Insights (?) from lattice QCD at finite baryo-chemical potential

(title given to me)

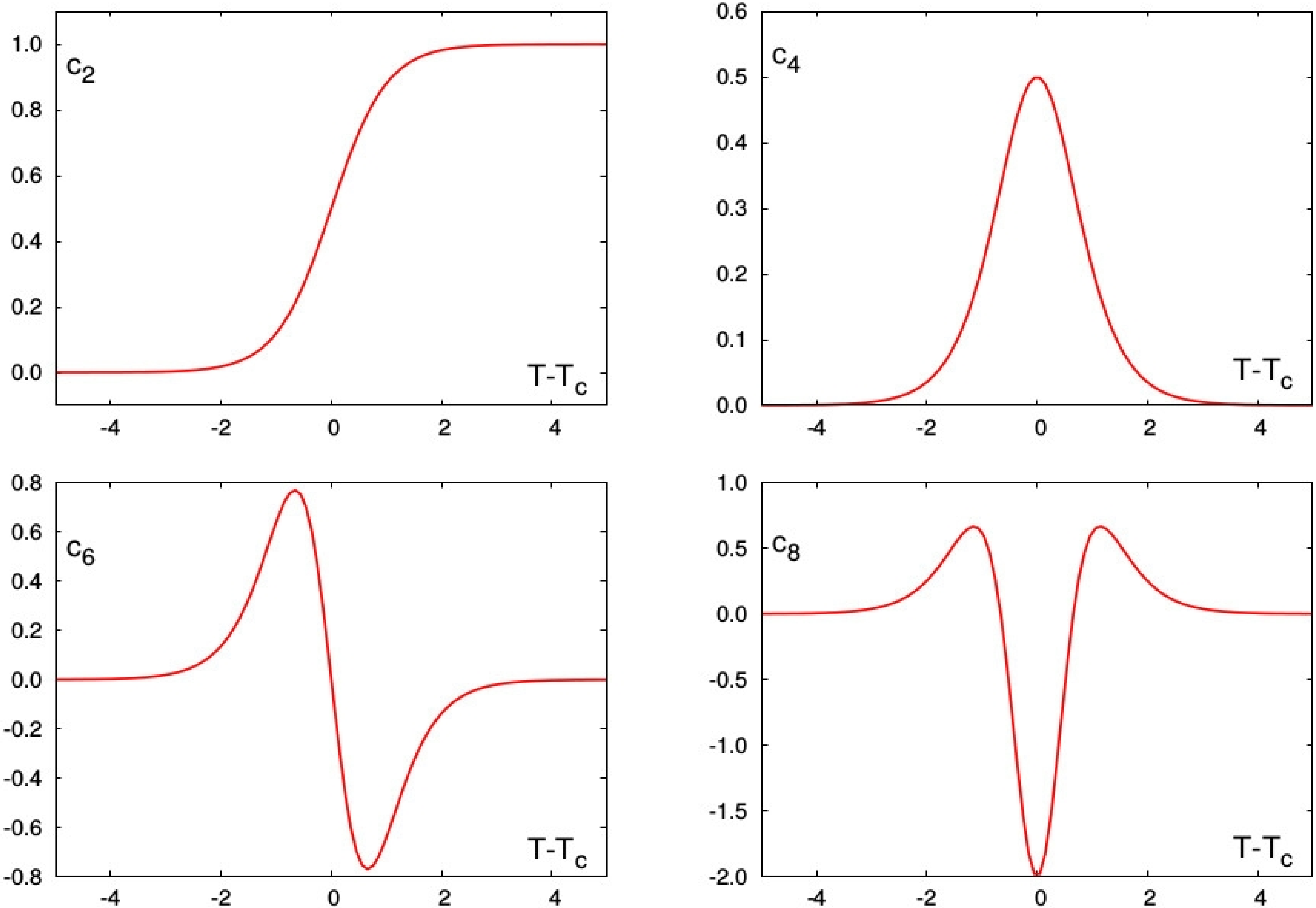
Frithjof Karsch
Bielefeld University & Brookhaven National Laboratory

- Taylor expansion of the equation of state
- characterization of bulk thermodynamics and fluctuations of conserved charges in the crossover region: QCD versus HRG
- constraining the location of the critical point
- skewness and kurtosis from QCD and BES&RHIC

Lattice results on the QCD critical point

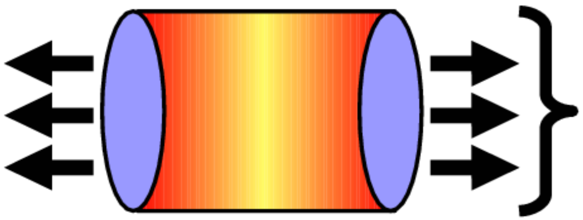
- Where is the critical point?

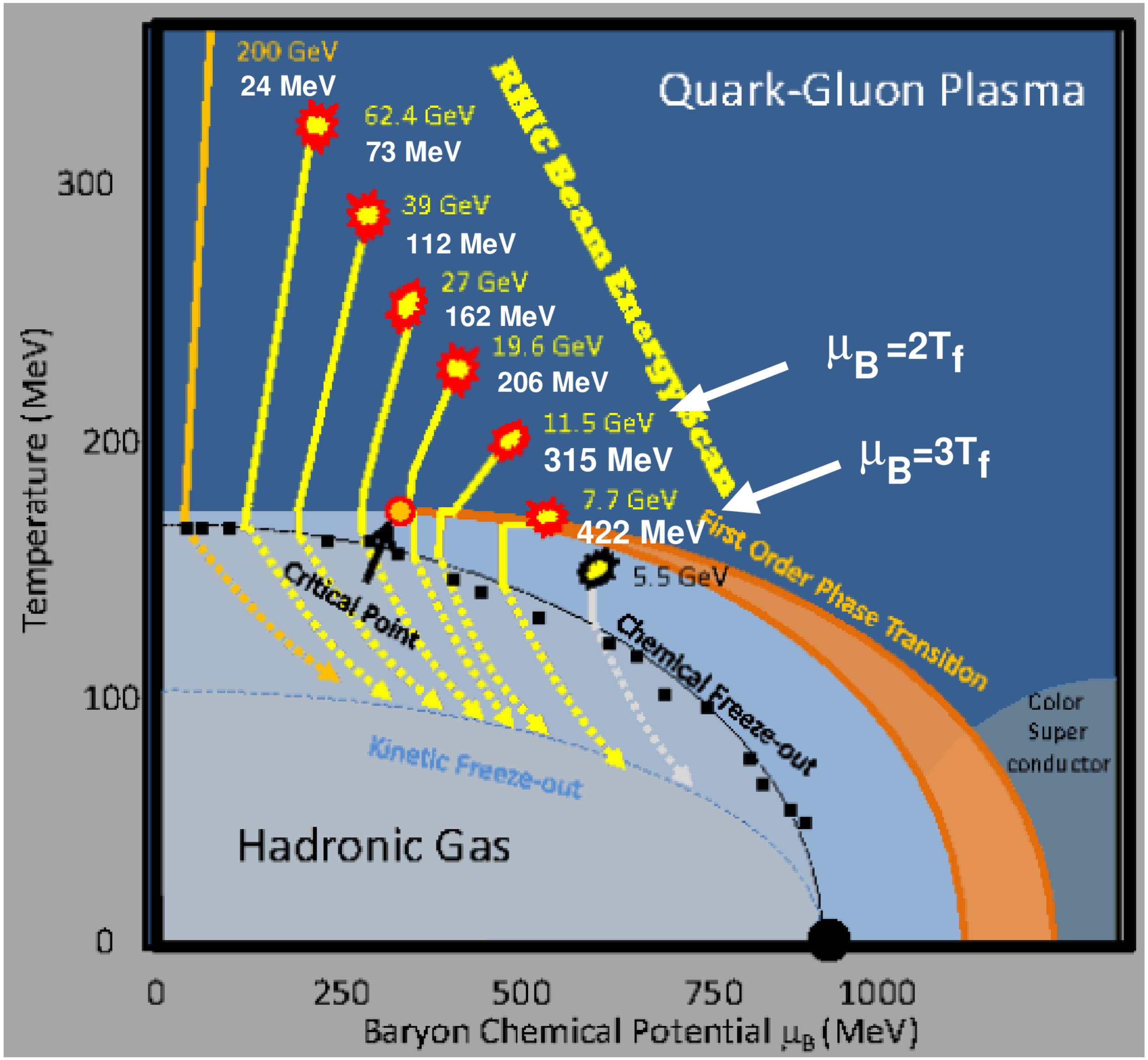
Generic expansion coefficients



similar in PNJL model: S. Roessner et al, PR D75 (2007) 034007; seminar by M. Wagner

QCD phase diagram and the beam energy scan (BES) at RHIC





estimates of the baryon chemical potentials at beam energies used in the BES@RHIC

based on comparison of data on particle yields with HRG model calculations

most relevant for current experiments is to reach knowledge on the EoS for

$$0 \leq \mu_B/T \leq 3$$

$$(T \simeq T_c \simeq 155\ \mathrm{MeV})$$

Probing the properties of matter through the analysis of conserved charge fluctuations

Taylor expansion of the **QCD** pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

cumulants of net-charge fluctuations and correlations:

$$\chi_{ijk}^{BQS} = \left.\frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}\right|_{\mu_{B,Q,S}=0} \quad , \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$

the pressure in hadron resonance gas (**HRG**) models:

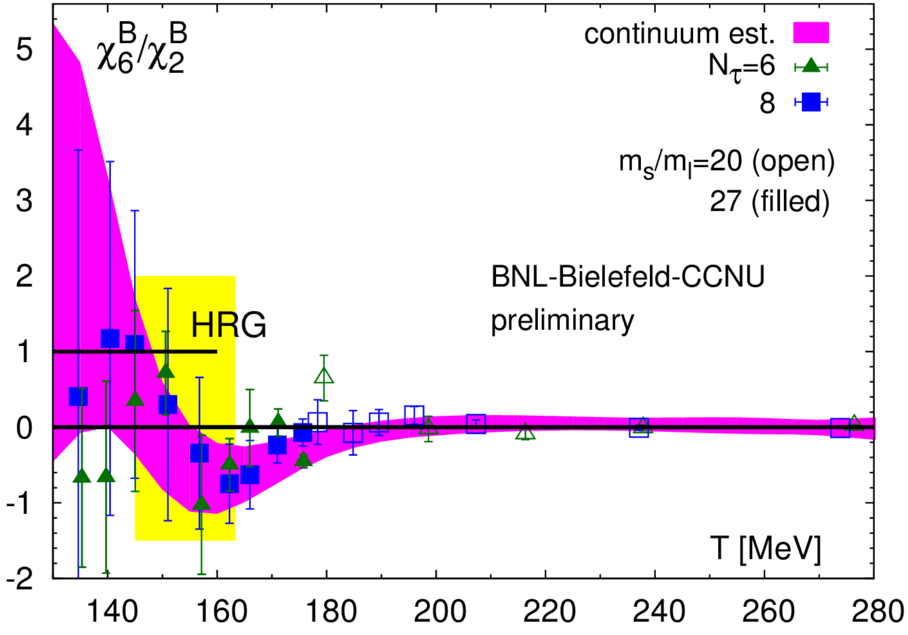
$$\frac{p}{T^4} = \sum_{m \in meson} \ln Z_m^b(T, V, \mu) + \sum_{m \in baryon} \ln Z_m^f(T, V, \mu)$$

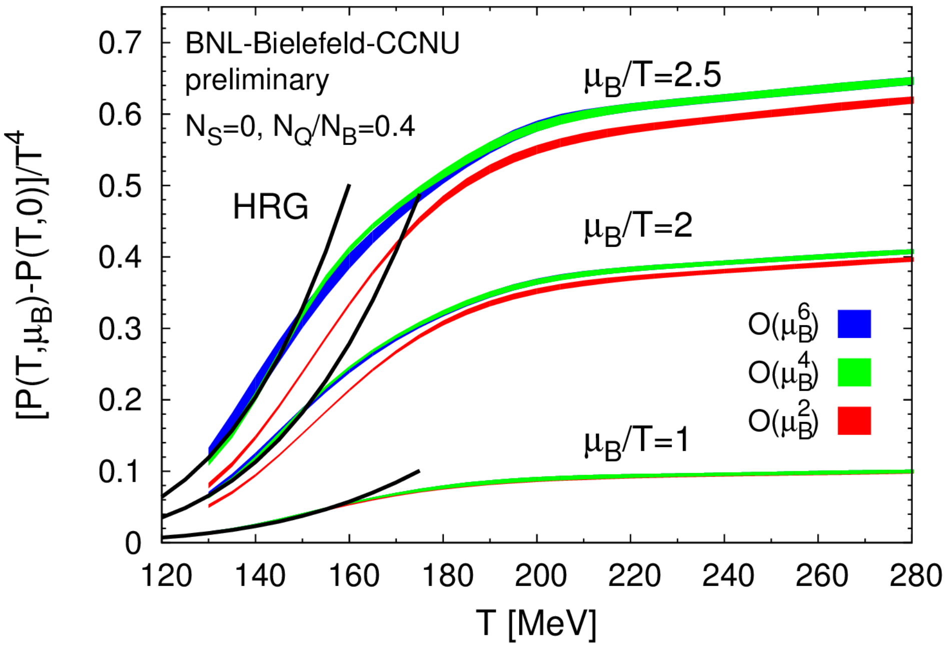
$$\sim \mathrm{e}^{-m_H/T} \mathrm{e}^{(B\mu_B + S\mu_S + Q\mu_Q)/T}$$

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B) - P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right)$$

estimating the $\mathcal{O}((\mu_B/T)^6)$ correction: $\sim \frac{1}{720}\frac{\chi_6^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^6$





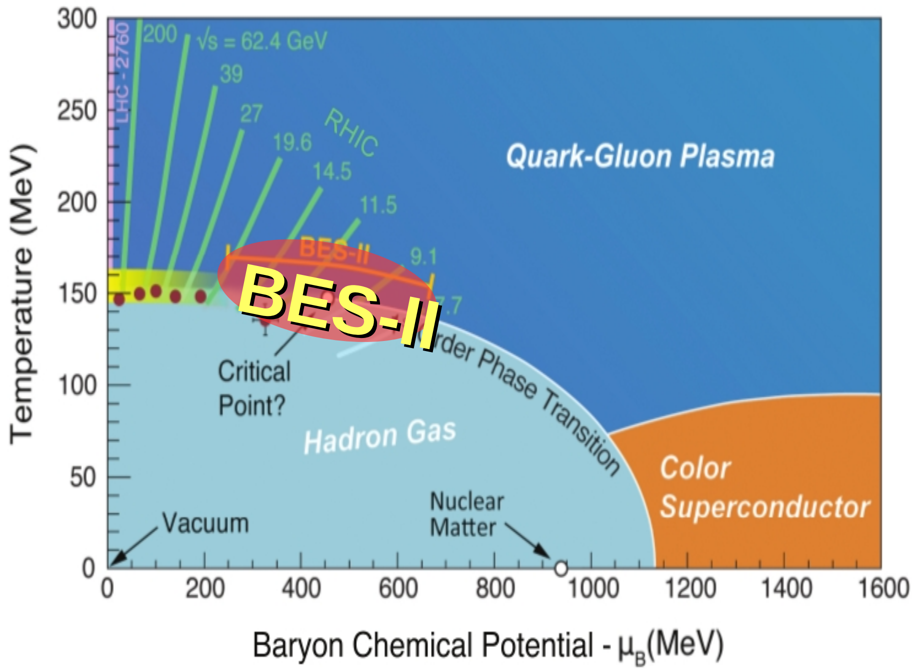
P. Hegde (Bielefeld-BNL-CCNU), arXiV:1412.6727;
Bielefeld-BNL-CCNU in preparation

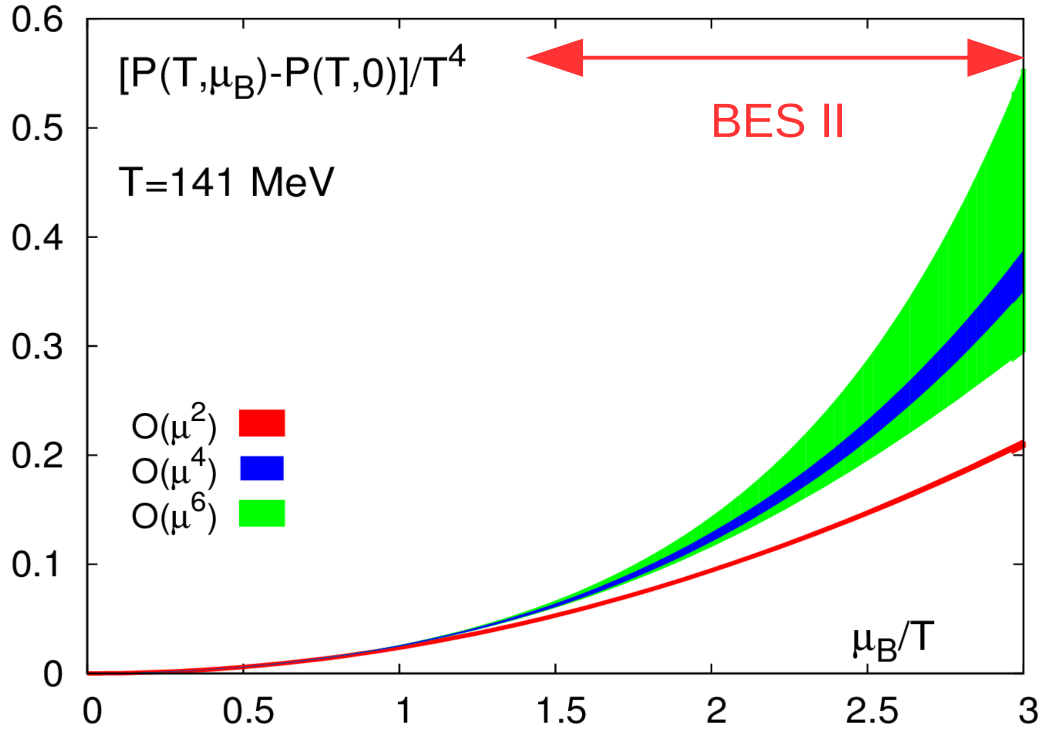
⇒ The EoS is well controlled for $\mu_B/T \leq 2$
or equivalently $\sqrt{s_{NN}} \geq 20$ GeV
talk by Sayantan Sharma

Equation of State and BES II in 2019/20

– aim at statistically significant predictions in the entire parameter range covered by BES II:

– need to improve 6th order correction



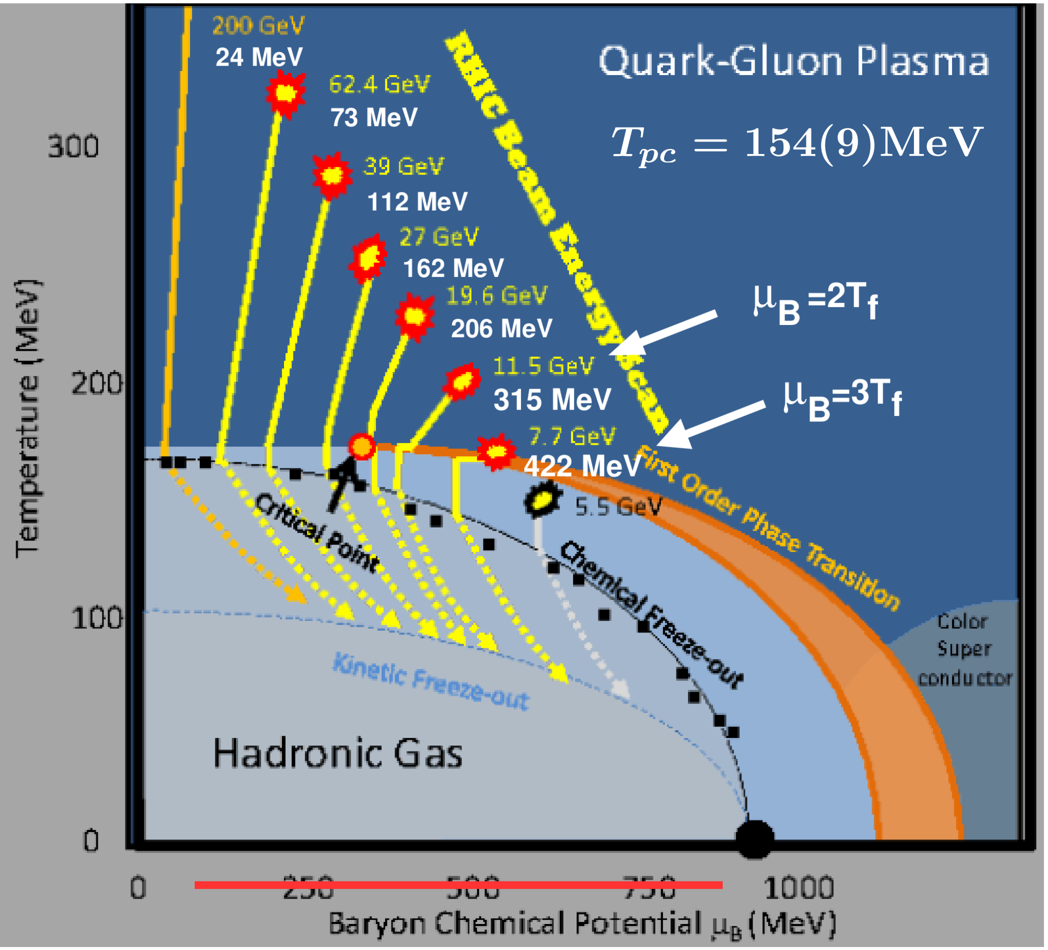


current accuracy

BI-BNL-CCNU, in preparation

in future we need to do better

Probing the properties of matter through the analysis of conserved charge fluctuations



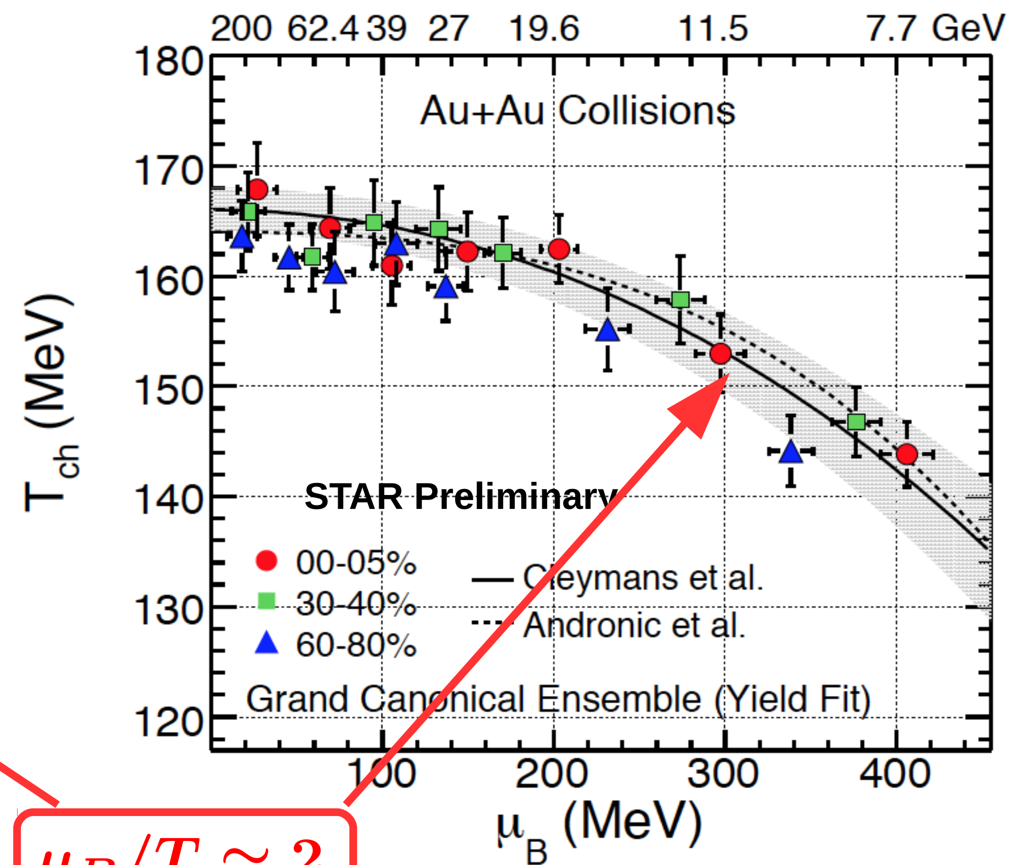
Where is the critical point?

I) need to understand the thermodynamics in the crossover region

RHIC Beam Energy Scan- I (2010-2014)

$\sqrt{s_{NN}}$ (GeV)	Events (10^6)	Year	*μ_B (MeV)	*T_{CH} (MeV)
200	350	2010	25	166
62.4	67	2010	73	165
39	39	2010	112	164
27	70	2011	156	162
19.6	36	2011	206	160
14.5	20	2014	264	156
11.5	12	2010	316	152
7.7	4	2010	422	140

*(μ_B, T_{CH}) : J. Cleymans et al., PR**C73**, 034905 (2006)



$\mu_B/T \simeq 2$

1)Access broad region of the QCD phase diagram.
2)STAR: Large and homogeneous acceptance, excellent PID capabilities.

STAR is a unique detector with huge discovery potential in exploring the QCD phase structure at high baryon density.

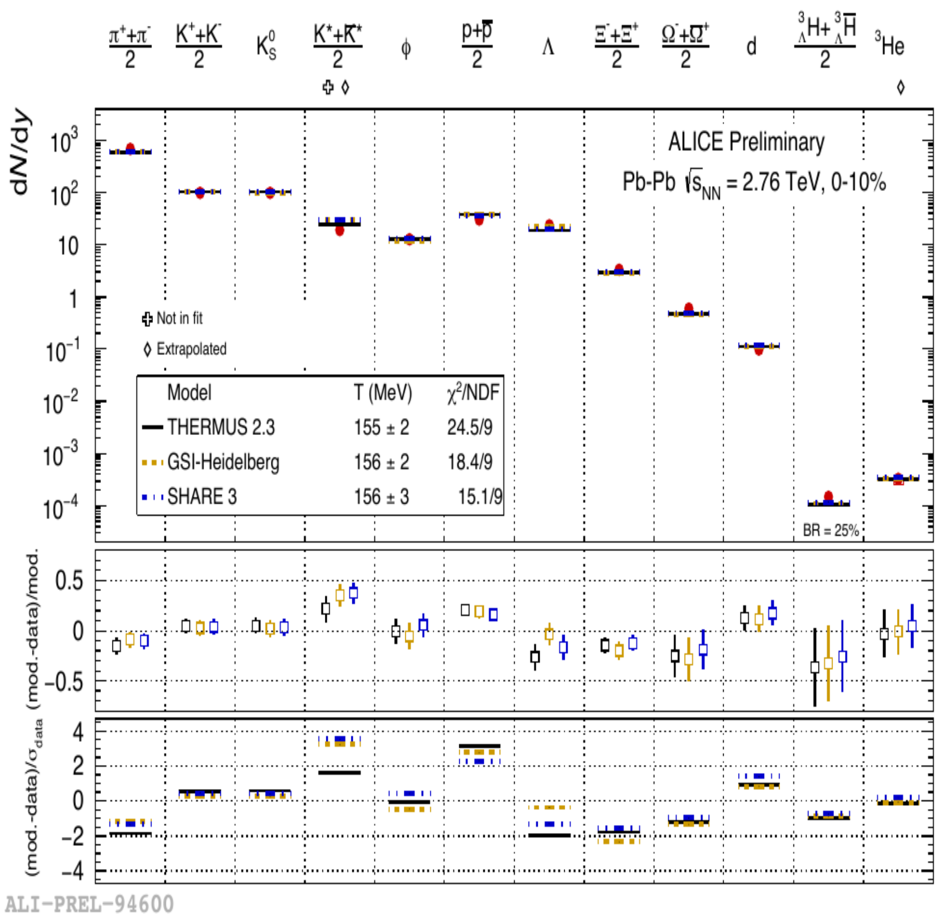
Particle yields, hadron resonance gas and freeze-out conditions

the pressure in hadron resonance gas (**HRG**) models:

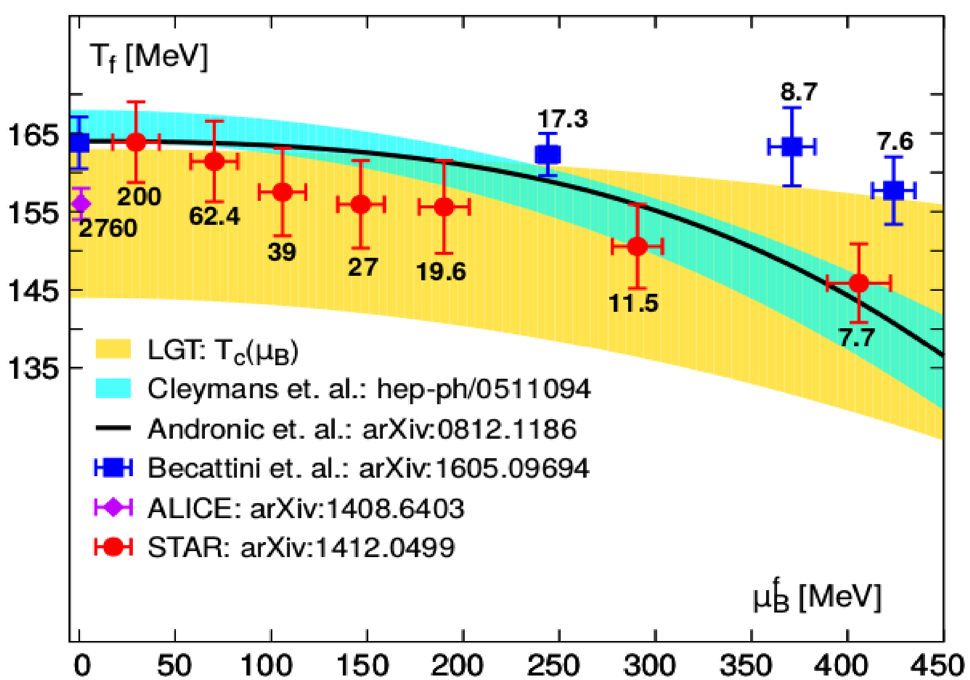
$$\frac{p}{T^4} = \sum_{m \in meson} \ln Z_m^b(T,V,\mu) + \sum_{m \in baryon} \ln Z_m^f(T,V,\mu)$$

$$\sim e^{-m_H/T} e^{(B\mu_B + S\mu_S + Q\mu_Q)/T}$$

$$(T_f, \mu_B^f)$$

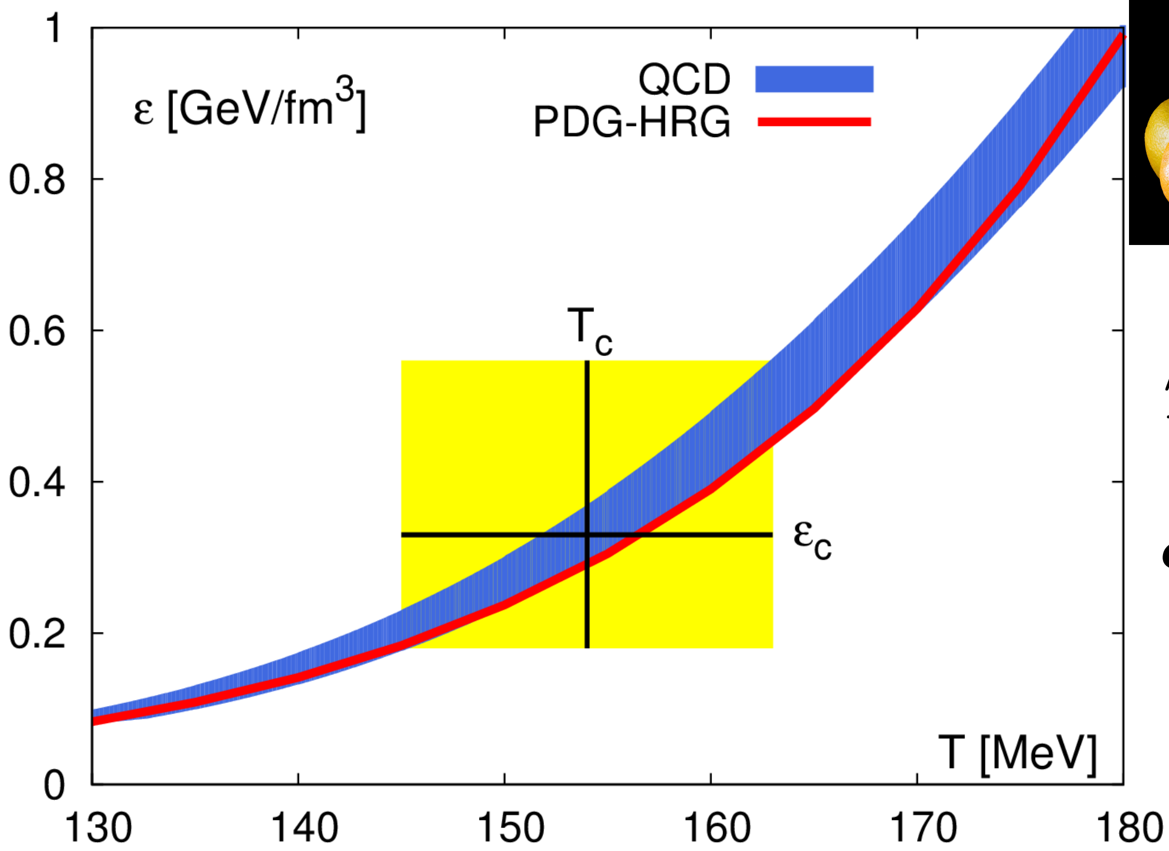


M. Floris, NP A931, 103 (2014), arXiv:1408.6403



Crossover transition parameters

PDG: Particle Data Group hadron spectrum



dense packing of spheres (DPS)

$$\epsilon^{\mathbf{DPS}} = 0.74\epsilon^{\mathbf{nucleon}}$$
$$\simeq 0.33\ \mathrm{GeV/fm^3}$$
$$(R_p \simeq 0.8\mathrm{fm})$$

$$T_c = (154 \pm 9)\ \mathrm{MeV}$$

$$\epsilon_c = (0.34 \pm 0.16)\mathrm{GeV/fm^3}$$

A. Bazavov et al. (hotQCD) ,
Phys. Rev. D90 (2014) 094503

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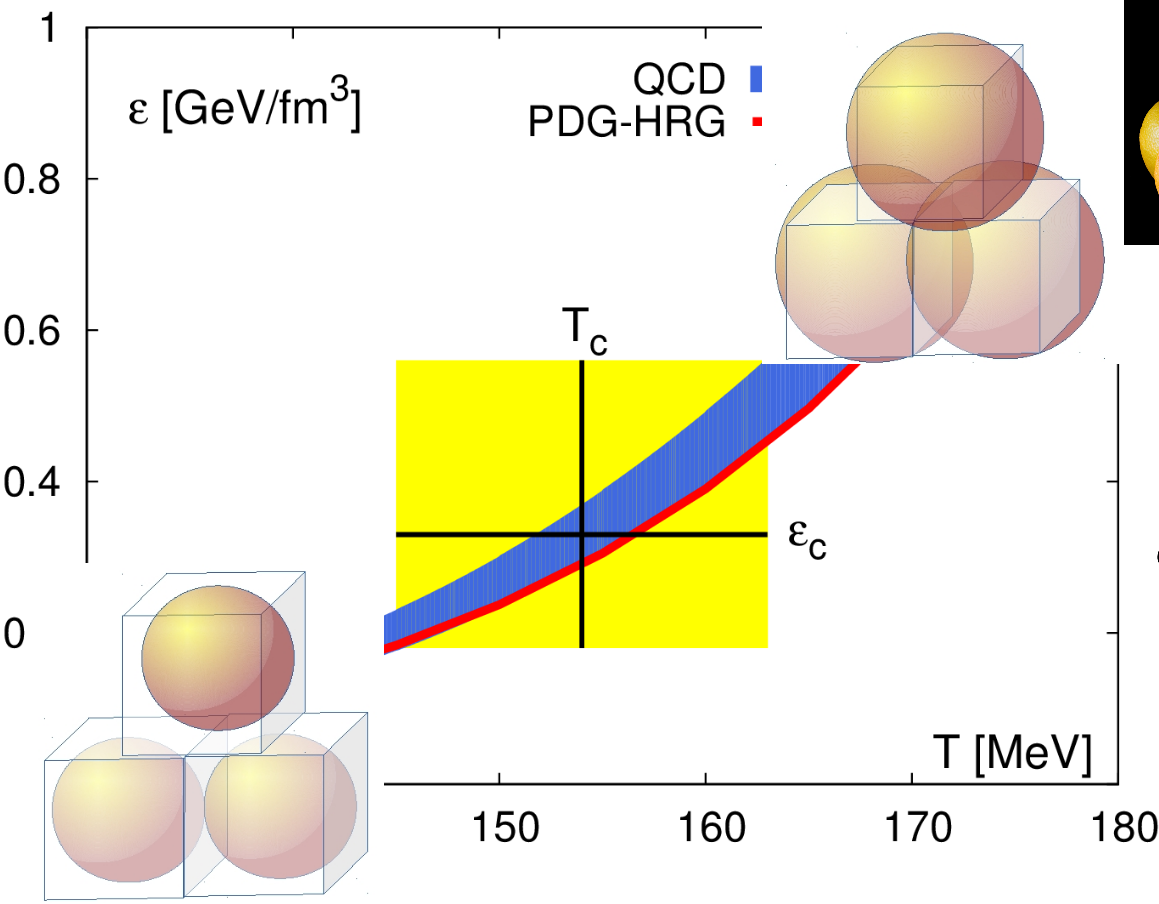


dilute hadron gas = QGP ??

dense packing of spheres (DPS)

$$\epsilon^{\mathrm{DPS}} = 0.74\epsilon^{\mathrm{nucleon}}$$
$$\simeq 0.33\ \mathrm{GeV/fm^3}$$
$$(R_p \simeq 0.8\mathrm{fm})$$

overlapping hadrons = QGP ??

$$T_c = (154 \pm 9)\ \mathrm{MeV}$$

$$\epsilon_c = (0.34 \pm 0.16)\mathrm{GeV/fm^3}$$

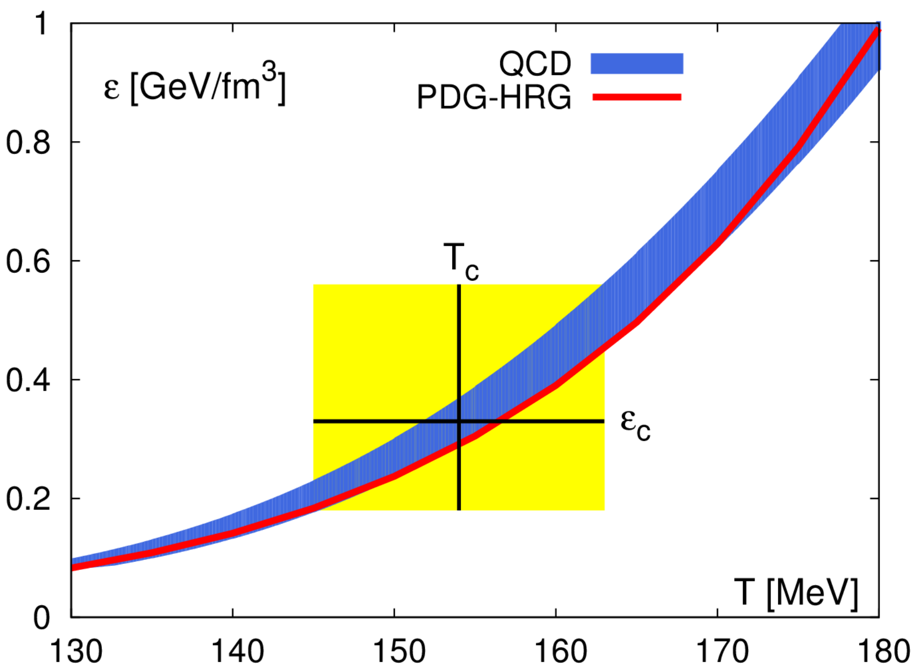
compare with:

$$\epsilon^{\mathrm{nucl.\ mat.}} \simeq 150\ \mathrm{MeV/fm^3}$$

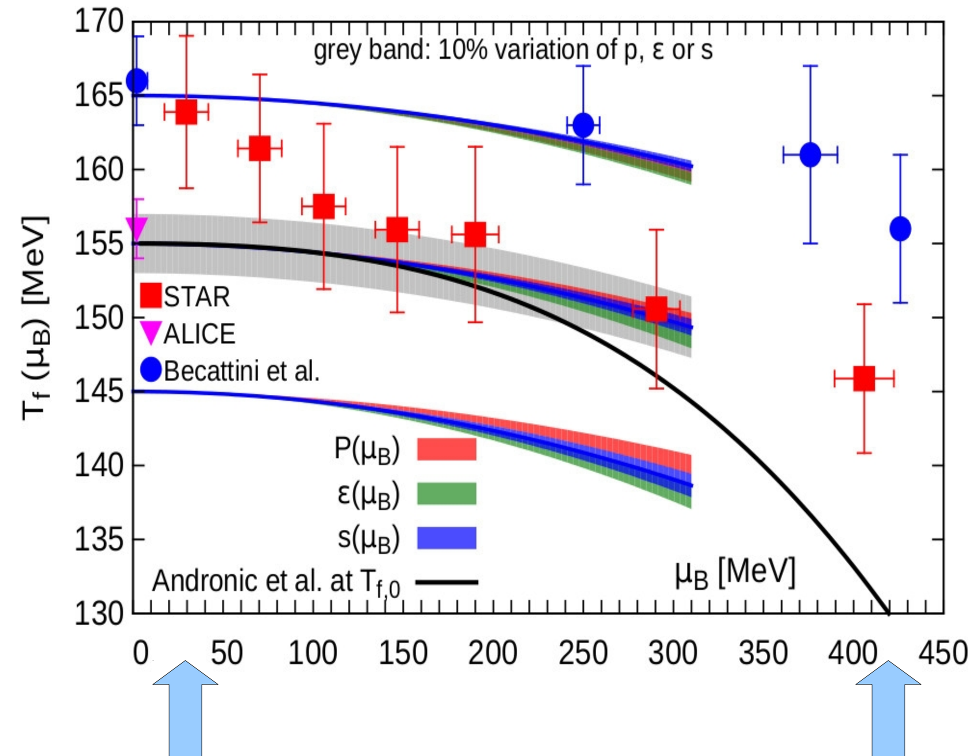
$$\epsilon^{\mathrm{nucleon}} \simeq 450\ \mathrm{MeV/fm^3}$$

A. Bazavov et al. (hotQCD) ,
Phys. Rev. D90 (2014) 094503

Lines of constant physics and freeze-out



$$T_f(\mu_B) = T_f(0)\left(1 - \kappa_{f,2}\hat{\mu}_B^2 - \kappa_{f,4}\hat{\mu}_B^4\right)$$



200 GeV $\sqrt{s_{NN}}$ **7.7 GeV**

$\mu_Q = \mu_S = 0$:

constant pressure: $\kappa_{2,p} \simeq 0.007 - 0.009$

constant energy density: $\kappa_{2,\epsilon} \simeq 0.009 - 0.011$

constant entropy density: $\kappa_{2,s} \simeq 0.008 - 0.010$

crossover line: $\kappa_{2,c} \simeq 0.006 - 0.015$

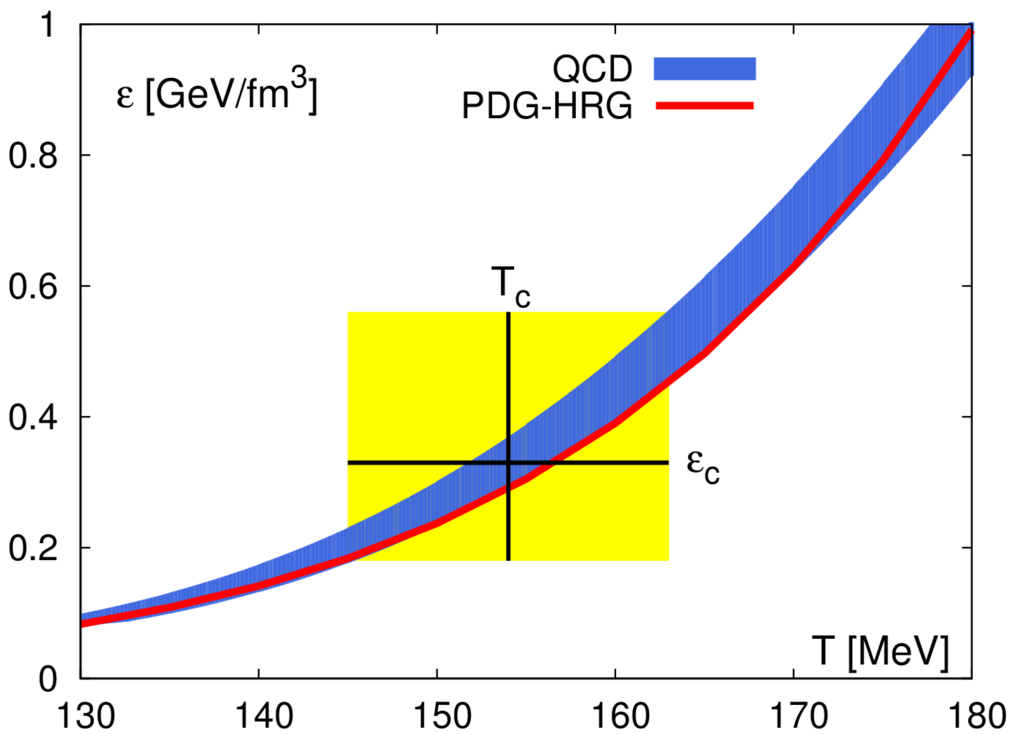
freeze-out at T=165 MeV is quite different from freeze-out at T-155 MeV:

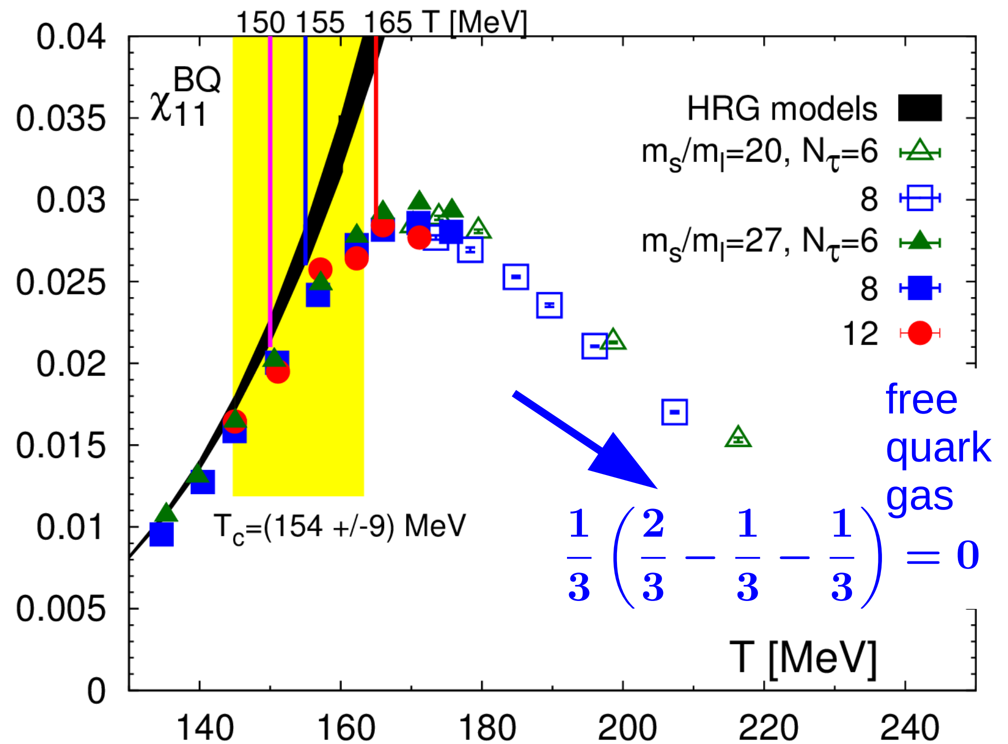
What happens to ordinary hadrons (as used in HRG models) in the transition region?

HRG vs. QCD
baryon number – electric charge correlations

$$\epsilon = -\frac{\partial P/T}{\partial 1/T}\Big|_{(\mu_B,\mu_Q,\mu_S)=0}$$

$$\chi_{11}^{BQ} = \frac{\partial^2 P/T^4}{\partial\hat{\mu}_B \partial\hat{\mu}_Q} = \langle B\cdot Q\rangle - \langle B\rangle\langle Q\rangle$$





– nothing dramatic happens when passing through the "transition" region ?

– change in composition of the thermal medium is detected through conserved charge correlations

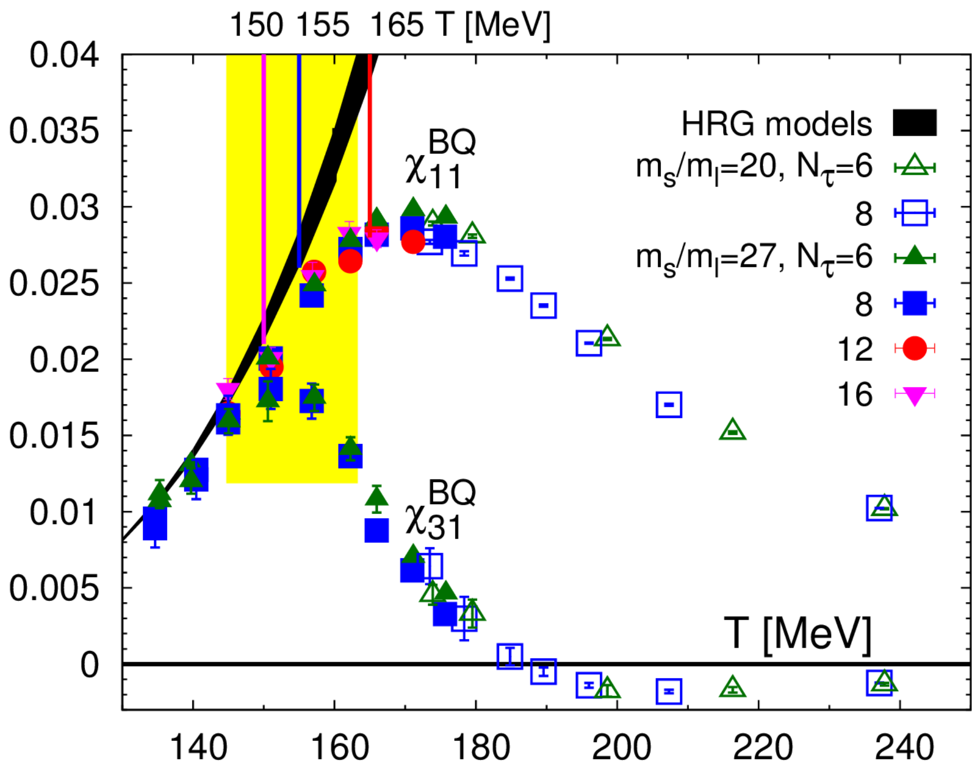
HRG vs. QCD
electric charge-baryon number correlations

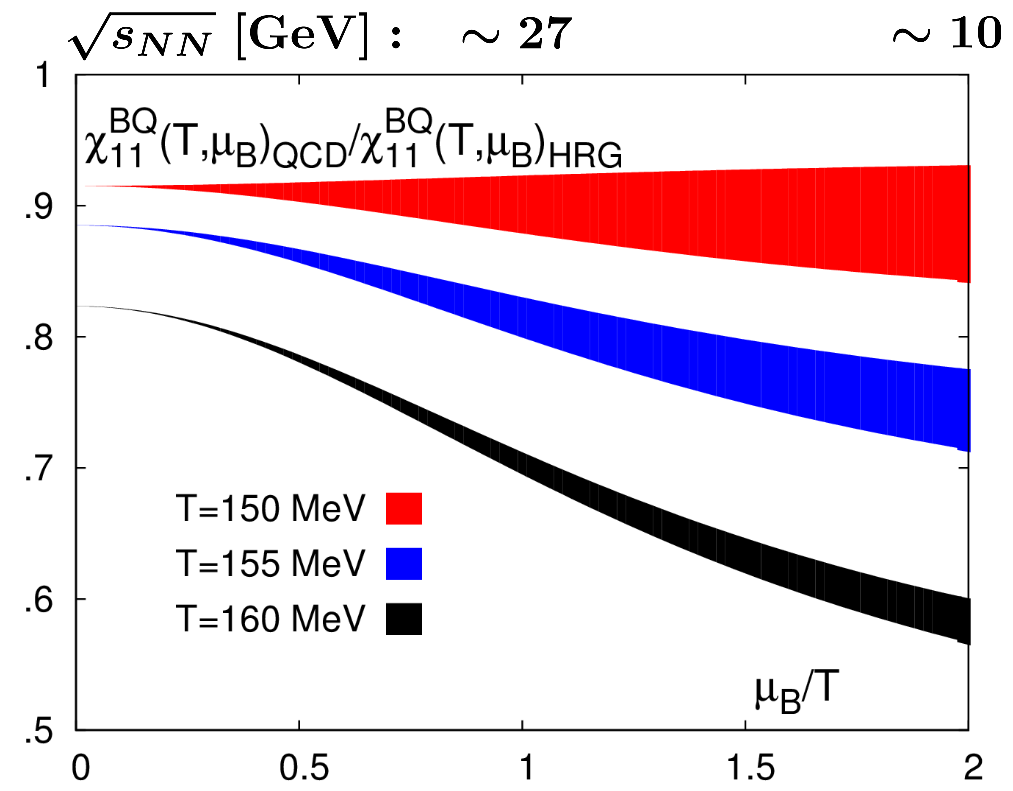
$$\mu_B/T > 0$$

for simplicity: $\mu_Q = \mu_S = 0$

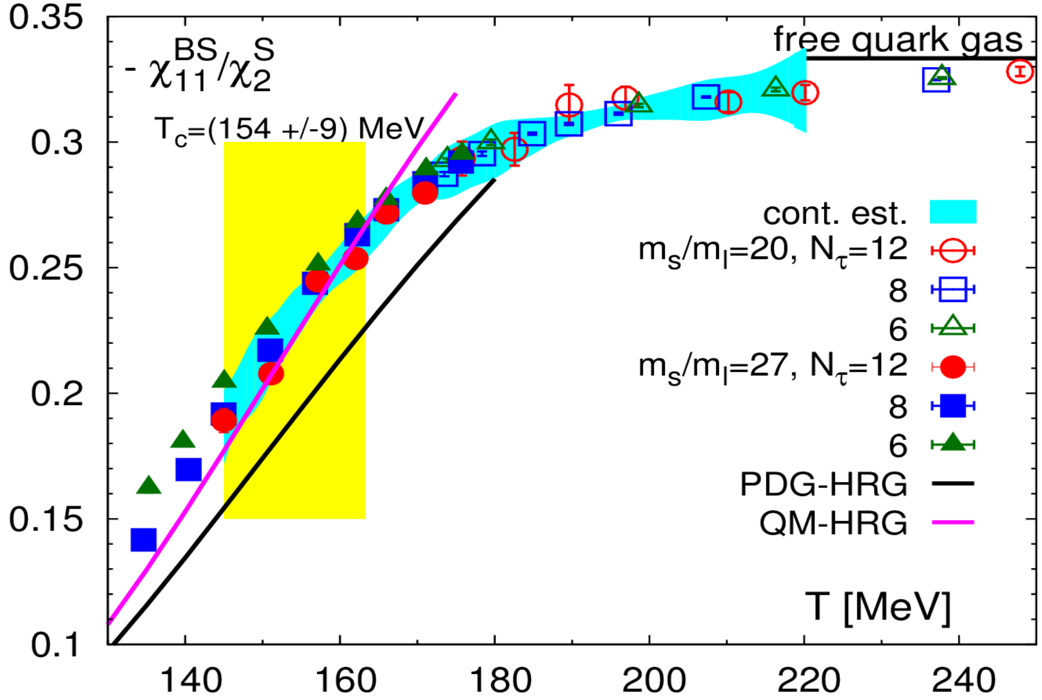
$$\chi_{11}^{BQ}(T,\mu_B) = \chi_{11}^{BQ} + \frac{1}{2}\chi_{31}^{BQ}\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$

– agreement between HRG and QCD will start to deteriorate for T>150 MeV





HRG vs. QCD
strangeness-baryon number correlations



$$\chi_{11}^{BS} = \frac{\partial^2 P/T^4}{\partial\hat{\mu}_B \partial\hat{\mu}_S} = \langle B \cdot S \rangle - \langle B \rangle \langle S \rangle$$

$$\chi_2^S = \frac{\partial^2 P/T^4}{\partial\hat{\mu}_S^2} = \langle S^2 \rangle - \langle S \rangle^2$$

S. Borsanyi et al., JHEP 1201 (2012) 138
A. Bazavov et al.,
PRL 113 (2014) 072001, arXiv:1404.6511

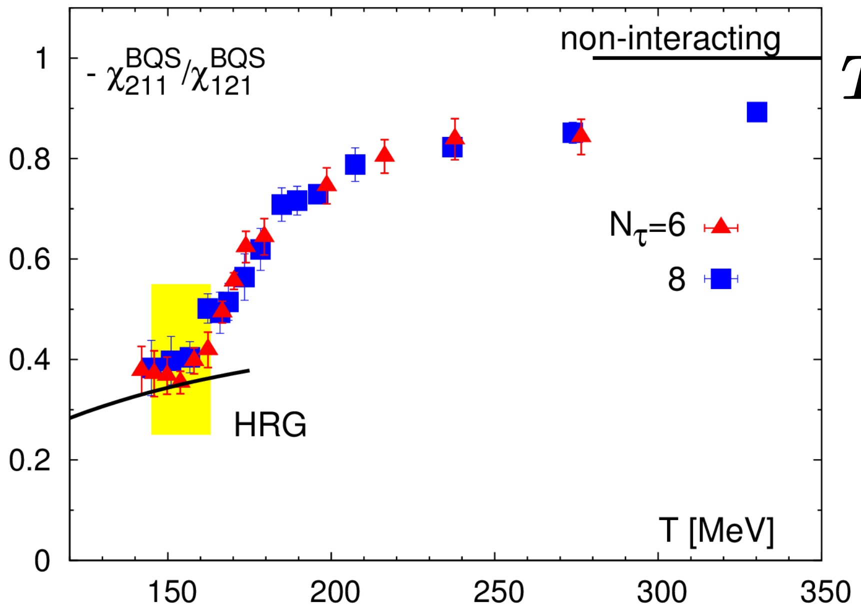
continuum extrapolated results on strangeness-baryon correlations **do NOT agree** with a conventional hadron resonance gas, based on experimentally known resonances listed in the particle data tables

in the crossover region (and above):

PDG-HRG $\neq$ QCD

QM-HRG does better

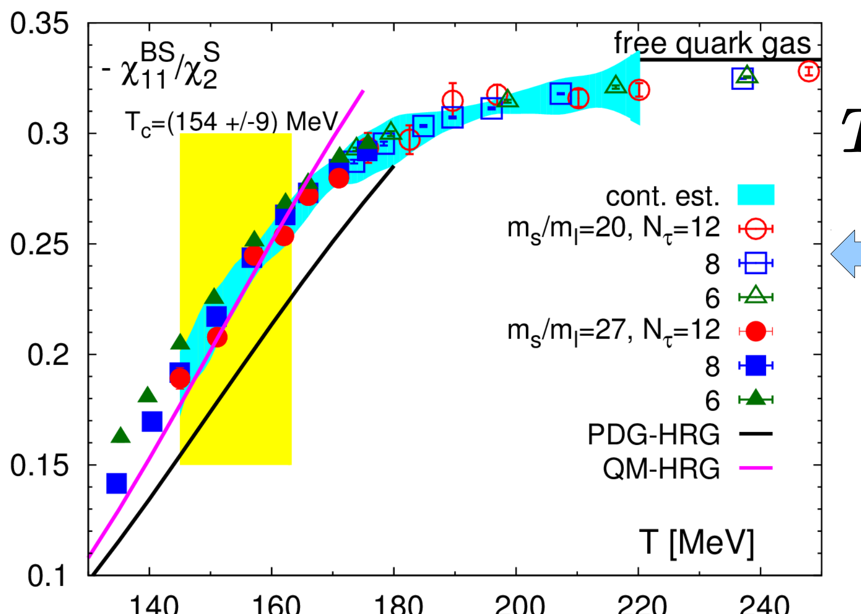
Analyzing **strangeness carrier** with higher order cumulants



$T > T_c$ **:** Is strangeness carried by quasi-particles with quantum numbers of quarks ?

$$\frac{\chi_{211}^{BQS}}{\chi_{121}^{BQS}} = \begin{cases} -1\ , & T \to \infty \\ \mathrm{HRG}\ , & T \le T_c \end{cases}$$

ratio of properly weighted charged, strange baryons



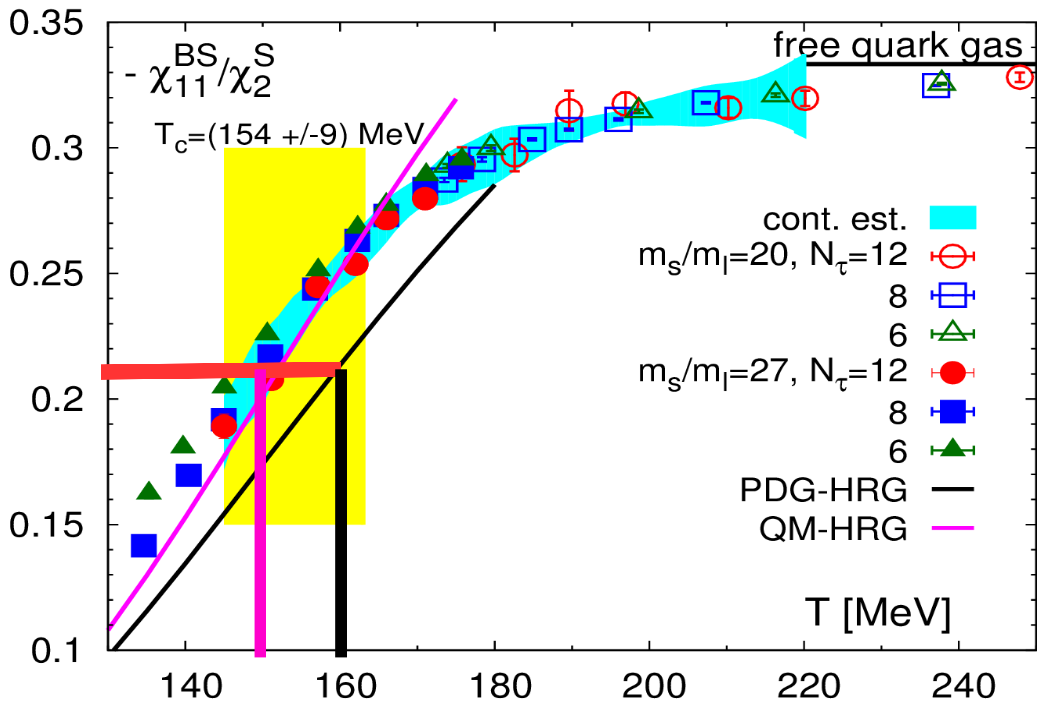
$T < T_c$ Who carries strangeness in the HRG?

significant deviations from a HRG based on known resonances (PDG-HRG)

Enhanced strange baryon fluctuations:
indications for more strange hadron resonances as predicted in quark model calculations (QM-HRG)

A. Bazavov et al.,
Phys. Rev. Lett. 113, 072001 (2014), arXiv:1404.6511

HRG vs. QCD
strangeness-baryon number correlations



$$\chi_{11}^{BS} = \frac{\partial^2 P/T^4}{\partial\hat{\mu}_B \partial\hat{\mu}_S} = \langle B\cdot S\rangle - \langle B\rangle\langle S\rangle$$

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S. Borsanyi et al., JHEP 1201 (2012) 138
A. Bazavov et al.,
PRL 113 (2014) 072001, arXiv:1404.6511

using ALICE data on particle yields,
combined with HRG assumptions: $\chi_{11}^{BS}/\chi_2^S \gtrsim 0.19(2)$

P. Braun-Munzinger et al.,
Phys. Lett. B747, 292 (2015)
arXiv:1412.8614

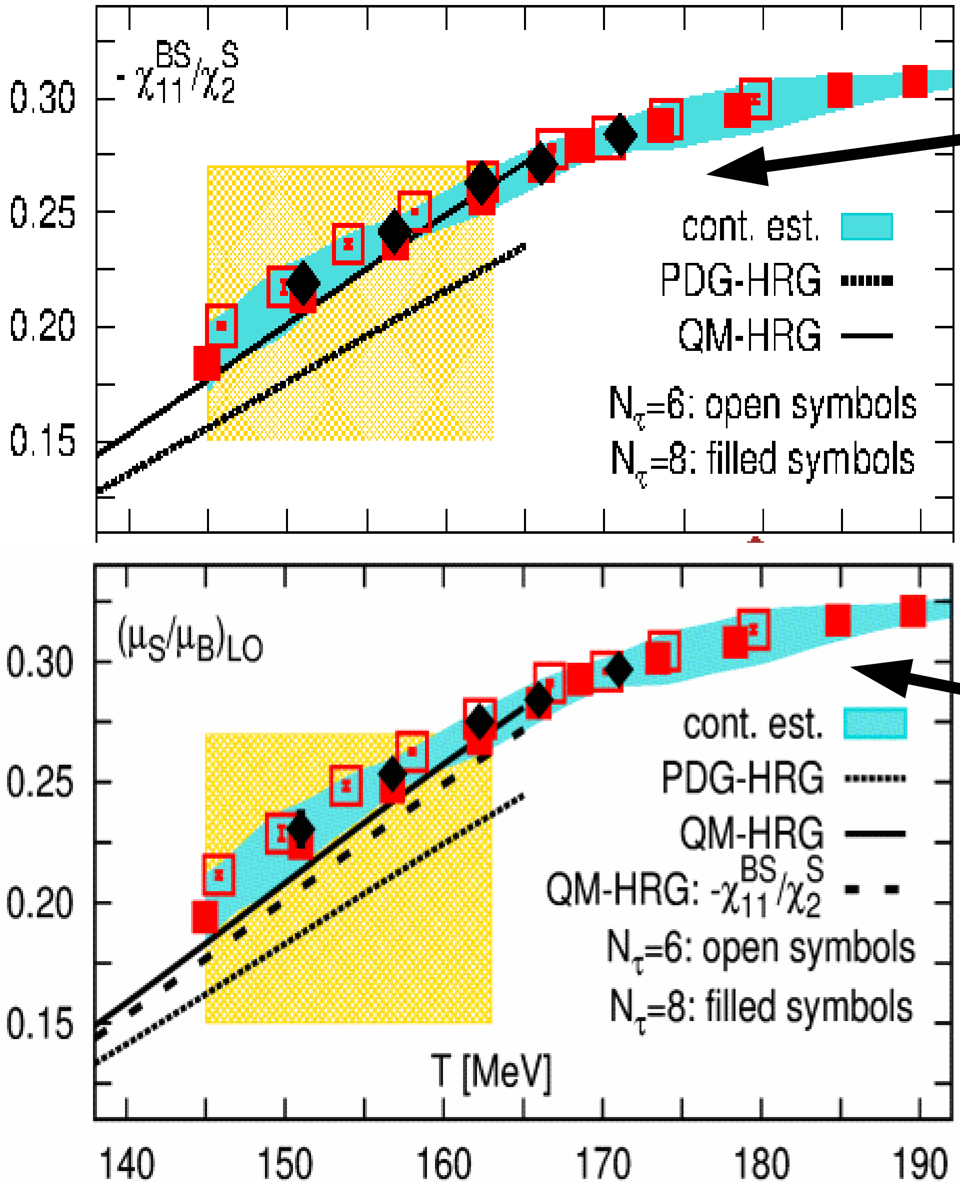
Gedankenexperiment: **ALICE measures** $\chi_{11}^{BS}/\chi_2^S = 0.21$

comparing the measurement with HRG gives: $T_f = 160\ \mathrm{MeV}$

comparing the measurement with QCD gives: $T_f = 150\ \mathrm{MeV}$

➡ **freeze-out param. from QCD**

Strangeness vs. baryon chemical potential
– the influence of additional strange hadrons –



A. Bazavov et al., PRL 113, 072001 (2014), arXiv:1404.6511

enhanced
strangeness-baryon correlation over strangeness fluctuations

strangeness neutrality
enforces relation between chemical potentials

$$\langle n_S \rangle = 0$$
$$= \chi_2^S \hat{\mu}_S^2 + \chi_{11}^{BS} \hat{\mu}_S \hat{\mu}_B + \mathcal{O}(\mu^4)$$

$$\frac{\mu_S}{\mu_B} = -\frac{\chi_{11}^{BS}}{\chi_2^S} + \mathcal{O}(\mu^2)$$

HRG provides good guidance for thermal conditions at freeze-out. However,

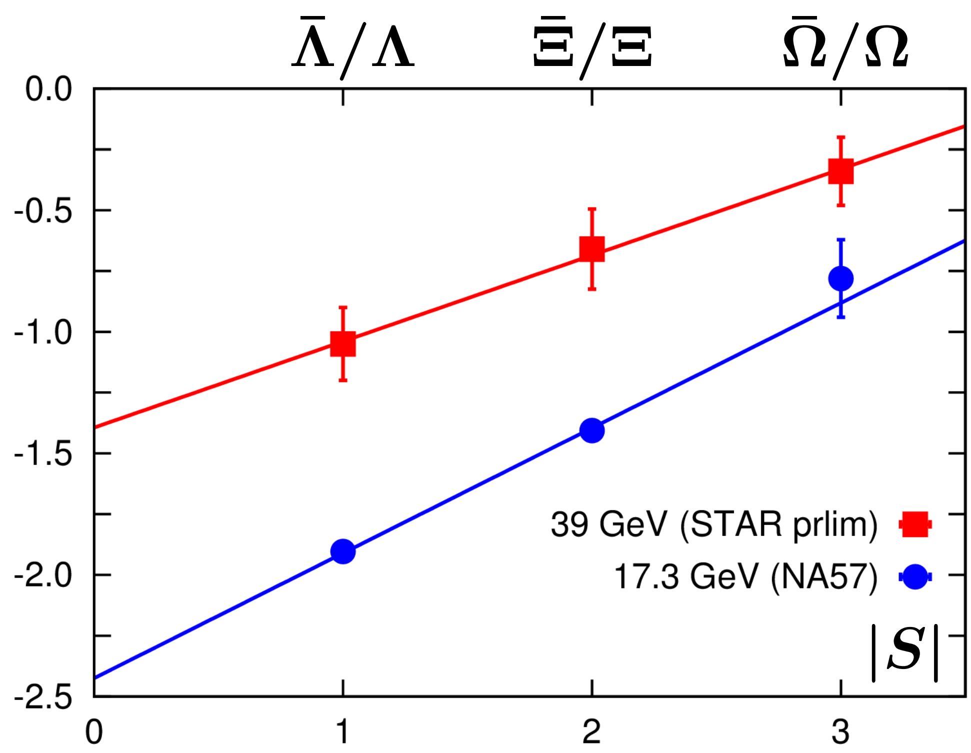
HRG is not QCD

we need a self-consistent determination of freeze-out parameters based on QCD

Strange hadron yields in HIC
Impact on determination of freeze-out parameter

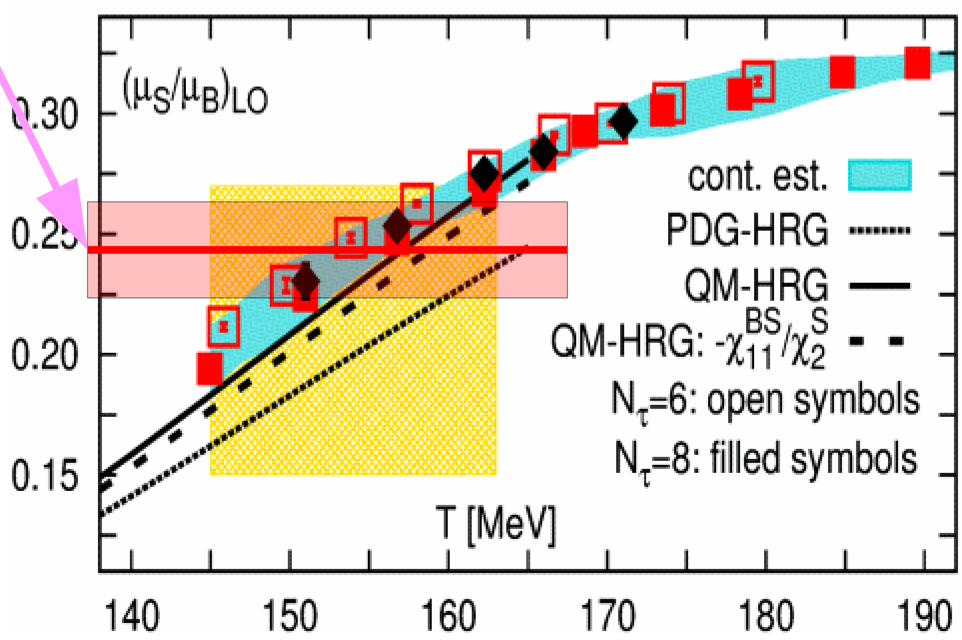
$$\frac{n_{\bar\Lambda}}{n_\Lambda}\ ,\ \frac{n_{\bar\Xi}}{n_\Xi}\ ,\ \frac{n_{\bar\Omega}}{n_\Omega} = \exp\left[-\frac{2\mu_B^f}{T_f} - \frac{2\mu_S^f}{T_f}|S|\right] = \exp\left[-\frac{2\mu_B^f}{T_f}\left(1-\frac{\mu_S^f}{\mu_B^f}|S|\right)\right]$$

STAR preliminary: $\mu_S/\mu_B = 0.241(19)$



no significant dependence on $\sqrt{s_{NN}}$

X. Zhuo (STAR), CPOD 2014

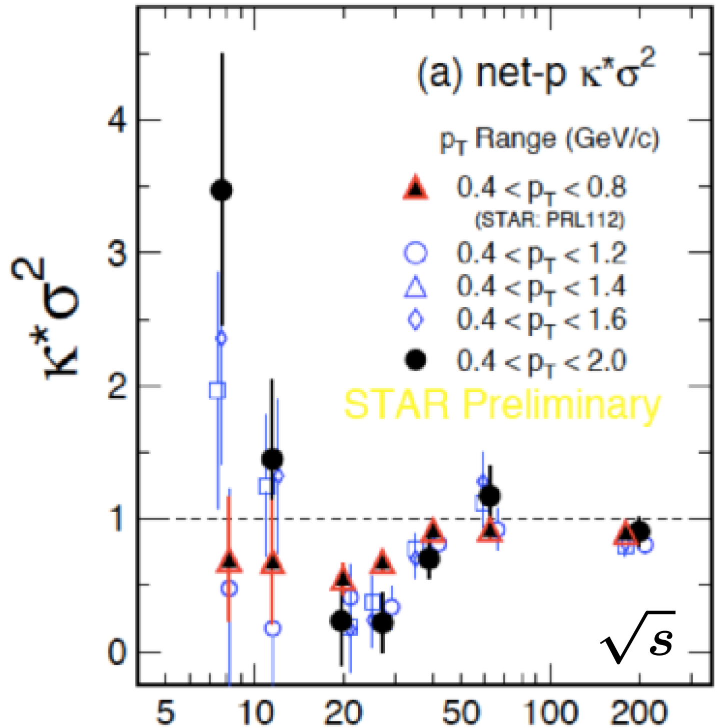


comparing data with QCD (rather than HRG) yields a smaller freeze-out temperature for strange hadrons

Constraining the location of the critical point

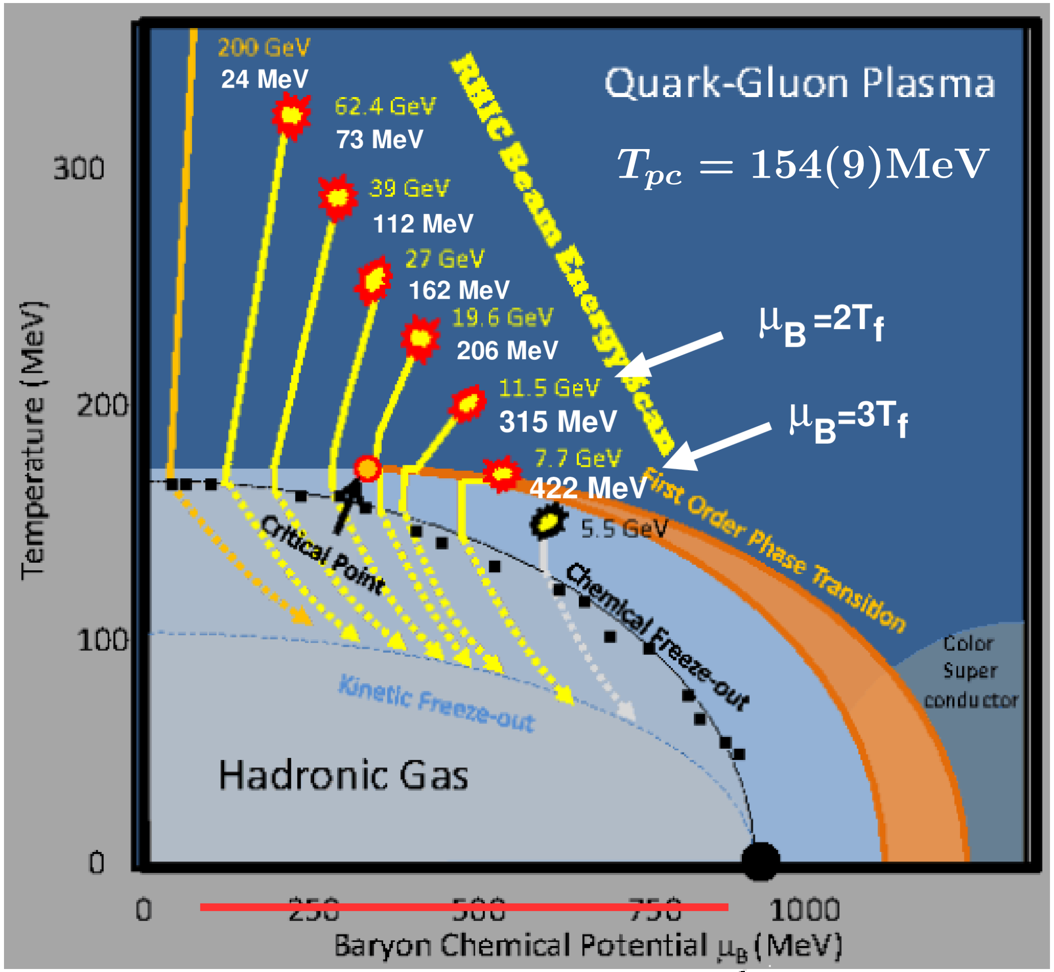
kurtosis*variance:

$$(\kappa\sigma^2)_X = \frac{\chi_4^X}{\chi_2^X}$$



3 2 1.2 0.7 μ_B/T

X. Luo (STAR Collaboration),
PoS CPOD2014 (2014) 019



Where is the
critical point?

Finite-size scaling and cumulants of conserved charge fluctuations

free energy density: $f(T, \mu_B, L) = f_s(t, h, L) + f_r(T, \mu_B, L) \quad , \quad V = L^3$

reduced temperature and external field variables: (t, h)

$$t = \frac{1}{t_0}\left[(T - T_c) + A(\mu - \mu_c)\right]$$

$$h = \frac{1}{h_0}\left[(T - T_c) + B(\mu - \mu_c)\right]$$

singular part of the free energy, RG arguments: $f_s = b^{-3} f(b^{y_t} t, b^{y_h} h, bL^{-1})$

$$b = L \quad \Rightarrow \quad f_s = L^{-3} f(L^{y_t} t, L^{y_h} h, 1)$$

$$y_t = 1/\nu \quad , \quad y_h = \beta\delta/\nu$$

$$y_h > y_t$$

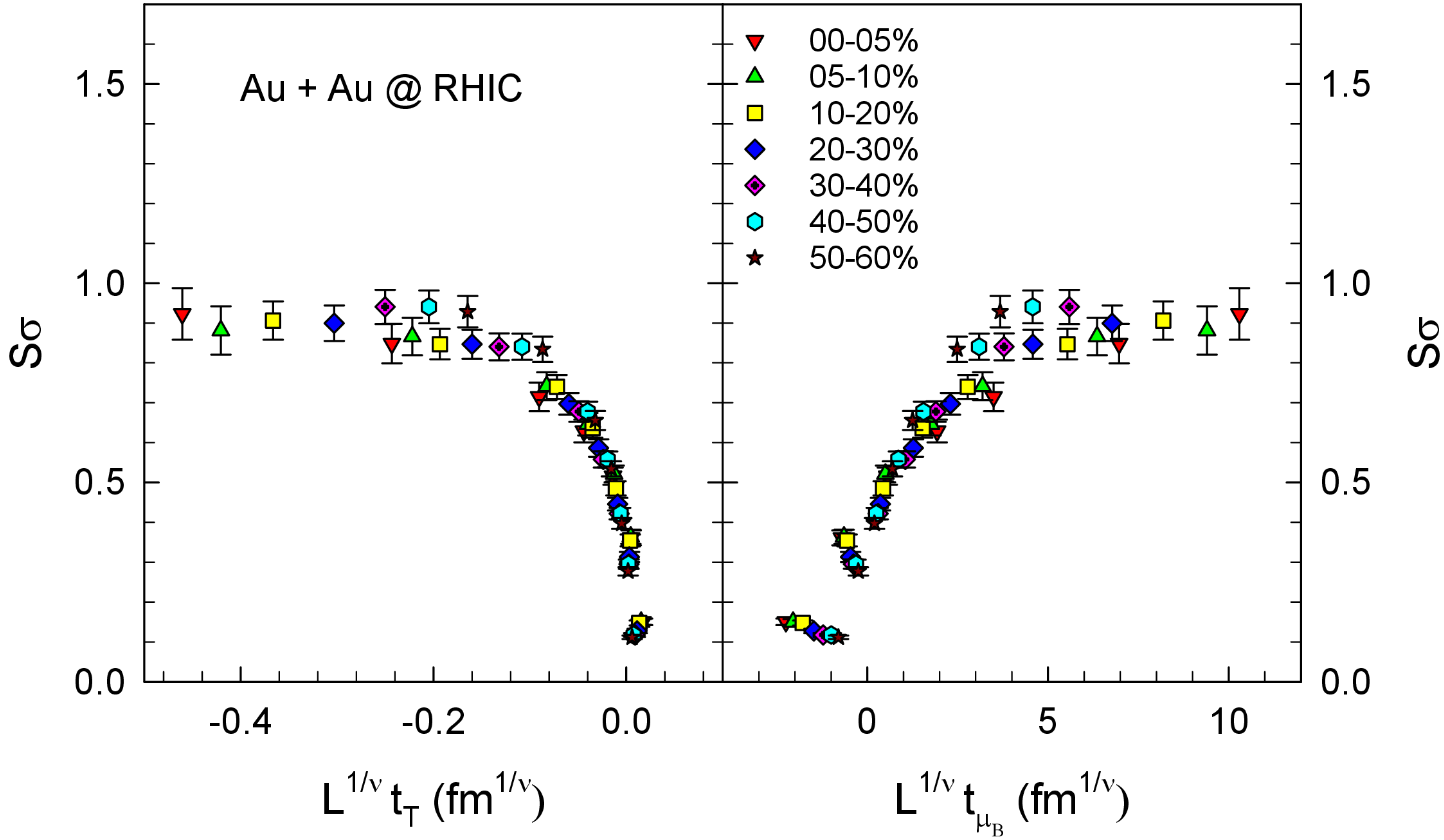
finite size scaling of cumulants

$$\chi_n^B = \frac{\mathrm{d}^n f_s}{\mathrm{d}\mu_B^n} \sim \frac{\partial^n f_s}{\partial h^n} \sim L^{-3+n\delta\beta/\nu} f_s^{(n)}(\ldots)$$

Finite-size scaling and cumulants of conserved charge fluctuations

finite size scaling of ratios of cumulants: $\frac{\chi_n^B}{\chi_m^B} \sim L^{(n-m)\beta\delta/\nu} \bar{f}_s(z_t, z_h, 1)$

$S\sigma \equiv \frac{\chi_3^P}{\chi_2^P}$ does NOT rise with volume (!!) – should rise like $L^{\beta\delta/\nu} = L^{2.48}$



R. Lacey et al., arXiv: 1606.08071

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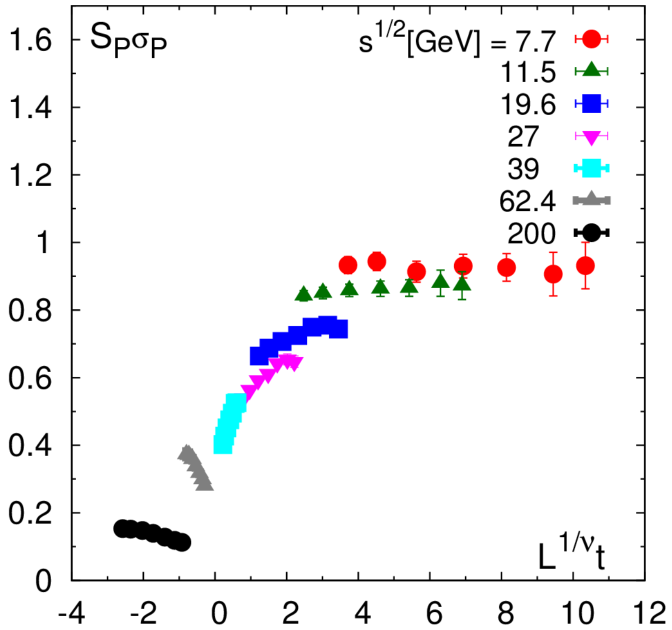
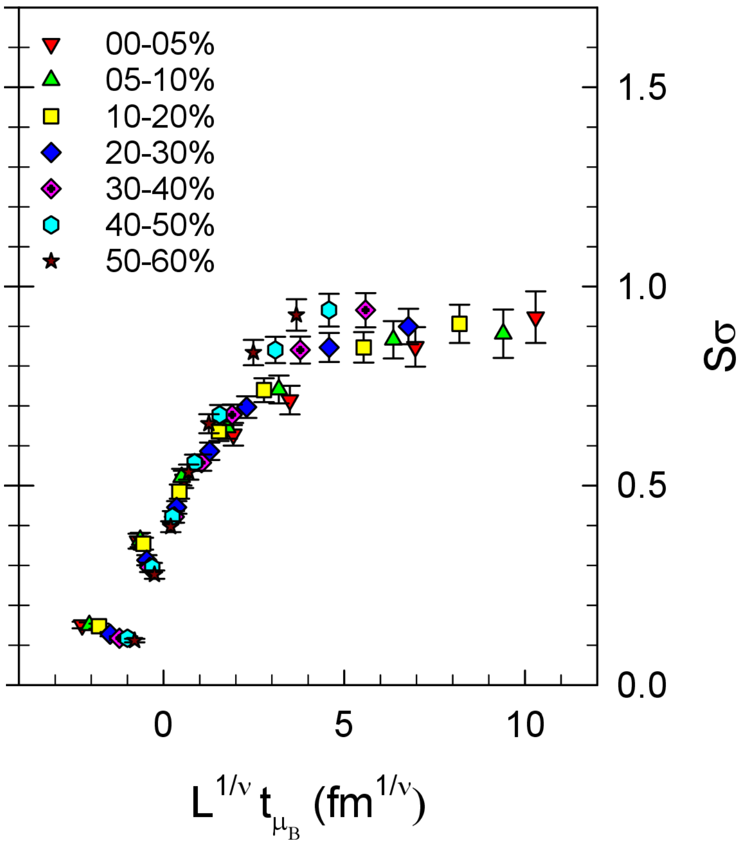
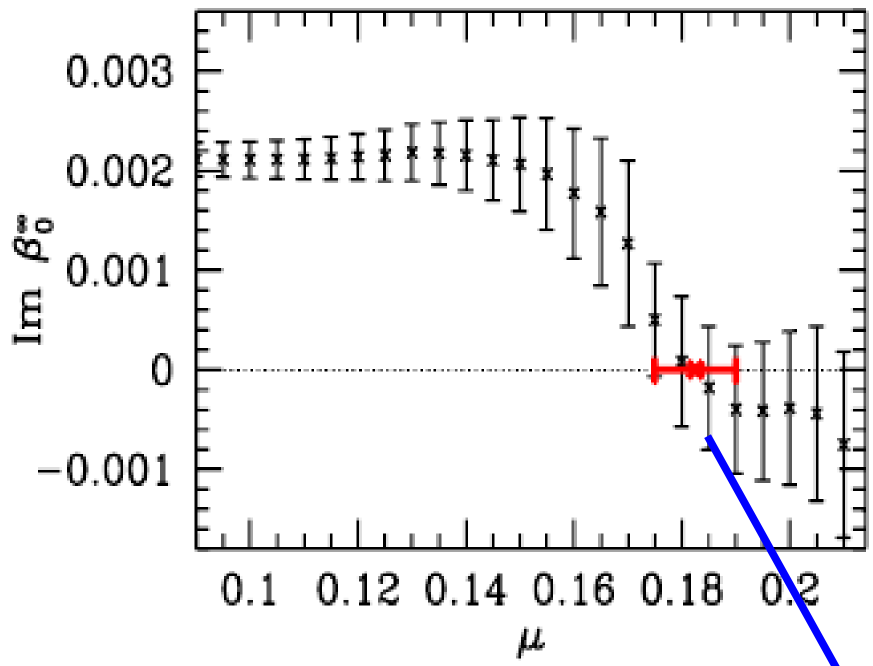


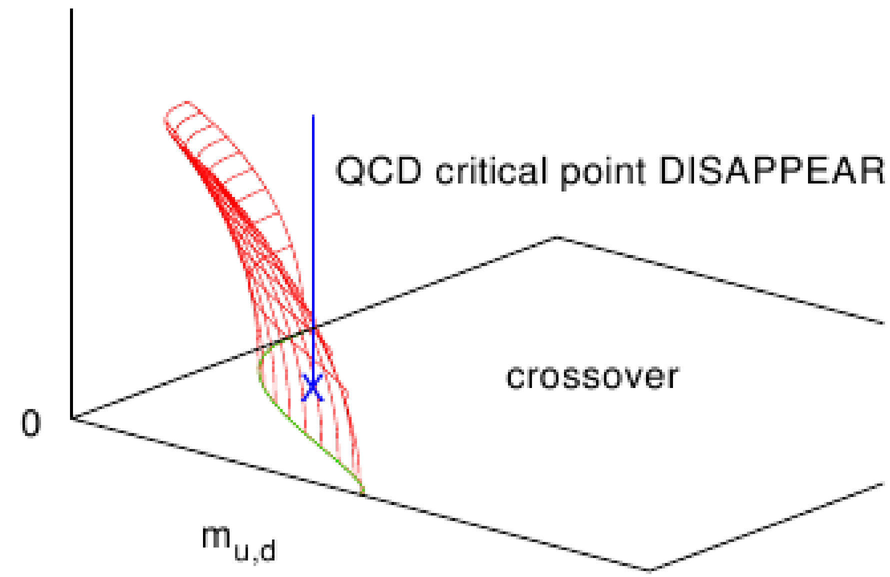


R. Lacey et al., arXiv: 1606.08071

LGT attempts to find the critical point



Z. Fodor, S. Katz. 2001, 2004



P. deForcrand, O. Philipsen, 2002

these calculations were possible because
(I) the lattices were coarse,
(II) the discretization schemes were crude

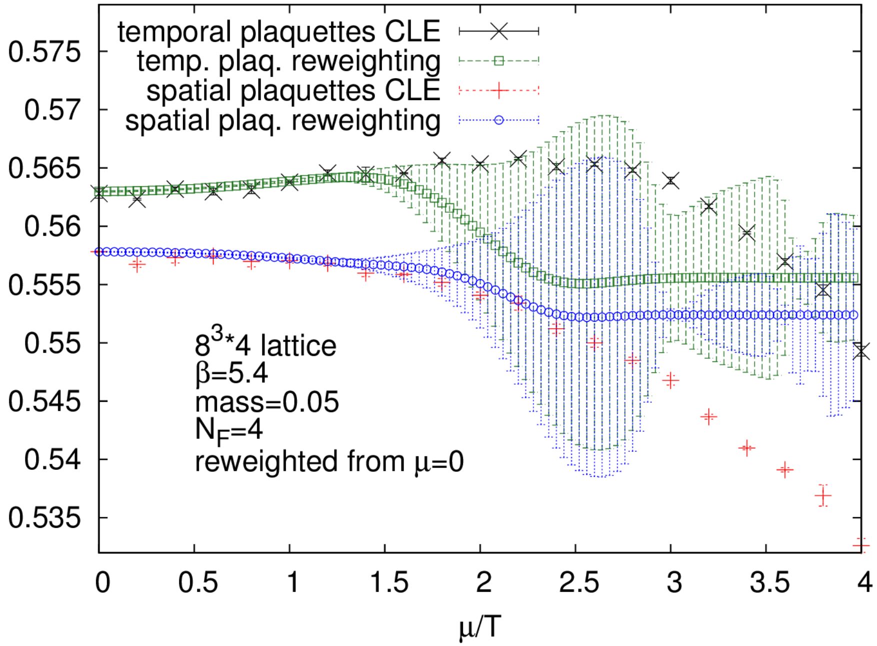
critical point or breakdown of the reweighting approach (loosing the overlap) ?

S. Ejiri, PRD69, 094506 (2004)

since 10 years no progress along this line

Complex Langevin vs. Reweighting

– the silent death of the Fodor/Katz critical point ? –



Z. Fodor, S. Katz. D Sexty, C. Torok, Phys. Rev. D 92 (2015) 094516

from Conclusion:

...reweighting from zero μ breaks down because of the overlap and sign problems around

$$\frac{\mu}{T} = 1 - 1.5$$

i.e.

$$\frac{\mu_B}{T} = 3 - 4.5$$

this should be compared to the first Fodor/Katz critical point estimate on lattices with comparable parameters:

$$\frac{\mu_B^{crit}}{T} = 4.5(3)$$

Z. Fodor, S. Katz. JHEP 0203 (2002) 014

(calculations with physical quark masses eventually lead to a twice smaller estimate for the critical chemical potential)

Taylor expansion of the pressure and critical point

$$\frac{P}{T^4} = \sum_{n=0}^{\infty} \frac{1}{n!} \chi_n^B(T) \left(\frac{\mu_B}{T}\right)^n$$

for simplicity : $\mu_Q = \mu_S = 0$

estimator for the radius of convergence:

$$\left(\frac{\mu_B}{T}\right)^{\chi}_{crit,n} \equiv r_n^{\chi} = \sqrt{\left|\frac{n(n-1)\chi_n^B}{\chi_{n+2}^B}\right|}$$

– radius of convergence corresponds to a critical point only, iff

$$\chi_n > 0 \text{ for all } n \geq n_0$$

forces P/T^4 and $\chi_n^B(T,\mu_B)$ to be monotonically growing with μ_B/T

↕

at T_{CP} : $\kappa_B \sigma_B^2 = \frac{\chi_4^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} > 1$

if not:

– radius of convergence does not determine the critical point

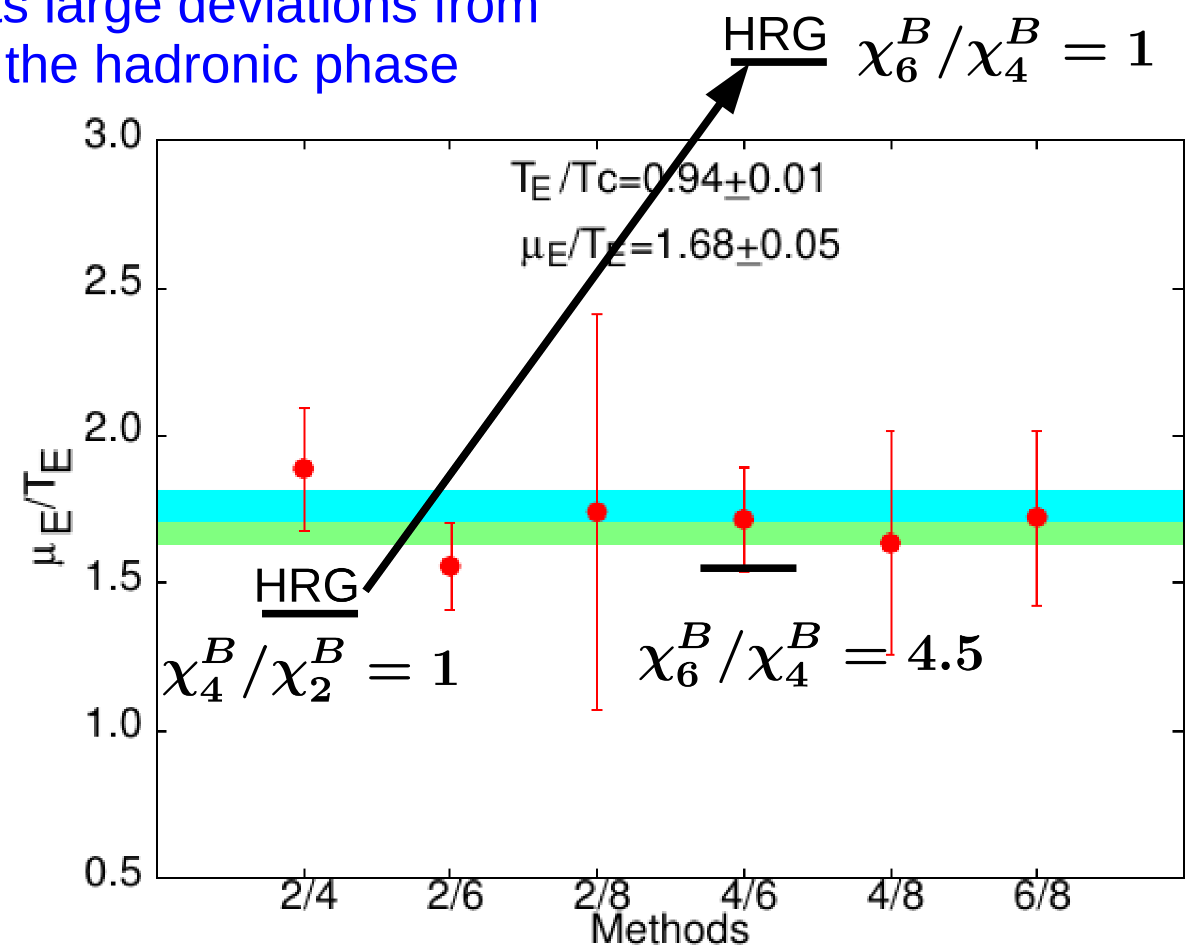
– Taylor expansion can not be used close to the critical point

Estimates of the radius of convergence

a challenging prediction from susceptibility series for standard staggered fermions:

$$\left(\frac{\mu_B}{T}\right)^{\chi}_{crit,n} \equiv r_n^{\chi} = \sqrt{\left|\frac{n(n-1)\chi_n^B}{\chi_{n+2}^B}\right|}$$

suggests large deviations from HRG in the hadronic phase



huge deviations from HRG in 6^{th} order cumulants!

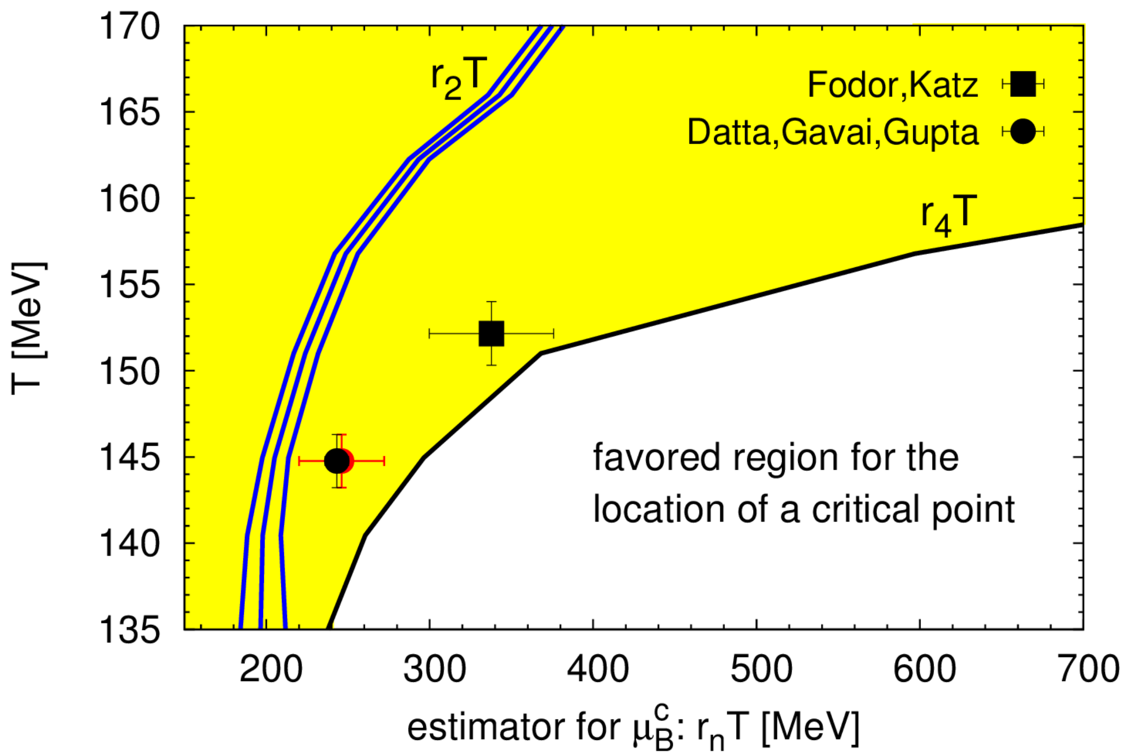
S. Datta et al., PoS Lattice2013 (2014) 202

suggests a critical point for $\mu_B/T < 2$

at present, we cannot rule it out!
however we strongly disfavor it

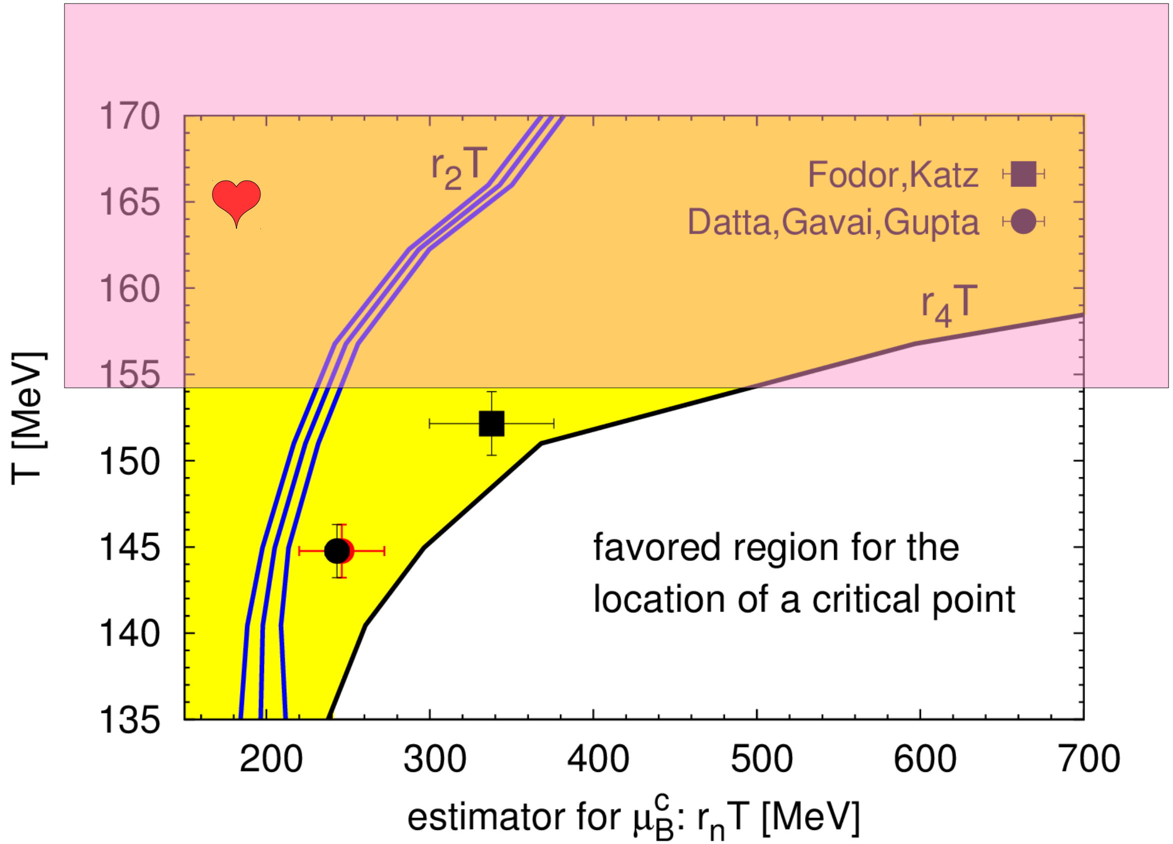
Bielefeld-BNL-CCNU

estimates/constraints on critical point location



05/30/16:
based on ongoing calculations of 6th order Taylor expansion coefficients performed by the Bielefeld-BNL-CCNU collaboration
FK, CPOD 2016

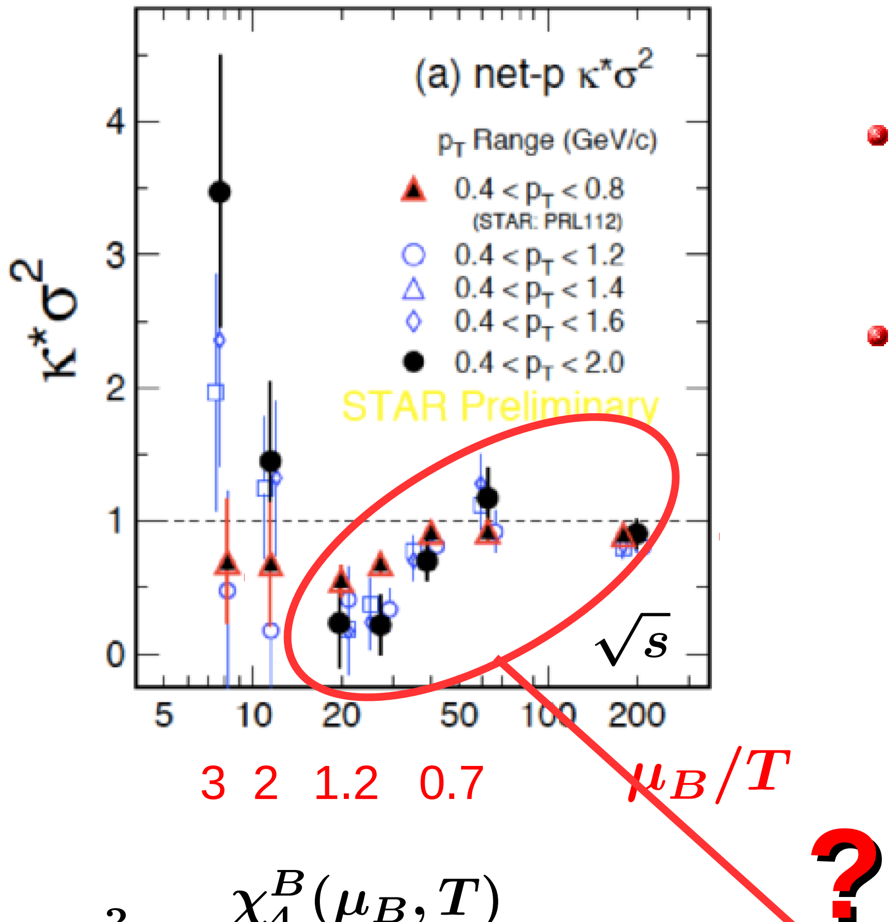
estimates/constraints on critical point location



ruled out from QCD thermodynamics at $\mu_B = 0$

05/30/16:
based on ongoing calculations of 6th order Taylor expansion coefficients performed by the Bielefeld-BNL-CCNU collaboration
FK, CPOD 2016

Exploring the QCD phase diagram



$$\kappa_B \sigma_B^2 = \frac{\chi_4^B(\mu_B, T)}{\chi_2^B(\mu_B, T)}$$

$$= \frac{\chi_4^B}{\chi_2^B} + r_{42}^{B,2}\left(\frac{\mu_B}{T}\right)^2 + \ldots$$

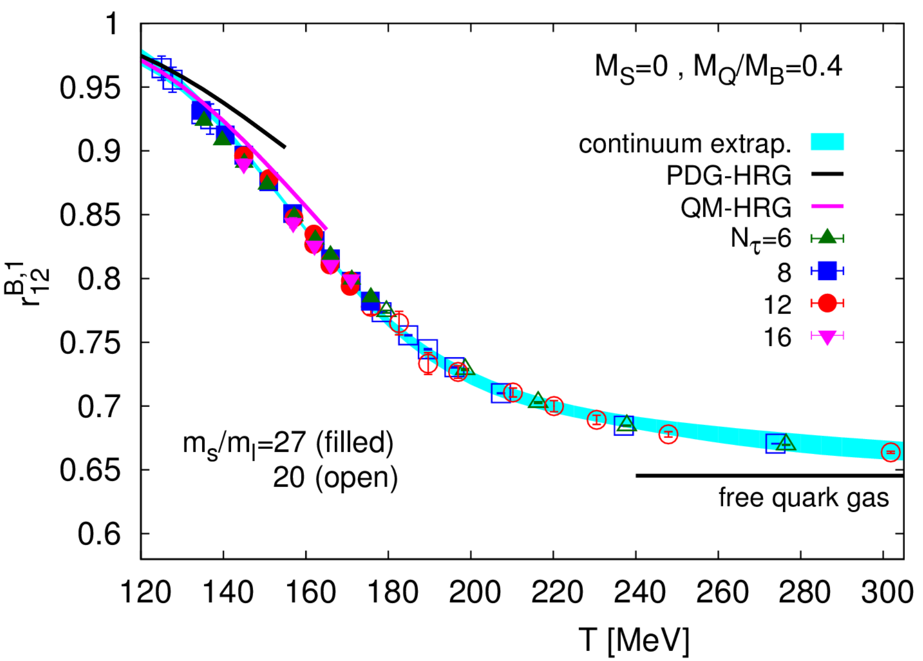
More moderate questions:

- Can we understand the systematics seen in cumulants of charge fluctuations in terms of QCD thermodynamics ?
- How far do we get with low order Taylor expansions of **QCD** in explaining the obvious deviations from HRG model behavior ?

- For $\sqrt{s} \geq 20$ GeV :
  Structure of (net-electric charge and) net-proton cumulants is inconsistent with HRG thermodynamics, but can eventually be understood in terms of **QCD thermodynamics in a next-to-leading order Taylor expansion**

Mean baryon number over variance of baryon number fluctuation

$$R_{12}^B(T,\mu_B) \equiv \frac{M_B}{\sigma_B^2} = \frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = r_{12}^{B,1}\hat{\mu}_B + \mathcal{O}(\mu_B^3)$$



$$r_{12}^{B,1} = \frac{\chi_2^B + s_1\chi_{11}^{BS} + q_1\chi_{11}^{BQ}}{\chi_2^B}$$

fixed through

$$\mu_S = s_1\mu_B + \mathcal{O}(\mu_B^3) \qquad M_S = 0$$

$$\mu_Q = q_1\mu_B + \mathcal{O}(\mu_B^3) \qquad \frac{M_Q}{M_B} = 0.4$$

$$\frac{\mu_B}{T} = \frac{1}{r_{12}^{B,1}} R_{12}^B + \mathcal{O}((R_{12}^B)^3)$$

$$R_{31}^B(T,\mu_B) \equiv \frac{S_B\sigma_B^3}{M_B} = \frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)} = r_{31}^{B,0} + r_{31}^{B,2}\hat{\mu}_B^2 + \ldots$$

eliminate μ_B

$$= r_{31}^{B,0} + \frac{r_{31}^{B,2}}{\left(r_{12}^{B,1}\right)^2}\left(R_{12}^B\right)^2$$

Conserved charge fluctuations and freeze-out
mean, variance and skewness

NLO Taylor expansion

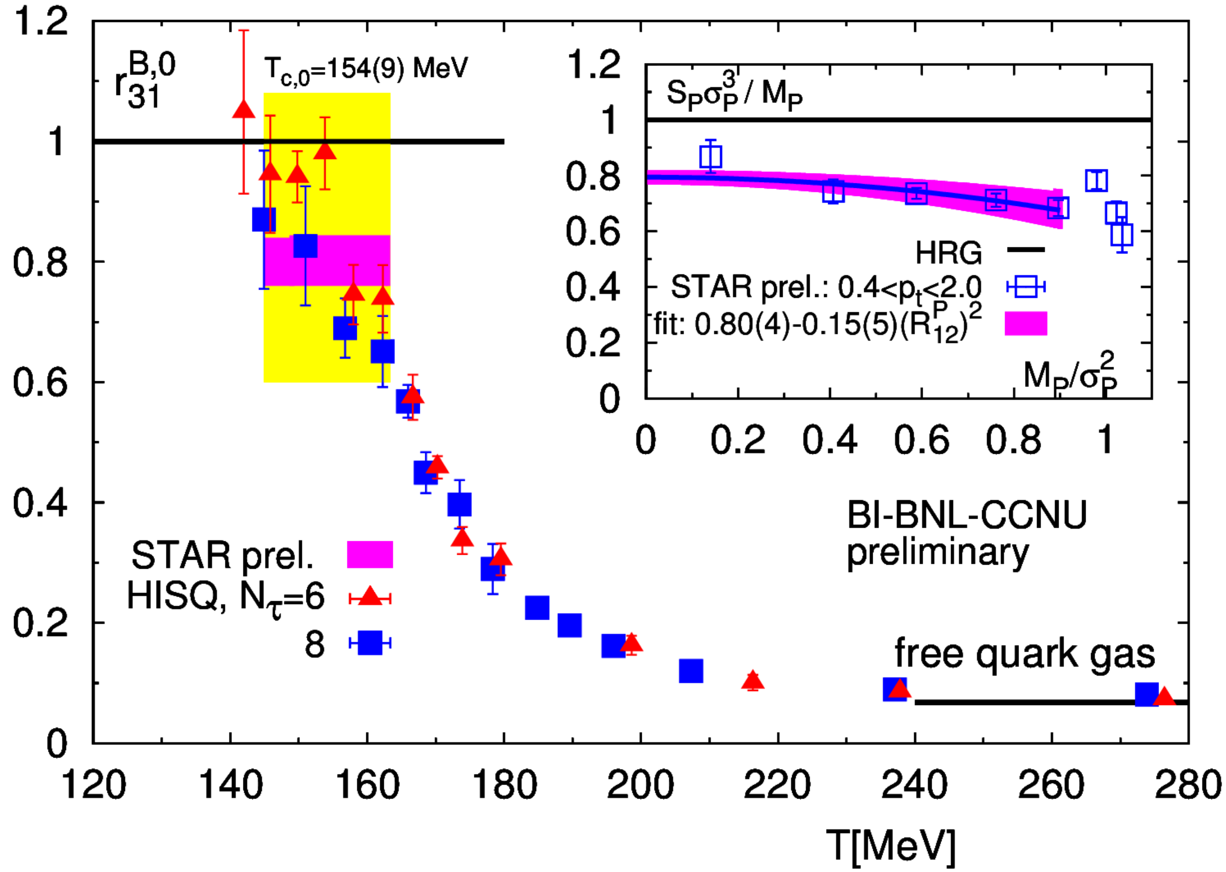
$$S_B\sigma_B = \frac{\chi_4^B}{\chi_2^B}\frac{M_B}{\sigma_B^2} + \frac{1}{6}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^3 + \ldots \qquad \mu_Q = \mu_S = 0$$

$$\Leftrightarrow \quad R_{31}^B \equiv \frac{S_B\sigma_B^3}{M_B} = r_{31}^{B,0} + r_{31}^{B,2}\left(\frac{M_B}{\sigma_B^2}\right)^2$$

e.g., $\mu_Q \neq \mu_S \neq 0$:

$$r_{31}^{B,0} = \frac{\chi_4^B + \frac{\mu_S}{\mu_B}\chi_{31}^{BS} + \frac{\mu_Q}{\mu_B}\chi_{31}^{BQ}}{\chi_2^B + \frac{\mu_S}{\mu_B}\chi_{11}^{BS} + \frac{\mu_Q}{\mu_B}\chi_{11}^{BQ}}$$

$$r_{31}^{B,2} = \ldots$$



F. Karsch et al., arXiv:1512.06987

Conserved charge fluctuations and freeze-out
mean, variance, skewness and kurtosis

in a NLO Taylor expansion $R_{31}^B \equiv S_B\sigma_B^3/M_B$, $R_{42}^B \equiv \kappa_B\sigma_B^2$ } **are closely related**

F. Karsch et al., arXiv:1512.06987

$$R_{31}^B = r_{31}^{B,0} + r_{31}^{B,2}\left(\frac{\mu_B}{T}\right)^2$$

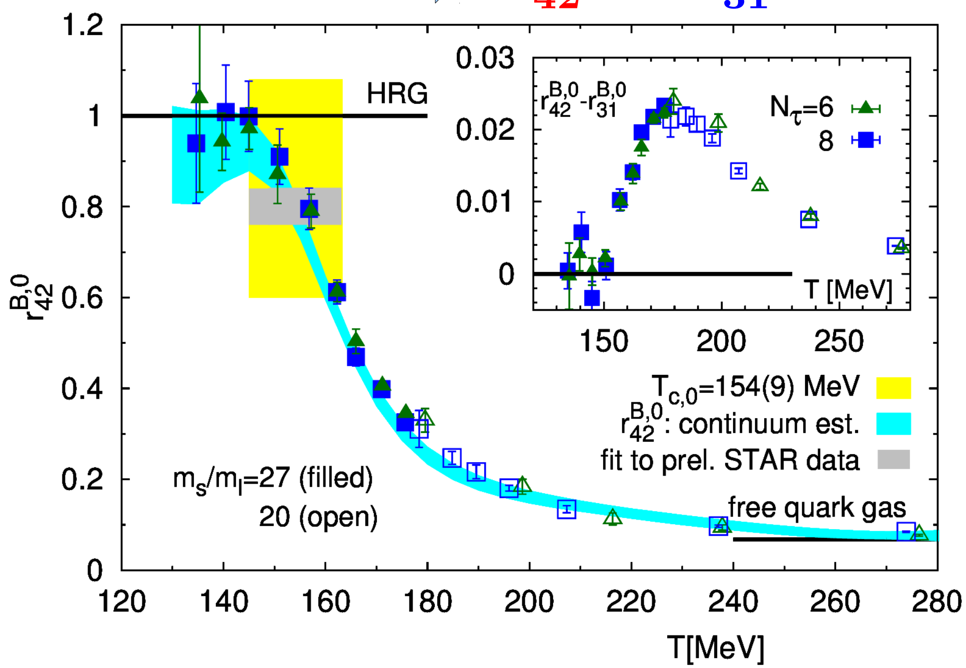
$$R_{42}^B = r_{42}^{B,0} + r_{42}^{B,2}\left(\frac{\mu_B}{T}\right)^2$$

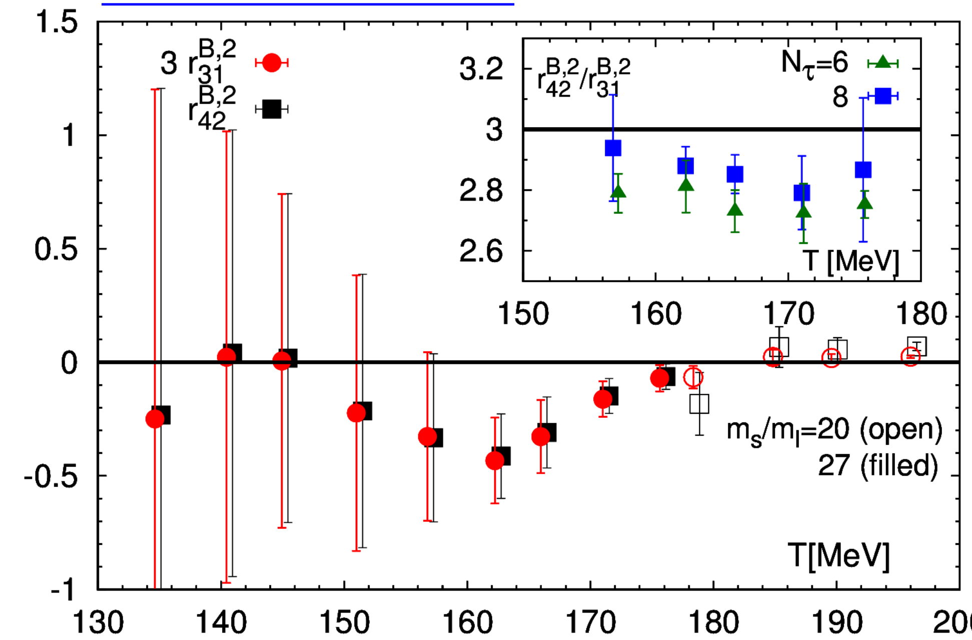
$\Rightarrow$ $r_{42}^{B,0} \simeq r_{31}^{B,0}$

$\mu_S = \mu_Q = 0:$

$$r_{42}^{B,2} = 3r_{31}^{B,2} = \frac{1}{2}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)$$

$\mu_S \neq \mu_Q \neq 0:$





lines of constant physics

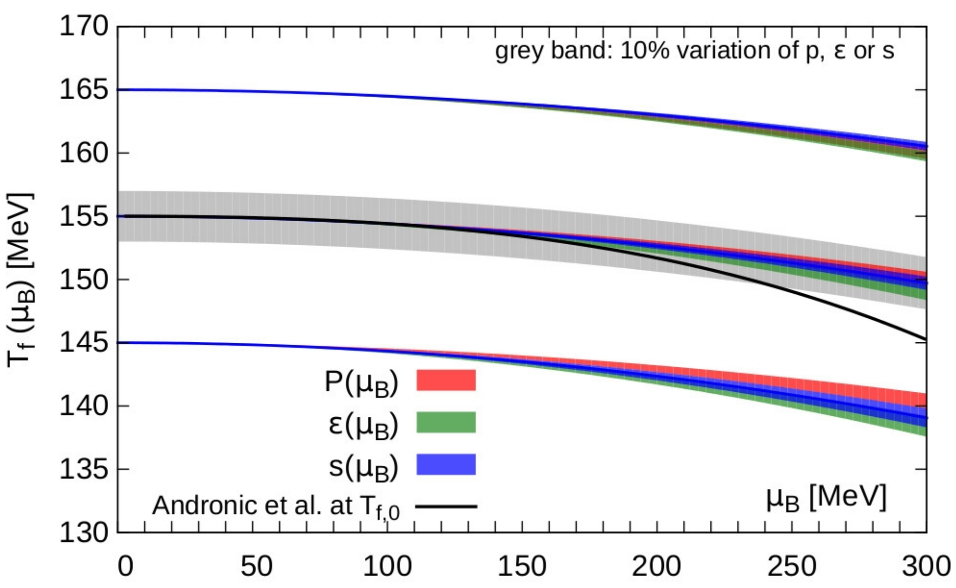
… one more complication in the comparison between QCD and data

$$R_{31}^B(T,\mu_B) \equiv \frac{S_B\sigma_B^3}{M_B} = \frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)} = r_{31}^{B,0} + r_{31}^{B,2}\hat{\mu}_B^2 + \ldots$$

freeze-out temperature T_f varies as μ_B increases

– Taylor expansion in $T_f(\mu_B) - T_f(\mu_B = 0)$

– parametrization of $T_f(\mu_B) = T_f(\mu_B = 0)(1 - \kappa_2^f \hat{\mu}_B^2)$



$$T_f(\mu_B) - T_f(\mu_B = 0) = -\kappa_2^f \hat{\mu}_B^2$$

yields additional contribution to $O(\hat{\mu}_B^2)$ expansion coefficient

$$r_{31}^{B,2} \rightarrow r_{31}^{B,2} - \kappa_2^f T_f(0) \left. \frac{\mathrm{d}r_{31}^{B,0}}{\mathrm{d}T} \right|_{T_f(0)}$$

lines of constant physics

… one more complication in the comparison between QCD and data

$$R_{31}^B(T,\mu_B) \equiv \frac{S_B\sigma_B^3}{M_B} = \frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)} = r_{31}^{B,0} + r_{31}^{B,2}\hat{\mu}_B^2 + \ldots$$

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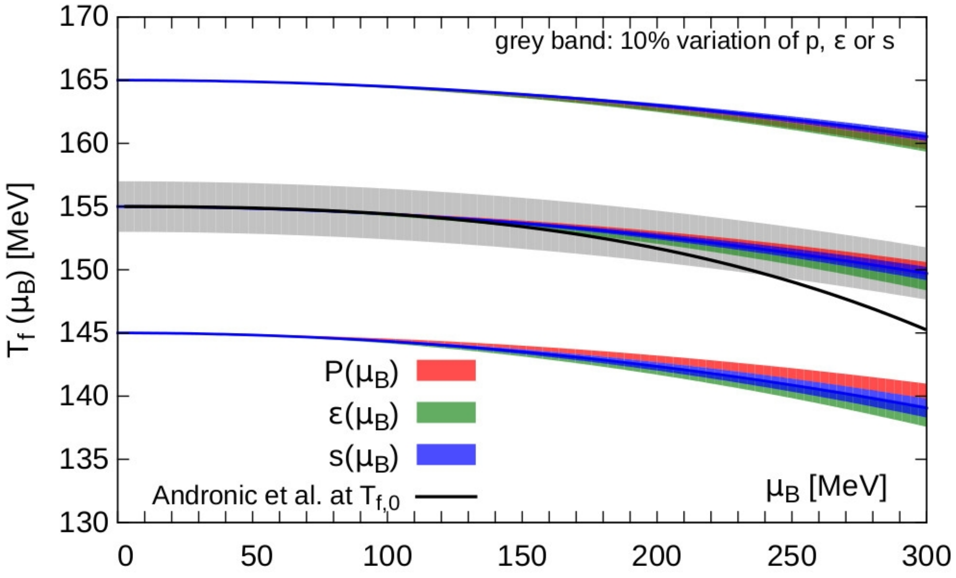
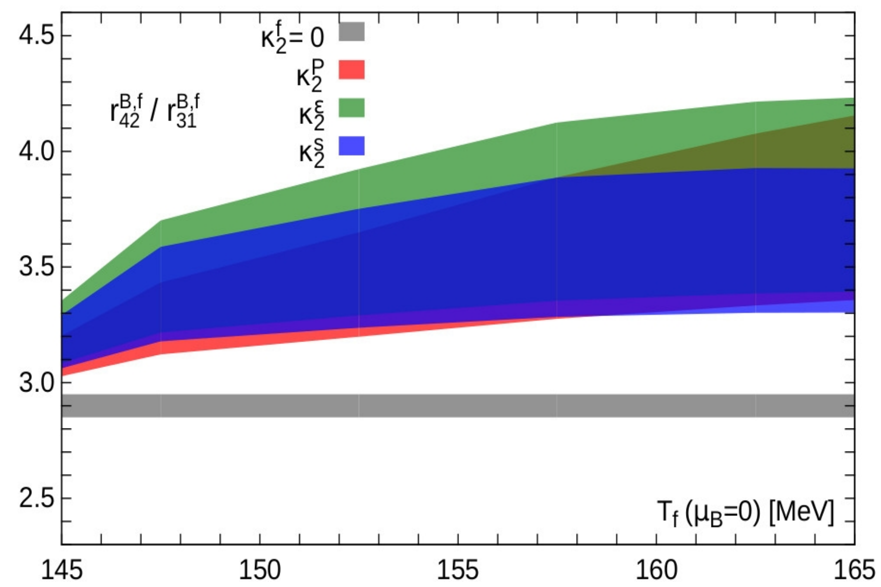
– Taylor expansion in $T_f(\mu_B) - T_f(\mu_B = 0)$

– parametrization of $T_f(\mu_B) = T_f(\mu_B = 0)(1 - \kappa_2^f \hat{\mu}_B^2)$

$r_{31}^{B,2} \rightarrow r_{31}^{B,f}$

$r_{42}^{B,2} \rightarrow r_{42}^{B,f}$





Conserved charge fluctuations and freeze-out
mean, variance, skewness and kurtosis

in a NLO Taylor expansion $R_{31}^{B} \equiv S_B\sigma_B^3/M_B$ and $R_{42}^{B} \equiv \kappa_B\sigma_B^2$ } **are closely related**

F. Karsch et al.,
arXiv:1512.06987

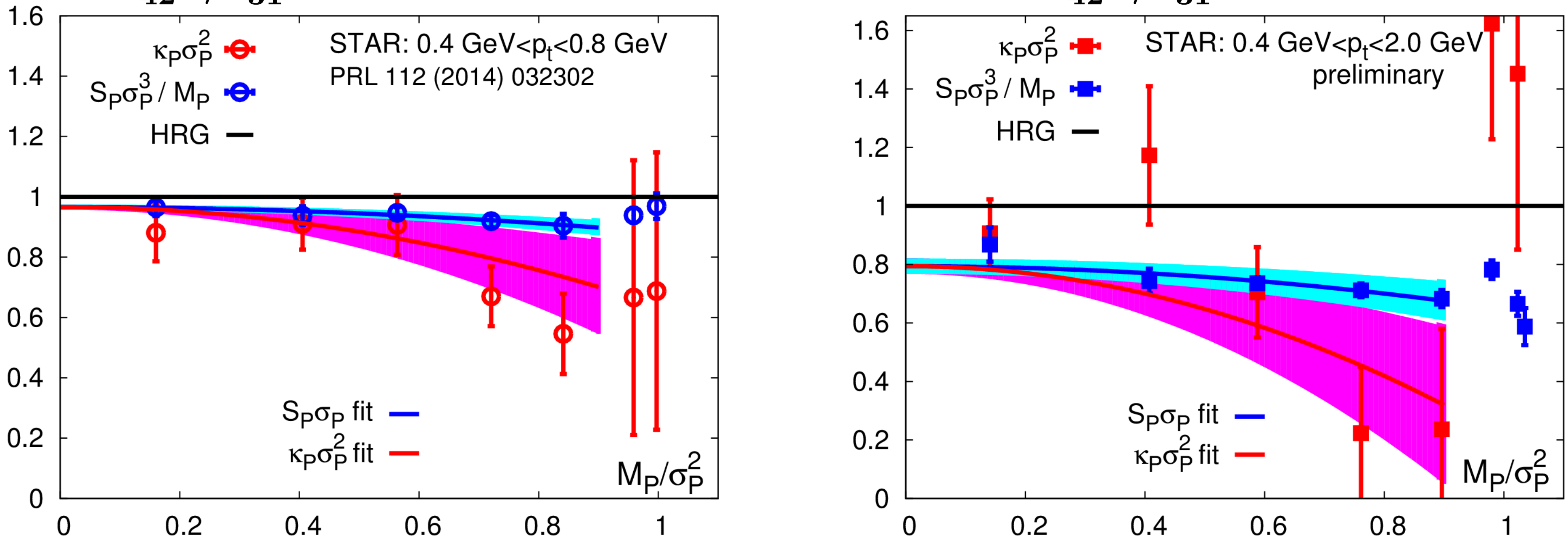
QCD thermodynamics gives:

$$I)\ \mu_B \to 0 \quad \Rightarrow \quad \kappa_B\sigma_B^2 \simeq \frac{S_B\sigma_B^3}{M_B}$$

$II)\ \mu_B > 0 \quad \Rightarrow$ slope of $\kappa_B\sigma_B^2$ is (3-4) times smaller and negative

fit to (prelim) STAR data:

$r_{42}^{B,f}/r_{31}^{B,f} = 3.9 \pm 2.1$ $\qquad$ $r_{42}^{B,f}/r_{31}^{B,f} = 4.0 \pm 2.1$



Conserved charge fluctuations and freeze-out
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in a NLO Taylor expansion $R_{31}^B \equiv S_B\sigma_B^3/M_B$, $R_{42}^B \equiv \kappa_B\sigma_B^2$ **are closely related**

F. Karsch et al., arXiv:1512.06987

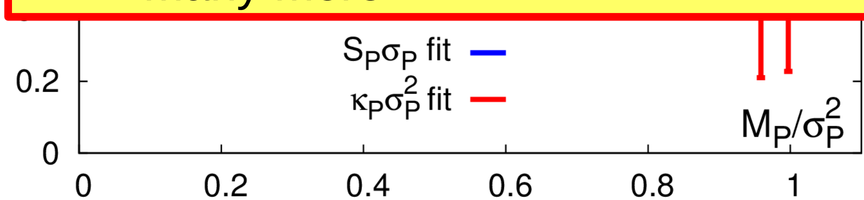
QCD thermodynamics gives: $I)\ \mu_B \rightarrow 0 \ \Rightarrow\ \kappa_B\sigma_B^2 \simeq \dfrac{S_B\sigma_B^3}{M_B}$

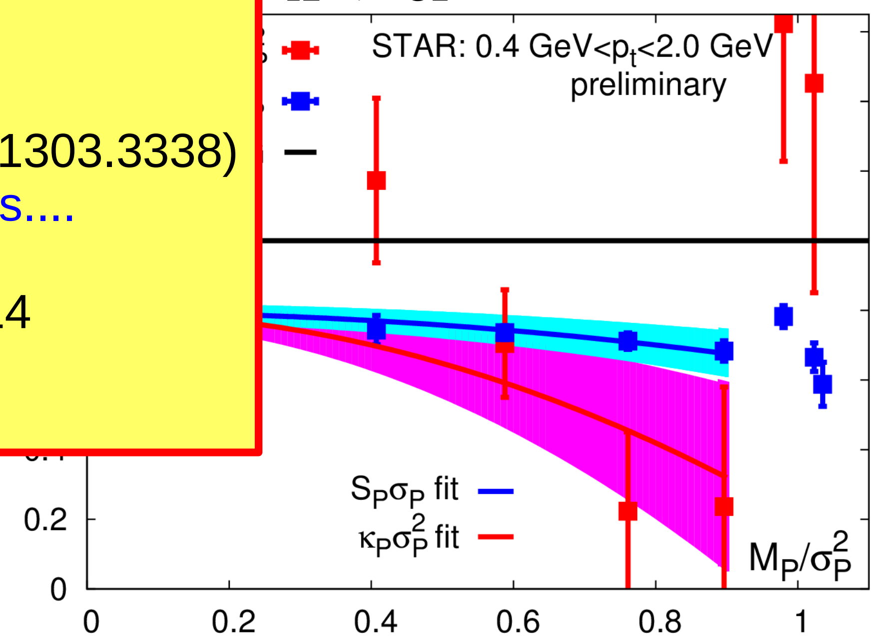
beyond equilibrium thermodynamics: $II)\ \mu_B > 0 \ \Rightarrow$ slope of $\kappa_B\sigma_B^2$ is (3-4) times smaller and negative

need to understand systematics :

– non-equilibrium effects
(S. Mukherjee et al., arXiv:1506.00645)
– proton vs. baryon number distributions
(M. Kitazawa et al, arXiv:1205.3292, arXiv:1303.3338)
– acceptance and pt-cuts, volume fluctuations....
(P. Garg et al, arXiv:1304.7133,
FK, K. Morita, K. Redlich, arXiv:1508.02614
A. Bzdak and V. Koch, arXiv:12064286)
+ many more

$r_{42}^{B,f}/r_{31}^{B,f} = 4.0 \pm 2.1$





Conclusions

– results on bulk thermodynamics coming from Taylor expansion of the QCD partition function are already now reliable in the range $0 \leq \mu_B/T \leq 2$

bulk QCD thermodynamics in the entire parameter range accessible to BES I and II may soon be accessible also through Taylor expansions

– attempts to understand freeze-out/hadronization in terms of HRG model based calculations at temperatures T > 160 MeV are difficult to conciliate with QCD;

QCD thermodynamics is quite different from HRG thermodynamics at T > 160 MeV

– properties of cumulants measured in BES-I for $\sqrt{s_{NN}} \leq 20\ \mathrm{GeV}$ clearly differ from HRG thermodynamics but are consistent with QCD thermodynamics close to the crossover transition temperature

$$S_B\sigma_B < M_B/\sigma_B^2 \ \ , \ \ \kappa_B\sigma_B^2 - S_B\sigma \sim \left(M_B/\sigma_B^2\right)^2$$

– with increasing statistical accuracy current LGT calculations seem to favor estimates for the location of the critical point (if it exists) at values of $\mu_B/T > 2$