Transport of Fluctuations

Masakiyo Kitazawa

(Osaka U.)

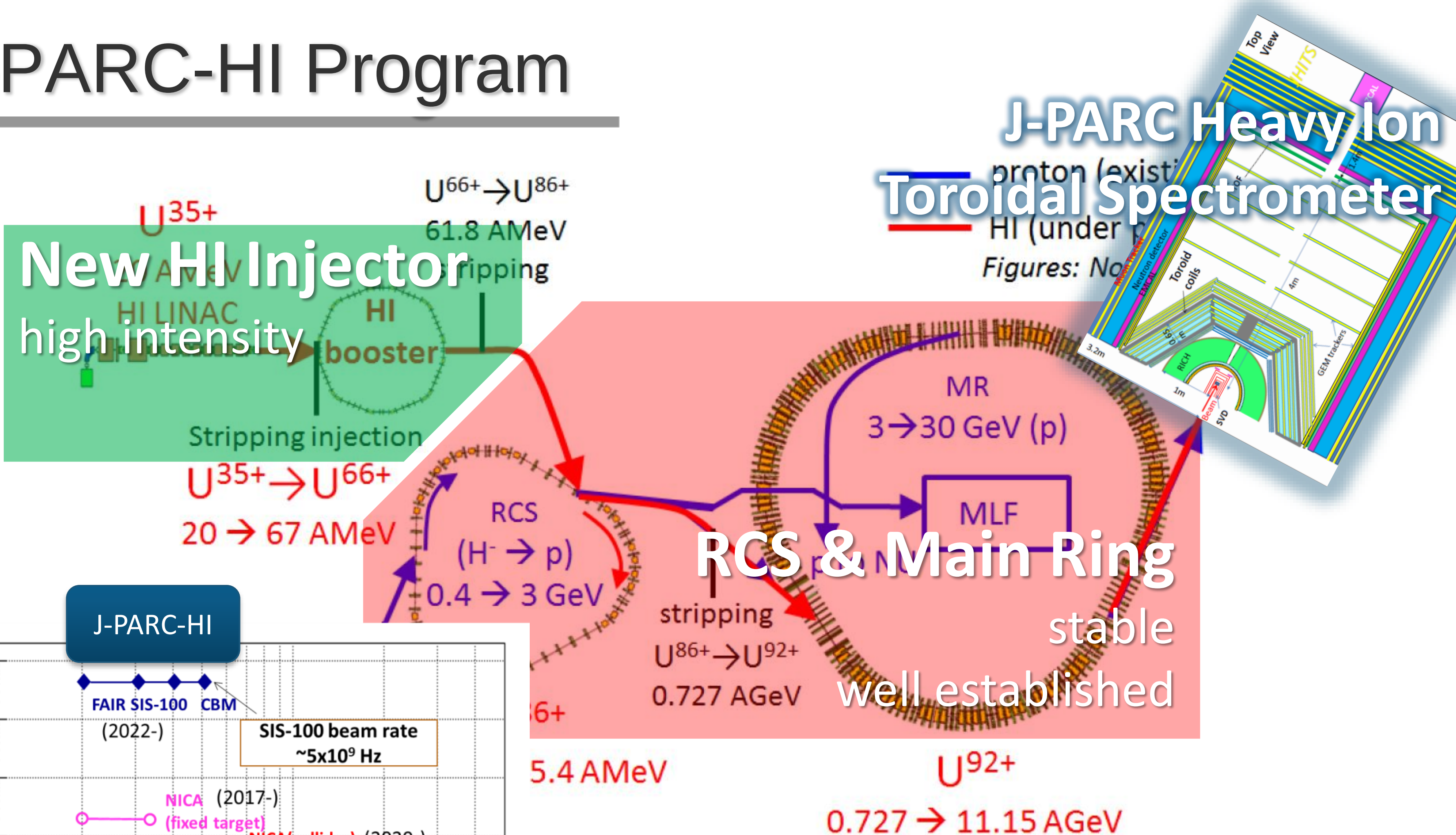
INT Workshop 16-3

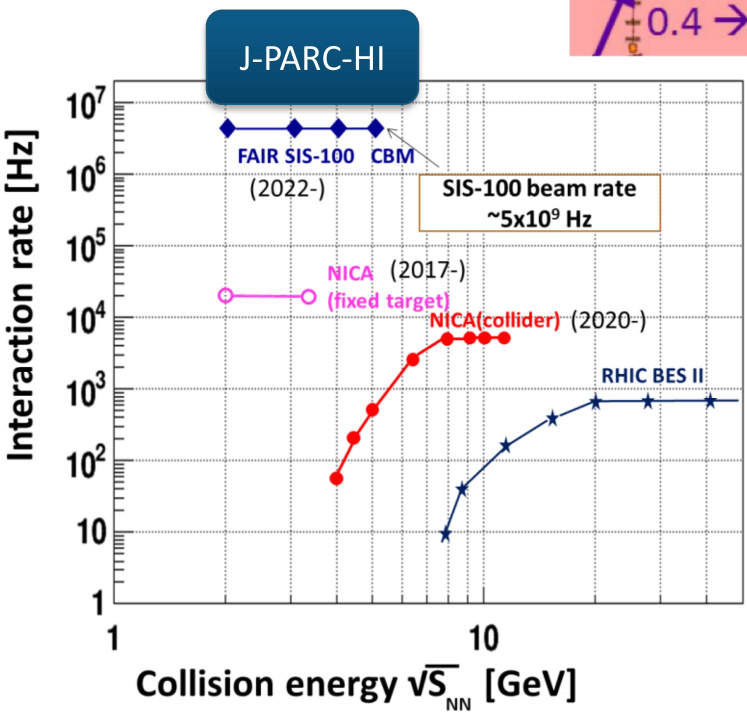
Exploring the QCD Phase Diagram through Energy Scan

INT, Seattle, 6/Oct./2016

J-PARC Heavy-Ion Program (J-PARC-HI)

J-PARC-HI Program





- fixed target HI experiment
- E_{lab}<20GeV/A ($\sqrt{s_{NN}}$<6.2 GeV)
- High luminosity beam

J-PARC-HI Program

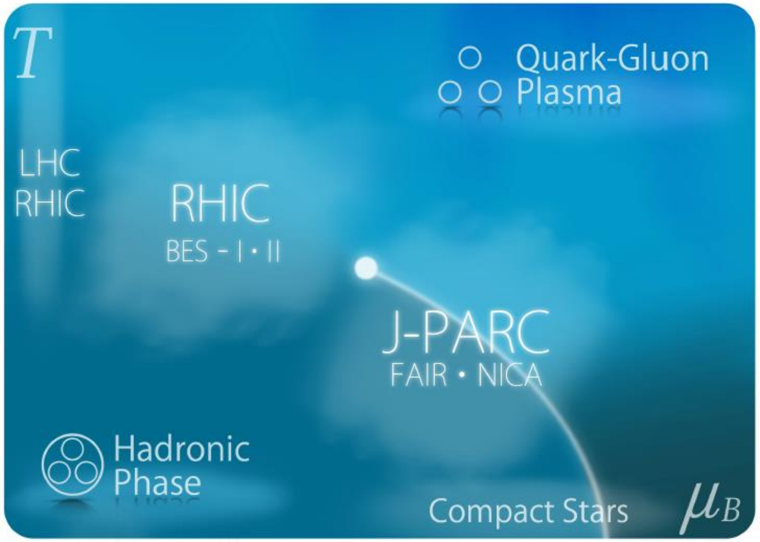
Recent Activities

June 2016	**White Paper** uploaded
July 2016	Submission of LOI
Aug. 2016	International Workshop
Sep. 2016	Symposium @ JPS meeting

Future Plan

2020	**Funding request to MEXT**
2021	**Earliest approval of funding**
2021-2022	Construction of HI Injector
2021-2023	Construction of HI injection system in RCS
2023-2024	Construction of HI spectrometer
2025	First collision

White Paper



Visit J-PARC-HI Web Page

http://asrc.jaea.go.jp/soshiki/gr/hadron/jparc-hi/

Fluctuations

Contents

1. Transport of fluctuations

   MK, Asakawa, Ono, PLB (2014); MK, NPA (2015)

2. Thermal blurring by rapidity conversion

   Ohnishi, MK, Asakawa, PRC (2016)

3. Transport near QCD critical point

   Sakaida, Asakawa, Fujii, MK, in prep.

Review: Asakawa, MK, PPNP (2016)

Before Main Topics…

Fluctuations = **Regular** + **Singular**

Before Main Topics…

Fluctuations = **Regular** Poisson Noise + **Singular** NON-Poisson Signal

Before Main Topics…

Fluctuations = **Regular** Poisson Noise + **Singular** NON-Poisson Signal

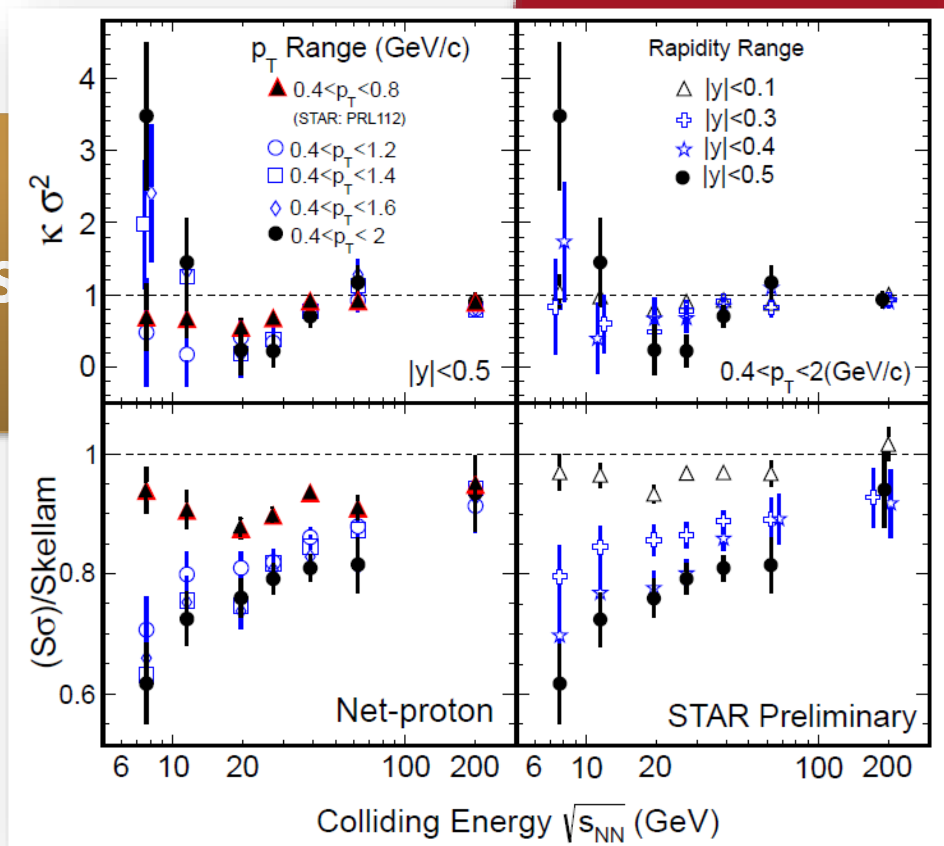
Before Main Topics…

Fluctuations = **Regular** Poisson Noise **Singular** NON-Poisson Signal

Before Main Topics…

Fluctuations

gular
-Poisson
gnal



Before Main Topics…

Fluctuations = **Regular** Poisson Noise + **Singular** NON-Poisson Signal

Off-equilibrium effects make the separation of two contributions impossible.

(diffusion, experimental cuts, efficiency, …)

Fragile Higher Orders

Fragile Higher Orders

Ex.: Relation b/w baryon & proton # cumulants

(with an approximation)

MK, Asakawa, 2012

$$\begin{cases} 2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^2\rangle_\mathrm{free} \\ 2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^3\rangle_\mathrm{free} \\ 2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^4\rangle_c + \cdots \end{cases}$$

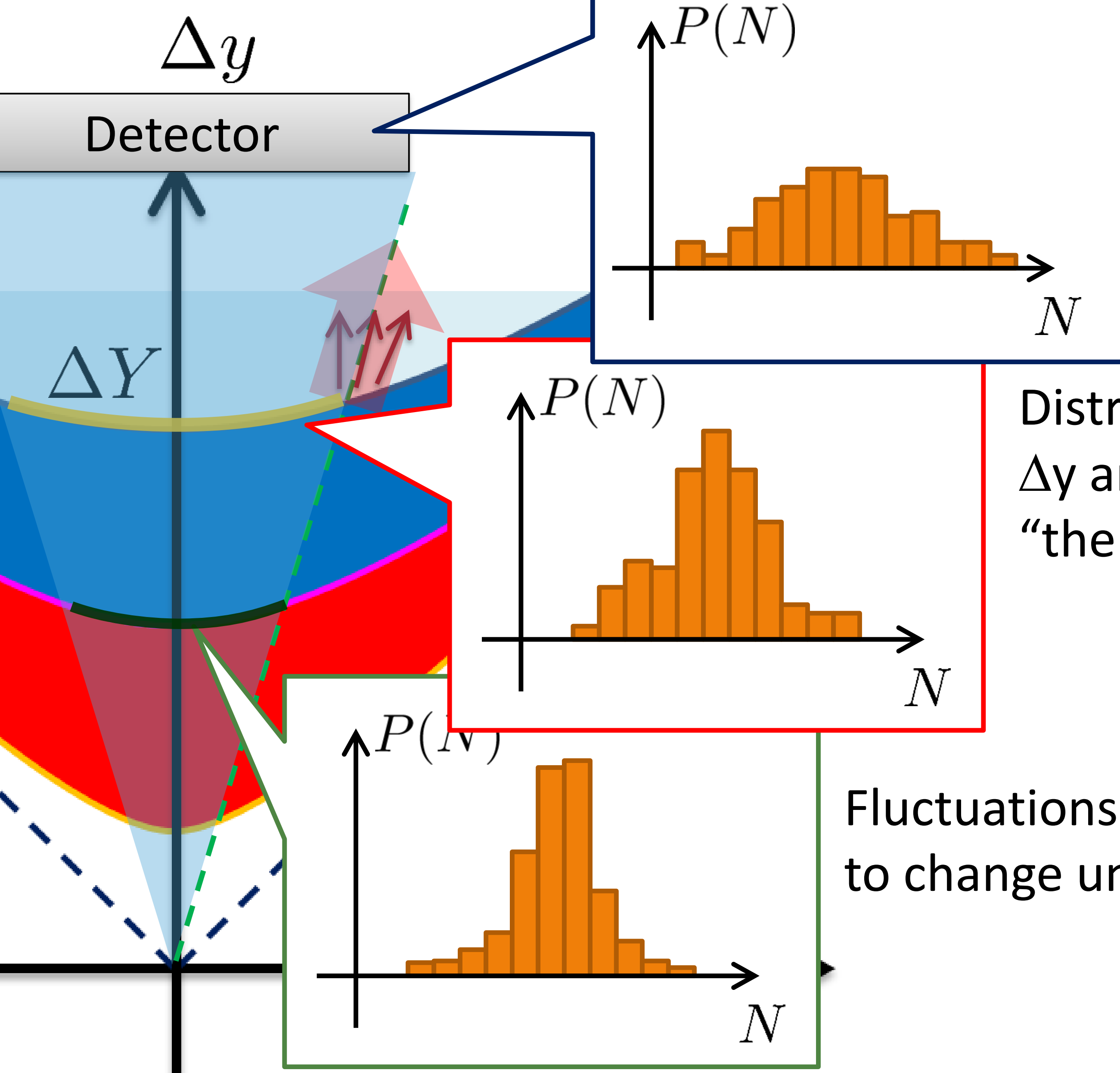
genuine info. Poisson noise

Higher orders are more seriously affected by efficiency loss.

Transport of Fluctuations

Diffusion After Chemical F.O.

Asakawa, Heinz, Muller (2000)
Jeon, Koch (2000)
Shuryak, Stephanov (2001)



Distributions in ΔY and Δy are different due to "thermal blurring".

Fluctuations in ΔY continue to change until kinetic f.o.

How to Describe Transport of Fluctuations?

A candidate

Stochastic diffusion equation

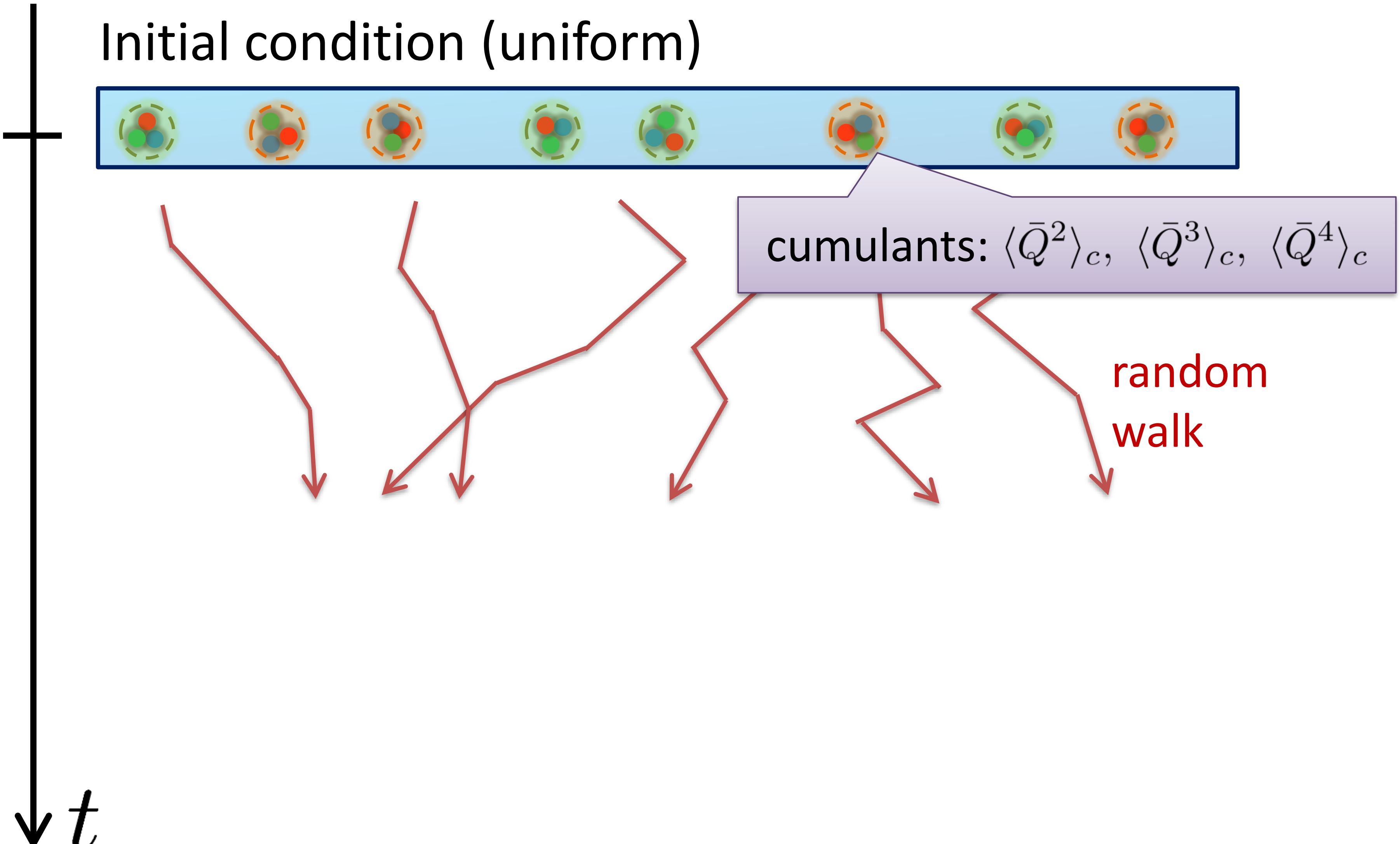
$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

Noise: Gaussian

Fluctuation in equil. is **Gaussian.**

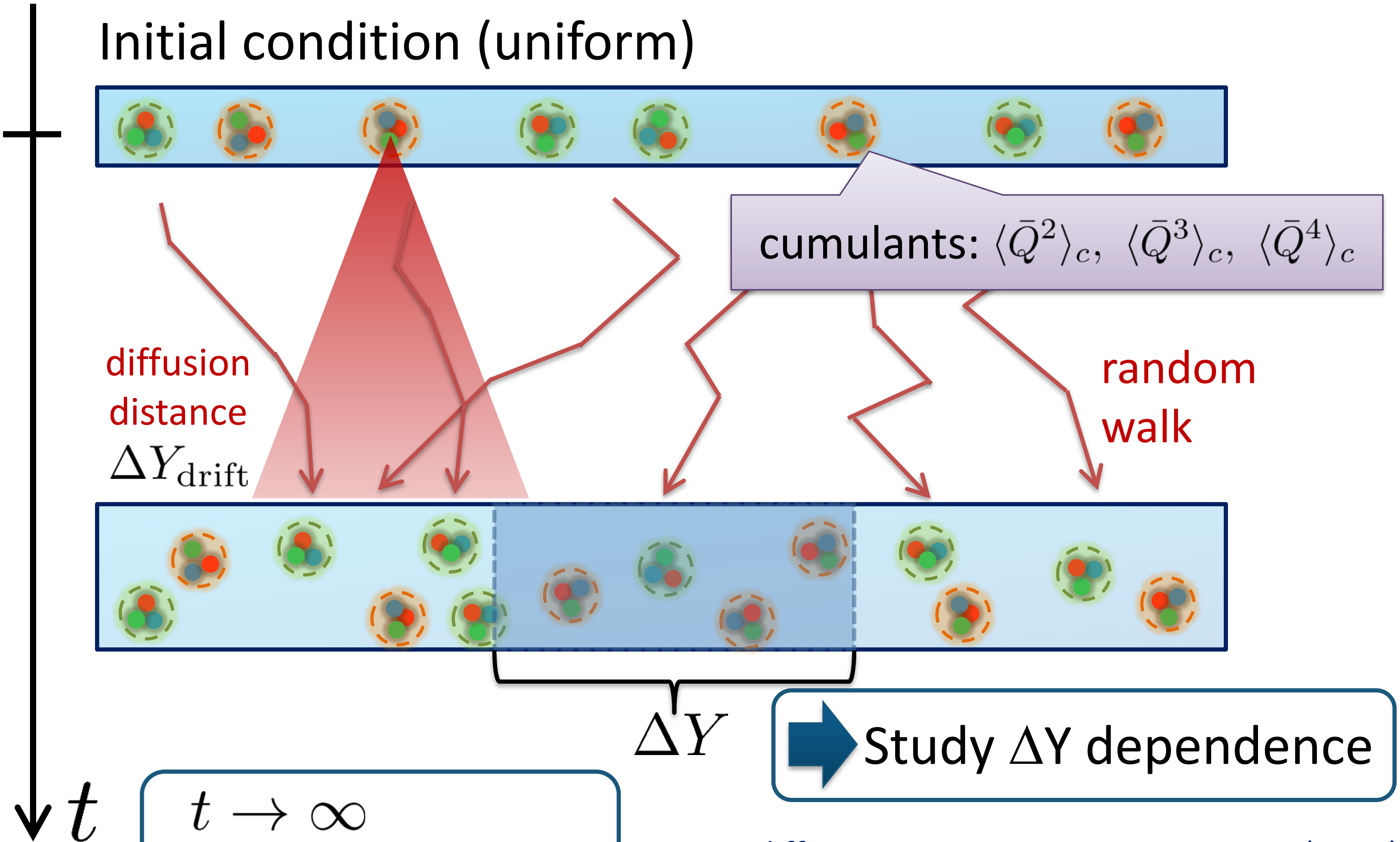
Undesirable to describe relaxation of cumulants toward Poisson value.

Non-Interacting Brownian Particle System



diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

Non-Interacting Brownian Particle System



Study ΔY dependence

$t \to \infty$

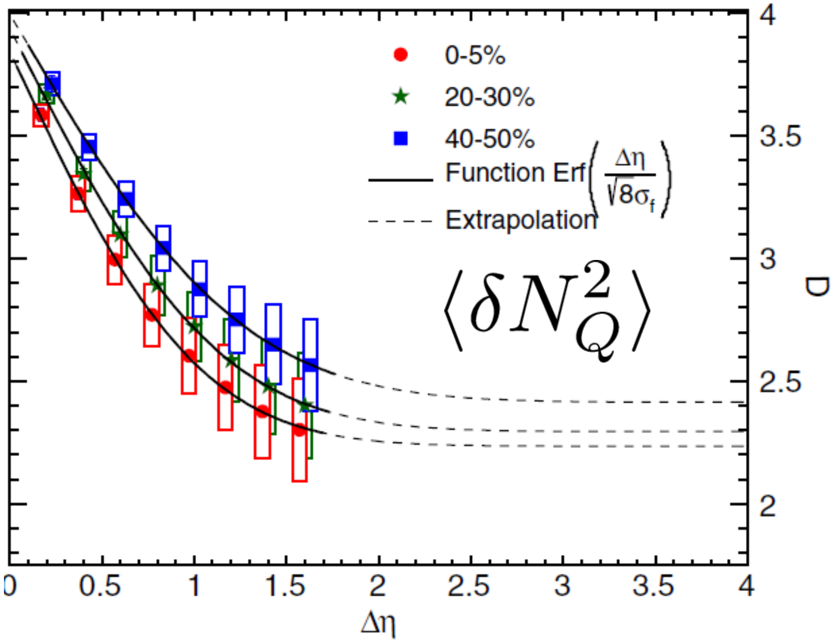
Poisson distribution

diffusion master equation: MK+, PLB(2014)

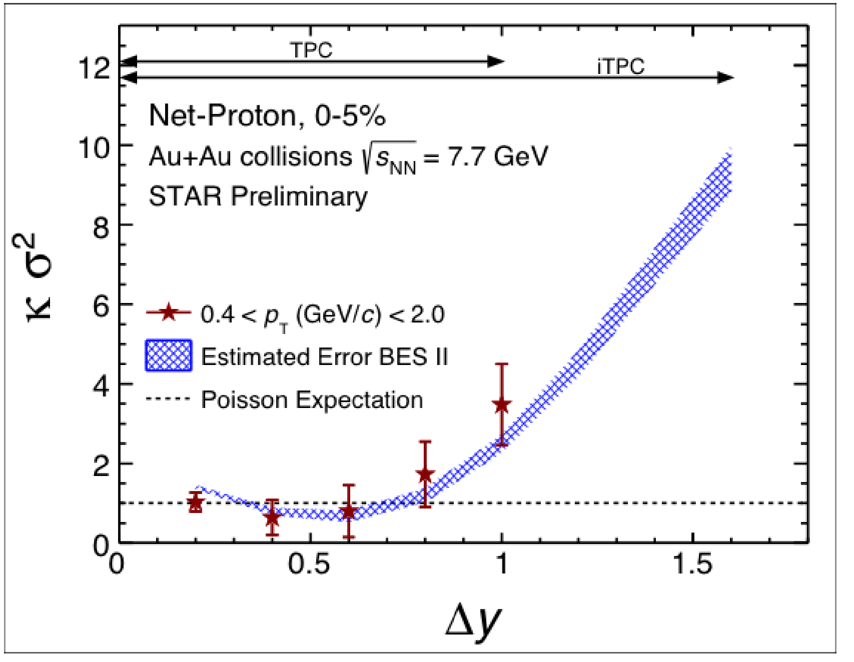
probabilistic argument: Ohnishi+, PRC(2016)

Rapidity Window Dependence

ALICE, PRL (2013)

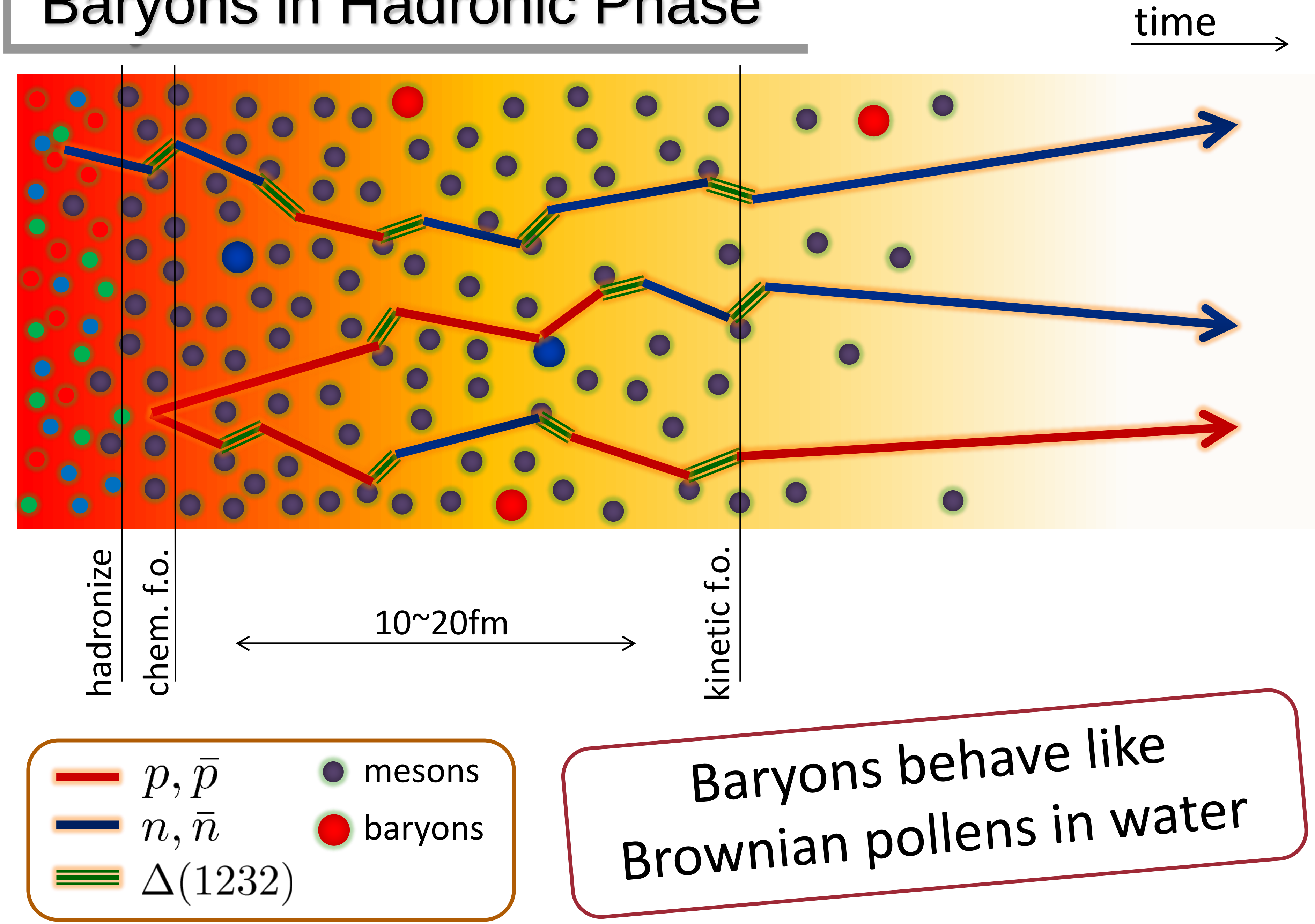


X. Luo, CPOD2014



Cumulants are dependent on rapidity window.

Baryons in Hadronic Phase

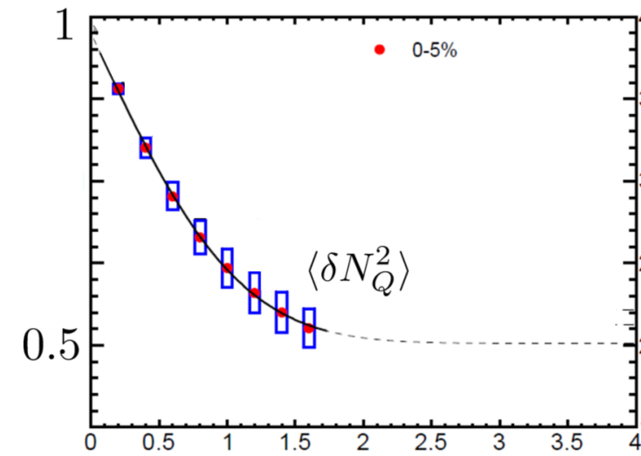


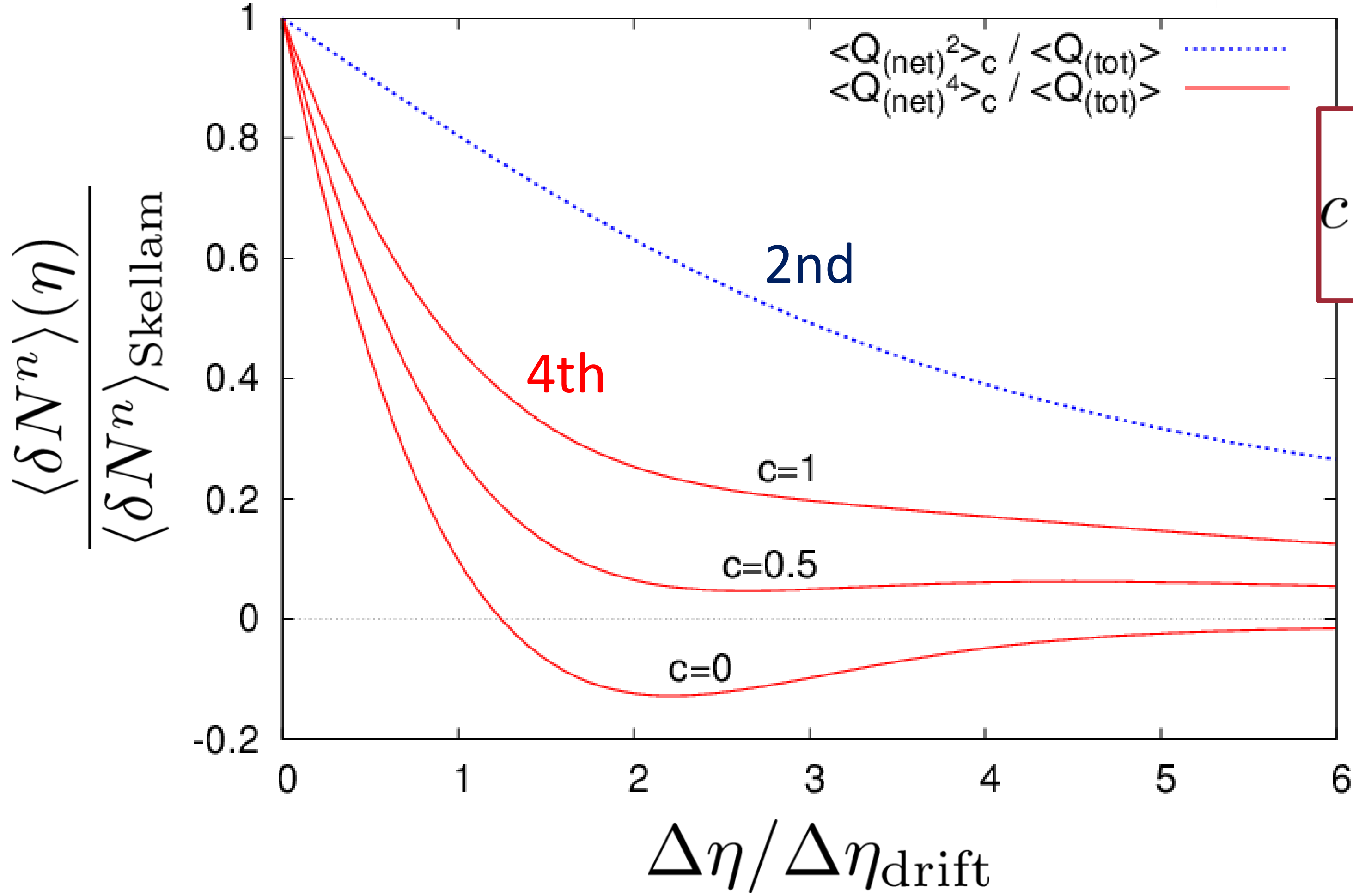
Baryons behave like Brownian pollens in water

Rapidity Window Dependence

No initial net fluctuation:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0$$



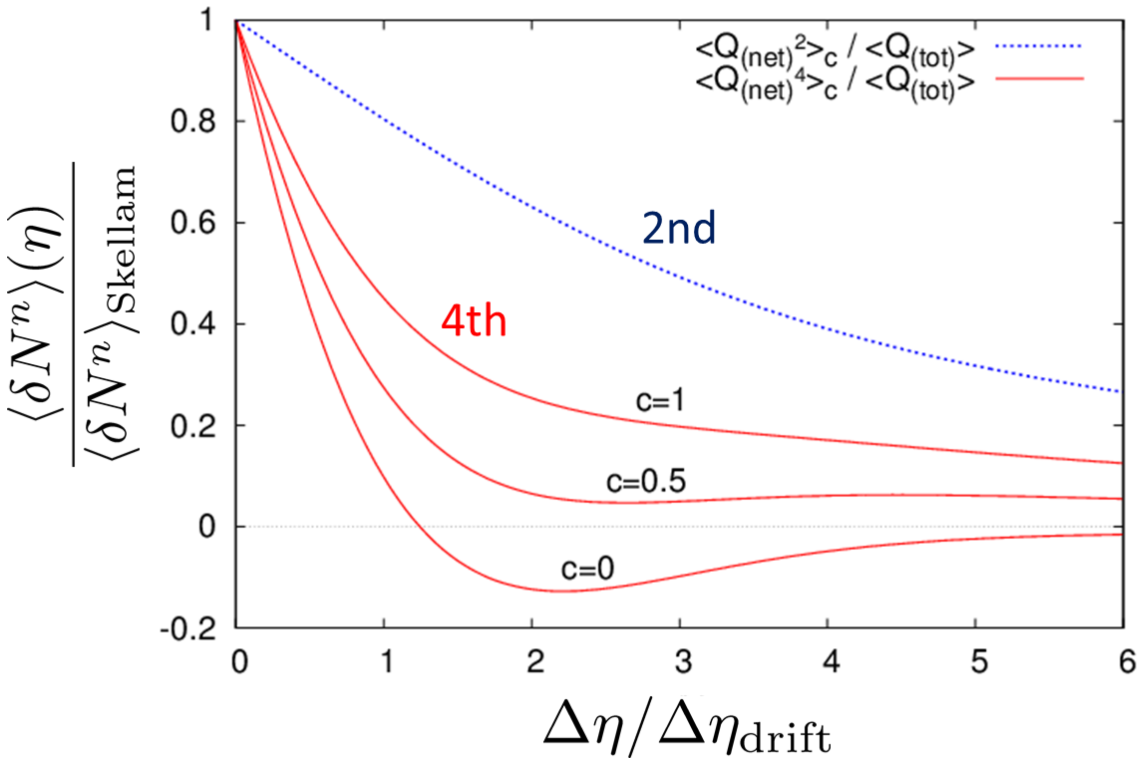


$$c = \frac{\langle Q_{(\mathrm{tot})}^2 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

parameter sensitive to hadronization

Rapidity Window Dependence

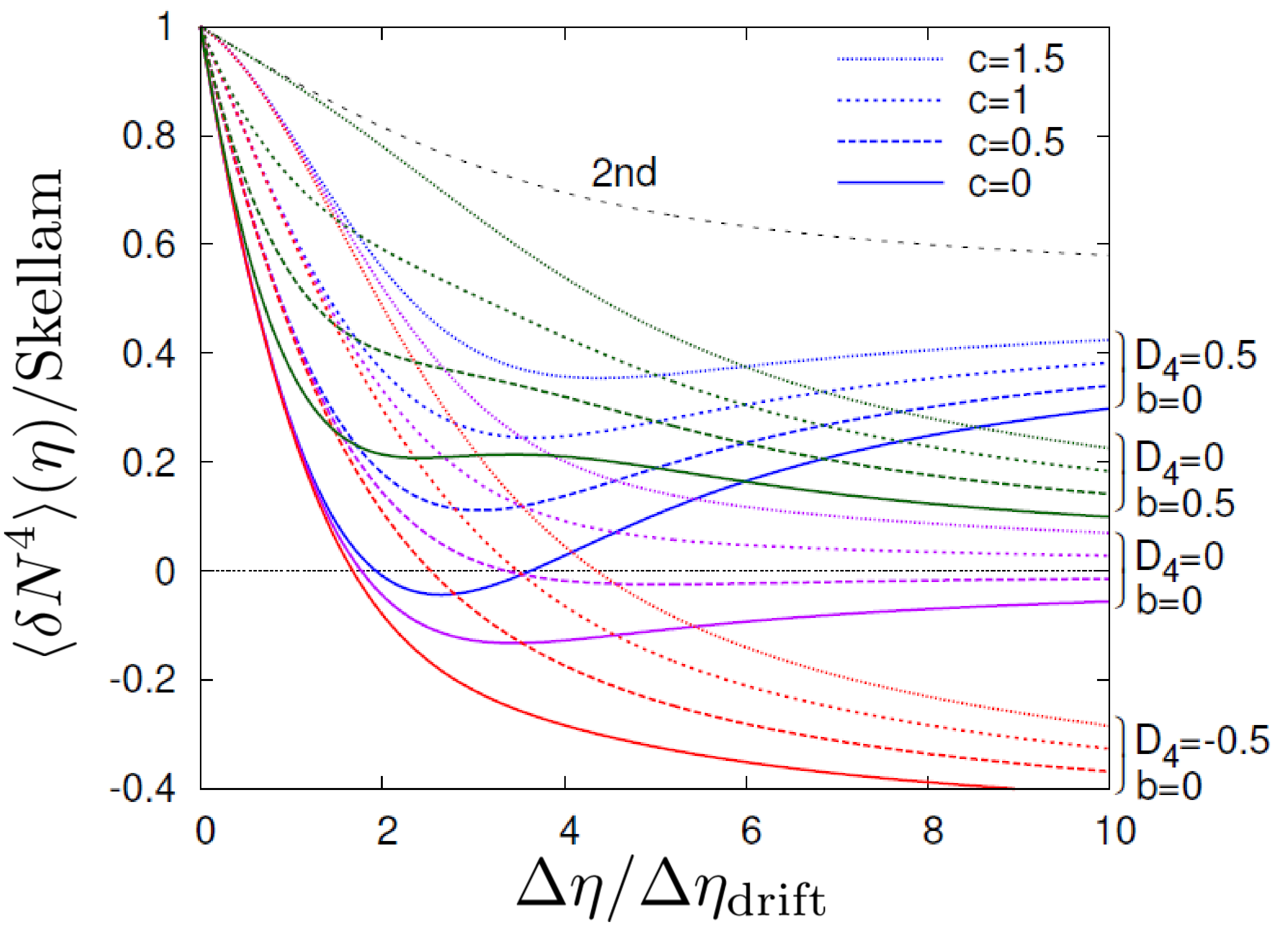
MK+, PLB(2014)



- ☐ Cumulants at finite Δy is different from initial value.
- ☐ 4^{th} cumulant can have a sign change.
- ☐ 4^{th} cumulant can have non-monotonic behavior.
- ☐ Poisson / non-Poisson : Not separable!

Δη Dependence: 4th order

MK, NPA(2015)



Initial Condition

$$D_4 = \frac{\langle Q_{(\mathrm{net})}^4 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$b = \frac{\langle Q_{(\mathrm{net})}^2 Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

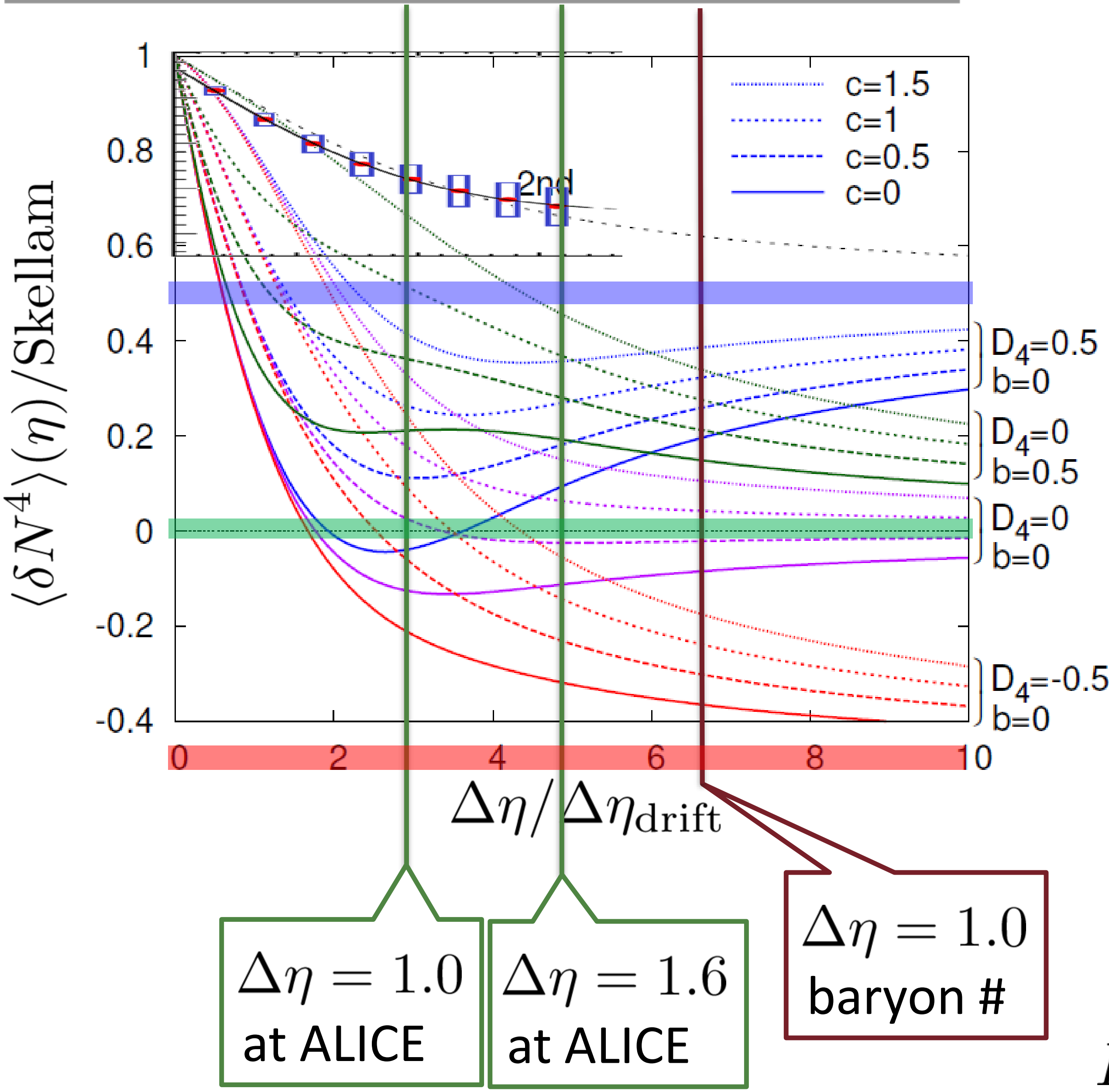
$$c = \frac{\langle Q_{(\mathrm{tot})}^2 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q_{(\mathrm{net})}^2 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

Characteristic Δη dependences!

Δη Dependence: 4th order

MK, NPA(2015)



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

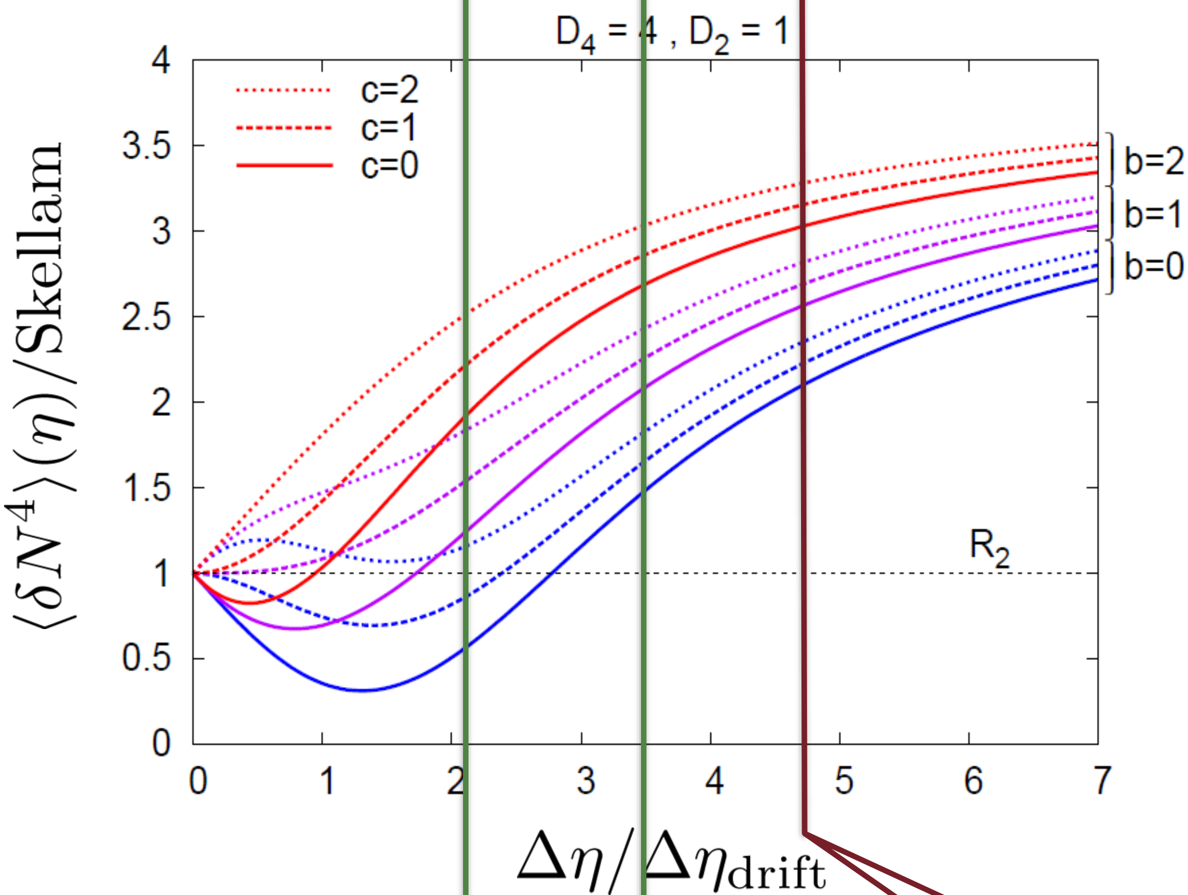
$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

$$D \sim M^{-1}$$

4^{th} order : w/ Critical Fluctuation



$\Delta\eta = 1.0$
at ALICE

$\Delta\eta = 1.6$
at ALICE

$\Delta\eta = 1.0$
baryon #

Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 4$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

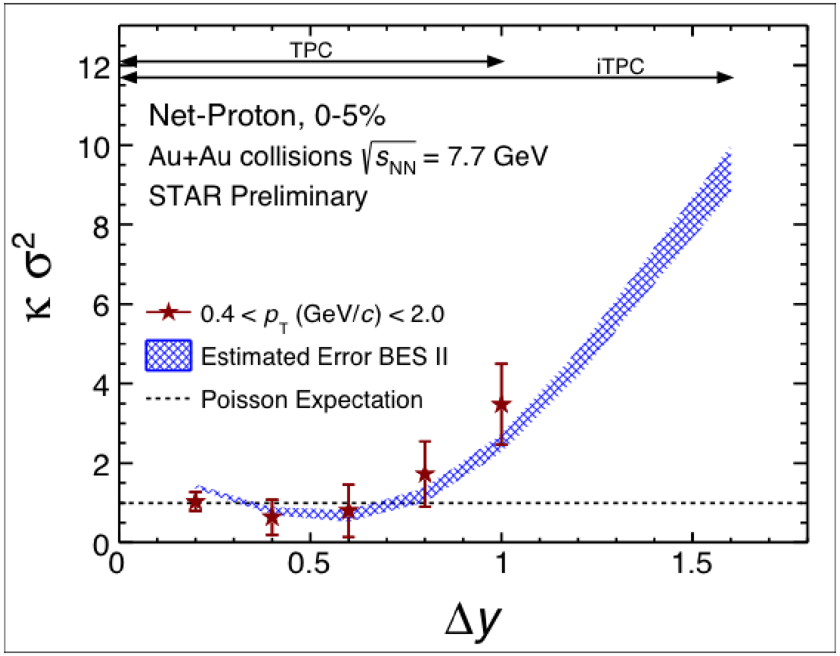
$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 1$$

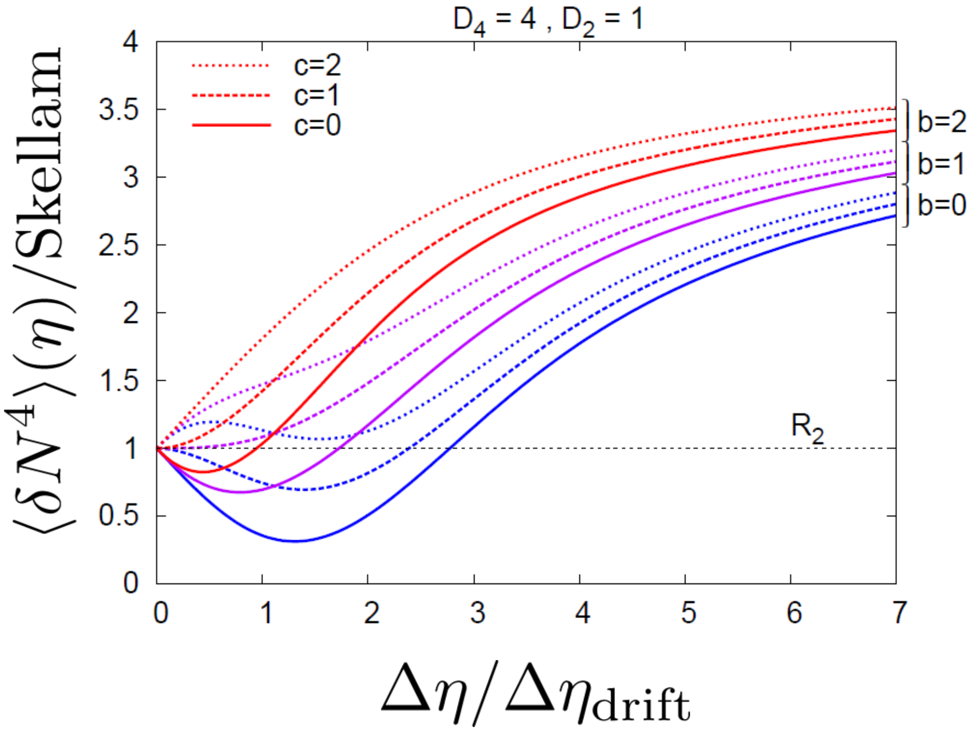
$$D \sim M^{-1}$$

$\Delta\eta$ Dependence @ STAR

MK+ (2014)
MK (2015)

X. Luo, CPOD2014

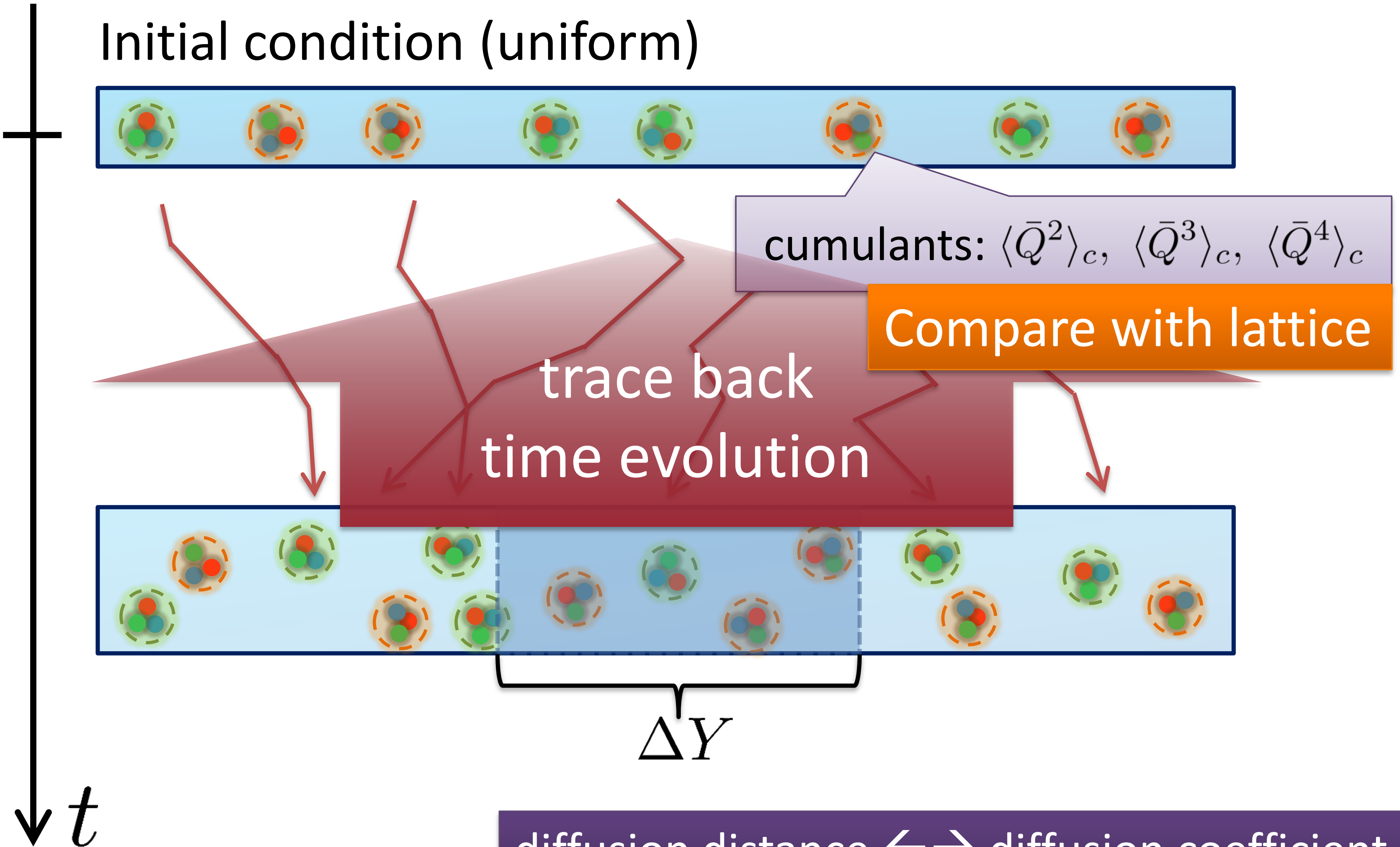




- Approach initial value as $\Delta y \rightarrow$ large Ling, Stephanov (2016)
- No power law $\sim(\Delta y)^4$ behavior at small Δy

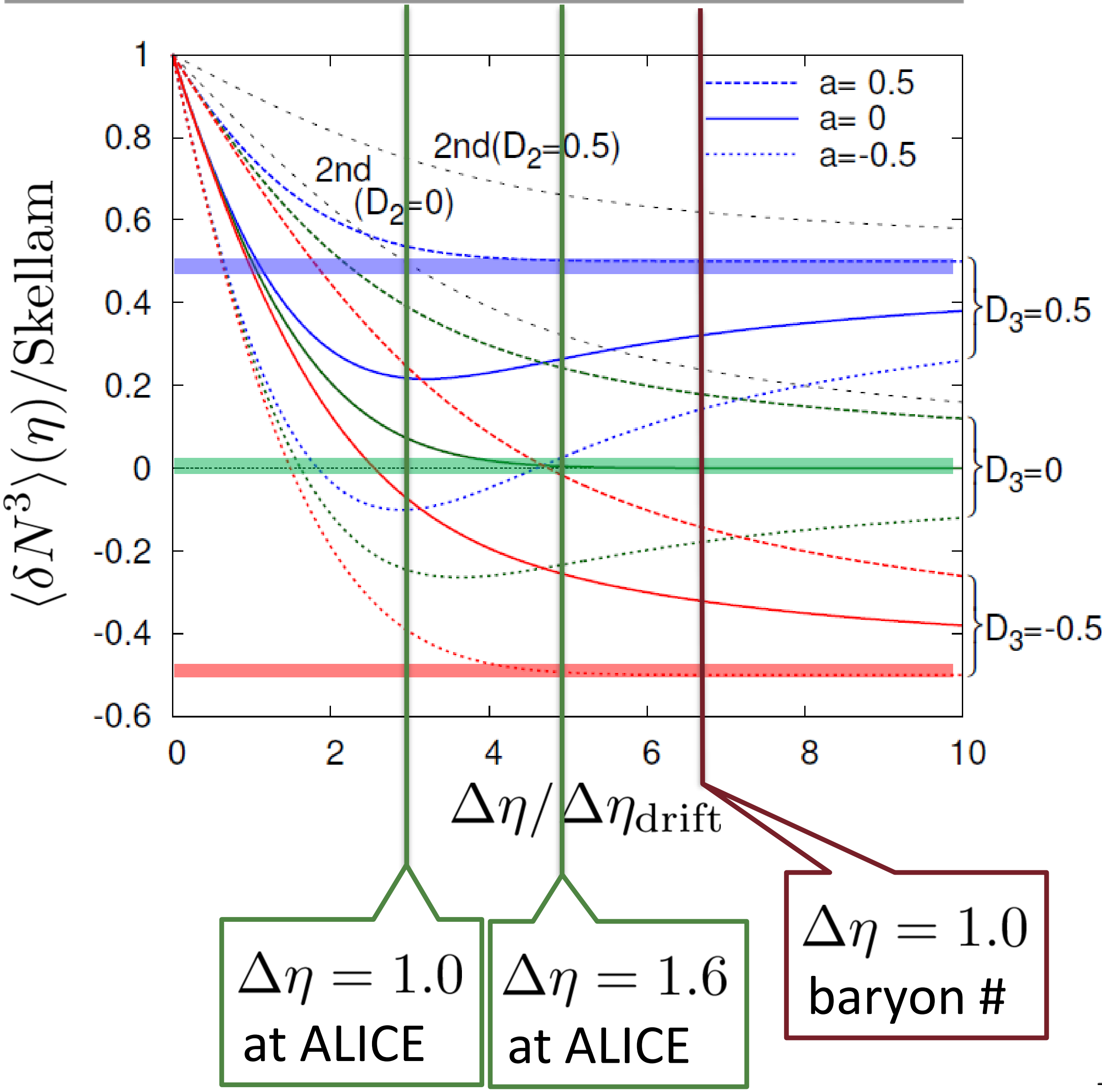
finite volume effect: Sakaida+, PRC064911(2014)

Non-Interacting Brownian Particle System



diffusion distance ←→ diffusion coefficient

Δη Dependence: 3rd order



Initial Condition

$$D_3 = \frac{\langle Q^3_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

$$a = \frac{\langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

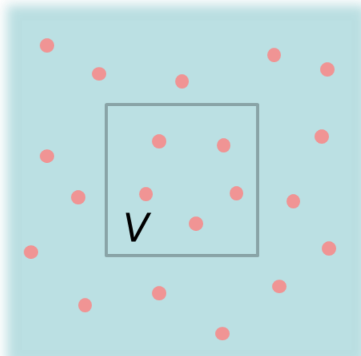
$$D \sim M^{-1}$$

Themal Blurring

Ohnishi, MK, Asakawa, PRC, in press

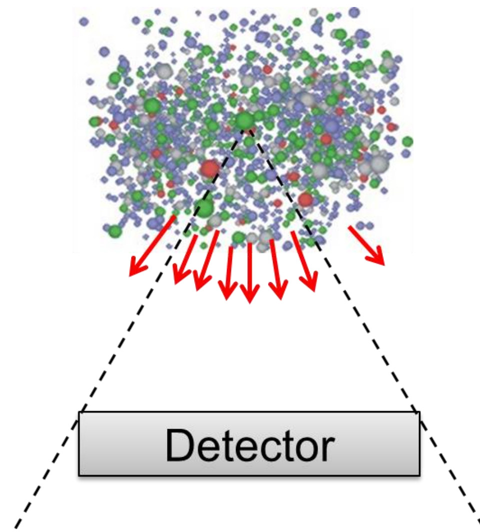
Fluctuations: Theory vs Experiment

Theoretical analyses
based on statistical mechanics



lattice, critical point,
effective models, …

Experiments



Fluctuation in a spatial volume

Fluctuations in a momentum space

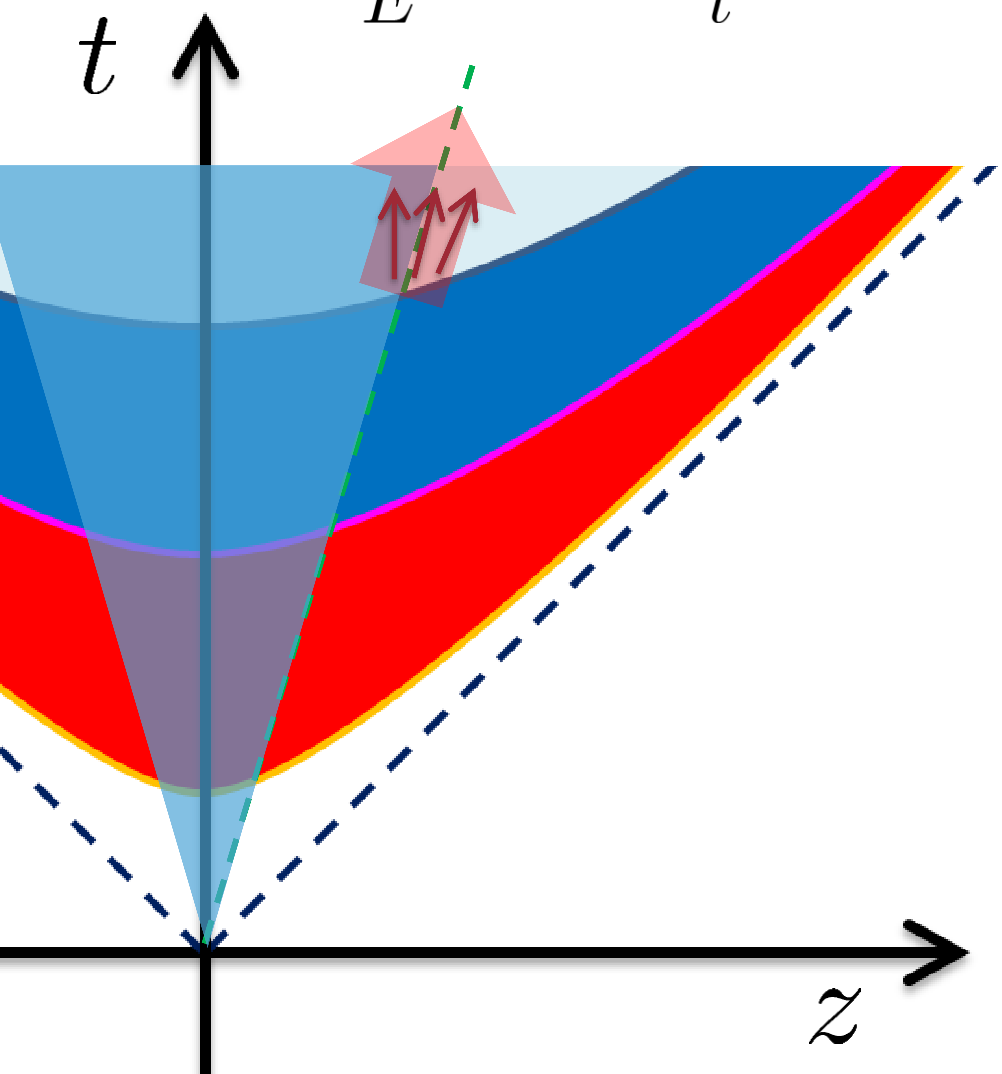
discrepancy in phase spaces

Asakawa, Heinz, Muller, 2000; Jeon, Koch, 2000; Shuryak, Stephanov, 2001

Connecting Phase Spaces

Asakawa, Heinz, Muller, 2000
Jeon, Koch, 2000

$$\frac{p_z}{E} = v_z = \frac{z}{t}$$



Under Bjorken picture,

coordinate-space rapidity *Y*

||

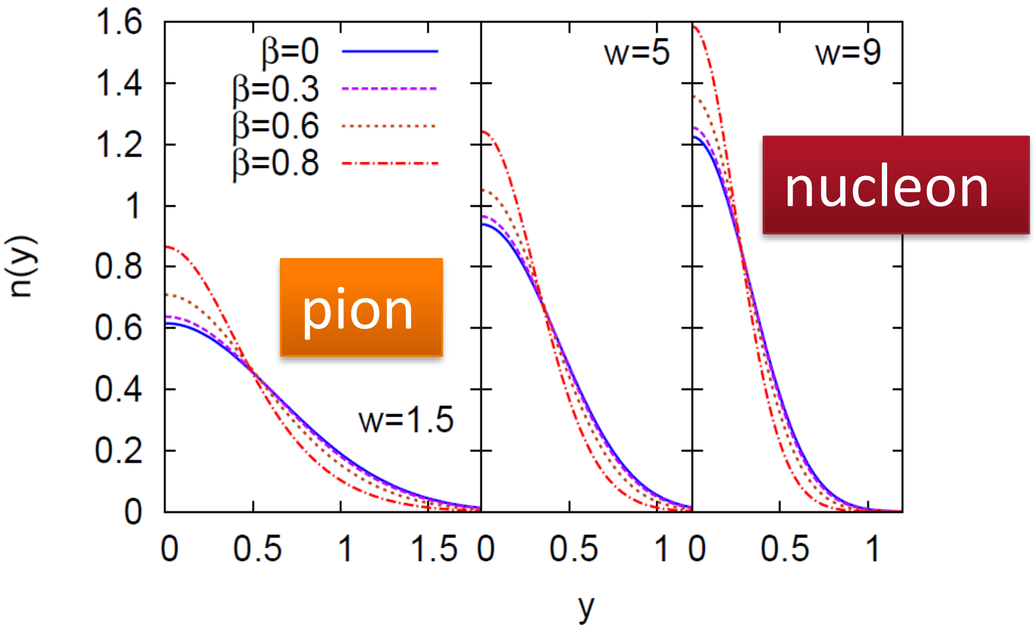
momentum-space rapidity *y*
of medium

$\wr$|

momentum-space rapidity *y*
of individual particles

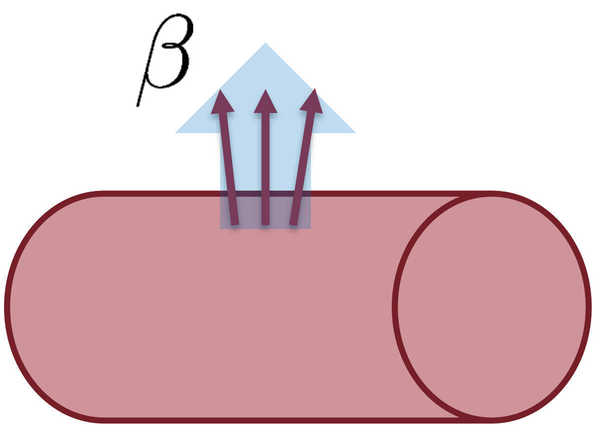
$$\Delta y \simeq \Delta Y$$

Thermal distribution in y space

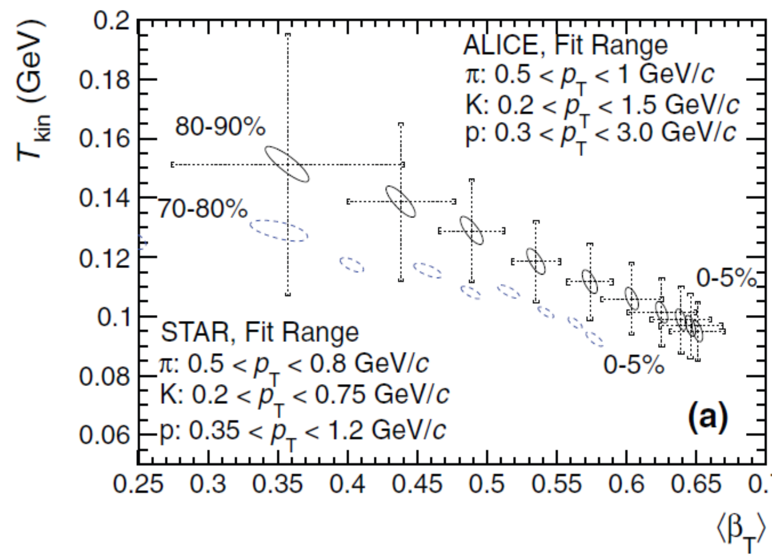


$$w = \frac{m}{T}$$

- pions $w \simeq 1.5$
- nucleons $w \simeq 9$

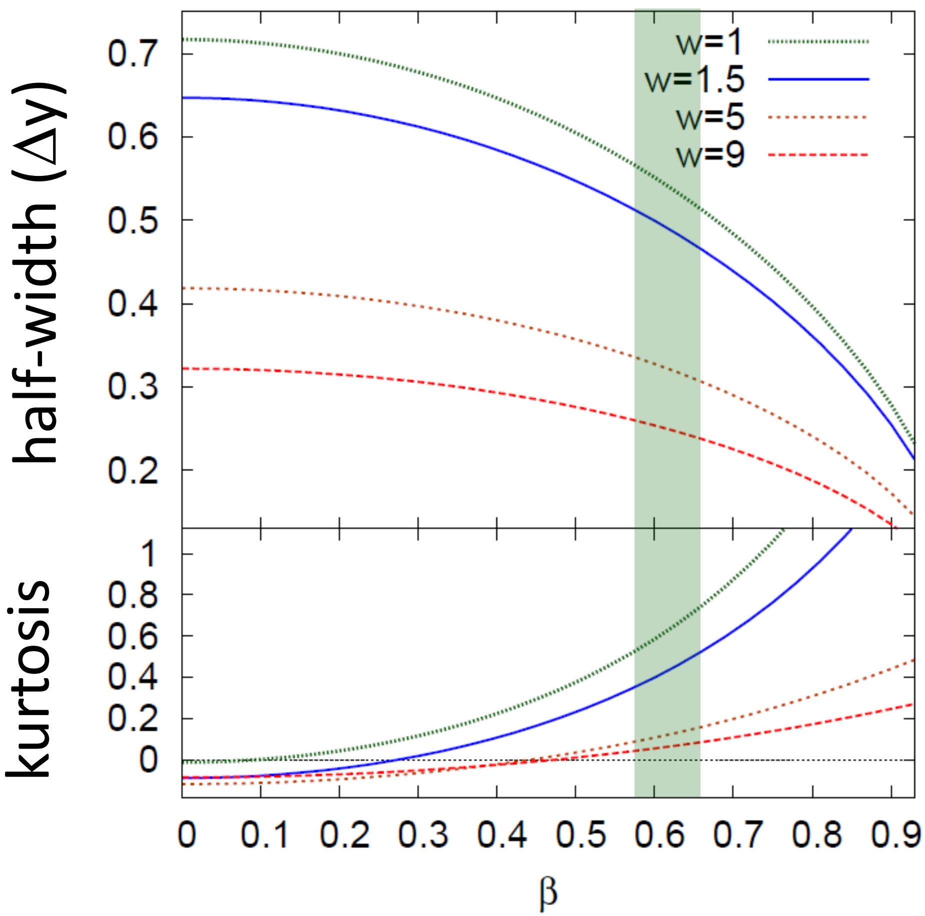


Blast wave squeezes the distribution in rapidity space



- blast wave
- flat freezeout surface

Thermal distribution in y space



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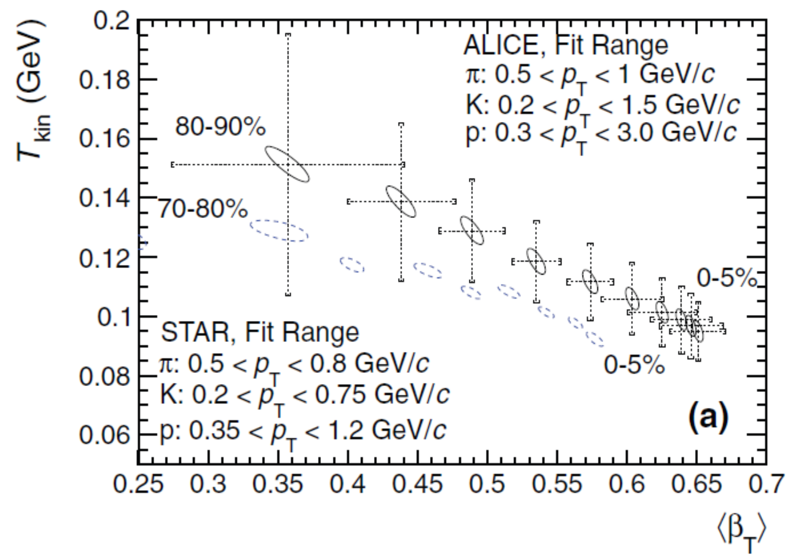


Rapidity distribution can be well approximated by Gaussian.

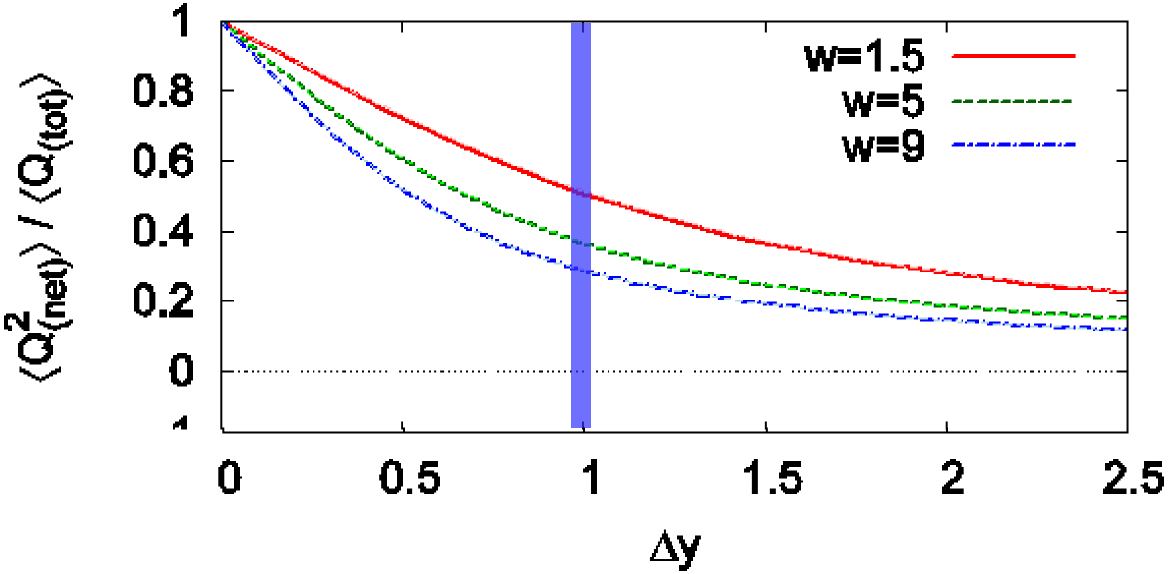
- blast wave
- flat freezeout surface

Δη Dependence

Initial condition
(before blurring)
no e-v-e fluctuations

Cumulants after blurring can take nonzero values

Cumulants after blurring



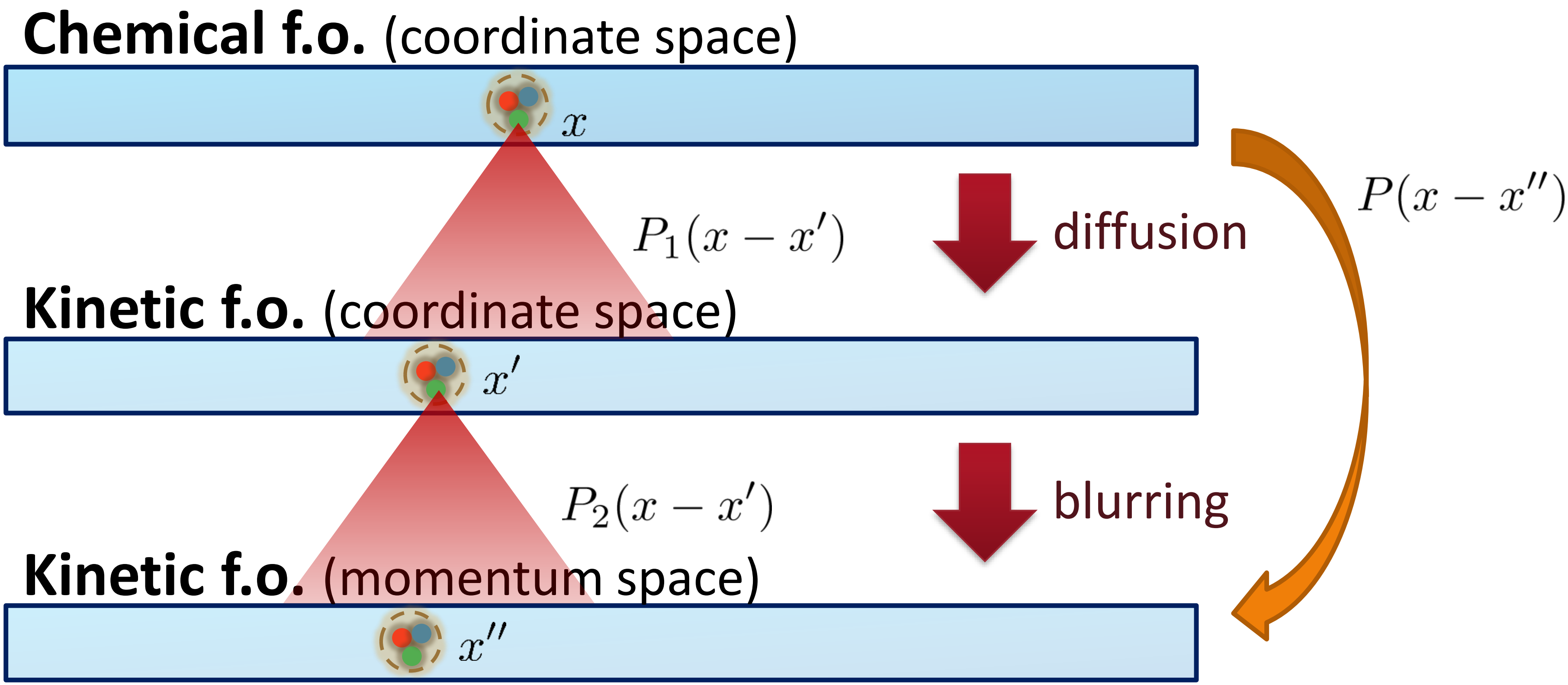
With Δy=1, the effect is **not** well suppressed

$$w = \frac{m}{T}$$

- pions $w \simeq 1.5$
- nucleons $w \simeq 9$

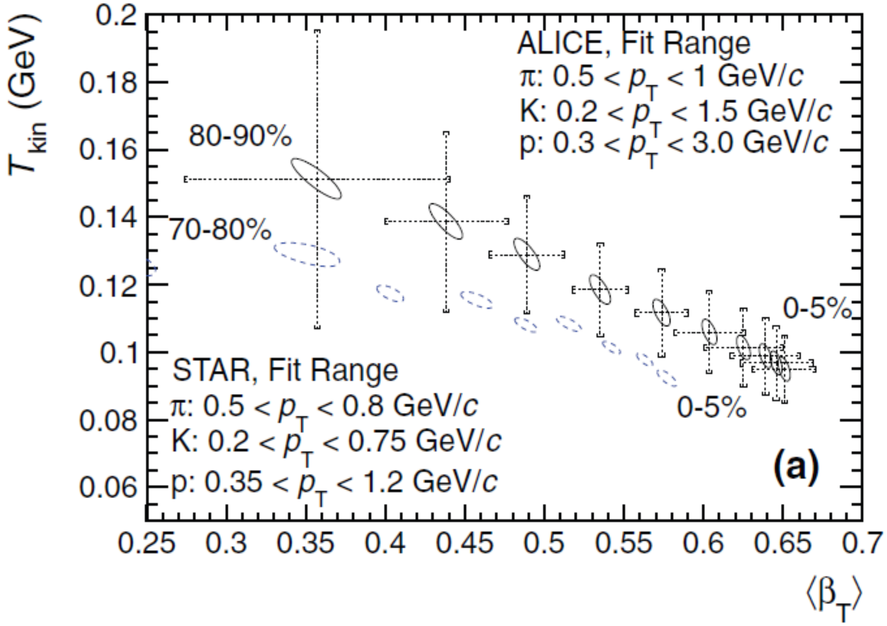
Diffusion + Thermal Blurring

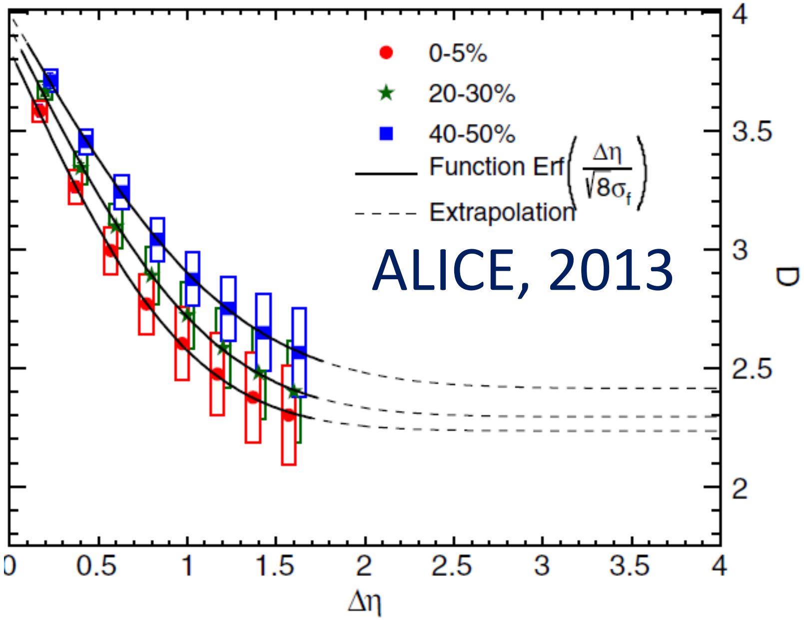
Thermal blurring can be regarded as a part of diffusion



Total diffusion: $P(x - x'') = \int dx' P_1(x - x') P_2(x' - x'')$

Centrality Dependence





More central ⇒ { lower T, larger β } ⇒ Weaker blurring

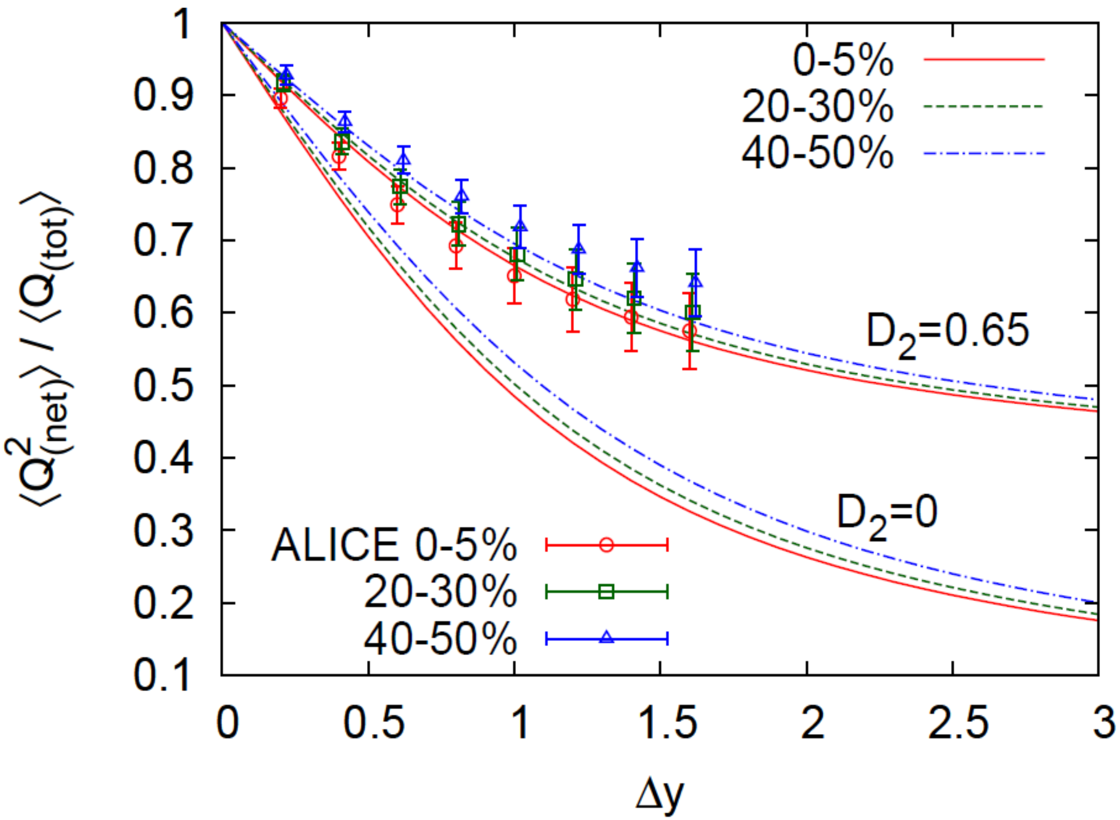
Is the centrality dependence understood solely by the thermal blurring at kinetic f.o.?

Centrality Dependence @ ALICE

$$D_2 = \frac{\langle \delta N_Q^2 \rangle}{\langle \delta N_Q^2 \rangle_{\rm eq.}}$$

Assumptions:

- Centrality independent cumulant at kinetic f.o.
- Thermal blurring at kinetic f.o.



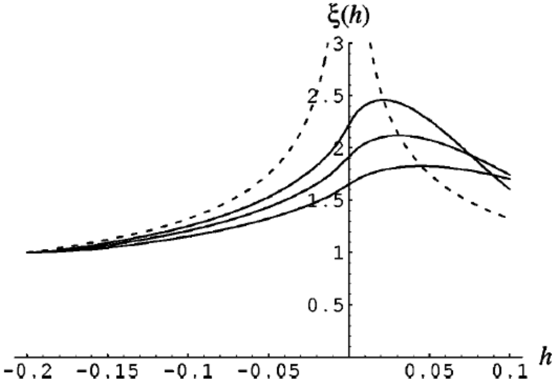
- ☐ Centrality dep. of fluctuation can be described by a simple thermal blurring picture.

Time Evolution of Fluctuation near QCD Critical Point

Sakaida, Asakawa, Fujii, MK, in preparation

Dynamical Evolution of Critical Fluctuations

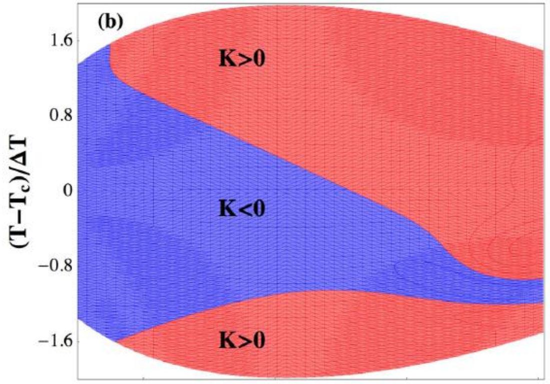
- Evolution of correlation length



Berdnikov, Rajagopal (2000)
Asakawa, Nonaka (2002)

- Higher orders (spatially uniform “σ” mode)

Mukherjee, Venugopalan, Yin (2015)

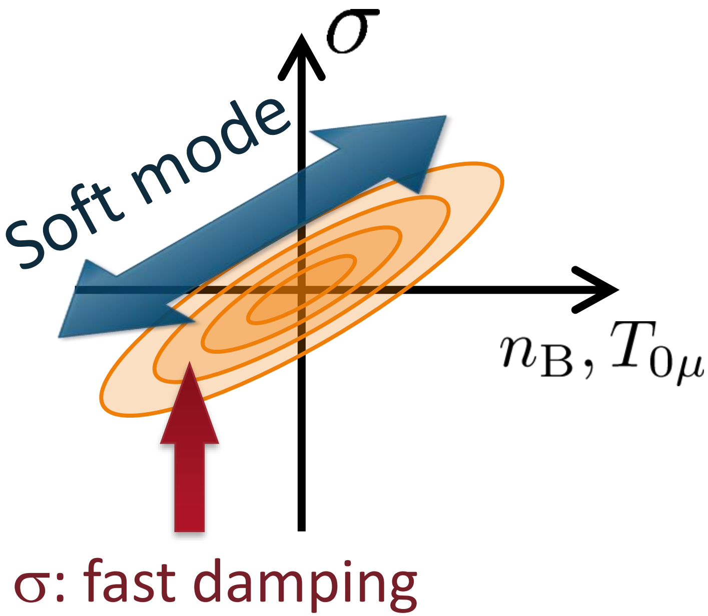


- Correlation functions

Kapusta, Torres-Rincon (2012)

Critical Mode = Diffusive Mode

Fujii (2004)
Fujii, Ohtani (2005)
Son, Stephanov (2005)



Soft mode of QCD CP

$$\begin{pmatrix} \dot{\sigma} \\ \dot{n} \end{pmatrix} = \begin{pmatrix} \Gamma_{\sigma\sigma} & \Gamma_{\sigma n} \\ \Gamma_{n\sigma} & \Gamma_{nn} \end{pmatrix} \begin{pmatrix} \sigma \\ n \end{pmatrix}$$

$\sim k^2$

$$\partial_t \sigma' = D(t) \partial_x^2 \sigma' + \partial_x \xi$$

Evolution of baryon number density

$$\partial_t n = D(t) \partial_x^2 n + \partial_x \xi$$

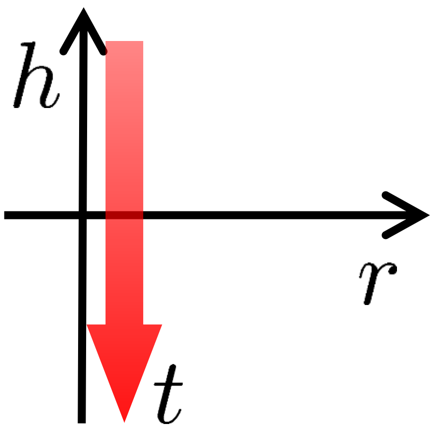
$$\langle \xi(x_1, t_1) \xi(x_2, t_2) \rangle = \chi_2(t) \delta^{(2)}(1-2)$$

$D(t),\ \chi_2(t)$:parameters characterizing criticality

Parametrization of D & χ_2

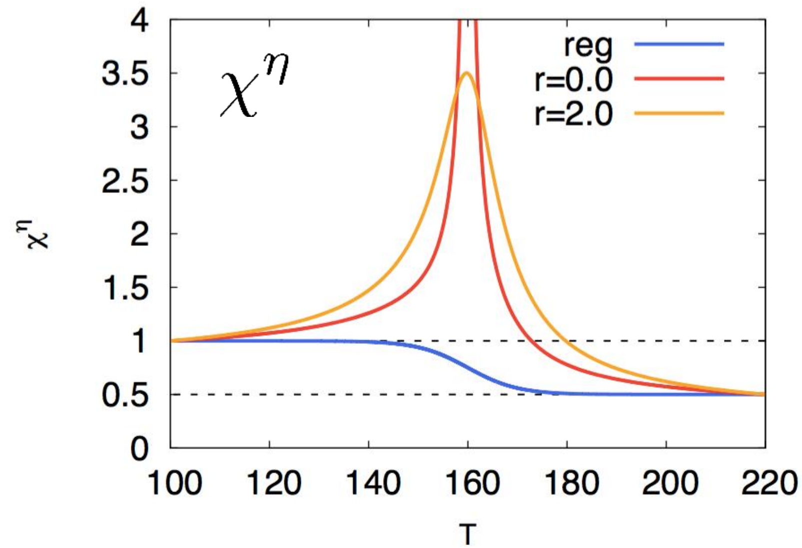
Berdnikov, Rajagopal (2000)
Stephanov (2011)
Mukherjee, Venugopalan, Yin (2015)

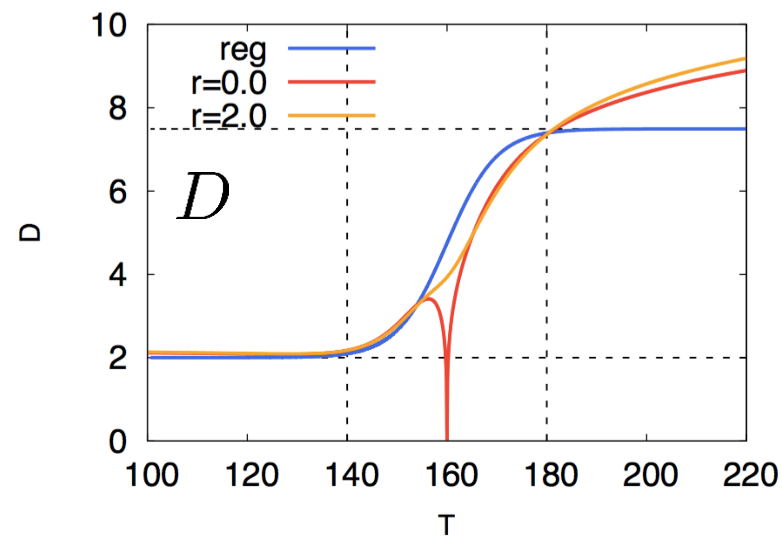
- model-H (3d-Ising)
- $\chi \sim \xi^{1.96}, \ D \sim \xi^{-1.044}$
- mapping to (T,μ) / time evolution



1D Bjorken expansion

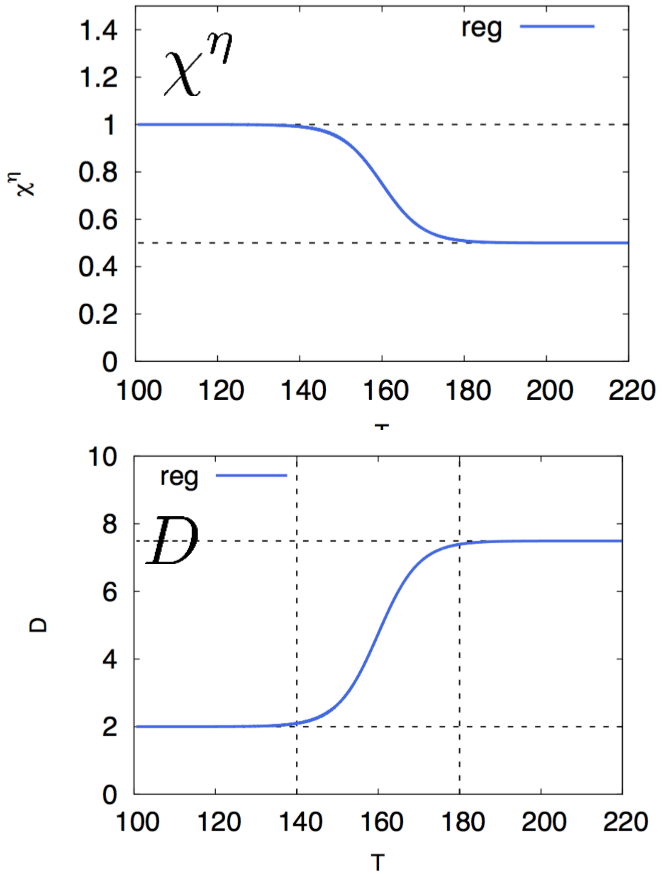
$c_s^2 = 0.2$

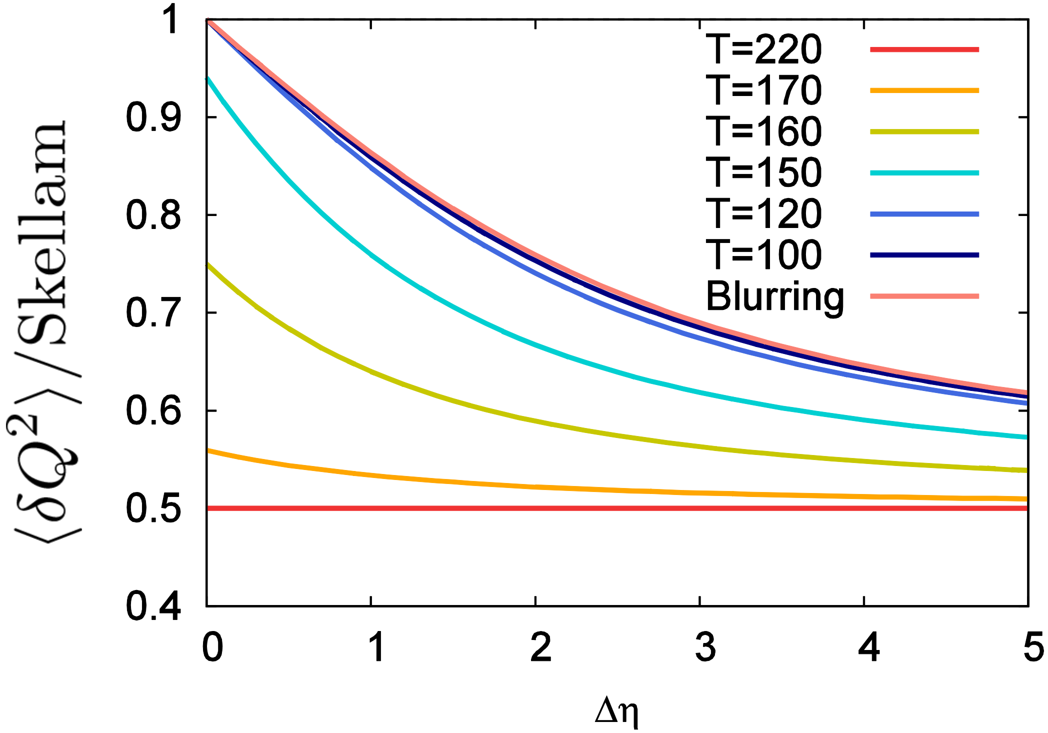




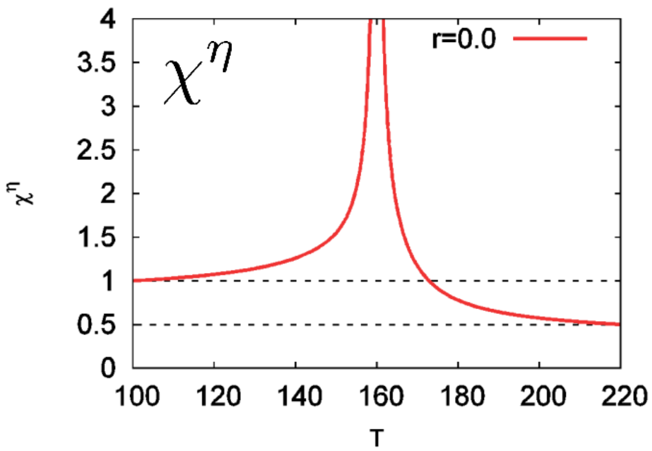
- $\chi^\eta_{\mathrm{QGP}}/\chi^\eta_{\mathrm{hadron}} = 0.5$
- QCD CP at T=160MeV
- kinetic f.o. at T=100MeV

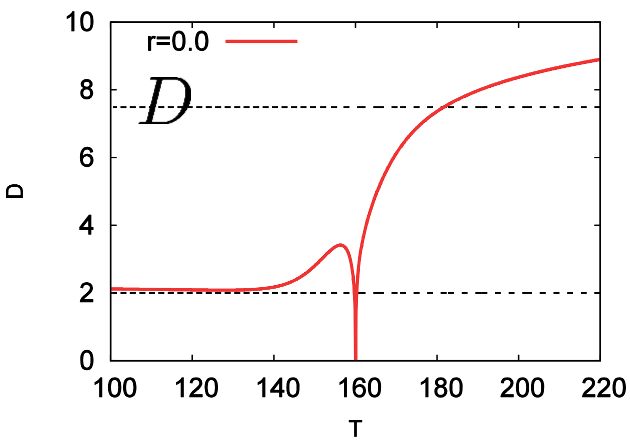
Time Evolution 1: No CP

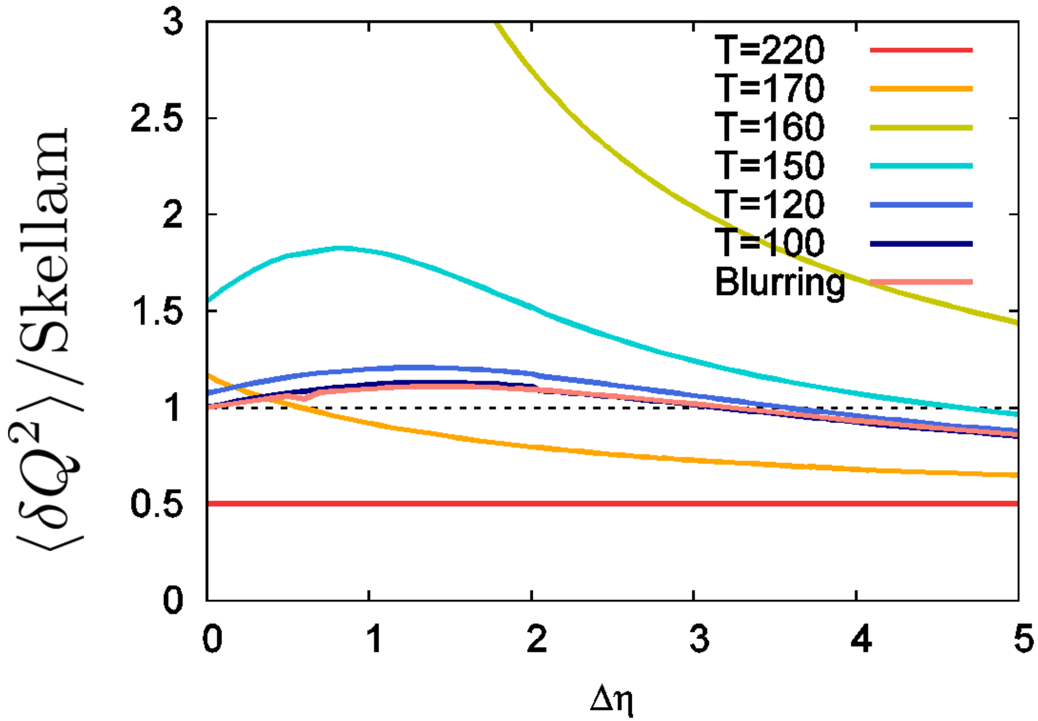




2: Critical Point



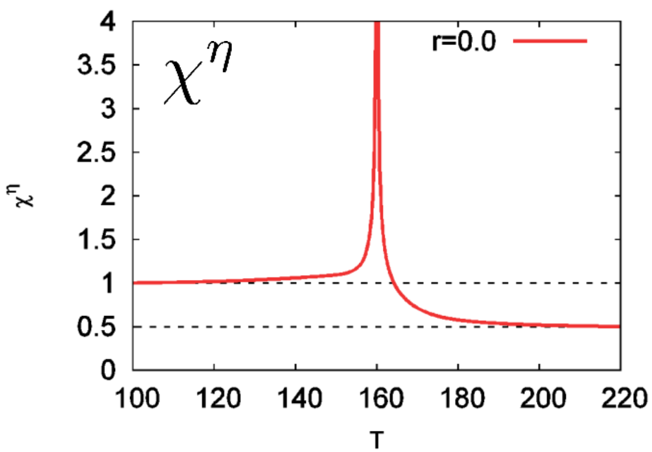


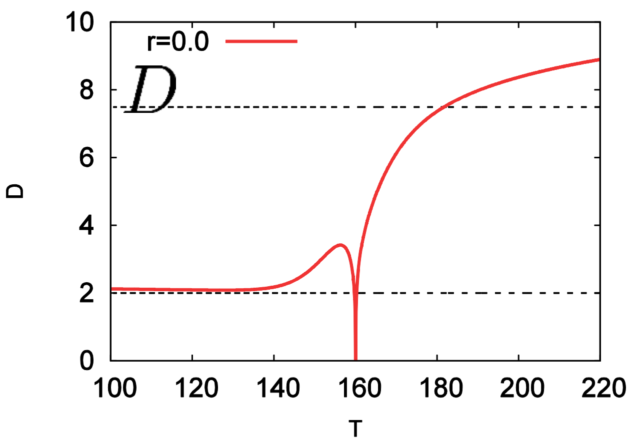


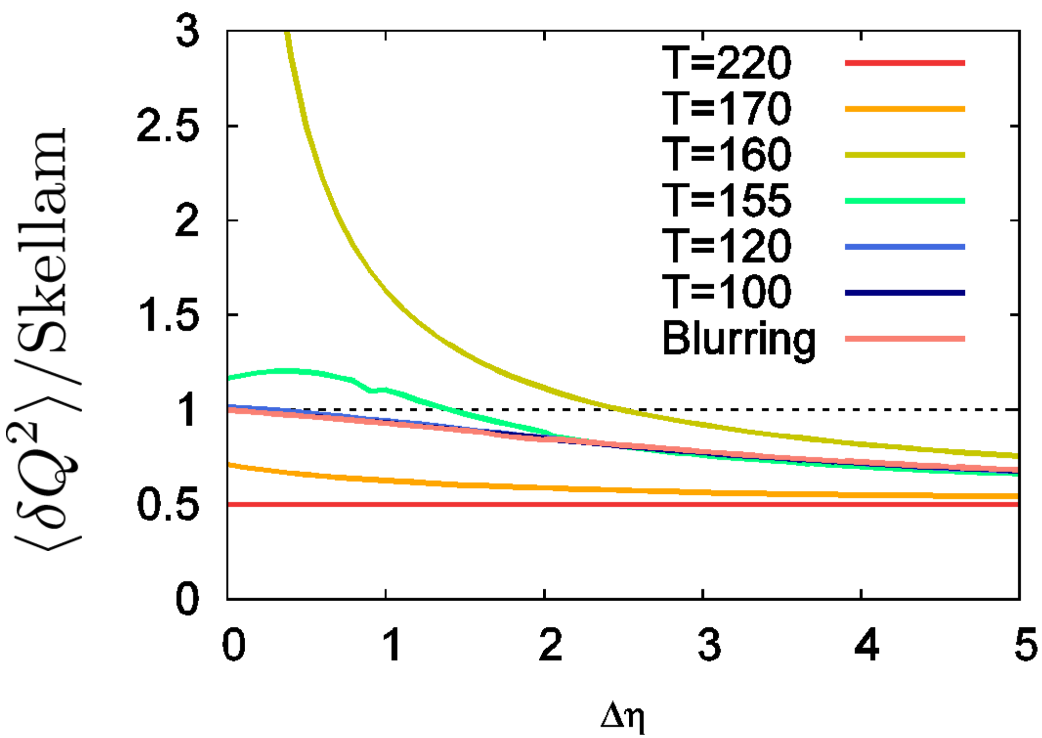
- Non-monotonic Δy dependence manifests itself.

  ➡ Robust experimental evidence of the existence of a peak in χ(T)

3: Critical Point (Narrower Critical Region)







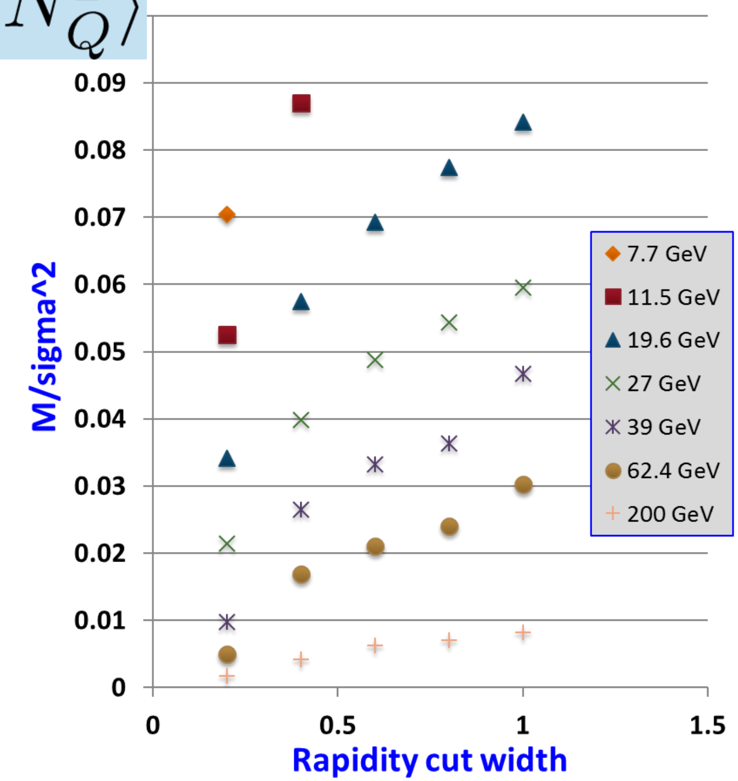
non-monotonic behavior

Peak in $\chi_2(T)$

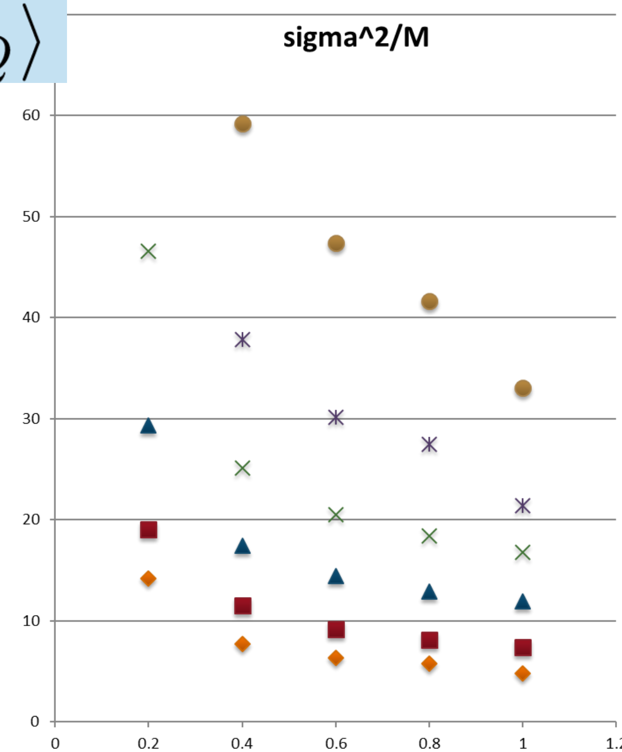
Net-Electric Charge

From N. Sahoo, Ph.D thesis

$$\frac{\langle N_Q \rangle}{\langle \delta N_Q^2 \rangle}$$



$$\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q \rangle}$$



- ☐ No non-monotonic dependence in net-electric charge fluc.
- ☐ How about net-proton number fluctuation??

Summary

Fragile Higher Orders

- Interpret experimental results carefully.

Plenty of information in $\Delta\eta$ dependences

- Cumulants at chemical freezeout
- Diffusion coefficients / thermal blurring
- Signal of QCD-CP as a non-monotonic behavior in 2^{nd} order

Future

- Δy dependence of $\langle \delta N_p^2 \rangle$
- Evolution of higher orders near CP with diffusive nature

Higher Order Cumulants??

- Relaxation of cumulants is slower for higher order.

  →
  - Longer survival?
  - Slower enhancement?

- Non-monotonic Δy dependence can appear only by diffusion → It's not the experimental evidence of peak in (higher order) susceptibility.

- Non-linear equation has to be solved.