A simple framework for multiplicity fluctuations near the QCD critical point

Maurício Hippert   Eduardo S. Fraga

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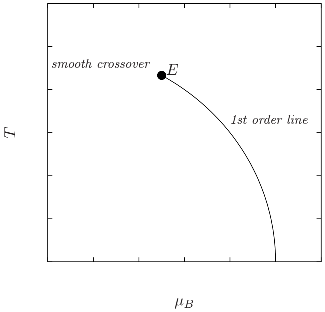
Instituto de Física
Universidade Federal do Rio de Janeiro

Outline

1. Quick motivation
2. Our treatment
    - Simulating fluctuations
    - Analytical calculations
3. Introducing limitations
    - Resonance decay effects
4. Further developments
    - Proton fluctuations/signatures
    - Non-Gaussian fluctuations/signatures
5. Final remarks

The QCD critical point

Is the QCD critical point there? Where is it? What are its properties?



Search in heavy ion collisions (BES):

- Freeze-out near the CEP.
- Non-monotonic behavior?
- Scaling?
- What could we see that would convince us?

Finding the CEP

Optimistic expectations

- "$\xi \sim t^{-\nu} \to \infty$": long range fluctuations (pions, protons ...).
- Fluctuation measures, e.g. $\langle(\Delta N)^4\rangle$, grow as ξ^{power}.
- Higher-order cumulants $\to$ stronger dependence in ξ.
- Clean universal scaling behavior (finite-size).

A little realism

HICs are a difficult environment.

- Finite size/duration: $\xi \nrightarrow \infty$, dynamical effects...
- Complicated physics, hard to control.
- Background contributions.

Tasks

- Need to test basic realistic ingredients! Do signatures “survive”?
- What is the role of different limitations in different signatures?
- Include fluctuations, finite size effects, resonance decay, acceptance cuts, error bars... (previous works)
- Study dependence in centrality and $\sqrt{s}$.

Mission

Construct a simple yet somewhat general framework for studying different effects/contributions.

Focus: Fluctuations of particle multiplicities.

MH, Fraga, Santos, PRD 93 (2016),
MH, Fraga, *in preparation*.

Outline

Mean field approximation

Local fluctuations

$$\Omega[\sigma] = \int d^3x \, \left\{ \frac{(\nabla\sigma)^2}{2} + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \cdots \right\} , \qquad (1)$$

$$m_\sigma = \xi^{-1} , \qquad \text{Ising: } \lambda_3 = \tilde{\lambda}_3 \, T \, (T\,\xi)^{-3/2} , \;\; \lambda_4 = \tilde{\lambda}_4 \, (T\,\xi)^{-1} . \qquad (2)$$

Assumptions

- Long-range fluctuations dominate: $\sigma_0 = \int d^3x \, \sigma(x)$.
  Integration over local fluctuations $\rightarrow \Omega_*(\sigma_0)$.
- Near CEP but Landau theory still ok/reasonable.
- For now, $\lambda_3 = 0 \, , \; \lambda_4 = 0$.
- Finite system.

Tsypin, PRB 55 (1996),
Stephanov, PRL 102 (2009).

Mean field approximation

Within assumptions, theory for σ_0, $\Omega[\sigma] \Rightarrow \Omega_*(\sigma_0)$.

Long range fluctuations

$$\mathcal{P}(\sigma_0) \propto \exp\left(\frac{-\Omega_*(\sigma_0)}{T}\right) , \tag{3}$$

$$\Omega_*(\sigma_0) = V\left(\frac{m_\sigma^2}{2}\sigma_0^2 + \frac{\lambda_3^*}{3}\sigma_0^3 + \frac{\lambda_4^*}{4}\sigma_0^4 + \cdots\right) . \tag{4}$$

Larger ξ $\rightarrow$ broader distribution.
ξ will be an <u>input</u>.

Framework

Interaction

- Mass correction (LO in σ_0)

$$\mathcal{L} \approx -G\,\sigma_0\,\vec{\pi}\cdot\vec{\pi} - g\,\sigma_0\,\bar{\psi}_p\psi_p \qquad (G \approx 300\text{MeV},\ g \approx 10?). \tag{5}$$

- $\delta\sigma_0 \to \delta m_\pi, \delta m_p \to$ fluctuations of observables.

Stephanov, Rajagopal, Shuryak, PRD 60 (1999).

Finite size / discrete modes

- Boundary conditions: $\vec{\pi}(R) = 0$, $\psi_p(R) = 0$.
- Momentum $p_i^\ell = \alpha_i^\ell / R$, $j_l(\alpha_i^\ell) = 0$.
- $2\ell + 1$ degeneracy.

Framework for Monte Carlo and analytic results.

Monte Carlo

Fluctuations can be simulated using Monte Carlo methods.

Algorithm:

1. Draw parameters from $\mathcal{P}(T)$, $\mathcal{P}(R)$ etc (spurious fluctuations).
2. Draw σ_0, m^2 from $\mathcal{P}(\sigma_0)$ (critical fluctuations).
3. Draw occupation numbers from Boltzmann factor $e^{-\beta\,(\omega_p-\mu)\,n_p}$ ("grand canonical" fluctuations).

- Different $\sigma_0, T, R, \ldots$ for each event $\rightarrow$ correlation.
- Event statistics $\rightarrow$ $\langle N\rangle$, $\langle \Delta N\,\Delta N\rangle$, ...
- Simplicity $\rightarrow$ large number of events.

Background can be systematically added!

Analytic expressions

Calculations by series and averages over fluctuations are possible.

Effective energy level fluctuations

- Critical: $\omega_0 + \delta\omega_\sigma = \sqrt{p^2 + m^2 + \delta m^2(\sigma_0)}$.
- System size: $p_i^\ell + \delta p_i^\ell = \alpha_i^\ell/(R + \delta R)$.
- T and μ fluctuations: $\dfrac{\omega + \delta\omega_{T,\mu} - \mu}{T} = \dfrac{\omega - (\mu + \delta\mu)}{T + \delta T}$.

Taylor expanding in $\delta\omega_i^\ell$ and averaging $\rightarrow$ *general formulae*,

$$\overline{\langle\cdots\rangle} \approx \langle(\cdots)_0\rangle + \sum_\omega \langle(\cdots)_\omega\rangle\,\overline{\delta\omega} + \sum_{\omega,\omega'}\langle(\cdots)_{\omega,\omega'}\rangle\,\overline{\delta\omega\delta\,\omega'} + \ldots\,, \qquad (6)$$

which is a function of $\overline{\delta\sigma_0^2}, \overline{\delta T^2}, \overline{\delta R^2}$ and further moments.

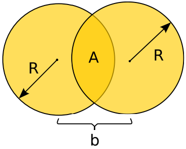
Outline

Spurious fluctuations

In HICs, thermodynamic parameters are not fully controlled. Model for spurious fluctuations needed:

1. Gaussian temperature fluctuations ($\sigma_T/T = 5\%$)
2. Geometrical fluctuations (below)

Geometric fluctuations

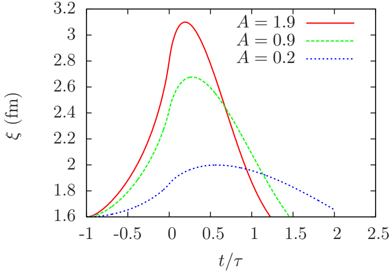


- Impact parameter distribution $\Rightarrow$ Overlap area.
- Assumption $V(b) = C(\sqrt{s})\ A(b)$.
- Fix $R_p = 6.8$ fm for $0 - 5\%$ centrality.
- Fluctuations of C missing!

Limiting ξ

Critical Slowing Down

- Non-equilibrium effects $\rightarrow \xi \nrightarrow \infty$.
- Evolution inspired by dynamical universality class.
- Parameters limited by cooling timescale, initial value and causality.
- Optimistically, cooling over critical point.



Berdnikov, Rajagopal, PRD 61 (2000),
MH, Fraga, Santos, PRD 93 (2016).

Acceptance range

A limited acceptance window can also be introduced.

Kinematic cuts

$$p_{min} < p_T < p_{max}\,, \qquad |\eta| < \eta_{max} \tag{7}$$

or, equivalently,

$$u_{min}(p) < |\cos\theta| < u_{max}(p)\,. \tag{8}$$

Particles of momentum p are accepted with probability

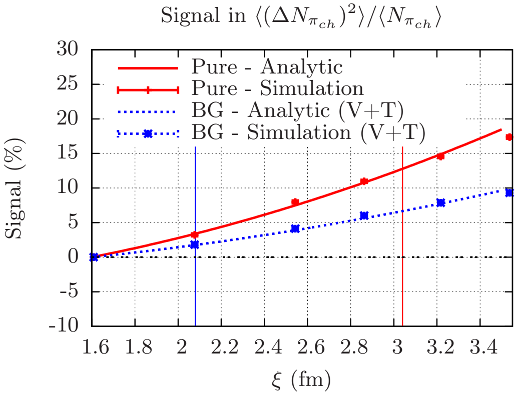
$$F(p) = \mathbf{e_0(p)}\cdot \max\big(u_{max}(p) - u_{min}(p), 0\big)\,. \tag{9}$$

From binomial distribution, for instance, $\langle n_p\rangle_{acc} = F(p)\,\langle n_p\rangle$ and

$$\langle(\Delta n_p)^2\rangle_{acc} = F(p)^2\,\langle(\Delta n_p)^2\rangle + F(p)\big(1 - F(p)\big)\,\langle n_p\rangle\,. \tag{10}$$

Results

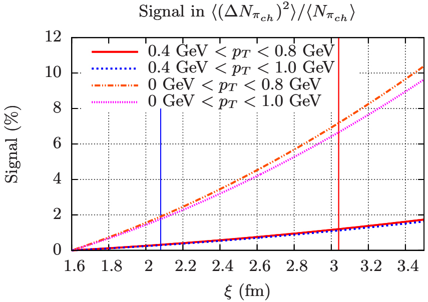
Peak height with relation to reference value:

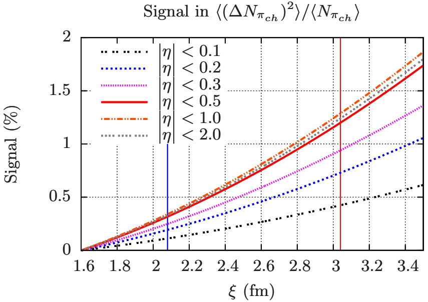


$$T = 130\,\text{MeV}\,, \qquad \mu = 420\,\text{MeV}\,, \qquad R_p = 6.8\,\text{fm}. \tag{11}$$

$\tau = 1\,\text{fm}\,,$ $\tau = 5.5\,\text{fm}$

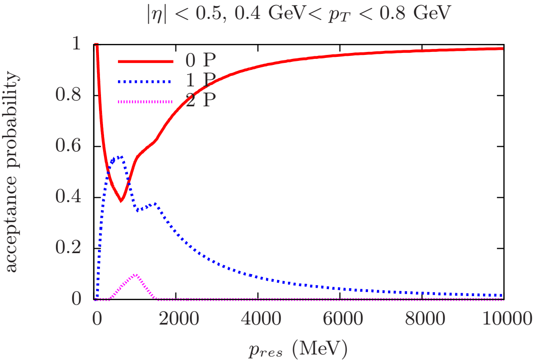
Results


Signal in $\langle(\Delta N_{\pi_{ch}})^2\rangle/\langle N_{\pi_{ch}}\rangle$
0.4 GeV < p_T < 0.8 GeV
0.4 GeV < p_T < 1.0 GeV
0 GeV < p_T < 0.8 GeV
0 GeV < p_T < 1.0 GeV
Signal (%)
ξ (fm)
12
10
8
6
4
2
0
1.6
1.8
2
2.2
2.4
2.6
2.8
3
3.2
3.4


Signal in $\langle(\Delta N_{\pi_{ch}})^2\rangle/\langle N_{\pi_{ch}}\rangle$
$|\eta| < 0.1$
$|\eta| < 0.2$
$|\eta| < 0.3$
$|\eta| < 0.5$
$|\eta| < 1.0$
$|\eta| < 2.0$
Signal (%)
ξ (fm)
2
1.5
1
0.5
0
1.6
1.8
2
2.2
2.4
2.6
2.8
3
3.2
3.4

Resonance decay within acceptance window

- $p_{res} \to p_1 + p_2$.
- One, both or none of particles accepted.
- Probability from phase space volume.
- Isotropy + energy-momentum conservation.
- Finite branching ratio: $P_{n\neq0} \to r_b \, P_{n\neq0}$.



"$\rho \to \pi\ \pi$" decays (BR: 100%).

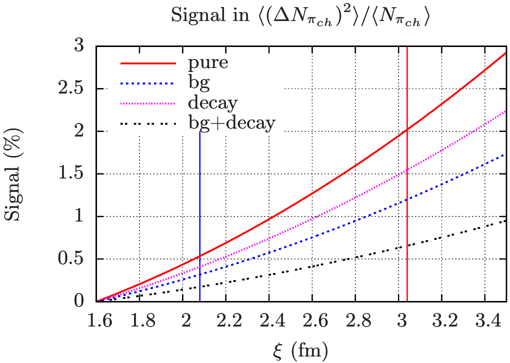
Resonance decay contributions

- Independent decays and $n_{p_{res}}$ distribution.
- For each decay,

$$\langle n_l^m \rangle = P_2 + P_l \,, \quad (12)$$

$$\langle n_1 \, n_2 \rangle = P_2 \,. \quad (13)$$

- "$\rho \to \pi \, \pi$" decays (BR: 100%).

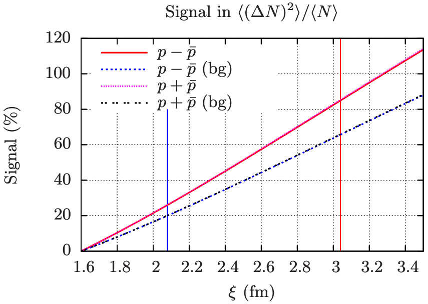


For effects on protons, up to higher-order, see Nahrgang et al, Eur. Phys. J. C 75 (2015).

Outline

Proton signatures

- Naive implementation yields very strong signal.
- Unlike pions, not much control over coupling g or mass m_p near CEP! Signal very sensitive to changes!
- **Caution:** very preliminary and unreliable!



Work in progress.

Higher-order moments

- Background calculations already possible.
- In theory, much stronger signal!
  Stephanov, PRL 102 (2009).
- Effective potential for σ_0, $\Omega[\sigma] \Rightarrow \Omega_*(\sigma_0)$

$$\Omega_*(\sigma_0) = V\left(\frac{m_\sigma^2}{2}\sigma_0^2 + \frac{\lambda_3^*}{3}\sigma_0^3 + \frac{\lambda_4^*}{4}\sigma_0^4 + \cdots\right), \qquad (14)$$

- Non-Gaussian fluctuations: λ_3, $\lambda_4 \Rightarrow \lambda_3^*$, $\lambda_4^* \Rightarrow \langle\sigma_0^3\rangle_c$, $\langle\sigma_0^4\rangle_c$.
- Tangled, non-linear evolution + wide range for $\tilde{\lambda}_3, \tilde{\lambda}_4$.
  $\Rightarrow$ In practice, less predictable!
  Mukherjee, Venugopalan, Yin, PRC 92 (2015),
  Tsypin, PRB 55 (1996).

Work in progress.

Outline

Summary and perspectives

- Initial approach in its first steps, but largely enhanceable.
- Evolution of fluctuations with $\xi(t), \lambda_3(t), \lambda_4(t)$ can be easily introduced $\rightarrow$ new limitations?
- Both simulations and analytical expressions (to be extended).
- Soon, results for non-Gaussian fluctuations and protons.
- New sources/models of fluctuations can be incorporated.
- Finite-efficiency effects can also be introduced.
- Future: use of given EoS?

Disclaimers

Caveats/Limitations

- Perfect equilibrium, no real dynamics → trend to overestimate signal, unreliable for p_T.
- Isotropy Assumption → effects of acceptance window should be taken with care!
- Homogeneous fluctuations → not realistic in relevant timescales.
- Background models still crude/incomplete → extra information and insight needed.
- Lack of control over some of the relevant parameters (protons and higher-order).

To keep in mind: still not exactly what we want!

But getting closer...

Acknowledgements

Thanks!

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