EQUATION OF STATE AND FLUCTUATIONS FROM THE LATTICE

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Lattice QCD

- Best first principle-tool to extract predictions for the theory of strong interactions in the non-perturbative regime

- Uncertainties:
  - Statistical: finite sample, error $\sim 1/\sqrt{\text{sample size}}$
  - Systematic: finite box size, unphysical quark masses

- Given enough computer power, uncertainties can be kept under control

- Results from different groups, adopting different discretizations, converge to consistent results

- Unprecedented level of accuracy in lattice data

Low temperature phase: HRG model

Dashen, Ma, Bernstein; Prakash, Venugopalan, Karsch, Tawfik, Redlich

- Interacting hadronic matter in the ground state can be well approximated by a non-interacting resonance gas
- The pressure can be written as:

$$p^{HRG}/T^4 = \frac{1}{VT^3}\sum_{i\in\ mesons} \ln \mathcal{Z}^M_{m_i}(T,V,\mu_{X^a}) + \frac{1}{VT^3}\sum_{i\in\ baryons} \ln \mathcal{Z}^B_{m_i}(T,V,\mu_{X^a})$$

where

$$\ln \mathcal{Z}^{M/B}_{m_i} = \mp \frac{Vd_i}{2\pi^2}\int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) \quad ,$$

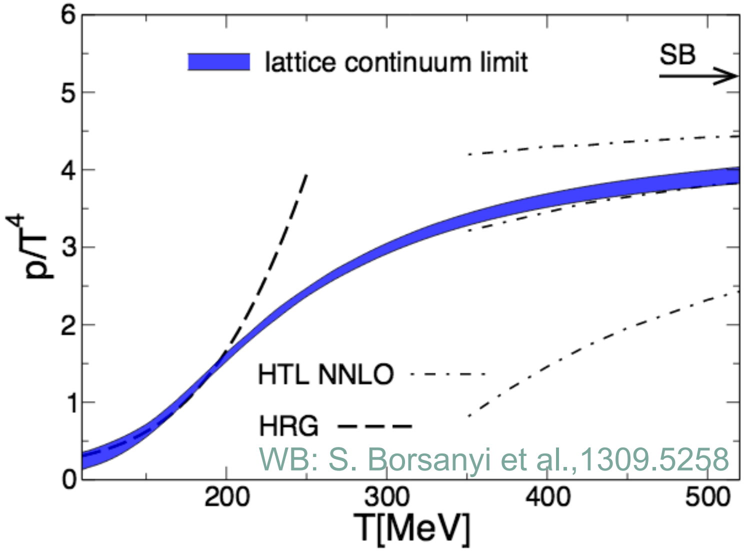
with energies $\varepsilon_i = \sqrt{k^2 + m_i^2}$, degeneracy factors d_i and fugacities

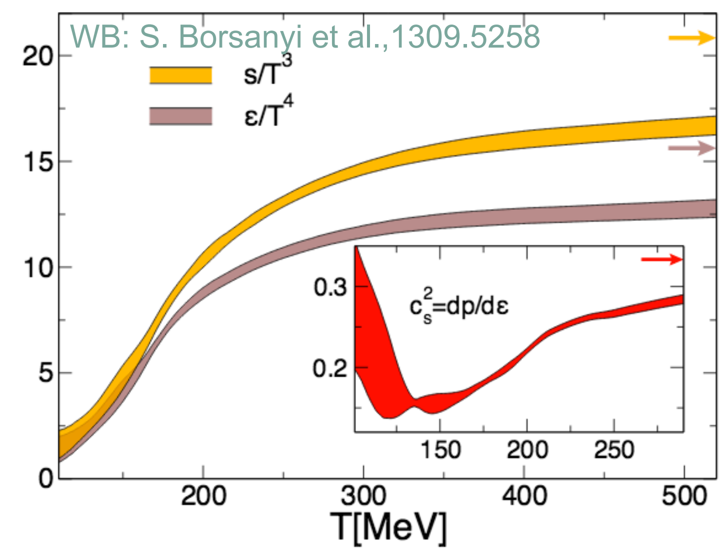
$$z_i = \exp\left((\sum_a X_i^a \mu_{X^a})/T \right) \quad .$$

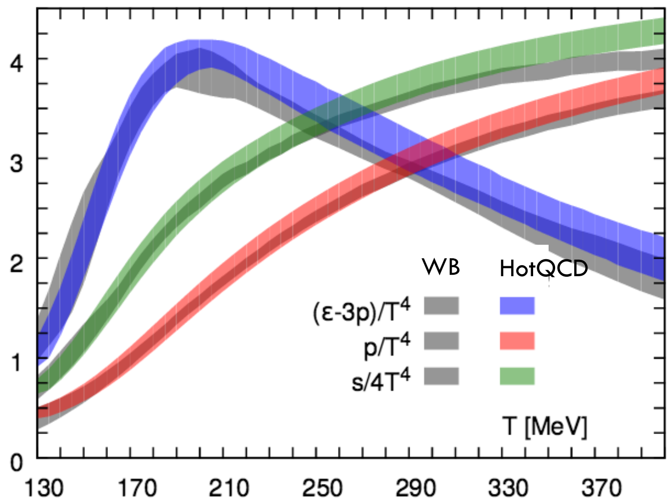
X^a: all possible conserved charges, including the baryon number B, electric charge Q, strangeness S.

- Needs knowledge of the hadronic spectrum

QCD Equation of state at μ_B=0





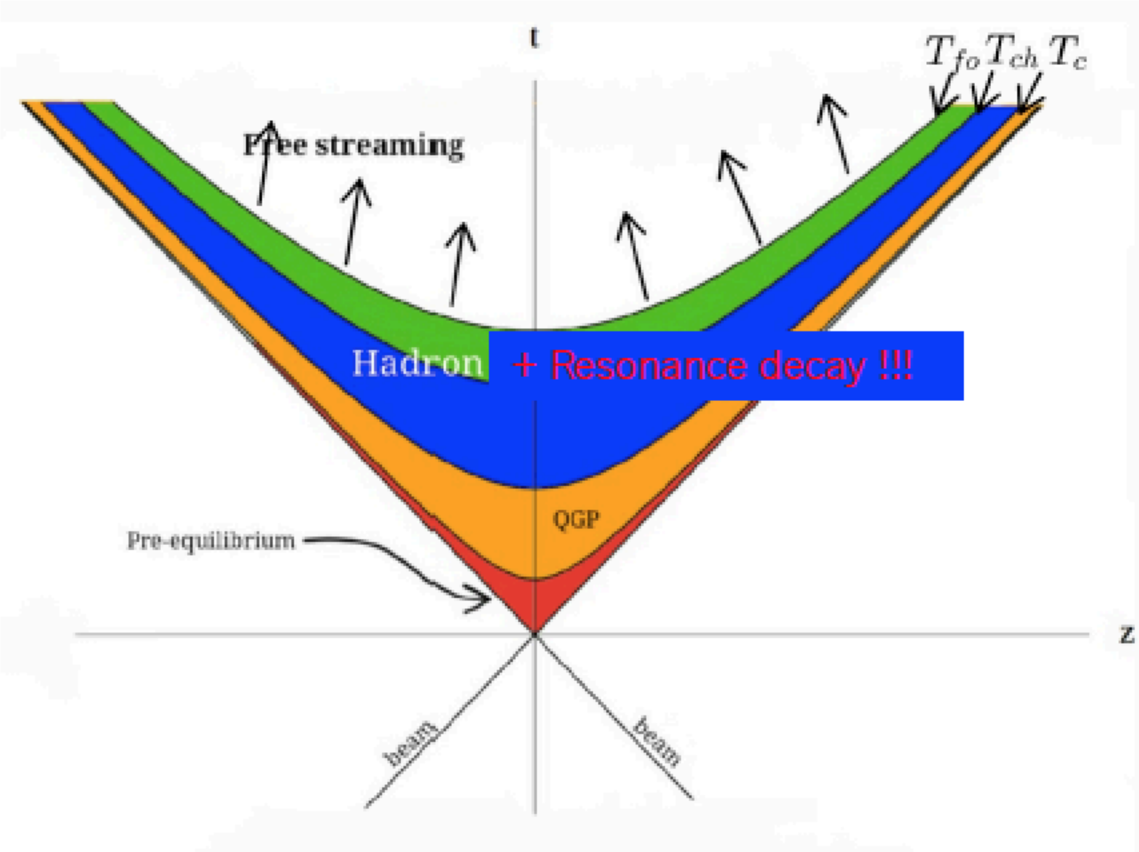


- EoS available in the continuum limit, with realistic quark masses
- Agreement between stout and HISQ action for all quantities

WB: S. Borsanyi et al., 1309.5258, PLB (2014)
HotQCD: A. Bazavov et al., 1407.6387, PRD (2014)

Full vs partial chemical equilibrium

The hadronic phase (blue and green areas) can be studied by means of the Hadron-Resonance Gas (HRG) model, where resonance formation and subsequent decay mediate the interaction among hadrons in the ground state.



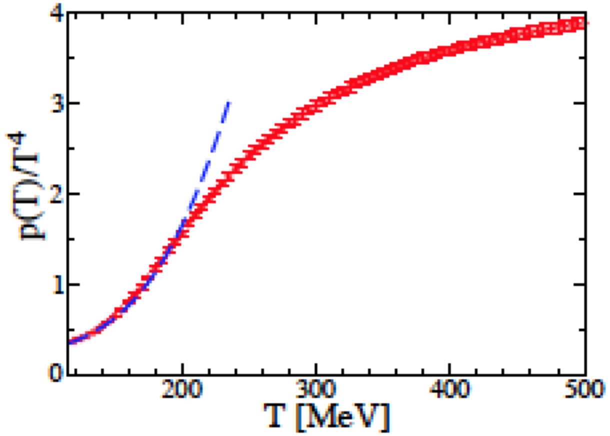
Resonances contribute to the effective number of stable species through their branching ratios $d_{r \to i}$.

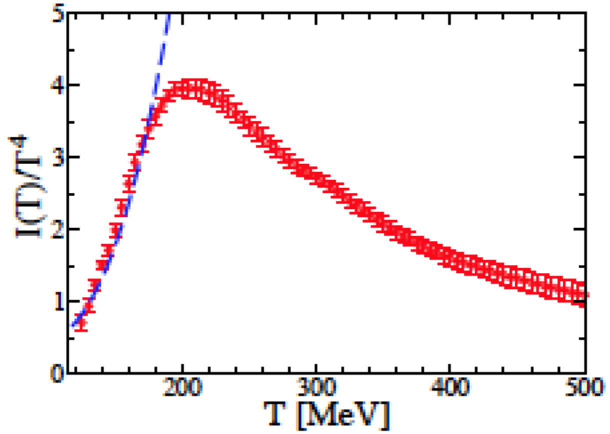
$$\bar{N}_i = N_i + \sum_r d_{r \to i} N_r$$

Equation of state in PCE

We want to provide a realistic EoS to describe the matter created in HICs.

Main ingredients:





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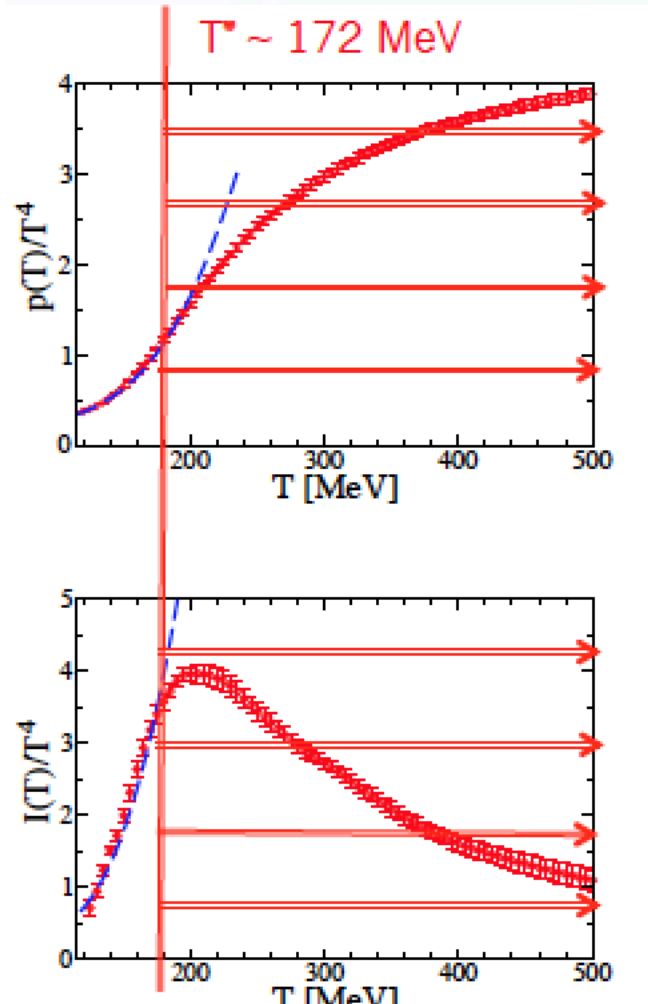

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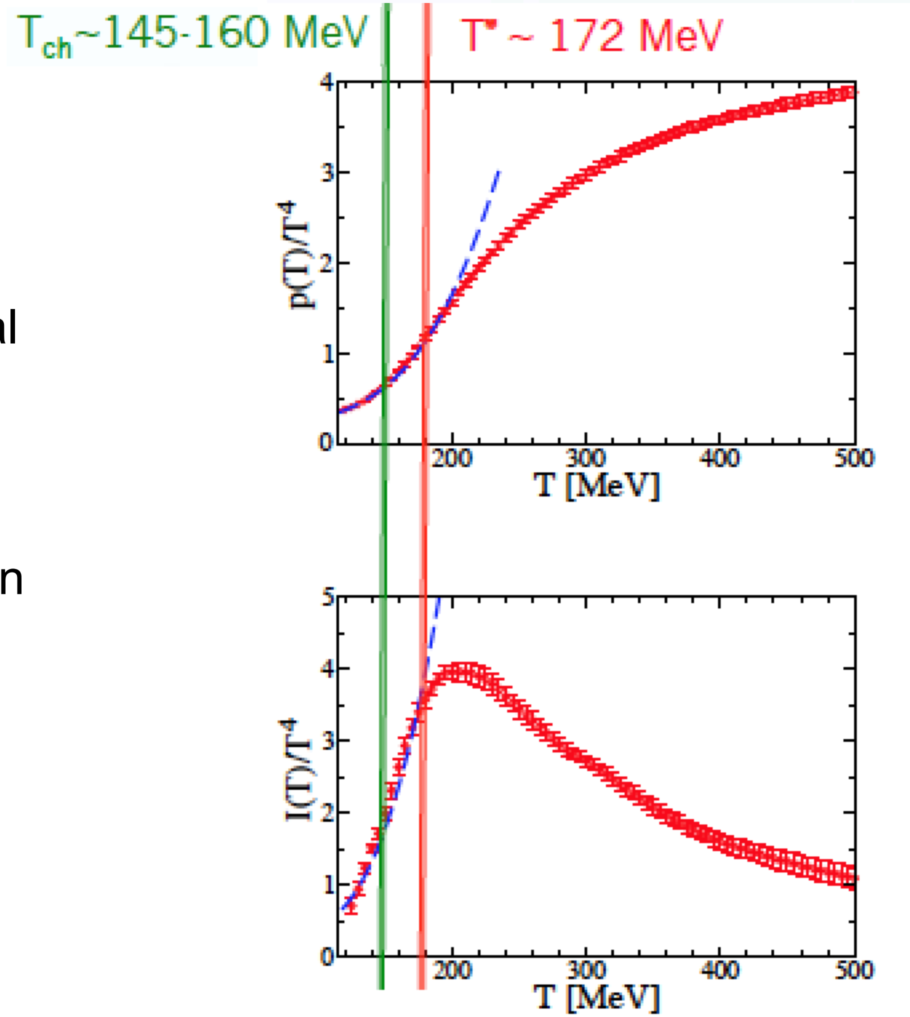
- For T_{ch}<T<T* : Hadron-Resonance Gas in chemical equilibrium (HRG in CE);



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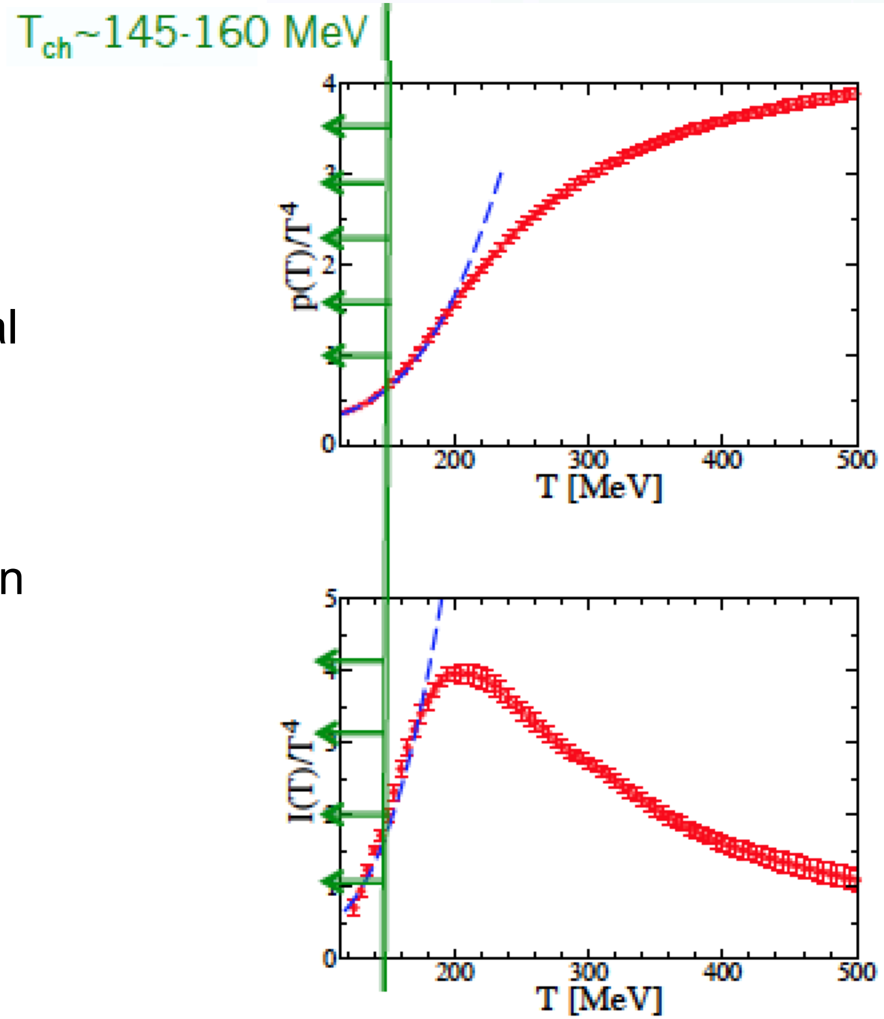
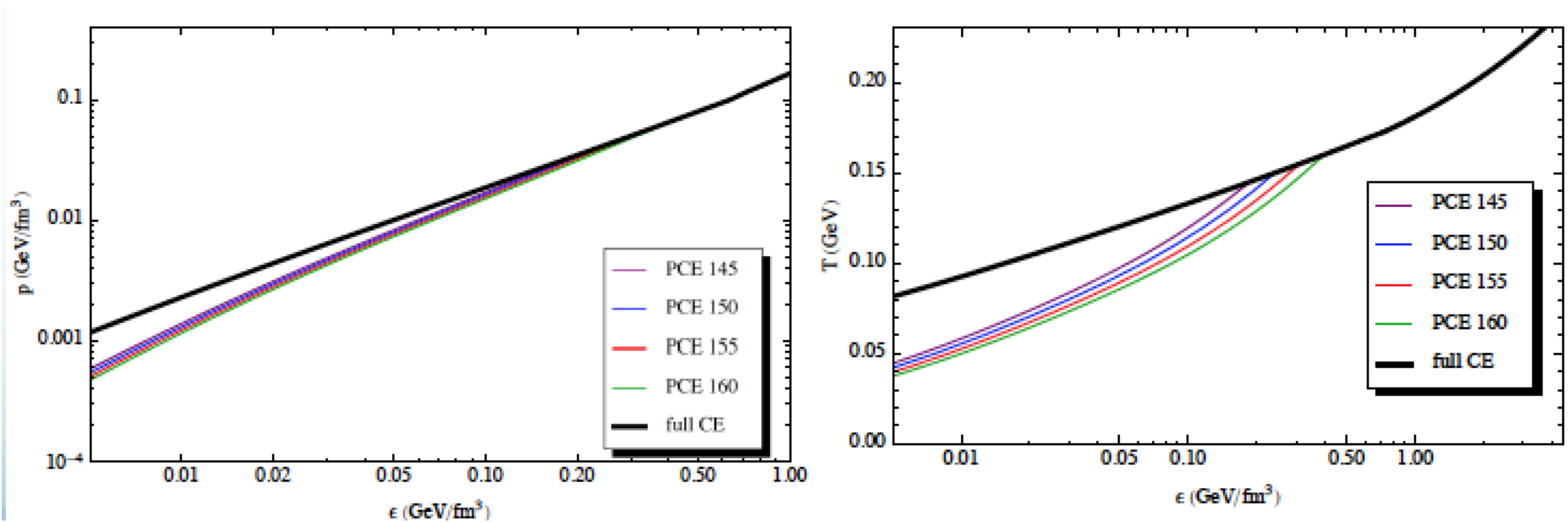
- For T<T_{ch} : HRG in partial chemical equilibrium (PCE)



Results

Pressure and temperature as functions of the energy density



M. Bluhm, C. R. et al., Nucl. Phys. A929 (2014) 157

Sign problem

- The QCD path integral is computed by Monte Carlo algorithms which samples field configurations with a weight proportional to the exponential of the action

$$Z(\mu_B, T) = \mathrm{Tr}\left(e^{-\frac{H_{\mathrm{QCD}} - \mu_B N_B}{T}}\right) = \int \mathcal{D}U e^{-S_G[U]} \det M[U, \mu_B]$$

- $\det M[\mu_B]$ complex → Monte Carlo simulations are not feasible

- We can rely on a few approximate methods, viable for small μ_B/T:
  - Taylor expansion of physical quantities around $\mu_B=0$ (Bielefeld-Swansea collaboration 2002; R. Gavai, S. Gupta 2003)
  - Simulations at imaginary chemical potentials (plus analytic continuation) (Alford, Kapustin, Wilczek, 1999; de Forcrand, Philipsen, 2002; D'Elia, Lombardo 2003)

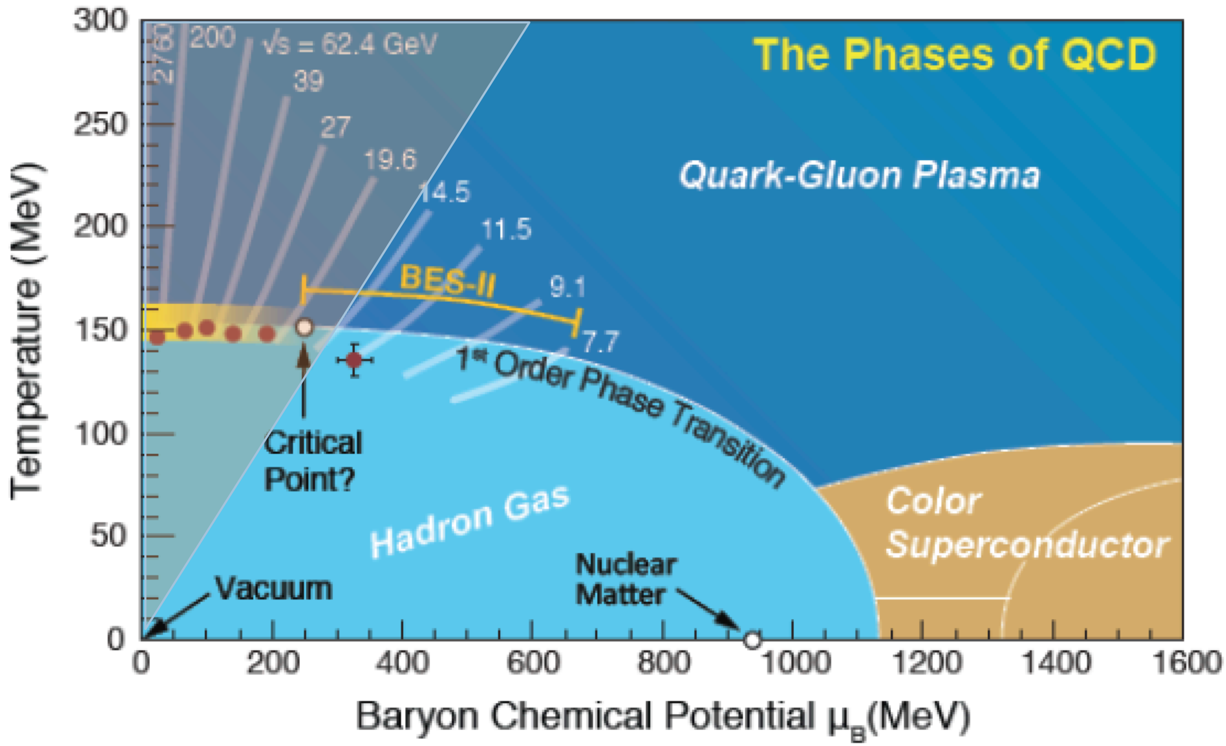
Equation of state as a Taylor expansion in μ_B

- Notation:

$$\hat{\mu}_B \equiv \mu_B/T \quad \hat{p} \equiv p/T^4 \quad \hat{n} \equiv n_B/T^3 \quad \hat{s} \equiv s/T^3$$

- Taylor expansion for the pressure:

$$\hat{p} = c_0(T) + c_2(T) \cdot \hat{\mu}_B^2 + c_4(T) \cdot \hat{\mu}_B^4 + c_6(T) \cdot \hat{\mu}_B^6 + \ldots$$



Physics at imaginary μ

- At imaginary μ there is no sign problem
- The partition function is periodic in μ_I with period 2πT

$$Z = \mathrm{Tr}\left(e^{-\beta\hat{H}+i\beta\mu_I\hat{N}}\right)$$

- For more chemical potentials: μ_B, μ_Q, μ_S, several trajectories are possible → useful for different physics
  - Here we use:

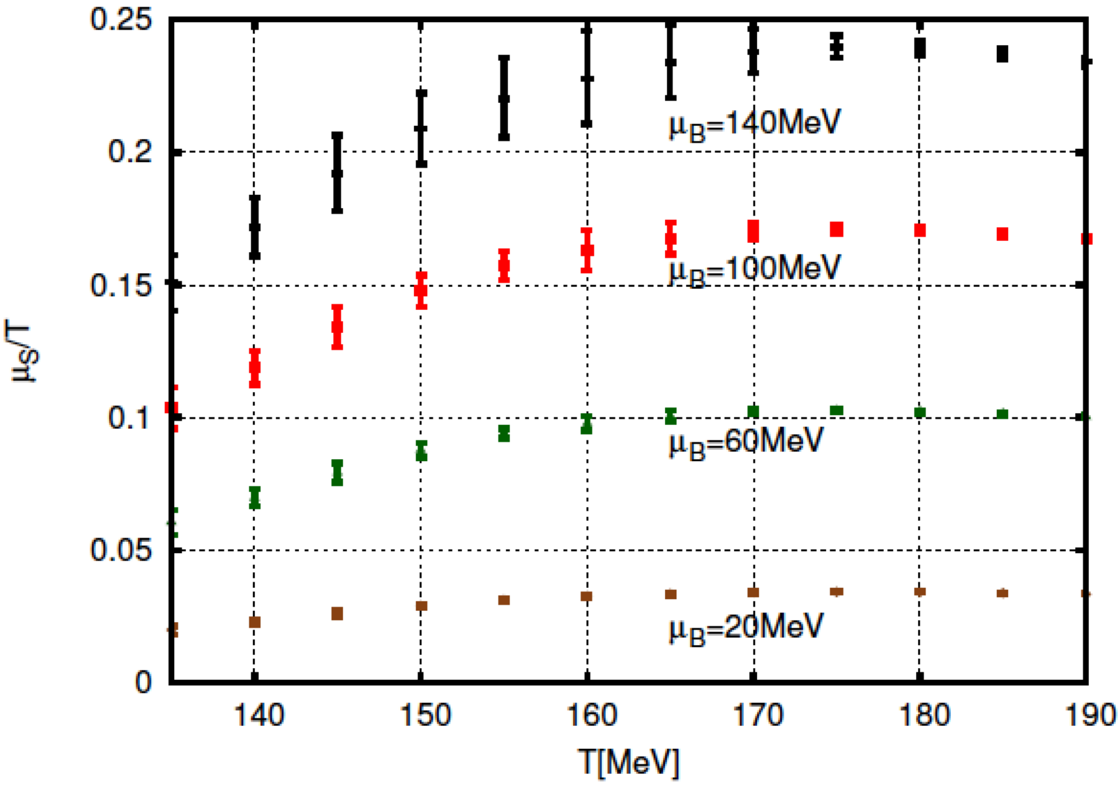
$\langle n_S \rangle = 0$ $\langle n_Q \rangle = 0.4 \langle n_B \rangle$

  - Other choices are possible, e.g.:

$\mu_S = 0$ $\mu_Q = 0$

Strangeness neutrality

- We simulate at μ_B, μ_S pairs such that $\langle n_S \rangle = 0$
- This requires a non-trivial fine tuning



Thermodynamic identities

- For the pressure we measure:

$$\frac{n}{\mu_B T^2} = \frac{T}{\mu_B}\frac{d\left(p/T^4\right)}{d\left(\mu_B/T\right)}\bigg|_{\langle n_S\rangle=0,\langle n_Q\rangle=0.4\langle n_B\rangle,T=\text{const.}}$$

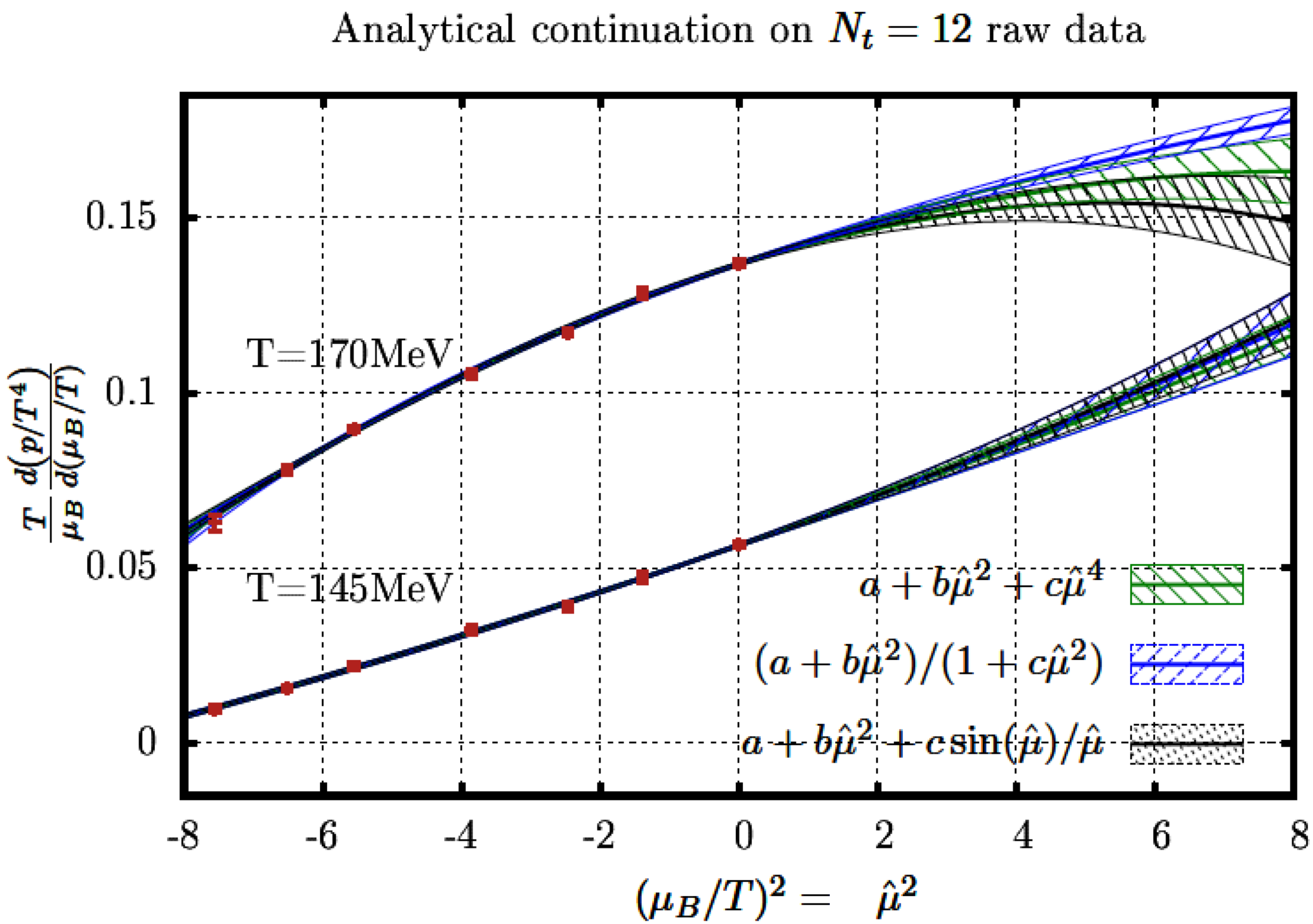
$$= n_B\left(1+0.4\frac{d\mu_Q}{d\mu_B}\right) = 2c_2 + 4c_4\left(\frac{\mu_B}{T}\right)^2 + 6c_6\left(\frac{\mu_B}{T}\right)^4 + \ldots$$

- For the entropy and energy:

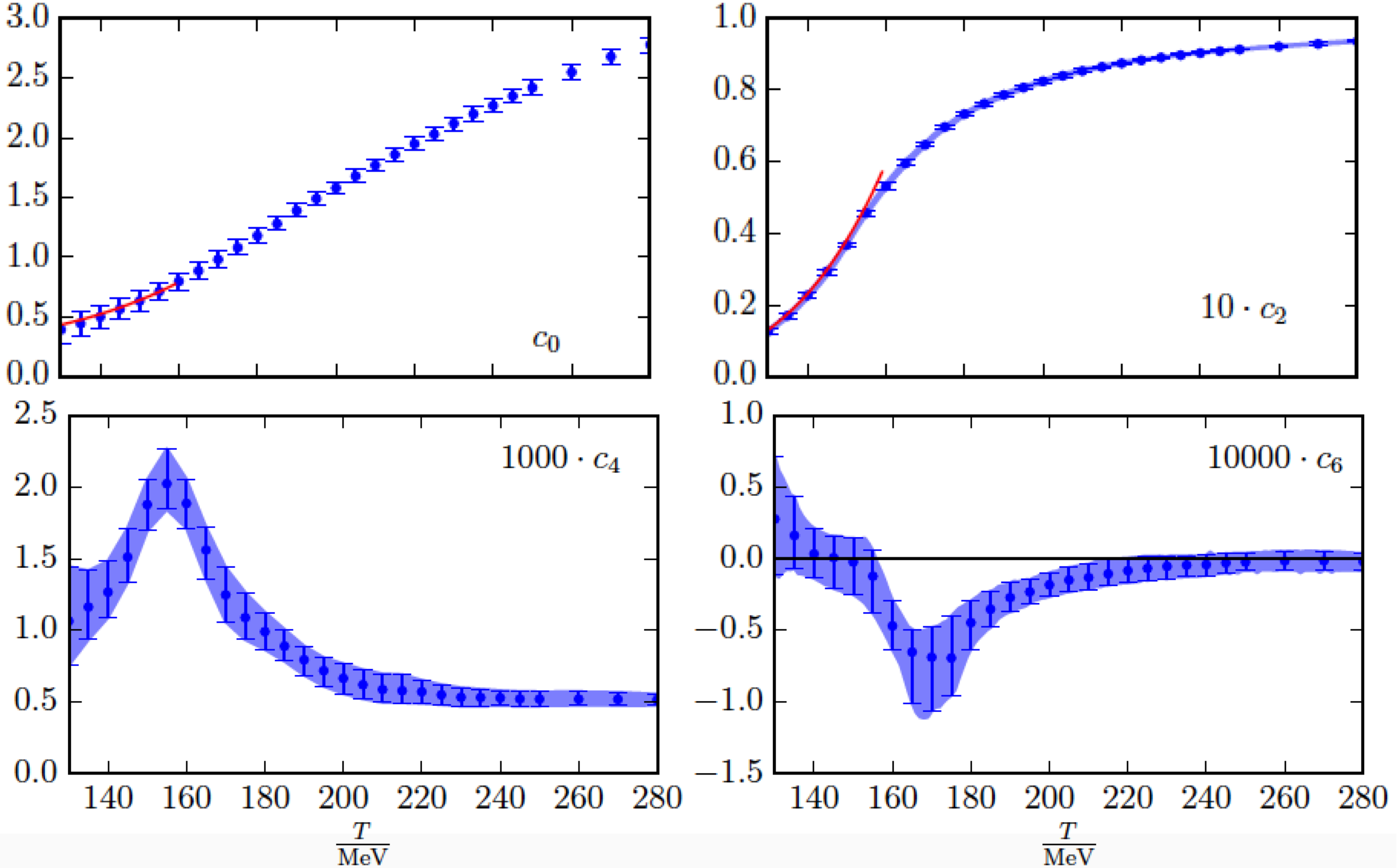
$$s = [T^4\partial/\partial T + 4T^3](p/T^4)$$

$$\hat{\epsilon} = \hat{s} - \hat{p} + \hat{\mu}_Q\hat{n}_Q + \hat{\mu}_B\hat{n}_B$$

Analytical continuation – illustration of systematics



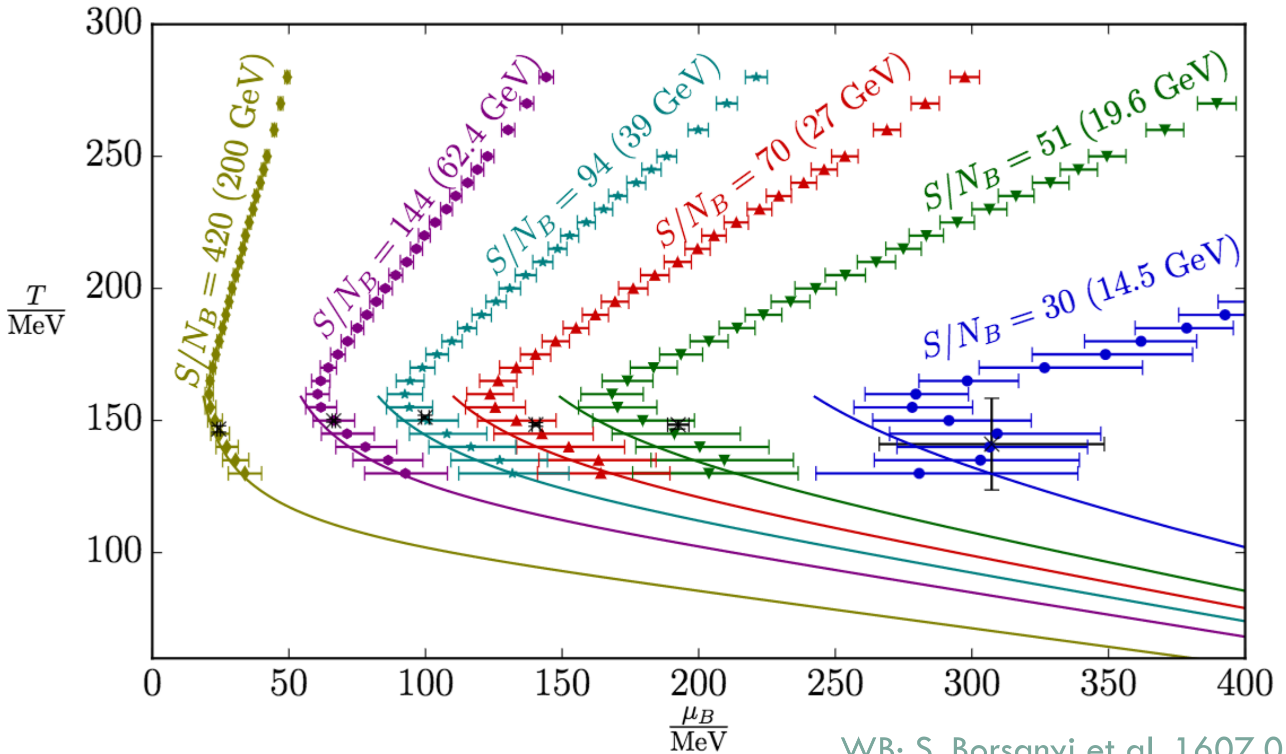
Taylor expansion of the pressure



WB: S. Borsanyi et al. 1607.02493 (2016)

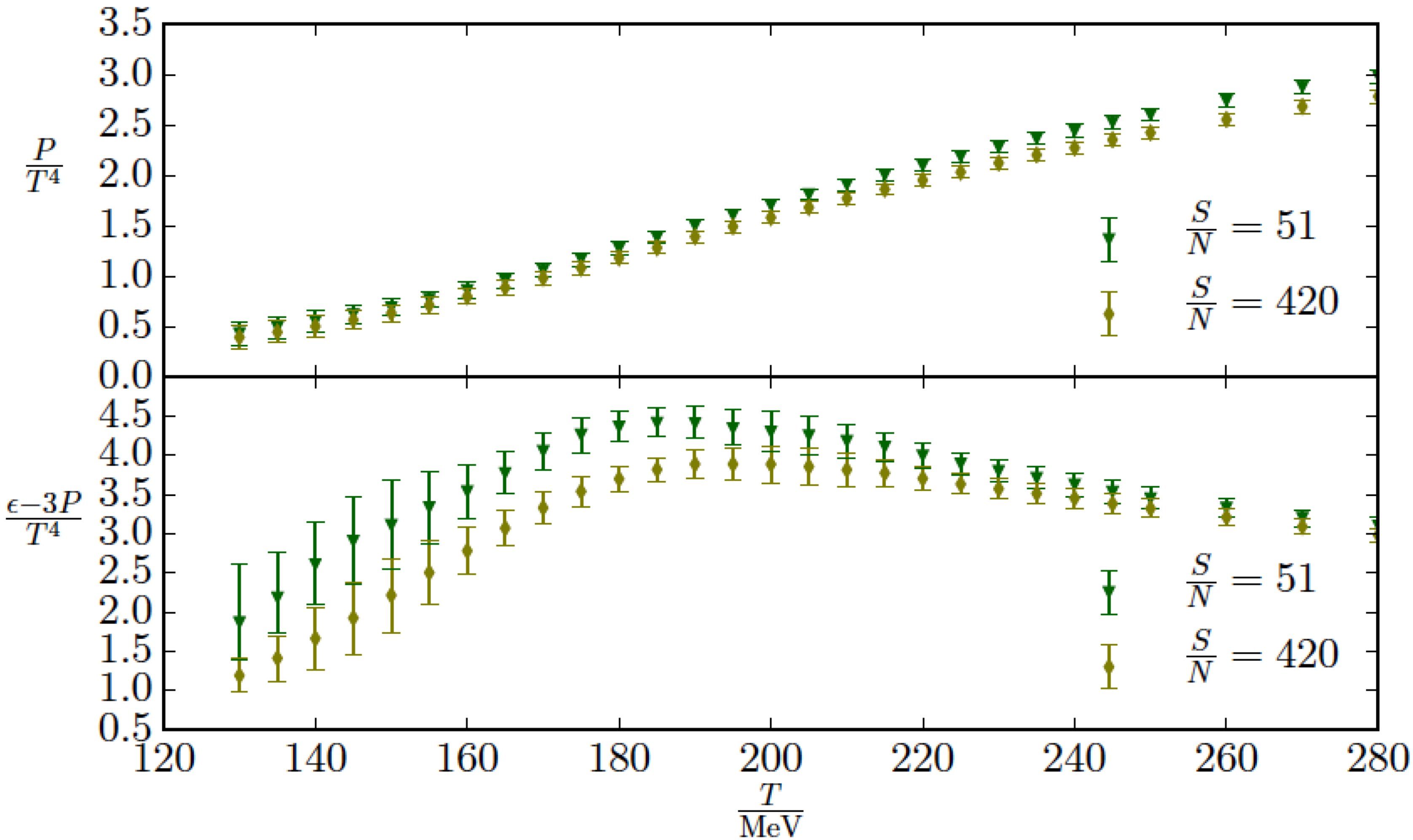
Equation of state at $\mu_B > 0$

- Extract the isentropic trajectory that the system follows in the absence of dissipation
- The freeze-out point estimates are from Alba et al., Phys. Lett. B738 (2014)



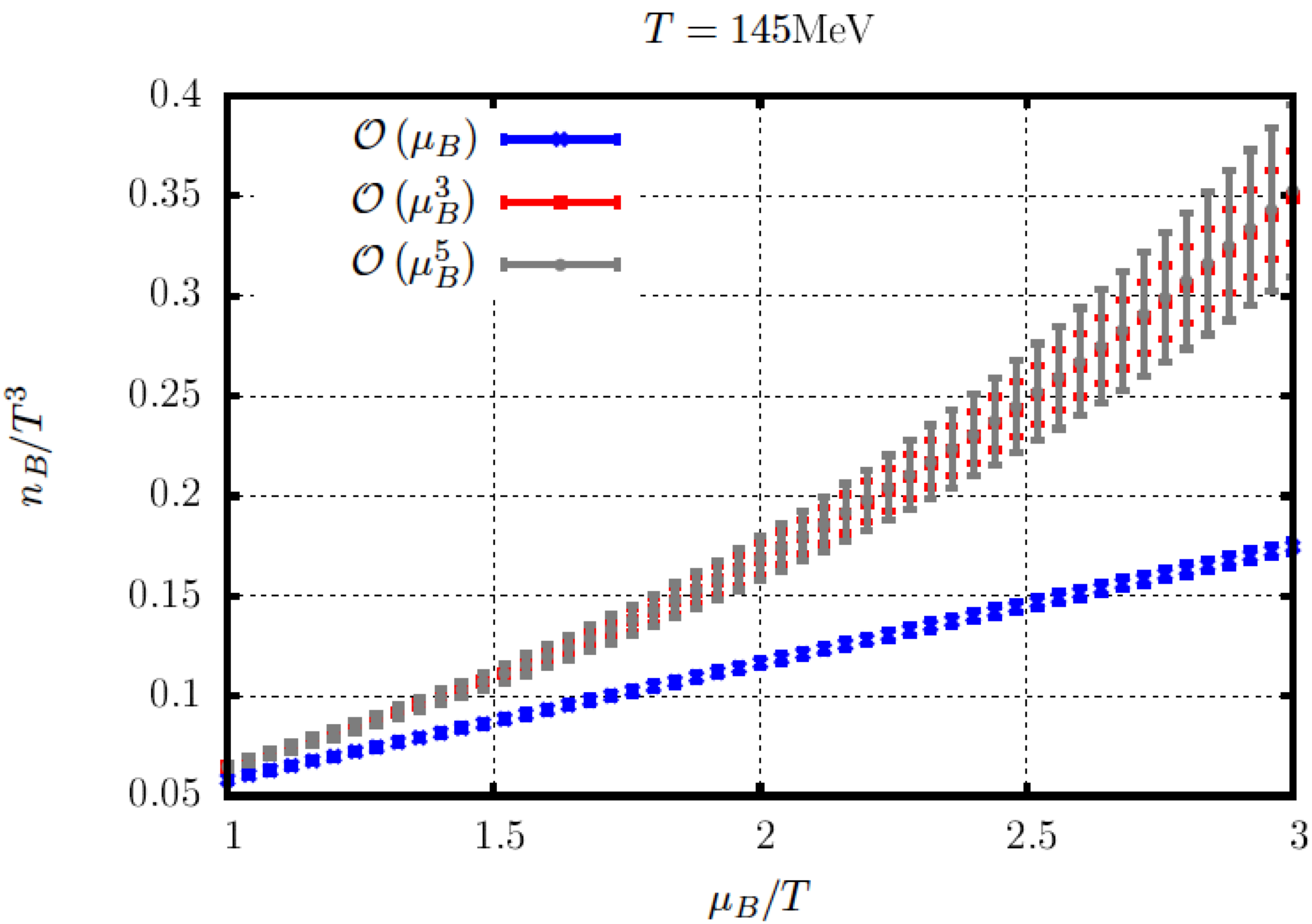
WB: S. Borsanyi et al. 1607.02493, (2016)

Equation of state along the trajectories



WB: S. Borsanyi et al. 1607.02493, (2016)

Different orders of μ_B expansion for n_B



Fluctuations of conserved charges

- Definition:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

- Relationship between chemical potentials:

$$\begin{aligned}
\mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q; \\
\mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q; \\
\mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S.
\end{aligned}$$

- They can be calculated on the lattice and compared to experiment

Connection to experiment

- Fluctuations of conserved charges are the cumulants of their event-by-event distribution

$$\text{mean}: M = \chi_1 \qquad \text{variance}: \sigma^2 = \chi_2$$

$$\text{skewness}: S = \chi_3/\chi_2^{3/2} \qquad \text{kurtosis}: \kappa = \chi_4/\chi_2^2$$

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma^3/M = \chi_3/\chi_1$$

F. Karsch: Centr. Eur. J. Phys. (2012)

- The chemical potentials are not independent: fixed to match the experimental conditions:

$$\langle n_S \rangle = 0 \qquad \langle n_Q \rangle = 0.4 \langle n_B \rangle$$

Things to keep in mind

- Effects due to volume variation because of finite centrality bin width
  - Experimentally corrected by centrality-bin-width correction method

V. Skokov et al., PRC (2013)

- Finite reconstruction efficiency
  - Experimentally corrected based on binomial distribution A.Bzdak,V.Koch, PRC (2012)
- Spallation protons
  - Experimentally removed with proper cuts in p_T
- Canonical vs Gran Canonical ensemble
  - Experimental cuts in the kinematics and acceptance V. Koch, S. Jeon, PRL (2000)
- Proton multiplicity distributions vs baryon number fluctuations
  - Recipes for treating proton fluctuations

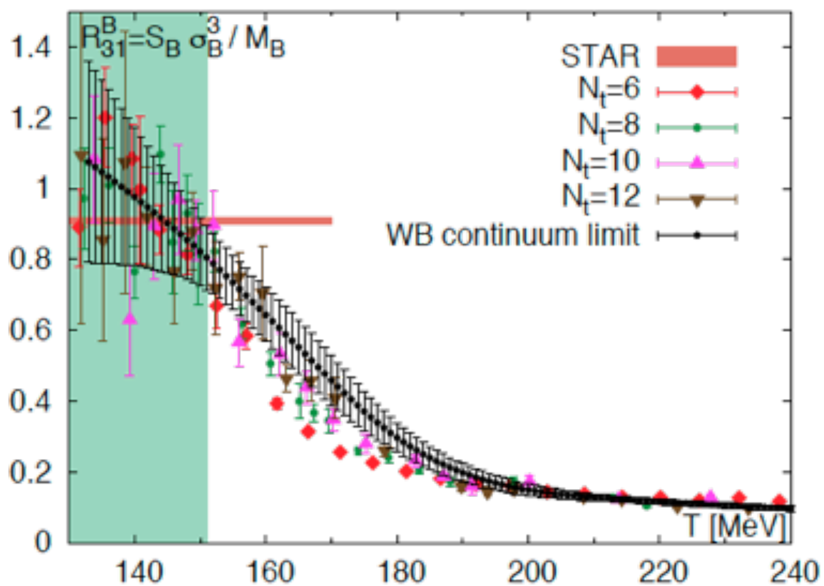
M. Asakawa and M. Kitazawa, PRC(2012), M. Nahrgang et al., 1402.1238

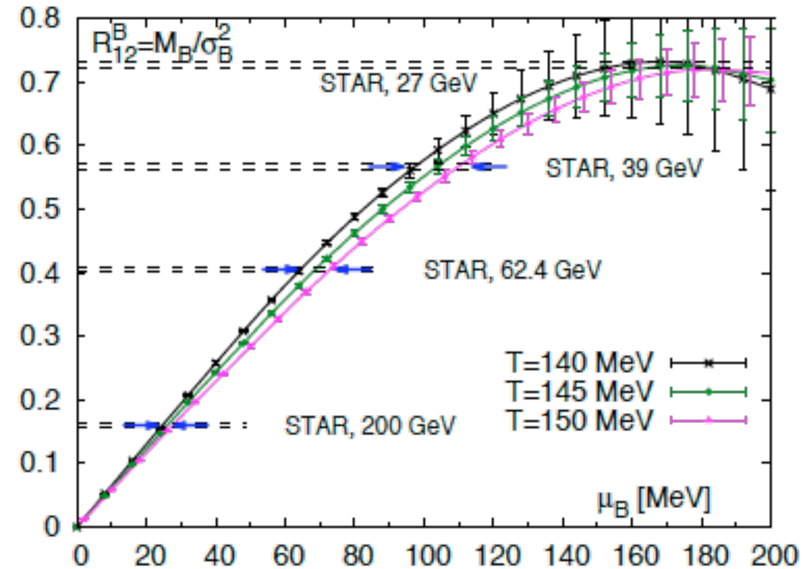
- Final-state interactions in the hadronic phase
  - Consistency between different charges = fundamental test

J.Steinheimer et al., PRL (2013)

Freeze-out parameters from B fluctuations

- Thermometer: $\dfrac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)}$ $=S_B\sigma_B^3/M_B$ Baryometer: $\dfrac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)}$ $=\sigma_B^2/M_B$





WB: S. Borsanyi et al., PRL (2014)
STAR collaboration, PRL (2014)

- Upper limit: $T_f \leq 151 \pm 4$ MeV
- Consistency between freeze-out chemical potential from electric charge and baryon number is found.

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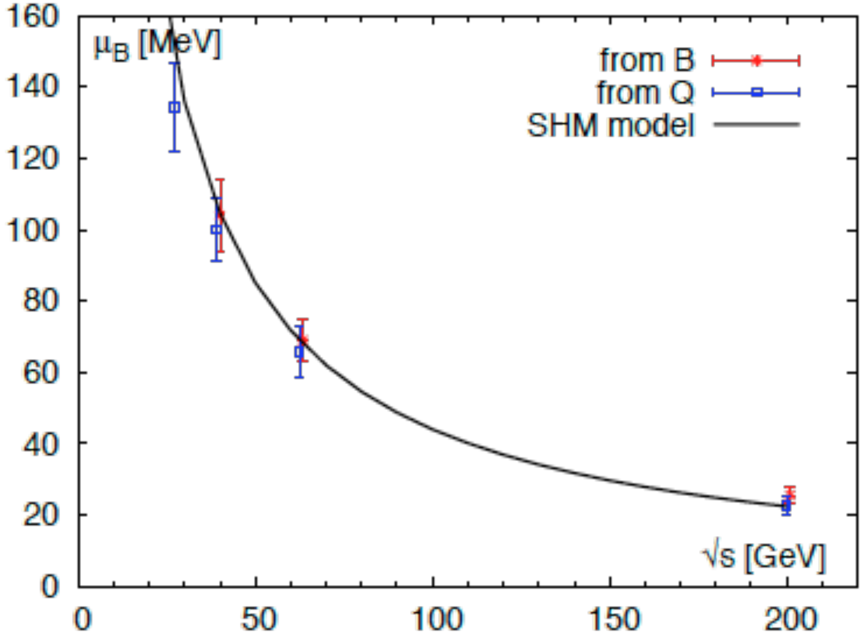

$\sqrt{s}[GeV]$	μ_B^f [MeV] (from B)	μ_B^f [MeV] (from Q)
200	25.8±2.7	22.8±2.6
62.4	69.7±6.4	66.6±7.9
39	105±11	101±10
27	-	136±13.8

WB: S. Borsanyi et al., PRL (2014)
STAR collaboration, PRL (2014)

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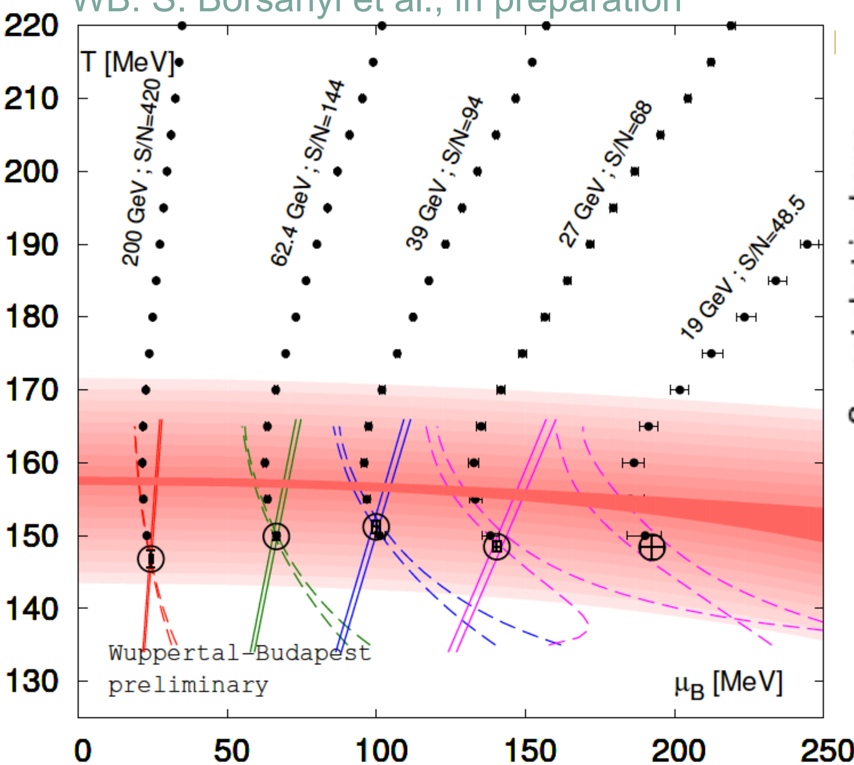
Freeze-out line from first principles

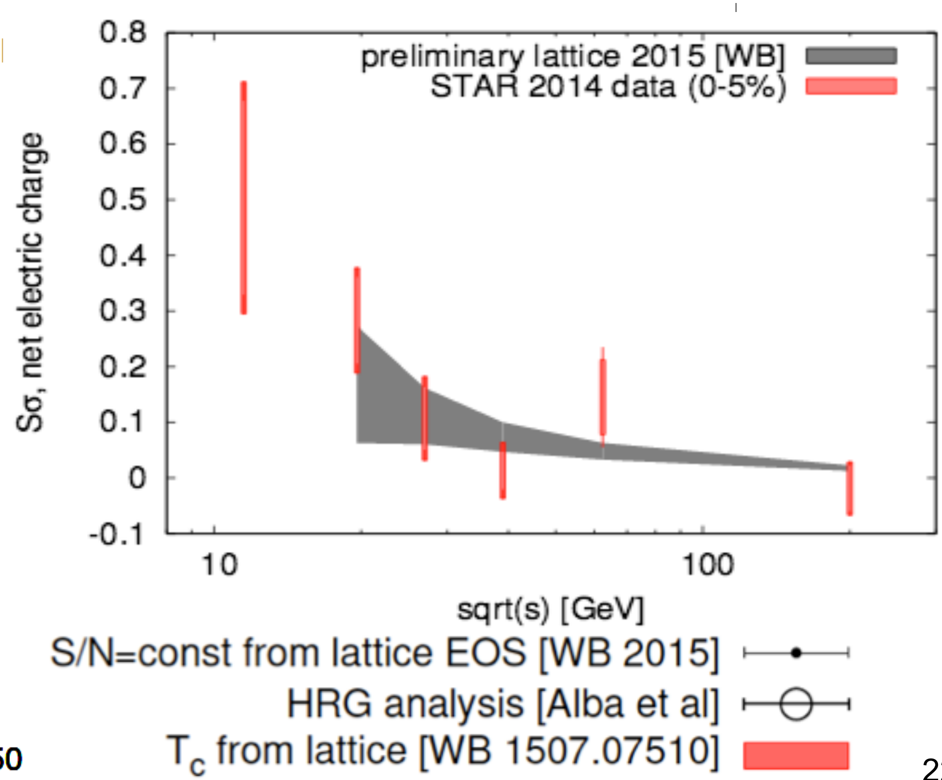
- Use T- and μ_B-dependence of R_{12}^Q and R_{12}^B for a combined fit:

$$R_{12}^Q(T,\mu_B) = \frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)} = \frac{\chi_{11}^{QB}(T,0) + \chi_2^Q(T,0)q_1(T) + \chi_{11}^{QS}(T,0)s_1(T)}{\chi_2^Q(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3).$$

$$R_{12}^B(T,\mu_B) = \frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{\chi_2^B(T,0) + \chi_{11}^{BQ}(T,0)q_1(T) + \chi_{11}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

WB: S. Borsanyi et al., in preparation

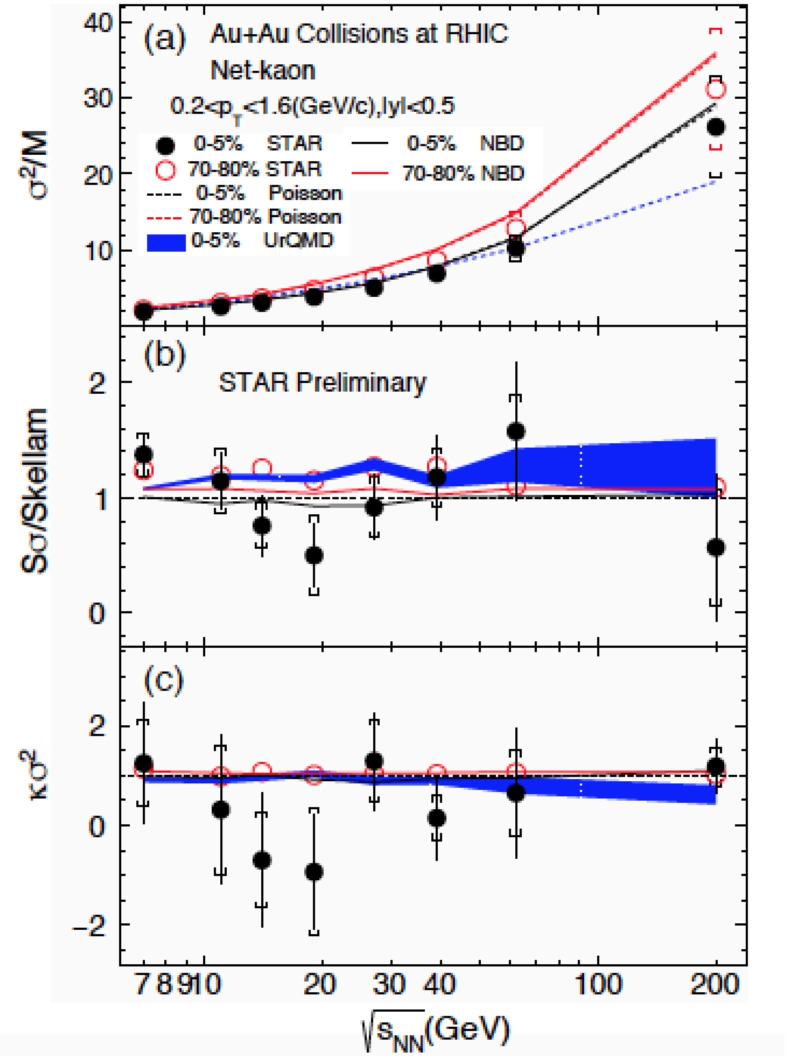




Kaon fluctuations

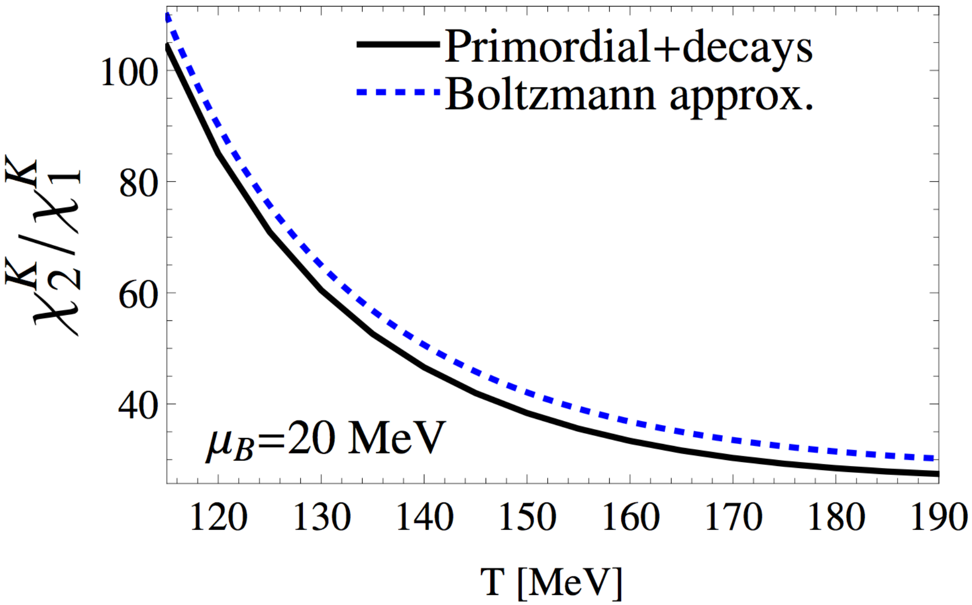
Talk by Ji XU at SQM 2016

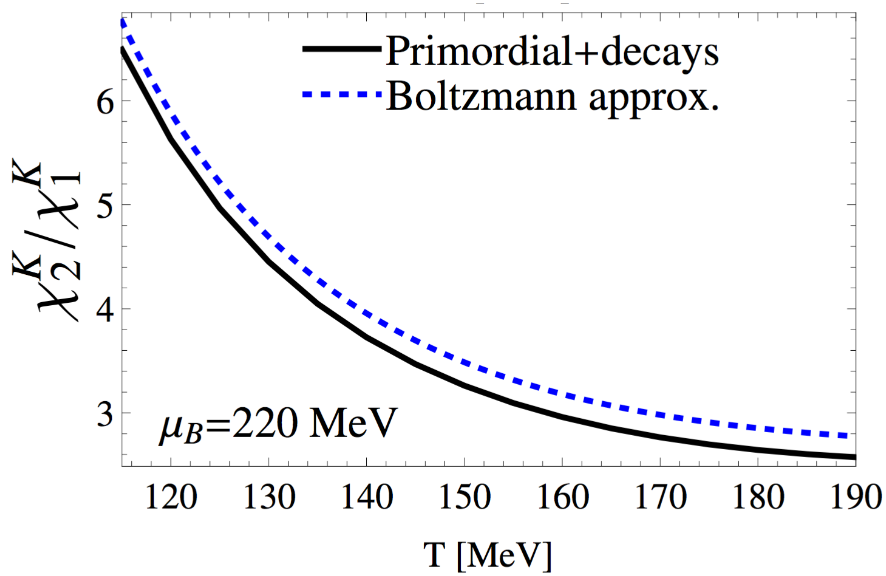
- Experimental data are becoming available.
- Exciting result but presently hampered by systematic errors
- BES-II will help
- Kaon fluctuations from HRG model will be affected by the hadronic spectrum and decays



Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al., 1607.02527





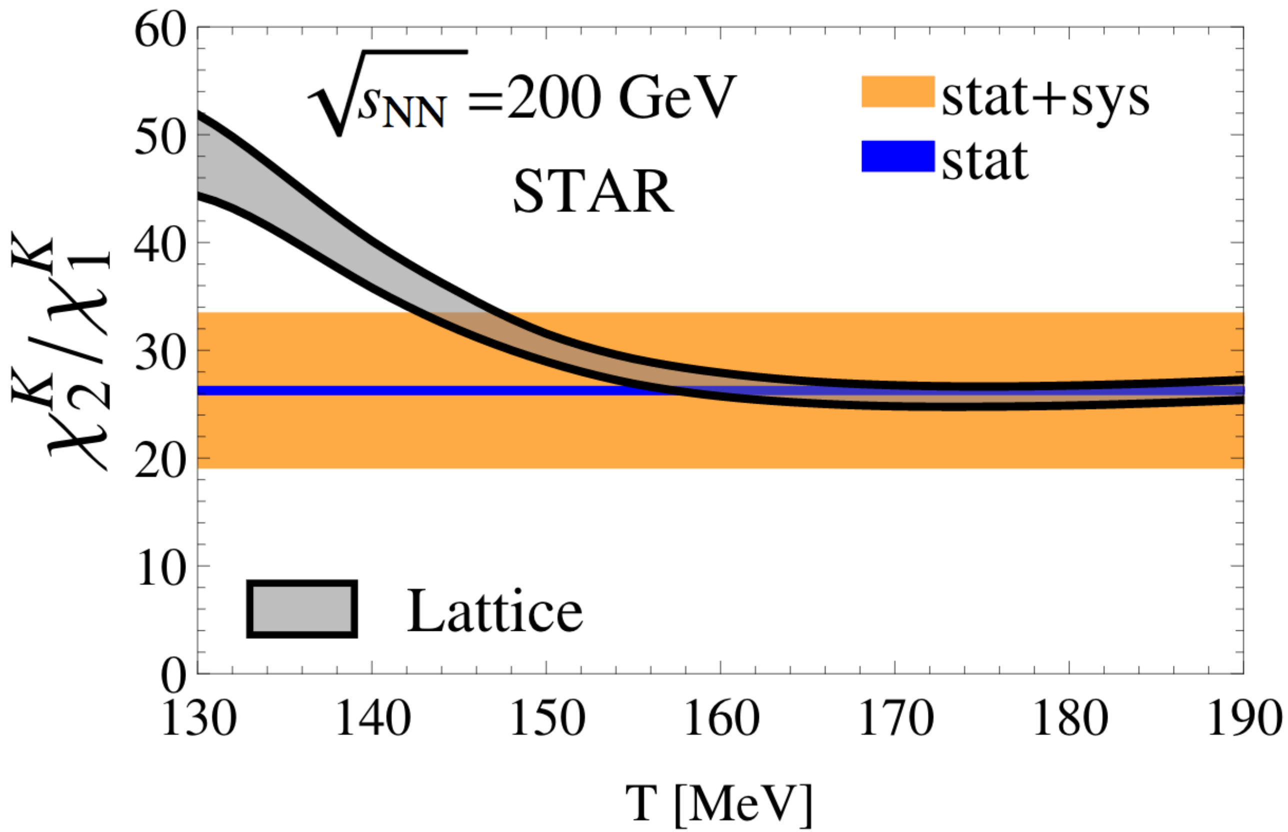
- Boltzmann approximation works well for lower order kaon fluctuations

$$\frac{\chi_2^K}{\chi_1^K} = \frac{\cosh(\hat{\mu}_S + \hat{\mu}_Q)}{\sinh(\hat{\mu}_S + \hat{\mu}_Q)}$$

- χ_2^K/χ_1^K from primordial kaons + decays is very close to the one in the Boltzmann approximation

Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al., 1607.02527



- Experimental uncertainty does not allow a precise determination of T_f^K

Conclusions

- Unprecedented precision in lattice QCD data allows a direct comparison to experiment for the first time
- QCD thermodynamics at μ_B=0 can be simulated with high accuracy
- Extensions to finite density are under control up to $O(\mu_B^6)$
- Comparison with experiment allows to determine properties of strongly interacting matter from first principles
- It is possible to identify kaon fluctuations in lattice QCD

Lattice details

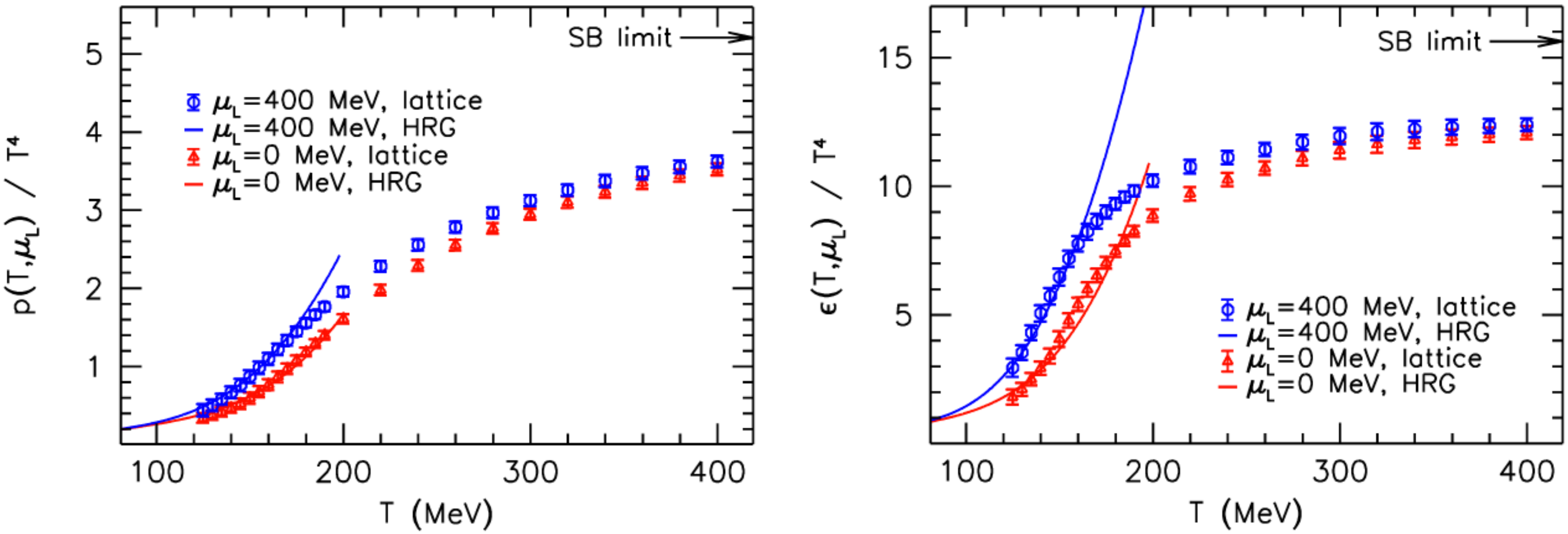
- **The 4stout staggered action**
  - 2+1+1 dynamical flavors
  - 4 levels of stout smearing in the fermionic action
  - The masses are set by bracketing both the pion and the kaon masses within a few percent, keeping $m_c/m_s=11.85$
  - The scale is set in two ways: f_π and w_0 (with Wilson flow). The scale setting procedure is one of the source of the systematic error in all of the plots

- **Ensembles**
  - Continuum limit from N_t=10, 12, 16
  - For imaginary μ we have $\mu_B=iT\pi j/8$, with j=3, 4, 5, 6, 6.5, 7

Equation of state at $\mu_B>0$

- Expand the pressure in powers of μ_B (or $\mu_L=3/2(\mu_u+\mu_d)$)

$$\frac{p(T,\{\mu_i\})}{T^4} = \frac{p(T,\{0\})}{T^4} + \frac{1}{2}\sum_{i,j}\frac{\mu_i\mu_j}{T^2}\chi_2^{ij} \quad \text{with} \quad \chi_2^{ij} \equiv \frac{T}{V}\frac{1}{T^2}\frac{\partial^2 \log \mathcal{Z}}{\partial\mu_i\partial\mu_j}\bigg|_{\mu_i=\mu_j=0}$$



S. Borsanyi et al., JHEP (2012)

- Continuum extrapolated results at the physical mass