Beam energy dependence of harmonic flow; a probe for the QCD phase Diagram

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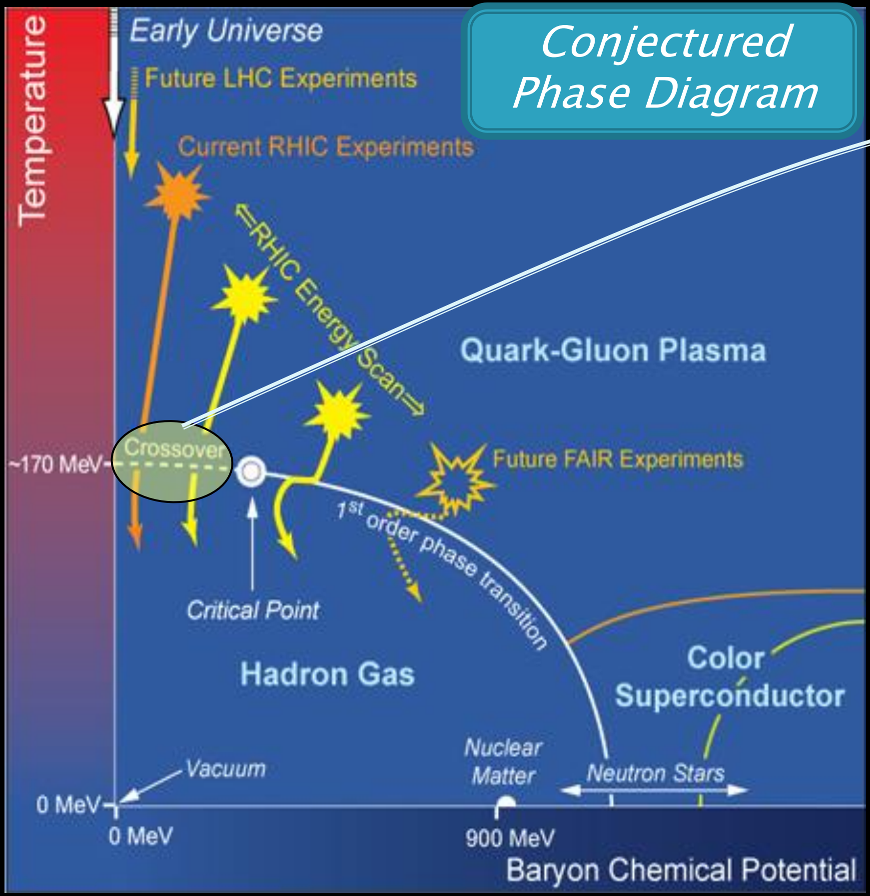
Quantitative study of the phase diagram for nuclear matter is a central current focus of our field



A Known known

- *Spectacular achievement:*

Validation of the crossover transition leading to the QGP
→ Necessary requirement for CEP

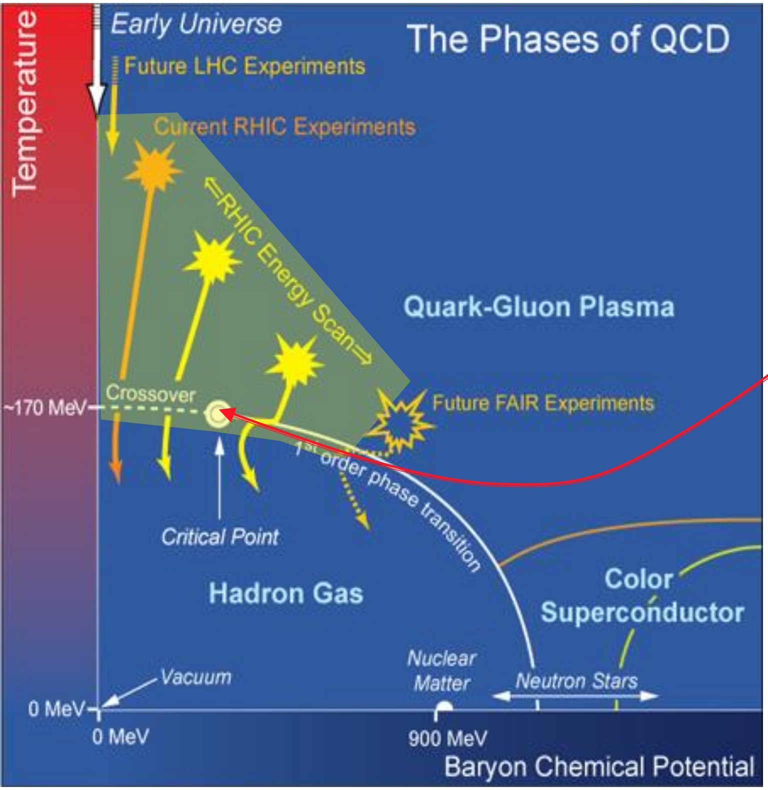
Known unknowns

- **Location of the critical End point (CEP)?**
- **Location of phase coexistence regions?**
- **Detailed properties of each phase?**

All are fundamental to charting the phase diagram

*Measurements which span a broad range of the **(T, μ_B)-plane** are ongoing/slated at RHIC and other facilities*

Remarks on the CEP



Requirements for characterization of the CEP!

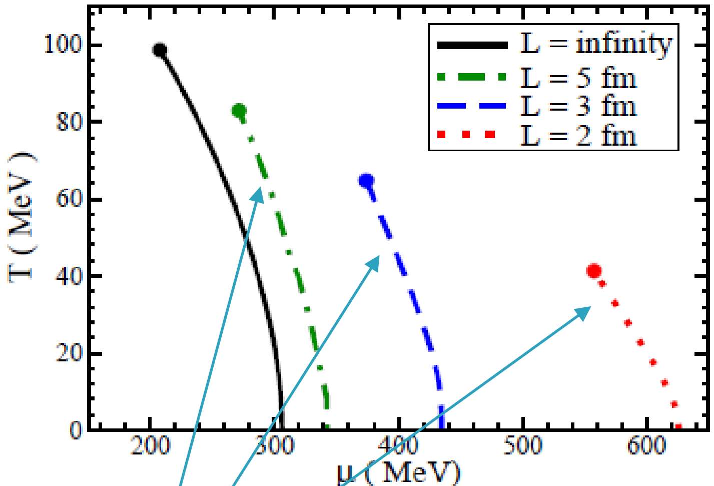
- *Its location* (T^{cep}, μ_B^{cep})*?*
- *Its static critical exponents -* ν, γ*?*
  - ✓ *Static universality class?*
  - ✓ *Order of the transition*
- *Dynamic critical exponent/s – z?*
  - ✓ *Is critical dynamics universal?*
  - ✓ *Dynamic universality class?*

All are required to fully characterize the CEP

Remarks on the CEP

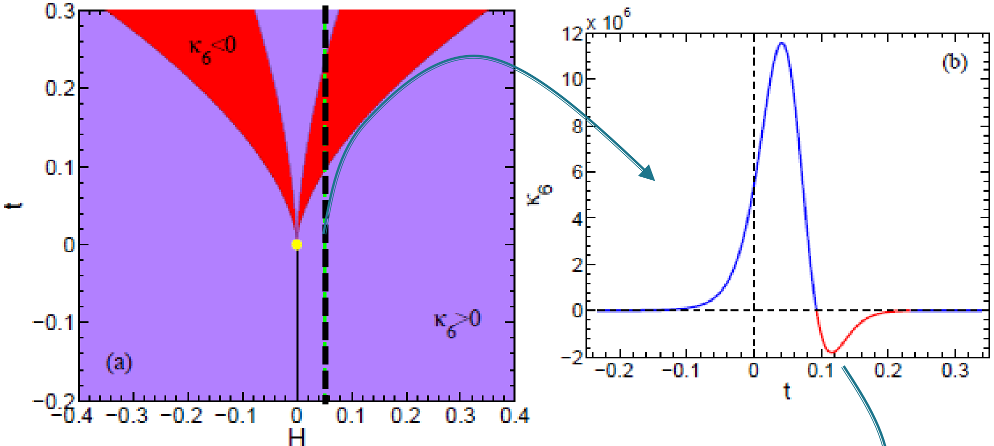
Finite-size effects on the sixth order cumulant -- 3D Ising model

Influence of FSE on the phase diagram

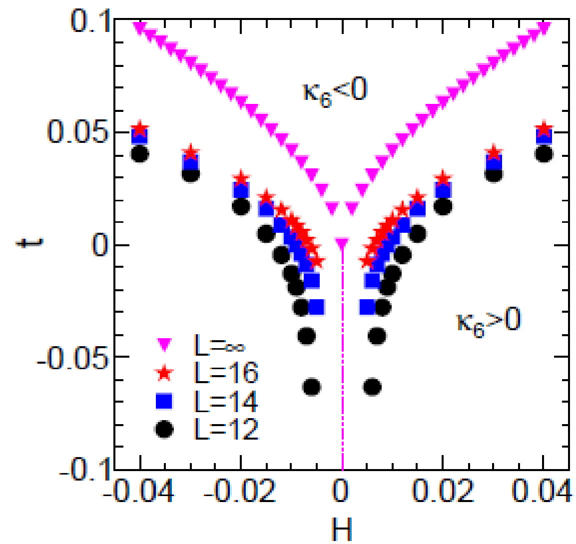


E. Fraga et. al.
J. Phys.G 38:085101, 2011

Displacement of pseudo-first-order transition lines and CEP due to finite-size



Pan Xue et al arXiv:1604.06858



FSE on temp dependence of minimum $\sim L^{2.5n}$

✓ ***A flawless measurement, sensitive to FSE, can <u>Not</u> locate and characterize the CEP directly***

✓ *One solution → exploit FSE*

Remarks on the CEP

Finite-Time Effects (FTE)?

χ_{op} diverges at the CEP

so relaxation of the order parameter could be anomalously slow

$\tau \sim \xi^z$ ← *dynamic critical exponent*

Non-linear dynamics → Multiple slow modes

$z_T \sim 3$, $z_v \sim 2$, $z_s \sim -0.8$

$z_s < 0$ - Critical speeding up

$z > 0$ - Critical slowing down

Y. Minami - **Phys.Rev. D83 (2011) 094019**

An important consequence

$$\xi \sim \tau^{1/z}$$

Significant signal attenuation for short-lived processes with $z_T \sim 3$ or $z_v \sim 2$

eg. $\langle(\delta n)\rangle \sim \xi^2$ **(without FTE)**

$\langle(\delta n)\rangle \sim \tau^{1/z} \ll \xi^2$ **(with FTE)**

****Note that observables driven by the sound mode would NOT be similarly attenuated****

FTE could depend on

- ✓ ***The specific observable***
  - ❖ ***Associated dynamic critical exponent/s***

FTE could also influence FSE

Remarks on the CEP

Inconvenient truths:

- *Finite-size and finite-time effects complicate the search and characterization of the CEP*
  - ***They impose non-negligible constraints on the magnitude of ξ.***

- *The observation of non-monotonic signatures, while helpful, is neither necessary nor sufficient for identification and characterization of the CEP.*
  - ***The prevailing practice to associate the onset of non-monotonic signatures with the actual location of the CEP is a ``gimmick”*** *.*

A Convenient Fact:

- *The **effects of finite size/time** lead to specific dependencies which can be leveraged, via scaling, to locate and characterize the CEP*

Extraction of the χ scaling function

- ***2^{nd} order phase transition***
- ***3D Ising Model (static) universality class for CEP***

$\nu \sim 0.66$ $\gamma \sim 1.2$

$T^{cep} \sim 165$ MeV, $\mu_B^{cep} \sim 95$ MeV

$$\chi(T,L) = L^{\gamma/\nu} P_\chi(tL^{1/\nu})$$

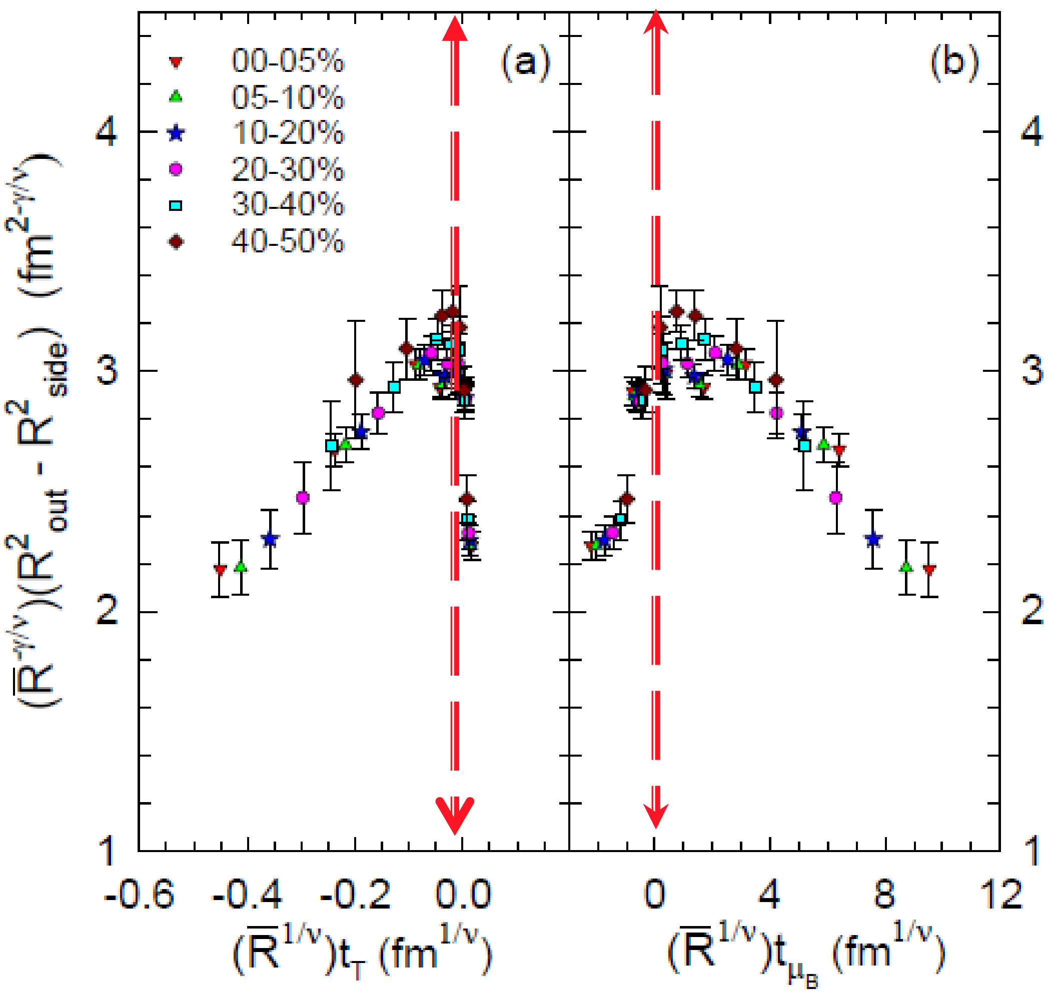
M. Suzuki,
Prog. Theor. Phys. 58, 1142, 1977

Use T^{cep}, μ_B^{cep}, ν and γ to obtain Scaling Function P_χ

$$\bar{R}^{-\gamma/\nu} \times (R^2_{\rm out} - R^2_{\rm side}) \text{ vs. } \bar{R}^{1/\nu} \times t_T,$$
$$\bar{R}^{-\gamma/\nu} \times (R^2_{\rm out} - R^2_{\rm side}) \text{ vs. } \bar{R}^{1/\nu} \times t_{\mu_B},$$
$$t_T = (T - T^{\rm cep})/T^{\rm cep}$$
$$t_{\mu_B} = (\mu_B - \mu_B^{\rm cep})/\mu_B^{\rm cep}$$

T anf μ_B *are* from $\sqrt{s_{NN}}$

Phys.Rev.Lett. 114 (2015) no.14, 142301



*****Scaling function validates the location of the CEP and the (static) critical exponents*****

Dynamic Finite – Size Scaling

➢ ***2nd order phase transition***

$\nu \sim 0.66$ $\gamma \sim 1.2$

$T^{cep} \sim 165$ MeV, $\mu_B^{cep} \sim 95$ MeV

DFSS ansatz

at time τ when T is near T_{cep}

$$\chi(L,T,\tau) = L^{\gamma/\nu} f\left(L^{1/\nu} t_T, \tau L^{-z}\right)$$

$$t_T = (T - T^{cep})/T^{cep}$$

For

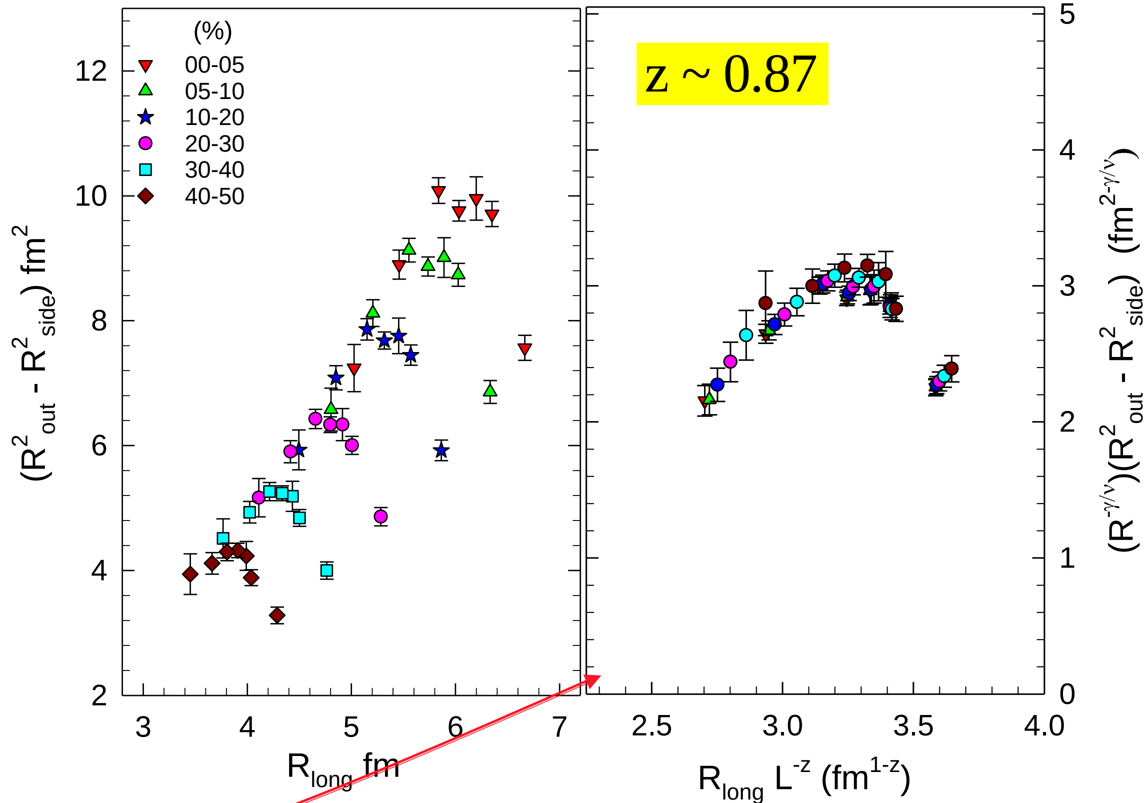
$T = T_c$

$$\chi(L,T_c,\tau) = L^{\gamma/\nu} f\left(\tau L^{-z}\right)$$

$R_{long} \propto \tau$

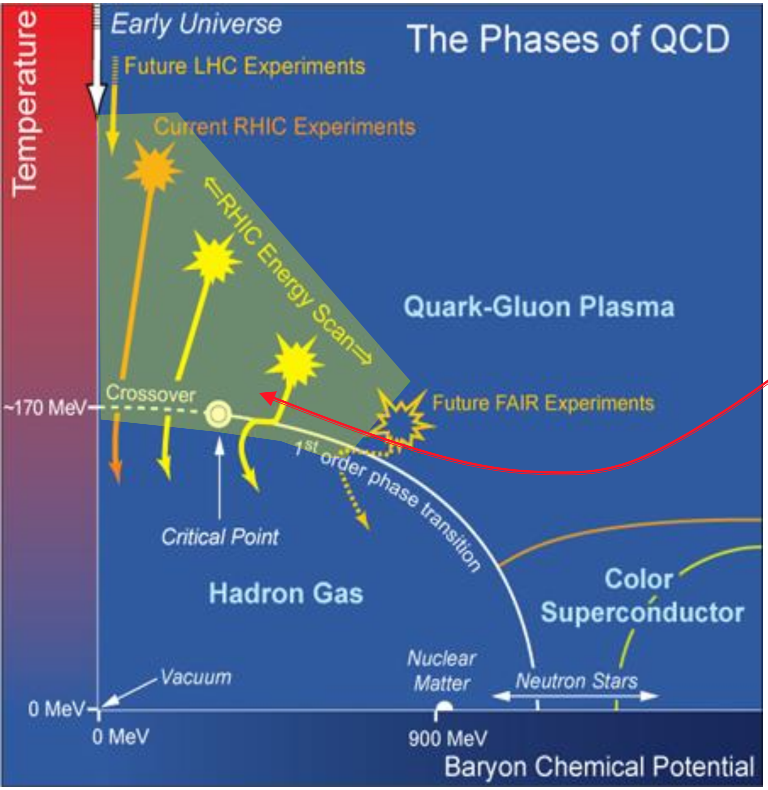
M. Suzuki,
Prog. Theor. Phys. 58, 1142, 1977

Experimental estimate of the dynamic critical exponent



The magnitude of z is similar to the predicted value for z_s but the sign is opposite

Characterizing the QCD phases



Requirements for characterization of the QCD phases!

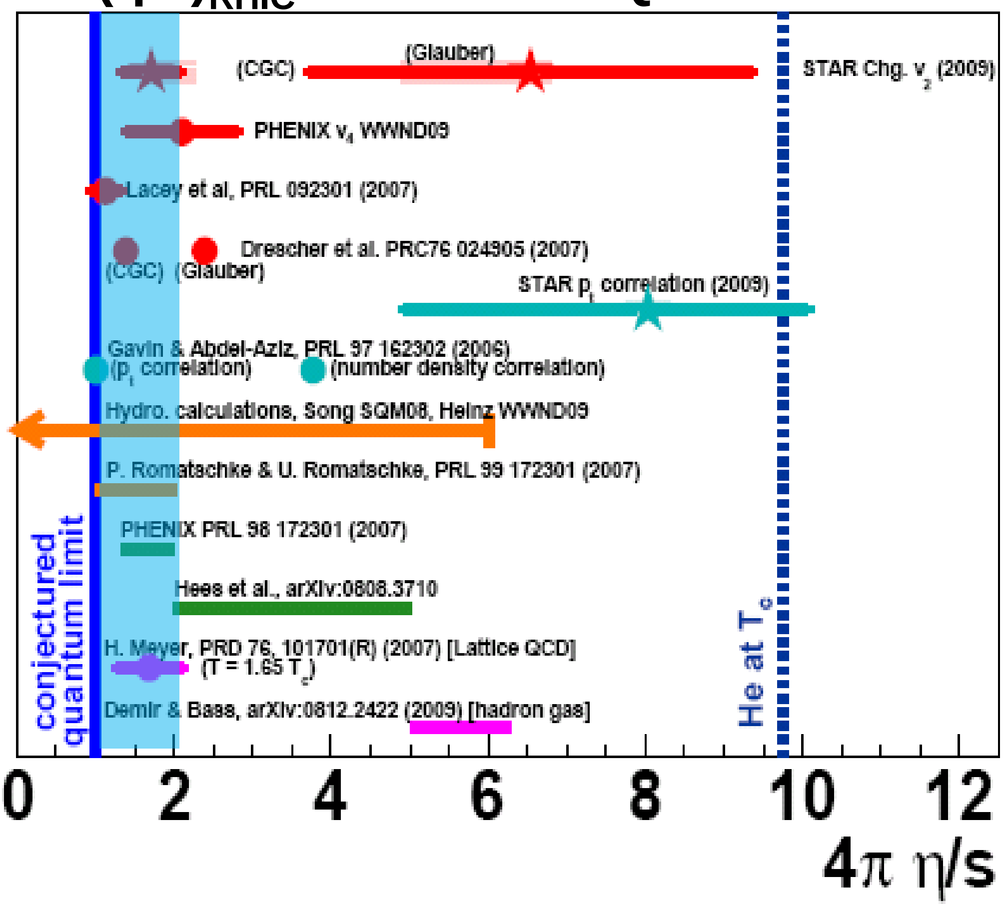
- *Specific shear viscosity?*
  - ✓ *T, μ_B dependence*
- *Specific bulk viscosity?*
  - ✓ *T, μ_B dependence*
- *Conductivity?*

Etc.

Flow measurements provide important constraints

Backdrop

$(\eta/s)_{RHIC}$ estimates – QM2009



Subsequently

- *Convergence on the magnitude of η/s at RHIC*

4πη/s ~ 1 - 2

- *T dependence of η/s?*
- *μ_B dependence of η/s?*
- *Interplay with ξ/s*

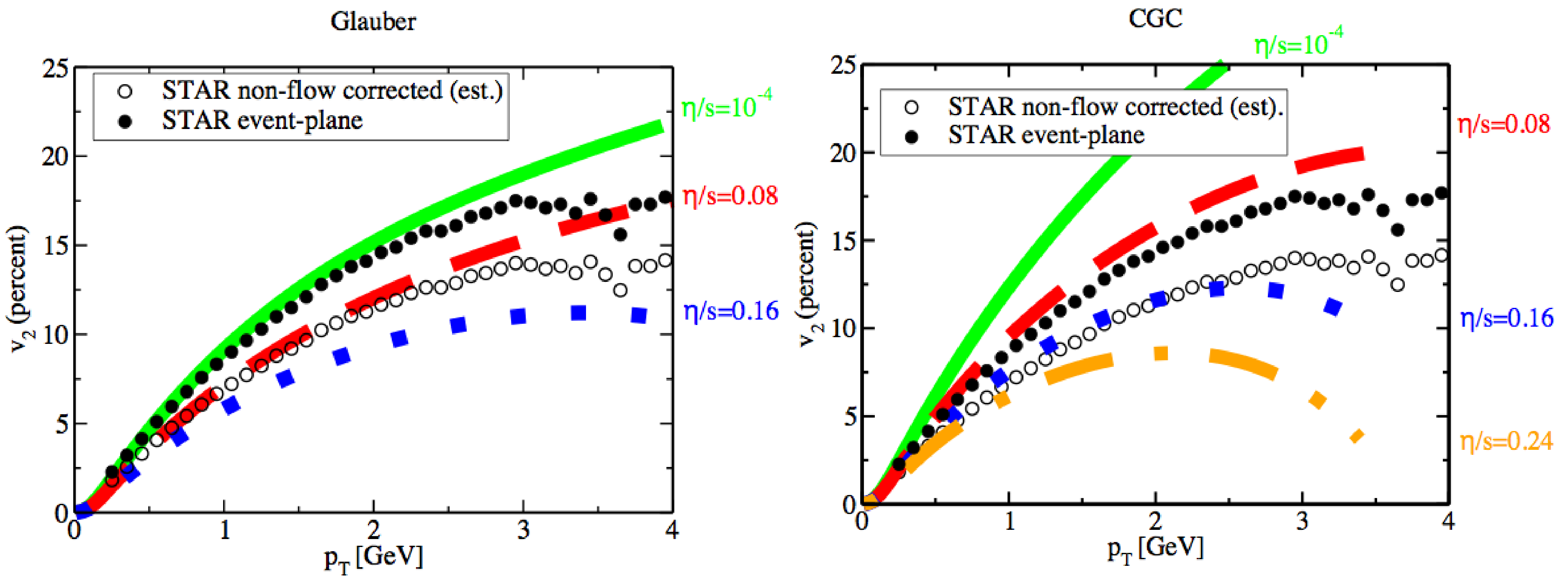
Status Quo

A major uncertainty in the extraction of η/s stems from Incomplete knowledge of the Initial-state eccentricity model

ε_n – η/s interplay?

Status Quo

Luzum et al. arXiv 0804.4015



<u>Note</u>

The Initial-state eccentricity difference between MC-KLN and MC-Glauber is ~ 20% due to fluctuation differences in the models!

ε_n – η/s interplay?

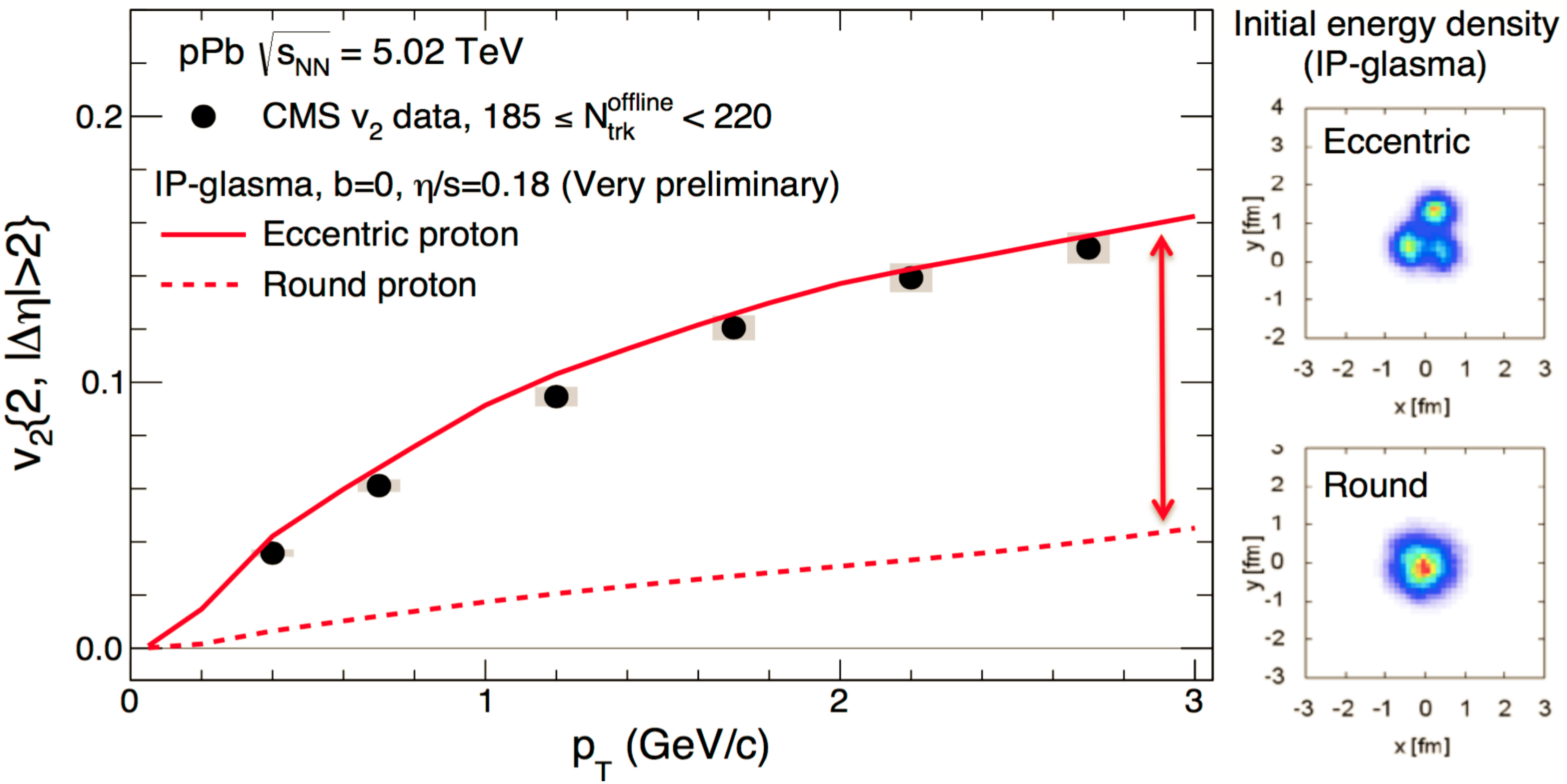
η/s is a property of the medium and should not depend on initial geometry!

This should <u>NOT</u> be treated as an uncertainty;

Additional specific constraints can be applied?

Backdrop

B. Schenke SQM16



➤ ***What is the relevant substructure of the nucleon?***
- valence quarks?

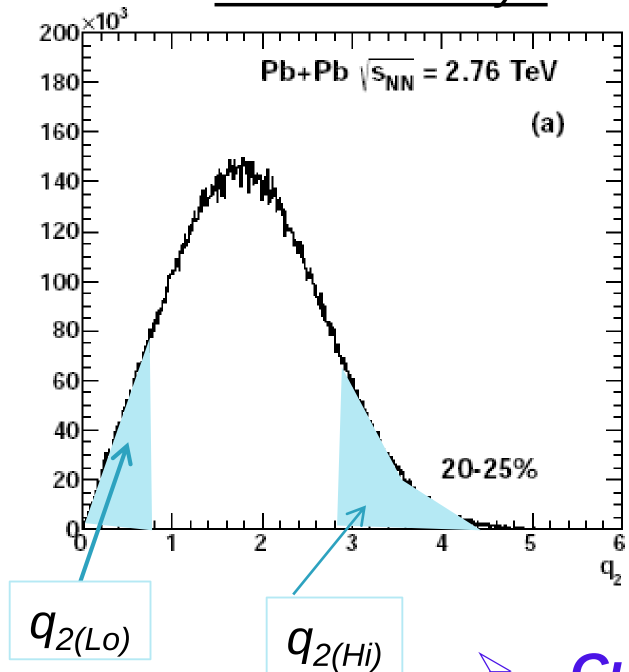
A constraint for initial-state fluctuations

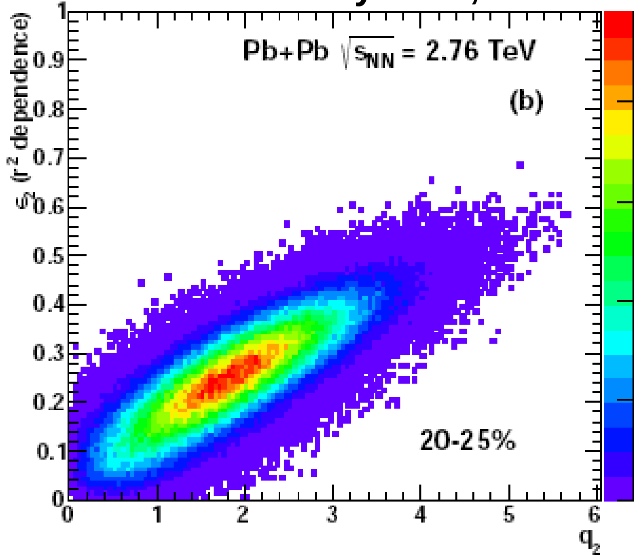
Shape fluctuations lead to a distribution of the Q vector at a <u>fixed centrality</u>

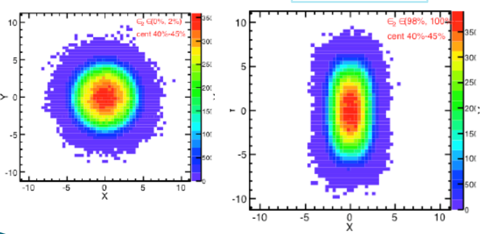
$$Q_{n,x} = \sum_{i}^{M} \cos(n\phi_i);\ Q_{n,y} = \sum_{i}^{M} \sin(n$$

$$q_n = Q_n/\sqrt{M},$$

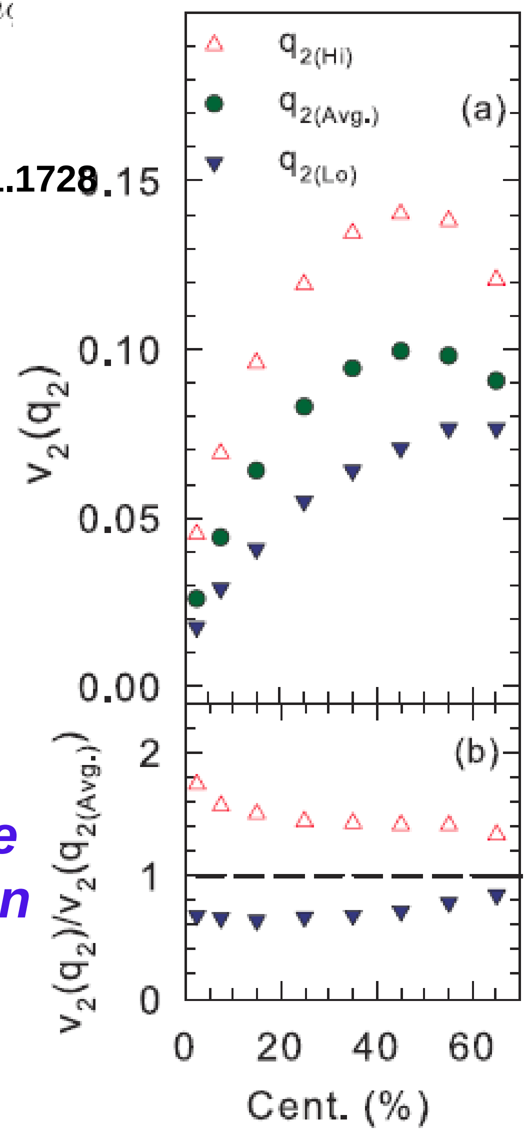
Lacey et. al, arxiv:1311.1728







ALICE data



- *Cuts on q_n should change the magnitudes $\langle \epsilon_n \rangle$, $\langle v_n \rangle$ at a given centrality due to fluctuations*

- *Viable constraint for initial state fluctations*

Compressibility & Bulk viscosity at the CEP

For an isothermal change

$$V dP = N d\mu$$

$$N\left(\frac{\partial \mu}{\partial N}\right)_{V,T} = V\left(\frac{\partial P}{\partial N}\right)_{V,T} = \frac{1}{\rho \kappa_T}$$

From partition function one can show that

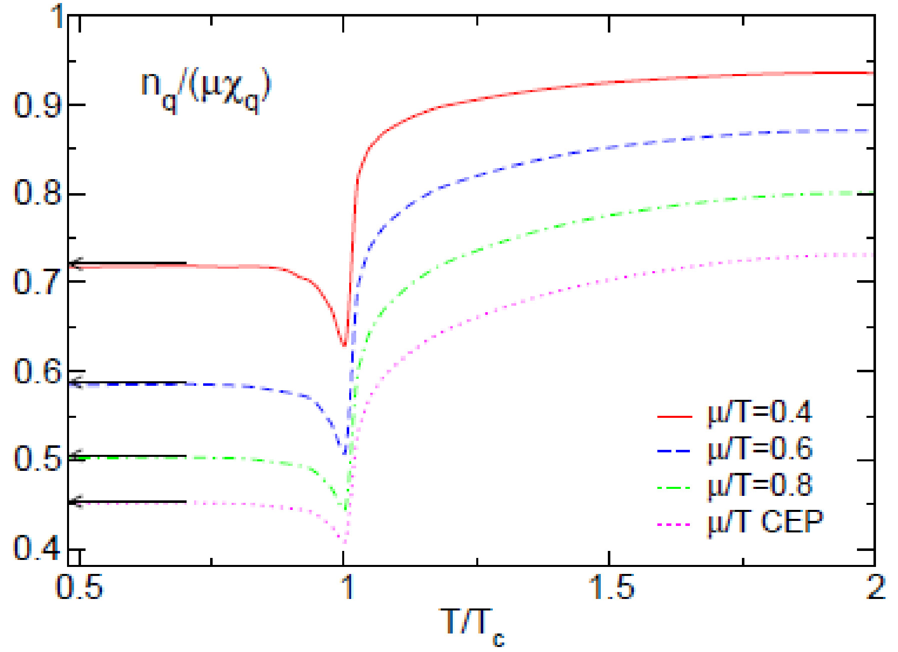
$$\frac{1}{\langle N \rangle}\left(\frac{\partial \langle N \rangle}{\partial \beta\mu}\right) = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$

$$\kappa_T^{-1} \propto \frac{\langle N \rangle}{\langle N^2 \rangle - \langle N \rangle^2} = \frac{C_1}{C_2}$$

The compressibility diverges at the CEP

Inverse compressibility

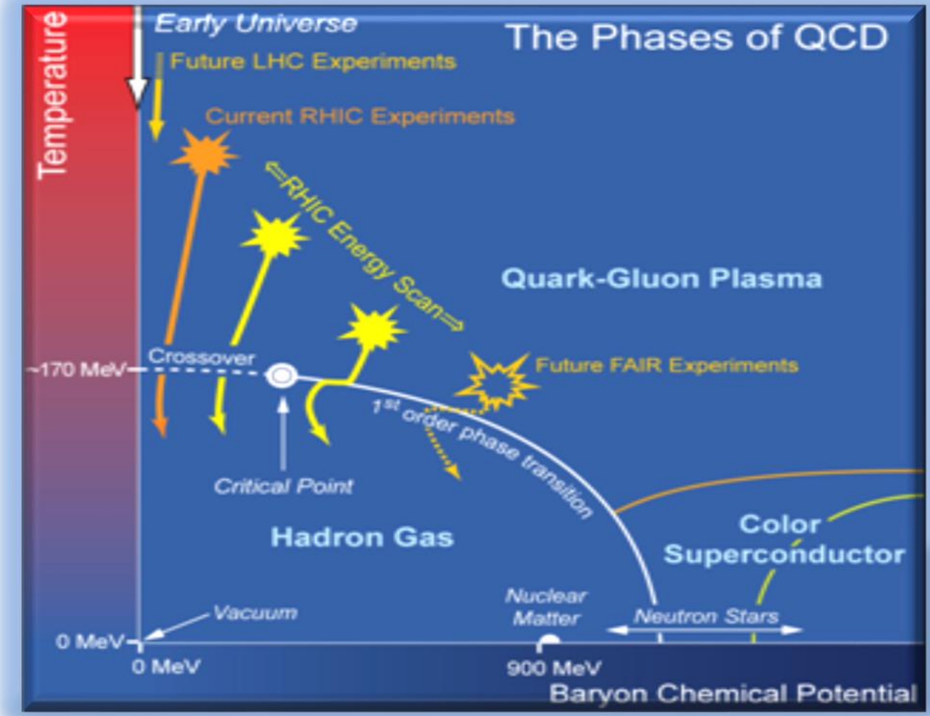
Stoki´c, Friman, and K. Redlich, **Phys.Lett.B673:192-196,2009**



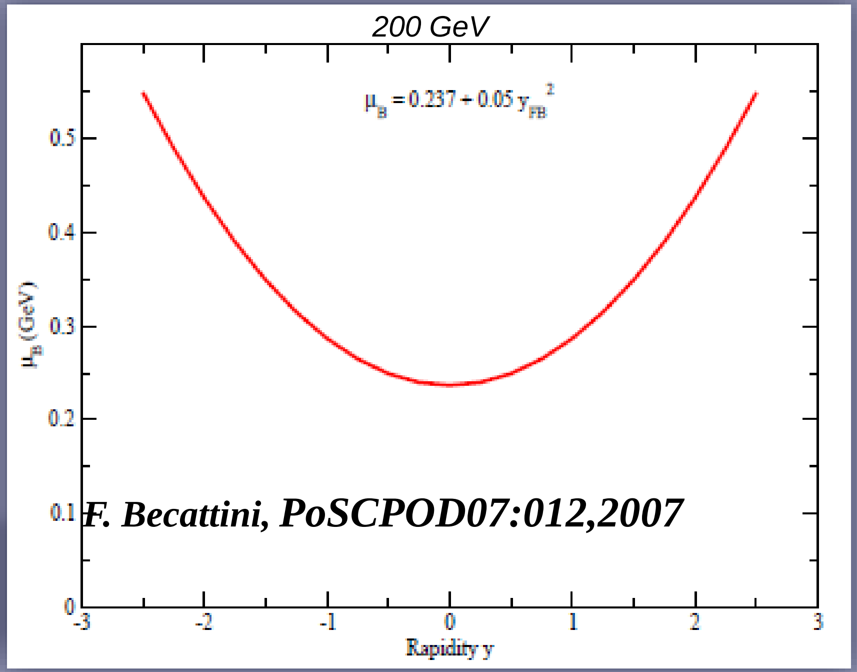
At the CEP the inverse compressibility → 0

Scaling function provides an independent handle

The BES at RHIC allows the study of a broad domain of $(\mu_B, T) -$ plane.

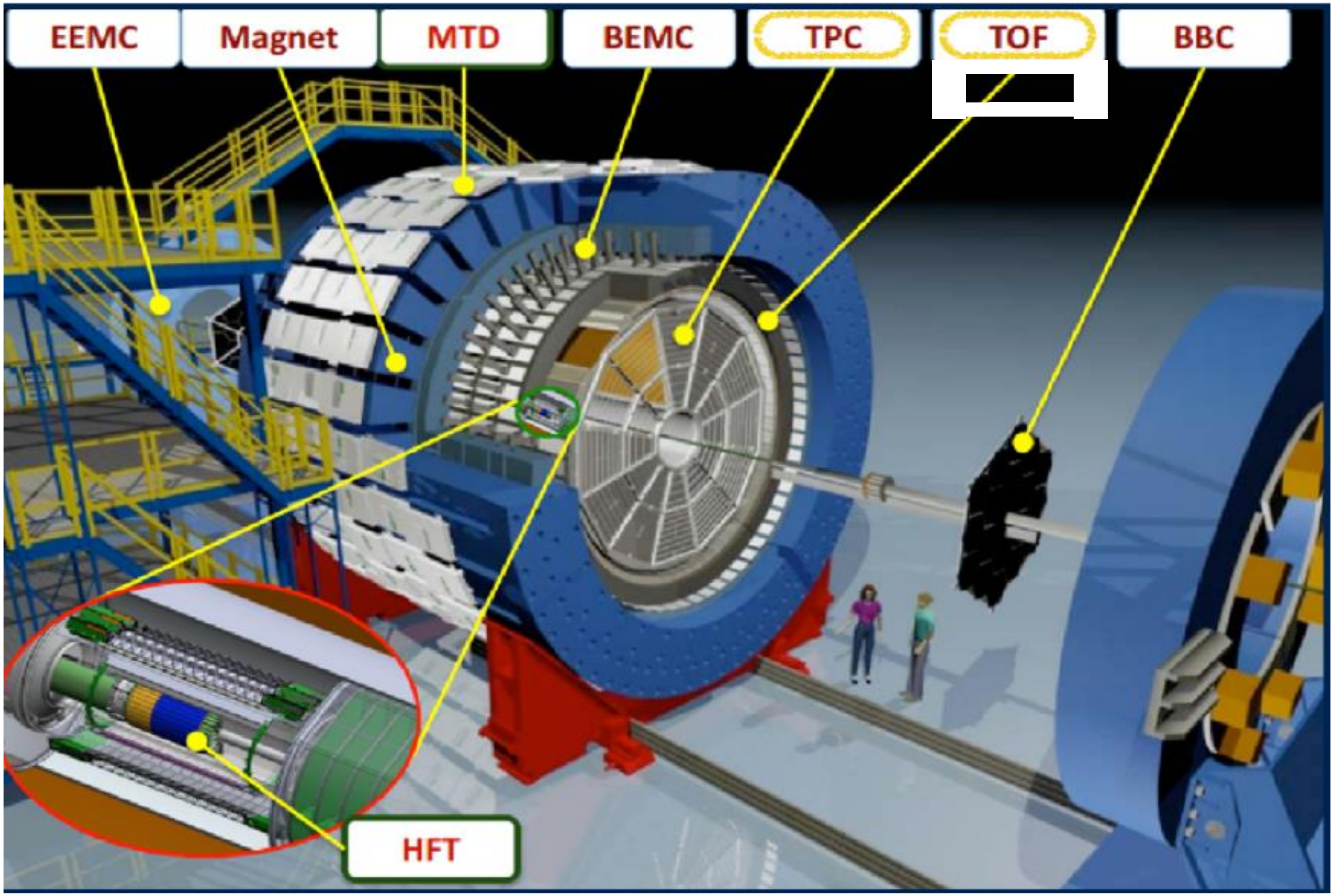


Rapidity dependence of (μ_B)



- ➢ *μ_B & T variations via beam energy or rapidity selections.*
- ➢ *Several systems for geometry and fluctuations*

STAR Detector at RHIC



- *TPC detector covers* $|\eta| < 1$
- *FTPC detector covers* $2.5 < |\eta| < 4$

Analysis technique

- **All current techniques used to study v_n are related to the correlation function.**
- ***Two particle correlation function*** $C(\Delta\varphi = \varphi_1 - \varphi_2)$ **used in this analysis,**

$$C(\Delta\varphi) = \frac{dN/d\Delta\varphi(same)}{dN/d\Delta\varphi(mix)} \quad and \quad v_n^2 = \frac{\sum_{\Delta\varphi} C(\Delta\varphi)\cos(n\,\Delta\varphi)}{\sum_{\Delta\varphi} C(\Delta\varphi)}$$

1

PLB 708, 249 (2012)

$$v_n(pT) = \frac{v_n^2(p_{T_{ref}}, p_T)}{\sqrt{v_n^2(p_{T_{ref}})}}$$

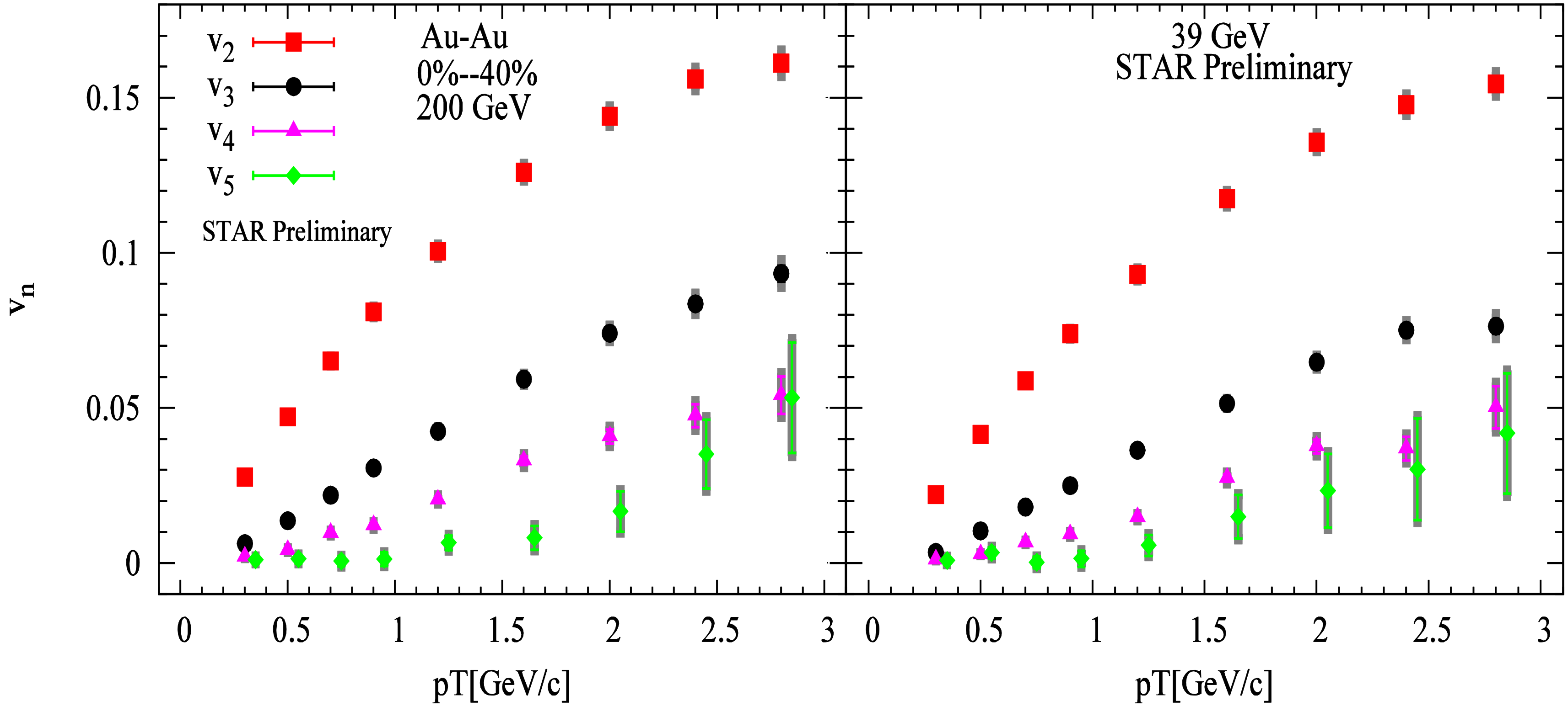
2

- ✓ ***Factorization ansatz for v_n verified.***
- ✓ ***Non-flow signals, as well as some residual detector effects (track merging/splitting) minimized with*** $|\Delta\eta = \eta_1 - \eta_2| > 0.7$ ***cut.***

Results

$v_n(p_T)$

$|\eta| < 1$ and $|\Delta\eta| > 0.7$

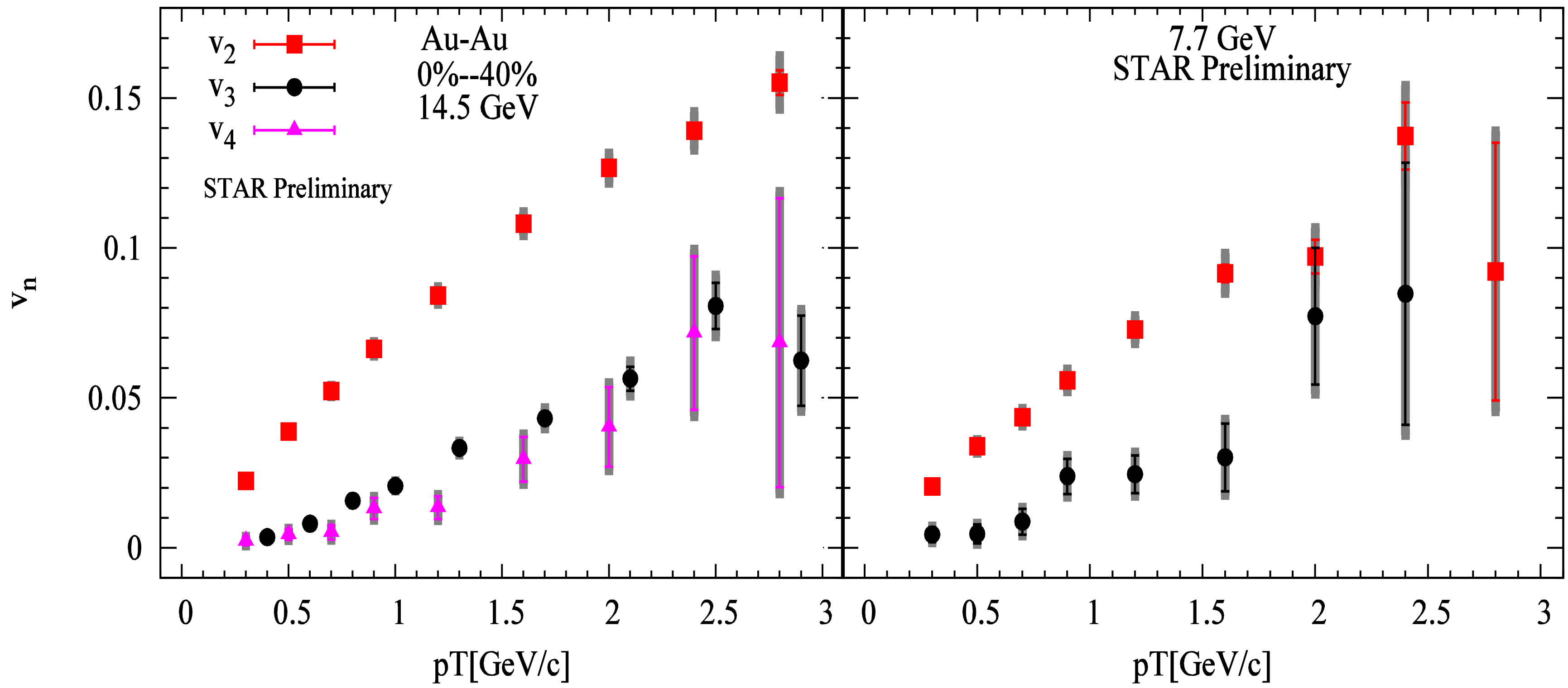


- $v_n(p_T)$ *indicate a similar trend for different beam energies.*
- $v_n(p_T)$ *decreases with harmonic order n.*

Results

$v_n(p_T)$

$|\eta| < 1$ and $|\Delta\eta| > 0.7$



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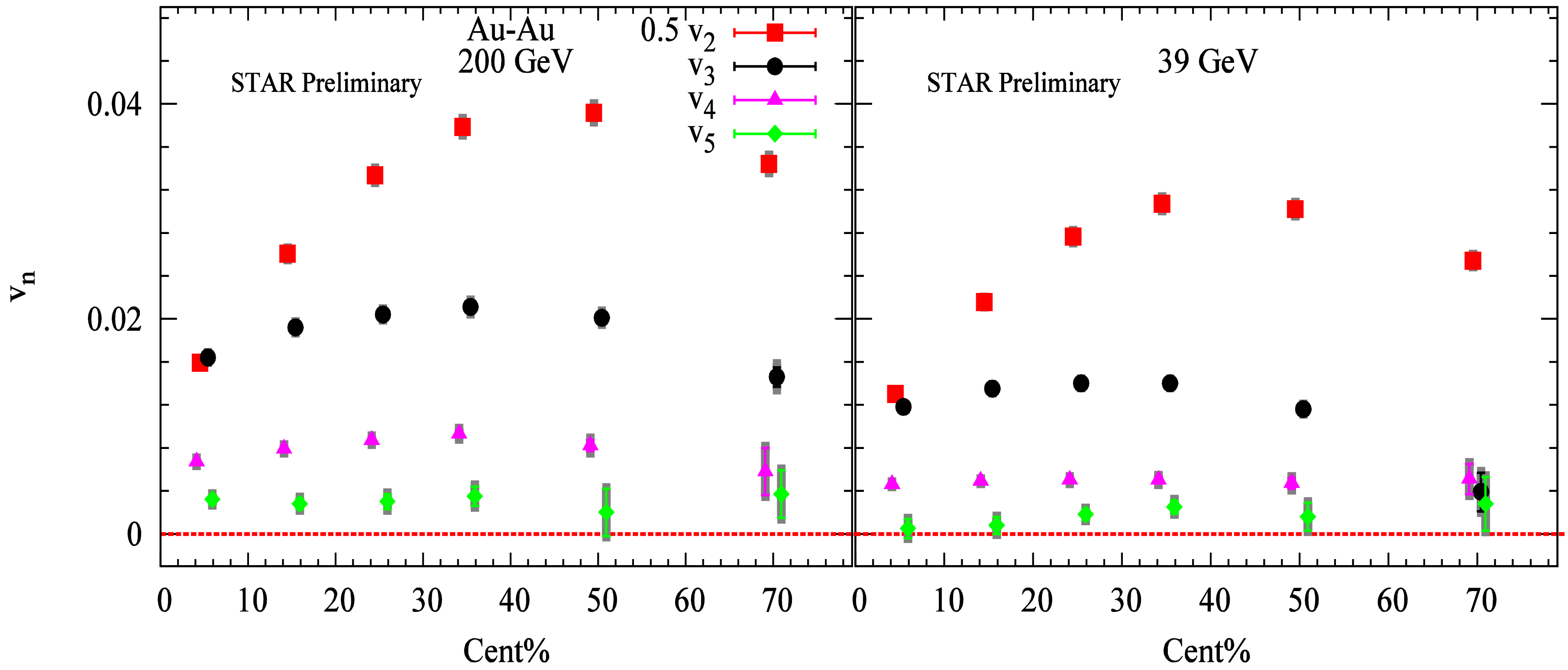
Results

$v_n(Cent)$

$|\eta| < 1$ and $|\Delta\eta| > 0.7$

$0.2 < p_T < 4\text{GeV/c}$



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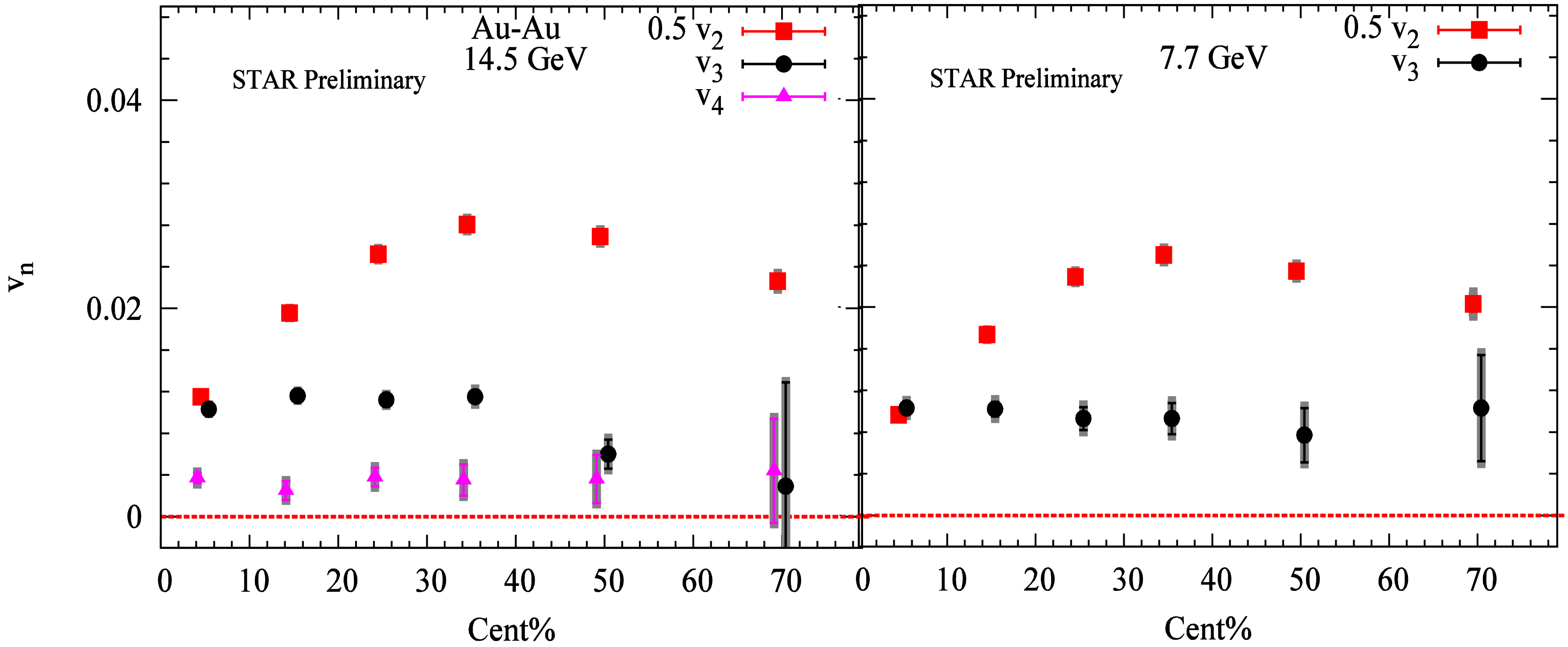


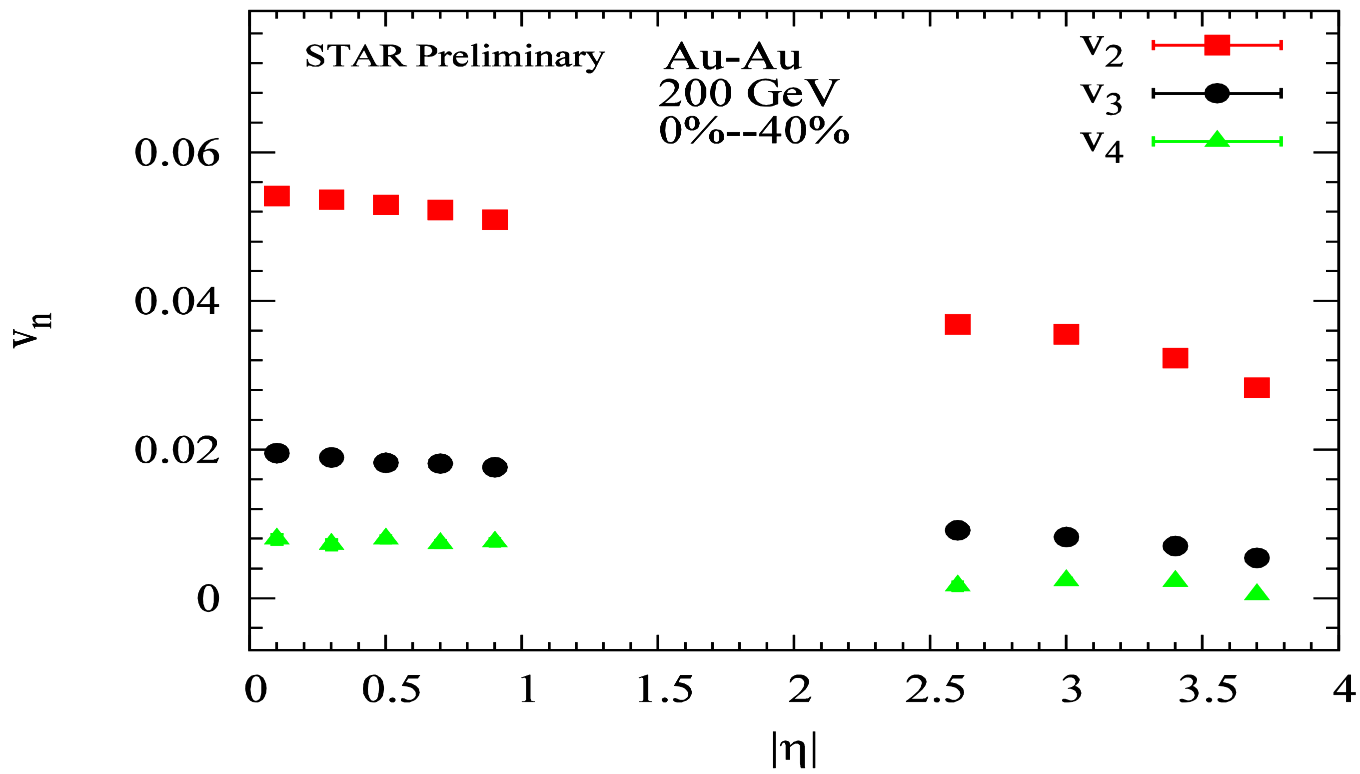
- $v_n(Cent)$ *indicate a similar trend for different beam energies.*
- $v_n(Cent)$*decreases with harmonic order n.*

Results

$v_n(\eta)$

$|\eta_{ref}| < 1 \; and \; |\eta| < 4$

$0.2 < p_T < 4\text{GeV/c}$



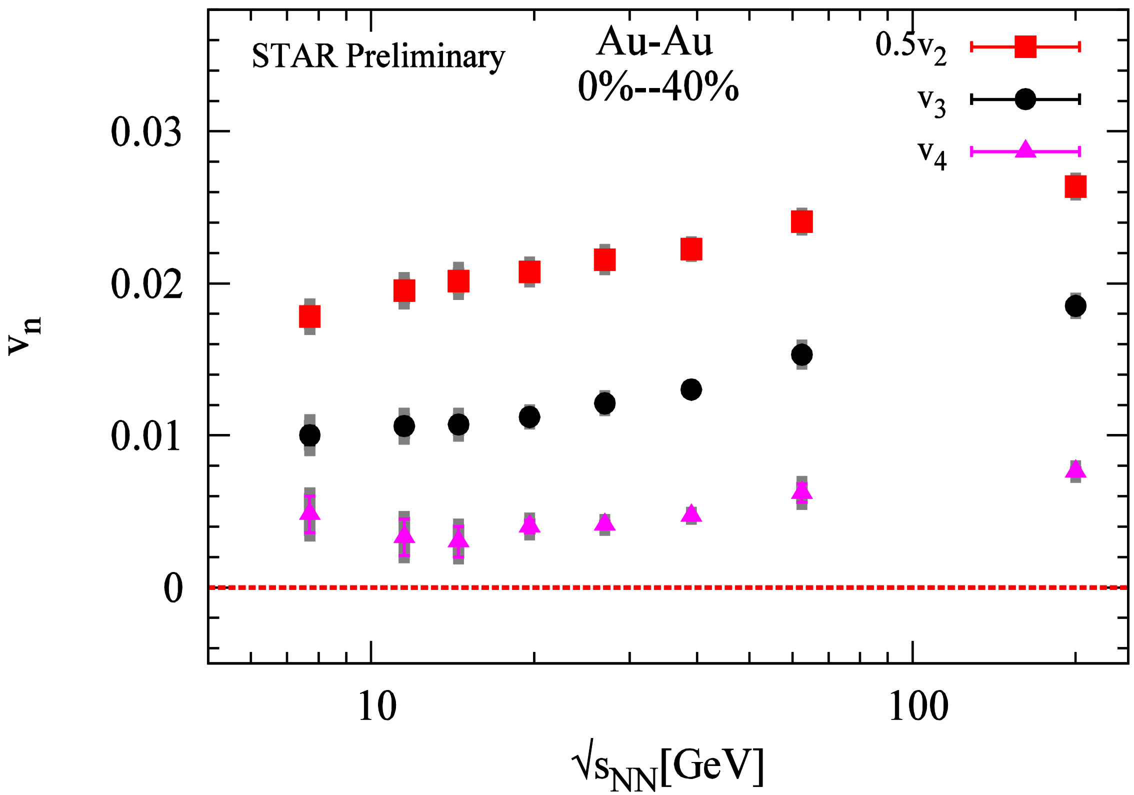
- *Mid and forward rapidity $v_n(\eta)$ decreases with harmonic order n.*

Results

$v_n(\sqrt{s_{NN}})$

$|\eta| < 1$ and $|\Delta\eta| > 0.7$

$0.2 < p_T < 4\mathrm{GeV/c}$



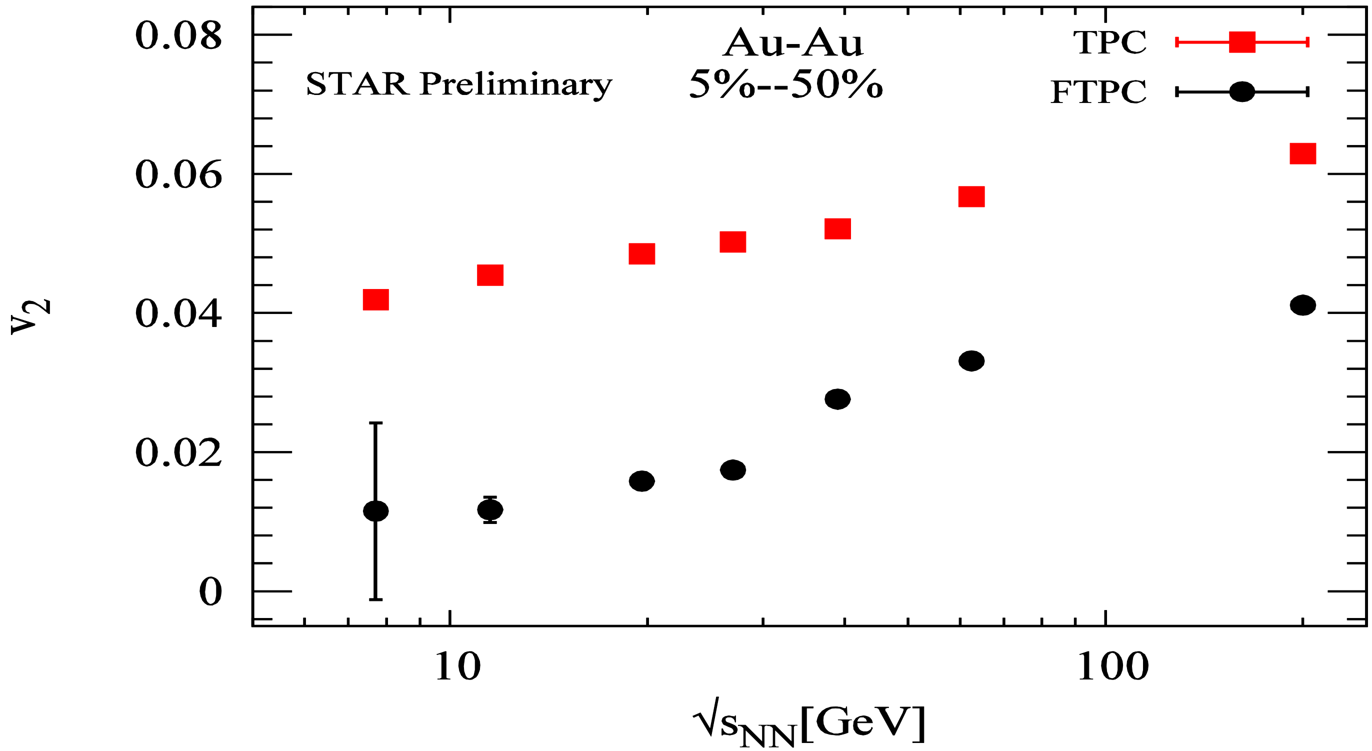
- *Mid rapidity $v_n(\sqrt{s_{NN}})$ shows a monotonic increase with beam energy.*
- *$v_n(\sqrt{s_{NN}})$ decreases with harmonic order n.*

Results

$v_2(\sqrt{s_{NN}})$

TPC and FTPC

$0.2 < p_T < 4\text{GeV/c}$



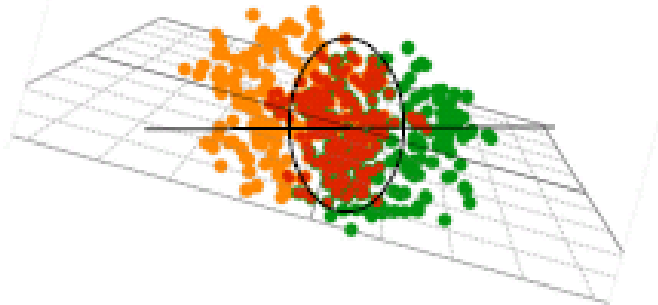
- Mid and forward rapidity $v_2(\sqrt{s_{NN}})$ shows a monotonic increase with beam energy.
- Forward rapidity $v_2(\sqrt{s_{NN}})$ shows a stronger dependence.

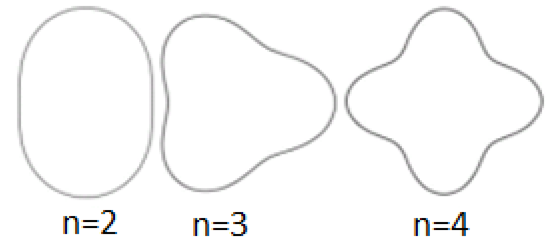
What indications do we have for the flow constraints?

Expansion Dynamics

Initial Geometry characterized by many shape harmonics (ε_n) → drive v_n

Flow is acoustic





$$\frac{dN}{d\phi} \propto \left(1 + 2\sum_{n=1} v_n \cos\left[n\left(\phi - \Psi_n\right)\right]\right)$$

Acoustic viscous modulation of v_n

$$\delta T_{\mu\nu}(t,k) = \exp\left(-\frac{2}{3}\frac{\eta}{s}k^2\frac{t}{T}\right)\delta T_{\mu\nu}(0)$$

Staig & Shuryak **arXiv:1008.3139**

$$k = n/\bar{R} \qquad t \propto \bar{R}$$

$$\delta T_{\mu\nu}(n,t) = \exp\left(-\beta n^2\right)\delta T_{\mu\nu}(0), \quad \beta = \frac{2}{3}\frac{\eta}{s}\frac{1}{\bar{R}^2}\frac{t}{T}$$

Scaling expectations:

Straightforward to include bulk viscosity

n^2 dependence

$$\left(\frac{v_n(p_T)}{\varepsilon_n}\right) \propto \exp\left(-\frac{\beta'}{RT}n^2\right)$$

v_n 's are related

$$\frac{\left(v_n(p_T)\right)^{1/n}}{\left(v_{n'}(p_T)\right)^{1/n'}} \sim \frac{\left(\varepsilon_{n'}\right)^{1/n'}}{\left(\varepsilon_n\right)^{1/n}} \cdot \exp\left(-\frac{\beta'}{RT}(n-n')\right)$$

System size dependence

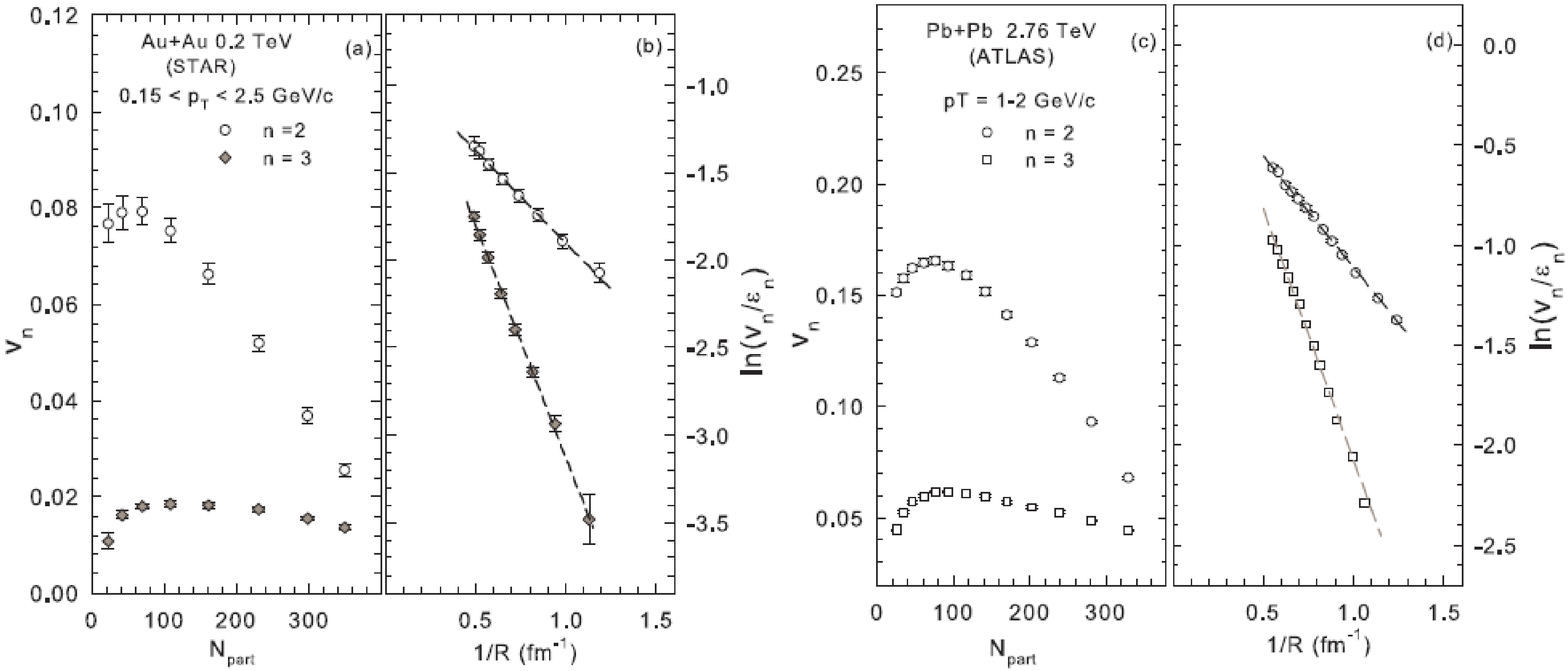
$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto \frac{-\beta''}{RT}$$

The factors which influence anisotropic flow – well understood

Scaling properties of flow

Acoustic Scaling – $\frac{1}{\bar{R}}$

$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto \frac{-\beta''}{\bar{R}}$$



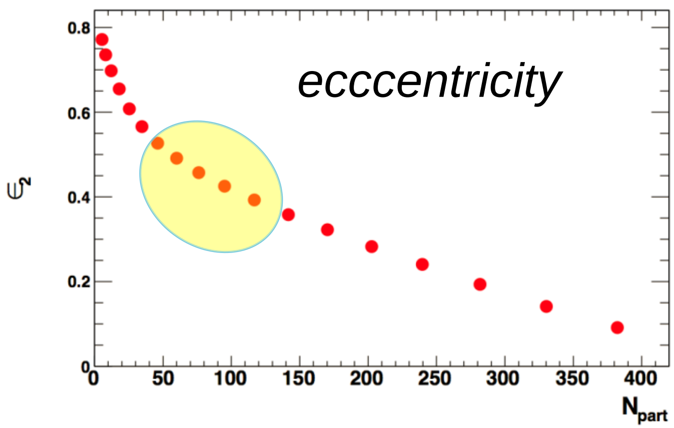
✓ **Characteristic $1/\bar{R}$ viscous damping validated with n^2 dependence at RHIC & the LHC**

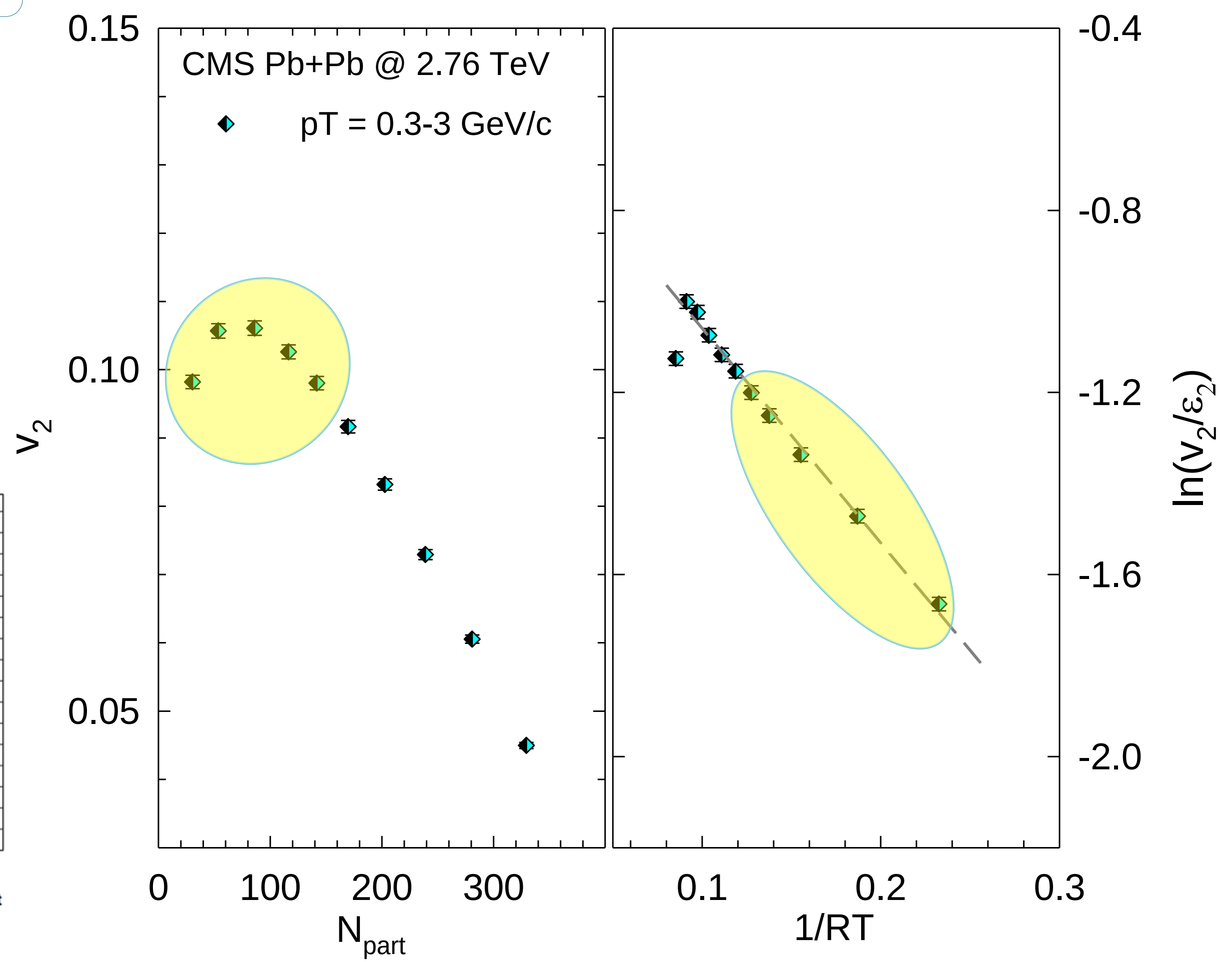
✓ ***Important constraint for η/s***

Acoustic Scaling – RT

$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto \frac{-\beta''}{RT}$$

$$RT \propto \left(\frac{dN_{chg}}{d\eta}\right)^{1/3}$$

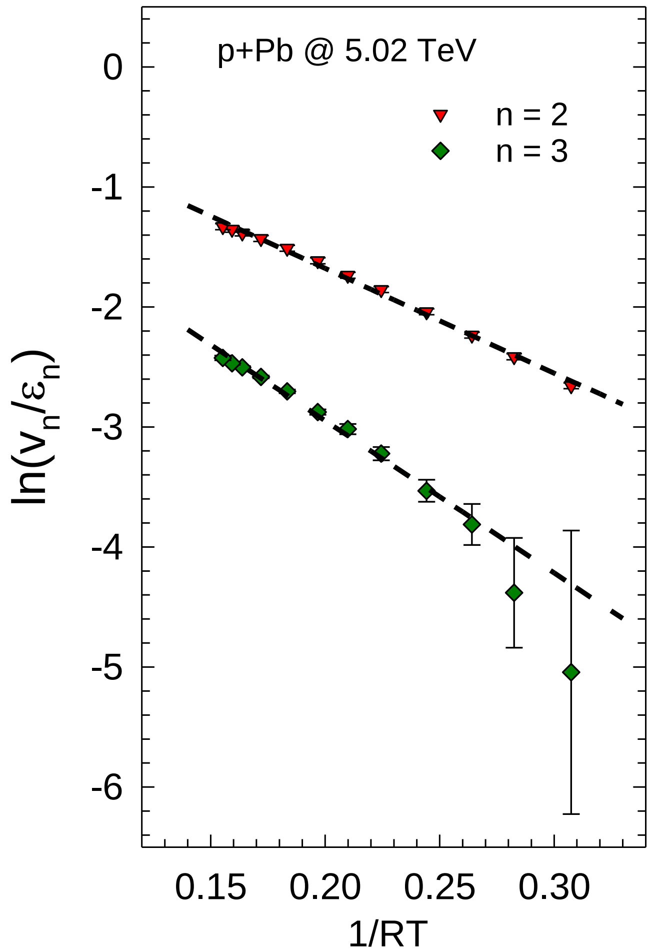




- **Eccentricity change <u>alone</u> is not sufficient**

- ✓ **Characteristic 1/(RT) viscous damping validated**
- ✓ **Similar patterns for other p_T selections**
- ✓ ***Important constraint for η/s & ζ/s***

Acoustic Scaling – RT



$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto \frac{-\beta''}{RT}$$

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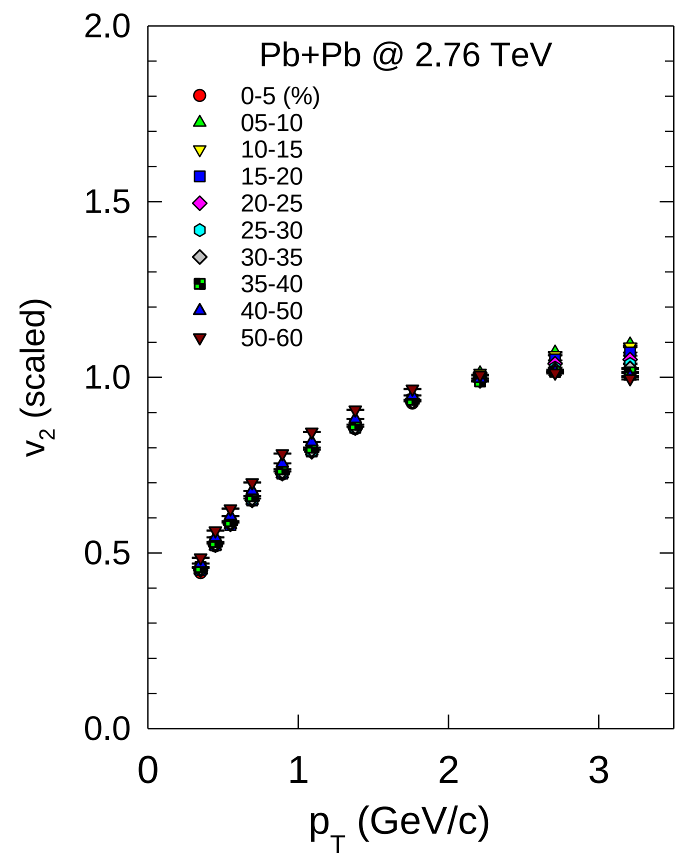


✓ ***Combined scaling understood***

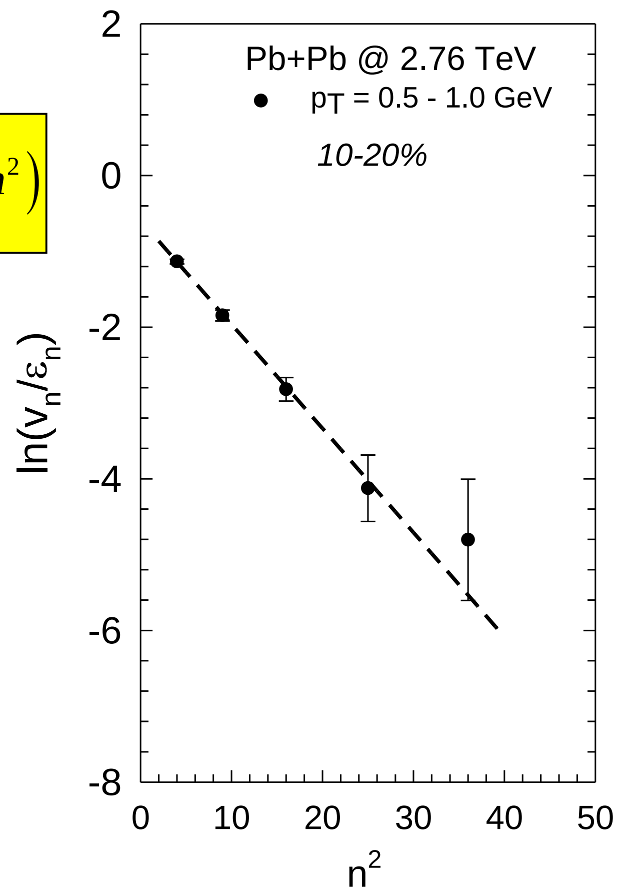
✓ **Characteristic 1/(RT) viscous damping validated**

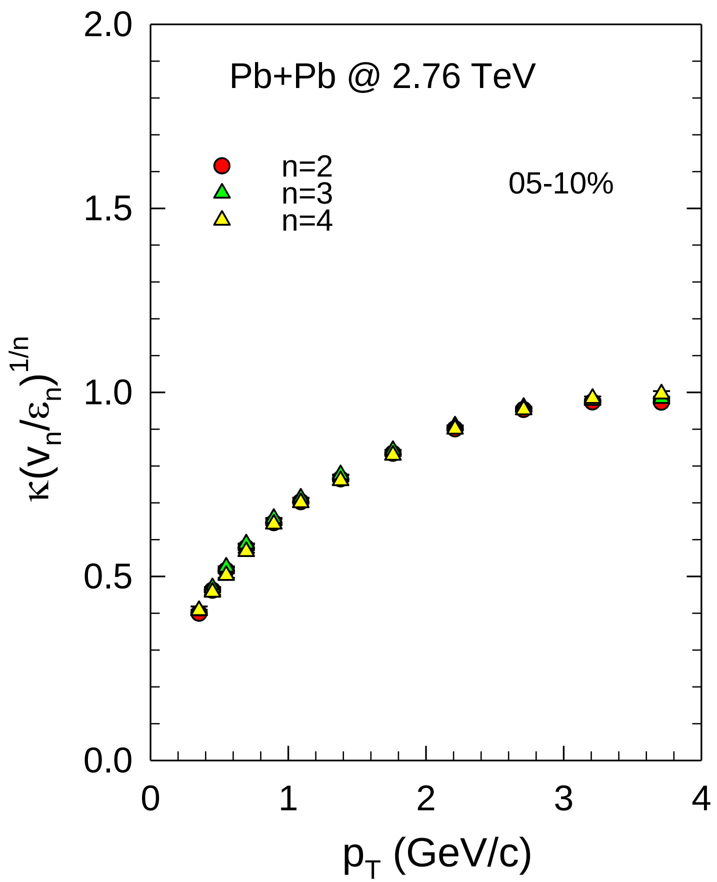
✓ ***Important constraint for η/s & ζ/s***

Acoustic Scaling – n^2

$$\left(\frac{v_n(p_T)}{\varepsilon_n}\right)^{1/n} \propto \exp(-\beta' n)$$

$$\left(\frac{v_n}{\varepsilon_n}\right) \propto \exp(-\beta' n^2)$$





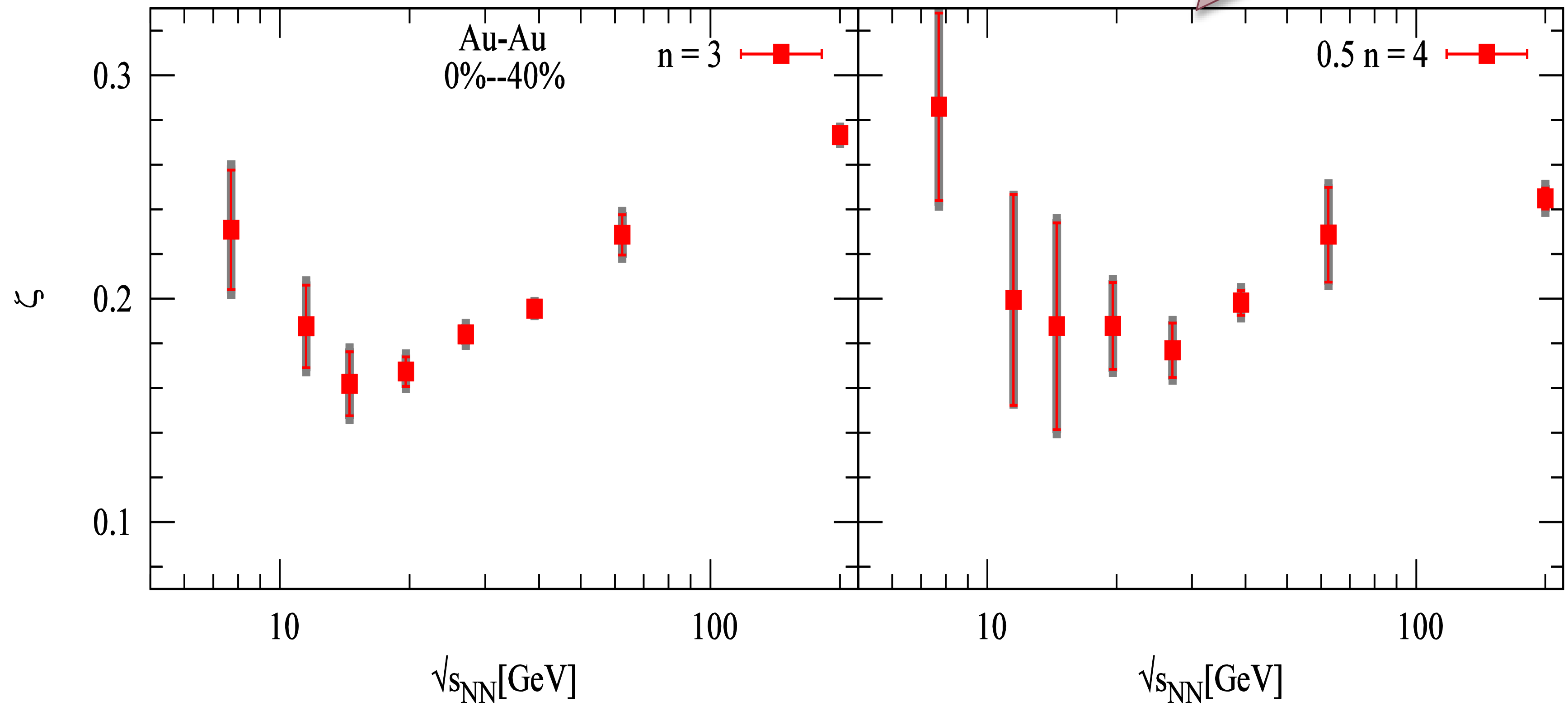
- ✓ **Characteristic n^2 viscous damping validated**
- ✓ **Similar patterns for other centrality selections**
- ✓ ***Important constraint for η/s & ζ/s***

Viscous coefficient

$|\eta| < 1$ and $|\Delta\eta| > 0.7$

$0.2 < p_T < 4\text{GeV/c}$

$$\xi = \ln\left(\frac{(v_n)^{\frac{1}{n}}}{(v_2)^{\frac{1}{2}}}\right)\left(\frac{dN}{d\eta}\right)^{\frac{1}{3}}$$

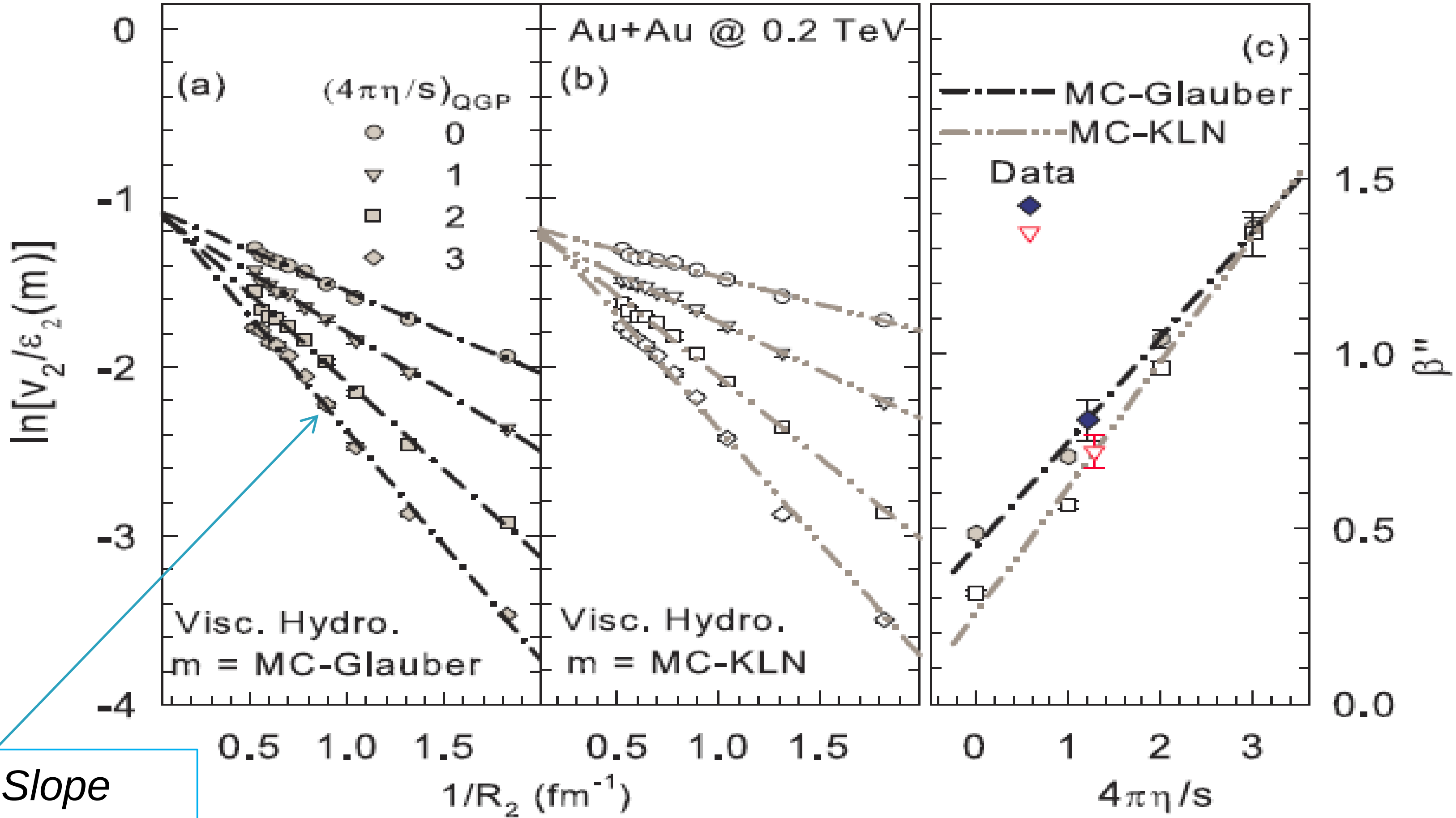


➢ ***The viscous coefficient ξ shows a non-monotonic behavior with beam energy***

Extraction of η/s

$$\ln\left(\frac{v_n}{\varepsilon_n}\right) \propto \frac{-\beta''}{\bar{R}}$$

Lacey et. al, arxiv:1311.1728



Slope sensitive to 4πη/s

Characteristic $1/\bar{R}$ viscous damping validated in viscous hydrodynamics; calibration → 4πη/s ~ 1.3 ± 0.2

Extracted η/s value insensitive to initial conditions

Epilogue

There is a wealth of data which can be leveraged to constrain the extraction of initial-state independent transport coefficients

End

Viscous coefficient

- The v_n *measurement are sensitive to* ε_n, *transport coefficient* η/s *and the expanding parameter RT*.
  - *Acoustic ansatz*

✓ *Sound attenuation in the viscous matter reduces the magnitude of* v_n.

- *Anisotropic flow attenuation,*

arXiv:1305.3341

$$\frac{v_n}{\varepsilon_n} \propto e^{-\beta n^2}, \ \beta \propto \frac{\eta}{s}\frac{1}{RT}$$

3

- *From macroscopic entropy considerations* $(RT)^3 \propto \frac{dN}{d\eta}$

arXiv:1601.06001

- *The viscous coefficient* ξ *encodes the transport coefficient* $\frac{\eta}{s}$,

$$\ln\left(\frac{(v_n)^{\frac{1}{n}}}{(v_2)^{\frac{1}{2}}}\right) - \ln\left(\frac{(\epsilon_n)^{\frac{1}{n}}}{(\epsilon_2)^{\frac{1}{2}}}\right) = A\left[-(n-2)\frac{\boldsymbol{\eta}}{\boldsymbol{s}}\right] / \left(\frac{dN}{d\eta}\right)^{\frac{1}{3}} + \ln(c)$$

4

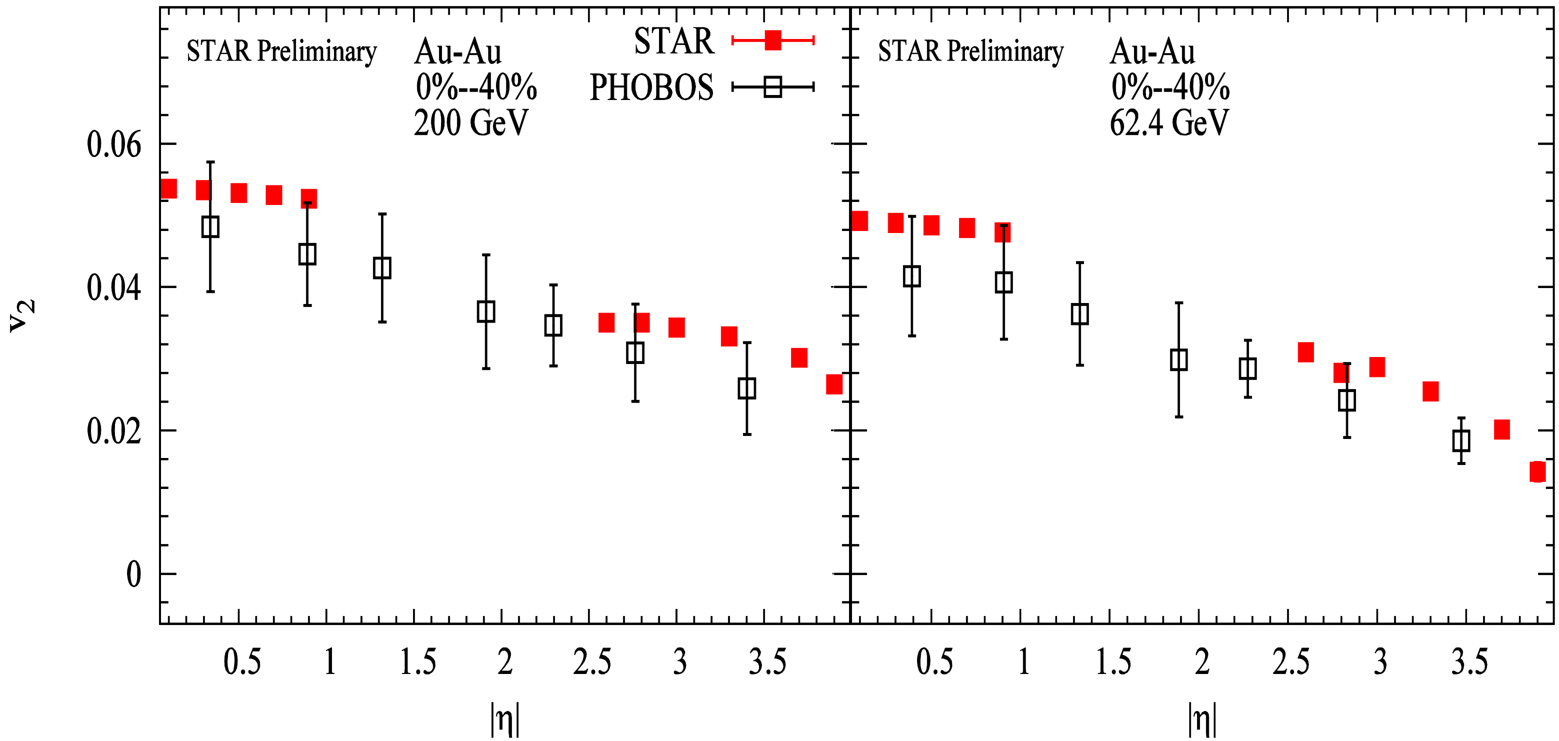
$$\boldsymbol{\xi} = \ln\left(\frac{(v_n)^{\frac{1}{n}}}{(v_2)^{\frac{1}{2}}}\right)\left(\frac{dN}{d\eta}\right)^{\frac{1}{3}}$$

5

$v_2(\eta)$

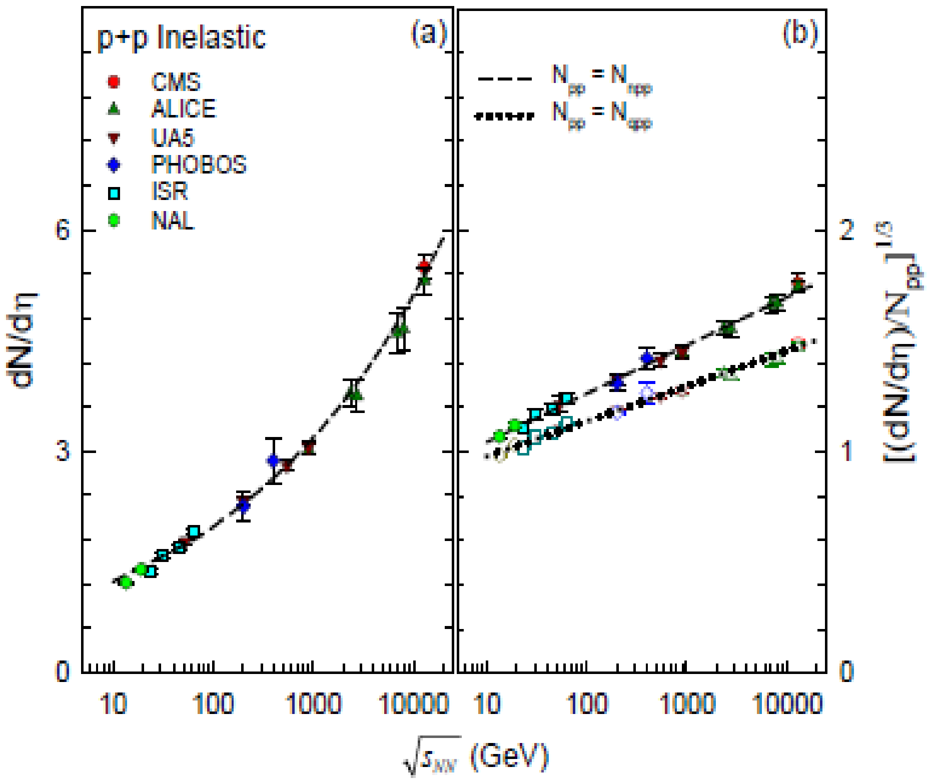
$|\eta_{ref}| < 1 \; and \; |\eta| < 4$

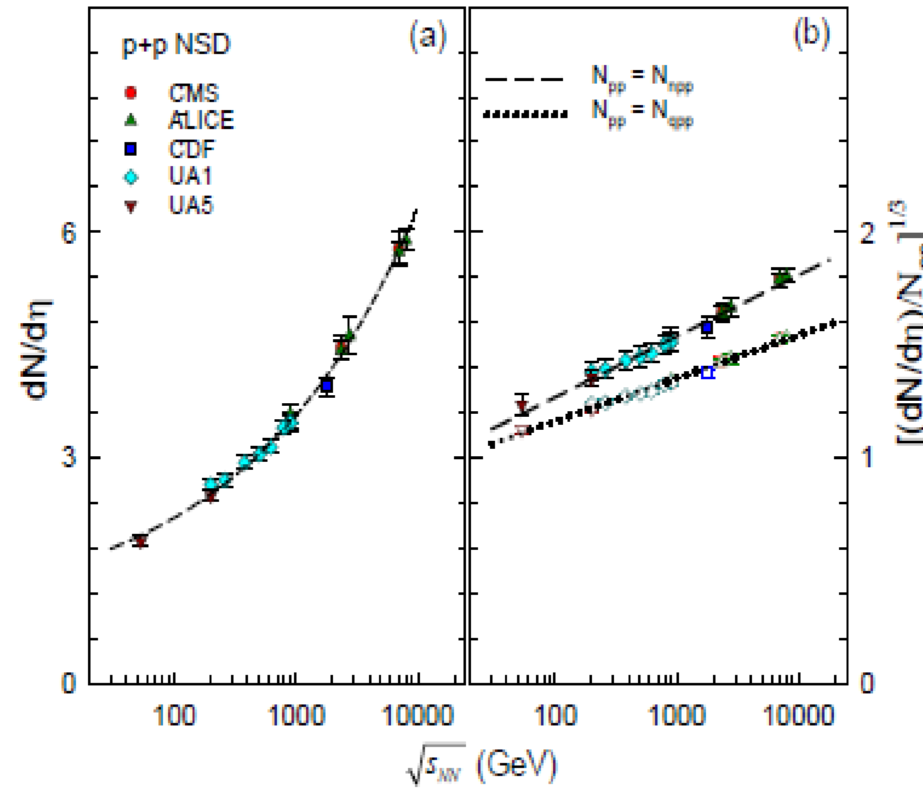
$0.2 < p_T < 4\text{GeV/c}$



- *Reasonable agreement between the STAR and PHOBOS measurements.*

$\frac{dN_{chg}}{d\eta}$ scaling p+p





$$\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta|_{\mathrm{INE}} = [b_{\mathrm{INE}} + m_{\mathrm{INE}} \log(\sqrt{s_{\mathrm{NN}}})]^3 ,$$
$$b_{\mathrm{INE}} = 0.826 \pm 0.008,\ m_{\mathrm{INE}} = 0.220 \pm 0.004,$$

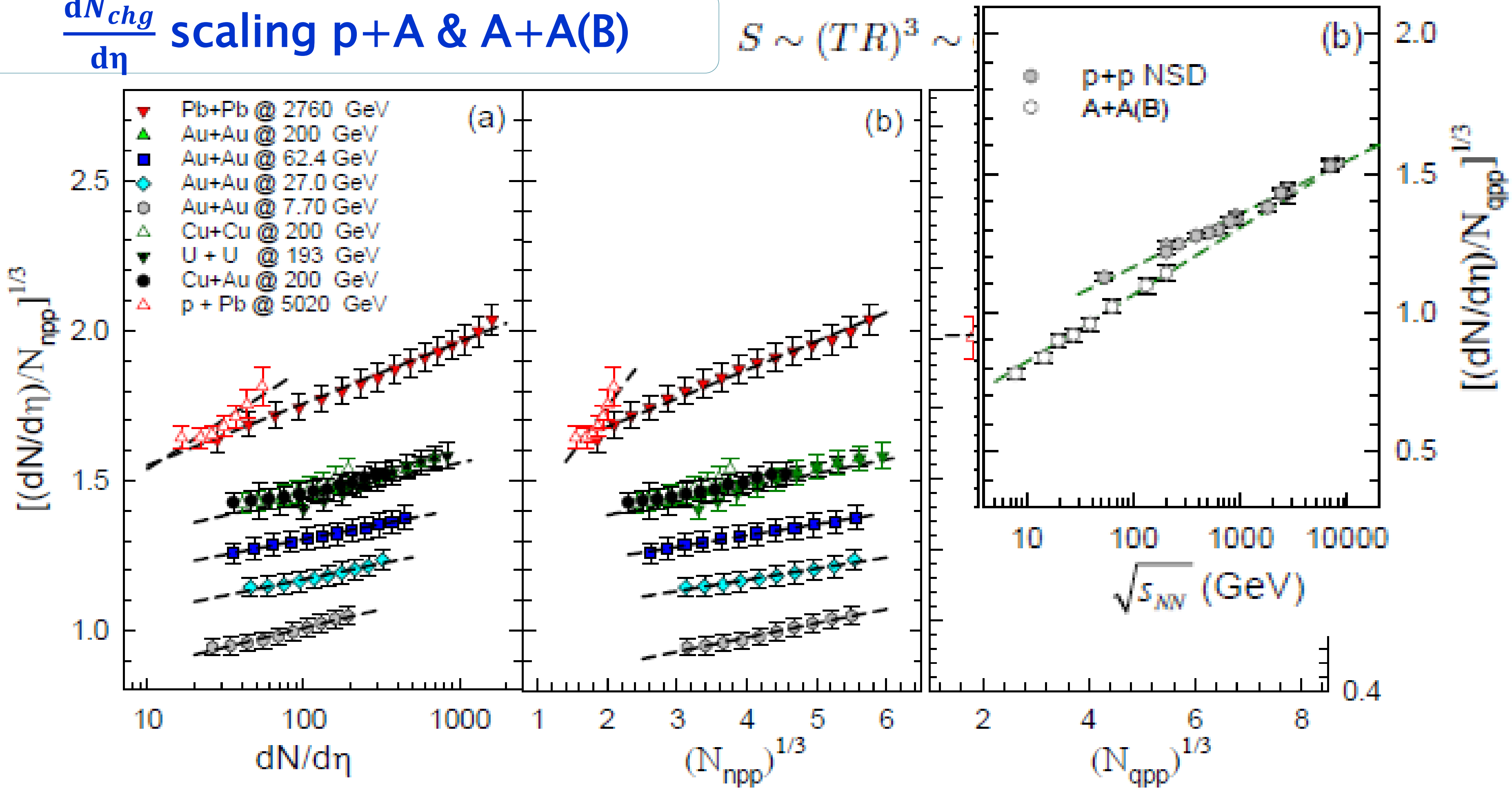
$$\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta|_{\mathrm{NSD}} = [b_{\mathrm{NSD}} + m_{\mathrm{NSD}} \log(\sqrt{s_{\mathrm{NN}}})]^3 ,$$
$$b_{\mathrm{NSD}} = 0.747 \pm 0.022,\ m_{\mathrm{NSD}} = 0.267 \pm 0.007,$$

Further scaling validation over full range of $\sqrt{s_{NN}}$ for p+p

- ✓ **Similar $\sqrt{s_{NN}}$ trend for quark and nucleon scaled multiplicity density**

$\frac{dN_{chg}}{d\eta}$ scaling p+A & A+A(B)

$S \sim (TR)^3 \sim$



$$dN_{ch}/d\eta|_{|\eta|=0.5} = N_{qpp}\left[b_{AA} + m_{AA}\log(\sqrt{s})\right]^3,$$

$$b_{AA} = 0.530 \pm 0.008,\ \ m_{AA} = 0.258 \pm 0.004,$$

Scaling validated for p+A & A+A(B) systems

- ✓ **Similar patterns for A+A(B) systems at the same $\sqrt{s_{NN}}$.**
- ✓ **Logarithmic dependence of $\langle p_T \rangle$ on multiplicity**