# Equation of state and transport coefficients at finite baryo-chemical potential

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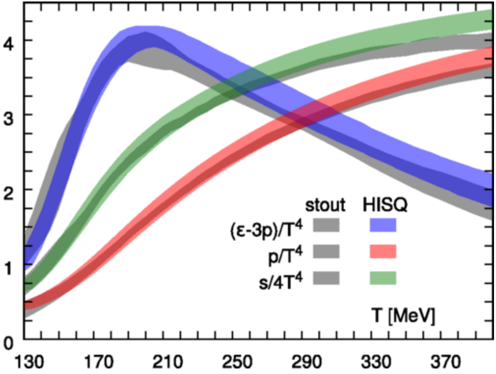
Exploring the QCD Phase Diagram through Energy Scans, INT
October $6^{th}$ 2016

# Outline

# The Success of Lattice QCD

Equation of State agrees for stout (WB) and HISQ (HotQCD) actions

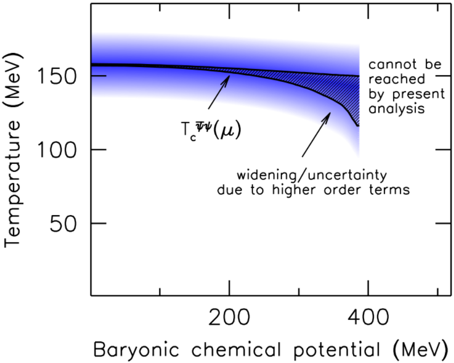



WB Phys.Lett. B730 (2014) 99-104
HotQCD Phys.Rev. D90 (2014) 094503

# Limitations at Large $\mu_B$ (Sign problem)

Taylor expand pressure in term of $\mu_B$, limits results for large $\mu_B$

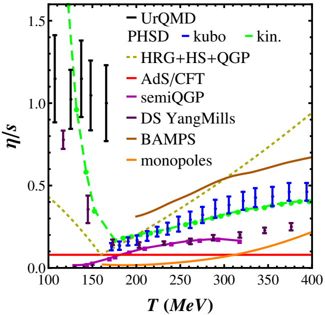
$$\frac{P(\mu_B)}{T^4} = c_0 + c_2 \left(\frac{\mu_B}{T}\right)^2 + c_4 \left(\frac{\mu_B}{T}\right)^4 + c_6 \left(\frac{\mu_B}{T}\right)^6 + \mathcal{O}(\mu_B^8)$$

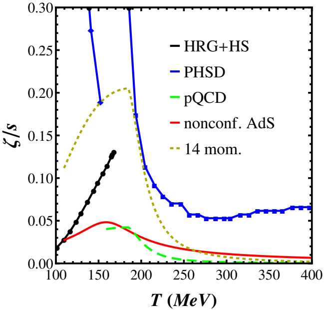



Phys.Lett. B751 (2015) 559-564

# Limitations for transport properties

- Lattice has technical difficulties to compute transport properties
- We're left with a number of models that don't converge...




JNH arXiv:1512.06315 (see for references)

# Filling in the gaps with Black Hole Engineering

What we need...

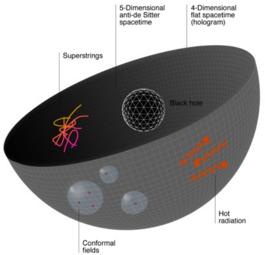
- Strongly coupled system
- Non-conformal equation of state
  - Equation of State at large baryon chemical potentials
  - Critical Point
- Perfect fluidity
  - Ability to compute transport coefficients near crossover and at large $\mu_B$

**One alternative**

Gauge/gravity duality - Black Hole Engineering

# Gauge/gravity duality

Maldacena, Gubser, Polyakov, Witten, 1998




Strong coupling limit of QFT in 4 dimensions (with many d.o.f.)

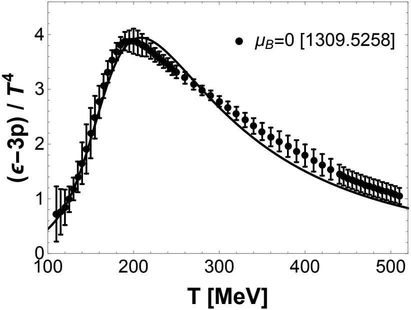
$\Updownarrow$

String Theory/Classical gravity in d>4 dimensions

curved spacetime $= (t, x, y, z, r)$ where $r$ is the holographic coordinate

# non-conformal Equation of State

$$S = \frac{1}{2\kappa^2}\int_{\mathcal{M}_5} d^5x\sqrt{-g}\left[\mathcal{R} - \frac{1}{2}(\partial_\mu\phi)^2 - \underbrace{V(\phi)}_{\substack{\phi\neq const \\ (nonconformal)}} - \underbrace{\frac{f(\phi)}{4}F_{\mu\nu}^2}_{\mu_B\neq 0}\right]$$



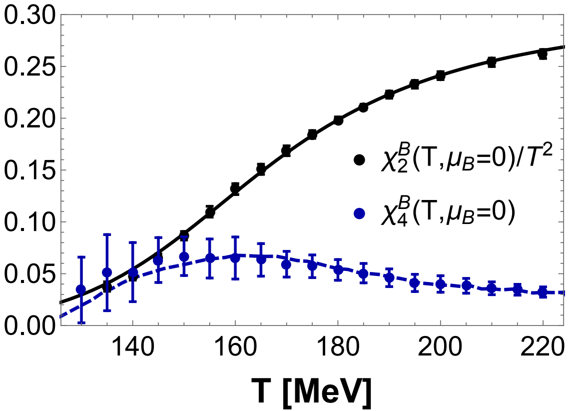

$\kappa^2$ gravitational constant
$V(\phi)$ dilaton potential
$f(\phi)$ Maxwell-Dilaton coupling
all fixed to lattice data at $\mu_B = 0$

$\rightarrow$ non-conformality!
Allows for $\zeta/s > 0$!

See also: DeWolfe, Gubser, Rosen
PRD83(2011)086005;PRD84(2011)126014
Rougemont et al JHEP1604(2016)102;
Rougemont,Noronha,JNH,PRL115(2015)no.20,202301

# Baryon susceptibilities

Derivatives of the pressure $\chi_n^B = \partial^n p/\partial\mu_B^n = \partial^{n-1}\rho/\partial\mu_B^{n-1}$




R. Rougemont,J. Noronha, JNH, PRL115(2015)no.20,202301

# Perfect Fluidity

Shear viscosity to entropy density*

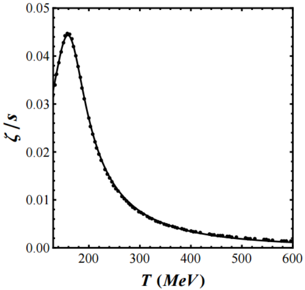
$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Kovtun,Son,Starinets, 2005

***The AdS/CFT bound that wasn't:**

- Magnetic field violates KSS limit
  PRD90(2014)no.6,066006
- $\eta/s(T)$ higher-order derivatives of the action
  PRD77(2008)126006

non-conformal EOS needed for bulk viscosity



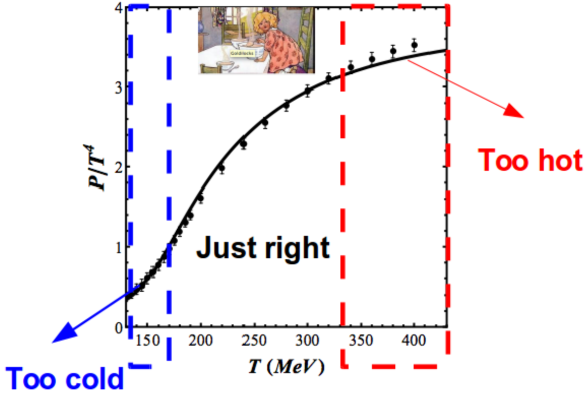

+13 more Israel-Stewart transport coefficients near Tc

S. Finazzo, R. Rougemont, H. Marrochio, J. Noronha, JHEP 1502 (2015) 051

# Transition Region-Goldilocks Zone

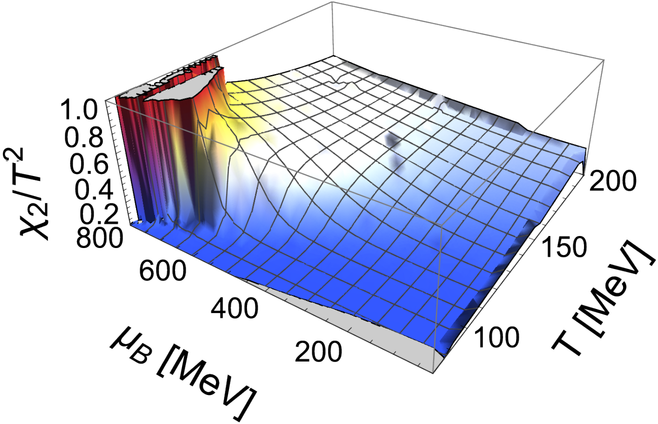
Only concerned with the transition region

"Holographic Goldilocks"



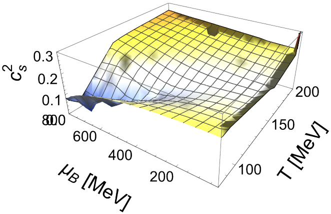
# Critical Point

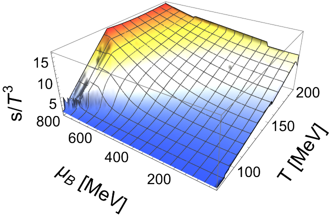
Critical Point ($T = 90$ MeV and $\mu_B = 725$ MeV) emerges naturally from the theory



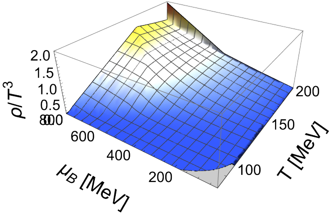
# Equation of State at finite $\mu_B$

Everything at $\mu_B > 0$ is a prediction



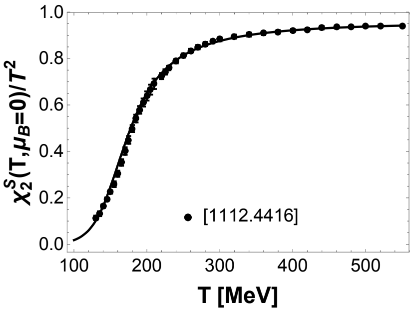



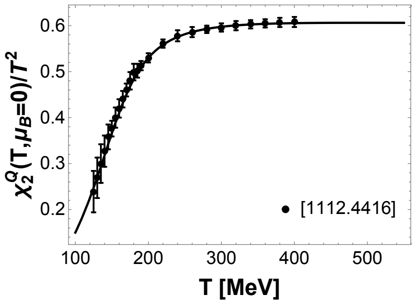

Critical behavior very sensitive to $\mu_B = 0$ Lattice QCD input



# Strangeness and Electric Charge

- Since $\mu_S < \mu_B$ and $\mu_Q << \mu_B$, assume $\mu_S \sim \mu_B \sim 0$
- Caveat: only valid when $\mu_S/\mu_B$ and $\mu_Q/\mu_B$ are small



# Viscosity vs. Dynamic Universality Class

## No Critical Point

- Calculations possible within HRG, Transport etc
- General ⇓ in viscosity as $\mu_B$ ⇑

## Dynamical CP phenomena Review

Hohenberg and Halperin,Rev. Mod. Phys. 49, 435

## Critical Point Universality Class H

- 3D Ising Model- Mixing between chiral condensate and baryon density

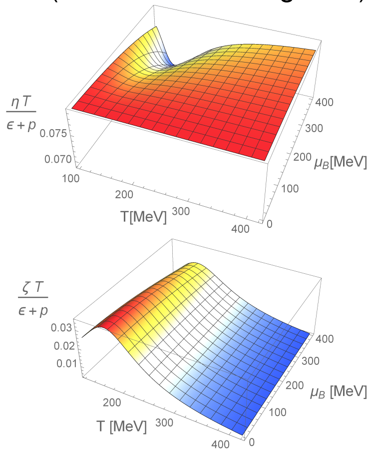
  Son and Stephanov PRD70 (2004) 056001
- Divergence ⇑ in viscosity as approaching CP
- See Stephanov and Yin's talks

## Critical Point Universality Class B

- Black Hole Engineering
- Currently B conserved, working on S & Q!
- ⇓ in viscosity as approaching CP
- Original AdS/CFT CP: Phys.Rev. D78 (2008) 106007

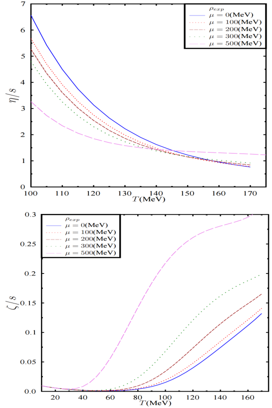
# Do we still have perfect fluidity at finite $\mu_B$?

## CP (Class B- no divergence)



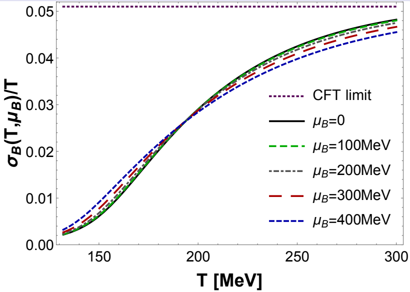

Rougemont, Noronha, JNH, Ratti to appear shortly

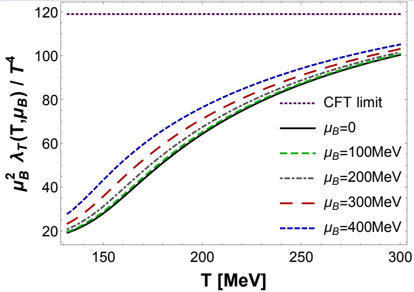
## HRG/No CP

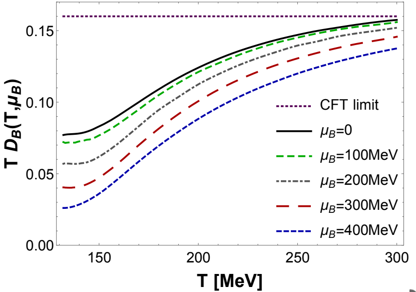



Kadam, Mishra Nucl.Phys. A934 (2014) 133-147
See also Denicol, Jeon, Gale, Noronha Phys.Rev. C88 (2013) no.6, 064901

# Baryon Transport Coefficients




DC Conductivity

$$\sigma_B = -\frac{\Lambda}{2\kappa^2\phi_A^{1/\nu}} \lim_{\omega\to 0} \frac{h(r)f(\phi)e^{2A(r)}\mathrm{Im}\left[a^*(r,\omega)a'(r,\omega)\right]}{\omega}$$

Baryon diffusion $D_B = \sigma_B/\chi_2^B$

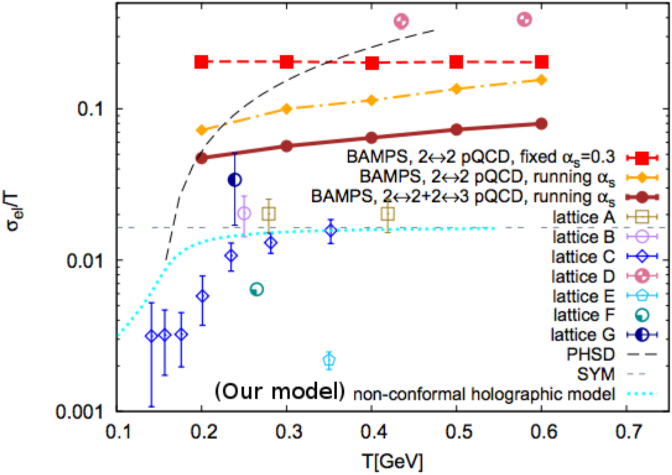
Thermal conductivity

$$\lambda_T = (\sigma_B/T)\left[(\epsilon+p)/\rho\right]^2$$

Rougemont, Noronha, JNH

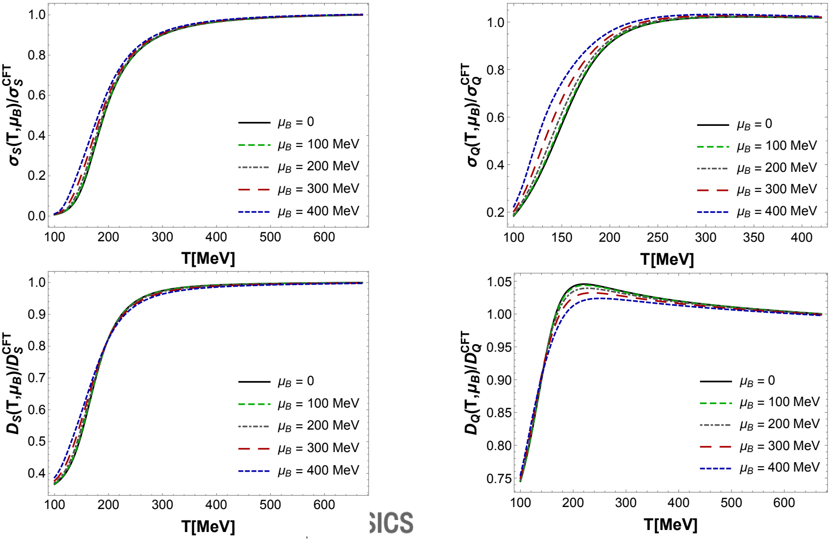
Phys.Rev.Lett. 115 (2015) no.20, 202301

# Electric Charge from various studies at $\mu_B = \mu_S = \mu_Q = 0$




Greif et al Phys.Rev. D90 (2014) no.9, 094014 (see citations within)

# Strange and Electric Conductivity Transport Coefficients
(to appear soon)



# Vorticity Viscous Coupling (only at $\mu_B = 0$ so far)

Israel-Stewart 2$^{nd}$ order terms:

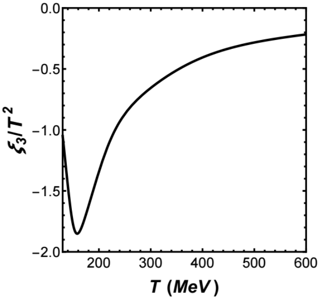
- Shear coupling terms:

$$-\frac{\lambda_2}{\eta}\pi_\lambda^{\langle\nu}\Omega^{\mu\rangle\lambda}$$

$$-\lambda_3\Omega_\lambda^{\langle\mu}\Omega^{\nu\rangle\lambda}$$

- Bulk coupling term:

$$+\lambda_3\Omega_{\mu\nu}\Omega^{\mu\nu}$$




$\lambda_2 = -\ln\frac{2\eta}{\pi T}$ and $\xi_2 = 2\eta\tau_\pi c_s^2\left(\frac{1}{3} - c_s^2\right)$

Finazzo et al JHEP 1502 (2015) 051

In red → violates causality

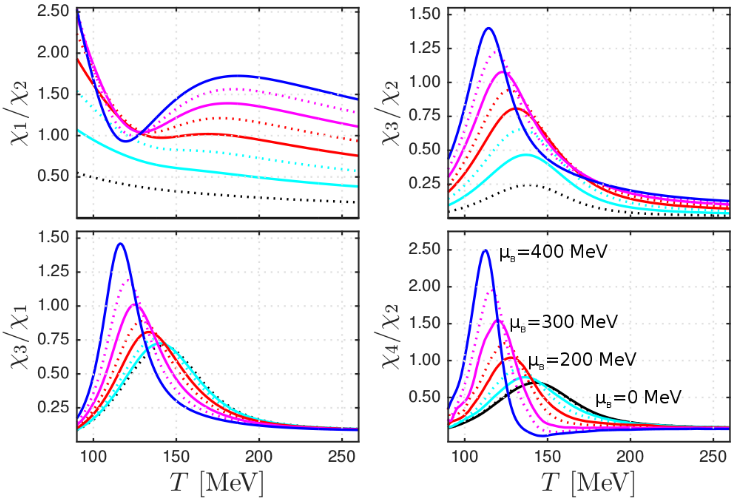
Denicol et al Phys.Rev. D85 (2012) 114047; Phys.Rev. D89 (2014) no.7, 074005; Finazzo et al JHEP 1502 (2015) 051

Vorticity puzzle: including vorticity in hydro while preserving causality? Models beyond hydro?

# What defines the transition region?

- Determine the **inflection point of the susceptibilities** $\chi_n^{BSQ}$'s across $\mu_B$
- Determine the **inflection point of the transport coefficients** across $\mu_B$
  - Should the inflection point of transport coefficients match the chemical freeze-out line?
- **Susceptibilities compared to experiments:** Compare derivatives of the pressure e.g. $\chi_2^B/\chi_1^B$ to experimental data
  - All caveats from Claudia Ratti's talk

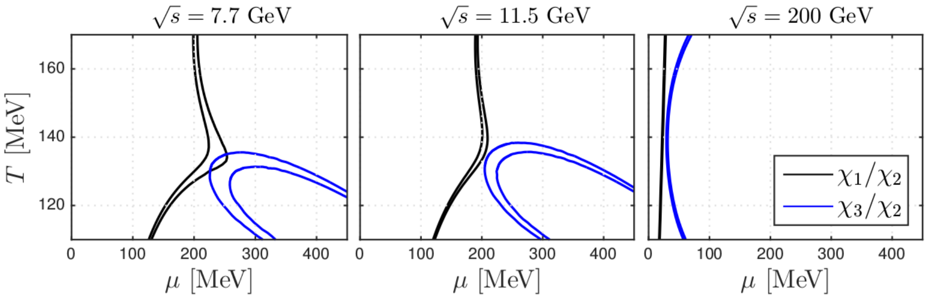
# Susceptibilities in the black hole model




- Compare $\chi_1/\chi_2$ and $\chi_3/\chi_2$ to (net-p) data from STAR

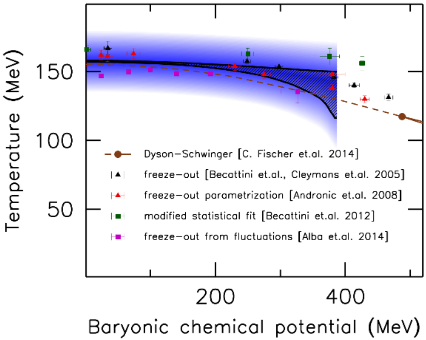
[STAR] Phys. Rev. Lett. 112 (2014) 032302

# Extraction of the freeze-out line from susceptibilities

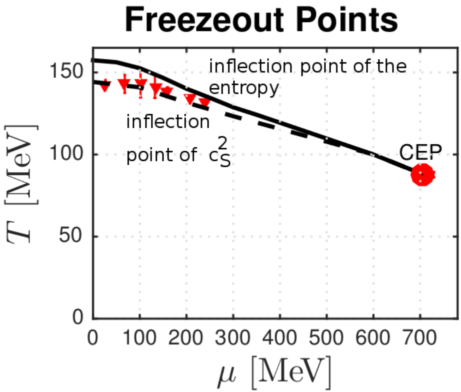



- Freeze out points $[T - \mu_B]$ are extracted from the line made by the closer points between $\chi_1/\chi_2$ and $\chi_3/\chi_2$

# Freeze-out line from the blackhole model



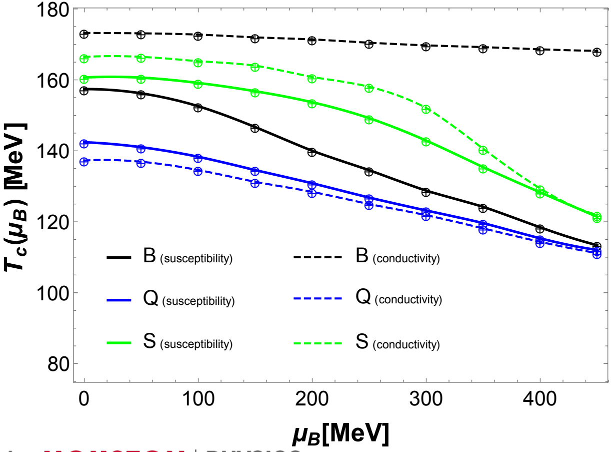

R. Bellwied *et. al.*, Phys. Lett. B **751** (2015) 053




Black Hole Model

# Inflection Points: Equilibrium vs. Dynamics

- $T^{eq}_{c,S}(\mu_B) \sim T^{dyn}_{c,S}(\mu_B)$ and $T^{eq}_{c,Q}(\mu_B) \sim T^{dyn}_{c,Q}(\mu_B)$
- $T^{eq}_{c,B}(\mu_B)$ decreases with $\mu_B$ whereas $T^{dyn}_{c,B}(\mu_B) = const$



# Conclusions and Outlook

- Black hole engineering provides a strongly interacting theory, non-conformal EoS that matches lattice, and calculable transport coefficients
- Critical Point arises at $T = 90$ MeV and $\mu_B = 725$ MeV sensitive to Lattice data at $\mu_B = 0$!
- Near crossover, $\mu_B \geq 0$, transport coefficients are suppressed compared to conformal field theory
- Freeze-out line compared to experimental data correlated with minimum of $c_s^2$
- Theory work is needed! Inclusion of multiple nonzero chemical potentials