

# Search for the QCD Critical Point -

# Fluctuations of Conserved Quantities in High-Energy Nuclear Collisions at RHIC

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# Outline

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## ➤ Introduction

## ➤ Data Analysis : **Net-P, Net-Q, Net-K**

- 1) *Centrality determination and Volume Fluctuations.*
- 2) *Efficiency correction, particle identification.*
- 3) *Error Estimation.*

## ➤ Results and Discussion: **CP Search**

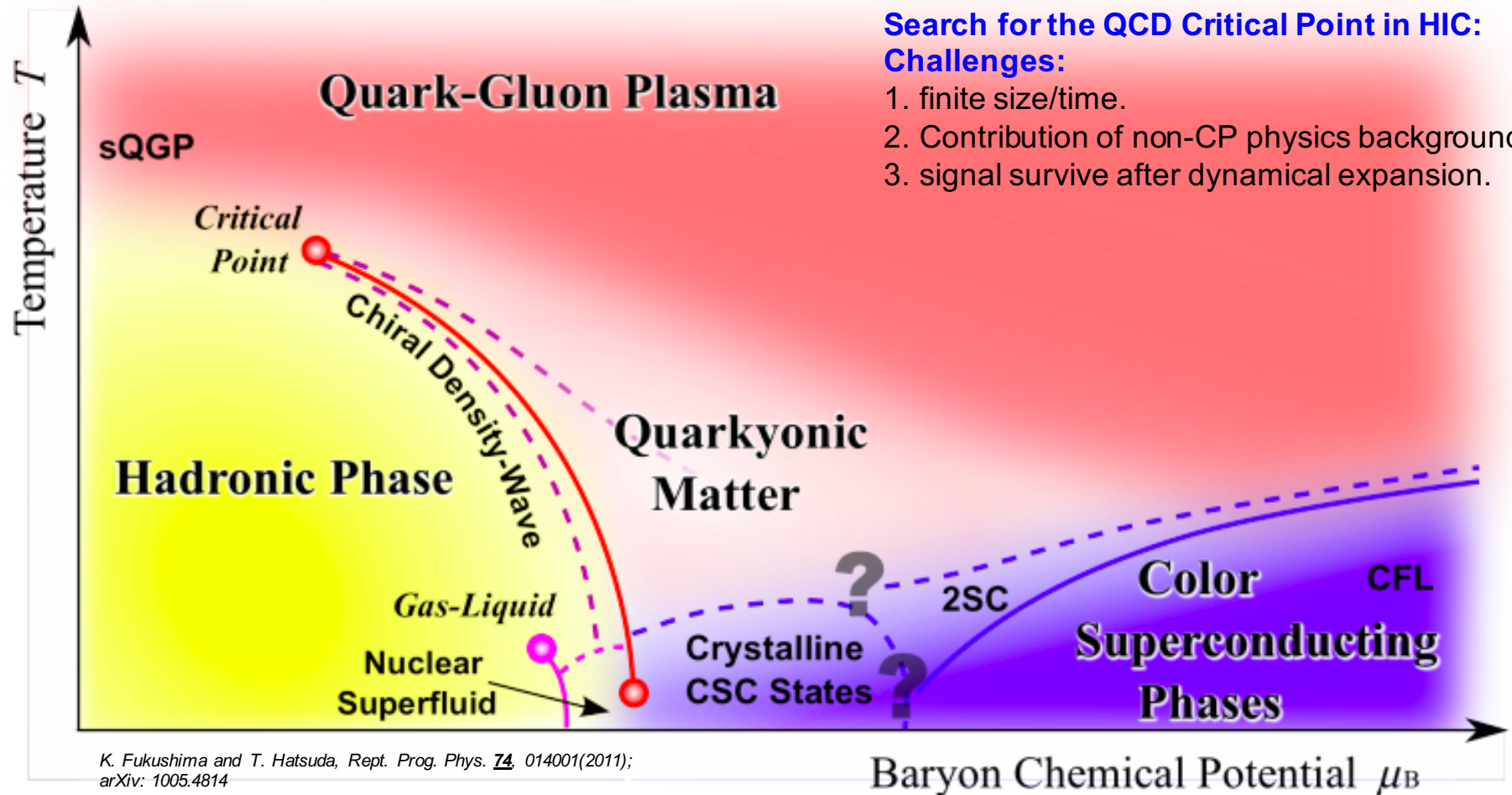
- 1) *Cumulants and Their Ratios: up to 4<sup>th</sup> order*
- 2) *Centrality, rapidity and energy dependence.*
- 3) *Deuteron Formation, UrQMD, JAM, NJL etc.*
- 4) *Correlation functions from data and model.*

## ➤ Summary and Outlook



# QCD Phase Diagram (Conjectured)

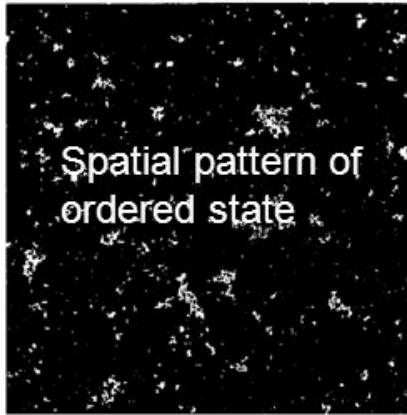
**QCD Phase Structure : Emergent properties of the strong interaction.**



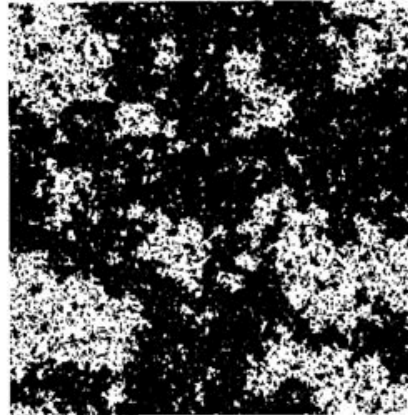


# Critical Point and Critical Phenomena

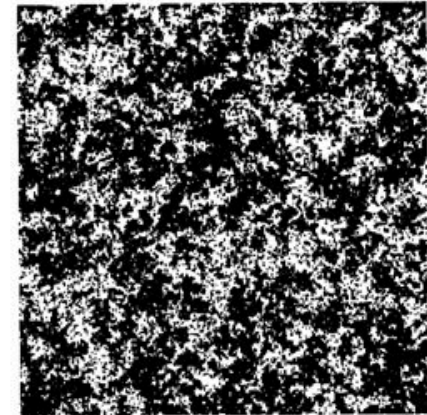
Ordered  $T=0.995T_c$



Critical  $T=T_c$



Disordered  $T=1.05T_c$



2D-Ising model simulation from ISBN4-563-02435-X C33421

## **Critical Phenomena :**

- Density fluctuations and cluster formations.
- Divergence of Correlation length ( $\xi$ ).
- Susceptibilities ( $\chi$ ), heat capacity ( $C_V$ ), Compressibility ( $\kappa$ ) etc.
- Critical opalescence.
- Universality and critical exponents determined by the symmetry and dimensions of underlying system.
- Finite Size and Finite time effects.

First CP is discovered in 1869 for  $\text{CO}_2$  by Andrews.

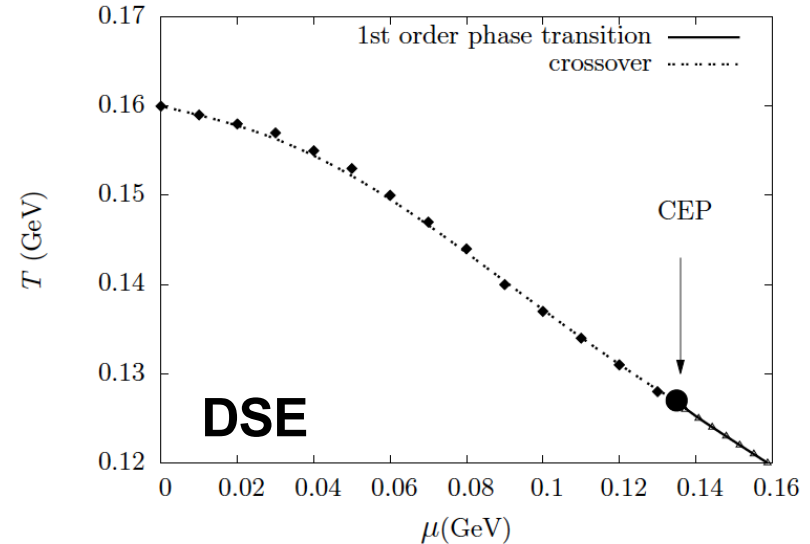
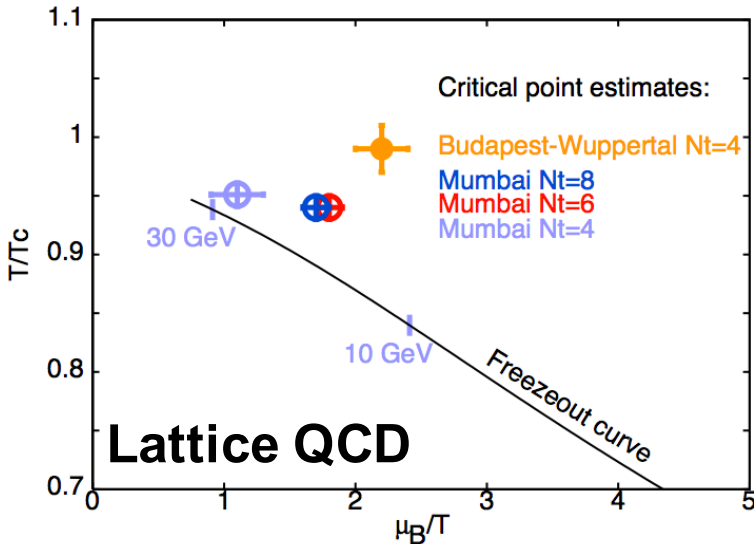
$$T_c = 31^\circ\text{C}$$

**Can we discovery the Critical Point of Quark Matter ?** (Put a permanent mark in the QCD phase diagram in text book. )

$$T_c \sim \text{Trillion } (10^{12}) \text{ } ^\circ\text{C}$$



# Location of CEP: Theoretical Prediction



## Lattice QCD:

1): Fodor&Katz, JHEP 0404,050 (2004).

$(\mu_B^E, T_E) = (360, 162)$  MeV (Reweighting)

2): Gavai&Gupta, NPA 904, 883c (2013)

$(\mu_B^E, T_E) = (279, 155)$  MeV (Taylor Expansion)

3): F. Karsch ( $\mu_B^E / T_E > 2$ , CPOD2016)

## DSE:

1): Y. X. Liu, et al., PRD90, 076006 (2014).

$(\mu_B^E, T_E) = (372, 129)$  MeV

2): Hong-shi Zong et al., JHEP 07, 014 (2014).

$(\mu_B^E, T_E) = (405, 127)$  MeV

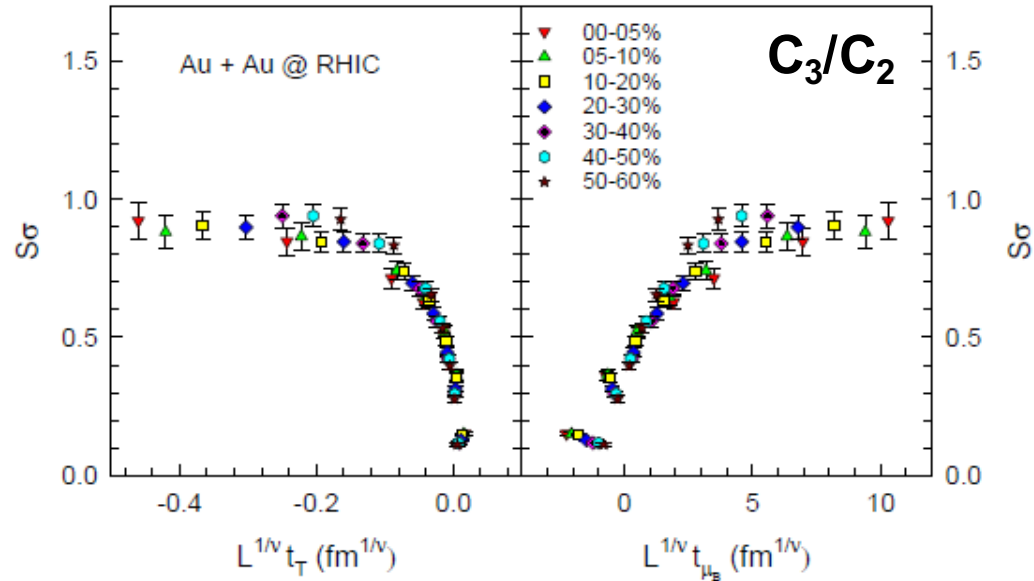
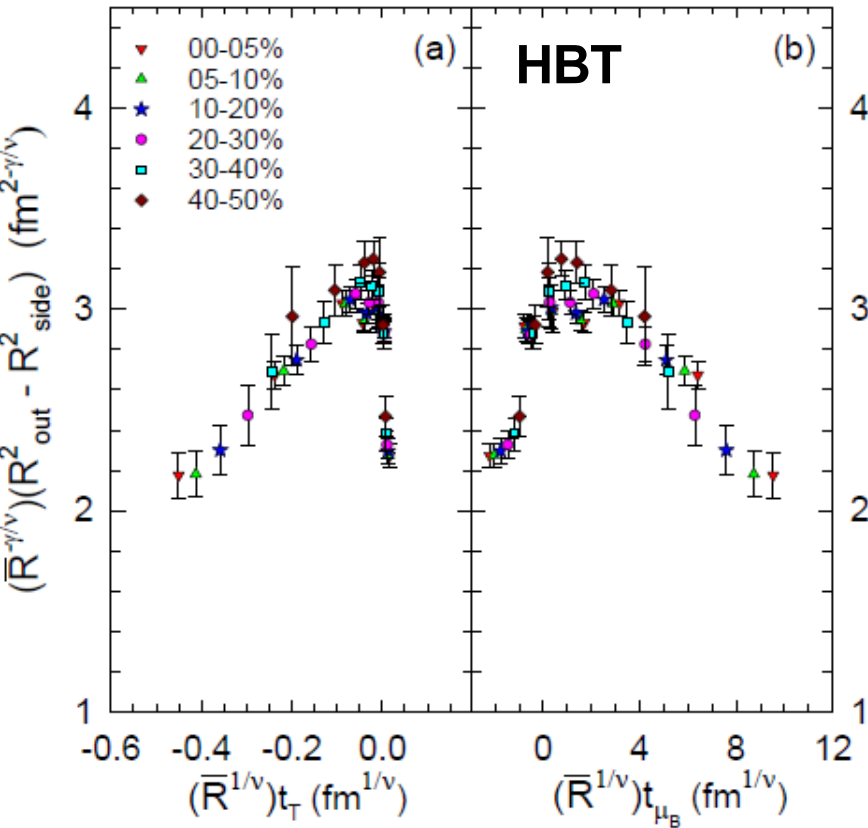
3): C. S. Fischer et al., PRD90, 034022 (2014).

$(\mu_B^E, T_E) = (504, 115)$  MeV

$$\mu_B^E = 266 \sim 504 \text{ MeV}, T_E = 115 \sim 162, \mu_B^E / T_E = 1.8 \sim 4.38$$



# Finite Size Scaling : HBT and Net-P Fluctuations



Roy Lacey et al., arXiv:1606.08071  
 M. Suzuki, Prog. Theor. Phys. 58, 1142, 1977  
 R. Lacey: Phys.Rev.Lett. 114 (2015), 142301

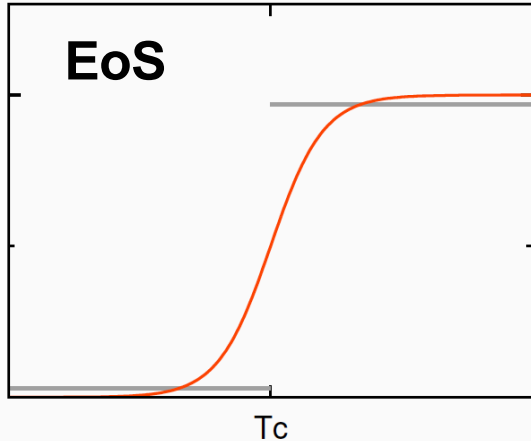
- 1) Scaling function validates the location of the CEP and the (static) critical exponents.
- 2) 2<sup>nd</sup> order PT (3D Ising Model):  $\nu=0.66$ ,  $\gamma=1.2$ ,  $T\sim 165$  MeV,  $\mu_B\sim 95$  MeV

***Do we understand the non-critical contributions ??***



# Fluctuations as Signature of Phase Transition

**Fluctuations are sensitive to the phase transition and critical point.**



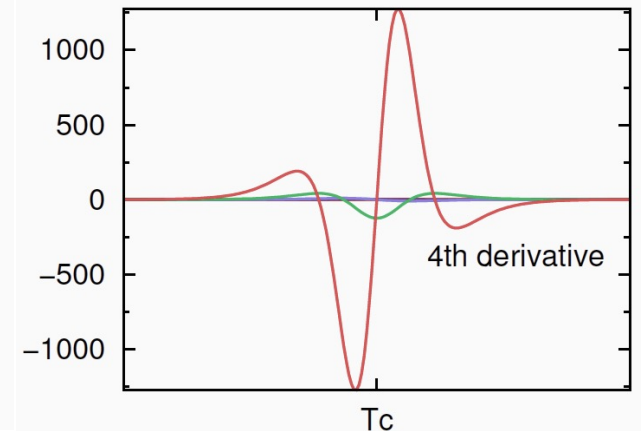
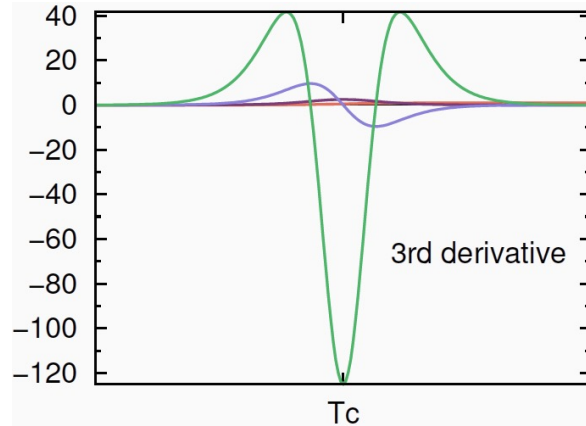
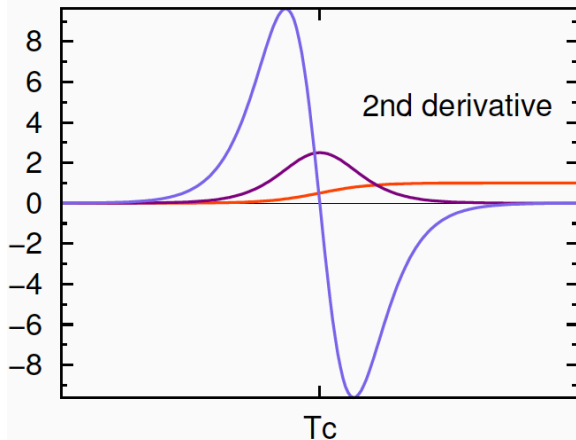
**1. Derivations of thermodynamic quantities (EoS) are related to E-by-E fluctuations in HIC.**

$$\chi_n = \left. \frac{\partial^n (P/T^4)}{\partial (\mu/T)^n} \right|_T$$

$$\chi_1 = \frac{1}{VT^3} \langle N \rangle, \quad \chi_2 = \frac{1}{VT^3} \langle (\Delta N)^2 \rangle, \quad \chi_3 = \frac{1}{VT^3} \langle (\Delta N)^3 \rangle,$$

$$\chi_4 = \frac{1}{VT^3} \langle (\Delta N)^4 \rangle_c \equiv \frac{1}{VT^3} (\langle (\Delta N)^4 \rangle - 3 \langle (\Delta N)^2 \rangle^2).$$

**2. It reveals more details: Sign change and diverge.**



M. Stephanov, K. Rajagopal, E. Shuryak, *Phys. Rev. Lett.* 81, 4816 (1998). M. Stephanov, K. Rajagopal, E. Shuryak, *Phys. Rev. D* 60, 114028 (1999). S. Jeon and V. Koch, *Phys. Rev. Lett.* 83, 5435 (1999). S. Jeon and V. Koch, *Phys. Rev. Lett.* 85, 2076 (2000). M. Asakawa, U. Heinz and B. Muller, *Phys. Rev. Lett.* 85, 2072 (2000). Y. Hatta, M. Stephanov, *Phys. Rev. Lett.* 91, 102003 (2003). V. Koch, A. Majumder, J. Randrup, *Phys. Rev. Lett.* 95, 182301 (2005). M. A. Stephanov, *Phys. Rev. Lett.* 102, 032301 (2009). M. Asakawa, S. Ejiri and M. Kitazawa, *Phys. Rev. Lett.* 103, 262301 (2009). M. A. Stephanov, *Phys. Rev. Lett.* 107, 052301 (2011).



# Higher Order Fluctuations of Conserved Quantities

## 1. Higher sensitivity to correlation length ( $\xi$ ) and probe non-gaussian fluctuations.

$$\langle (\delta N)^3 \rangle_c \approx \xi^{4.5}, \quad \langle (\delta N)^4 \rangle_c \approx \xi^7$$

$$C_{1,x} = \langle x \rangle, C_{2,x} = \langle (\delta x)^2 \rangle,$$

$$C_{3,x} = \langle (\delta x)^3 \rangle, C_{4,x} = \langle (\delta x)^4 \rangle - 3 \langle (\delta x)^2 \rangle^2$$

M. A. Stephanov, *Phys. Rev. Lett.* 102, 032301 (2009).

M. A. Stephanov, *Phys. Rev. Lett.* 107, 052301 (2011).

M. Asakawa, S. Ejiri and M. Kitazawa, *Phys. Rev. Lett.* 103, 262301 (2009).

Y. Hatta, M. Stephanov, *Phys. Rev. Lett.* 91, 102003 (2003).

## 2. Connection to the susceptibility of the system.

$$\frac{\chi_q^4}{\chi_q^2} = \kappa \sigma^2 = \frac{C_{4,q}}{C_{2,q}} \quad \frac{\chi_q^3}{\chi_q^2} = S \sigma = \frac{C_{3,q}}{C_{2,q}},$$

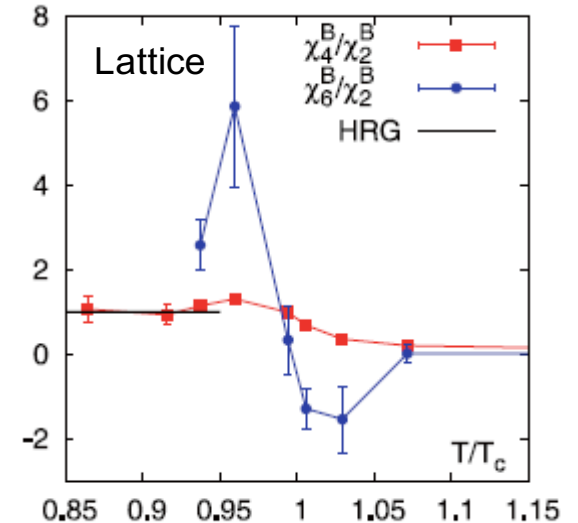
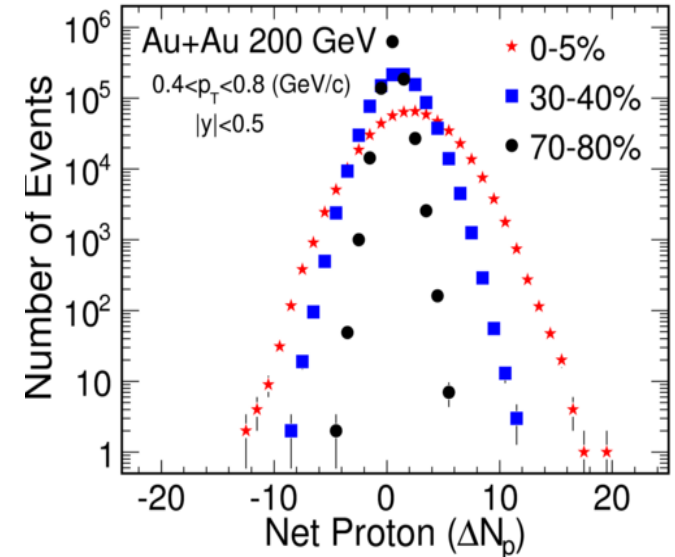
$$\chi_q^{(n)} = \frac{1}{VT^3} \times C_{n,q} = \frac{\partial^n (p/T^4)}{\partial (\mu_q)^n}, q = B, Q, S$$

S. Ejiri et al, *Phys. Lett. B* 633 (2006) 275. Cheng et al, *PRD* (2009) 074505. B.

Friman et al., *EPJC* 71 (2011) 1694. F. Karsch and K. Redlich, *PLB* 695, 136 (2011).

S. Gupta, et al., *Science*, 332, 1525(2012). A. Bazavov et al., *PRL* 109, 192302(12) // S.

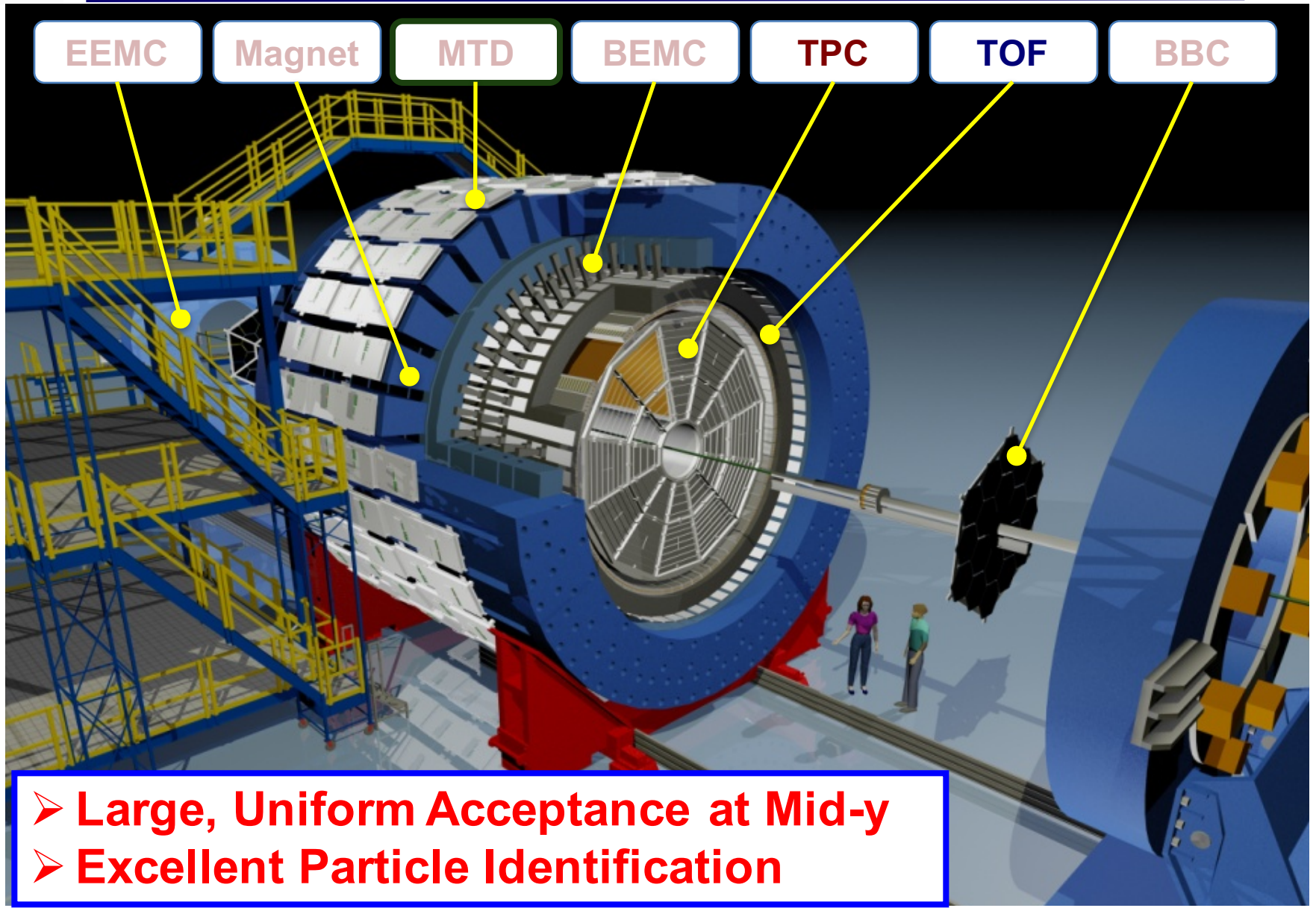
Borsanyi et al., *PRL* 111, 062005(13) // P. Alba et al., *arXiv:1403.4903*







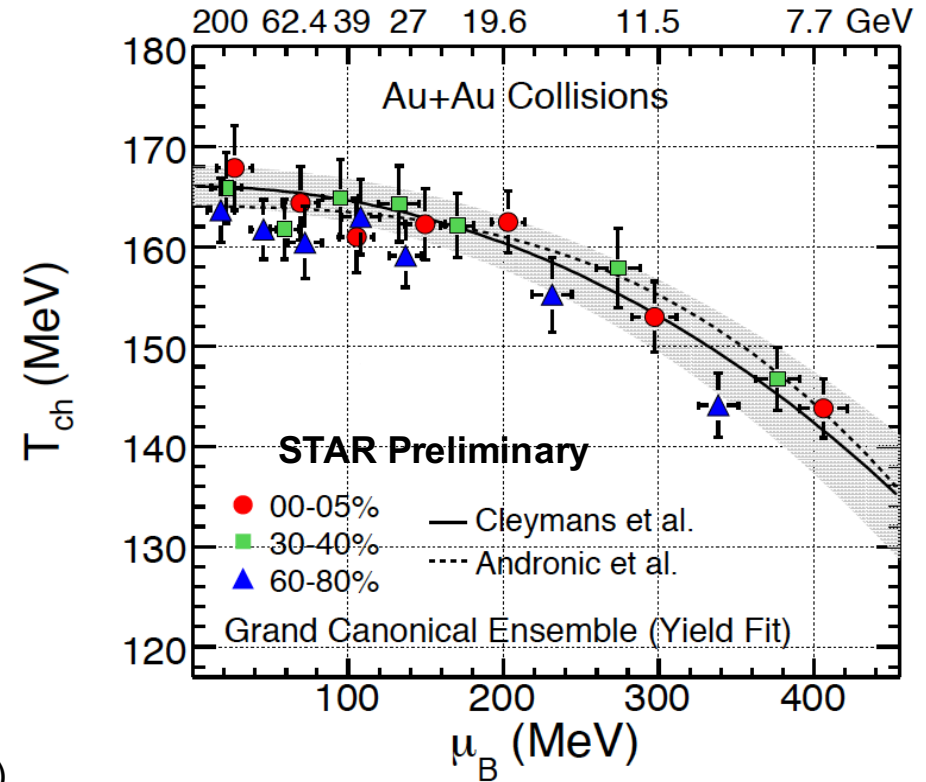
# STAR Detector System





# RHIC Beam Energy Scan- I (2010-2014)

$\sqrt{s_{NN}}$ (GeV)	Events ( $10^6$ )	Year	* $\mu_B$ (MeV)	* $T_{CH}$ (MeV)
200	350	2010	25	166
62.4	67	2010	73	165
39	39	2010	112	164
27	70	2011	156	162
19.6	36	2011	206	160
14.5	20	2014	264	156
11.5	12	2010	316	152
7.7	4	2010	422	140



\*( $\mu_B$ ,  $T_{CH}$ ) : J. Cleymans et al., PRC73, 034905 (2006)

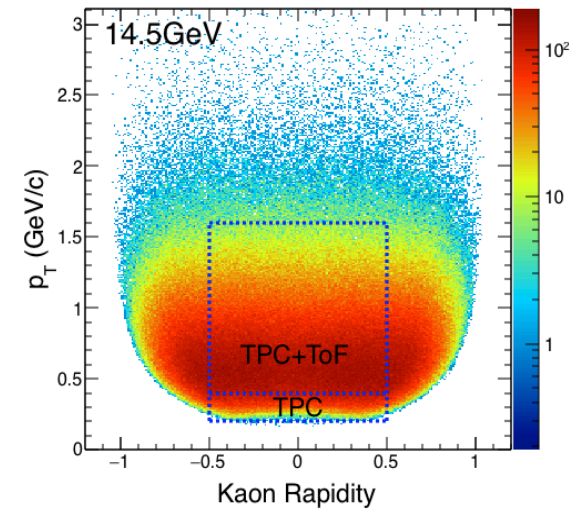
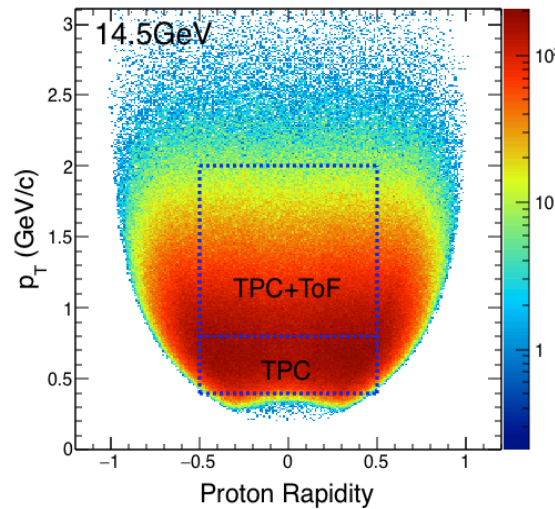
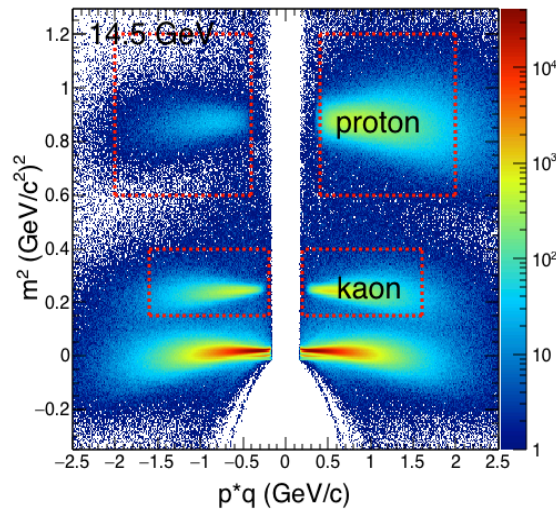
- 1) Access broad region of the QCD phase diagram.
- 2) STAR: Large and homogeneous acceptance, excellent PID capabilities.

***STAR is a unique detector with huge discovery potential in exploring the QCD phase structure at high baryon density.***



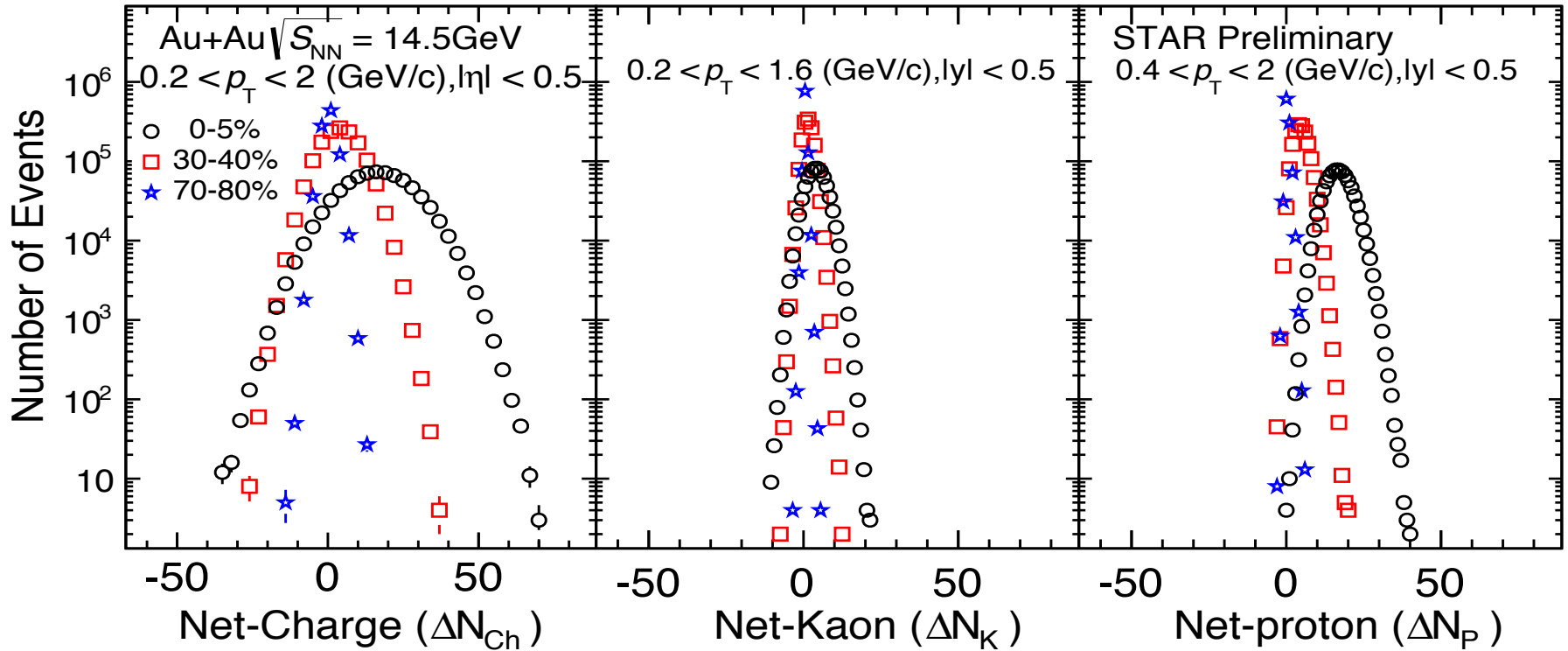
# Analysis Details

	Net-Charge	Net-Proton	Net-Kaon
Kinematic cuts	$0.2 < p_T \text{ (GeV/c)} < 2.0$ $ \eta  < 0.5$	$0.4 < p_T \text{ (GeV/c)} < 2.0$ $ y  < 0.5$	$0.2 < p_T \text{ (GeV/c)} < 1.6$ $ y  < 0.5$
Particle Identification	Reject protons from spallation for $p_T < 0.4 \text{ GeV/c}$	$0.4 < p_T \text{ (GeV/c)} < 0.8 \rightarrow \text{TPC}$ $0.8 < p_T \text{ (GeV/c)} < 2.0 \rightarrow \text{TPC+TOF}$	$0.2 < p_T \text{ (GeV/c)} < 0.4 \rightarrow \text{TPC}$ $0.4 < p_T \text{ (GeV/c)} < 1.6 \rightarrow \text{TPC+TOF}$
Centrality definition, <i>→ to avoid auto-correlations</i>	Uncorrected charged primary particles multiplicity distribution	Uncorrected charged primary particles multiplicity distribution, without (anti-)protons	Uncorrected charged primary particles multiplicity distribution, without (anti-)kaons
	$0.5 <  \eta  < 1.0$	$ \eta  < 1.0$	$ \eta  < 1.0$





# Raw Event-By-Event Net-Particle Multiplicity Distribution



Effects needed to be addressed to get final moments/cumulants:

1. Auto-correlation effects: Centrality definition.
2. Effects of volume fluctuation: Centrality bin width correction
3. Finite detector efficiency: Factorial moments

A. Bzdak and V. Koch, PRC86, 044904 (2012); X.Luo, et al. J. Phys. G40,105104(2013); X.Luo, Phys. Rev. C 91, 034907 (2015); A. Bzdak and V. Koch, PRC91, 027901 (2015). V. Skokov et al., Phys. Rev. C 88, 034911 (2013).



# Efficiency Correlation and Error Estimation

- We can express the cumulants in terms of the factorial moments, which can be easily efficiency corrected by assuming **binomial response function for efficiency**.

$$F_{u,v,j,k}(N_{p_1}, N_{p_2}, N_{\bar{p}_1}, N_{\bar{p}_2}) = \frac{f_{u,v,j,k}(n_{p_1}, n_{p_2}, n_{\bar{p}_1}, n_{\bar{p}_2})}{(\varepsilon_{p_1})^u (\varepsilon_{p_2})^v (\varepsilon_{\bar{p}_1})^j (\varepsilon_{\bar{p}_2})^k}$$

A. Bzdak and V. Koch, PRC91, 027901 (2015).

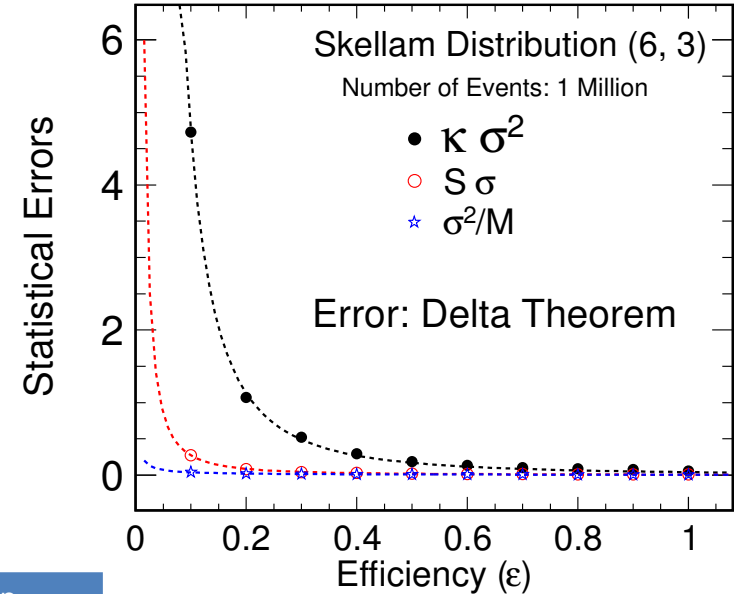
X. Luo, PRC91, 034907 (2015);

- Statistical Errors based on Delta Theorem.  
With same N events: error(net-charge) > error(net-kaon) > error(net-proton)

Au+Au 14.5GeV	Net-Charge	Net-Proton	Net-Kaon
Typical Width( $\sigma$ )	12.2	4.2	3.4
Average efficiency( $\varepsilon$ )	65%	75%	38%
$\sigma^2/\varepsilon^2$	355	32	82

Those numbers are for illustration purpose and not used in actual analysis

$$f(\varepsilon) = \frac{1}{\sqrt{n}} \frac{a}{\varepsilon^b}$$



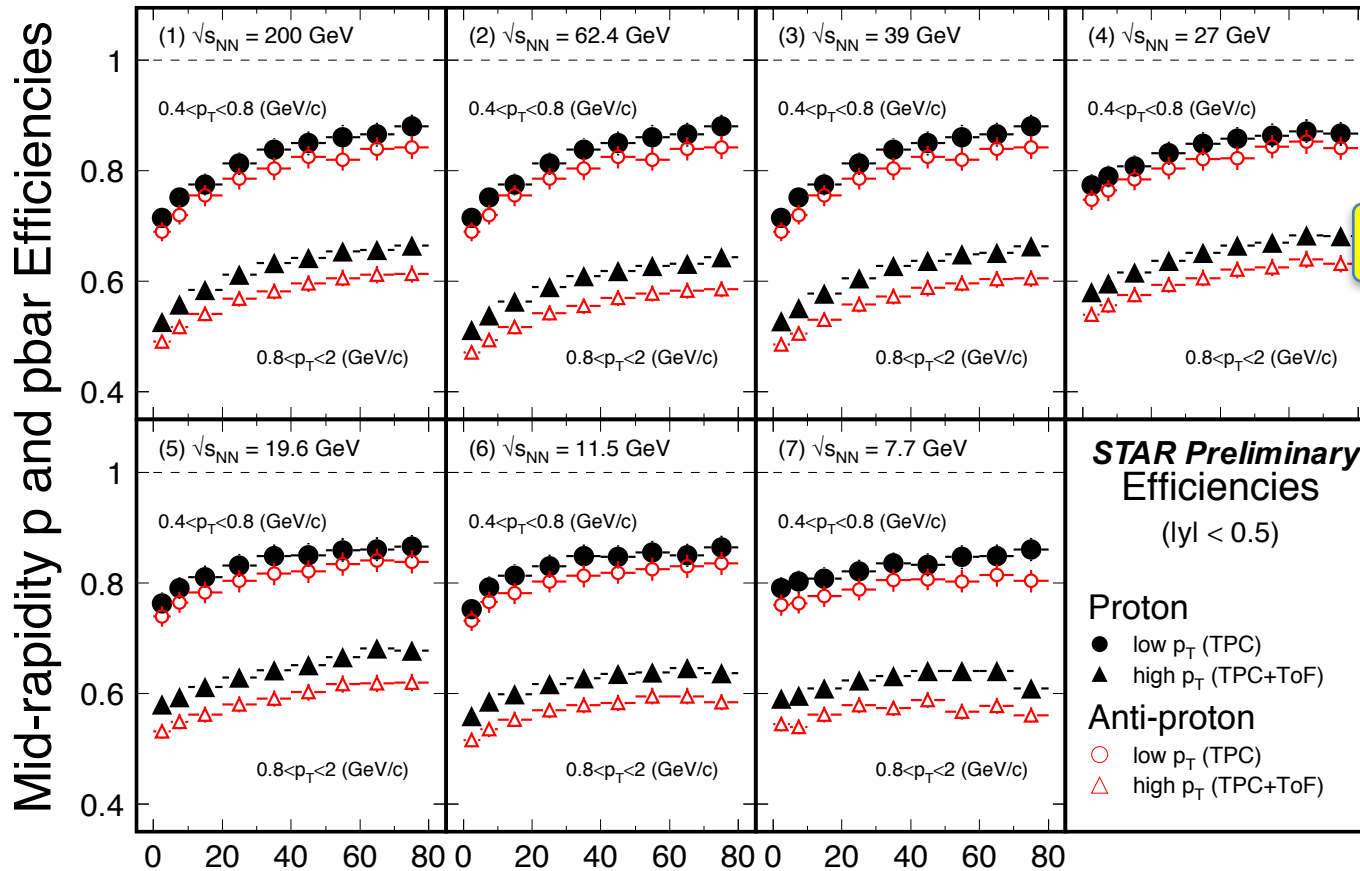
$$error(S\sigma) \propto \frac{\sigma}{\varepsilon^{3/2}}$$

$$error(\kappa\sigma^2) \propto \frac{\sigma^2}{\varepsilon^2}$$



# Efficiencies for Protons and Anti-protons

## Au + Au Collisions at RHIC

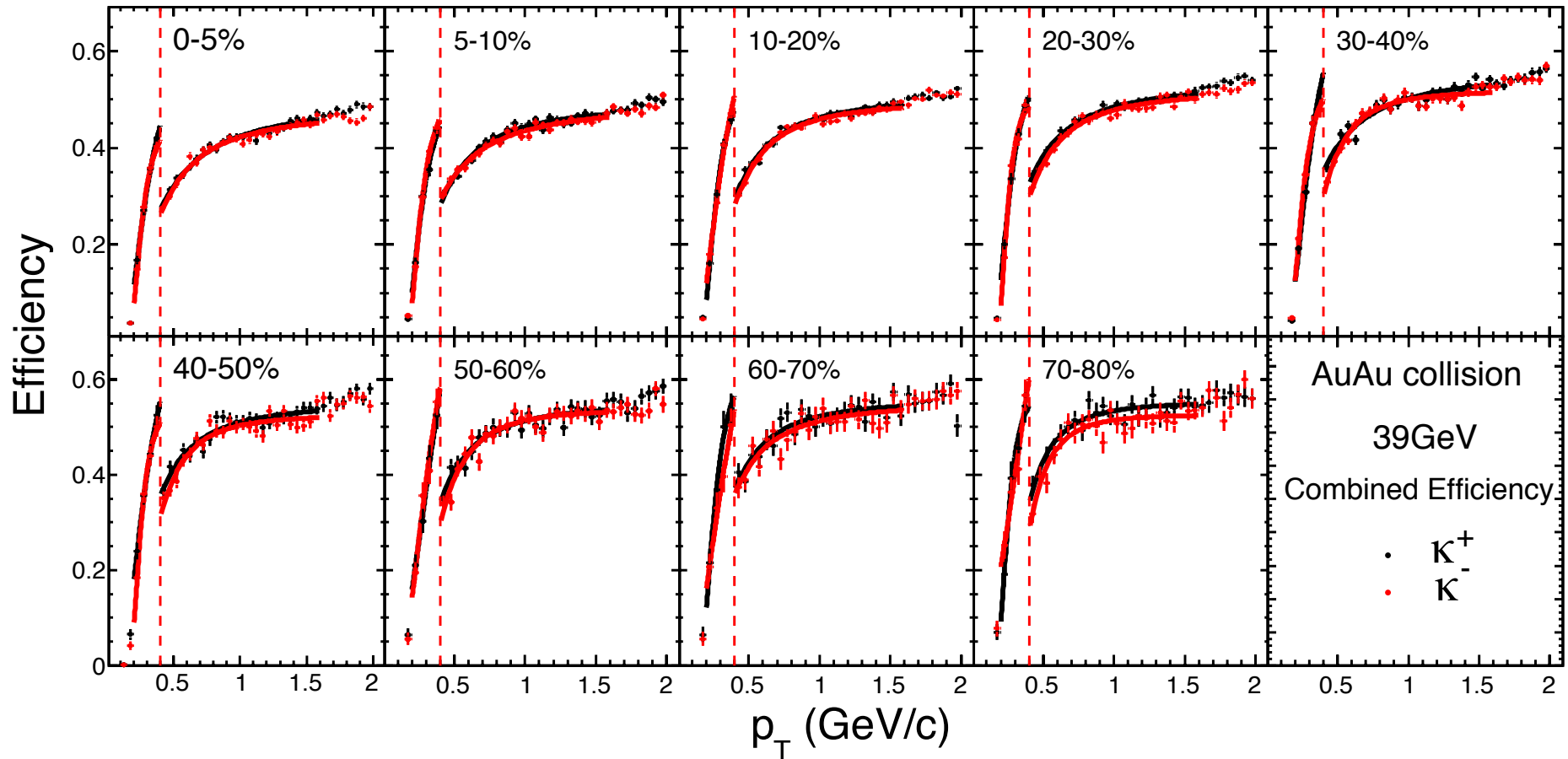


## Fraction of Collision Centralities (%)

- Due to TOF matching eff., high  $p_T$  efficiency (~50%) are smaller than low  $p_T$  (~80%).
- Efficiency decrease with increasing energies and centralities.
- Proton Efficiency > Anti-proton Efficiency



# Efficiencies for $K^+$ and $K^-$

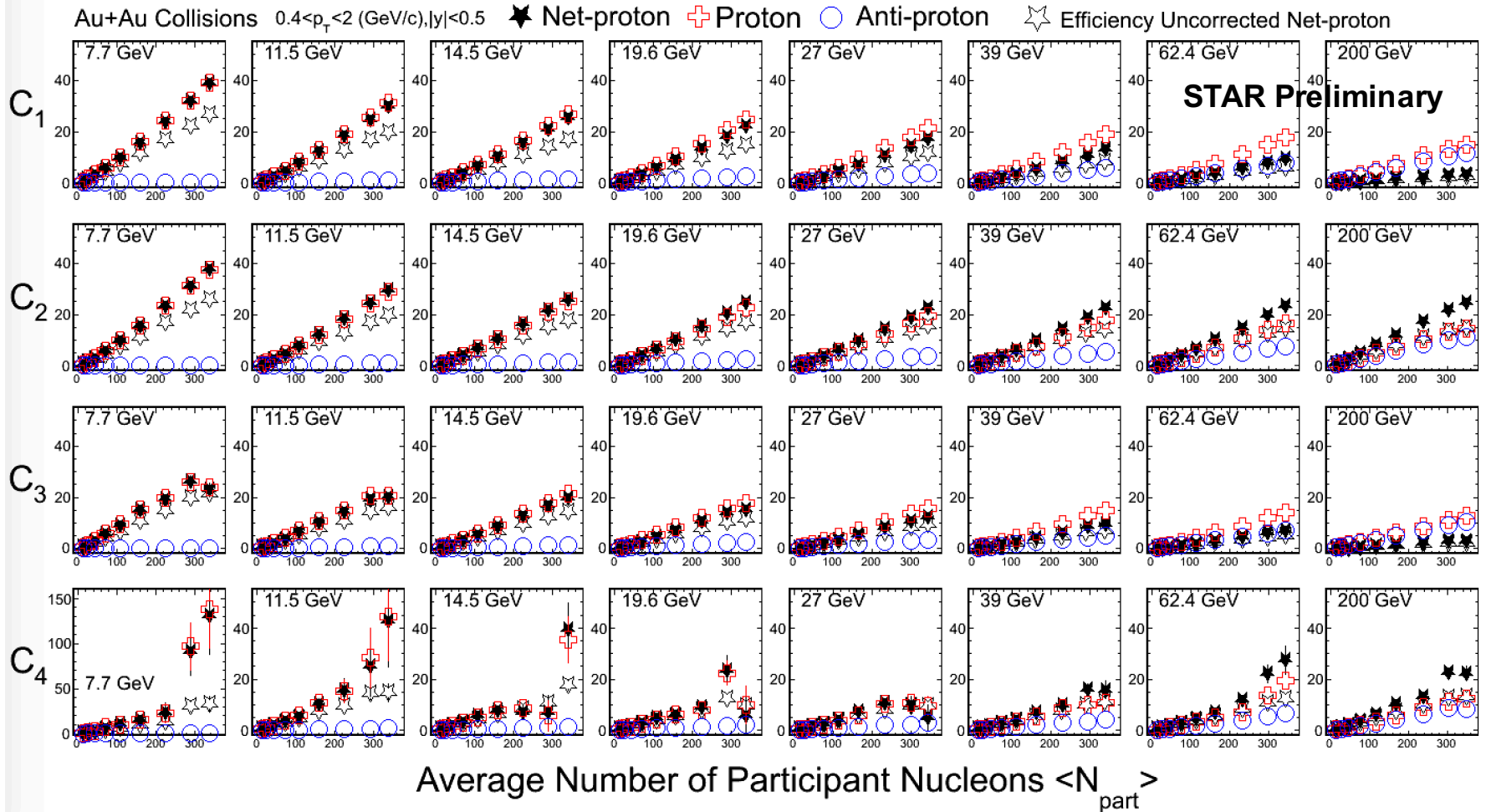


- $0.2 < p_T < 0.4$  (GeV/c), TPC only
  - $0.4 < p_T < 1.6$  (GeV/c), TPC+TOF
- Efficiency=Efficiency(Tracking)\*Efficiency(TOF match)

**Ji Xu, SQM 2016.**



# Net-Proton Cumulants ( $C_1 \sim C_4$ ) Vs. Centrality

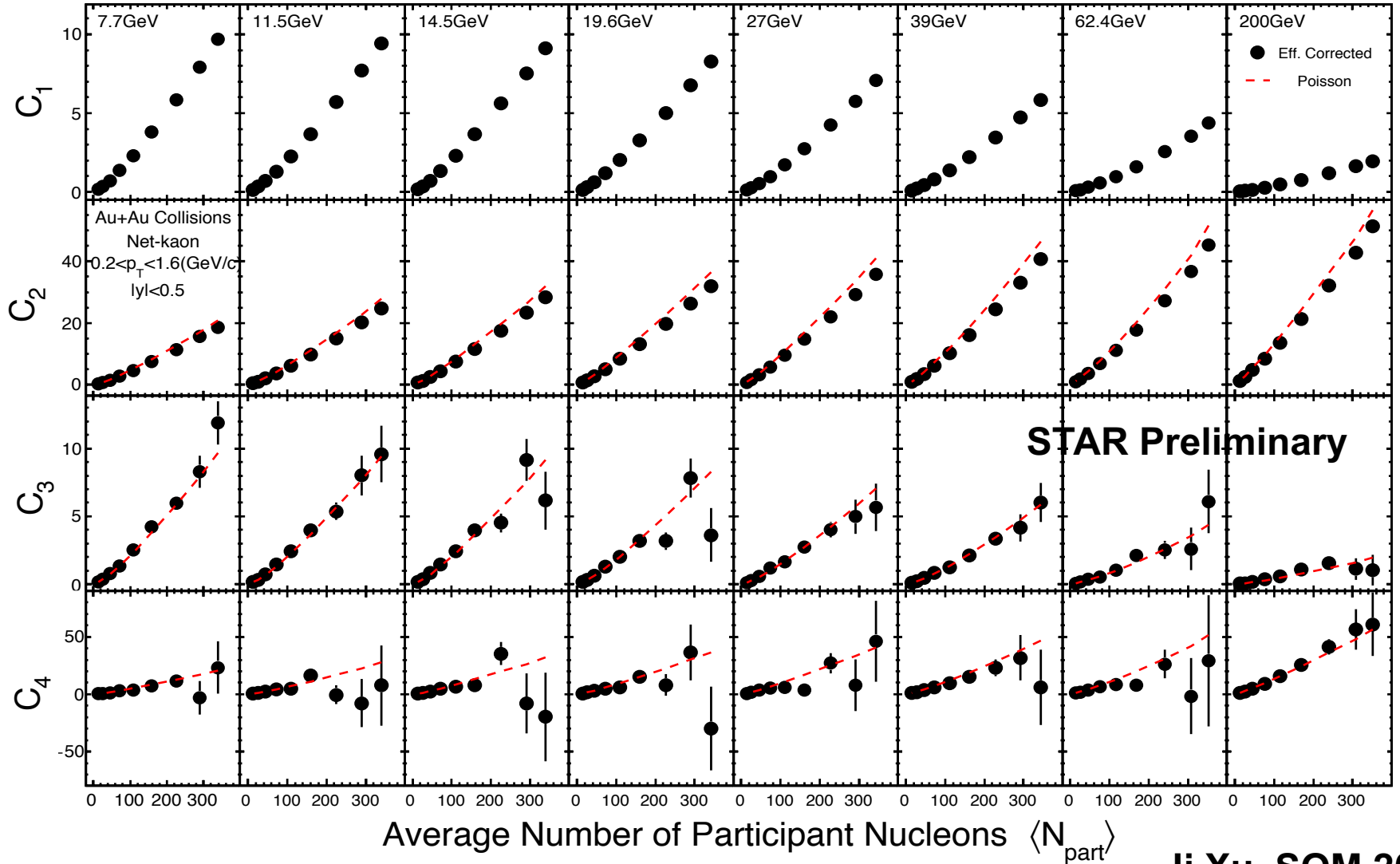


1. In general, cumulants are linearly increasing with  $\langle N_{part} \rangle$ .
2. Efficiency corrections are important.
3. At low energies, the proton cumulants are close to net-proton.





# Cumulants for Net-Kaon



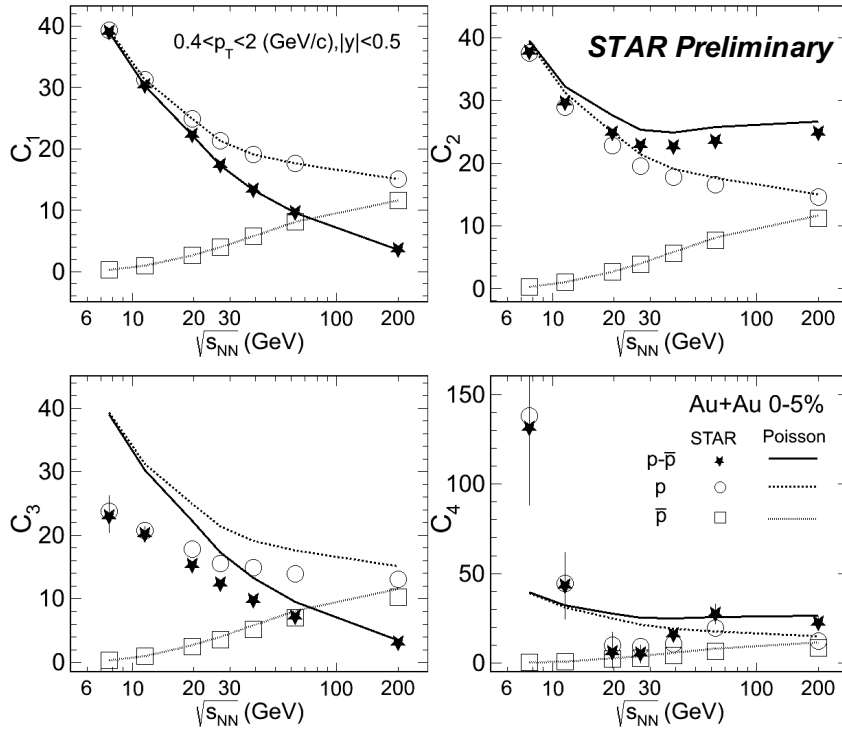
Ji Xu, SQM 2016.

$C_3$  and  $C_4$  generally consistent with Poisson expectation.

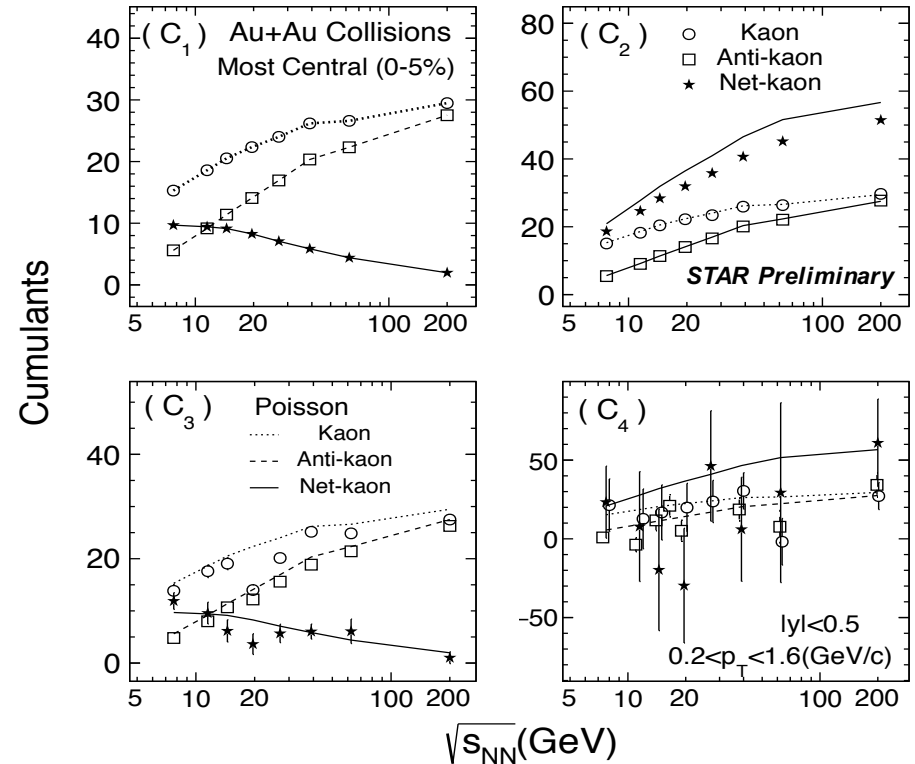


# Cumulants vs. Baselines

## Cumulants vs. Poisson (Protons)

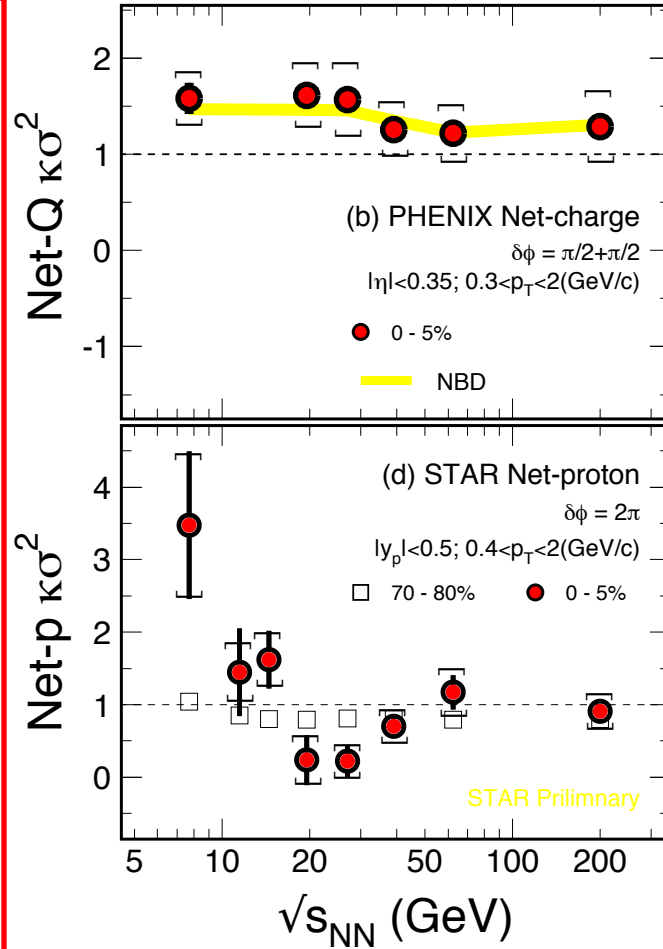
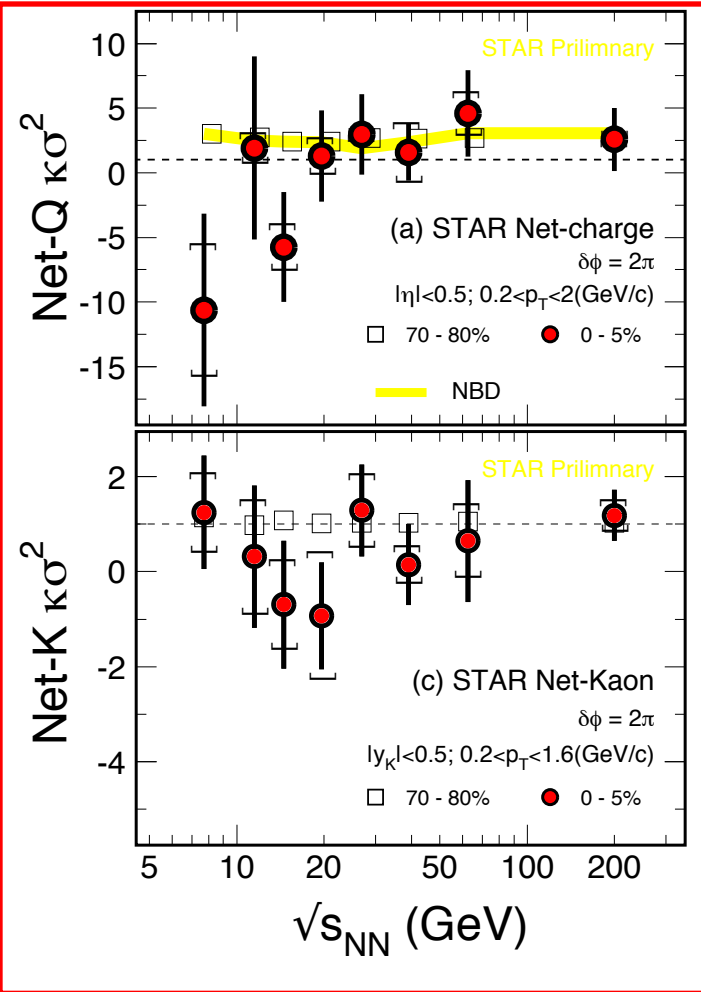


## Cumulants vs. Poisson (Kaons)



- The higher the order of cumulants, the larger deviations from Poisson expectations for net-proton and proton.
- In general, the cumulants for net-kaon, kaon and antikaon are consistent with Poisson baseline within uncertainties.

# Forth Order Fluctuations of Net-P, Net-Q, Net-K



$$\text{error}(\kappa * \sigma^2) \propto$$

$$\frac{1}{\sqrt{N}} \frac{\sigma^2}{\epsilon^2}$$

In STAR:

$$\sigma(Q) > \sigma(K) > \sigma(p)$$

- 1) Within errors, the results of net-Q and net-Kaon show flat energy dependence.
- 2) More statistics are needed at low energies.



# Theoretical and Model Calculations

Motivation:

## 1) Signals from Criticality.

- NJL, VDW liquid-gas EoS, PQM,  $\sigma$  model etc.
- DSE, Lattice QCD.

Theoretical predictions are important for us !

## 2) Study non-CP background in HIC.

Transport model: UrQMD, AMPT, JAM etc.

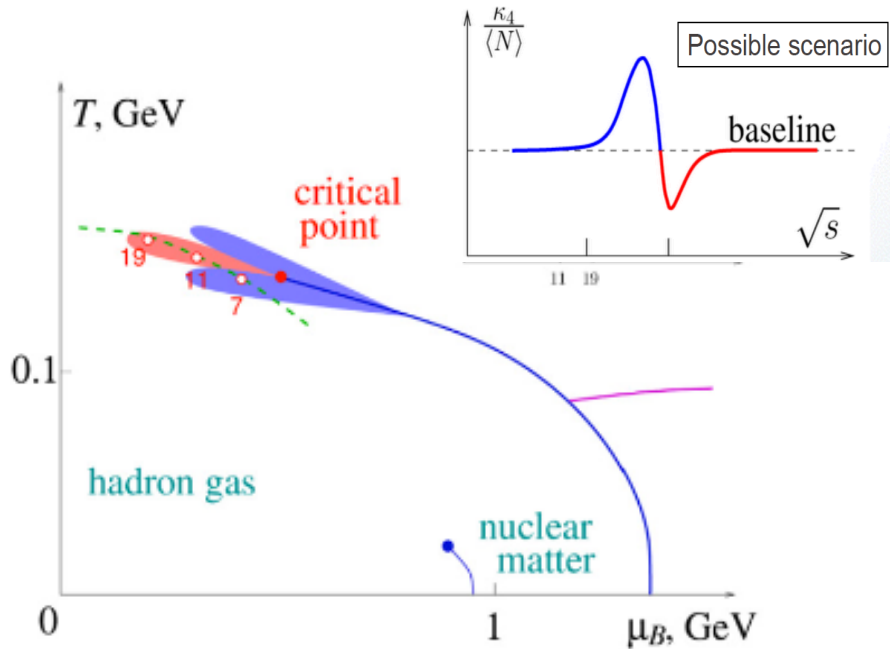


# Forth Order Fluctuations: Net-proton

Model

$$\kappa\sigma^2 = C_4/C_2$$

STAR BES Data



M.A. Stephanov, PRL107, 052301 (2011).

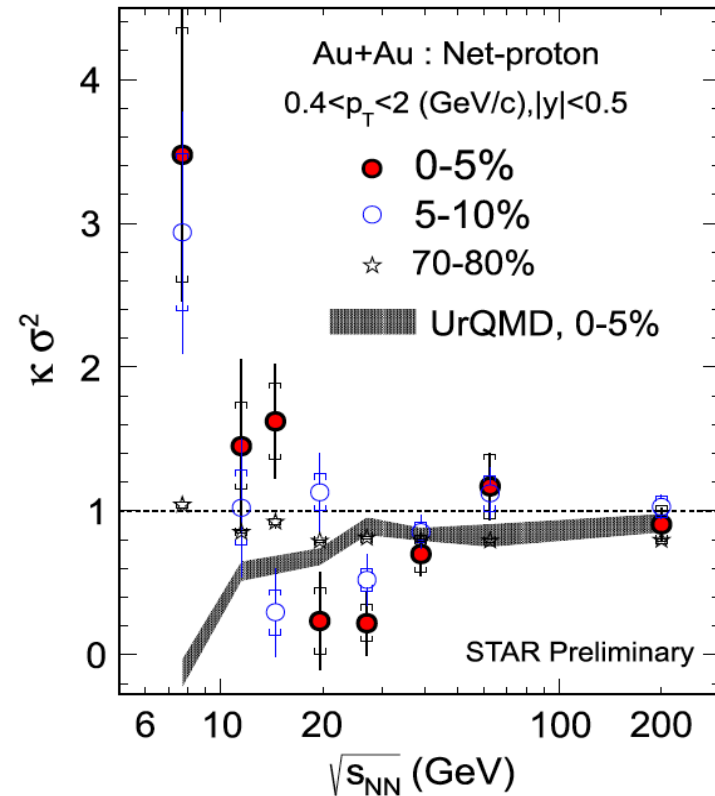
Schaefer&Wanger, PRD 85, 034027 (2012)

Vovchenko et al., PRC92, 054901 (2015)

JW Chen et al., PRD93, 034037 (2016)

arXiv: 1603.05198.

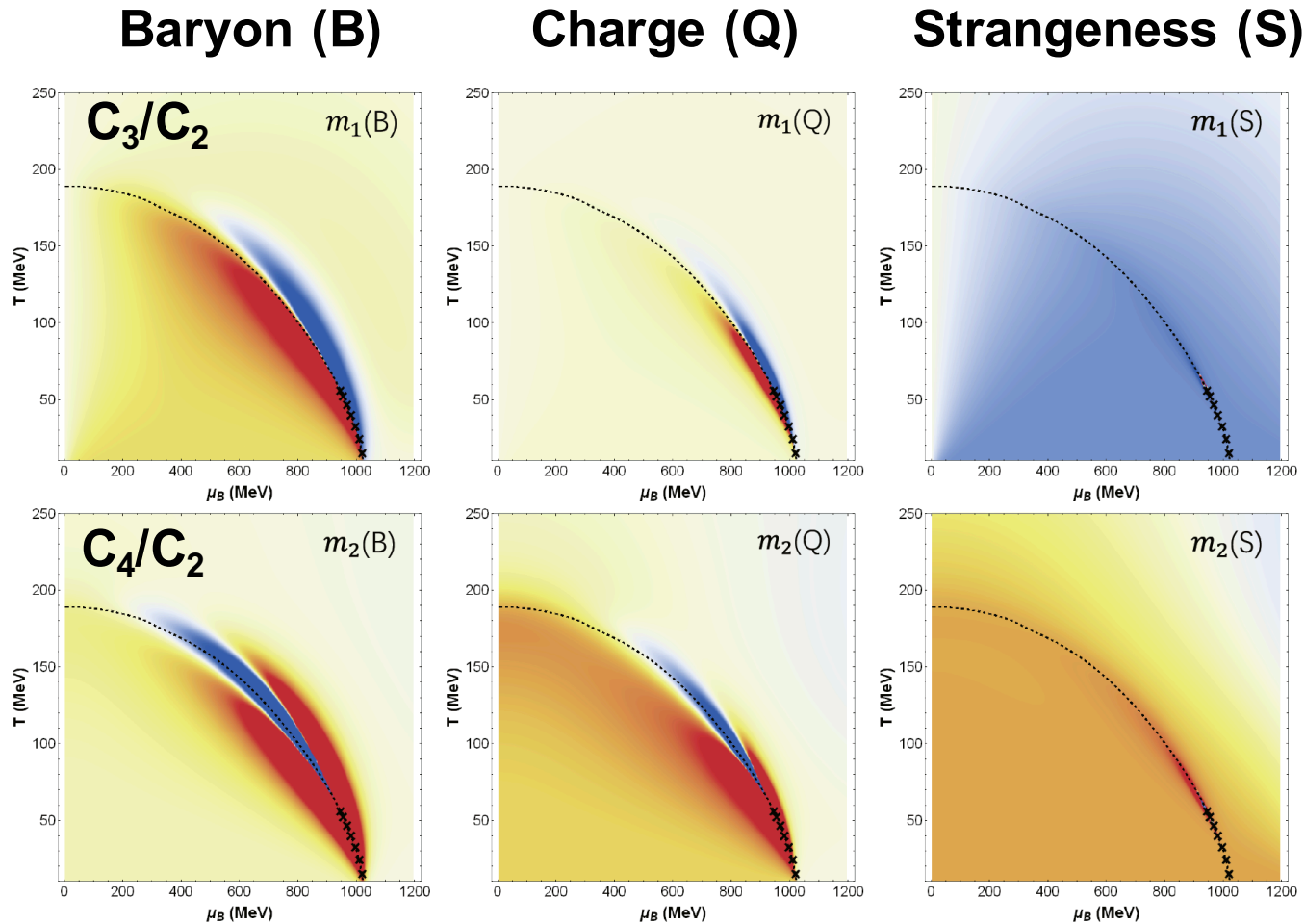
STAR, PoS(CPOD14)019; QM (15).



**Non-monotonic energy dependence is observed for 4<sup>th</sup> order net-proton fluctuations in most central Au+Au collisions.**



# NJL Model Calculations

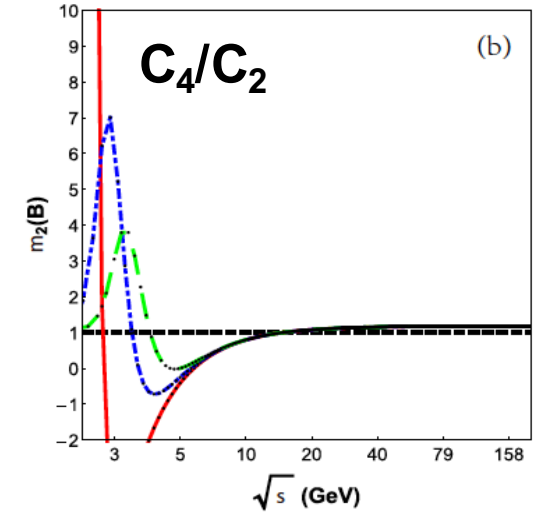
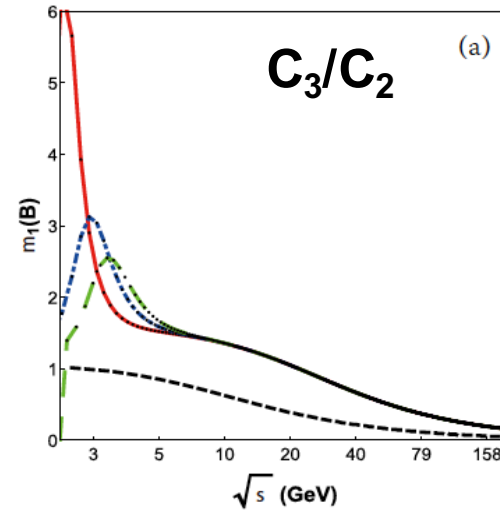
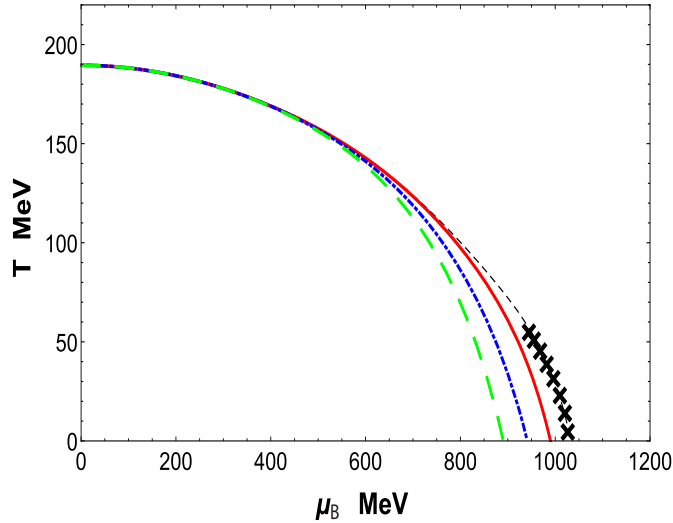


- 1) CP Signals from baryon fluctuations are much stronger than Q and S.
- 2) Forth and third order fluctuations have very different behavior.

W. Fan, X. Luo, H. Zong, arXiv: 1608. 07903 JW Chen et al., PRD93, 034037 (2016); arXiv: 1603.05198.



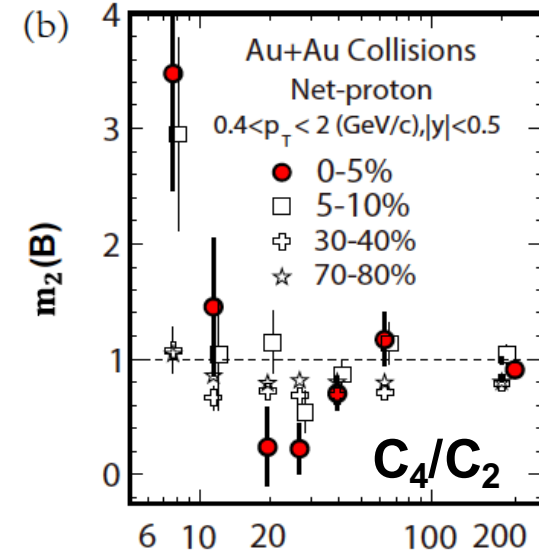
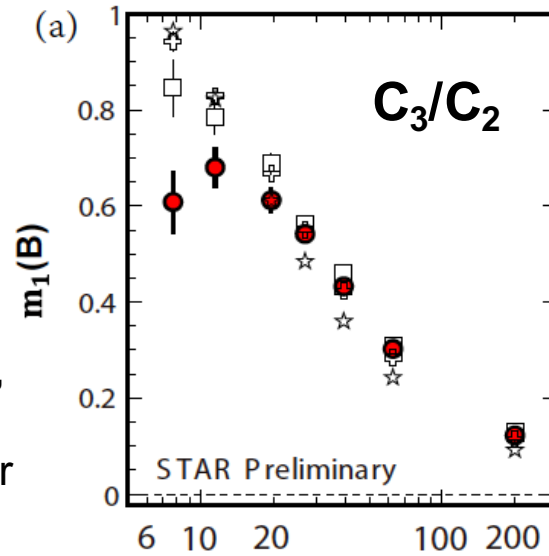
# Comparison Between NJL Model and STAR Data



$$m_1 = C_3/C_2$$

$$m_2 = C_4/C_2$$

Along the assumed freeze-out curve, the NJL Model can qualitatively describe the non-monotonic behavior observed in data.



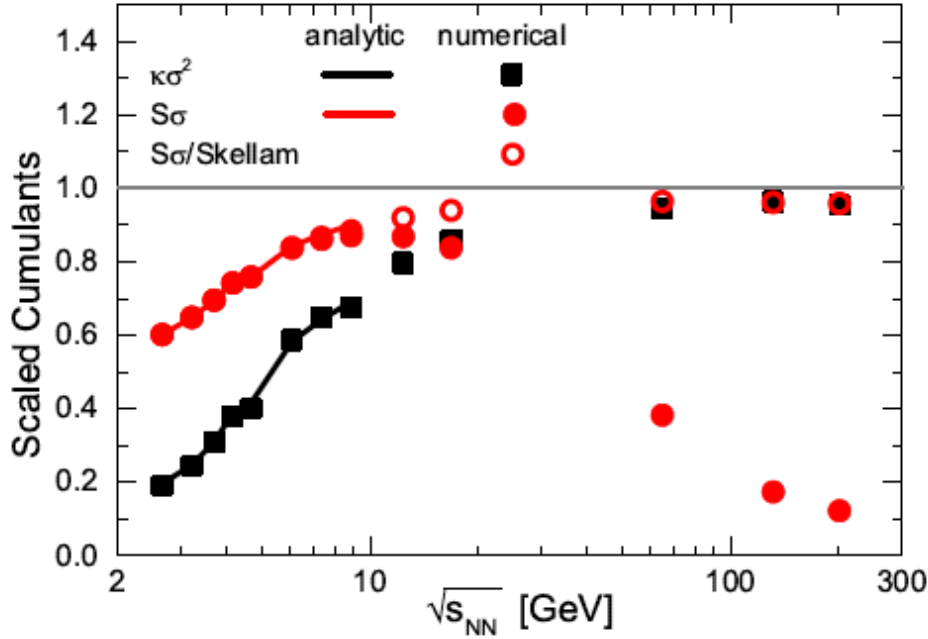
W. Fan, X. Luo, H. Zong, arXiv: 1608. 07903

Colliding Energy  $\sqrt{s}$  (GeV)

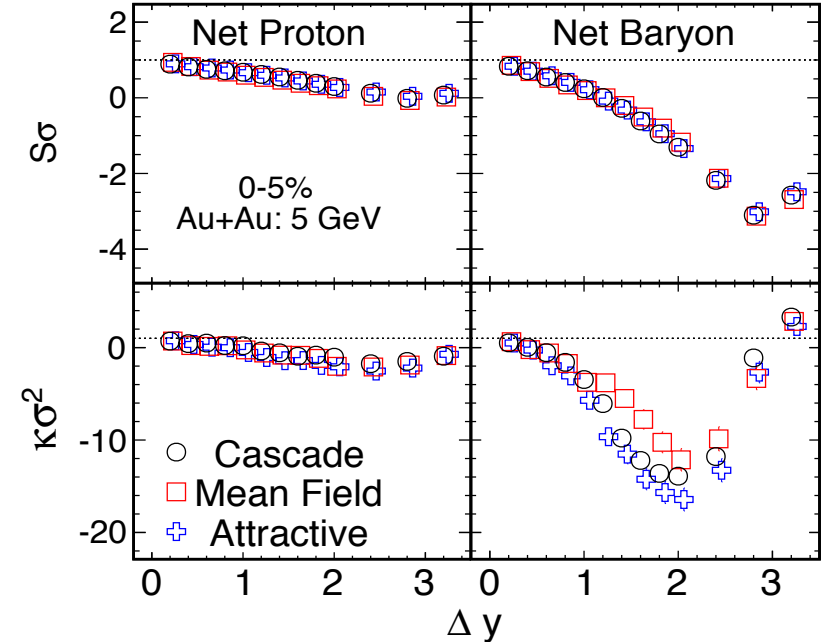


# Model Calculations

## Effects of Deuteron Formation



## JAM



At  $\sqrt{s_{NN}} \leq 10$  GeV: Data:  $\kappa\sigma^2 > 1$  Model:  $\kappa\sigma^2 < 1$

➤ Model simulation indicates: *Baryon conservations, Mean-field potential, Deuteron formation, Softening of EOS. All suppress the net-proton fluctuations.*

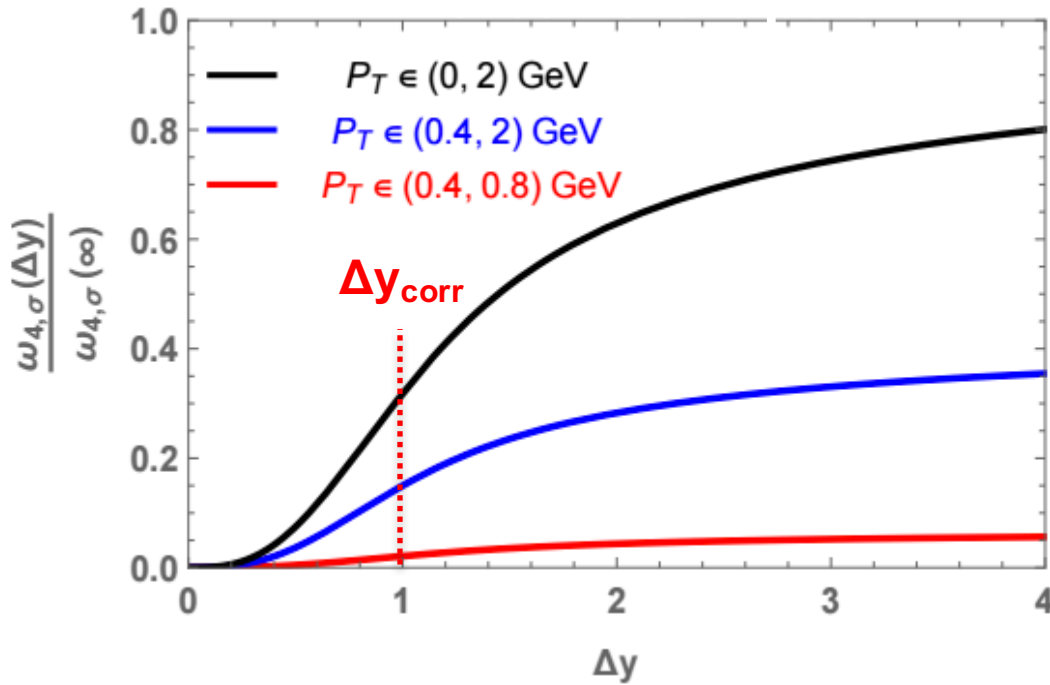
- 1) Z. Feckova, J. Steinhilber, B. Tomasik, M. Bleicher, *PRC***92**, 064908(2015). J. Xu, S. Yu, F. Liu, X. Luo, *PRC***94**, 024901(2016). X. Luo *et al*, *NPA***931**, 808(14), P.K. Netrakanti *et al*. 1405.4617, *NPA***947**, 248(2016), P. Garg *et al*. *Phys. Lett.* **B726**, 691(2013).
- 2) S. He, X. Luo, Y. Nara, S. Esumi, N. Xu, *PLB* 2016, arXiv: 1607.06376.





# Acceptance Dependence : Test Power Law Behavior

Acceptance dependence of the critical contribution



$\Delta y_{\text{corr}}$  : The correlation range in rapidity

B. Ling, M. Stephanov, Phys. Rev. C 93, 034915 (2016).  
Adam&Volker, arXiv:1607.07375

$$C_1 = \langle N \rangle$$

$$C_2 = \langle N \rangle + \hat{K}_2$$

$$C_3 = \langle N \rangle + 3\hat{K}_2 + \hat{K}_3$$

$$C_4 = \langle N \rangle + 7\hat{K}_2 + 6\hat{K}_3 + \hat{K}_4$$

$\hat{K}_2, \hat{K}_3, \hat{K}_4$  : 2,3,4-particle correlation function

Generating function for the factorial cumulants:  $\hat{K}_n$  (corr. fun.):

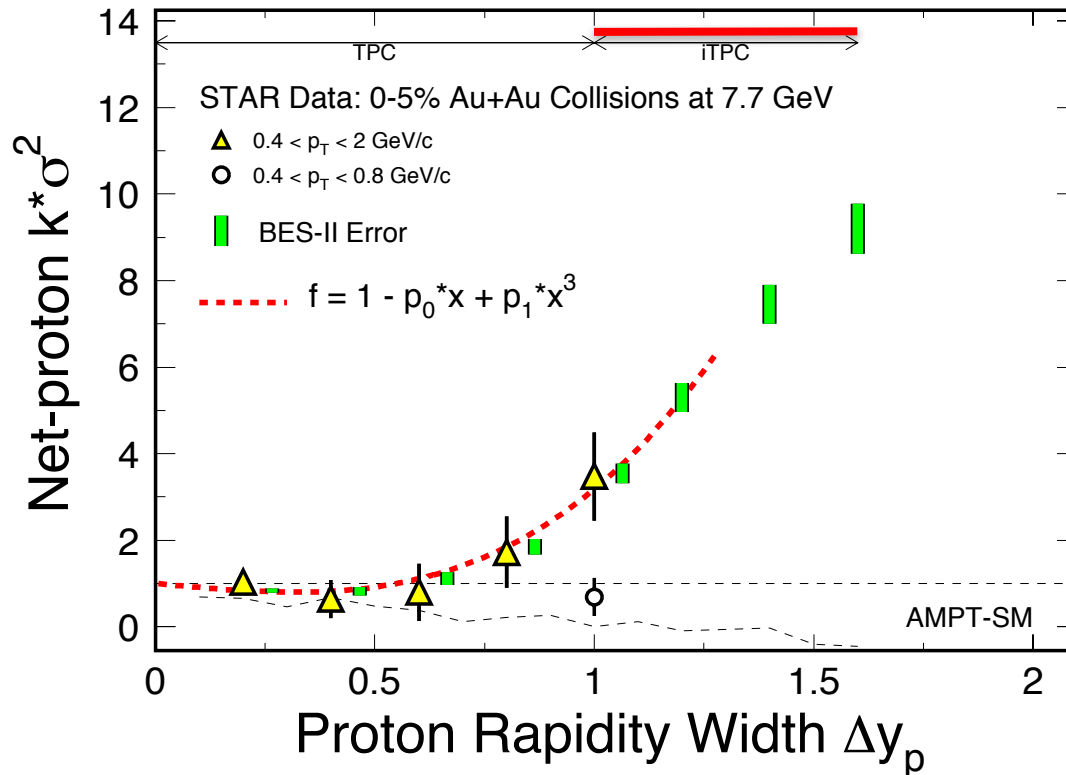
$$g(x) \equiv \sum_{k=1}^{\infty} \hat{k}_k \frac{x^k}{k!} = \ln \langle (1+x)^N \rangle.$$

$$\text{If } \Delta y \ll \Delta y_{\text{corr}} : C_n \propto \hat{K}_n \propto \langle N \rangle^n \sim (\Delta y)^n$$

$$\text{If } \Delta y \gg \Delta y_{\text{corr}} : C_n \propto \hat{K}_n \propto \langle N \rangle \sim (\Delta y)$$



# Net-Proton Fluctuations vs. Rapidity



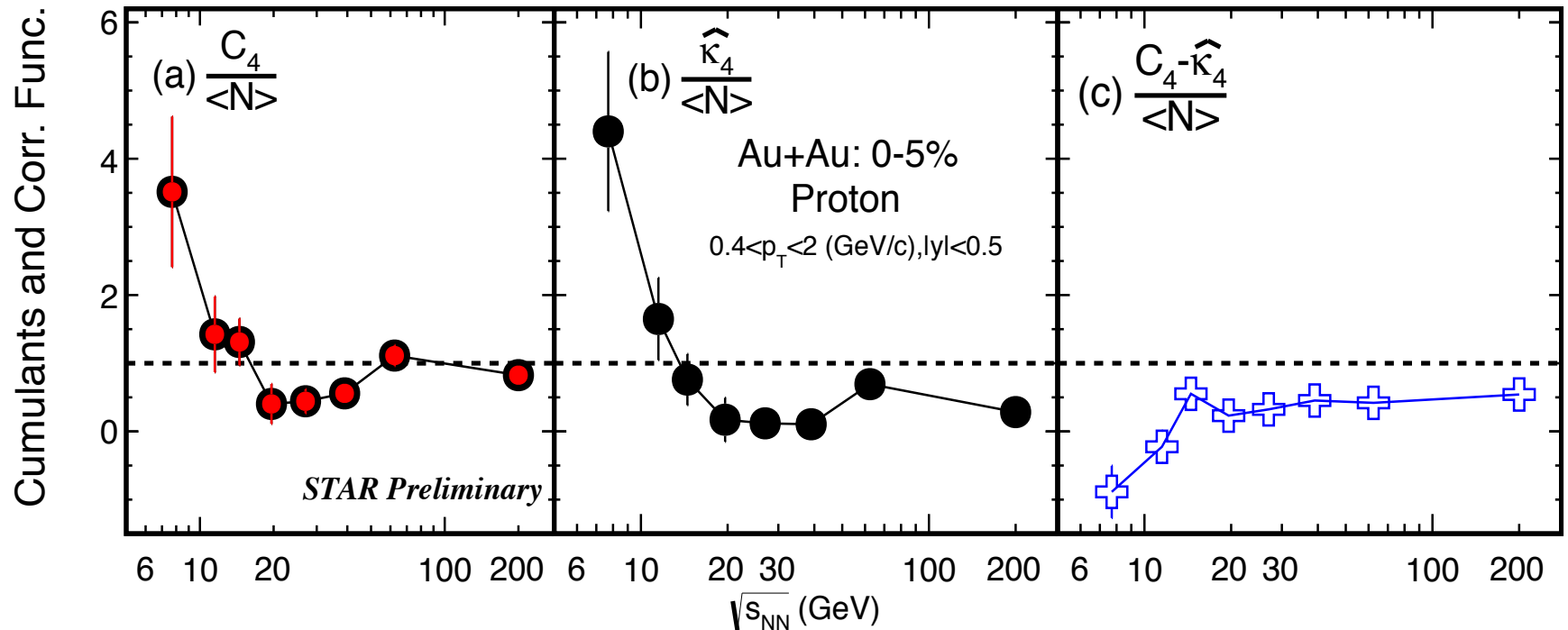
STAR BES-II Whitepaper:  
<https://drupal.star.bnl.gov/STAR/starnotes/public/sn0598>

- 1) BES-I results: Poisson + Baryon conservation +  $v^3$ , **criticality?**
- 2) BES-II: iTPC extend the rapidity coverage to  $\Delta y = 1.6$ , allowing to studying kinematic dependence and precision measurement of higher moments



# Contributions from Four- Particle Correlation

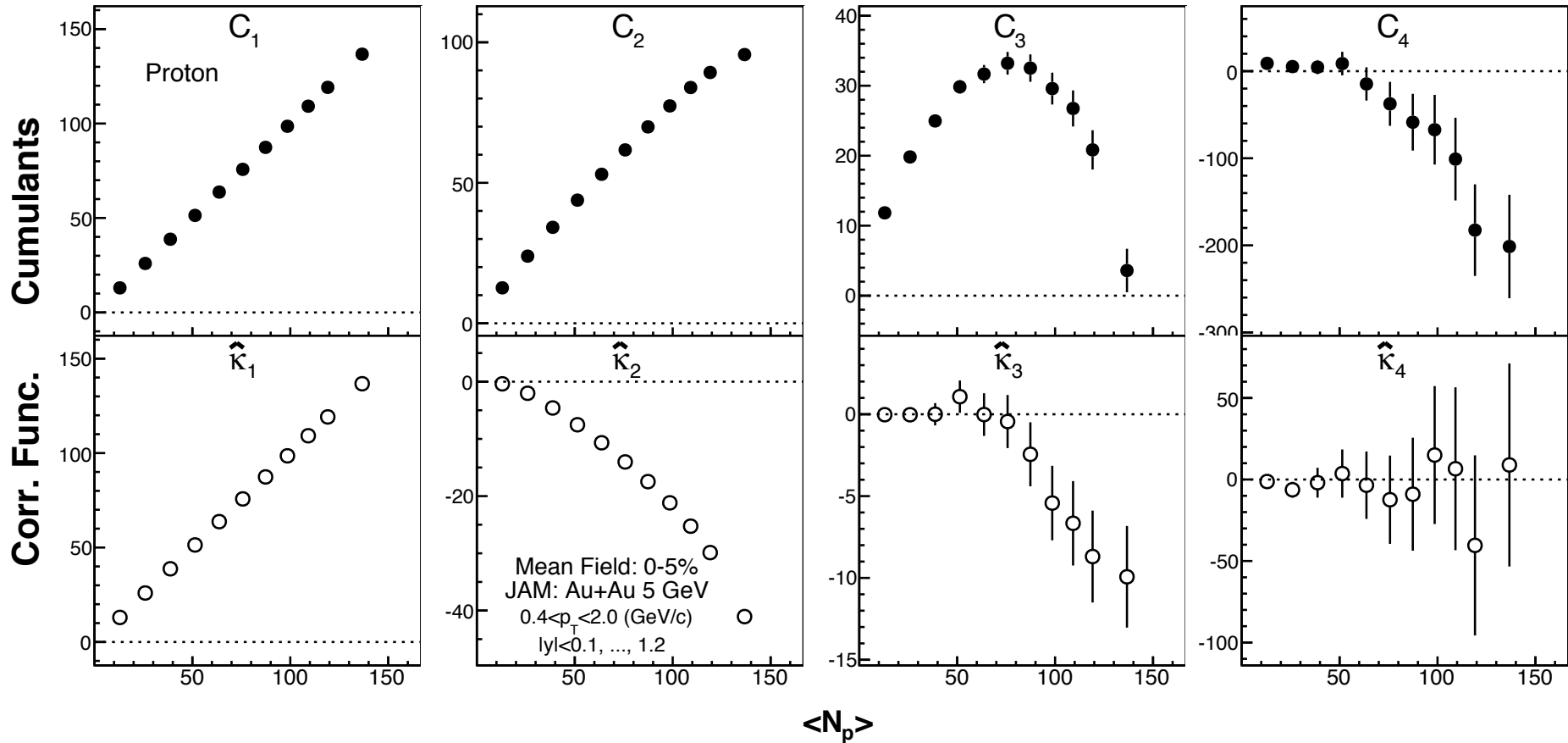
X. Luo (for the STAR Collaboration), PoS(CPOD2014)019 [arXiv:1503.02558].



Without the four particle correlation, the non-monotonic behavior observed in fourth order net-proton fluctuations disappears.



# Proton Cumulants and Correlation Functions from JAM Model



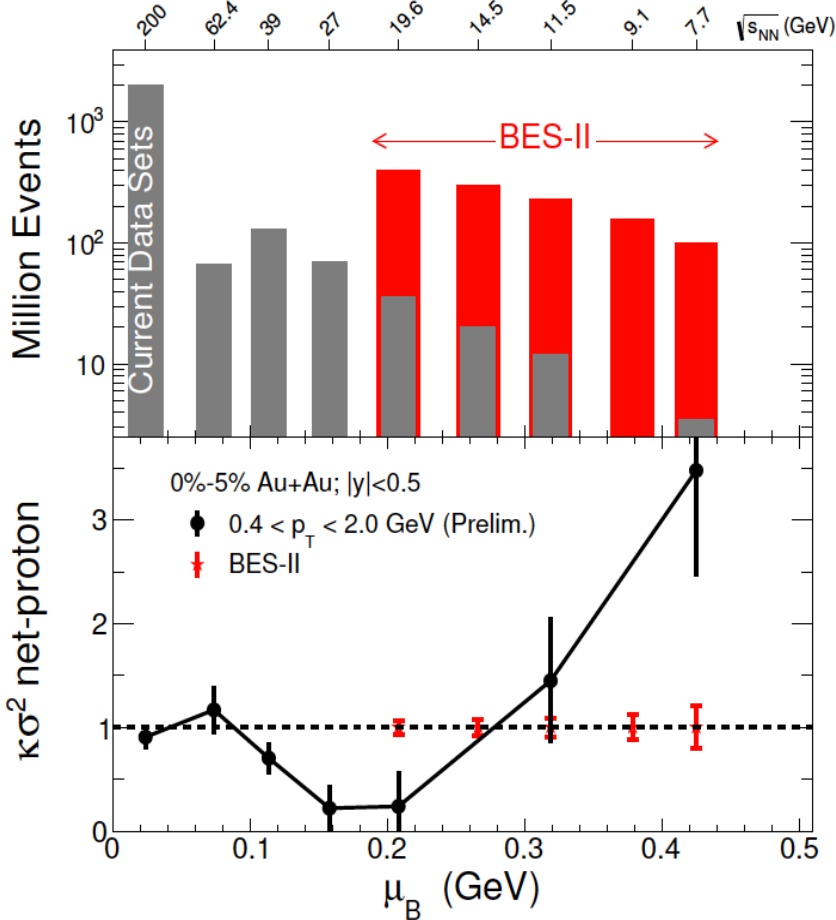
- The cumulants can be strongly suppressed due to baryon conservations  
[A. Bzdak, V. Koch, V. Skokov, Phys. Rev. C 87 \(2013\) 014901.](#)
- The two and three- proton correlation function are negative.
- How about the experimental data ? Stay tuned for QM 2017.



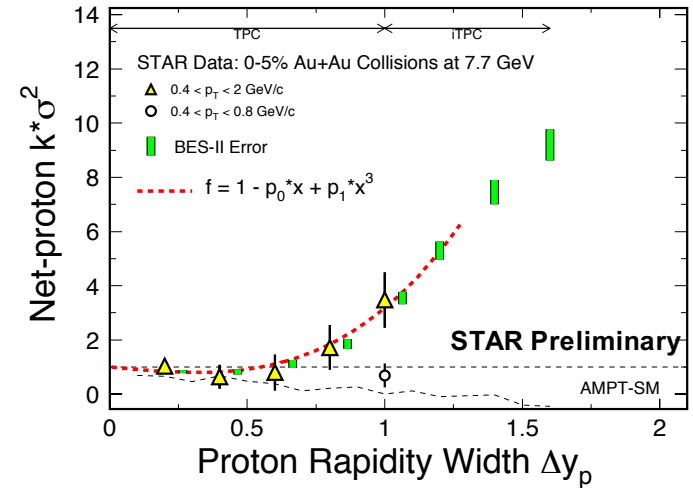
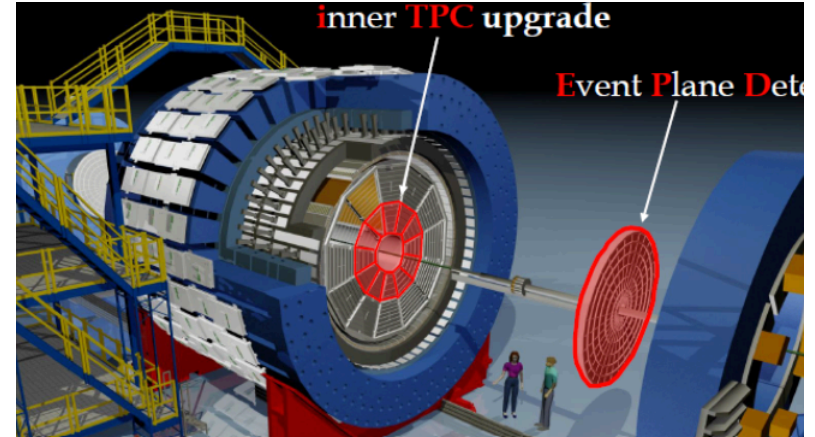
# BES II at RHIC (2019-2020)

## More Data

RHIC Luminosity Upgrade for Low Energies



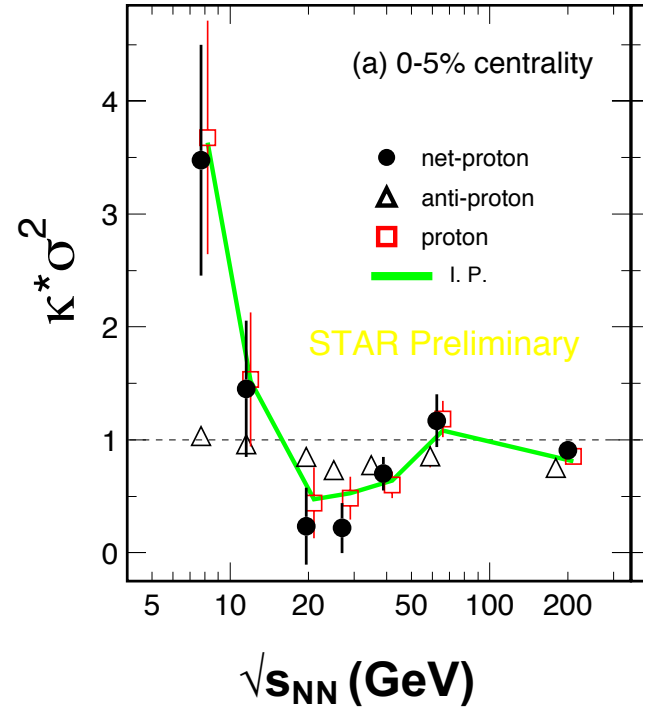
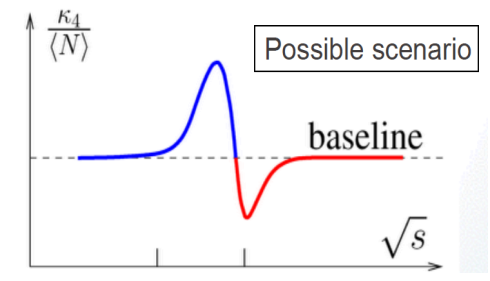
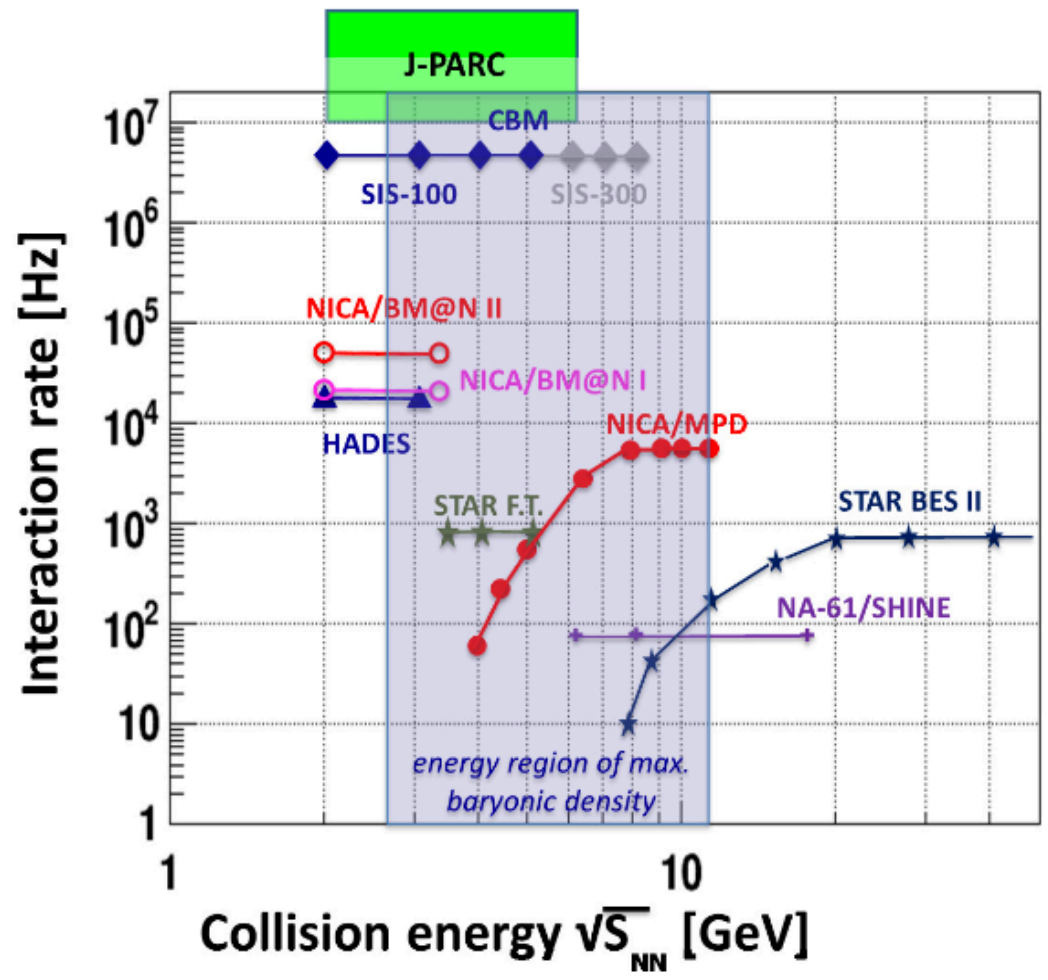
iTPC upgrade extends the rapidity coverage to  $\Delta y = 1.6$



- 1) Event statistics driven by QCD CP search and di-electron measurements.
- 2) The STAR Fix-target mode is also planned in BESII. ( $\sqrt{s_{NN}}$ : 4.5, 3.9, 3.6, 3.0 GeV)



# Future Experiments for High Baryon Density



Longer future: search for the “peak structure” at lower energies  
 $350 < \mu_B < 750$  MeV (  $2 < \sqrt{s_{NN}} < 8$  GeV ). **FXT experiment is more effective.**



# Summary

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- We show cumulants of net-P, net-K and net-Q for Au+Au collisions at 7.7, 11.5, 14.5, 19.6, 27, 39, 62.4 and 200 GeV.
- *Non-monotonic energy dependence is observed at central Au+Au collisions for net-proton kurtosis, which is consistent with the presence of critical point. Observation of the criticality ?*
- Acceptance Studies : *Looking for the power law behavior. Understand the background contribution to corr. func.*
- Study the QCD phase structure at high baryon density with high precision:
  - (1) BES-II at RHIC (2019-2020, both collider and fix target mode).
  - (2) Fix-target at low energies: : FAIR/CBM(starting at 2022), JPARC.

***Thank you !***

