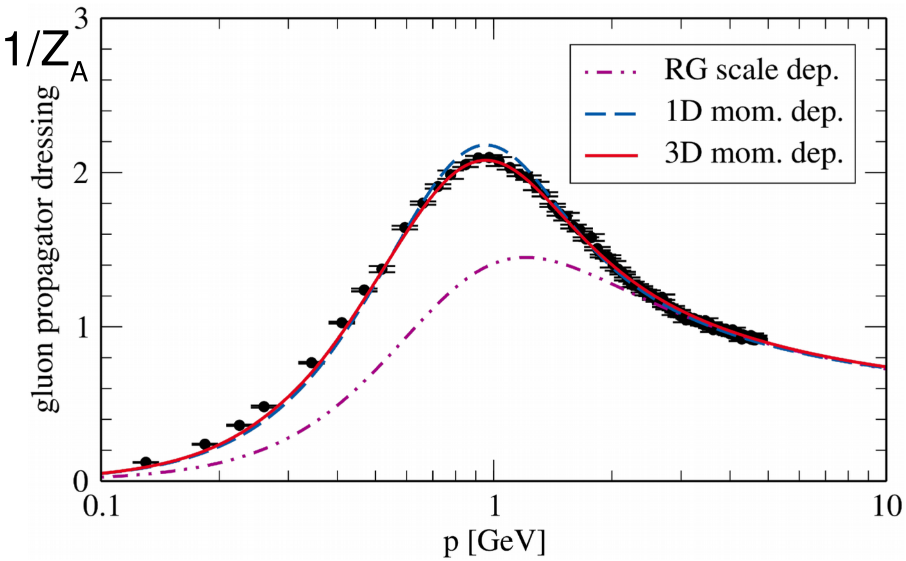
YM propagators (T=0)

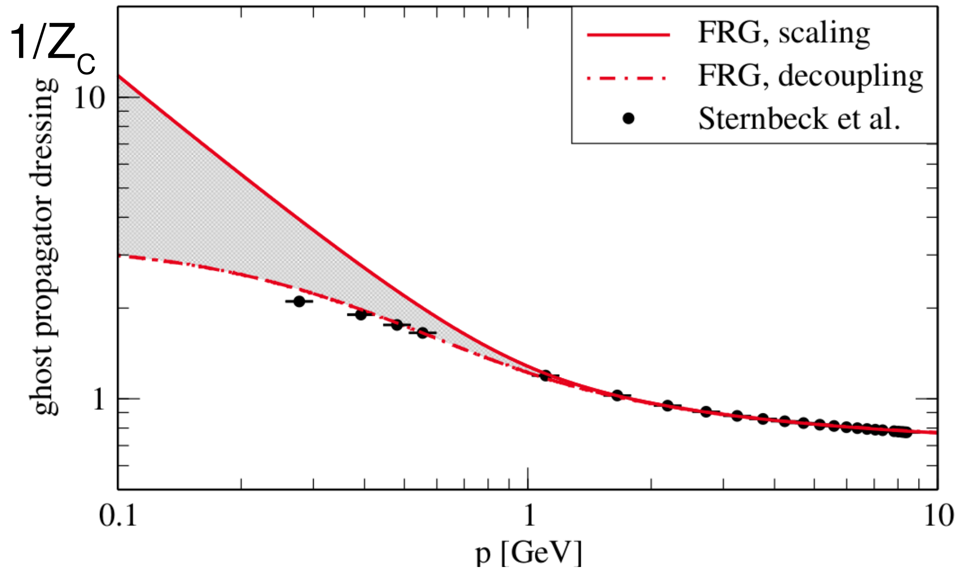
Self-consistent solution of the system of transversal 2-,3- and 4-point functions

- Cyrol, Fister, Mitter, Pawlowski, NSt Phys.Rev. **D94** (2016) no.5, 054005

$$[\Gamma^{(2)}_{A^2}]^{\mu\nu}_{ab}(p) = Z_A(p)p^2\Pi^{\mu\nu}_T(p)\delta_{ab}$$

$$[\Gamma^{(2)}_{\bar{c}c}]_{ab}(p) = Z_c(p)p^2\delta_{ab}$$





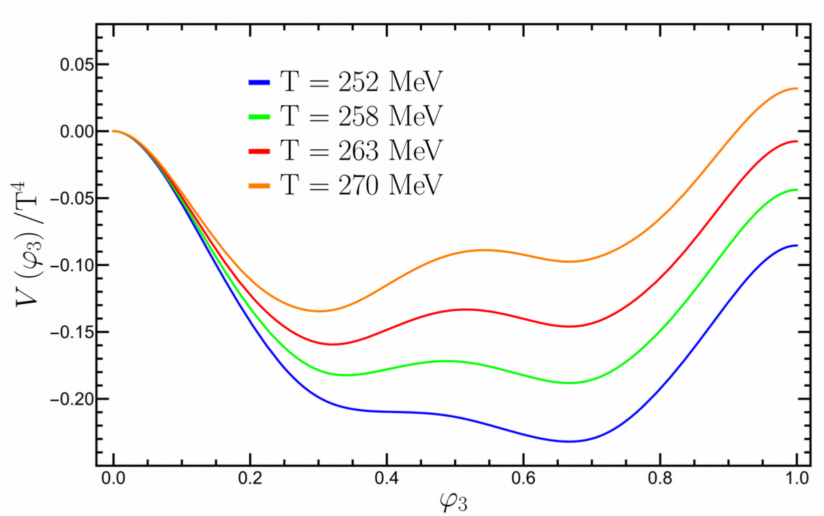
Finite T results in preparation
- Cyrol, Mitter, Pawlowski, NSt in prep

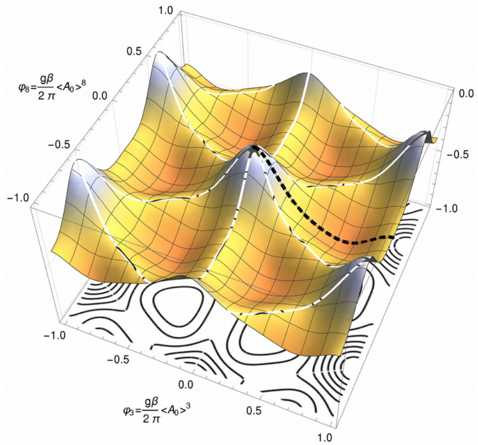
Confinement from correlation functions:

- Fister, Pawlowski, Phys.Rev. **D88** (2013) 045010
- Braun, Gies, Pawlowski, Phys.Lett. **B684** (2010) 262-267

Confinement

$V(\langle A_0 \rangle)$ from YM propagators





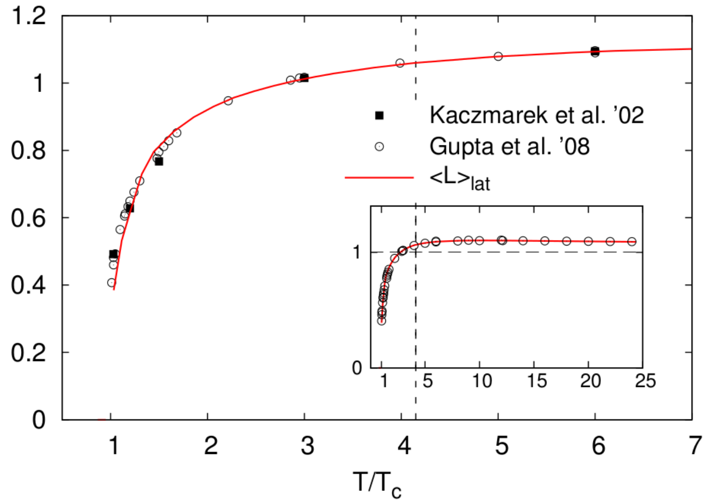
Confined:

$$\bar{\varphi}_3 = \frac{2}{3}$$

$$L(\bar{\varphi}_3, 0) = 0$$

- Pawlowski, Scherzer, Strodthoff, Wink in prep.
- Herbst, Luecker, Pawlowski, (2015), 1510.03830
- Fister, Pawlowski, Phys.Rev. D88 045010 (2013)
- Braun, Gies, Pawlowski, Phys.Lett. B684 (2010)

$\langle L(A_0) \rangle$ from $L(\langle A_0 \rangle)$



Order parameters:

$\bar{\varphi}_3$, $L(\langle A_0 \rangle)$ most easily computed in functional methods

$\langle L(A_0) \rangle$ computed on the lattice; now also in the FRG

$$\langle L(A_0) \rangle \leq L(\langle A_0 \rangle)$$

$$\langle L(A_0) \rangle = 0 \iff L(\langle A_0 \rangle) = 0$$

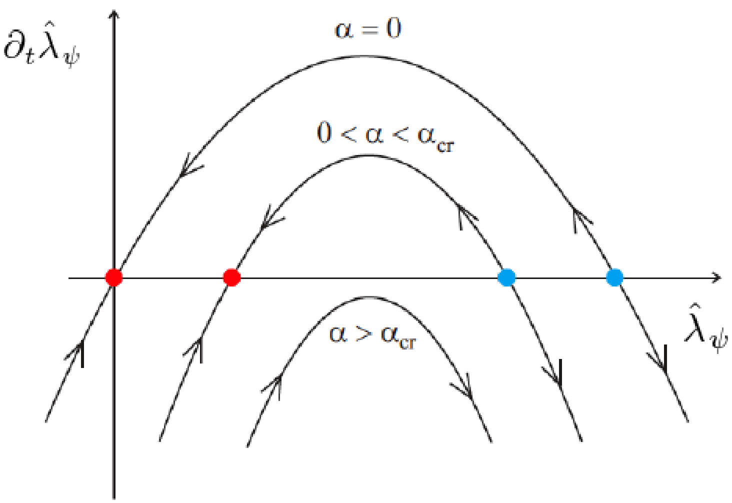
- Herbst, Luecker, Pawlowski, (2015), 1510.03830

Towards full QCD...

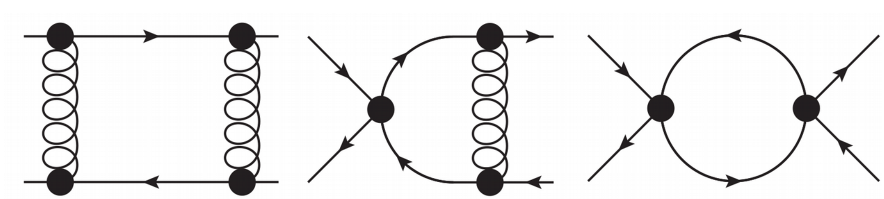
Chiral symmetry breaking

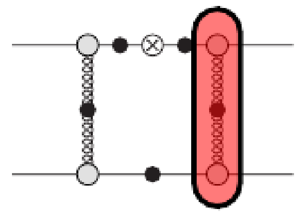
χSB <-> resonance in 4-quark interaction (pion pole)

β-function:



$$k\partial_k \hat{\lambda}_\psi = (d-2)\hat{\lambda}_\psi - a\hat{\lambda}_\psi^2 - b\hat{\lambda}_\psi g^2 - cg^4$$

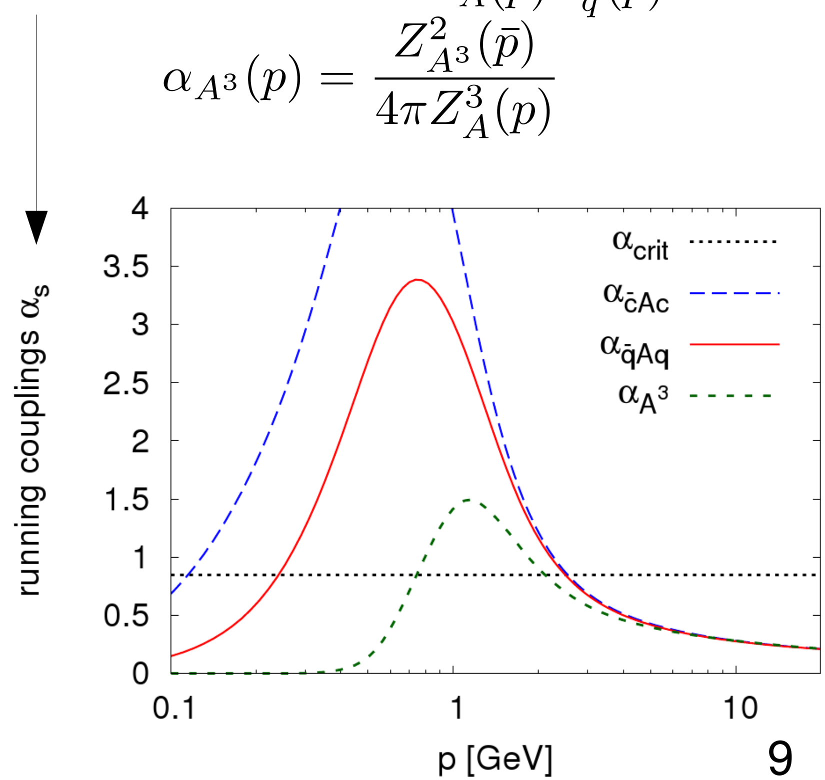




$$\alpha_{\bar{c}Ac}(p) = \frac{Z^2_{\bar{c}Ac}(\bar{p})}{4\pi Z_A(p) Z_c^2(p)}$$

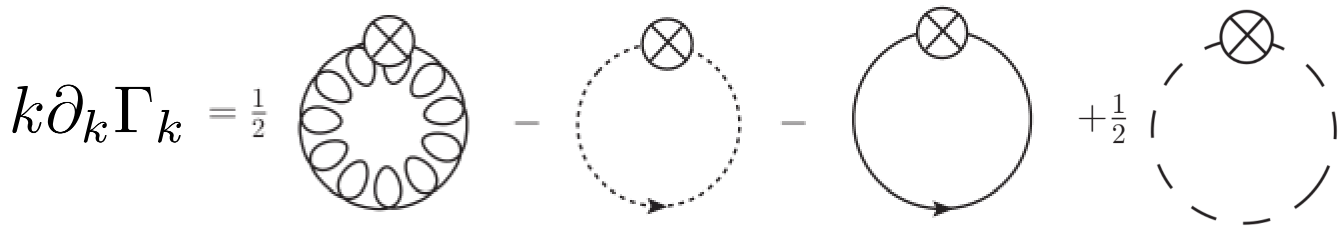
$$\alpha_{\bar{q}Aq}(p) = \frac{Z^2_{\bar{q}Aq}(\bar{p})}{4\pi Z_A(p) Z_q^2(p)}$$

$$\alpha_{A^3}(p) = \frac{Z^2_{A^3}(\bar{p})}{4\pi Z_A^3(p)}$$



- Review: Braun J.Phys. **G39** (2012) 033001

Dynamical Hadronization

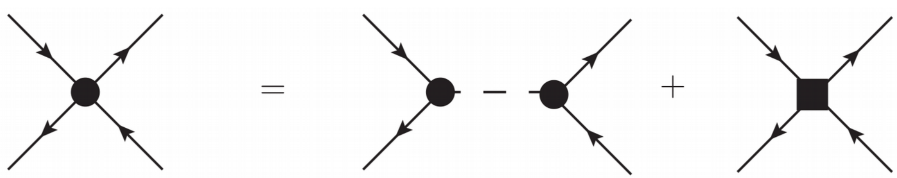


Free energy/ Grand potential | Gluon fluctuations | Ghost fluctuations | Quark fluctuations | Mesonic (σ,π) fluctuations

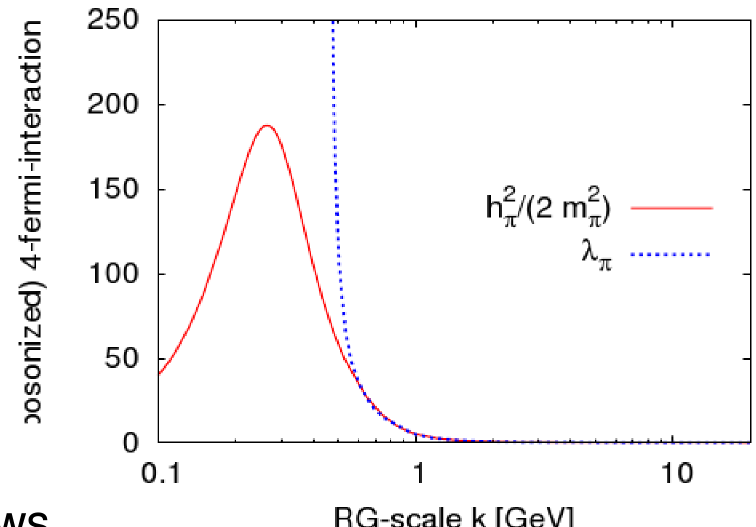
Efficient bookkeeping
no double counting

Dynamical hadronization
Store resonant 4-Fermi structures in terms of effective mesonic interactions

➢ Gies, Wetterich Phys.Rev. **D65** (2002) 065001



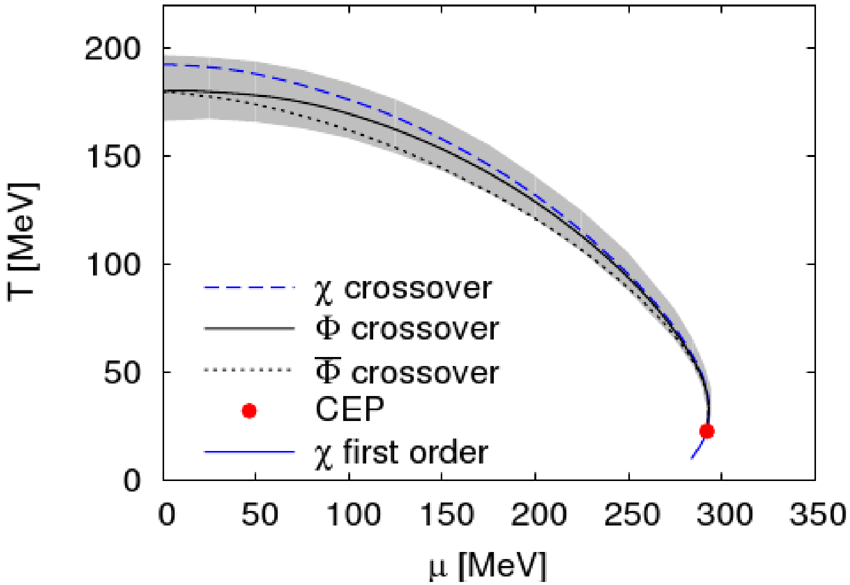
full 4-Fermi | res. 4-Fermi



✔ Effective models incorporated
- initial conditions determined by QCD-flows

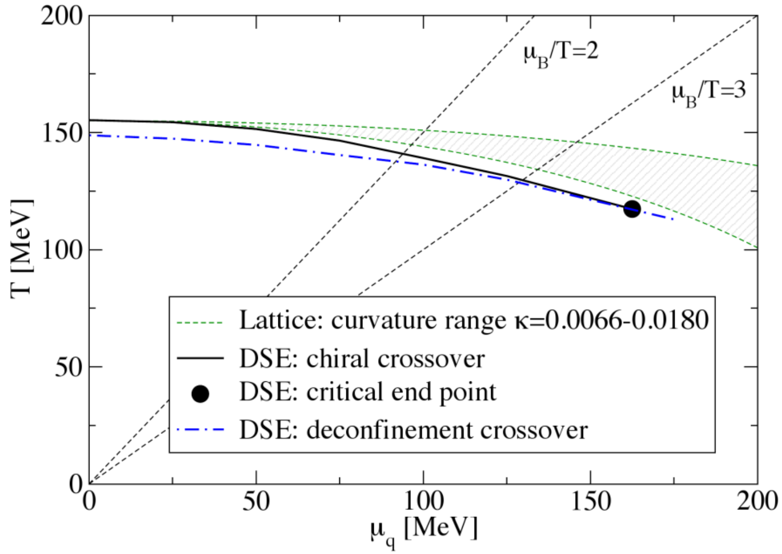
Functional methods at T,μ>0

PQM model, Nf=2, FRG



- Herbst, Pawlowski, Schaefer
  Phys.Lett. **B696** (2011)

Quark+Gluon propagator DSE, Nf=2+1



- Fischer, Luecker, Welzbacher
  Nucl.Phys. **A931** (2014) 774-779

But: so far all require additional phenomenological input

Aim: Quantitative framework for continuum QCD
fundamental parameters of QCD as only input parameters

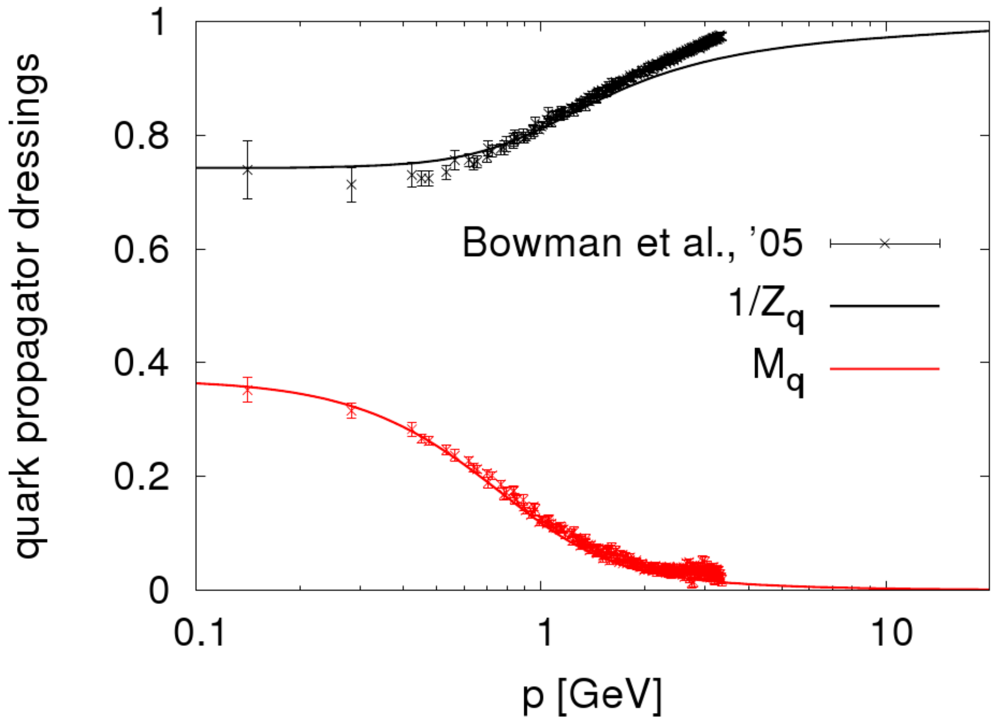
fQCD collaboration
J. Braun, L. Corell, A. K. Cyrol, L. Fister, W. J. Fu, M. Leonhardt, M. Mitter, J. M. Pawlowski, M. Pospiech, F. Rennecke, NSt, N. Wink

Quark propagator (T=0)

Quenched quark propagator

From the full matter system using quenched gluon propagator as only input

$$\Gamma_{\bar{q}q}(p) = Z_q(p)(\mathrm{i}\not{p} + M_q(p))$$



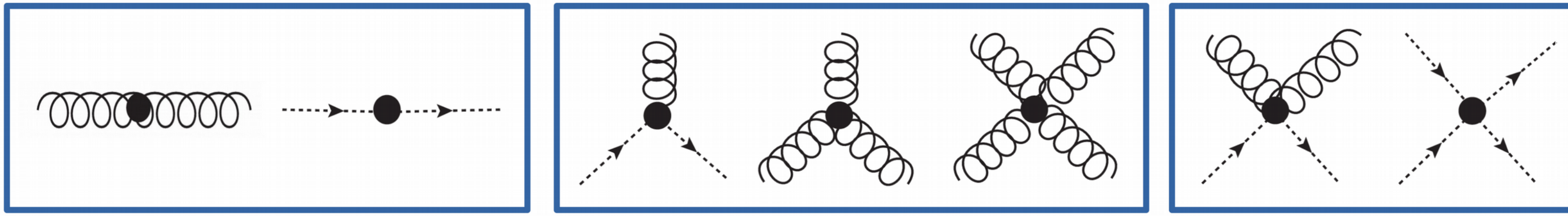
Very good agreement with (quenched) lattice results!

- Mitter, Pawlowski, NSt Phys.Rev. **D91** (2015) 054035

Truncation

Vertex expansion: systematic expansion in terms of 1PI vertices

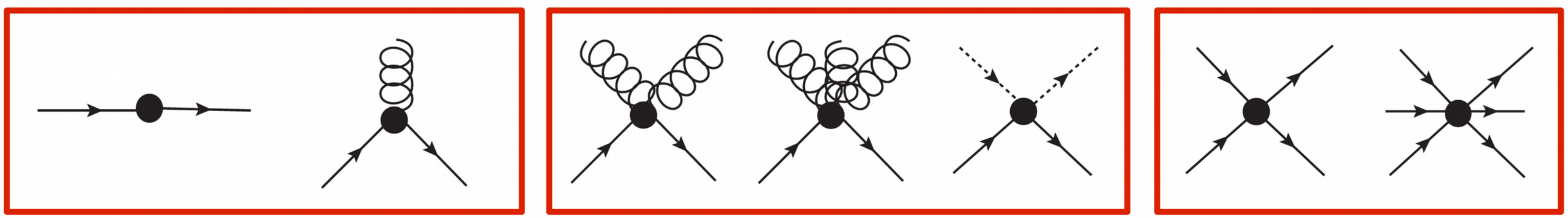
Perturbative relevance counting no longer valid



- full mom. dep.

- classical tensor structure
- mom. dep. (sym. channel)

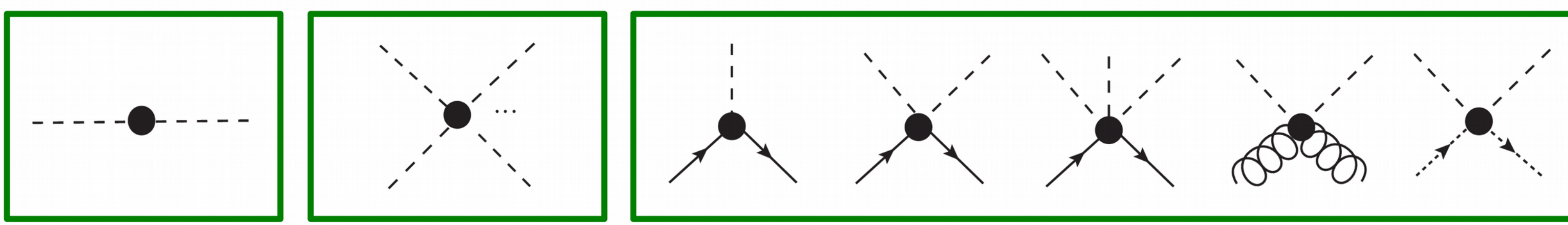
under investigation:
- full tensor structure
- mom. dep. (sym. channel)



- full tensor structure
- full mom. dep.

- partial tensor structure
- mom. dep. (sym. channel)

- full tensor structure
- mom. dep. (single channel)



- full mom. dep.

- via effective potential

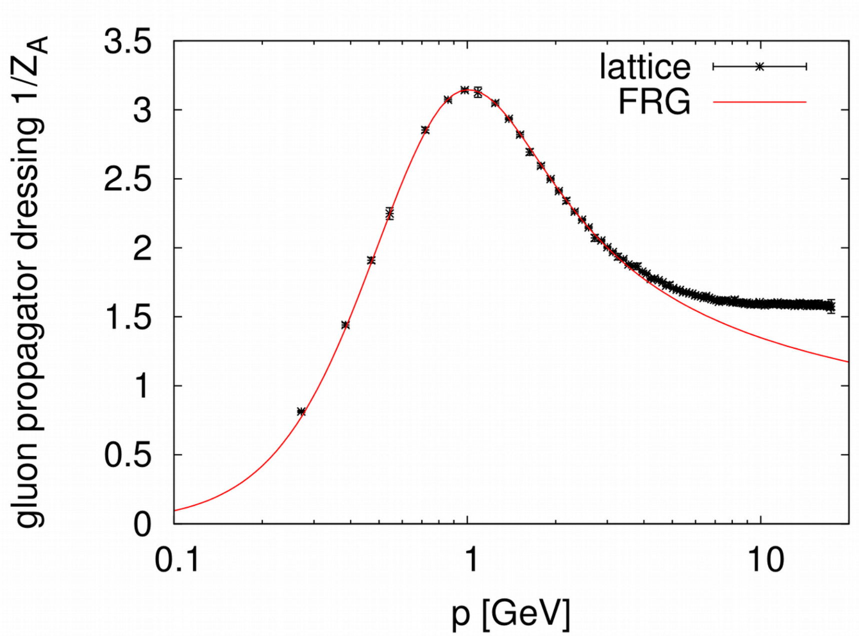
- full tensor structure
- mom. dep. (sym. channel)

Unquenching

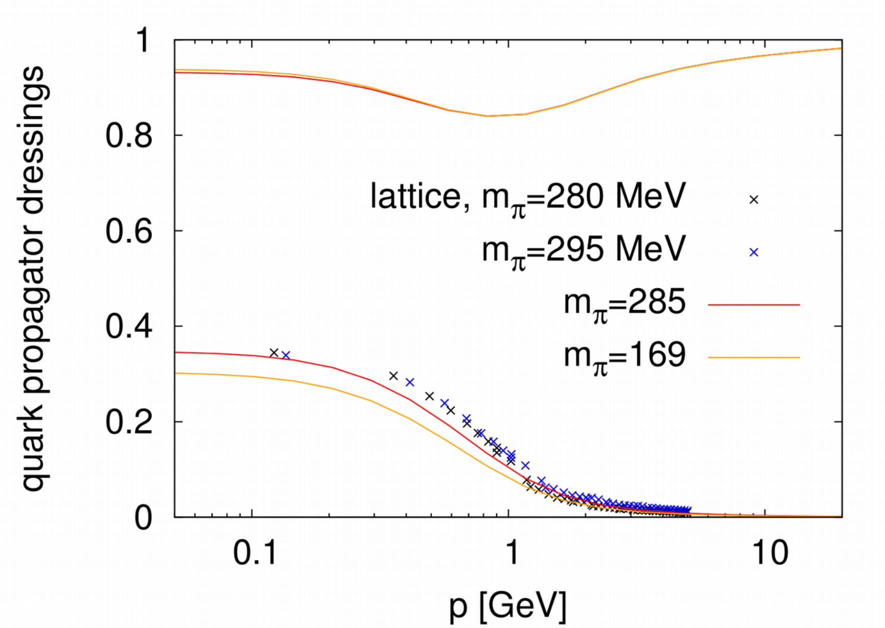
Unquenched gluon and quark propagators

Self-consistent, parameter-free solution of the coupled matter-glue system

- Cyrol, Mitter, Pawlowski, NSt in prep



- Lattice: Sternbeck et al
  PoS LATTICE2012, 243 (2012)

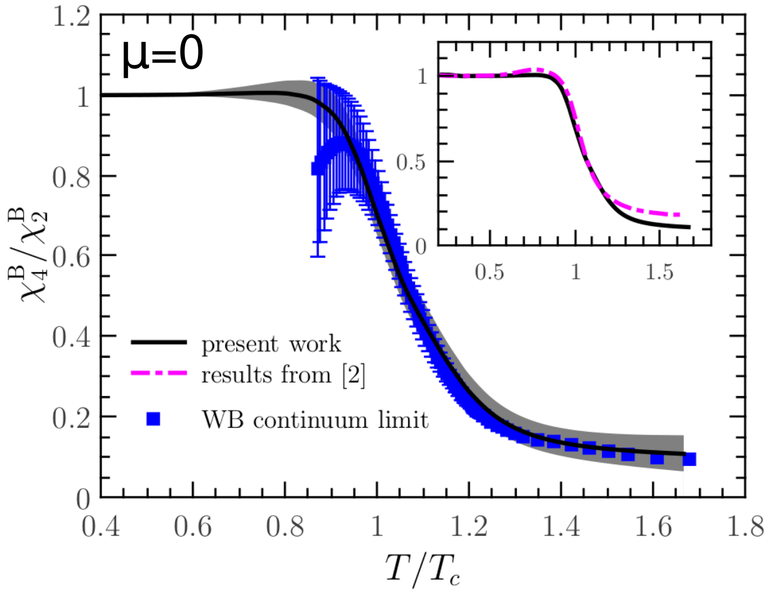


- Lattice: Oliveira et al 1605.09632

Summary

✓ Everything in place for first quantitative results of the full system at finite T and μ

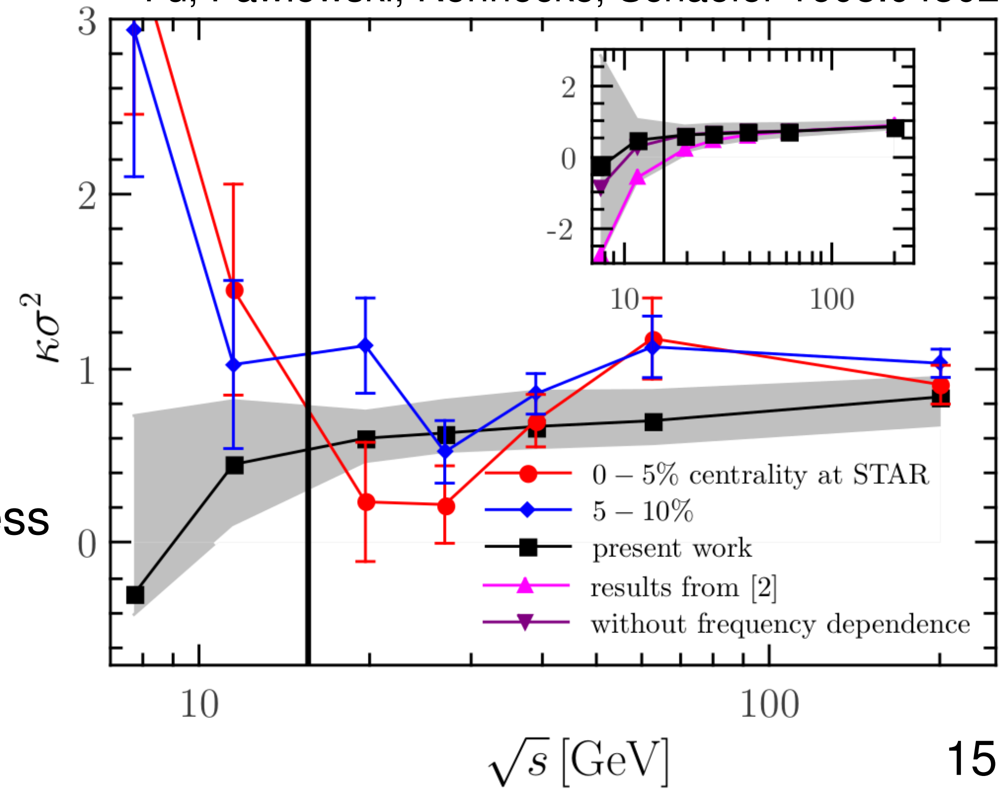
Fluctuation observables



- Fu, Pawlowski, Rennecke, Schaefer 1608.04302
- Fu, Pawlowski, Phys.Rev. **D93** (2016) no.9, 091501
- Fu, Pawlowski, Phys.Rev. **D92** (2015) no.11, 116006

$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4}$$

- Fu, Pawlowski, Rennecke, Schaefer 1608.04302



Nf=2 PQM model
(QCD-improved PL Pot;
PL fluctuations from lattice YM pot;
analytical frequency dep.)

- μ(√s) from freeze out curve; rescaled to Nf=2
- T_F from matching experimental skewness

→ kurtosis is prediction

Realtime observables

Spectral Functions

Real-time observables from Euclidean framework

$$\Gamma_R^{(2)}(\omega,\vec{p}) = -\lim_{\epsilon\to 0}\Gamma_E^{(2)}(-\mathrm{i}(\omega+\mathrm{i}\epsilon),\vec{p})$$

$$\rho(\omega,\vec{p}) = \frac{\mathrm{Im}\,\Gamma_R^{(2)}(\omega,\vec{p})}{\mathrm{Im}\,\Gamma_R^{(2)}(\omega,\vec{p})^2+\mathrm{Re}\,\Gamma_R^{(2)}(\omega,\vec{p})^2}$$

requires analytical continuation from Euclidean to Minkowski signature, a numerically hard problem

Popular approaches (based on Euclidean data)

- Maximum Entropy Method (MEM)
- Padé Approximants

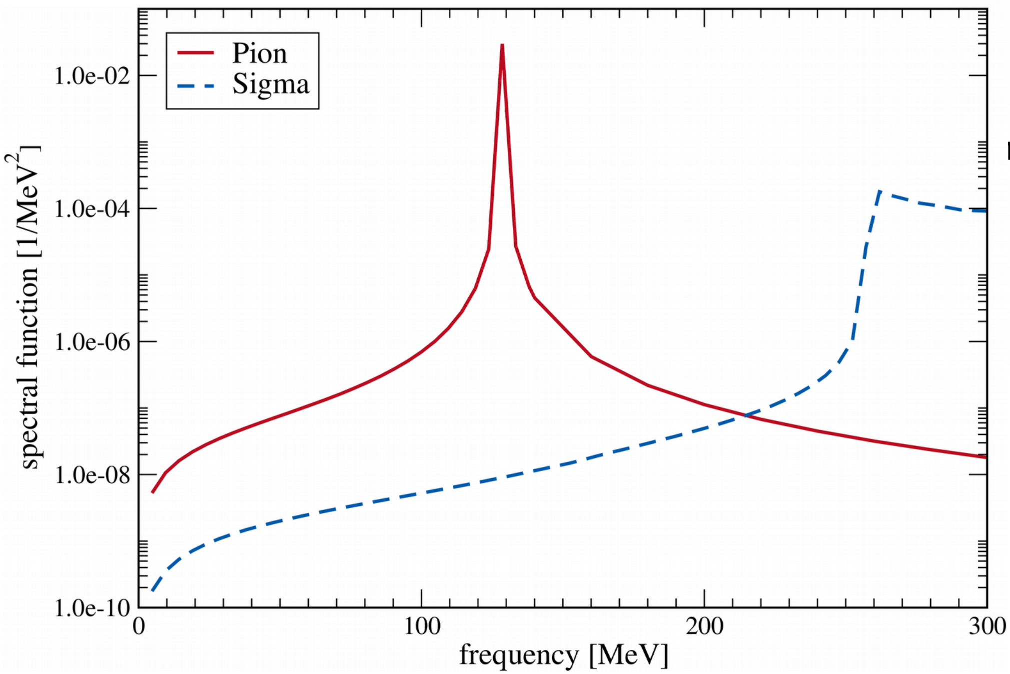
Alternative: analytic continuation on the level of the functional equation

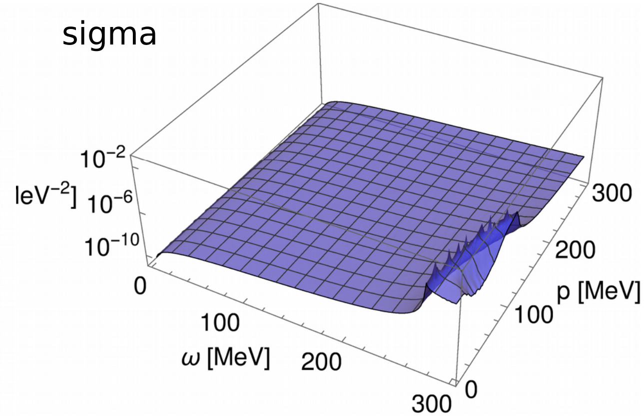
- Kamikado, NSt, von Smekal, Wambach Eur.Phys.J. **C74** (2014) 2806
- Floerchinger JHEP 1205 (2012) 021
- Strauss, Fischer, Kellermann PRL **109** (2012) 252001

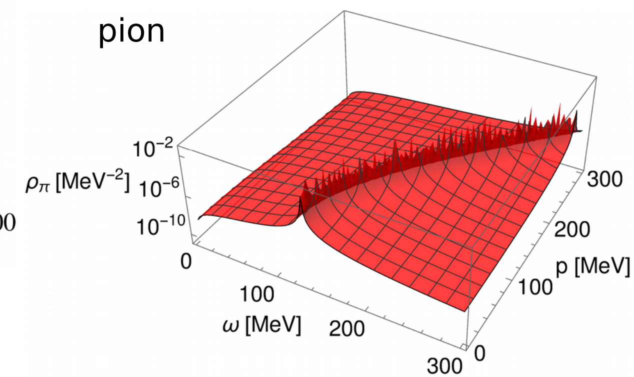
Here Minkowski external momenta appear as external parameters

Spectral Functions

O(N) at T=0, full momentum dependence







- NSt, in prep

Summary

- ✔ Directly calculated spectral functions
- ✔ Tested in scalar and Yukawa models at T,μ>0
- ✔ Allows the inclusion of full momentum dependence

❑ Quark & gluon spectral functions in full QCD

- Pawlowski, NSt
  Phys.Rev. **D92** (2015) 9, 094009
- Tripolt, NSt, von Smekal, Wambach
  Phys.Rev. **D89** (2014) 034010
- Kamikado, NSt, von Smekal, Wambach
  Eur.Phys.J. **C74** (2014) 2806

Transport Coefficients

Kubo formula for the shear viscosity

$$\eta = \lim_{\omega\to 0} \frac{1}{20} \frac{\rho_{\pi\pi}(\omega,\vec{0})}{\omega}$$

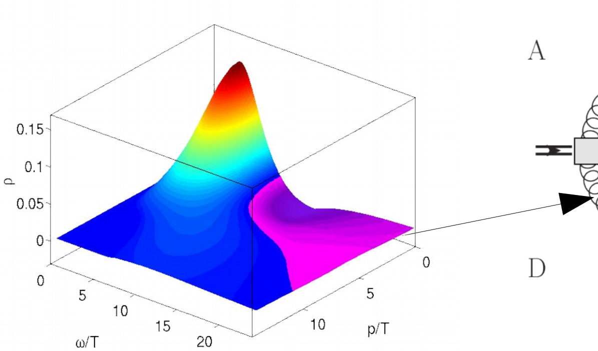
Require $\rho_{\pi\pi}(\omega,\vec{p}) = \int_x e^{-\mathrm{i}\omega x_0 + \mathrm{i}\vec{p}\vec{x}} \langle [\pi_{ij}(x), \pi_{ij}(0)] \rangle$

Expansion formula

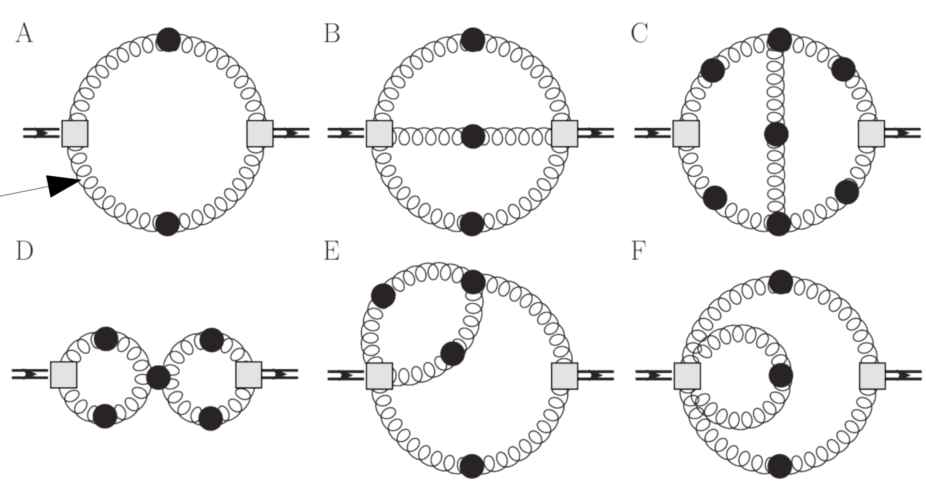
- Pawlowski Annals Phys. 322 (2007) 2831-2915

$$\langle \pi_{ij}[\hat{A}]\pi_{ij}[\hat{A}] \rangle = \pi_{ij}[G_{A\phi_k}\frac{\delta}{\delta\phi_k} + A]\pi_{ij}[G_{A\phi_k}\frac{\delta}{\delta\phi_k} + A]$$

Finite number of diagrams involving **full** propagators/vertices

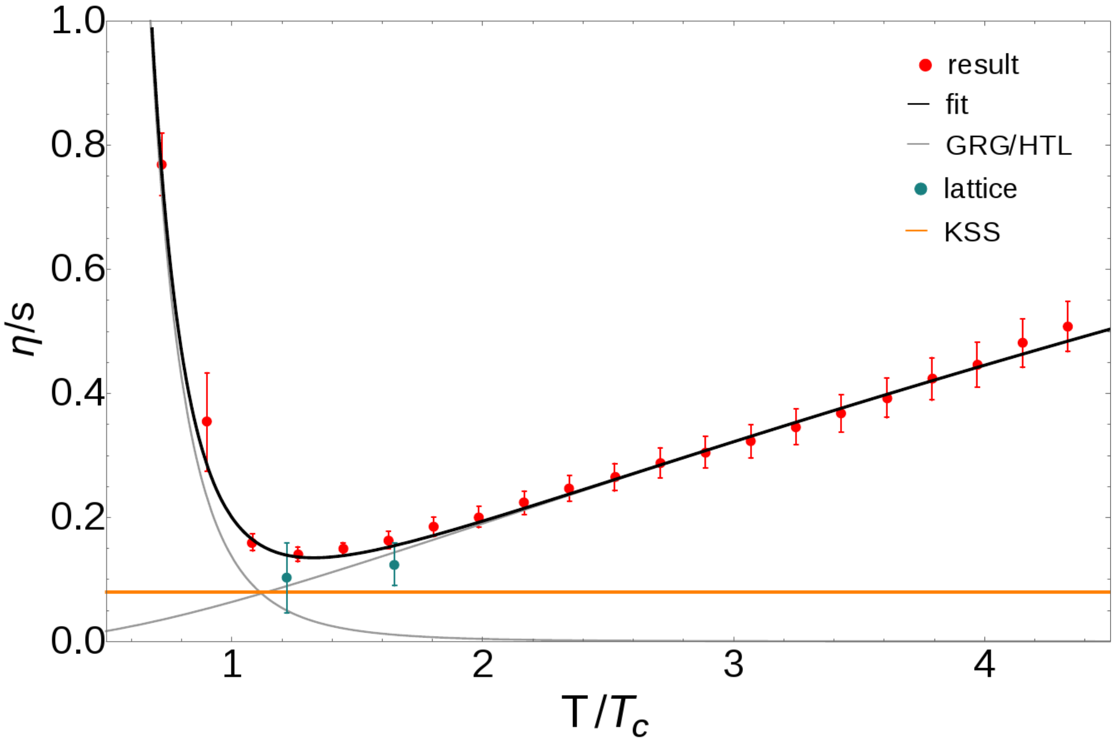


- Haas, Fister, Pawlowski Phys. Rev. D90 091501 (2014)



All diagrams to 2-loop order

η/s in Yang-Mills Theory



T_{min}=1.26 T_C
Value=0.14

- Christiansen, Haas, Pawlowski, NSt PRL **115** (2015) 11, 112002

Direct sum: $$\frac{\eta}{s}(T) = \frac{a}{\alpha_s(cT/T_c)^\gamma} + \frac{b}{(T/T_c)^\delta}$$

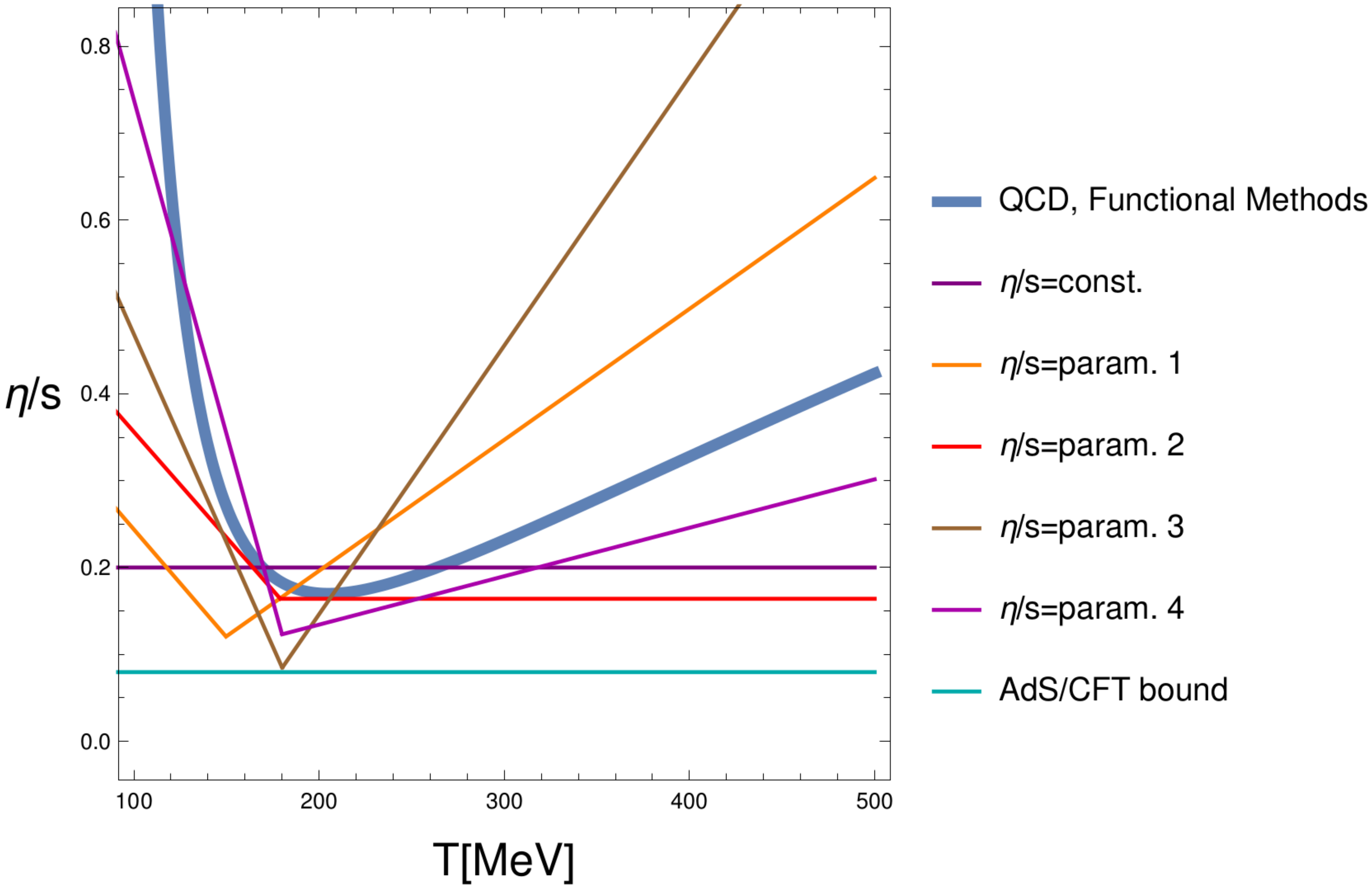
$$\gamma = 1.6 \quad a = 0.15 \quad b = 0.14 \quad c = 0.66 \quad \delta = 5.1$$

High T: **consistent with HTL-resummed pert. theory** (fixing γ) supporting quasiparticle picture

Small T: algebraic decay **glueball resonance gas**

Estimate: η/s in QCD

- c.f. Niemi, Eskola, Paatelainen 1505.02677



Summary

- **YM theory & QCD phase structure**

  towards a quantitative continuum approach to QCD
  - ✔ Quantitative grip on fluctuation physics in the vacuum
  - ❑ Finite temperature and density
  - ❑ Nuclear matter: nuclear binding energy...

- **Elementary spectral functions**

  new approach to analytical continuation problem
  - ✔ tested in low energy eff. models (O(N), QM model)
  - ❑ quark & gluon, vector meson, ...

- **Transport Coefficients**

  from loop expansion involving full propagators and vertices
  - ✔ Global quantitative prediction for η/s in YM theory
  - ❑ full QCD, bulk viscosity, relaxation times

Thank you for your attention!