Features of the QCD phase diagram from small, noisy, fluctuating systems

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Preliminary remarks:

- The ultimate goal of the BES program is mapping the phase diagram of strong interactions. Or, at least, finding clear fingerprints of its major features.
- It is amusing to see that a Google search (images) for something like "successful phase diagram mapping physics" produces lots of QCD phase diagrams, even though we only have cartoons…
- We can generously interpret this as our community being very active and/or optimistic.
- On the other hand, several phase diagrams have been obtained in different realms of physics, experimentally and theoretically.

- However, we are still light-years away from something like:

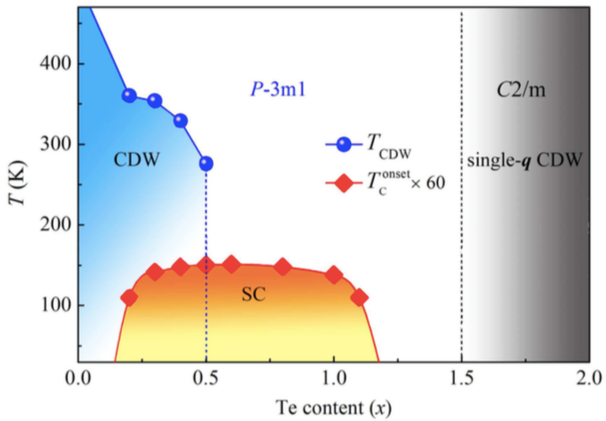


FIG. 4. (Color online) Electronic phase diagram of $1T$-$TaSe_{2-x}Te_x$ as a function of temperature and Te content.

case of $1T$-$TaTe_2$. However, according to the experimentally obtained phase diagram of Fig. 4, one should notice

[Liu et al (2014)]

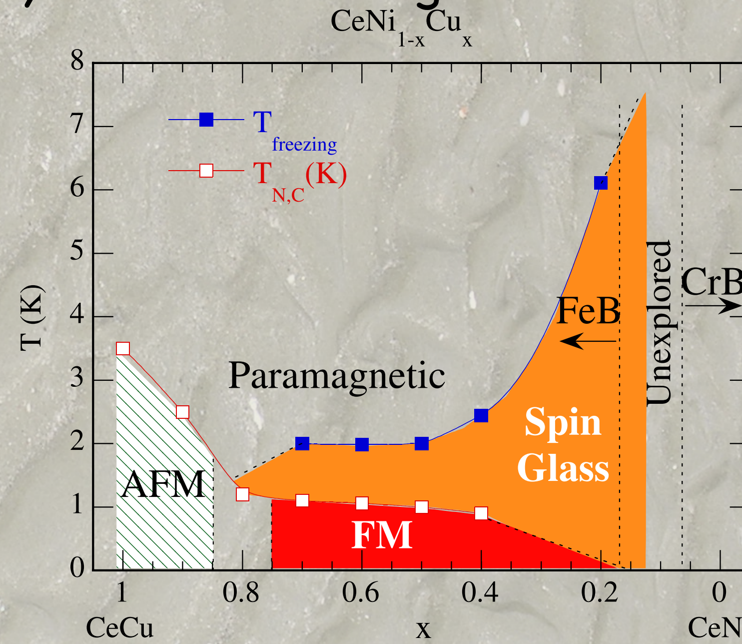


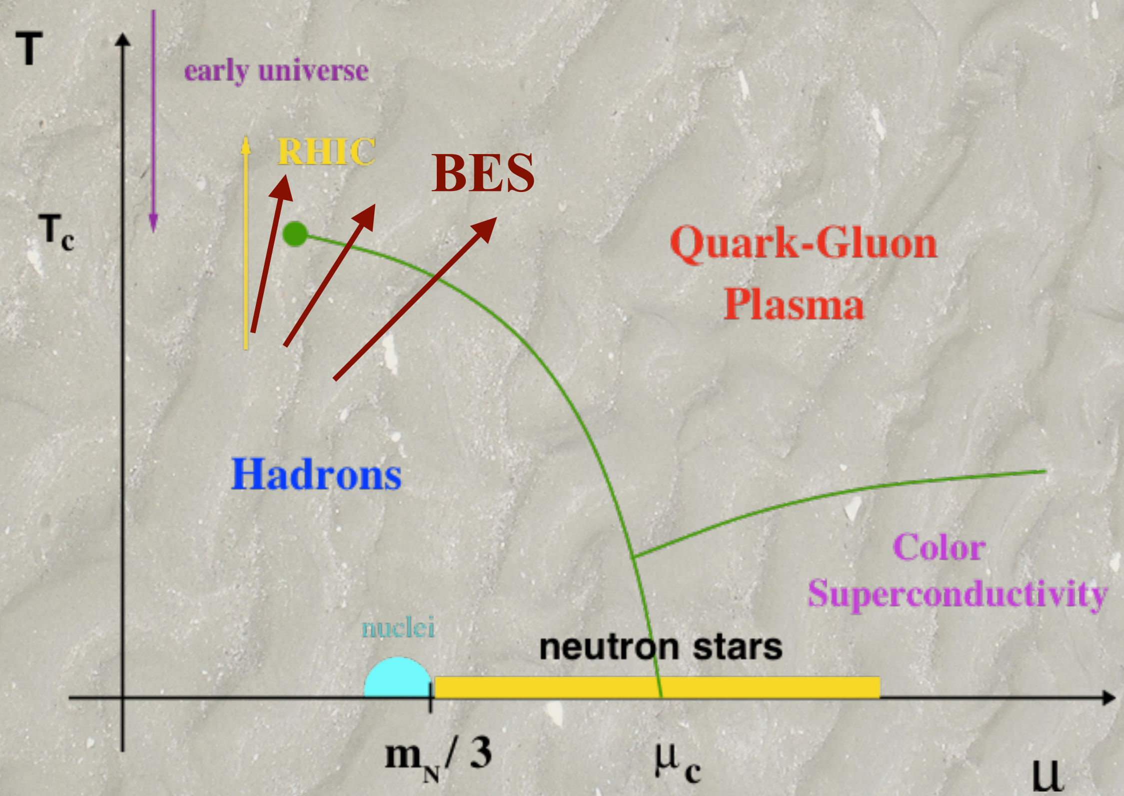
Figure 1. Magnetic phase diagram for the $CeNi_{1-x}Cu_x$ series as a function of Cu concentration from [14]. Open squares represent the long-range magnetic ordering temperature $T_{C,N}$ and full squares represent the spin-glass freezing temperature T_f.

[Marcano et al (2013)]

- The problem, as also mentioned previously in this program, is that we have a very small, short-lived, noisy, fast-evolving system that is (very) indirectly probed in the BES search [see talk by Steinheimer].

- So, we need all we can use!

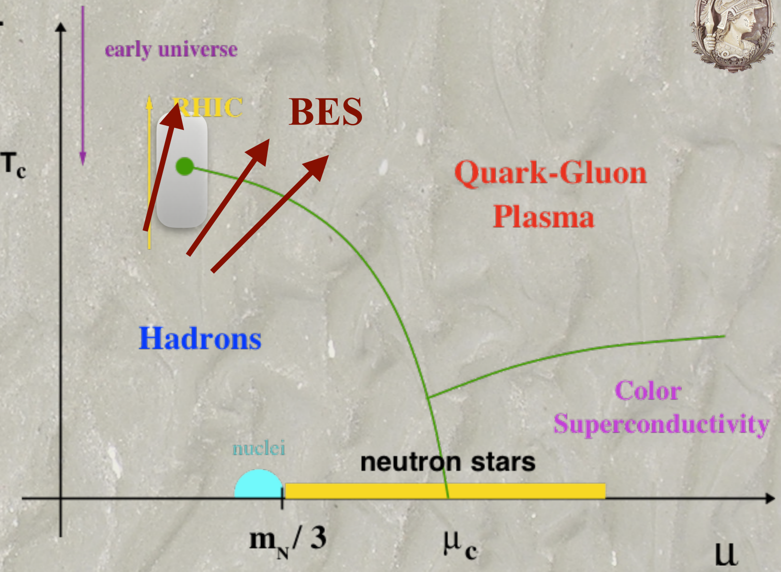
The phase diagram for QCD can be schematically divided, assuming it behaves roughly as a simplified cartoon of this sort (as suggested by several model descriptions):

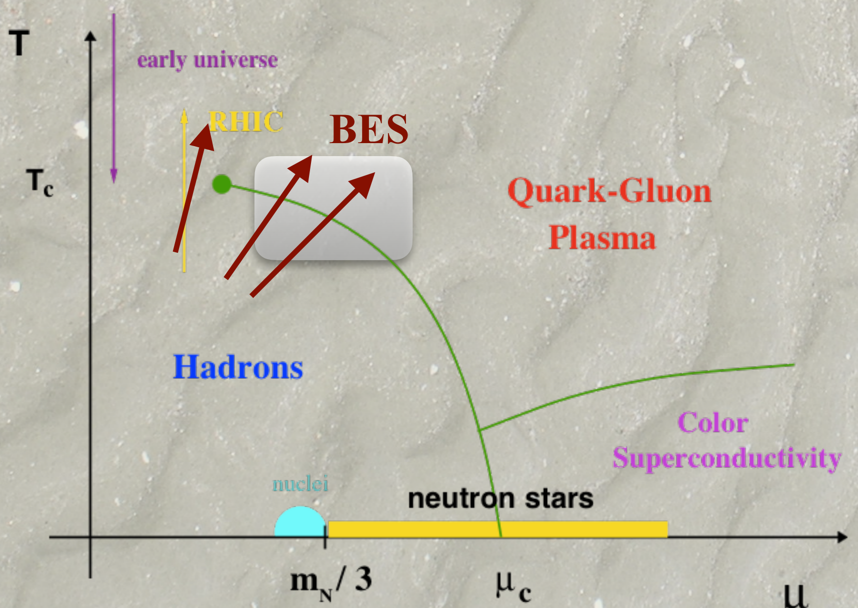


Also assuming the BES spans a large enough region, so that a CEP and a 1st-order line can be, in principle, probed.

CEP region (near criticality):

- $(\xi)^{power}$-dominated signatures
- background/noise -> spurious fluctuations
- finite size, finite lifetime, critical slowing down
- fast dynamics





1st-order transition region:

- finite size, finite lifetime
- bubble nucleation & spinodal decomposition
- two-peak structure
- structure formation (patterns)
- nonzero (conserved) baryon number
- fast dynamics

So

- 2 distinct regions, even if there is no clear boundary in practice
- 2 sets of problems, features, needed techniques, etc
- related sometimes, but different

Remark - direct comparison to lattice QCD can be dangerous:

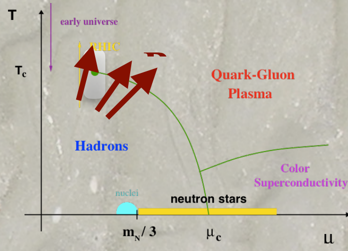
- sign problem in this region (no benchmark EoS):
  we do not have the "correct" EoS!
- the actual systems are finite and come in different sizes
  (lattice results are extrapolated to the thermodynamic limit)
- the QGP formed is very noisy, fluctuates a lot, and is indirectly measured within a given acceptance
- dynamics is crucial
  (totally absent on the lattice)

Nevertheless: several statistical mechanics techniques successfully used in lattice simulations can be useful in analyzing the BES data

Outline

- ★ CEP region (near criticality) - quick recap
- ★ In real life — including spurious effects
- ★ Critical slowing down and finite lifetime
- ★ Geometric & temperature fluctuations
- ★ Finite size of the system
- ★ Resonance decays and acceptance constraints
- ★ 1st-order transition region
- ★ Final remarks

CEP region (near criticality) - quick recap:



In Wonderland:

- Correlation length becomes divergent ⇒ system scale invariant (conformal):

- For the zero mode:

$$\mathcal{P}_{\sigma_0}[\sigma_0] \propto \exp\left(\frac{-\Omega[\sigma_0]}{T}\right)$$

$$\Omega[\sigma_0] = V\left(\frac{m_\sigma^2}{2}\sigma_0^2 + \frac{\lambda_3}{3}\sigma_0^3 + \frac{\lambda_4}{4}\sigma_0^4 + \cdots\right)$$

$$\mathcal{L}_{\sigma\pi\pi,\sigma pp} = 2G\sigma\pi^+\pi^- + g_p\sigma\bar{p}p$$

(couplings in the effective pot. renormalized by local fluctuations)

- Moments of observables as signatures:
  - ★ fluctuations of σ_0 affect particles that couple to it (pions, protons), e.g. fluctuations of the occupation numbers, etc.
  - ★ This contributes to the moments of fluctuations

- Freeze-out near the CEP & correlated fluctuations of observables ⇒ possible signature

[Stephanov, Rajagopal & Shuryak (1999); Stephanov (2002, 2009)]

[Tsypin (1994, 1996)]

Higher (non-gaussian) cumulants grow faster with the correlation length:

[Stephanov (2009); Athanasiou, Rajagopal & Stephanov (2010)]

- Within tree level:

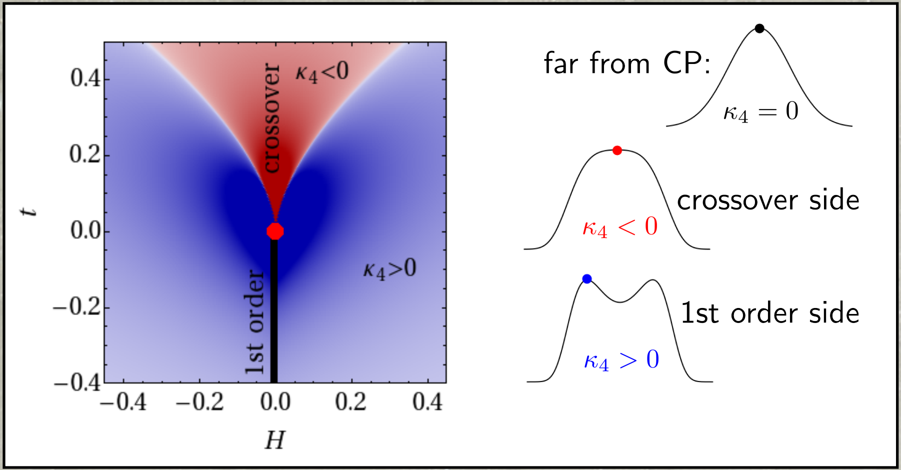
$$\kappa_2 = \langle \sigma_0^2 \rangle = \frac{T}{V}\,\xi^2\,; \qquad \kappa_3 = \langle \sigma_0^3 \rangle = \frac{2\lambda_3 T}{V}\,\xi^6\,;$$

$$\kappa_4 = \langle \sigma_0^4 \rangle_c \equiv \langle \sigma_0^4 \rangle - \langle \sigma_0^2 \rangle^2 = \frac{6T}{V}\left[2(\lambda_3 \xi)^2 - \lambda_4\right]\xi^8$$

$$m_\sigma \sim \xi^{-1}, \qquad \lambda_3 \sim T\,(T\,\xi)^{-3/2}, \qquad \lambda_4 \sim (T\,\xi)^{-1}$$

[Tsypin (1994, 1996)]

- Dependence on ξ and on which side of the CEP one is are universal
  $\Rightarrow$ possible signature in the sign of kurtosis [Stephanov (2011)]



- However: this picture can be totally modified by nonequilibrium evolution
  [Mukherjee, Venugopalan & Yin (2015)]

In real life — including spurious effects:

- Critical slowing down and finite lifetime

- Some spurious sources of fluctuations and noise

  Geometric fluctuations

  Temperature fluctuations

  Finite size of the system

  Resonance decays & acceptance constraints

Alternative — Monte Carlo simulation with

[Hippert, ESF & Santos (2015); Hippert & ESF (to appear)]

★ Interaction via mass correction:

$$\delta m_\pi^2 = 2\,G\,\delta\sigma_0\,, \qquad \delta m_p = g\,\delta\sigma_0 \qquad (G \approx 300\text{MeV},\ g \approx 10)$$

★ Fluctuations of σ_0 change m ⇒ modifies distribution of particles.

★ Freeze-out near the CEP.

[details: see talk by Hippert]

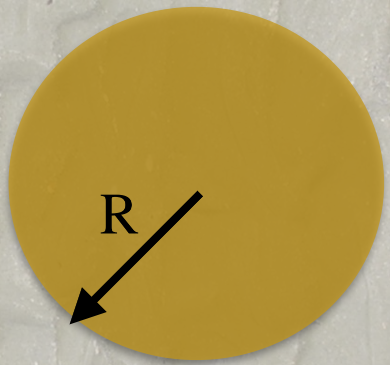
Framework

★ Monte Carlo with probability distribution:

$$\mathcal{P}_{\sigma_0}[\sigma_0] \propto \exp\left(\frac{-\Omega[\sigma_0]}{T}\right)$$

$$\Omega[\sigma_0] = V\left(\frac{m_\sigma^2}{2}\sigma_0^2 + \frac{\lambda_3}{3}\sigma_0^3 + \frac{\lambda_4}{4}\sigma_0^4 + \cdots\right)$$

$$\mathcal{L}_{\sigma\pi\pi,\sigma pp} = 2G\sigma\pi^+\pi^- + g_p\sigma\bar{p}p$$



★ Finite system

★ Dirichlet boundary conditions: more natural than cubic, yet simple

$$p_i^{(l)} = \alpha_i^{(l)}/R \qquad j_l(\alpha_i^{(l)}) = 0$$

★ Or, analytically approx. (systematic):

$$\begin{aligned}\overline{\langle(\Delta N)^2\rangle} &= \sum_{\substack{l_1,m_1,i_1\\ l_2,m_2,i_2}} \overline{\langle \Delta n_{l_1,i_1}\, \Delta n_{l_2,i_2}\rangle} \\ &= \sum_{\substack{l_1,i_1\\ l_2,i_2}} (2\,l_1+1)(2\,l_2+1)\; A_{(l_1,i_1),(l_2,i_2)} + \\ &\quad + \sum_{l_1,i_1} (2\,l+1)\; B_{(l,i)}\,,\end{aligned}$$

★ Caveats: equilibrium, no expansion, long-range fluctuations dominate.

★ $\langle\cdots\rangle$: average over a grand-canonical ensemble with δσ, δR, δT, etc fixed.

★ $\overline{\cdots}$: average over δσ, δR, δT, etc.

★ A and B: in terms of boson/fermion distributions & fluctuations in energy levels.

★ Advantages: can include all sorts of spurious fluctuations; can be systematically improved; can include dynamics.

• Critical slowing down and finite lifetime τ

⋆ Near the CEP, ξ grows

⇒ regions that represent fluctuations around equilibrium get larger

⇒ relaxation is slower and slower near the CEP: $\tau_{relax} \sim \xi^z$

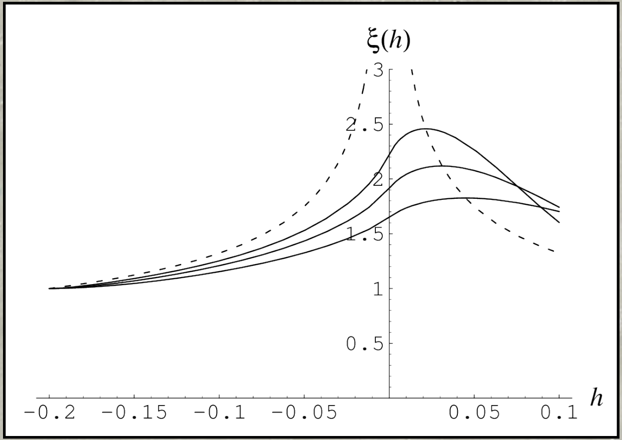
⋆ the value of z depends on the dynamic universality class of the system

[Guida & Zinn-Justin (1997)]

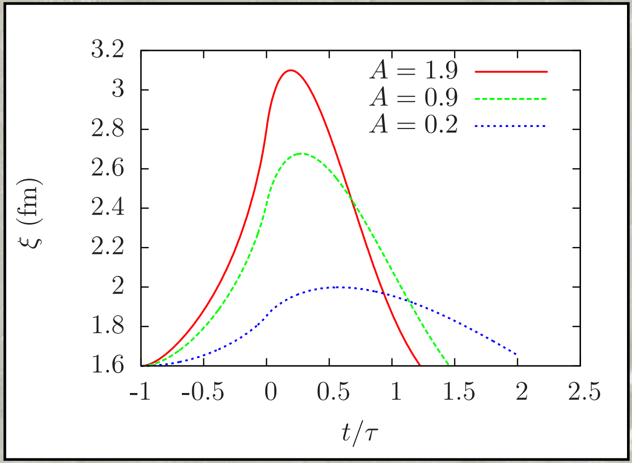
⋆ ξ does not reach its equilibrium value, and is limited by $\tau_{relax} \lesssim \tau$

⋆ $\xi(t)$ and cumulants also follow Kibble-Zurek scaling [Mukherjee, Venugopalan & Yin (2016)]

[see talk by Mukherjee]



[Berdnikov & Rajagopal (2000)]



⋆ best scenario, free parameter limited by the speed of light!

[Hippert, ESF & Santos (2015)]

• Some spurious sources of fluctuations

Geometric fluctuations

[Hippert, ESF & Santos (2015)]

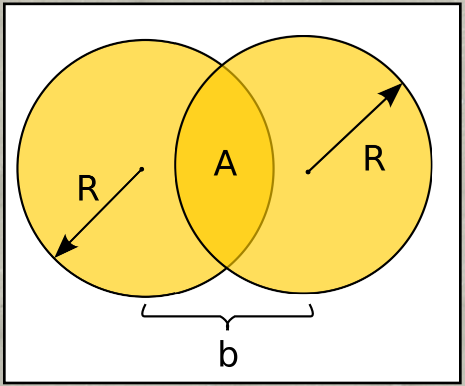
★ Volume fluctuations from a centrality bin width effect (CBWE) — variations of the volume within a centrality bin — not initial volume fluctuations (from dynamical initial conditions).

★ Since the resulting distribution turns out not to be Gaussian, volume fluctuations will also affect higher-order cumulants.

★ Impact parameter distribution ⇒ overlap area.

★ Assumption $V(b) = C\, A(b, R_N)$; fix $\langle R_{plasma} \rangle = 6.8$ fm for 0 – 5% centrality.

★ $R_N = 6.38$ fm taken from Woods-Saxon nuclear density profile.



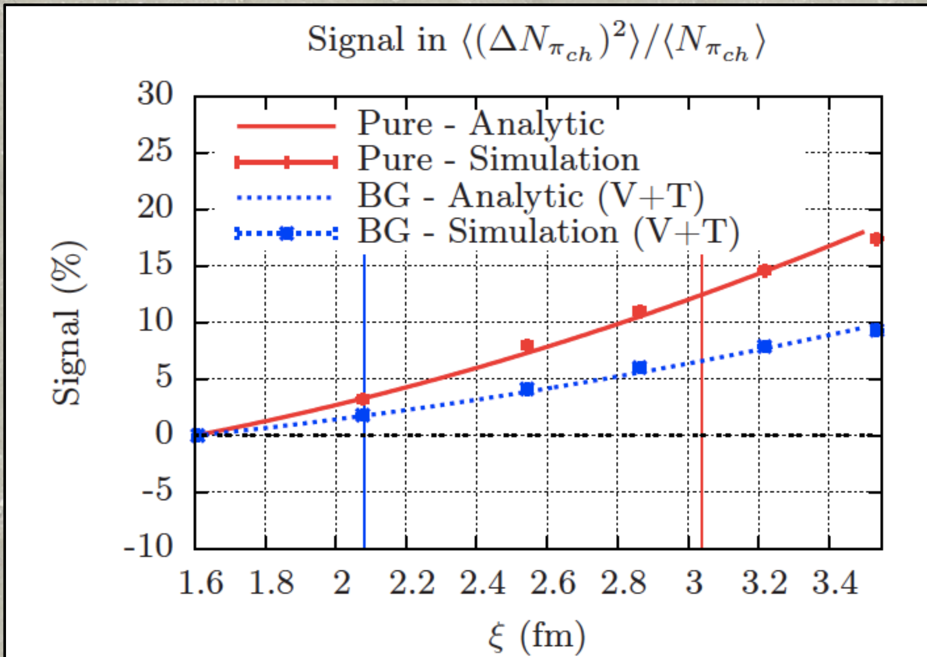
$$A(b, R_N) = 2R_N^2 \cos^{-1}\left(\frac{b}{2R_N}\right) - b\sqrt{R_N^2 - \frac{b^2}{4}}$$

$$p_i = \frac{\alpha_i}{R_p + \delta R_p}$$

Temperature fluctuations: gaussian distribution of temperatures, with a 5% standard deviation.

Results taking into account volume and temperature fluctuations as background + finite lifetime (critical slowing down)

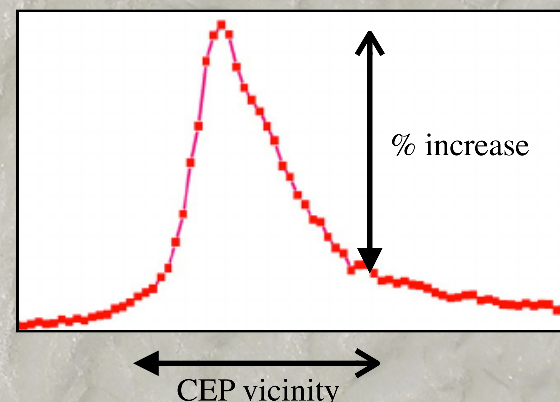
[Hippert, ESF & Santos (2015); Hippert & ESF (to appear)]



★ 2nd-order moment (for simplicity); doable for higher orders (soon!).

$$\text{Signal} = \frac{\sigma_N(\xi) - \sigma_N(\xi_0)}{\sigma_N(\xi_0)}$$

$$\sigma_N(\xi) = \frac{\langle(\Delta N_{\pi_{\rm ch}})^2\rangle}{\langle N_{\pi_{\rm ch}}\rangle}$$

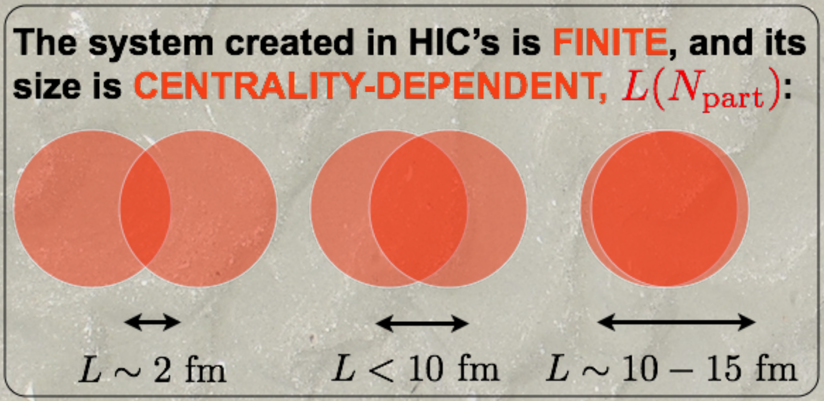


- ★ Systems goes through the CEP (very optimistic!).
- ★ Signal above reference ("far from the CEP", $\xi=\xi_0=1.6$fm).
- ★ Ising universality class assumed.
- ★ 10^6 events in the MC simulations.
- ★ acceptance: $0 < p_T < 1$ GeV & $\eta < 0.5$
- ★ Time spent near CEP: τ = 1fm (optimistic) ,
  (vertical lines) τ = 5.5fm (overly optimistic)

[details & preliminary results for protons & higher-order cumulants: see talk by Hippert]

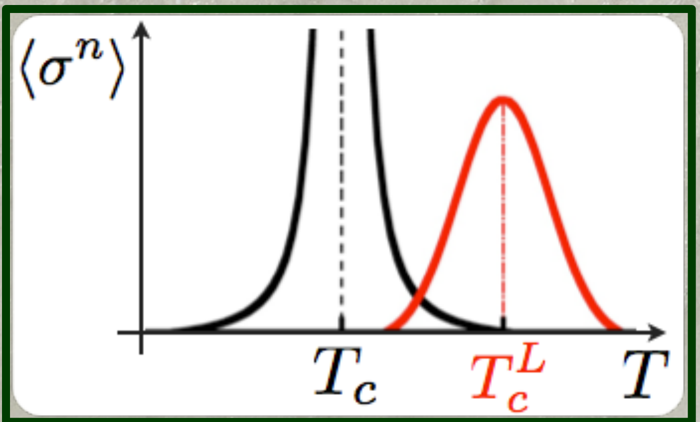
Finite size of the system

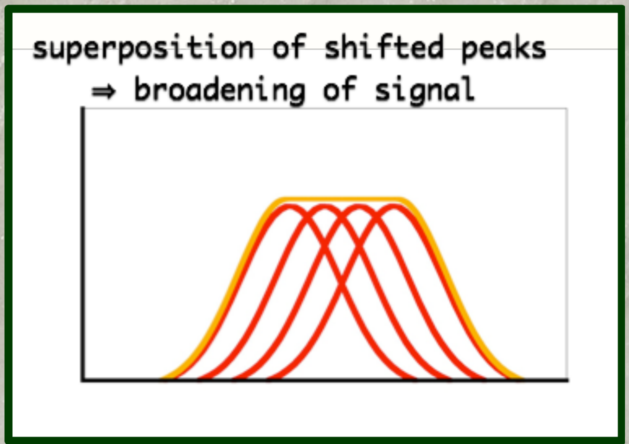
[Palhares, ESF & Kodama (2009)]



- In BES even smaller systems (plasmas) are expected! Not only a centrality effect!

- Measurements will generally probe pseudocritical, smoothened, shifted thermodynamic quantities. Ex. – cumulants:



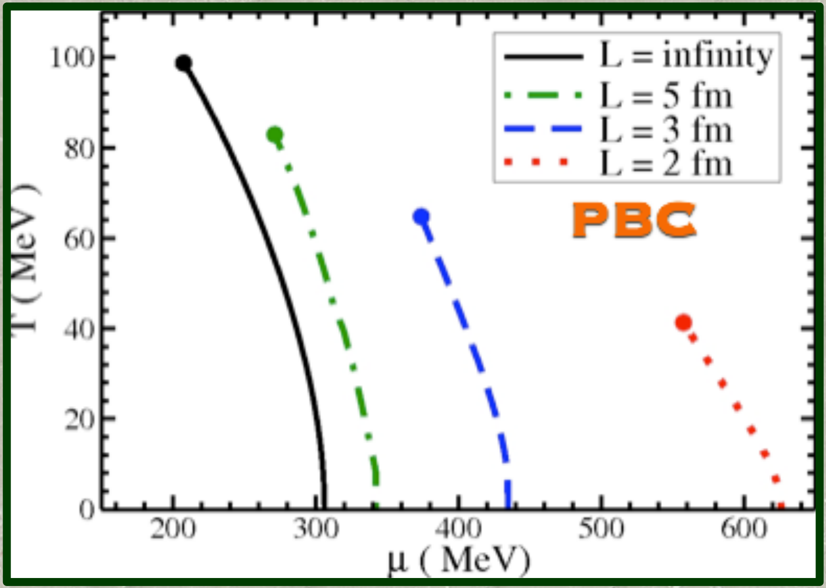


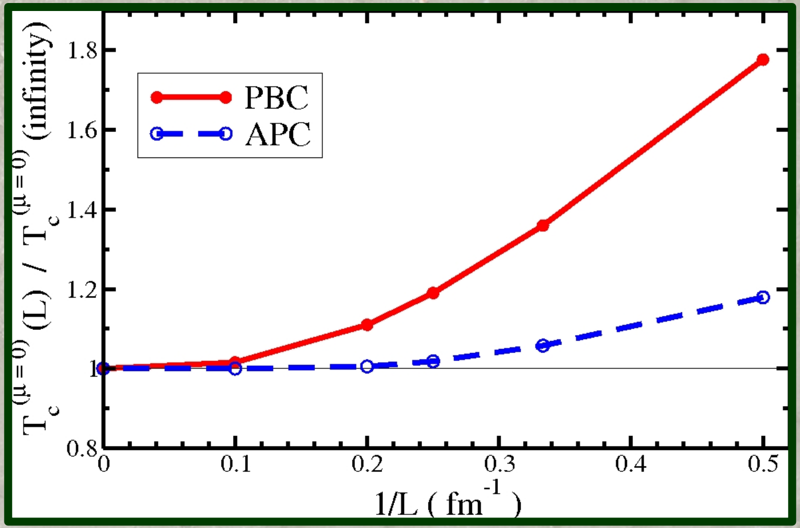
- Most (≈all) signatures based on non-monotonic behavior of observables partially hidden by background + shifts and smoothening.

$$\langle \sigma^n \rangle_L \sim \xi^{p_n} \; f_n(\xi/L)$$

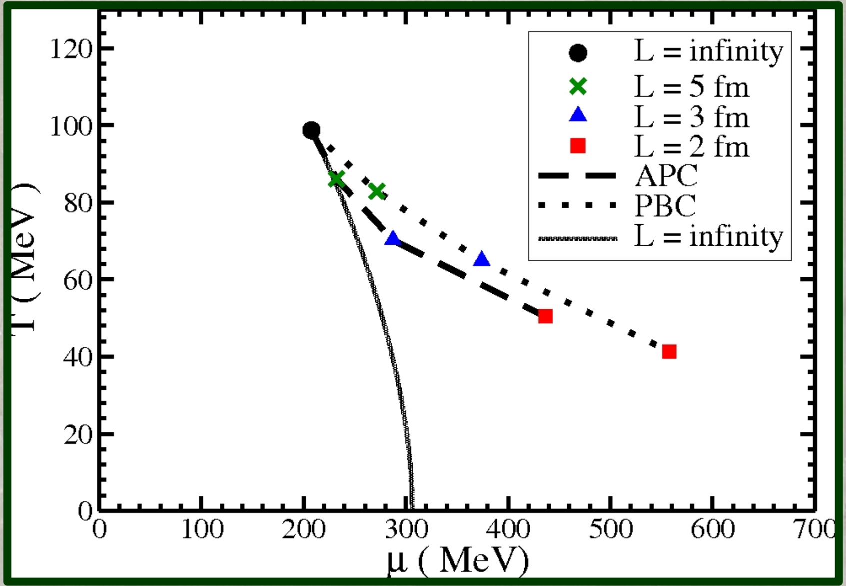
Shifts in the phase diagram (equilibrium)

[Palhares, ESF & Kodama (2009)]





- ★ Effect that adds up to finite lifetime and critical slowing down.
- ★ Direct use of lattice data can be dangerous also for this reason.



Finite-size scaling as a tool for searching the CEP

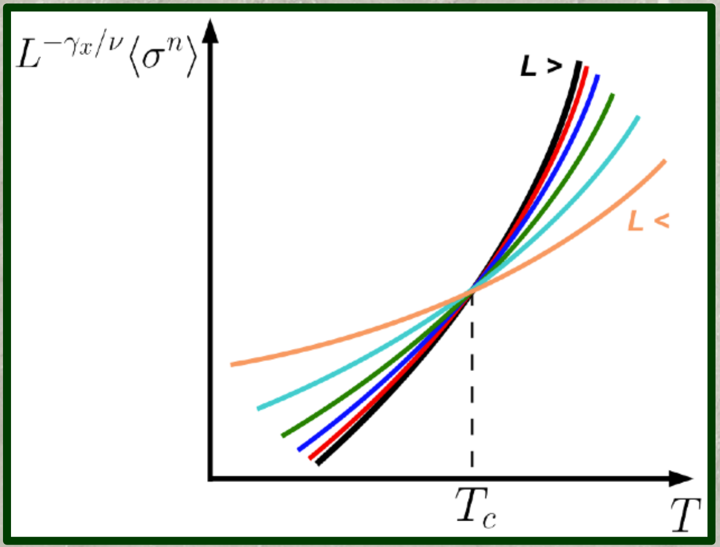
[ESF, Palhares & Sorensen (2011)]

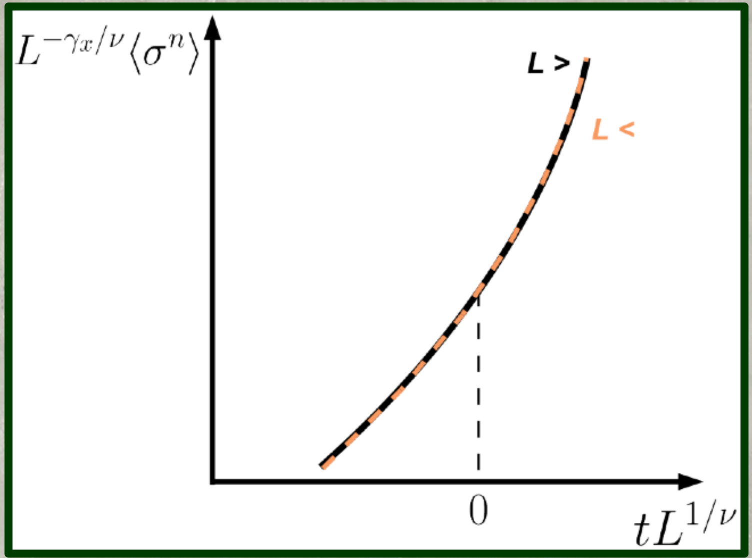
In the vicinity of the CEP:

- FSS applies as can be demonstrated by a RG analysis.
- For any correlation function of the order parameter all lines should collapse in a full scaling plot:

$$X(t,L) = L^{\gamma_X/\nu} f_X(tL^{1/\nu})$$

Ex.: cumulant scaling plots



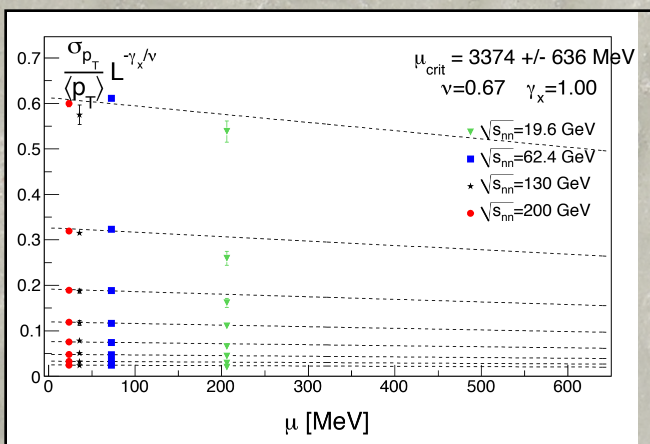


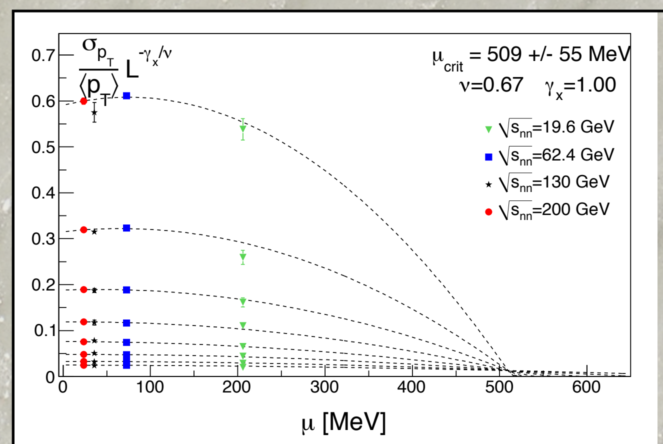
✓ Size (L): difference defined by centrality; estimated by HBT.

✓ Distance to the CEP ("t"): constrained by freeze-out curve, parametrized either by μ or by center-of-mass energy

✓ Observables (X): transverse momentum fluctuations, pion multiplicity fluctuations (soft pions), etc.

✓ Caveat: method hindered by critical slowing down effects.





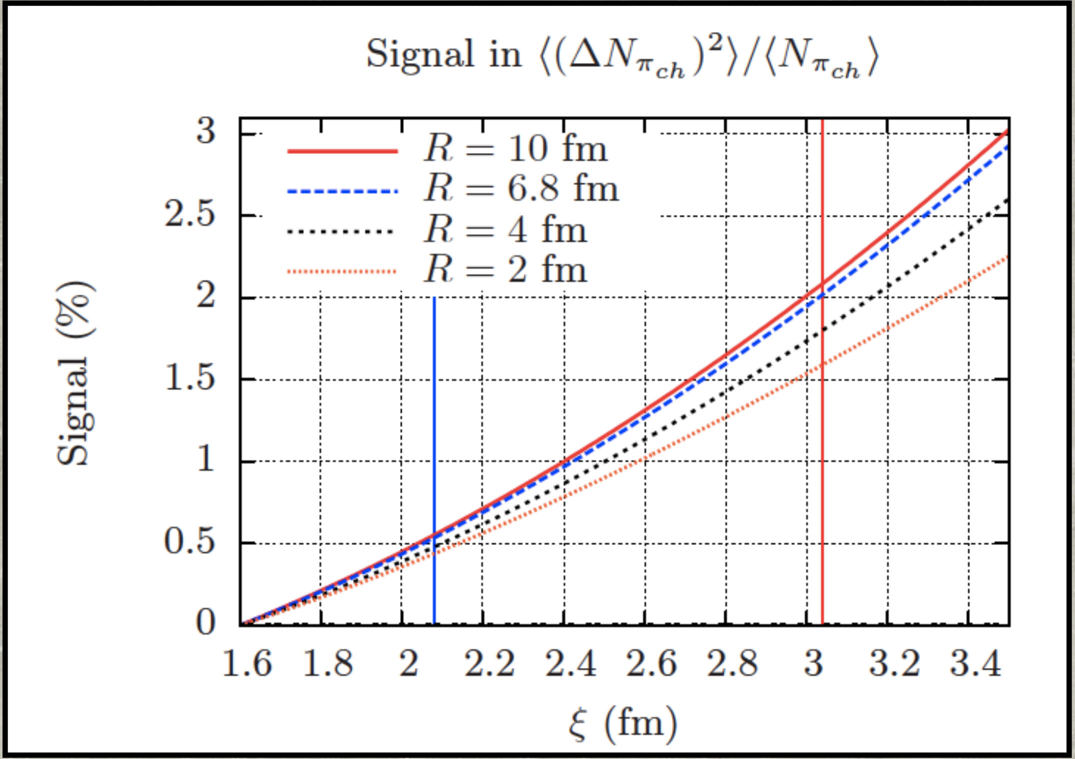
[see talk by Lacey]

[ESF, Palhares & Sorensen (2011)]

- Very restricted data -> extrapolations using fits.
- Scaling function should be smooth -> polynomial fit for each L.
- Enforce the condition that all the curves cross at some critical μ (adjustable parameter).
- Data at the time seemed to favor values of the critical chemical potential above 450 MeV.

Finite-size effects on cumulants

[Hippert & ESF (to appear)]

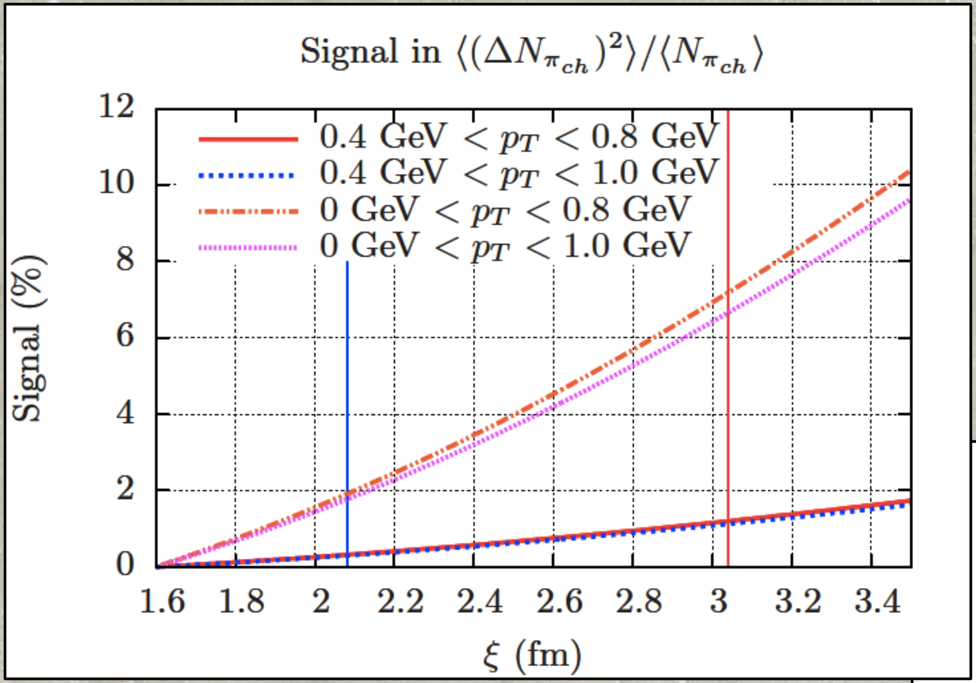


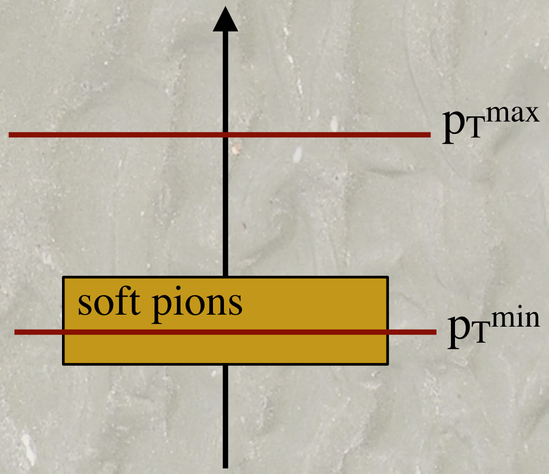
- ★ Overall volume factor cancels out.
- ★ Signal above reference ("far from the CEP", $\xi=\xi_0=1.6$fm).
- ★ acceptance: $0.4 < p_T < 0.8$ GeV & $\eta < 0.5$
- ★ Time spent near CEP: τ = 1fm (optimistic) ,
  (vertical lines) τ = 5.5fm (overly optimistic)

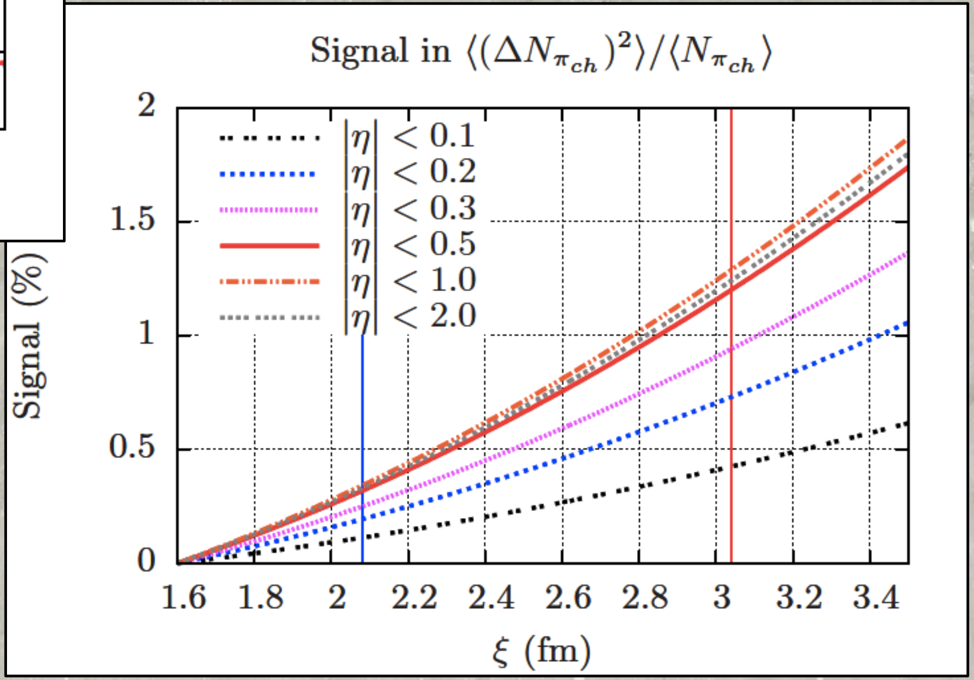
[details & results for protons: see talk by Hippert]

Acceptance constraints

[Hippert & ESF (to appear)]





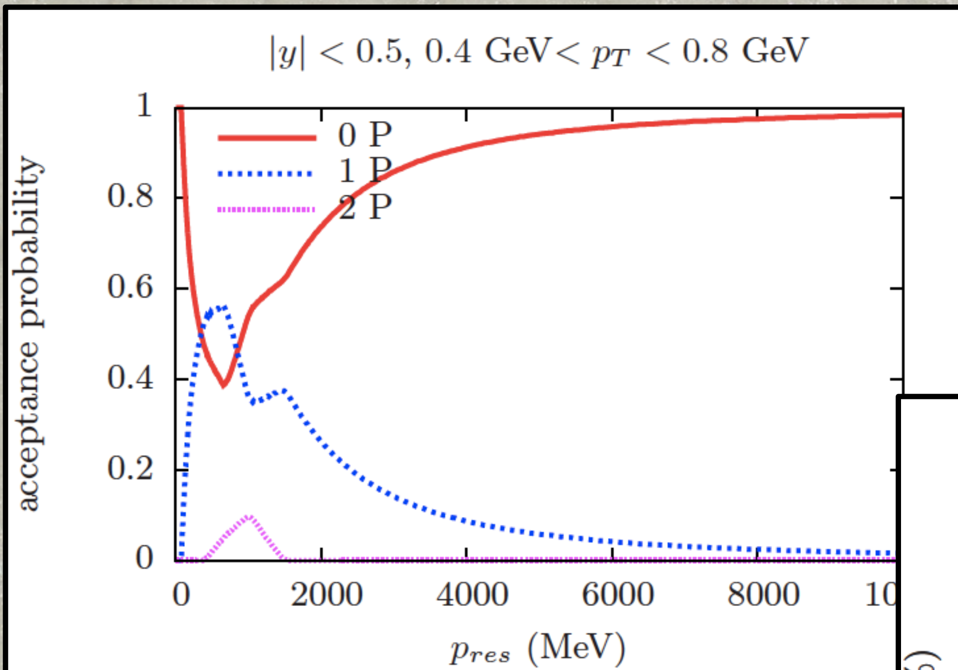


- ★ Signal above reference ("far from the CEP", $\xi=\xi_0=1.6$fm).
- ★ acceptance: $0.4 < p_T < 0.8$ GeV for diff. η
- ★ Small effect for larger values of η (saturates near $\eta=2$)
- ★ Less momentum modes pairs due to "directional cut"

[details & results for protons: see talk by Hippert]

Resonance decays and acceptance constraints

[Hippert & ESF (to appear)]

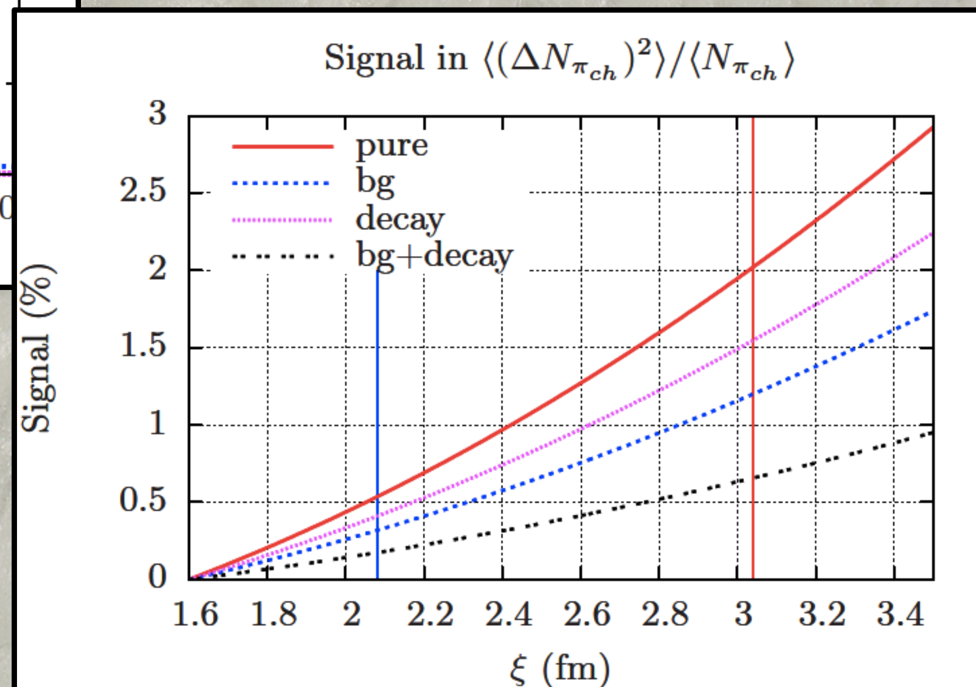


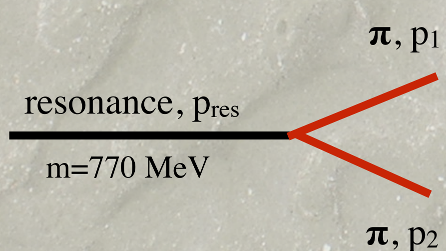
★ 0 P: no particle detected

1 P: only 1 particle detected

2 P: 2 particles detected

★ Abrupt cut for 2P given by twice p_T^{max}

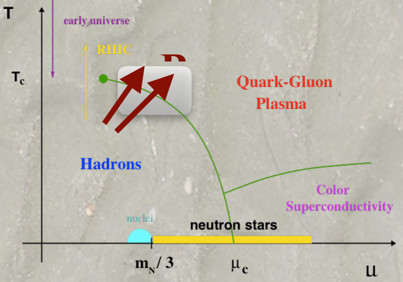




[details & results for protons: see talk by Hippert]

[acceptance: $0.4 < p_T < 0.8$ GeV]

1st-order transition region



- Finite size, finite lifetime
  - ★ Smaller systems (due to smaller collision energies).
  - ★ Smaller expansion rates (due to smaller pressure gradients).
  - ★ No critical slowing down.

- Bubble nucleation & spinodal decomposition
  ⇒ structure formation (inhomogeneous patterns).

- Two-peak structure due to competing phases (lattice-inspired method)
  ⇒ probability histograms of bulk quantities (not correlation functions).

  ⇒ correlation functions will not distinguish crossover from 1st order PT.

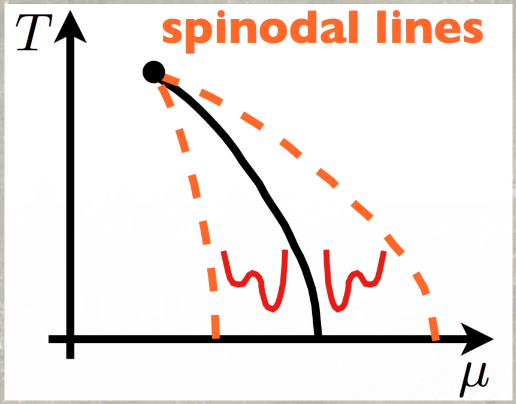
- Nonzero (conserved) baryon number
  ⇒ constraint on phase conversion dynamics

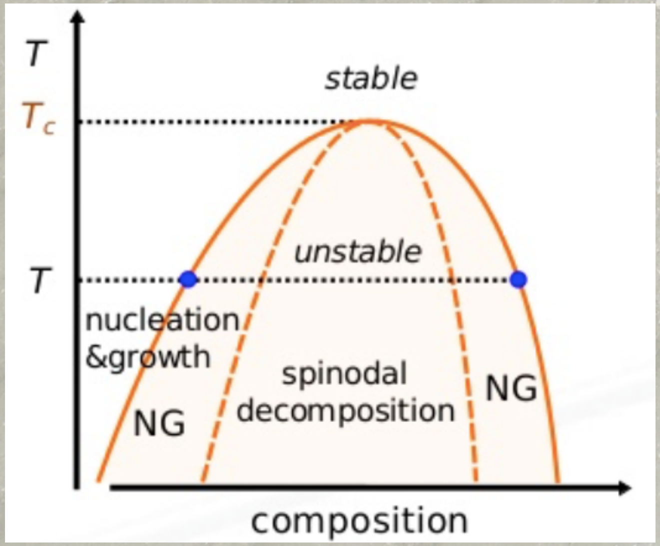
- Fast dynamics

[Chernodub, ESF, Palhares & Sorensen (unpubl., 2011)]

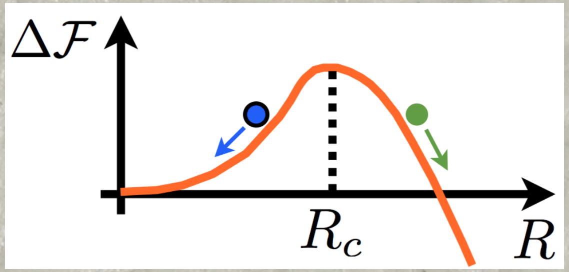
Bubble nucleation & spinodal decomposition
⇒ structure formation (patterns)

★ Mechanically unstable regions in the phase diagram





★ Free energy in the unstable direction



★ Different patterns

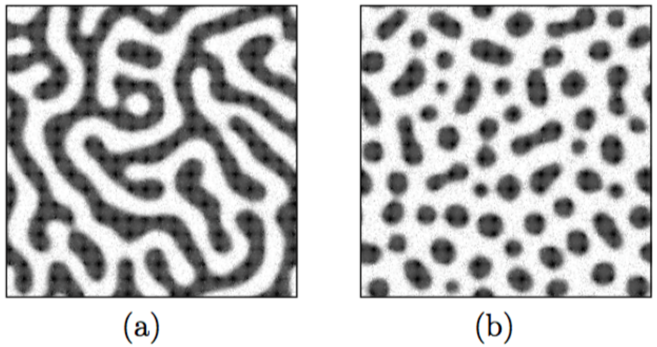


FIG. 2. Spatial patterns of model B: a) Spinodal decomposition and b) Nucleation. ($t = 2500$, $\lambda = 0$, $\sigma^2 = 0.1$ and $\varepsilon = 10^{-4}$)

[Garcia-Ojalvo et al (1998)]

How will such structures affect hydro, transport, etc?

Final remarks

➢ We have 2 distinct regions, even if there is no clear boundary in practice: criticality effects x pattern formation signatures.

➢ Near the critical region, one can systematically incorporate spurious contributions (resonances, acceptance limitations, finite size, finite lifetime and critical slowing down) expected to affect the fluctuations in the BES. It is a method that can be improved or adapted.

➢ Results from the second moment for pions are small (as expected). But now we have estimated how this signal is diminished by the background. Results for higher moments (and also for protons) soon!

➢ Dynamics is still missing in this approach. But the evolution of cumulants can be incorporated. Also, it can in principle be coupled to hydro.

➢ Comparison to lattice results are useful, but should be done with extra caution.

➢ The 1st-order region might offer a different class of signatures to be explored.