From Susceptibilites to Correlations

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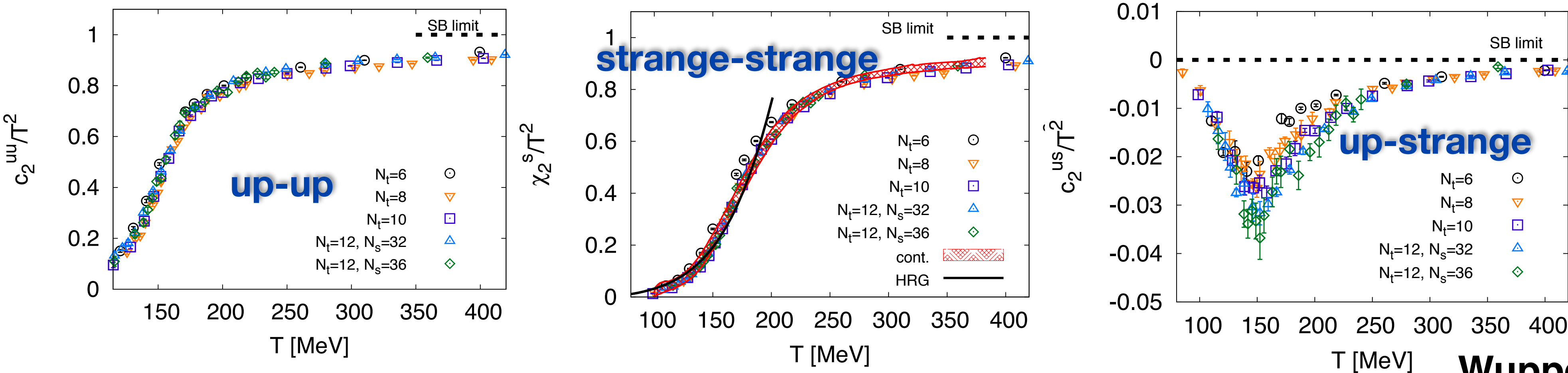
PARADIGM

Lattice: Susceptibilites & Fluctuations



Wuppertal

$$\frac{1}{V}\langle \delta E \delta E \rangle = T^2 \frac{\partial \epsilon}{\partial T} = T^2 C_V$$

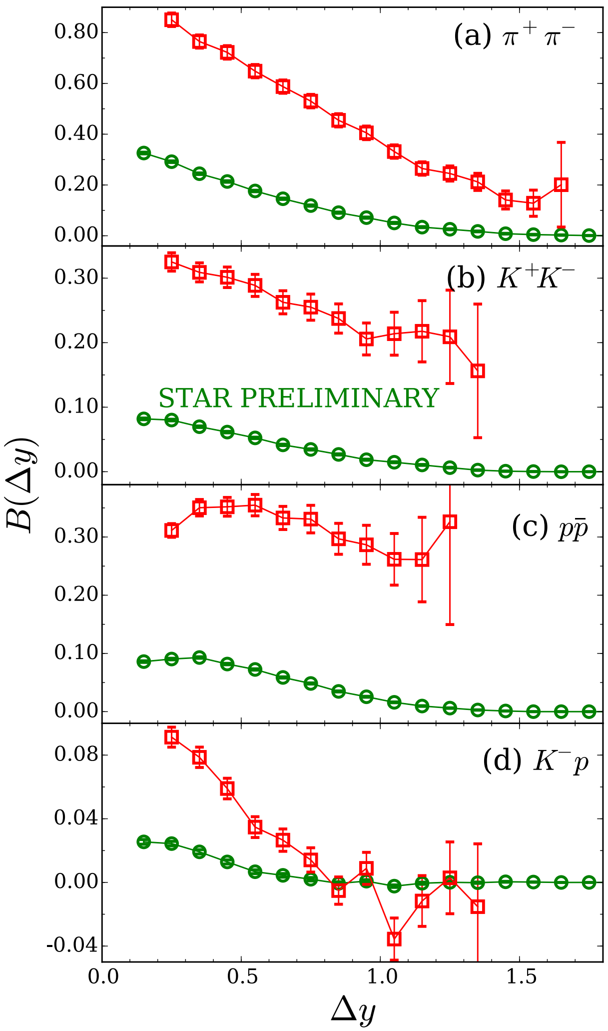
$$\frac{1}{V}\langle \delta P_z \delta P_z \rangle = (P+\epsilon)T = h$$

$$\frac{1}{V}\langle \delta Q_a \delta Q_b \rangle = T \frac{\partial \rho_a}{\partial \mu_b} = \chi_{ab}$$

$$\frac{1}{V}\langle \delta Q_a \delta V \rangle = T \frac{\partial \epsilon}{\partial \mu_a}$$

Grand Canonical Ensemble:

- **Energy/charge fluctuate**
- **heat/particle bath**
- **infinite equilibration time**



Experiment: Correlations

$$C(\delta E(0)\delta E(y))$$

$$C(\delta N_i(0)\delta N_j(y))$$

$$C(\delta E(0)\delta Q_i(y))$$

Final State:

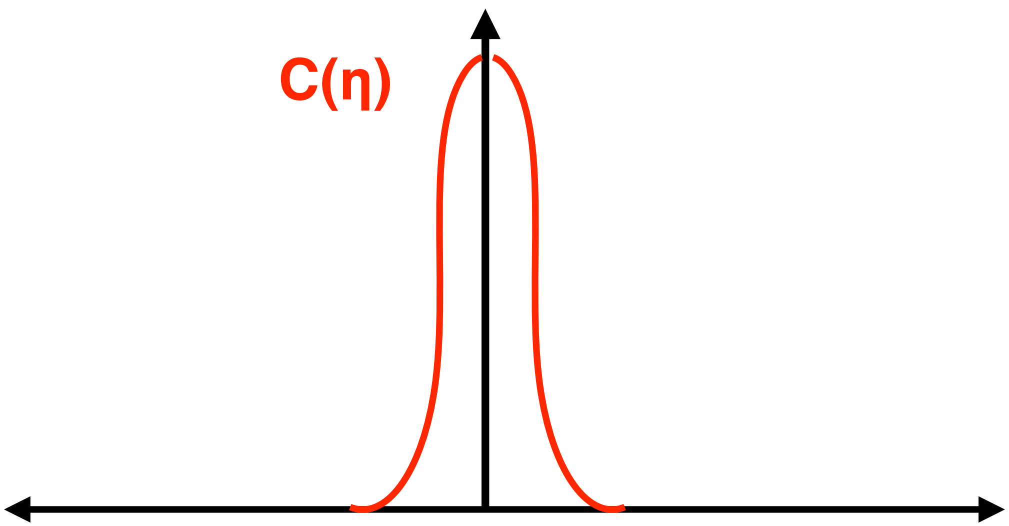
- **Charge/Energy/Momentum conserved**
- **Finite time for quantities to move**
- **Must translate to momentum space**
- ***δP_z* not accessible**

Correlations in Equilibrated System

$$\int d\eta \frac{1}{V} \langle \delta Q_a(0) \delta Q_b(\eta) \rangle = \chi_{ab}$$

$$\int d\eta \frac{1}{V} \langle \delta E(0) \delta E(\eta) \rangle = T^2 C_V$$

$$\int d\eta \frac{1}{V} \langle \delta P_z(0) \delta P_z(\eta) \rangle = (P + \epsilon) T$$



Correlations are *LOCAL*

$$\langle \delta Q_a(0) \delta Q_b(\eta) \rangle = \chi_{ab} \delta(\eta)$$

$$\langle \delta E(0) \delta E(\eta) \rangle = T^2 C_V \delta(\eta)$$

$$\langle \delta P_z(0) \delta P_z(\eta) \rangle = (P + \epsilon) T \delta(\eta)$$

Correlations in HI Environment

$$\int d\eta \frac{1}{V} \langle \delta Q_a(0) \delta Q_b(\eta) \rangle = 0$$

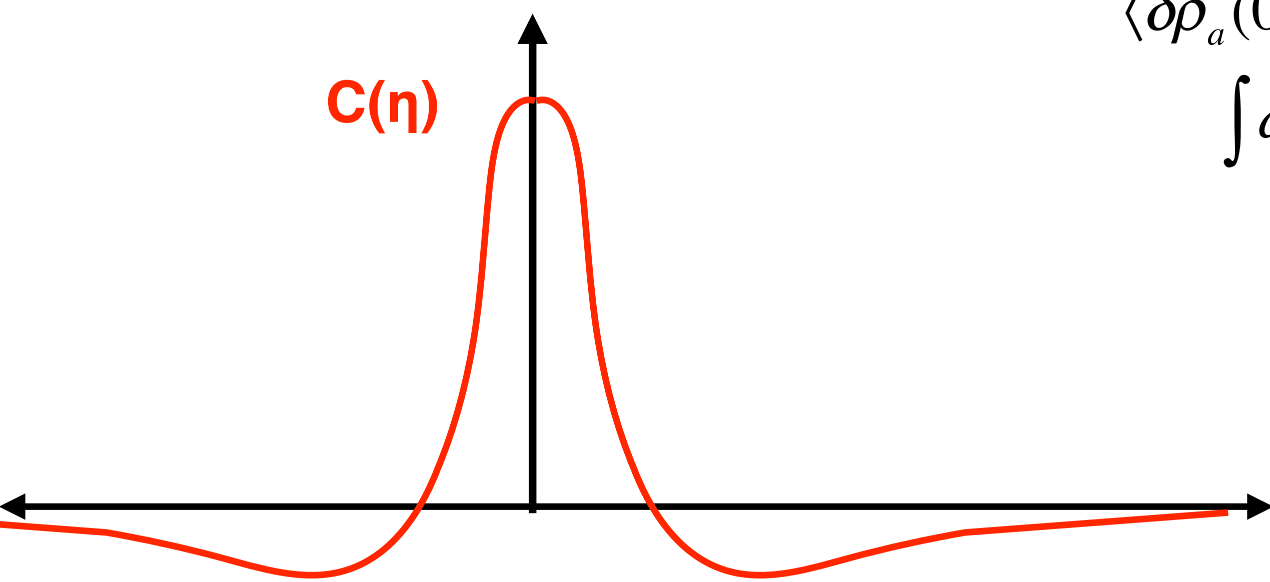
$$\int d\eta \frac{1}{V} \langle \delta E(0) \delta E(\eta) \rangle = 0$$

$$\int d\eta \frac{1}{V} \langle \delta P_z(0) \delta P_z(\eta) \rangle = 0$$

Non-local correlation: *C'*

$$\langle \delta\rho_a(0) \delta\rho_b(\eta) \rangle = \chi_{ab} \delta(\eta) + C'_{ab}(\eta)$$

$$\int d\eta\, C'_{ab}(\eta) = -\chi_{ab}$$



Evolution of C′

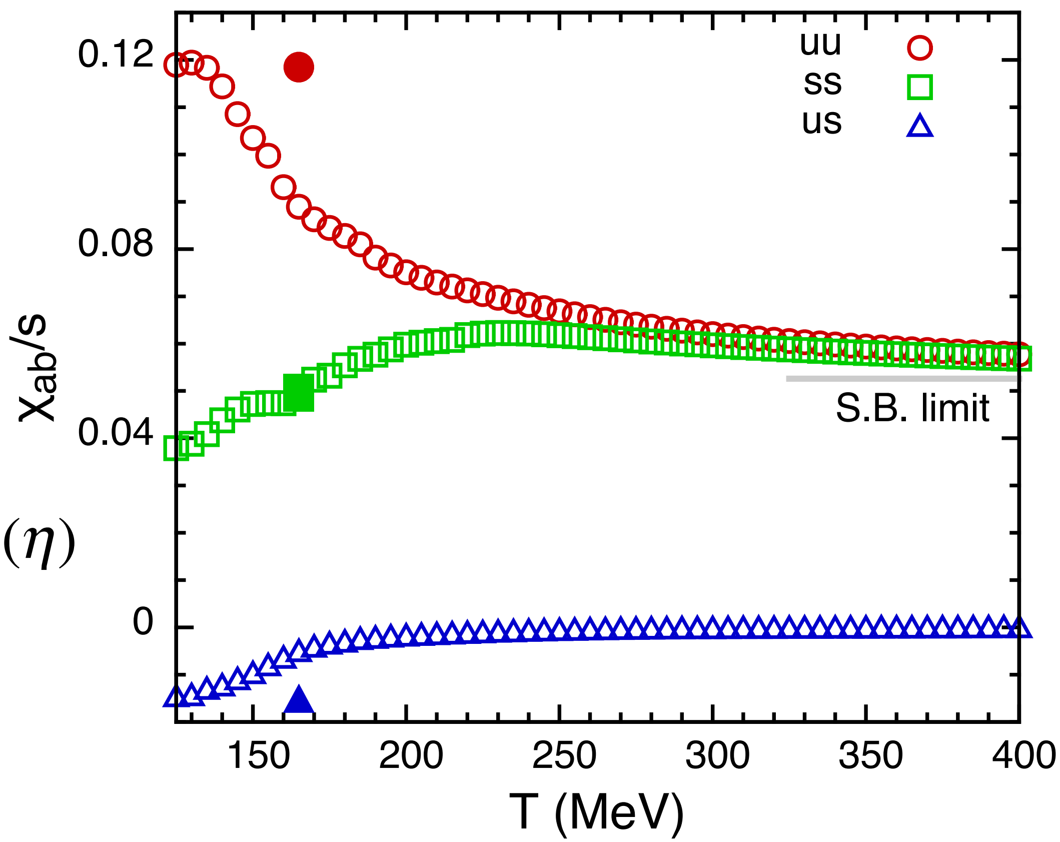
$$\partial C'(\eta,\tau) = \frac{1}{\tau}\partial_\eta(.......)\ \{\text{local cons.}\}$$

Charge-Charge

charge moves diffusively

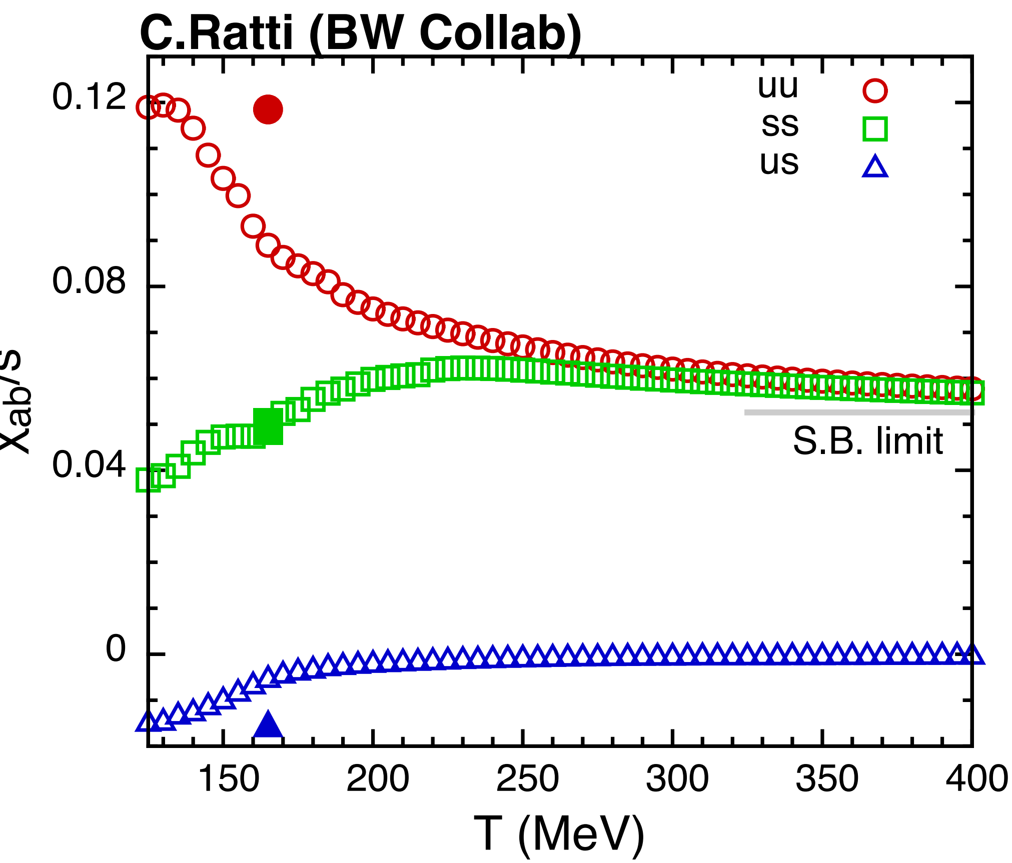
$$\partial C'_{ab}(\eta,\tau) = \frac{1}{\tau}\partial_\eta\left\{2D\,\partial_\eta\, C'(\eta,\tau)\right\} + S(\tau)\delta(\eta)$$

$$S(\tau) = -\partial_\tau\left(\frac{\chi_{ab}}{s}\right)$$



PRODUCTION

Charge Sources



$$\partial_\tau C_{ab}(\tau,\eta) - \frac{2D}{\tau^2}\partial_\eta^2 C_{ab}(\tau,\eta) = \delta(\eta) S_{ab}(\eta)$$

$$S_{ab}(\eta) = -\frac{dS}{d\eta}\frac{d}{d\tau}\left(\frac{\chi_{ab}}{s}\right)$$

Source determined by changing susceptibility

P_x-P_x (transverse momentum)

Source defined by change in *T*

$$C_{xx}(\eta) = \langle \delta P_x(0)\delta P_x(\eta)\rangle$$

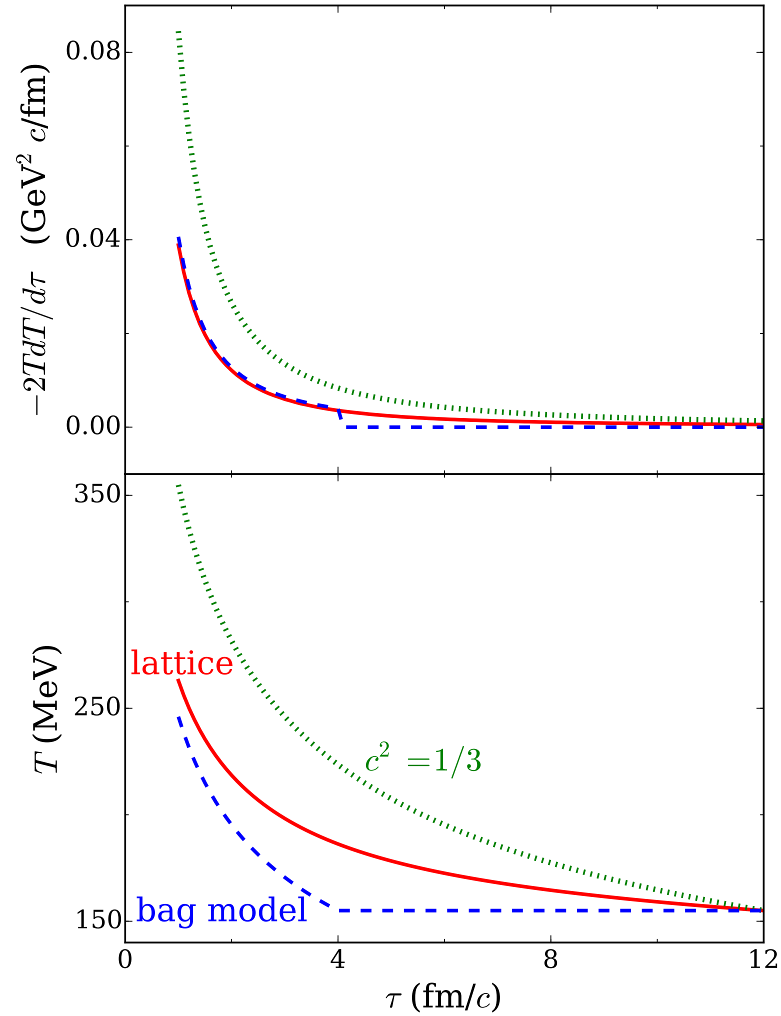
$$\delta P_x(\eta) \equiv \int dxdy\ \delta T_{0x}(\eta)$$

$$\partial_\tau C'_{xx}(\eta,\tau) = \frac{2D}{\tau^2}\partial_\eta^2 C'_{xx}(\eta,\tau) + S_{xx}(\tau)\delta(\eta),$$

$$S_{xx}(\tau) = -\pi R^2 \frac{d}{d\tau}\left[(P+\epsilon)T\tau\right]$$

$$= -\frac{dS}{d\eta}\frac{d}{d\tau}T^2.$$

(no transverse flow)



E_t-E_t & P_z-P_z

P_z and E_t evolve together

becomes E_t -E_t correlation

$$C_{TT}(\tau,\eta) = \tau^2 \cosh\eta \langle \delta\tilde{T}_{00}(\tau,0)\delta\tilde{T}_{00}(\tau,\eta) - \delta\tilde{T}_{0z}(\tau,0)\delta\tilde{T}_{0z}(\tau,\eta)\rangle$$
$$+ 2\tau^2 \sinh\eta \langle \delta\tilde{T}_{00}(\tau,0)\delta\tilde{T}_{0z}(\tau,\eta)\rangle.$$
$$C_{LT}(\tau,\eta) = \tau^2 \langle \delta\tilde{T}_{00}(\tau,0)\delta\tilde{T}_{00}(\tau,\eta) - \delta\tilde{T}_{0z}(\tau,0)\delta\tilde{T}_{0z}(\tau,\eta)\rangle \sinh\eta + 2\tau^2 \langle \delta\tilde{T}_{00}(\tau,0)\delta\tilde{T}_{0z}(\tau,\eta)\rangle \cosh\eta$$
$$C_{LL}(\tau,\eta) = \tau^4 \langle \delta\tilde{T}_{0z}(\tau,0)\delta\tilde{T}_{0z}(\tau,\eta)\rangle$$

becomes P_z-P_z correlation

E_t-E_t & P_z-P_z

T^2/c^2 related to specific heat

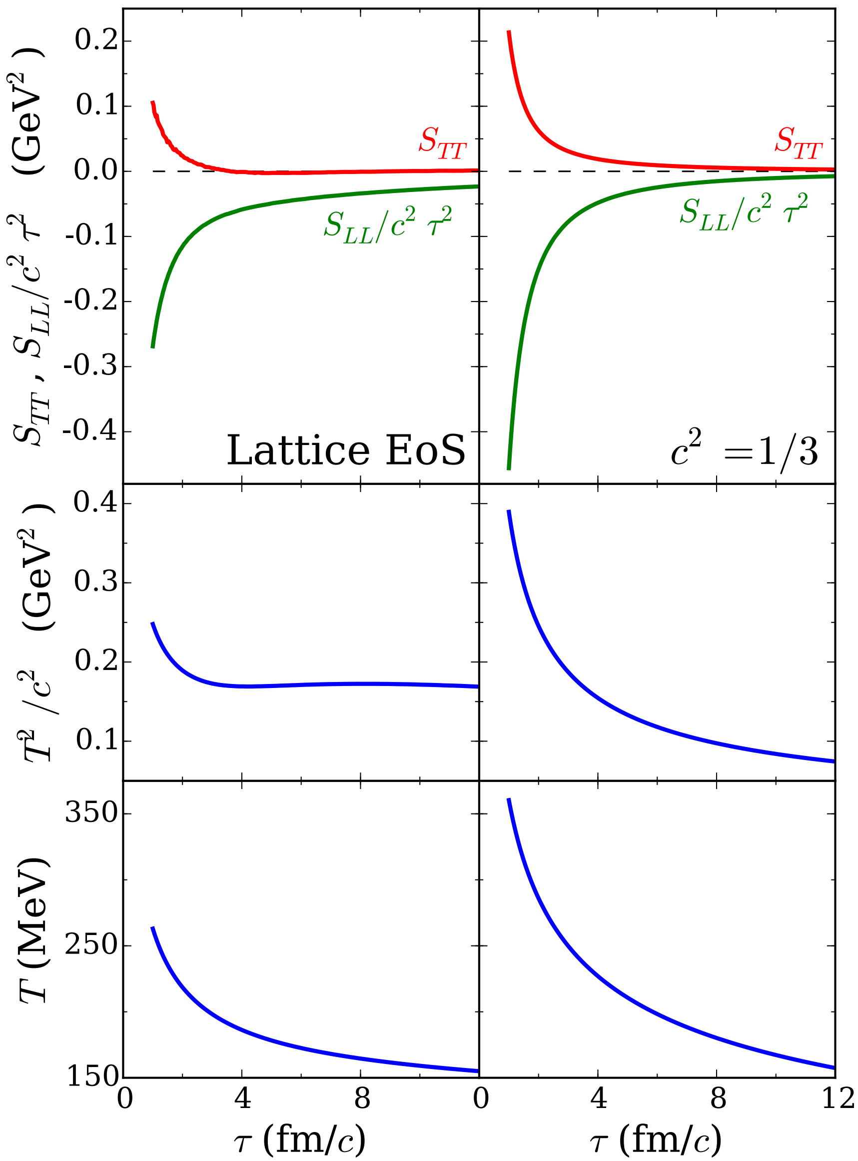
$$S_{TT}(\tau) = -\frac{dS}{d\eta}\partial_\tau \left\{ T^2 \left(\frac{1}{c^2} - 1 \right) \right\}$$

$$S_{LL}(\tau) = -\frac{dS}{d\eta}\partial_\tau \left\{ \tau^2 T^2 \right\},$$

$$S_{LT} = 0$$

Same as P_x-P_x source

E_t-E_t & P_z-P_z



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$$S_{LT} = 0$$

Same as P_x-P_x source

PROPAGATION

P_x-P_x (transverse momentum)

Spread is diffusive, *D* related to shear viscosity

$$C_{xx}(\eta) = \langle \delta P_x(0) \delta P_x(\eta) \rangle$$

$$\delta P_x(\eta) \equiv \int dxdy\ \delta T_{0x}(\eta)$$

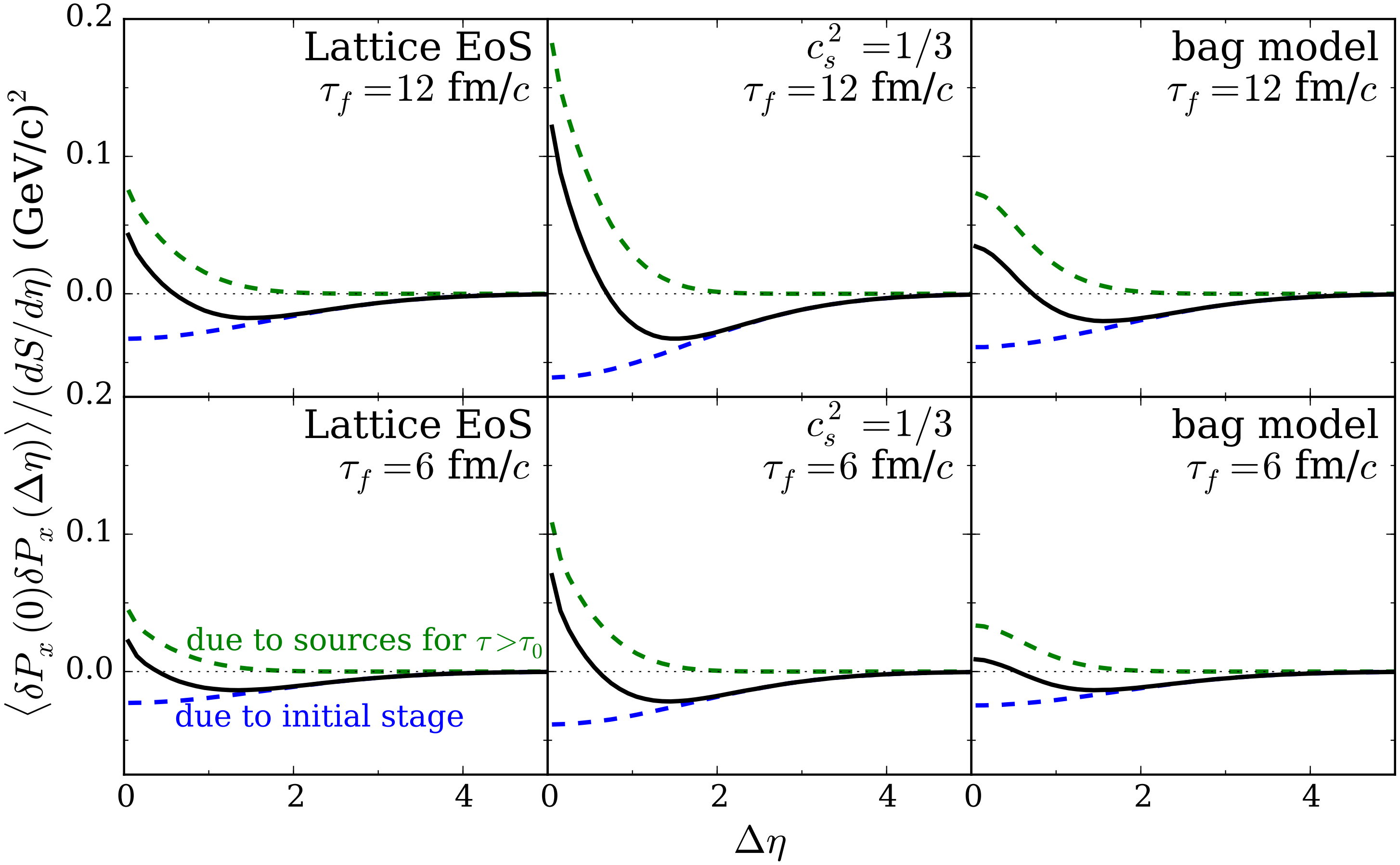
$$\partial_\tau C'_{xx}(\eta,\tau) = \frac{2D}{\tau^2} \partial_\eta^2 C'_{xx}(\eta,\tau) + S_{xx}(\tau)\delta(\eta),$$

$$S_{xx}(\tau) = -\pi R^2 \frac{d}{d\tau}\left[(P+\epsilon)T\tau\right]$$

$$= -\frac{dS}{d\eta}\frac{d}{d\tau}T^2.$$

$$D = \frac{\eta_s}{P+\epsilon}$$

P_x-P_x (transverse momentum)



Result depends on:

- **Initial *C′***
- **Eq. of State**
- **Viscosity**

E_t-E_t & P_z-P_z

$$C_{TT}(\tau,\eta) = \tau^2 \cosh\eta \langle \delta\tilde{T}_{00}(\tau,0)\delta\tilde{T}_{00}(\tau,\eta) - \delta\tilde{T}_{0z}(\tau,0)\delta\tilde{T}_{0z}(\tau,\eta)\rangle$$
$$+ 2\tau^2 \sinh\eta \langle \delta\tilde{T}_{00}(\tau,0)\delta\tilde{T}_{0z}(\tau,\eta)\rangle.$$
$$C_{LT}(\tau,\eta) = \tau^2 \langle \delta\tilde{T}_{00}(\tau,0)\delta\tilde{T}_{00}(\tau,\eta) - \delta\tilde{T}_{0z}(\tau,0)\delta\tilde{T}_{0z}(\tau,\eta)\rangle \sinh\eta + 2\tau^2 \langle \delta\tilde{T}_{00}(\tau,0)\delta\tilde{T}_{0z}(\tau,\eta)\rangle \cosh\eta$$
$$C_{LL}(\tau,\eta) = \tau^4 \langle \delta\tilde{T}_{0z}(\tau,0)\delta\tilde{T}_{0z}(\tau,\eta)\rangle$$

Correlation mixes up various components

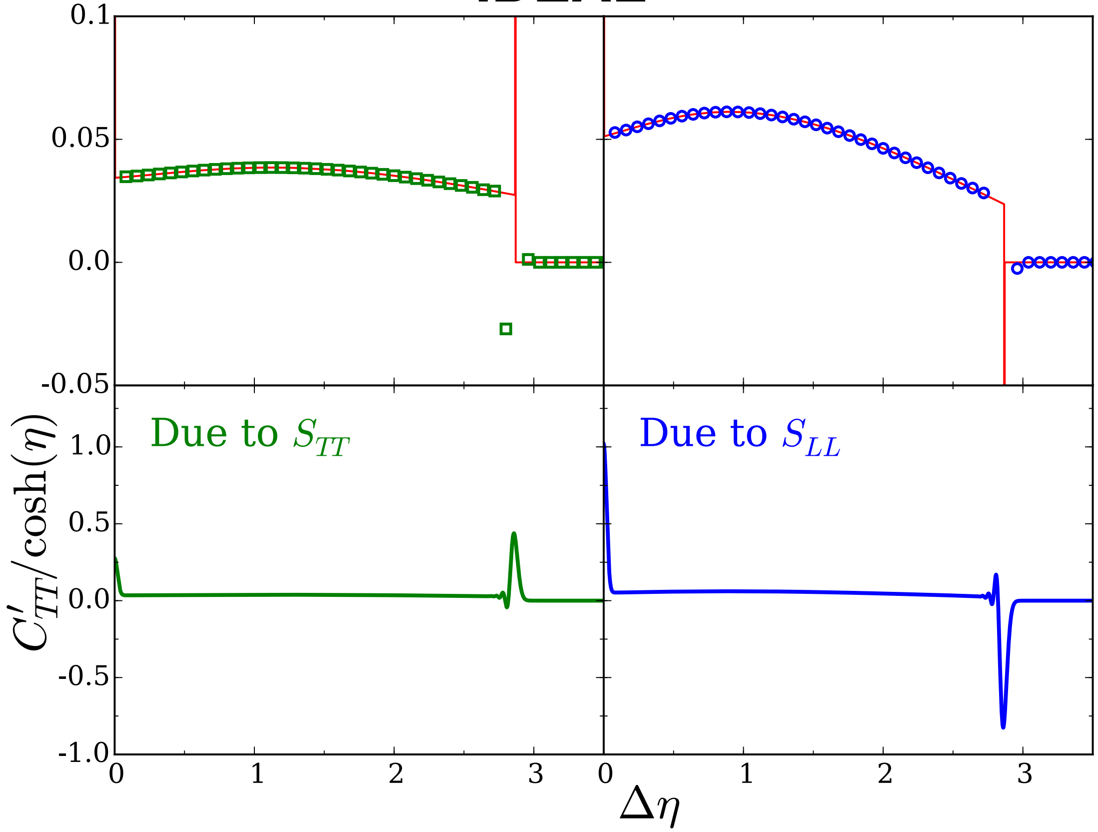
$$\tau\partial_\tau C'_{TT} + 2\partial_\eta \Big\{ \frac{(1+c^2)}{2} \cosh\eta(\cosh\eta\; C_{LT} - \sinh\eta\; C_{TT}) + c^2 \sinh\eta(\cosh\eta\; C_{TT} - \sinh\eta\; C_{LT} + (1/\tau^2)C_{LL})$$
$$- \sinh\eta(1/\tau^2)C_{LL} \Big\} = (S_{TT}/A)\delta(\eta),$$

$$\tau\partial_\tau C'_{LT} + 2\partial_\eta \Big\{ c^2 \cosh\eta(\cosh\eta\; C'_{TT} - \sinh\eta\; C_{LT} + (1/\tau^2)C'_{LL}) - \cosh\eta\; (1/\tau^2)C'_{LL}$$
$$+ \frac{(1+c^2)}{2} \sinh\eta(\cosh\eta\; C'_{LT} - \sinh\eta\; C'_{TT}) \Big\} = (S_{LT}/A)\delta(\eta),$$
$$\partial_\tau C'_{LL} + \tau^2 c^2 \partial_\eta \left\{ -\cosh\eta\; C'_{LT} + \sinh\eta\; C'_{TT} \right\} = (S_{LL}/A)\delta(\eta).$$

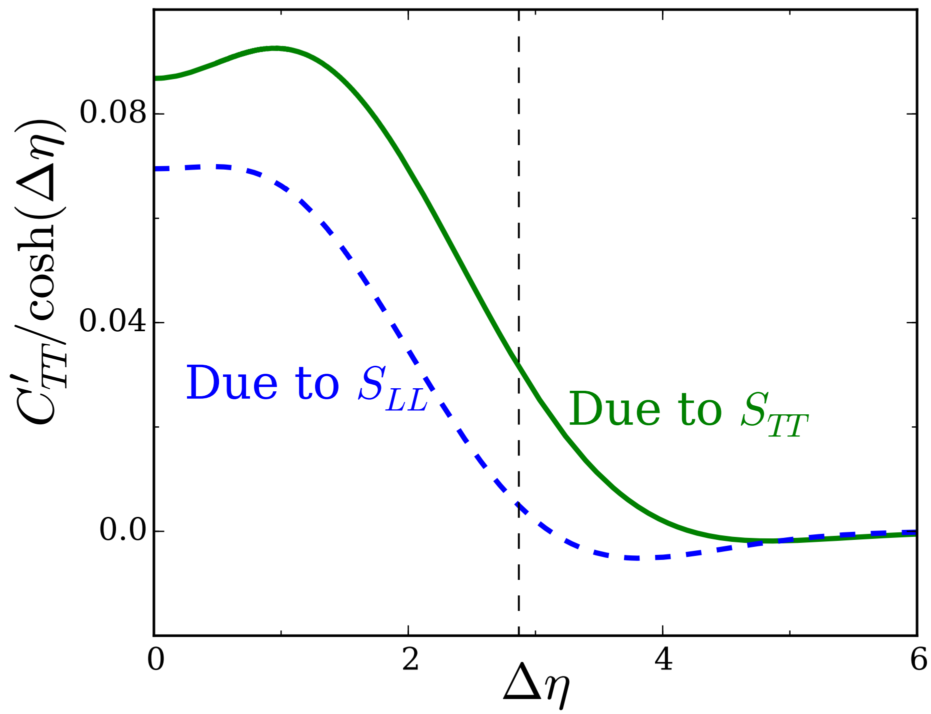
E/P_z Evolution is COMPLICATED

Evolve Pulse from τ=1 fm/c to τ=12 fm/*c*

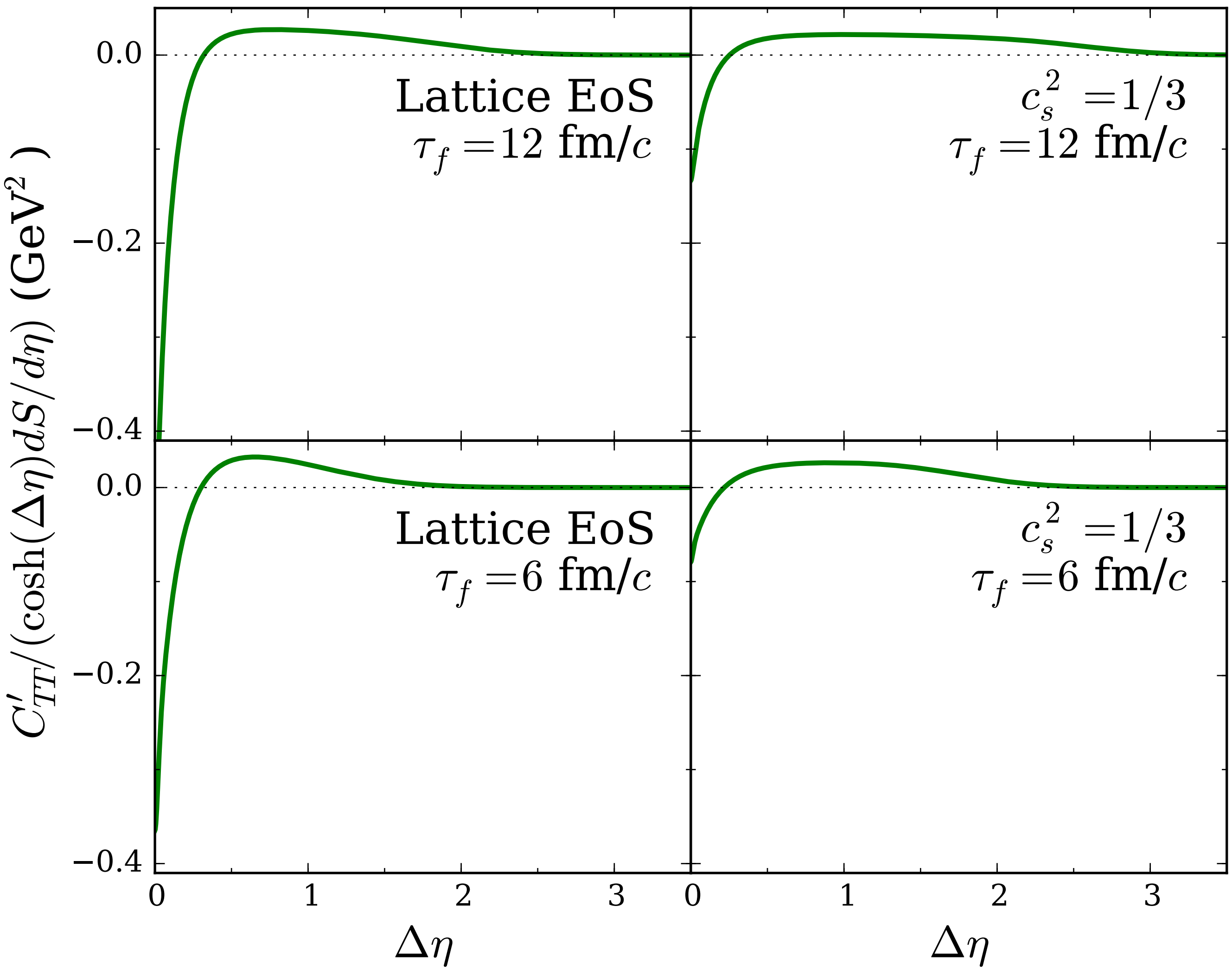
IDEAL



With Viscosity



E_t-E_t & P_z-P_z



- **Ignores initial source & transverse expansion, What about jets?**

- **Sensitive to EoS, but COMPLICATED**

PROJECTION

Projecting C(Δη) to C(Δy)
either final state or cascade

$$f_h(\mathbf{p}) = f_h^{(0)} e^{\lambda_{a,\mu}\delta j^{\mu}_{h,a}(\mathbf{p})},$$

$$j^{\mu}_{h,a}(\mathbf{p}) = q_{h,a}\frac{p^{\mu}}{E_p},$$

Alter *f(p)* by Lagrange Multipliers λ_a

$$\delta f_h(\mathbf{p}) \approx f_h^{(0)}(\mathbf{p})\ \lambda_{a,\mu}\delta j^{\mu}_{h,a}(\mathbf{p})$$

q_a* could refer to any "charge" including *E* or *P

$$\chi^{\mu\nu}_{ab} = \sum_h \int \frac{d^3p}{(2\pi)^3} f_h^{(0)}(\mathbf{p}) q_{h,a}\frac{p^{\mu}}{E_p}\frac{p^{\nu}}{E_p} q_{h,b}$$

$$\lambda^{\mu}_a = (\chi^{-1})^{\mu\nu}_{ab}\delta j_{\nu,b}$$

Projecting C(Δη) to C(Δy)
either final state or cascade

$\delta\rho\ d\Omega_0 + \delta j_i d\Omega_i$ is charge that moves through hypersurface $d\Omega^\mu$

$$\delta dN_h = d\Omega_0 \frac{d^3p}{(2\pi)^3} f^{(0)}(\mathbf{p}) q_{h,a} \chi^{-1}_{ab} \delta\rho_b,$$

$$+ d\Omega_i \frac{d^3p}{(2\pi)^3 E_p} f^{(0)}(\mathbf{p}) q_{h,a} \frac{p_i}{E_p} \left(\chi^{(\mathrm{J})}\right)^{-1}_{ij,ab} \delta j_{b,j}$$

$$\chi^{(\mathrm{J})}_{ij,ab} = \delta_{ij} \chi^{(\mathrm{J})}_{ab},$$

$$= \delta_{ij} \frac{1}{3} \sum_h \int \frac{d^3p}{(2\pi)^3} f^{(\mathrm{eq})}_h q_{h,a} q_{h,b} \frac{|\mathbf{p}|^2}{E_p^2}$$

Works for all species, any hyper-surface, *dΩ*

PRACTICE

Algorithm I: Solve Correlations

$$\partial_\tau C_{ab}(\tau,\eta) - \frac{1}{\tau}\partial_\eta\{.......\} = S_{ab}(\tau)\delta(\eta)$$

- **Reasonable for Bjorken geometry**
- **Impractical for transverse flow or boost invariance**
- **For BES, Q_a, P_x, P_z, E_t all mix**

Algorithm II: Propagate Qa & Qb separately

$$d\sigma_{ab} = d^4x S_{ab}(x)$$

- **Find source at each point in space-time**

$$d\sigma_{ab} \rightarrow d\tilde{\sigma}_{ab} = \begin{pmatrix} \tilde{\sigma}_{11} & 0 & 0 \\ 0 & \tilde{\sigma}_{22} & 0 \\ 0 & 0 & \tilde{\sigma}_{33} \end{pmatrix}$$

- **diagonalize**

Make charges in new basis with random signs

$$d\tilde{q}_a^{(1)} = (\pm)\sqrt{|d\tilde{\sigma}_{aa}|},$$
$$d\tilde{q}_b^{(2)} = d\tilde{\sigma}_{aa}/d\tilde{q}_a^{(1)}$$

- **Return to original basis**

$$dq_a = U_{ab}^{-1} d\tilde{q}_b$$

- **Propagate (1) and (2) in separate evolutions**
- **Correlate (1) with (2)**
- **Add correlations from smooth distribution to account for local correlation**

Algorithm II: Propagate Qa & Qb separately

SOME CHOICES

- **All source points together or one-at-a-time**
  one-at-a-time, no noise but need to sample S(x)
- **For purely diffusive, random walk works great,**
  label partners for (1) and (2)
  no need to rerun hydro evolution

Algorithm III: Thermal Noise

Kapusta, Mueller, Young, Stephanov…

- **Add external "currents" that are correlated**

$$\langle j_i^{(\mathrm{n})}(x_1) j_k^{(\mathrm{n})}(x_2)\rangle = 2\sigma T \delta_{ik}\delta^4(x_1 - x_2)$$

conductivity

- **Solve one-particle evolution**
- **Generates local "delta" function**

$$C(r_1 - r_2) = \chi\delta(r_1 - r_2) + ...$$

- **Correlate event with itself**
- **Must avoid double counting of local correlation**
- **Identical to previous methods for small mesh size**

What's best?

For purely diffusive evolution of charges:

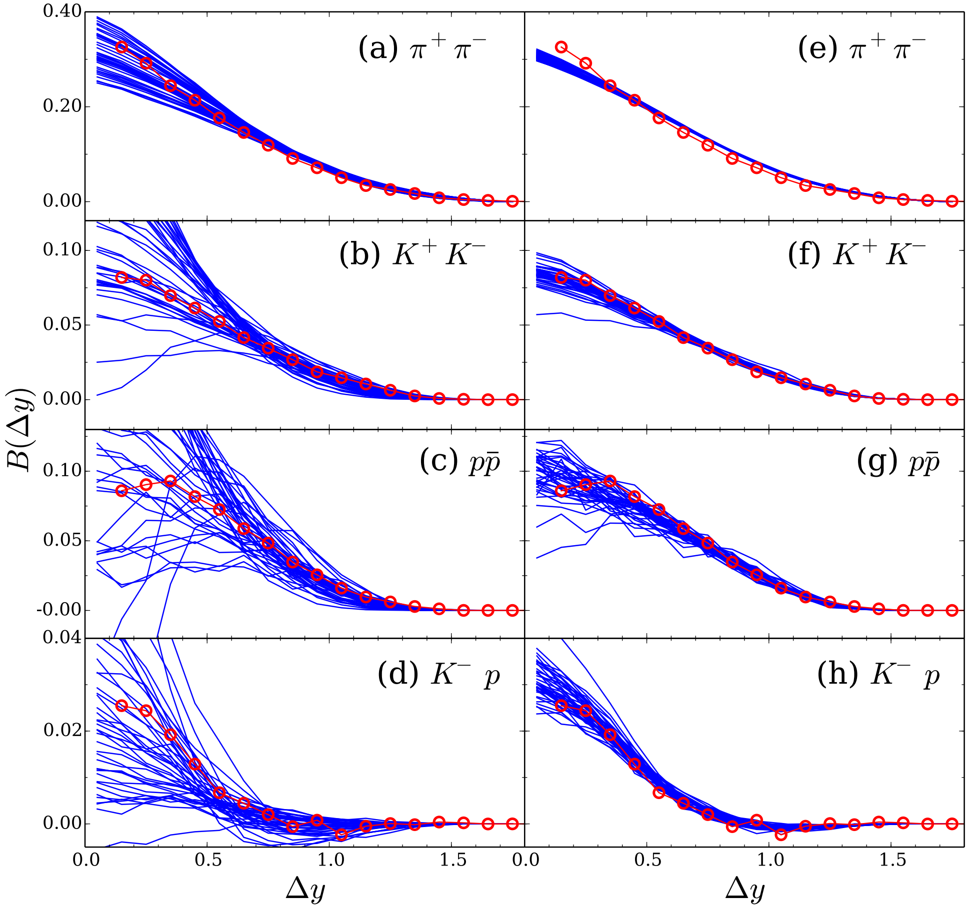
- **Algorithm II with random walk**

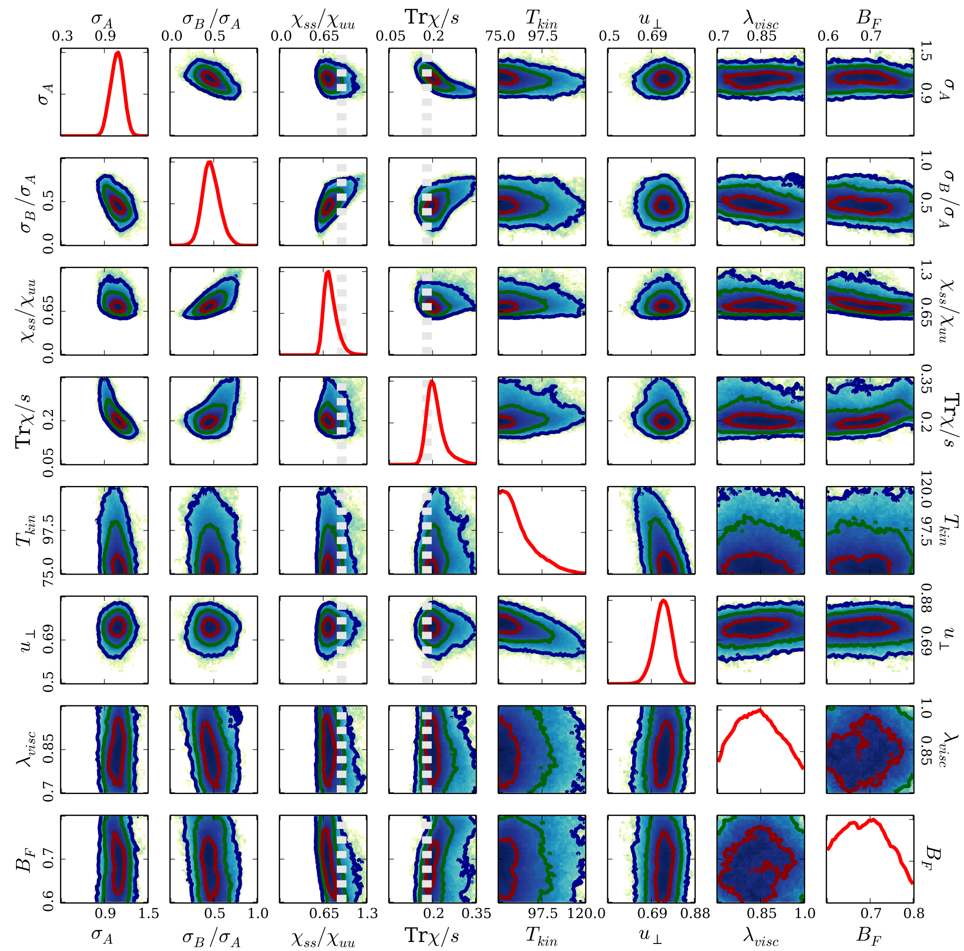
For energy/momentum:

- **Not obvious**
- **Both II and III require thousands of calculations to overcome noise or sample *S*.**

PREDICTIONS

STAR BALANCE FUNCTIONS VS. TWO-SURGE MODEL





Demonstrates ~ chemical equilibrium of QGP phase!!!

PROMISE

Charge Correlations

- 6-D measurement, C(P,Q)
- Chemical evolution
- Determine Diffusion Constant
- Understand initial thermalization of quarks

Energy Momentum

- Much more difficult to model
- Extra handle on EoS and viscosity
- Phase transition signal
- Understand initial-state correlations / jets

Limitations

- Assume instantaneous local equilibration
- What about N>2 correlations?