QCD thermodynamics from fluctuations of conserved charges on the lattice

Sayantan Sharma

BROOKHAVEN NATIONAL LABORATORY

October 7, 2016

Bielefeld-BNL-CCNU collaboration

A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, S. Mukherjee, H. Ohno, P. Petreczky, H. Sandmeyer, C. Schmidt, S. Sharma, W. Soeldner, P. Steinbrecher, M. Wagner

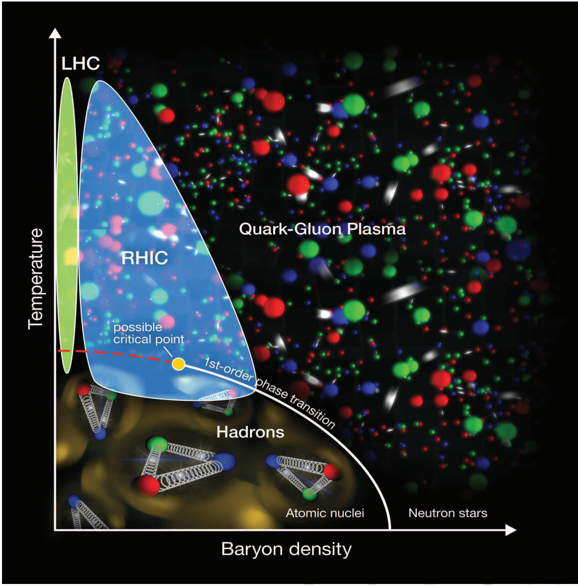
Outline

1. The QCD phase diagram: outstanding issues on lattice
2. Fluctuations and Equation of state at finite μ_B
3. QCD medium for $T > T_c$ from fluctuations

Outline

1. The QCD phase diagram: outstanding issues on lattice
2. Fluctuations and Equation of state at finite μ_B
3. QCD medium for $T > T_c$ from fluctuations

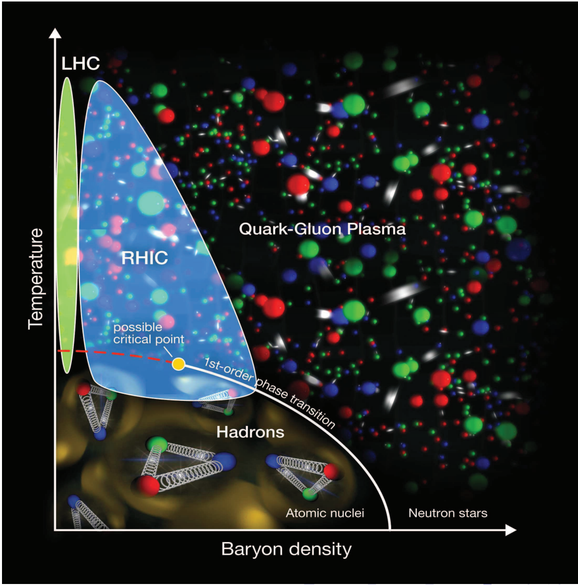
QCD phase diagram: some outstanding issues



QCD phase diagram: some outstanding issues

- Existence of critical end-point in thermal QCD?

  See Frithjof Karsch's talk

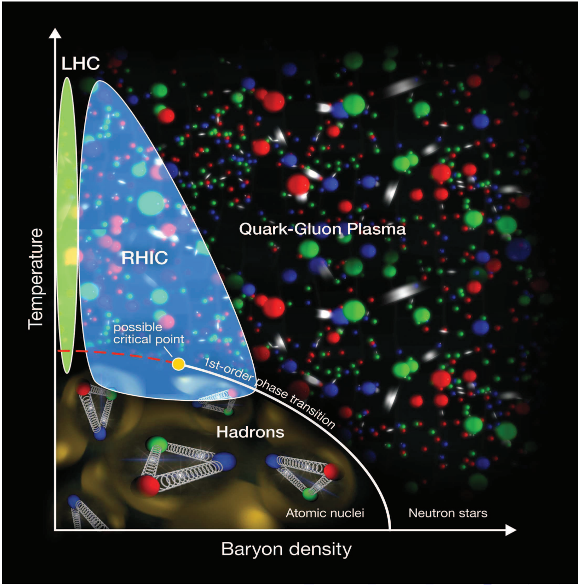


QCD phase diagram: some outstanding issues

- Existence of critical end-point in thermal QCD?

  See Frithjof Karsch's talk

- Can we observe the criticality at $\mu_B = 0$?



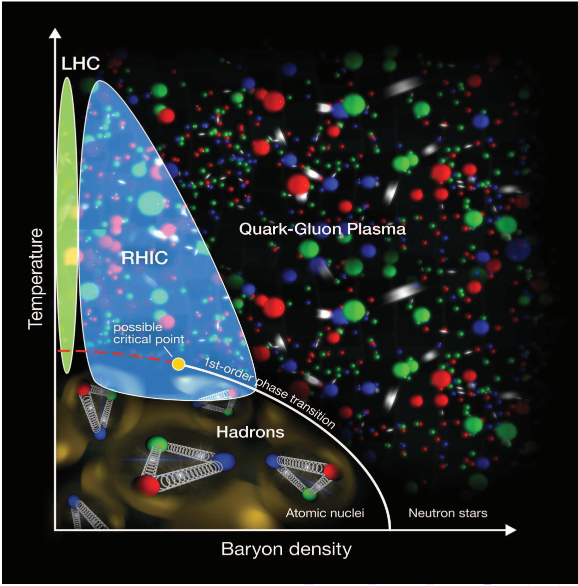
QCD phase diagram: some outstanding issues

- Existence of critical end-point in thermal QCD?

  See Frithjof Karsch's talk

- Can we observe the criticality at $\mu_B = 0$?
- Calculate χ in thermal QCD $\rightarrow$ dynamical modeling in rapidity.

  See Masakiyo Kitazawa's talk



QCD phase diagram: some outstanding issues

- Existence of critical end-point in thermal QCD?

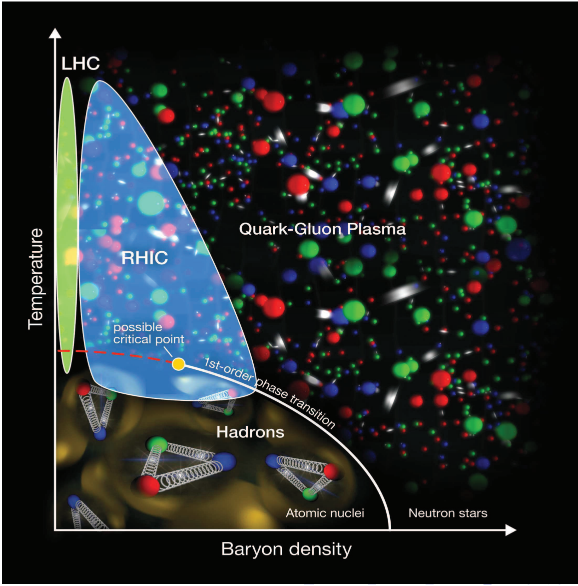
  See Frithjof Karsch's talk

- Can we observe the criticality at $\mu_B = 0$?
- Calculate χ in thermal QCD $\rightarrow$ dynamical modeling in rapidity.

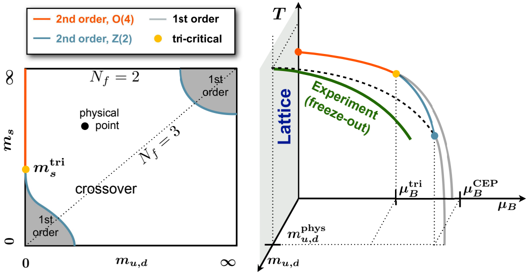
  See Masakiyo Kitazawa's talk

- How to realistically characterize fireball created in Heavy-Ion collisions? hydrodynamic evolution $\rightarrow$ freezeout surface, EoS

[courtesy: www.bnl.gov/news]

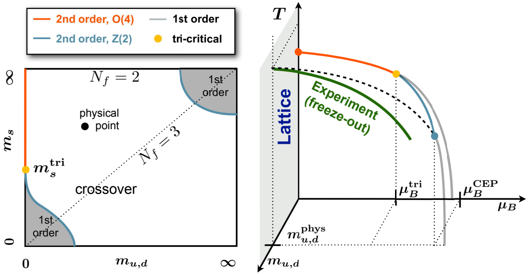


Status of Phase diagram



- Need to understand both critical properties and

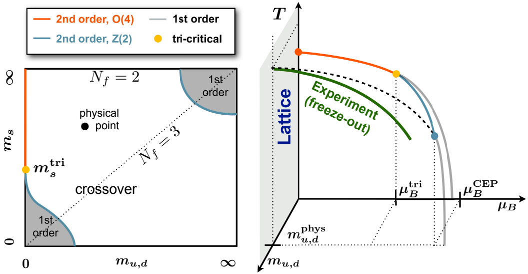
Status of Phase diagram



- Need to understand both critical properties and
- Understand microscopic degrees of freedom

  Example: $-3\frac{\chi_{11}^{BS}}{\chi_2^S}$ [Koch, Majumder, Randrup, 05]

Status of Phase diagram



- Need to understand both critical properties and
- Understand microscopic degrees of freedom

  Example: $-3\frac{\chi_{11}^{BS}}{\chi_2^S}$ [Koch, Majumder, Randrup, 05]
- Both LHC and RHIC experiments would enable to explore different parts of the phase diagram.

Basic observables on the lattice

- One of the methods to circumvent sign problem at finite μ:
  Taylor expansion of physical observables around $\mu = 0$ in powers of μ/T [Bielefeld, Swansea collaboration, 02]

$$\frac{P(\mu_B, T)}{T^4} = \frac{P(0, T)}{T^4} + \frac{1}{2}\left(\frac{\mu_B}{T}\right)^2 \chi_2^B(0, T) + \frac{1}{4!}\left(\frac{\mu_B}{T}\right)^4 \chi_4^B(0) + \dots$$

Basic observables on the lattice

- One of the methods to circumvent sign problem at finite μ:
  Taylor expansion of physical observables around $\mu = 0$ in powers of μ/T [Bielefeld, Swansea collaboration, 02]

$$\frac{P(\mu_B, T)}{T^4} = \frac{P(0,T)}{T^4} + \frac{1}{2}\left(\frac{\mu_B}{T}\right)^2 \chi_2^B(0,T) + \frac{1}{4!}\left(\frac{\mu_B}{T}\right)^4 \chi_4^B(0) + \ldots$$

- The series for χ_2^B should diverge at the critical point. On finite lattice χ_2^B peaks, ratios of Taylor coefficients equal, indep. of volume
  [Gavai& Gupta, 03]

Basic observables on the lattice

- One of the methods to circumvent sign problem at finite μ: Taylor expansion of physical observables around $\mu = 0$ in powers of μ/T [Bielefeld, Swansea collaboration, 02]

$$\frac{P(\mu_B, T)}{T^4} = \frac{P(0, T)}{T^4} + \frac{1}{2}\left(\frac{\mu_B}{T}\right)^2 \chi_2^B(0, T) + \frac{1}{4!}\left(\frac{\mu_B}{T}\right)^4 \chi_4^B(0) + \ldots$$

- The series for χ_2^B should diverge at the critical point. On finite lattice χ_2^B peaks, ratios of Taylor coefficients equal, indep. of volume [Gavai& Gupta, 03]
- Current state of the art:
  χ_8^B for $N_\tau = 8$ pure staggered fermions[Gavai& Gupta, 08].
  χ_6^B for $N_\tau = 6, 8, 12, 16$ HISQ fermions
  [BNL-Bielefeld-CCNU, HotQCD, Budapest-Wuppertal collaborations, 16].

Challenges for Lattice computations

- The Baryon no./Charge fluctuations $\rightarrow$ expressed in terms of Quark no. susceptibilities.

Challenges for Lattice computations

- The Baryon no./Charge fluctuations $\rightarrow$ expressed in terms of Quark no. susceptibilities.
- QNS χ_{ij}'s can be written as derivatives of the Dirac operator. Example :$\chi_2^u = \frac{T}{V}\langle Tr(D_u^{-1}D_u'' - (D_u^{-1}D_u')^2) + (Tr(D_u^{-1}D_u'))^2\rangle$.
  $\chi_{11}^{us} = \frac{T}{V}\langle Tr(D_u^{-1}D_u' D_s^{-1}D_s')\rangle$.

Challenges for Lattice computations

- The Baryon no./Charge fluctuations $\rightarrow$ expressed in terms of Quark no. susceptibilities.
- QNS χ_{ij}'s can be written as derivatives of the Dirac operator. Example :$\chi_2^u = \frac{T}{V}\langle Tr(D_u^{-1} D_u^{''} - (D_u^{-1} D_u^{'})^2) + (Tr(D_u^{-1} D_u^{'}))^2 \rangle$.
  $\chi_{11}^{us} = \frac{T}{V}\langle Tr(D_u^{-1} D_u^{'} D_s^{-1} D_s^{'})\rangle$.
- Higher derivatives $\rightarrow$ more inversions
  Inversion is the most expensive step on the lattice !

Challenges for Lattice computations

- The Baryon no./Charge fluctuations $\rightarrow$ expressed in terms of Quark no. susceptibilities.
- QNS χ_{ij}'s can be written as derivatives of the Dirac operator.
  Example :$\chi_2^u = \frac{T}{V}\langle Tr(D_u^{-1} D_u^{''} - (D_u^{-1} D_u^{'})^2) + (Tr(D_u^{-1} D_u^{'}))^2\rangle$.
  $\chi_{11}^{us} = \frac{T}{V}\langle Tr(D_u^{-1} D_u^{'} D_s^{-1} D_s^{'})\rangle$.
- Higher derivatives $\rightarrow$ more inversions
  Inversion is the most expensive step on the lattice !
- Extending to higher orders?

Challenges for Lattice computations

- The Baryon no./Charge fluctuations $\rightarrow$ expressed in terms of Quark no. susceptibilities.
- QNS χ_{ij}'s can be written as derivatives of the Dirac operator.
  Example :$\chi_2^u = \frac{T}{V}\langle Tr(D_u^{-1}D_u^{''} - (D_u^{-1}D_u^{'})^2) + (Tr(D_u^{-1}D_u^{'}))^2\rangle$.
  $\chi_{11}^{us} = \frac{T}{V}\langle Tr(D_u^{-1}D_u^{'}D_s^{-1}D_s^{'})\rangle$.
- Higher derivatives $\rightarrow$ more inversions
  Inversion is the most expensive step on the lattice !
- Extending to higher orders?
  - Matrix inversions increasing with the order

Challenges for Lattice computations

- The Baryon no./Charge fluctuations $\rightarrow$ expressed in terms of Quark no. susceptibilities.
- QNS χ_{ij}'s can be written as derivatives of the Dirac operator.
  Example :$\chi_2^u = \frac{T}{V}\langle Tr(D_u^{-1}D_u^{''} - (D_u^{-1}D_u^{'})^2) + (Tr(D_u^{-1}D_u^{'}))^2\rangle$.
  $\chi_{11}^{us} = \frac{T}{V}\langle Tr(D_u^{-1}D_u^{'}D_s^{-1}D_s^{'})\rangle$.
- Higher derivatives $\rightarrow$ more inversions
  Inversion is the most expensive step on the lattice !
- Extending to higher orders?
  - Matrix inversions increasing with the order
  - Delicate cancellation between a large number of terms for higher order QNS.

A new method to introduce μ

- The staggered fermion matrix used at finite μ [Hasenfratz, Karsch ,83]

$$
\begin{aligned}
D(\mu)_{xy} &= \sum_{i=1}^{3} \eta_i(x) \left[U_i^\dagger(y)\delta_{x,y+\hat{i}} - U_i(x)\delta_{x,y-\hat{i}} \right] \\
&+ \eta_4(x) \left[\mathrm{e}^{\mu \mathrm{a}} U_4^\dagger(y)\delta_{x,y+\hat{4}} - \mathrm{e}^{-\mu \mathrm{a}} U_4(x)\delta_{x,y-\hat{4}} \right]
\end{aligned}
$$

A new method to introduce μ

- The staggered fermion matrix used at finite μ [Hasenfratz, Karsch ,83]

$$
\begin{aligned}
D(\mu)_{xy} &= \sum_{i=1}^{3} \eta_i(x) \left[U_i^\dagger(y)\delta_{x,y+\hat{i}} - U_i(x)\delta_{x,y-\hat{i}} \right] \\
&+ \eta_4(x) \left[\mathrm{e}^{\mu a} U_4^\dagger(y)\delta_{x,y+\hat{4}} - \mathrm{e}^{-\mu a} U_4(x)\delta_{x,y-\hat{4}} \right]
\end{aligned}
$$

- One can also add μ coupled to the conserved number density as in the continuum.

$$
D(0)_{xy} - \frac{\mu a}{2} \eta_4(x) \left[U_4^\dagger(y)\delta_{x,y+\hat{4}} + U_4(x)\delta_{x,y-\hat{4}} \right] .
$$

Pros and Cons

- Linear method: $D' = \sum_{x,y} N(x, y)$, and
  $D'' = D''' = D''''... = 0$

Pros and Cons

- Linear method: $D' = \sum_{x,y} N(x,y)$, and
  $D'' = D''' = D''''... = 0$
- No. of inversions significantly reduced for higher orders in linear method.
  For 8th order QNS the no. of matrix inversions reduced from 20 to 8.
  [Gavai & Sharma, 12]

Pros and Cons

- Linear method: $D' = \sum_{x,y} N(x,y)$, and
  $D'' = D''' = D''''... = 0$
- No. of inversions significantly reduced for higher orders in linear method. For 8th order QNS the no. of matrix inversions reduced from 20 to 8.
  [Gavai & Sharma, 12]
- Linear method: χ_n have additional zero-T artifacts. $\rightarrow$ explicit counter terms needed for $\chi_{2,4}$, [Gavai & Sharma, 15]

Pros and Cons

- Linear method: $D' = \sum_{x,y} N(x,y)$, and
  $D'' = D''' = D'''' ... = 0$
- No. of inversions significantly reduced for higher orders in linear method. For 8th order QNS the no. of matrix inversions reduced from 20 to 8.
  [Gavai & Sharma, 12]
- Linear method: χ_n have additional zero-T artifacts. $\rightarrow$ explicit counter terms needed for $\chi_{2,4}$, [Gavai & Sharma, 15]
- Good news!
  The artifacts $\sim \mathcal{O}(a^{n-4}) \rightarrow$ irrelevant for $n \geq 6$.

Pros and Cons

- Linear method: $D' = \sum_{x,y} N(x,y)$, and
  $D'' = D''' = D''''... = 0$
- No. of inversions significantly reduced for higher orders in linear method. For 8th order QNS the no. of matrix inversions reduced from 20 to 8.
  [Gavai & Sharma, 12]
- Linear method: χ_n have additional zero-T artifacts. $\rightarrow$ explicit counter terms needed for $\chi_{2,4}$, [Gavai & Sharma, 15]
- Good news!
  The artifacts $\sim \mathcal{O}(a^{n-4}) \rightarrow$ irrelevant for $n \geq 6$.
- In Exp method: counter terms already at the Lagrangian level. We use this method for χ_n^B, $n = 2, 4$.

Recent developments in algorithms

- Calculating explicitly the lowest eigenvalues improves performance of the fermion inverter
  $D^{-1}|R\rangle = \sum_{i=1}^{N} 1/\lambda_i|\psi_i\rangle\langle\psi_i|R\rangle$+Explicit Inversion of Dirac Operator.

- Efficient codes based on modern computer architectures are being developed. [O. Kaczmarek, C. Schmidt, P. Steinbrecher, M. Wagner, 14]

Outline

1. The QCD phase diagram: outstanding issues on lattice
2. Fluctuations and Equation of state at finite μ_B
3. QCD medium for $T > T_c$ from fluctuations

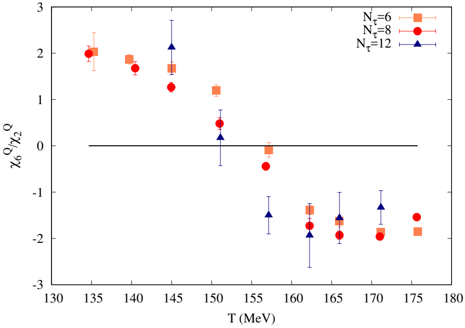
Criticality at $\mu_B = 0$

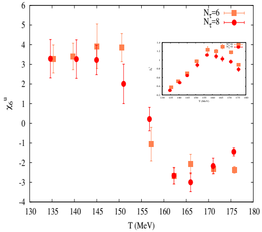
- $m_u, m_d << \Lambda_{QCD}$
- $SU_L(2) \times SU_R(2) \times U_B(1) \times U_A(1)$ is fairly good symmetry
- For $N_f = 2$ light flavors:
  2nd order phase transition at $\mu = 0$ with $O(4)$ critical exponent if $U_A(1)$ is not effectively restored at T_c. [Pisarski& Wilczek, 83].
- Growing evidence for $O(4)$ scaling from lattice studies of scaling of chiral condensate and Dirac eigenvalue spectrum.
  [BNL-Bielefeld Collaboration, 09,10, BNL-Columbia-LLNL, 13, H. Ohno et. al, 12, V. Dick et. al, 15].
- Verified also in QCD inspired models. [G. Almasi et. al, 16].
- Effects should be visible in higher order fluctuations measured in LHC
  [Friman, Karsch & Redlich, 11].

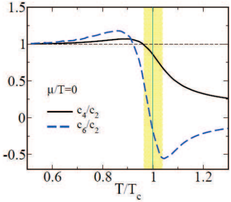
Sixth order quark number fluctuations

- For the first time observe negative dip in χ_6^Q just above T_c $\rightarrow$ signal of $O(4)$ criticality?

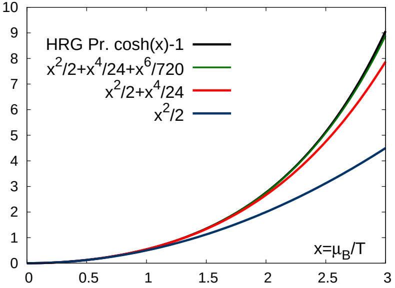
[bottom inset plot from Friman, Karsch & Redlich, 11]

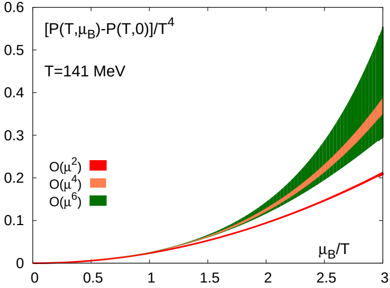






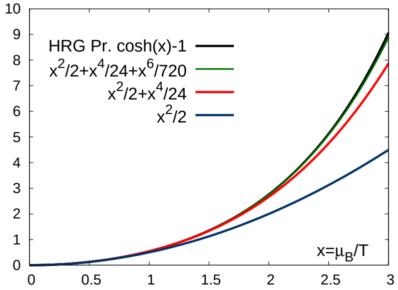
- In a regime where Hadron Resonance gas is anticipated to be a good description of QCD, including χ_6^B term already reproduces $P(\mu_B)$ within 5% accuracy.

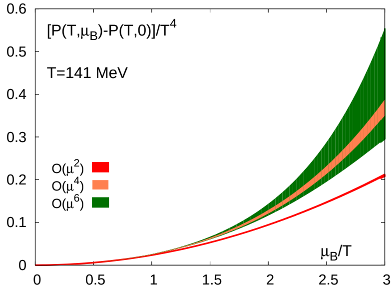




Constraining EoS for $\mu_S = \mu_Q = 0$

- In a regime where Hadron Resonance gas is anticipated to be a good description of QCD, including χ_6^B term already reproduces $P(\mu_B)$ within 5% accuracy.
- We need to improve the errors on χ_6^B $\rightarrow$ work in progress.





EoS at finite μ_B for $\mu_S = \mu_Q = 0$

- Pressure for $T > 160$ MeV already constrained by χ_B^6 for $\mu_B/T \leq 2 \rightarrow$ input for hydrodynamic modeling of QGP at finite μ.

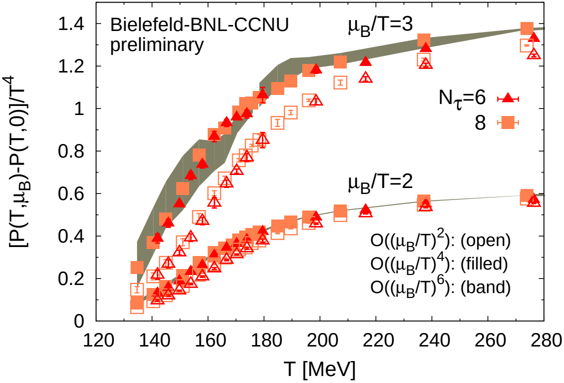
  [A. Jaiswal, B. Friman, K. Redlich, 15; A. Monnai and B. Schenke, 15]

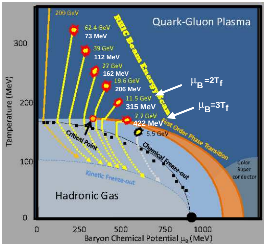
EoS at finite μ_B for $\mu_S = \mu_Q = 0$

- Pressure for $T > 160$ MeV already constrained by χ_B^6 for $\mu_B/T \leq 2 \rightarrow$ input for hydrodynamic modeling of QGP at finite μ.

  [A. Jaiswal, B. Friman, K. Redlich, 15; A. Monnai and B. Schenke, 15]

- Extension to $\mu_B/T \sim 3$ is in progress.





EoS for strangeness neutrality

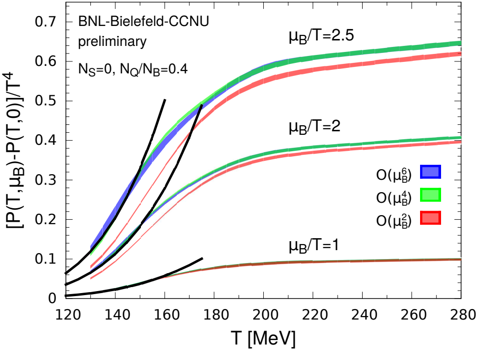
- In HIC: $n_S = 0$, $\frac{n_Q}{n_B} = 0.4$ mimics Pb-Pb, Au-Au.

EoS for strangeness neutrality

- In HIC: $n_S = 0$, $\frac{n_Q}{n_B} = 0.4$ mimics Pb-Pb, Au-Au.
- The pressure for $T < 250$ MeV already constrained by χ_B^6 for $\mu_B/T \leq 2.5$.

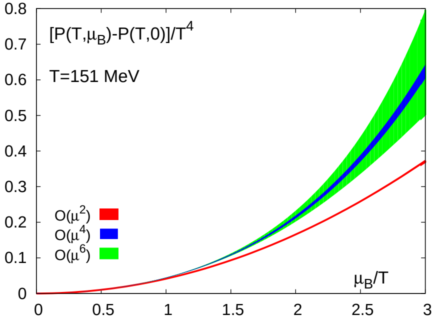
EoS for strangeness neutrality

- In HIC: $n_S = 0$, $\frac{n_Q}{n_B} = 0.4$ mimics Pb-Pb, Au-Au.
- The pressure for $T < 250$ MeV already constrained by χ_B^6 for $\mu_B/T \leq 2.5$.
- Errors for $\mu_B/T \sim 3$ to be determined by χ_8^B.



Breakdown of HRG at finite μ_B

- Breakdown of HRG+ onset of criticality can be already constrained with χ_6^B.
- Near critical point all terms in the Taylor expansion nearly equal $\rightarrow$ need to improve the errors to observe!
- At CEP: $\chi_n > 0$.



Can HRG a good approximation for QCD near chemical freezeout?

- Hadron Resonance Gas model: residual hadron interactions at the freezeout taken into account by considering all known resonances
  [Braun-Munzinger, Cleymans, Oeschler, Redlich, 02].

Can HRG a good approximation for QCD near chemical freezeout?

- Hadron Resonance Gas model: residual hadron interactions at the freezeout taken into account by considering all known resonances
  [Braun-Munzinger, Cleymans, Oeschler, Redlich, 02].

- Other motivation: At low $T, \mu_B = 0$, thermodynamics dominated by pions

Can HRG a good approximation for QCD near chemical freezeout?

- Hadron Resonance Gas model: residual hadron interactions at the freezeout taken into account by considering all known resonances [Braun-Munzinger, Cleymans, Oeschler, Redlich, 02].

- Other motivation: At low $T, \mu_B = 0$, thermodynamics dominated by pions
- The interaction from 3 loop χPT within 15% of ideal gas results [Gerber & Leutwyler, 89]

Can HRG a good approximation for QCD near chemical freezeout?

- Hadron Resonance Gas model: residual hadron interactions at the freezeout taken into account by considering all known resonances [Braun-Munzinger, Cleymans, Oeschler, Redlich, 02].

- Other motivation: At low $T, \mu_B = 0$, thermodynamics dominated by pions
- The interaction from 3 loop χPT within 15% of ideal gas results [Gerber & Leutwyler, 89]

- Residual interactions $\propto n_i n_j \sim \mathrm{e}^{-(\mathrm{m_i}+\mathrm{m_j})/\mathrm{T}}$.

Can HRG a good approximation for QCD near chemical freezeout?

- Hadron Resonance Gas model: residual hadron interactions at the freezeout taken into account by considering all known resonances [Braun-Munzinger, Cleymans, Oeschler, Redlich, 02].

- Other motivation: At low $T, \mu_B = 0$, thermodynamics dominated by pions
- The interaction from 3 loop χPT within 15% of ideal gas results [Gerber & Leutwyler, 89]

- Residual interactions $\propto n_i n_j \sim \mathrm{e}^{-(\mathrm{m_i}+\mathrm{m_j})/\mathrm{T}}$.
- Virial expansion can be used to estimate the effect of interactions.

Can HRG a good approximation for QCD near chemical freezeout?

- Hadron Resonance Gas model: residual hadron interactions at the freezeout taken into account by considering all known resonances [Braun-Munzinger, Cleymans, Oeschler, Redlich, 02].

- Other motivation: At low $T, \mu_B = 0$, thermodynamics dominated by pions
- The interaction from 3 loop χPT within 15% of ideal gas results [Gerber & Leutwyler, 89]

- Residual interactions $\propto n_i n_j \sim \mathrm{e}^{-(\mathrm{m_i}+\mathrm{m_j})/\mathrm{T}}$.
- Virial expansion can be used to estimate the effect of interactions.
- Scattering phase shifts from expt used to calculate interaction cross-section.

Can HRG a good approximation for QCD near chemical freezeout?

- Hadron Resonance Gas model: residual hadron interactions at the freezeout taken into account by considering all known resonances [Braun-Munzinger, Cleymans, Oeschler, Redlich, 02].

- Other motivation: At low $T, \mu_B = 0$, thermodynamics dominated by pions
- The interaction from 3 loop χPT within 15% of ideal gas results [Gerber & Leutwyler, 89]

- Residual interactions $\propto n_i n_j \sim \mathrm{e}^{-(\mathrm{m_i}+\mathrm{m_j})/\mathrm{T}}$.
- Virial expansion can be used to estimate the effect of interactions.
- Scattering phase shifts from expt used to calculate interaction cross-section.
- HRG a good approximation if resonances not very near to two particle threshold. [Prakash & Venugopalan, 92]

Can HRG a good approximation for QCD near chemical freezeout?

- Hadron Resonance Gas model: residual hadron interactions at the freezeout taken into account by considering all known resonances [Braun-Munzinger, Cleymans, Oeschler, Redlich, 02].

- Other motivation: At low $T, \mu_B = 0$, thermodynamics dominated by pions
- The interaction from 3 loop χPT within 15% of ideal gas results [Gerber & Leutwyler, 89]

- Residual interactions $\propto n_i n_j \sim \mathrm{e}^{-(m_i+m_j)/T}$.
- Virial expansion can be used to estimate the effect of interactions.
- Scattering phase shifts from expt used to calculate interaction cross-section.
- HRG a good approximation if resonances not very near to two particle threshold. [Prakash & Venugopalan, 92]
- For light hadrons validity of HRG needs to be checked! For charm it is expected to work.

Characterizing freezeout curve

- Using Line of constant energy per particle [J. Cleymans et al., 05].
  Freezeout curve parameterized as $T = T_{f,0}(1 - \kappa_2^f \mu_B^2 / T_{f,0}^2)$.

Characterizing freezeout curve

- Using Line of constant energy per particle [J. Cleymans et al., 05].
  Freezeout curve parameterized as $T = T_{f,0}(1 - \kappa_2^f \mu_B^2 / T_{f,0}^2)$.
- With this ansatz comparison of HRG to experiment: $\kappa_2^f = 0.023(3)$.

Characterizing freezeout curve

- Using Line of constant energy per particle [J. Cleymans et al., 05].
  Freezeout curve parameterized as $T = T_{f,0}(1 - \kappa_2^f \mu_B^2 / T_{f,0}^2)$.
- With this ansatz comparison of HRG to experiment: $\kappa_2^f = 0.023(3)$.
- To accurately mimic T_f at $\mu_B \to 0$:
  $T(\mu_B) - T_{f,0} = \mathrm{e}^{-\mathrm{a}/\mu_\mathrm{B}} \Rightarrow \kappa_2^\mathrm{f} \sim 0$.
  [A. Andronic, P. Braun-Munzinger, J. Stachel , 06]

Characterizing freezeout curve

- Using Line of constant energy per particle [J. Cleymans et al., 05].
  Freezeout curve parameterized as $T = T_{f,0}(1 - \kappa_2^f \mu_B^2 / T_{f,0}^2)$.
- With this ansatz comparison of HRG to experiment: $\kappa_2^f = 0.023(3)$.
- To accurately mimic T_f at $\mu_B \to 0$:
  $T(\mu_B) - T_{f,0} = \mathrm{e}^{-\mathrm{a}/\mu_\mathrm{B}} \Rightarrow \kappa_2^\mathrm{f} \sim 0$.
  [A. Andronic, P. Braun-Munzinger, J. Stachel , 06]
- However all phenomenological curves give more weight to low energy collision data.

Characterizing freezeout curve

- Using Line of constant energy per particle [J. Cleymans et al., 05].
  Freezeout curve parameterized as $T = T_{f,0}(1 - \kappa_2^f \mu_B^2 / T_{f,0}^2)$.
- With this ansatz comparison of HRG to experiment: $\kappa_2^f = 0.023(3)$.
- To accurately mimic T_f at $\mu_B \to 0$:
  $T(\mu_B) - T_{f,0} = \mathrm{e}^{-\mathrm{a}/\mu_\mathrm{B}} \Rightarrow \kappa_2^\mathrm{f} \sim 0$.
  [A. Andronic, P. Braun-Munzinger, J. Stachel , 06]
- However all phenomenological curves give more weight to low energy collision data.
- Can we go beyond HRG?

Freezeout curve: exercise from lattice

- Freezeout curve at leading order in μ_B: $T = T_{f,0}(1 - \kappa_2^f \mu_B^2 / T_{f,0}^2)$.

Freezeout curve: exercise from lattice

- Freezeout curve at leading order in μ_B: $T = T_{f,0}(1 - \kappa_2^f \mu_B^2 / T_{f,0}^2)$.
- Basic observables $\Sigma_r^{QB} = \frac{R_{12}^Q}{R_{12}^B}$, $R_{12}^X = \frac{\chi_1^X}{\chi_2^X}$.

Freezeout curve: exercise from lattice

- Freezeout curve at leading order in μ_B: $T = T_{f,0}(1 - \kappa_2^f \mu_B^2 / T_{f,0}^2)$.
- Basic observables $\Sigma_r^{QB} = \frac{R_{12}^Q}{R_{12}^B}$, $R_{12}^X = \frac{\chi_1^X}{\chi_2^X}$.
- Expanding the observable about the freezeout surface at $\mu_B = 0, n_Q/n_B = 0.4$,
  $\Sigma_r^{QB}(\mu_B) = \Sigma_r^{QB}(0) + \left[\Sigma_r^{QB,2} - \kappa_2^f\ T_{f,0} \frac{d\Sigma_r^{QB,0}}{dT}\Big|_{T_{f,0}}\right] \frac{\mu_B^2}{T^2}$.

Freezeout curve: exercise from lattice

- Freezeout curve at leading order in μ_B: $T = T_{f,0}(1 - \kappa_2^f \mu_B^2 / T_{f,0}^2)$.
- Basic observables $\Sigma_r^{QB} = \frac{R_{12}^Q}{R_{12}^B}$, $R_{12}^X = \frac{\chi_1^X}{\chi_2^X}$.
- Expanding the observable about the freezeout surface at $\mu_B = 0, n_Q/n_B = 0.4$,
  $$\Sigma_r^{QB}(\mu_B) = \Sigma_r^{QB}(0) + \left[\Sigma_r^{QB,2} - \kappa_2^f \; T_{f,0} \frac{d\Sigma_r^{QB,0}}{dT}\Big|_{T_{f,0}}\right] \frac{\mu_B^2}{T^2} \; .$$
- Instead from experimental parameterization, get μ_B from the first order Taylor expansion of R_{12}^B as $\mu_B = T \frac{R_{12}^{B,0}}{R_{12}^{B,1}}$.

Freezeout curve: exercise from lattice

- Freezeout curve at leading order in μ_B: $T = T_{f,0}(1 - \kappa_2^f \mu_B^2 / T_{f,0}^2)$.
- Basic observables $\Sigma_r^{QB} = \frac{R_{12}^Q}{R_{12}^B}$, $R_{12}^X = \frac{\chi_1^X}{\chi_2^X}$.
- Expanding the observable about the freezeout surface at $\mu_B = 0, n_Q/n_B = 0.4$, $\Sigma_r^{QB}(\mu_B) = \Sigma_r^{QB}(0) + \left[\Sigma_r^{QB,2} - \kappa_2^f \; T_{f,0} \frac{d\Sigma_r^{QB,0}}{dT}\Big|_{T_{f,0}}\right] \frac{\mu_B^2}{T^2}$.
- Instead from experimental parameterization, get μ_B from the first order Taylor expansion of R_{12}^B as $\mu_B = T \frac{R_{12}^{B,0}}{R_{12}^{B,1}}$.
- As a result $\Sigma_r^{QB}(\mu_B) = \Sigma_r^{QB}(0) \left[1 + c_{12} \; \left(R_{12}^B\right)^2\right] + \mathcal{O}\left(R_{12}^B\right)^4$.

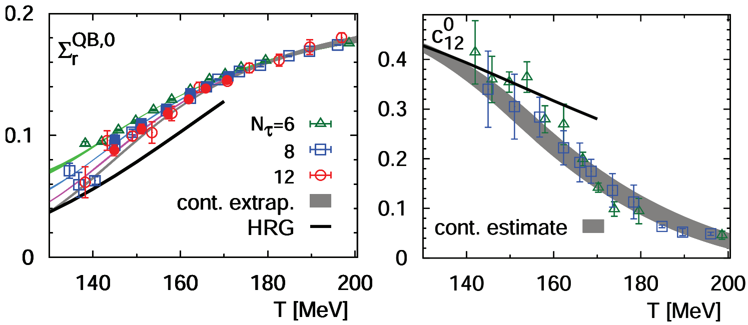
Freezeout curve: exercise from lattice

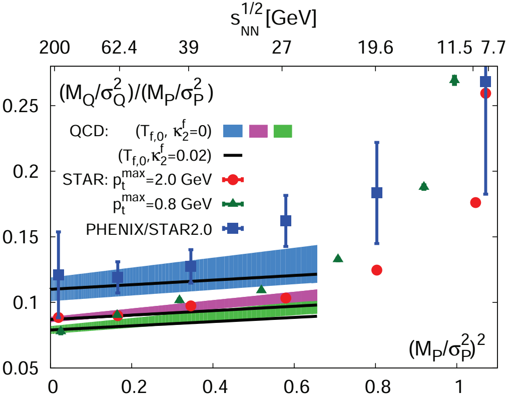
- Freezeout curve at leading order in μ_B: $T = T_{f,0}(1 - \kappa_2^f \mu_B^2 / T_{f,0}^2)$.
- Basic observables $\Sigma_r^{QB} = \frac{R_{12}^Q}{R_{12}^B}$, $R_{12}^X = \frac{\chi_1^X}{\chi_2^X}$.
- Expanding the observable about the freezeout surface at $\mu_B = 0, n_Q/n_B = 0.4$,
$\Sigma_r^{QB}(\mu_B) = \Sigma_r^{QB}(0) + \left[\Sigma_r^{QB,2} - \kappa_2^f \; T_{f,0} \frac{d\Sigma_r^{QB,0}}{dT}\Big|_{T_{f,0}}\right] \frac{\mu_B^2}{T^2}$.
- Instead from experimental parameterization, get μ_B from the first order Taylor expansion of R_{12}^B as $\mu_B = T \frac{R_{12}^{B,0}}{R_{12}^{B,1}}$.
- As a result $\Sigma_r^{QB}(\mu_B) = \Sigma_r^{QB}(0) \left[1 + c_{12} \; \left(R_{12}^B\right)^2\right] + \mathcal{O}\left(R_{12}^B\right)^4$.
- An estimate of Σ_r^{QB} and R_{12}^B from experiments allows us to calculate c_{12}.

[Bielefeld-BNL-CCNU collaboration, 15]

- Caveat: In experiments one measures protons Σ_r^{Qp}, R_{12}^p. Need to understand proton vs baryon number distributions. [Asakawa & Kitazawa, 12].
  Within HRG at least R_{12}^B is mimicked by R_{12}^P within 10%.
- Additionally take into account also corrections due to finite range of momenta of detected particles.

  [Karsch, Morita and Redlich, 15, P Garg et. al., 13, Bzdak & Koch, 12].
- From the 2 independent expressions of Σ_r^{QB} we extract
  $c_{12}(T_{f,0},\kappa_2^f) = c_{12}^0(T_{f,0}) - \kappa_2^f D_{12}$.

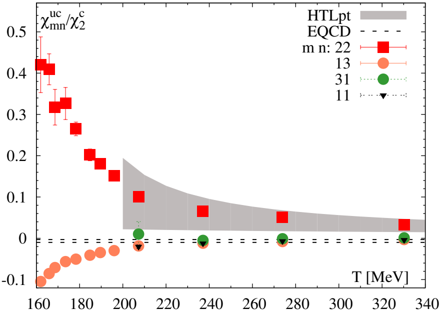




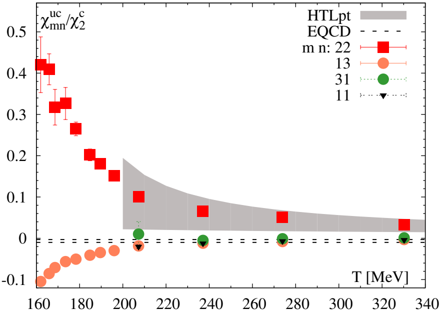
- This exercise give $T_{f,0} = 147(2)$ MeV consistent with expectation that its at or below T_c. Consistent with recent analysis of ALICE data which gives $T_{f,0} \sim T_c$. [P. Braun-Munzinger, A. Kalweit, K. Redlich, J. Stachel, 15]
Curvature: $\kappa_2^f < -0.012(15) \rightarrow$ near to chiral curvature $\kappa_2^B = 0.0066(7)$.
[Bielefeld-BNL-CCNU collaboration, 15]

Outline

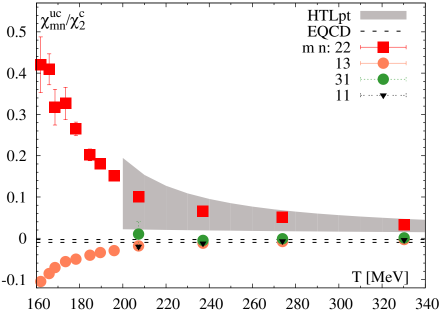
1. The QCD phase diagram: outstanding issues on lattice
2. Fluctuations and Equation of state at finite μ_B
3. QCD medium for $T > T_c$ from fluctuations



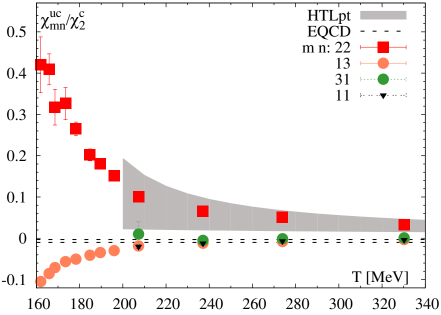
- Looking at simpler system → charm fluctuations



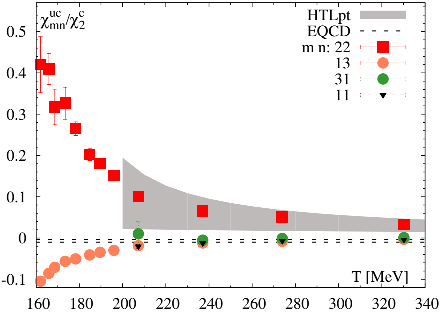
- Looking at simpler system $\rightarrow$ charm fluctuations
- Deviation from Hard Thermal Loop results between 160 – 200 MeV



- Looking at simpler system $\rightarrow$ charm fluctuations
- Deviation from Hard Thermal Loop results between $160 - 200$ MeV
- We saw earlier that open charm hadrons melt at T_c. [BI-BNL collaboration, 14]



- Looking at simpler system $\rightarrow$ charm fluctuations
- Deviation from Hard Thermal Loop results between $160 - 200$ MeV
- We saw earlier that open charm hadrons melt at T_c. [BI-BNL collaboration, 14]
- Pressure for broad "resonances" considerably lower than sharp width QP. [Biro & Jakovac, 14]



- Looking at simpler system $\rightarrow$ charm fluctuations
- Deviation from Hard Thermal Loop results between 160 – 200 MeV
- We saw earlier that open charm hadrons melt at T_c. [BI-BNL collaboration, 14]
- Pressure for broad "resonances" considerably lower than sharp width QP. [Biro & Jakovac, 14]
- Charm spectral function may have a broad asymmetric peak $\rightarrow$ not a good quasi-particle below 200 MeV.

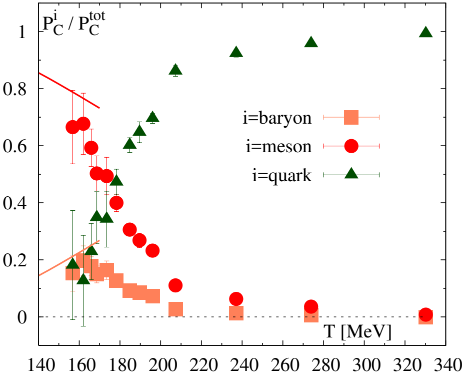
Charm d.o.f at deconfinement

- Considering charm mesons+baryon+quark-like excitations

$$p_C(T,\mu_B,\mu_C) = p_M(T)\cosh\left(\frac{\mu_C}{T}\right) + p_{B,C=1}(T)\cosh\left(\frac{\mu_C+\mu_B}{T}\right) + p_q(T)\cosh\left(\frac{\mu_C+\mu_B/3}{T}\right).$$

- Considering fluctuations upto 4th order we have 2 trivial constraints $\chi_4^C = \chi_2^C$, $\chi_{11}^{BC} = \chi_{13}^{BC}$.
- A more non-trivial constraint: $c_1 \equiv \chi_{13}^{BC} - 4\chi_{22}^{BC} + 3\chi_{31}^{BC} = 0$.

Charm d.o.f at deconfinement



- Meson and baryon like excitations survive upto $1.2\,T_c$.
- Quark-quasiparticles start dominating the pressure beyond $T \gtrsim 200$ MeV $\Rightarrow$ hints of strongly coupled QGP [Mukherjee, Petreczky, SS, 15]

Do diquarks exist beyond T_c?

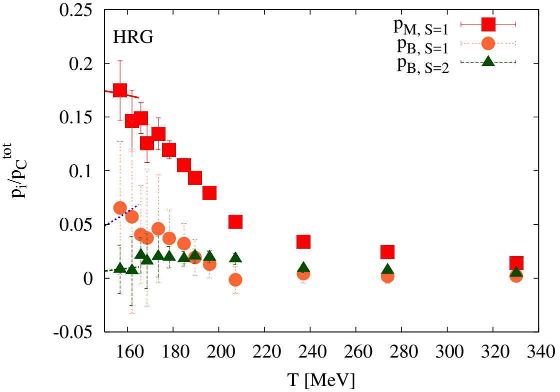
- We look specifically at the sector of strange and charm hadrons.
- Upto 4th order derivatives additionally one has 3 more measurements $\chi^{BSC}_{[112]}$

$$p_{SC}(T,\mu_B,\mu_C) = \sum_{j=0}^{1} p_{B,S=j}(T)\cosh\left(\frac{\mu_C+\mu_B-j\mu_S}{T}\right) + p_M(T)\cosh\left(\frac{\mu_C+\mu_S}{T}\right) + p_D(T)\cosh\left(\frac{\mu_C+\mu_B/3-\mu_S}{T}\right).$$

- Di-quarks carry color quantum number...should disappear when quark d.o.f start dominating around 200 MeV.

Do diquarks exist beyond T_c?

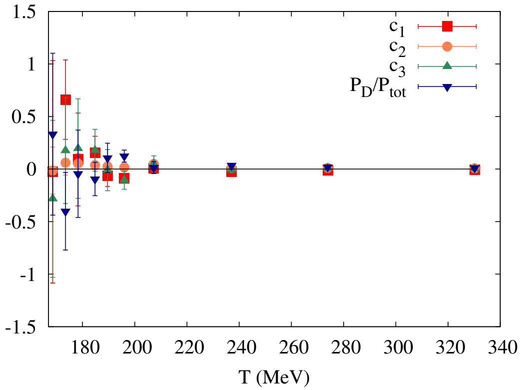
- $p_D = \chi^{BSC}_{[211]} - \chi^{BSC}_{[112]} = 0$ for our data.



- Strange baryon-like excitations suppressed than meson-like excitations.
- These studies consistent with screening mass of sc-mesons [Y. Maezawa et. al., PRD 2015].

Do diquarks exist beyond T_c?

- For these calculations to be valid one should satisfy constraint relations $\rightarrow$ smoothly connect to HRG and free gas at low and high T.



- LQCD data agree with the constraints imposed by our proposed model.

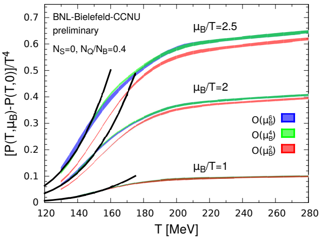
Modeling of heavy quarks in QGP

- Open charm hadrons melt at T_c [BI-BNL collaboration, 14] $\Rightarrow$ freezeout temperature for D_s is now well known
  Input for heavy flavour transport models [A. Beraudo et. al., 12]

- Additional baryons may contribute to hadronic interactions near the freezeout $\rightarrow$ can it explain the discrepancy for between flow and suppression for D mesons?

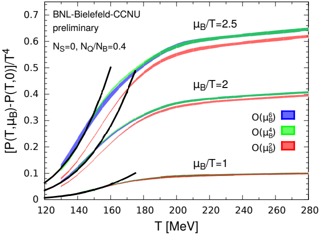
- Charm baryon and meson-like excitations surviving in the medium till $1.2\ T_c$.
- Our study more in favour for resonant scattering of heavy quarks in the medium [M. He, R. J. Fries, R. Rapp, 12].

Outlook



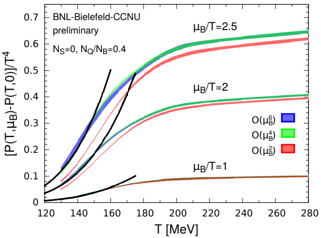
- Fluctuation measurements on lattice has made tremendous progress in last years.

Outlook



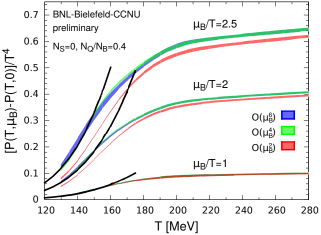
- Fluctuation measurements on lattice has made tremendous progress in last years.
- Imp. for realistic modeling of dynamics of hot fireball in HIC

Outlook



- Fluctuation measurements on lattice has made tremendous progress in last years.
- Imp. for realistic modeling of dynamics of hot fireball in HIC
- LQCD EoS for $\mu_B/T <= 2 \rightarrow \sqrt{s}_{NN} \geq 20$ GeV already under control.

Outlook



- Fluctuation measurements on lattice has made tremendous progress in last years.
- Imp. for realistic modeling of dynamics of hot fireball in HIC
- LQCD EoS for $\mu_B/T <= 2 \to \sqrt{s}_{NN} \geq 20$ GeV already under control.
- χ_6^B measured with improved precision: progress towards constraining EoS for $\mu_B/T \sim 3$. Analysis of χ_8^B ongoing.