Multi-particle correlations, baryon stopping and non-binomial efficiency

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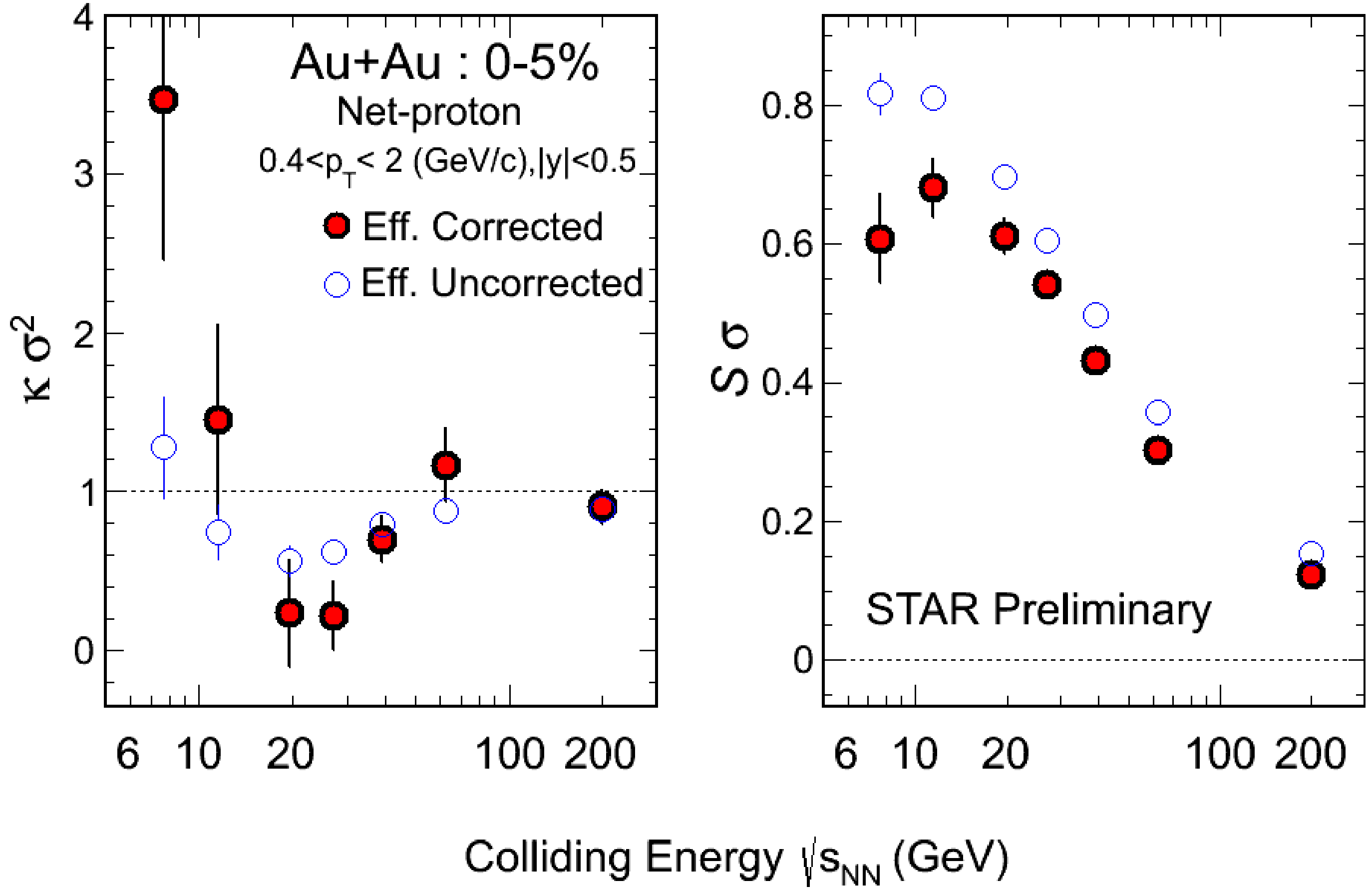
AB, R. Holzmann, V. Koch, 1603.09057

AB, V. Koch, N. Strodthoff , 1607.07375

A. Bialas, AB, V. Koch, 1608.07041

Efficiency correction is important

STAR (thanks to X. Luo)



K_4/K_2 **my notation** K_3/K_2

If efficiency is driven by binomial with p (or ϵ)

true measured

$$\left\langle \frac{N!}{(N-i)!} \right\rangle = \frac{1}{p^i} \left\langle \frac{n!}{(n-i)!} \right\rangle$$

$$F_i = \frac{1}{p^i} f_i$$

So we express true cumulants through factorial moments F_i, which are known from the above equality (f_i is measured, p is known)

AB, V. Koch,
PRC 86 (2012) 044904; PRC 91 (2015) 027901

R. Holzmann,
talk at HIC for FAIR

If ϵ **depends on N** the method brakes down.

Let's test it. Suppose that

$$P(N) = \frac{\langle N \rangle^N}{N!} e^{-\langle N \rangle},$$

$$\epsilon(N) = \epsilon_0 + \epsilon'(N - \langle N \rangle)$$

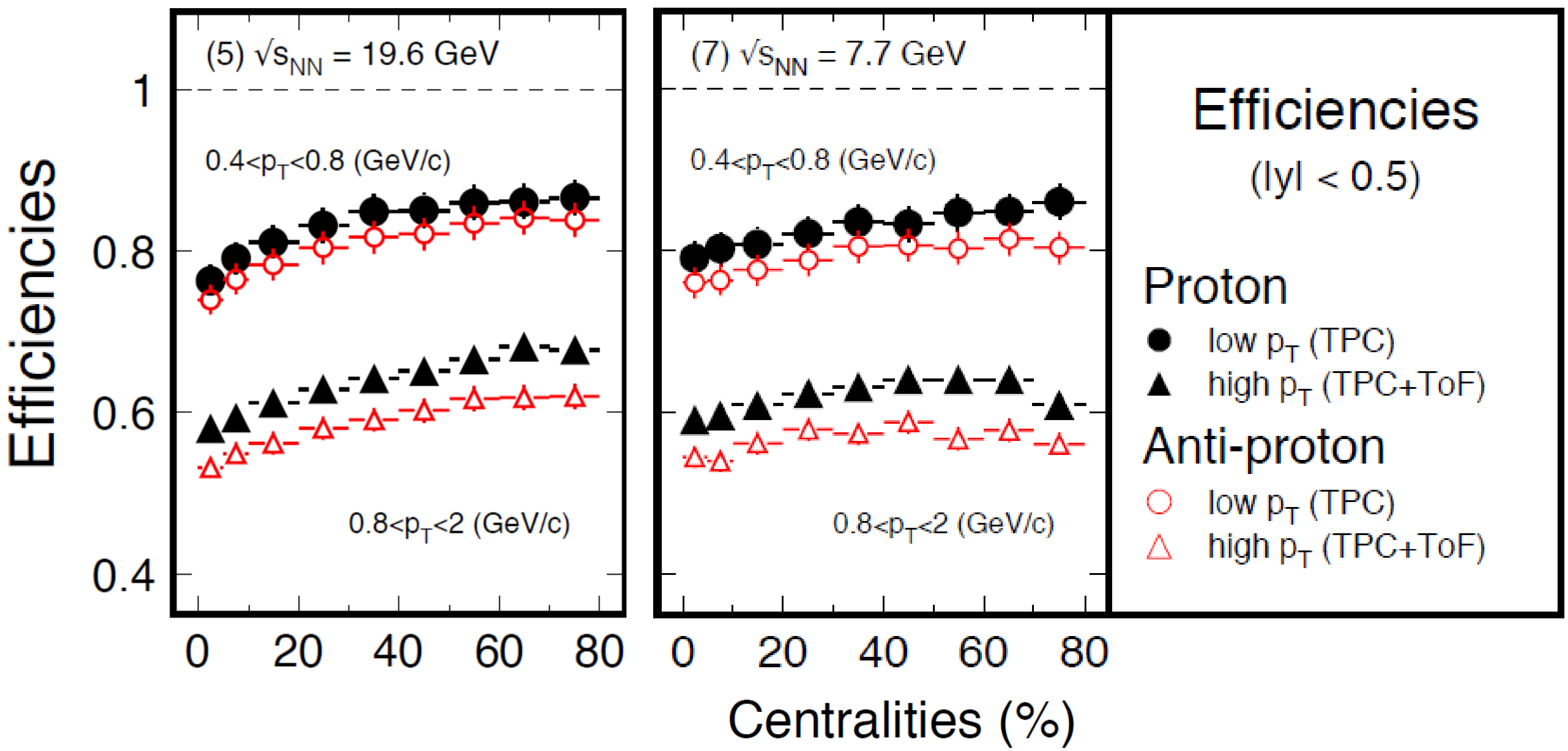
$$B(n, N) = \frac{N!}{n!(N-n)!} \epsilon(N)^n \left[1 - \epsilon(N)\right]^{N-n}$$

We calculate exact f_i and correct using constant efficiency $F_i = f_i/\epsilon_0^i$.

We use $\langle N \rangle = 40, \epsilon_0 = 0.65$ and plot K_n/K_2 as a function of ϵ'.

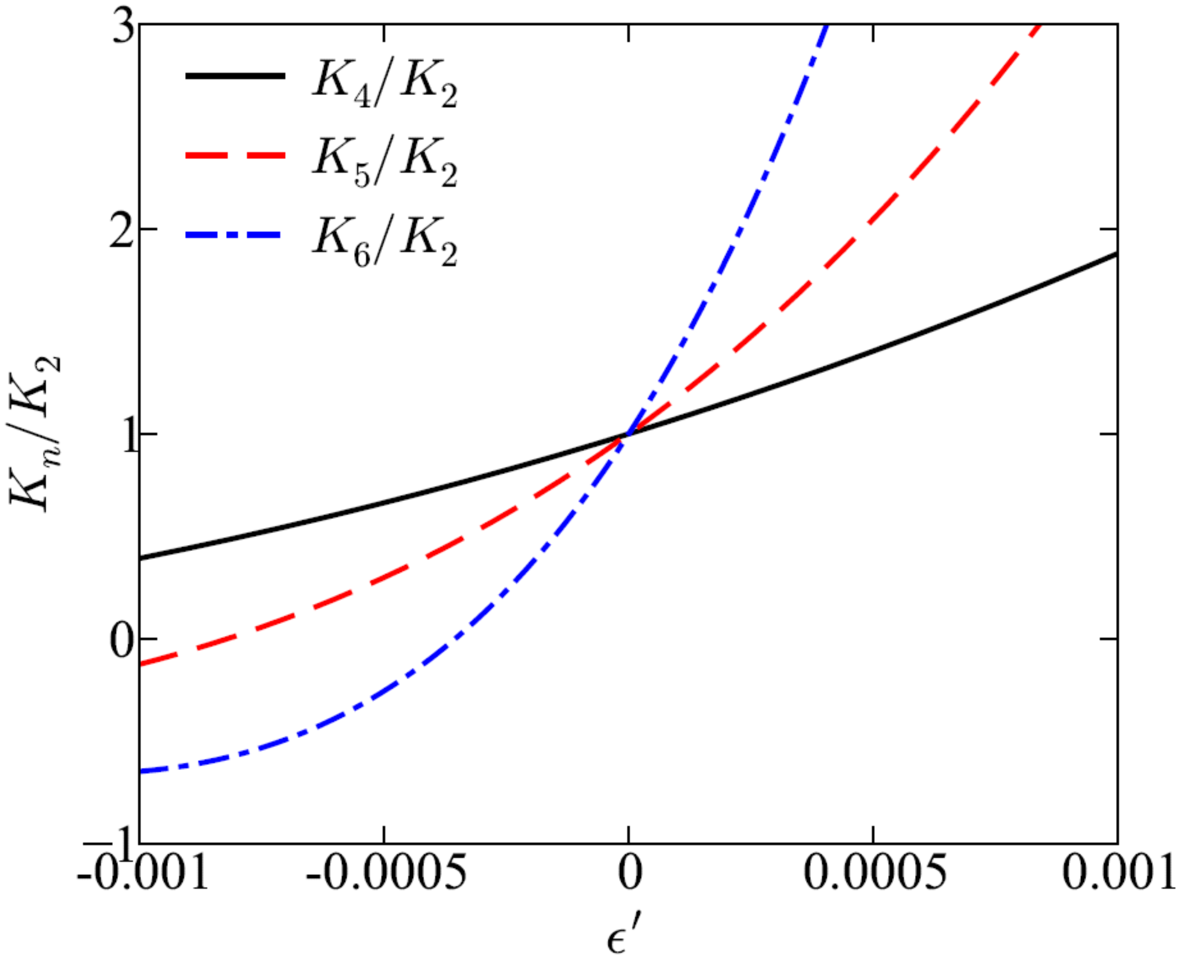
STAR efficiencies at 19.6 GeV and 7.7 GeV

X. Luo [STAR Collaboration] arXiv:1503.02558 [nucl-ex]].



We obtain

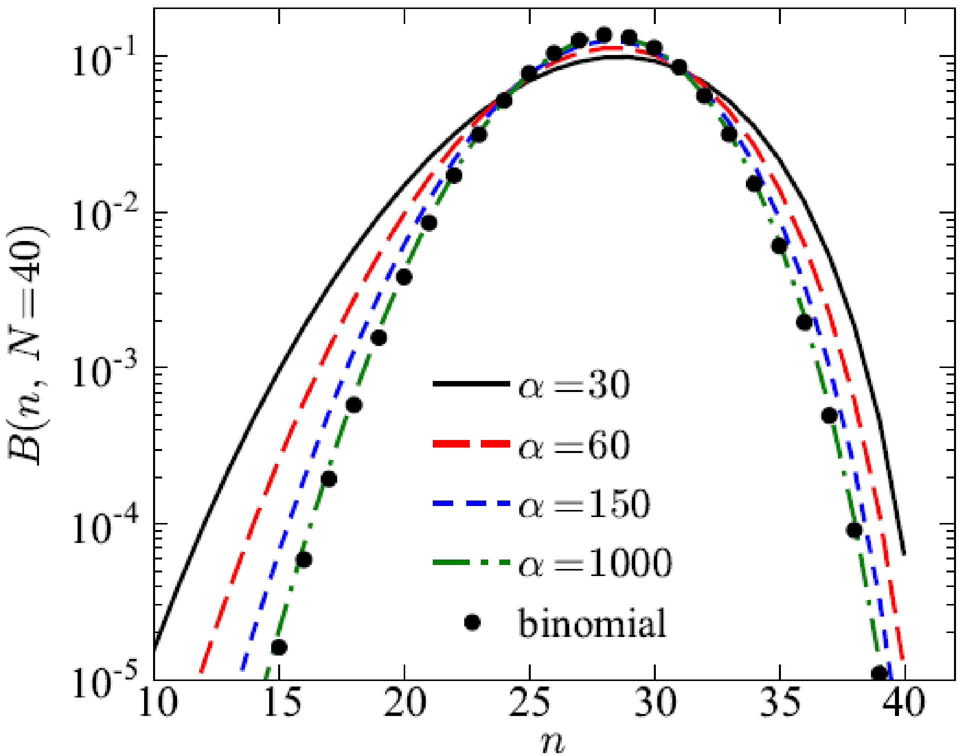
AB, R.Holzmann, V.Koch
arXiv:1603.09057



Large corrections for small ϵ'

Non-binomial distribution, e.g., beta-binomial distribution (we return 2 balls)

AB, R.Holzmann, V.Koch
arXiv:1603.09057



Beta-binomial	$\alpha = 30$	$\alpha = 60$	$\alpha = 150$	$\alpha = 1000$
K_3/K_2	1.28	1.24	1.13	1.02
K_4/K_2	0.82	1.45	1.35	1.07
K_5/K_2	-1.11	1.15	1.63	1.16
K_6/K_2	5.71	-0.44	1.80	1.32

Take-home message

- Multiplicity dependent efficiency and non-binomial efficiency is (most likely) important

- Technique based on correcting factorial moments is not good enough

- Proper unfolding is warranted (see backup)

Multi-particle correlation functions

based on **preliminary** STAR data

See also:
B.Ling, M.Stephanov,
PRC 93 (2016) no.3, 034915

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \boldsymbol{C_2}(y_1, y_2)$$

correlation function

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2)[1 + c_2(y_1, y_2)]$$

reduced correlation function

$$\langle N(N-1)\rangle = \langle N\rangle^2 + \langle N\rangle^2 c_2$$

$$c_2 = \frac{\int \rho(y_1)\rho(y_2)c_2(y_1, y_2)dy_1 dy_2}{\int \rho(y_1)\rho(y_2)dy_1 dy_2}$$

coupling

and the second order cumulant

$$K_2 = \langle N\rangle + \underbrace{\langle N\rangle^2 c_2}_{\boldsymbol{C_2}}$$

In the same way

$$\rho_3(y_1, y_2, y_3) = \rho(y_1)\rho(y_2)\rho(y_3)[1 + c_2(y_1, y_2\ ) + \cdots + c_3(y_1, y_2, y_3)]$$

$$F_3 = \langle N(N-1)(N-2)\rangle = \langle N\rangle^3 + 3\langle N\rangle^3 c_2 + \langle N\rangle^3 c_3$$

$$c_3 = \frac{\int \rho(y_1)\rho(y_2)\rho(y_3)c_3(y_1, y_2, y_3)dy_1 dy_2 dy_3}{\int \rho(y_1)\rho(y_2)\rho(y_3)dy_1 dy_2 dy_3}$$

coupling

and the third order cumulant

$$K_3 = \langle N\rangle + 3\langle N\rangle^2 c_2 + \langle N\rangle^3 c_3$$

$$3\boldsymbol{C_2} \qquad \boldsymbol{C_3}$$

Finally we obtain

$$c_2 = \frac{\int \rho(y_1)\rho(y_2)c_2(y_1,y_2)dy_1dy_2}{\int \rho(y_1)\rho(y_2)dy_1dy_2}$$

$$\boldsymbol{C_2} = \int C_2(y_1,y_2)dy_1dy_2$$

$$K_2 = \langle N \rangle + \langle N \rangle^2 c_2$$

$$K_3 = \langle N \rangle + 3\langle N \rangle^2 c_2 + \langle N \rangle^3 c_3$$

$$K_4 = \langle N \rangle + 7\langle N \rangle^2 c_2 + 6\langle N \rangle^3 c_3 + \langle N \rangle^4 c_4$$

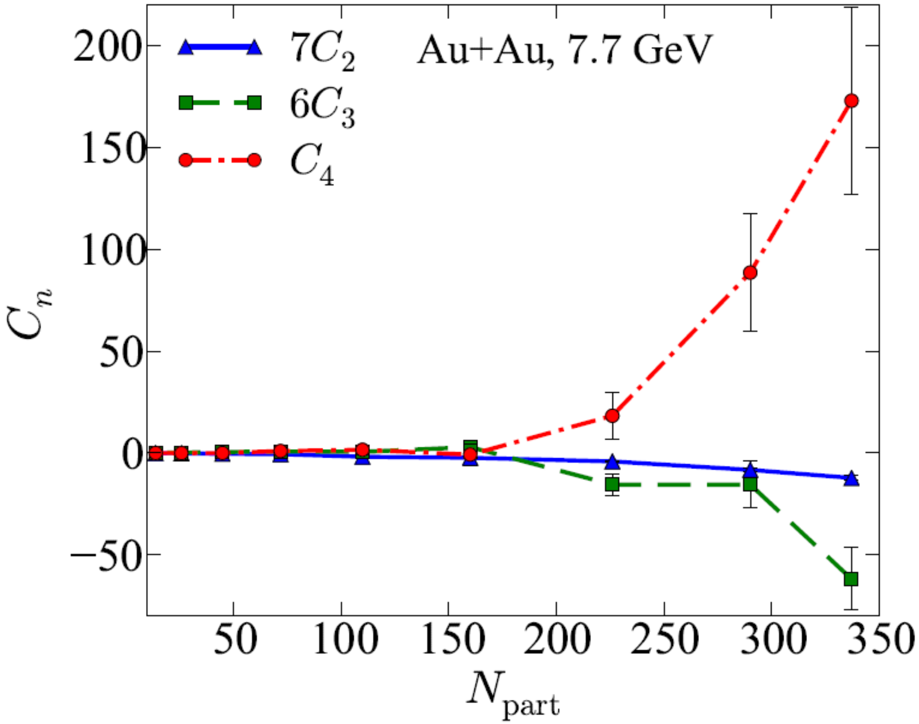
or, e.g.,

cumulants mix correlation functions

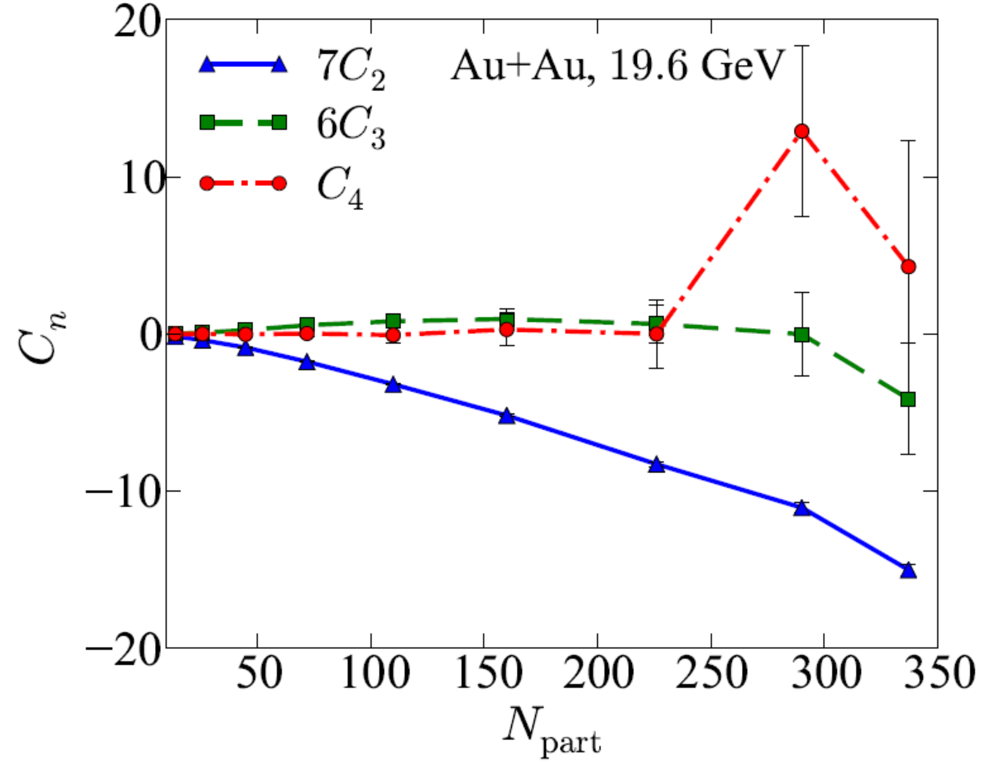
$$K_4 = \langle N \rangle + 7\boldsymbol{C_2} + 6\boldsymbol{C_3} + \boldsymbol{C_4}$$

results for C_n

central signal at **7.7** GeV is driven by **4-particle** correlations



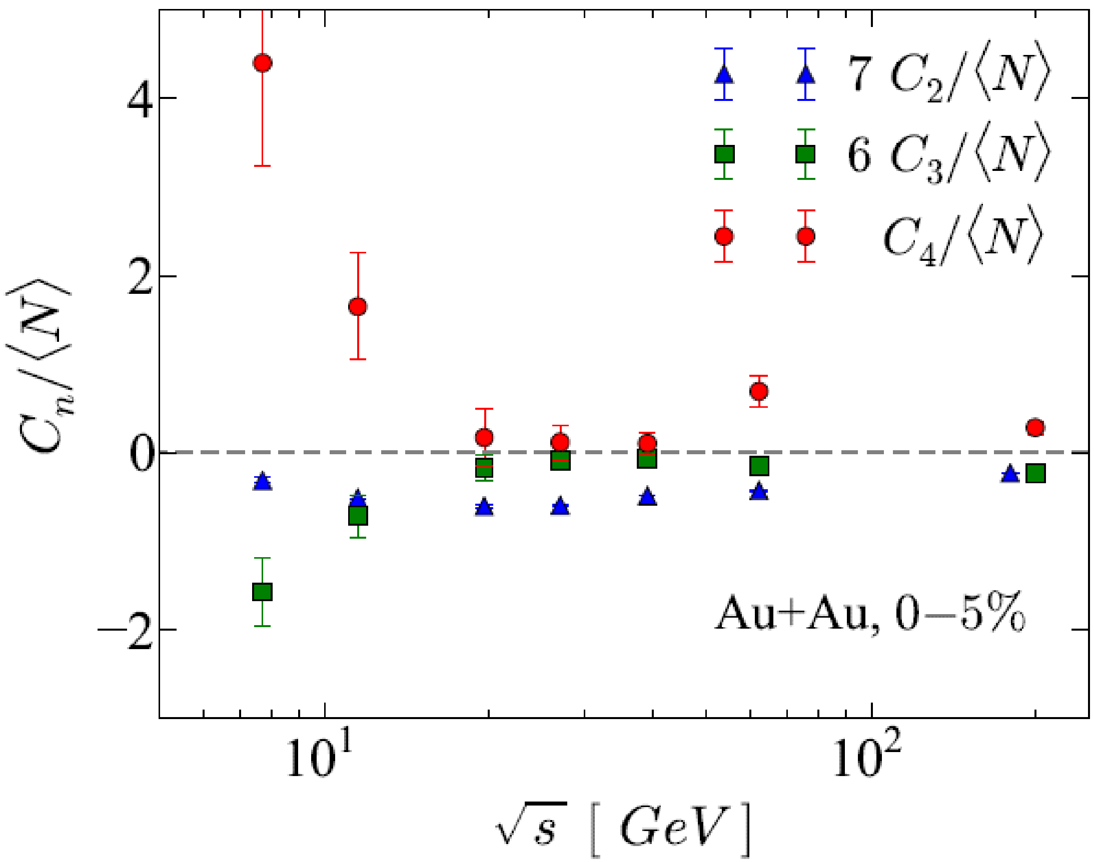
central signal at **19.6** GeV is driven by **2-particle** correlations



$$K_2 = \langle N \rangle + C_2$$

$$K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

AB, V. Koch, N. Strodthoff, arXiv:1607.07375



C_4 at 62 GeV !

$$K_4 = \langle N\rangle + 7\boldsymbol{C_2} + 6\boldsymbol{C_3} + \boldsymbol{C_4}$$

cumulant correlation functions

Observations (i)

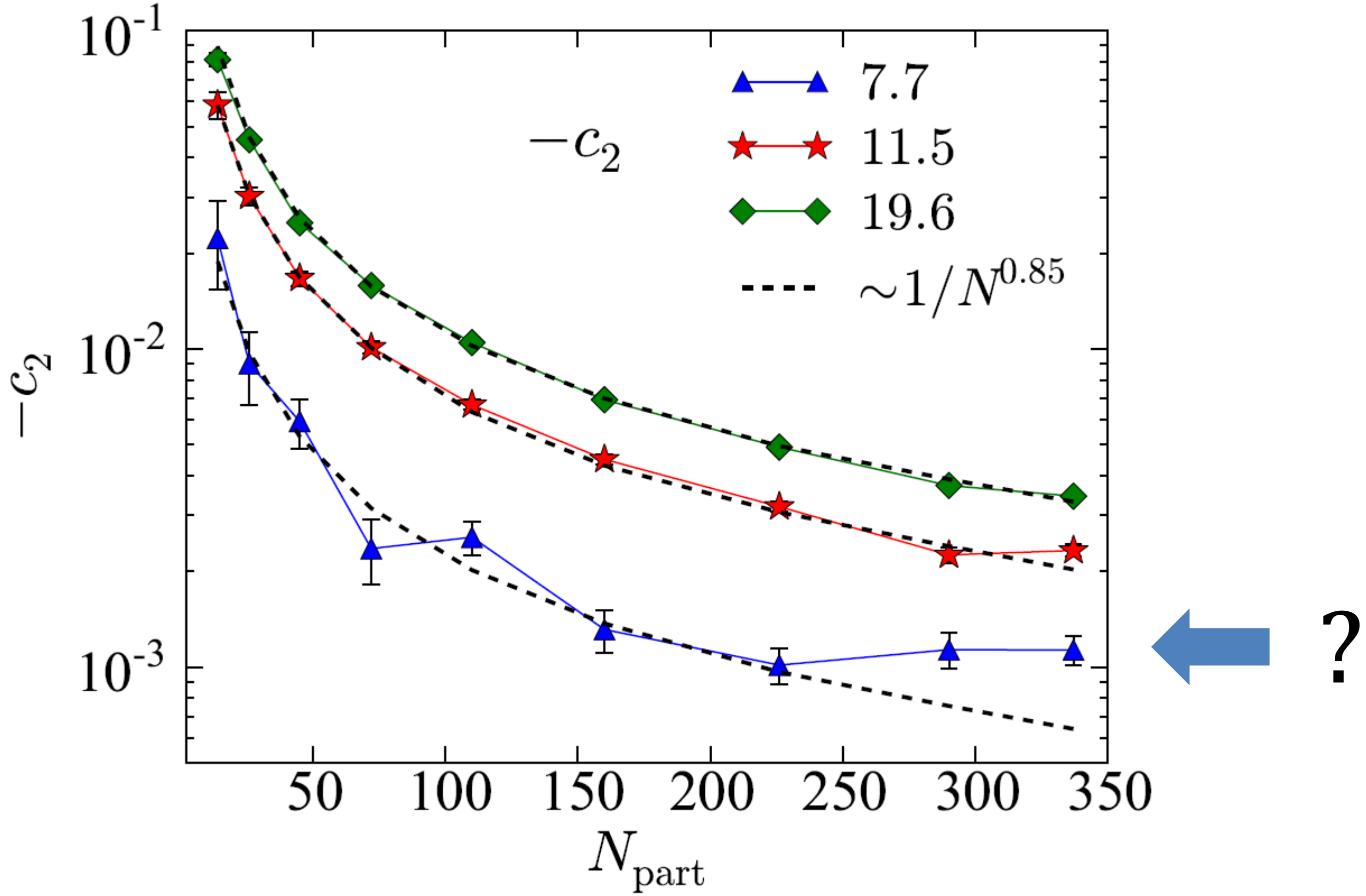
$$K_2 = \langle N \rangle + \langle N \rangle^2 c_2$$

$$K_4 = \langle N \rangle + 7\langle N \rangle^2 c_2 + 6\langle N \rangle^3 c_3 + \langle N \rangle^4 c_4$$

Suppose we have N_s **independent sources** of correlations (resonances, superposition of p+p etc.)

$$c_k \sim \frac{N_s}{N^k} \sim \frac{1}{N^{k-1}}$$

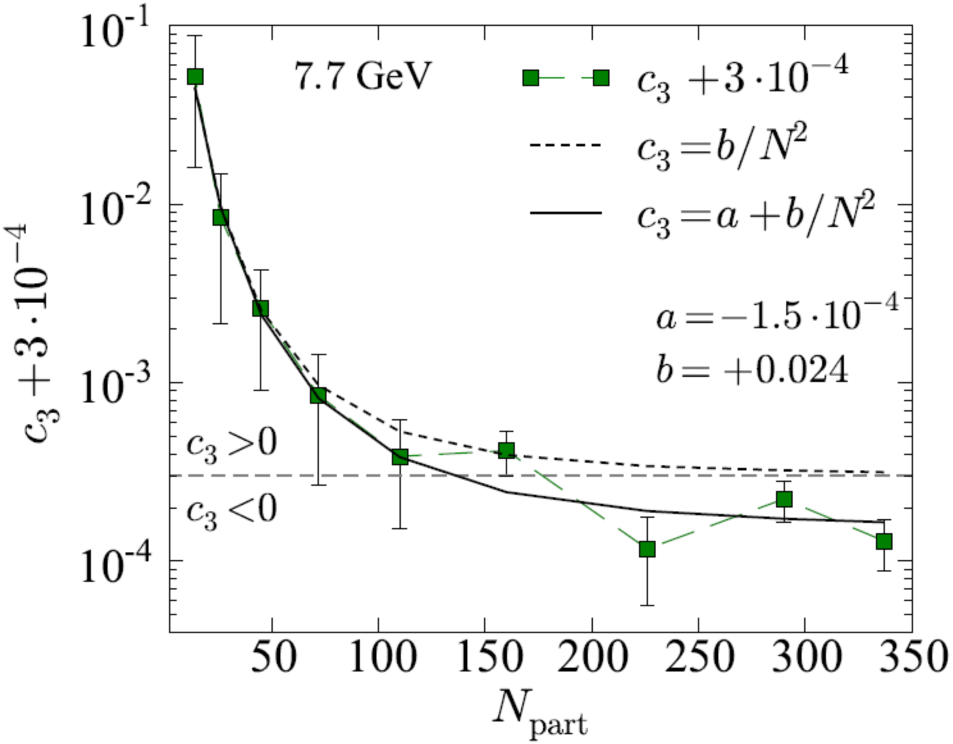
results for c_2

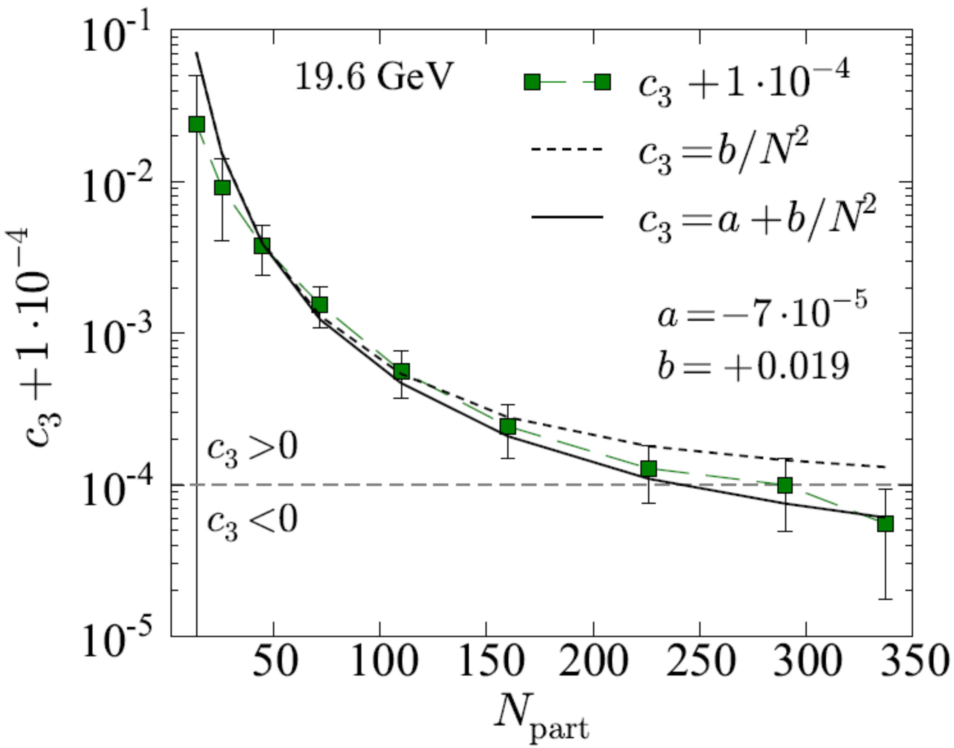


central 7 GeV points are somehow special

results for c_3

AB, V. Koch, N. Strodthoff, arXiv:1607.07375





At 7 GeV c_3 changes sign and is roughly constant

Similar stuff for c_4 (backup)

Observations (ii)

$$c_2 = \frac{\int \rho(y_1)\rho(y_2)c_2(y_1,y_2)dy_1dy_2}{\int \rho(y_1)\rho(y_2)dy_1dy_2}$$

$$K_2 = \langle N\rangle + \langle N\rangle^2 c_2$$

$$K_4 = \langle N\rangle + 7\langle N\rangle^2 c_2 + 6\langle N\rangle^3 c_3 + \langle N\rangle^4 c_4$$

Rapidity dependence:

long-range correlation

$$c_n(y_1, \dots, y_n) = c_n^0$$

$$c_n = c_n^0$$

short-range correlation

$$c_2(y_1, y_2) = c_2^0 \delta(y_1 - y_2)$$

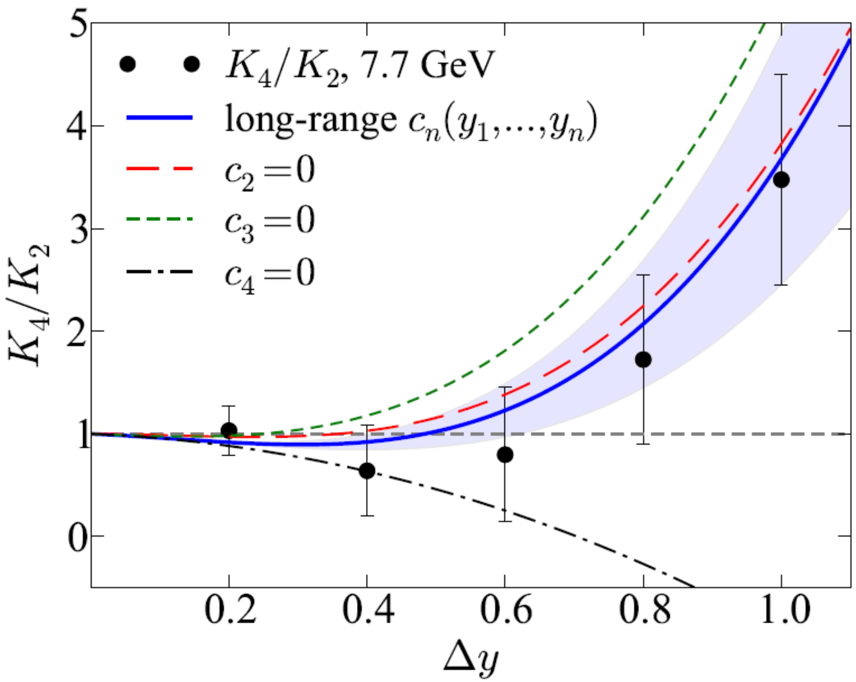
$$c_2 \sim 1/(\Delta y)$$

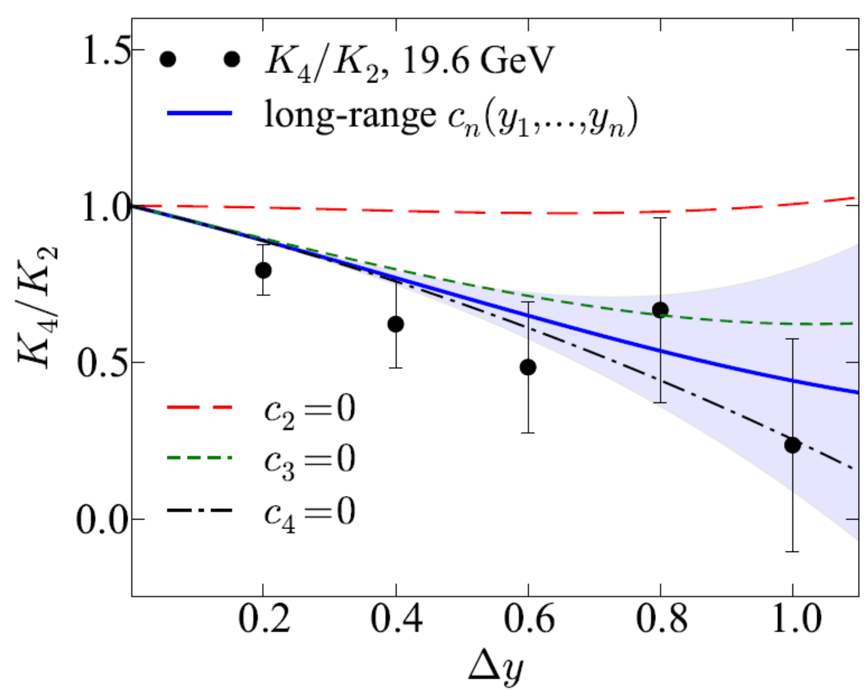
$$K_n \sim \Delta y$$

$$K_2 = \langle N\rangle + c_2^0\langle N\rangle^2, \qquad \langle N\rangle \sim \Delta y$$

$$K_4 = \langle N\rangle + 7c_2^0\langle N\rangle^2 + 6c_3^0\langle N\rangle^3 + c_4^0\langle N\rangle^4$$

Rapidity dependence consistent with long-range correlations





$|y| < 0.5$ is not particularly large

Initial state effect? (e.g., volume fluctuation)

Volume fluctuation has some interesting and promising properties for central collisions, see talk by V. Skokov

It would be great to see Δy dependence of $\boldsymbol{C_n}$ (separately for protons and anti-protons) for all energies and centralities.

Suppose that always $\boldsymbol{C_n} \sim (\Delta y)^n$

What does that mean? Most likely initial state effect

$$K_4 = \langle N \rangle + 7\boldsymbol{C_2} + 6\boldsymbol{C_3} + \boldsymbol{C_4}$$

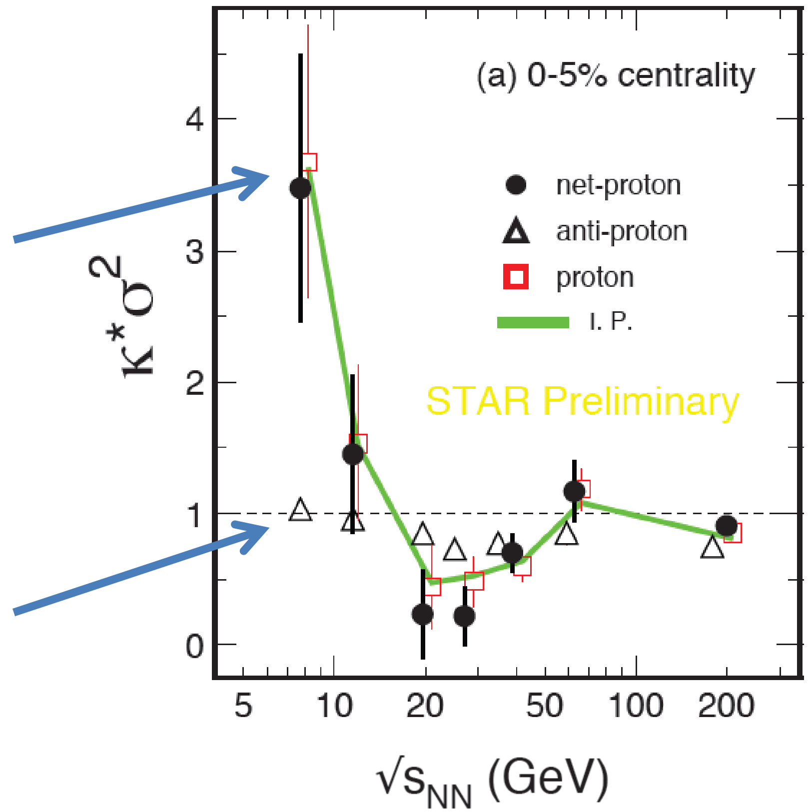
cumulant correlation functions

Observations (iii)

$$K_2 = \langle N \rangle + \langle N \rangle^2 c_2$$

$$K_4 = \langle N \rangle + 7\langle N \rangle^2 c_2 + 6\langle N \rangle^3 c_3 + \langle N \rangle^4 c_4$$

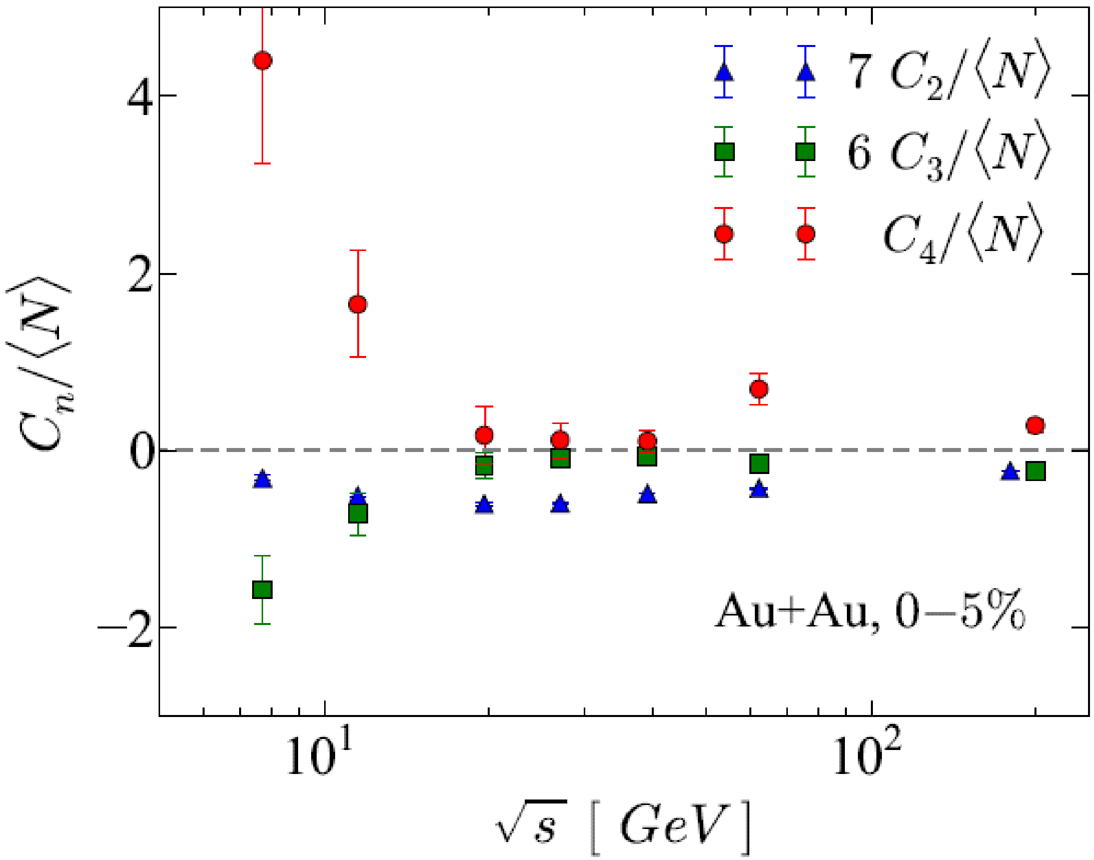
If c_n weakly depends on N than for $\langle N \rangle \ll 1$ (**anti-protons**) $K_n \approx \langle N \rangle$



$$\langle N_{\rm proton} \rangle \sim 40$$

$$\langle N_{\rm anti-proton} \rangle \sim 0.25$$

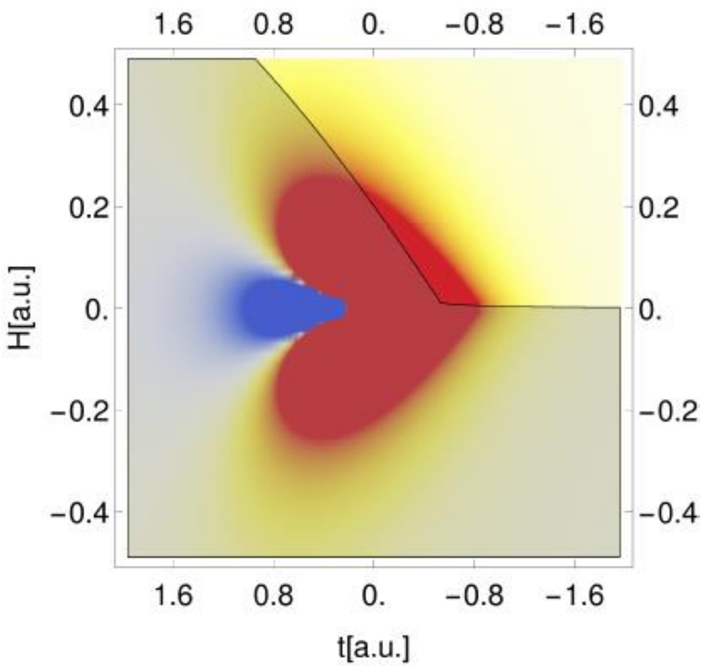
Exclusions plots



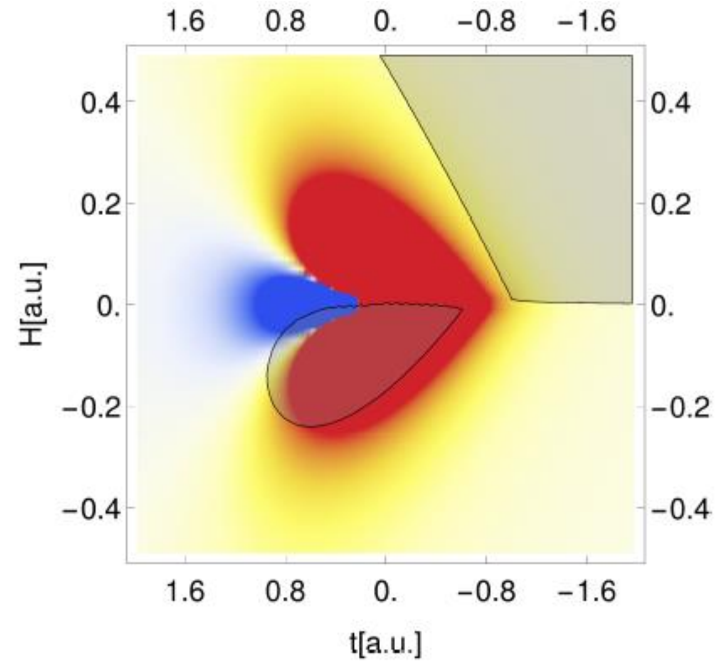
$$K_4 = \langle N\rangle + 7\boldsymbol{C_2} + 6\boldsymbol{C_3} + \boldsymbol{C_4}$$

cumulant correlation functions

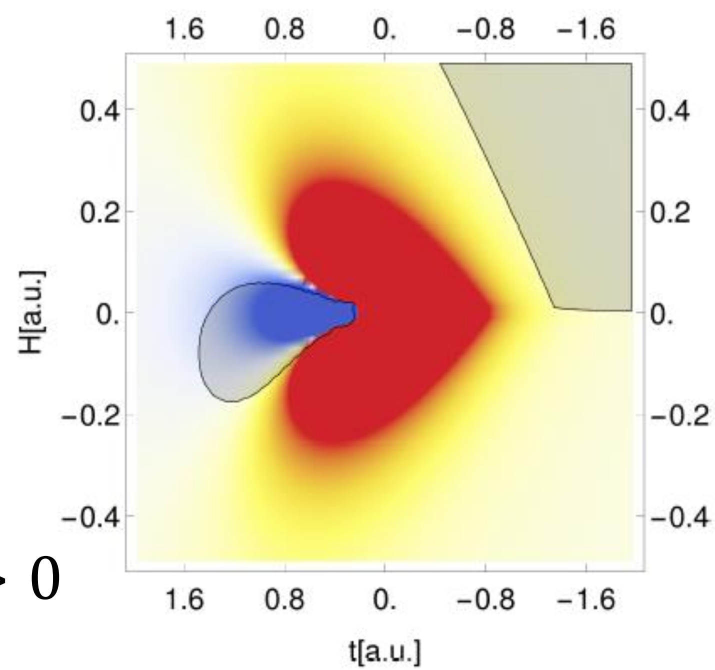
Exclusions plots (Ising model)



$C_2 < 0$



$C_3 < 0$



$C_4 > 0$

Take-home message

- Cumulants are rather tricky to interpret
- Multi-particle correlations seems to be more natural
- Independent sources vs collective sources
- Long-range rapidity vs short-range rapidity
- Let’s do **exclusions plots**

Baryon stopping

At low energy protons are not produced. They are transferred from incoming nucleus.

There is no infinite deceleration. It take some time and length to slow down or stop a proton.

$$E_z = E_i - \sigma(z - z_c)$$

E_i – initial energy

z_c – collision point

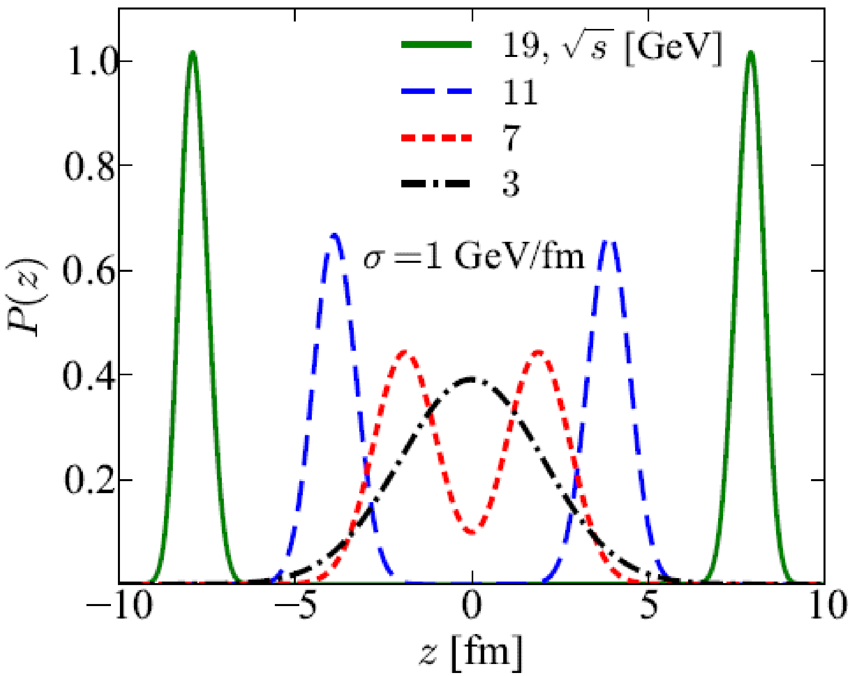
E_z – energy at a point z

$$E_z \to M_t \cosh(y)$$

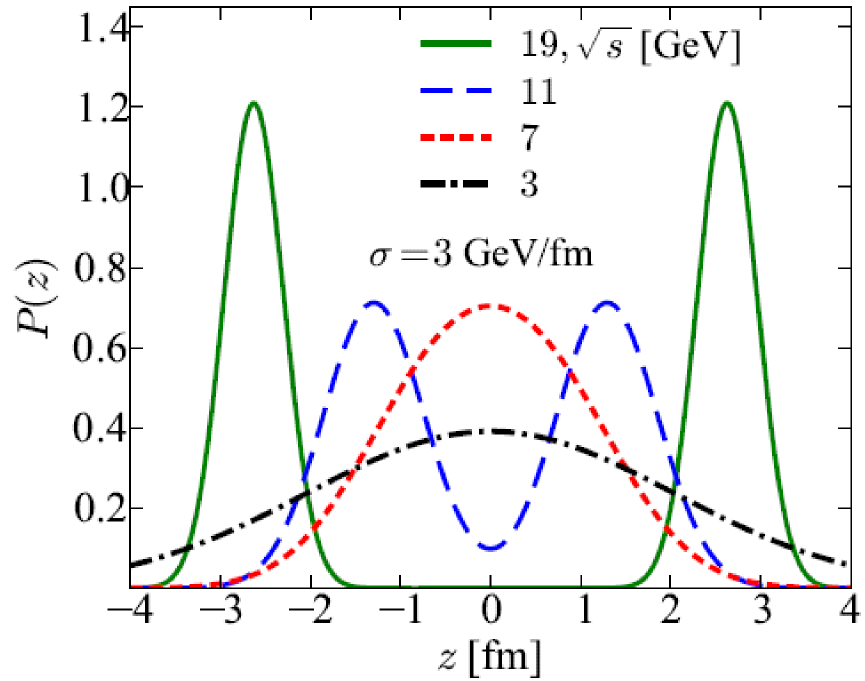
σ – energy loss per unit length

Distribution of protons in z

A.Bialas, AB, V.Koch, 1608.07041



wounded nucleon model



wounded quark model

Are protons stopped in pairs, triplets etc.?

Correlation between pions and protons from stopping?

Conclusions

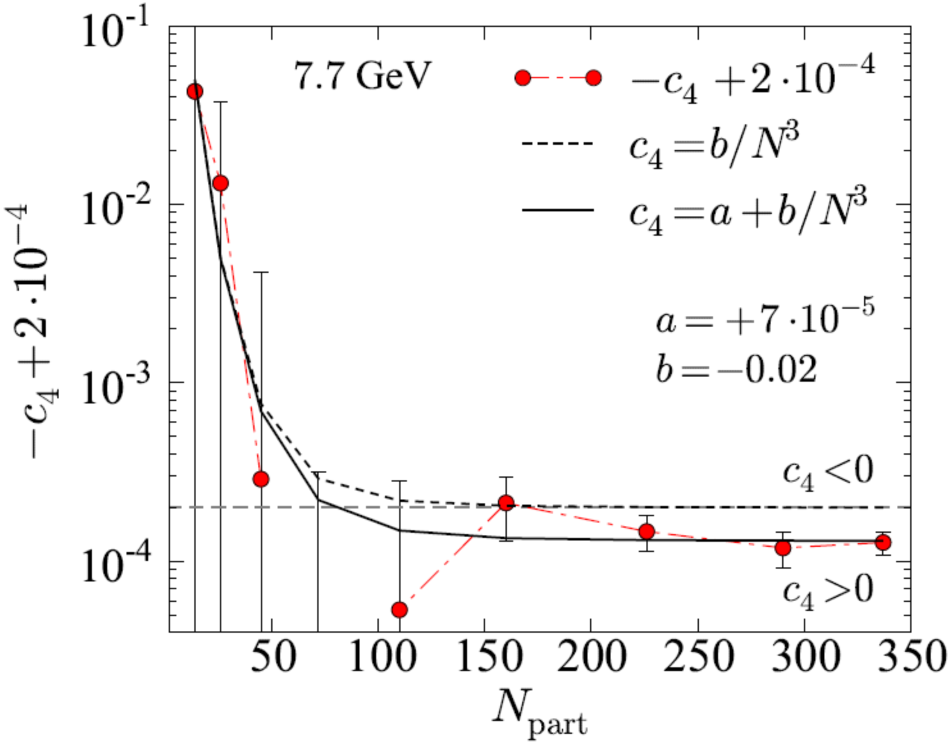
Efficiency story not yet over, non-binomial corrections are surprisingly strong

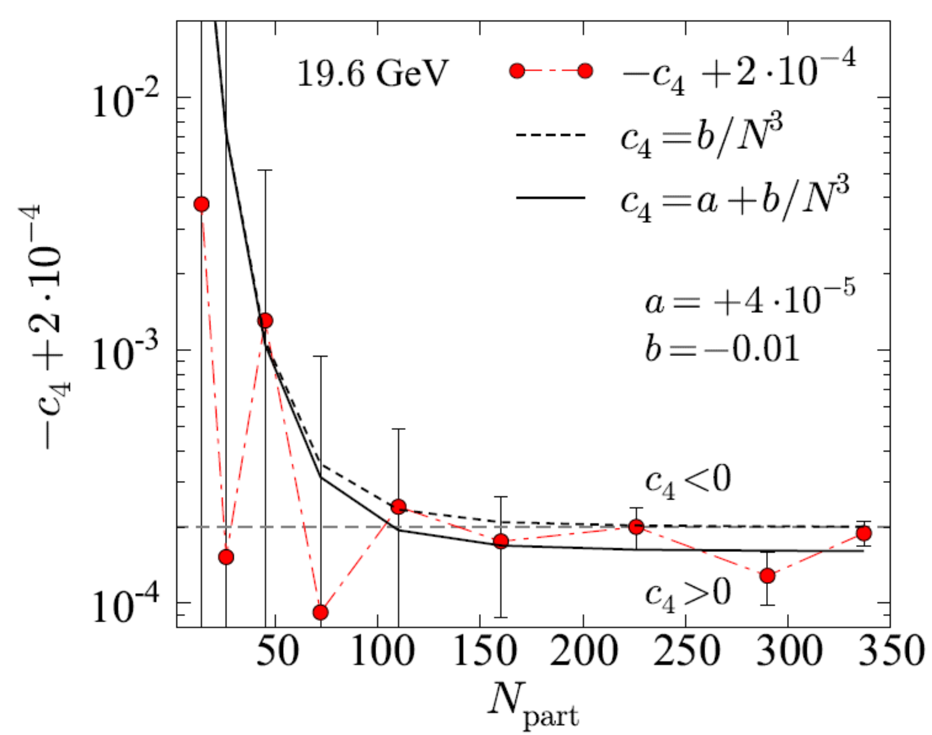
Multi-particle correlations carry interesting information. Independent vs collective sources, short- vs long-range correlations, exclusion plots …

The effect of baryon stopping not yet understood. Disconnected stopped protons in the z direction ?

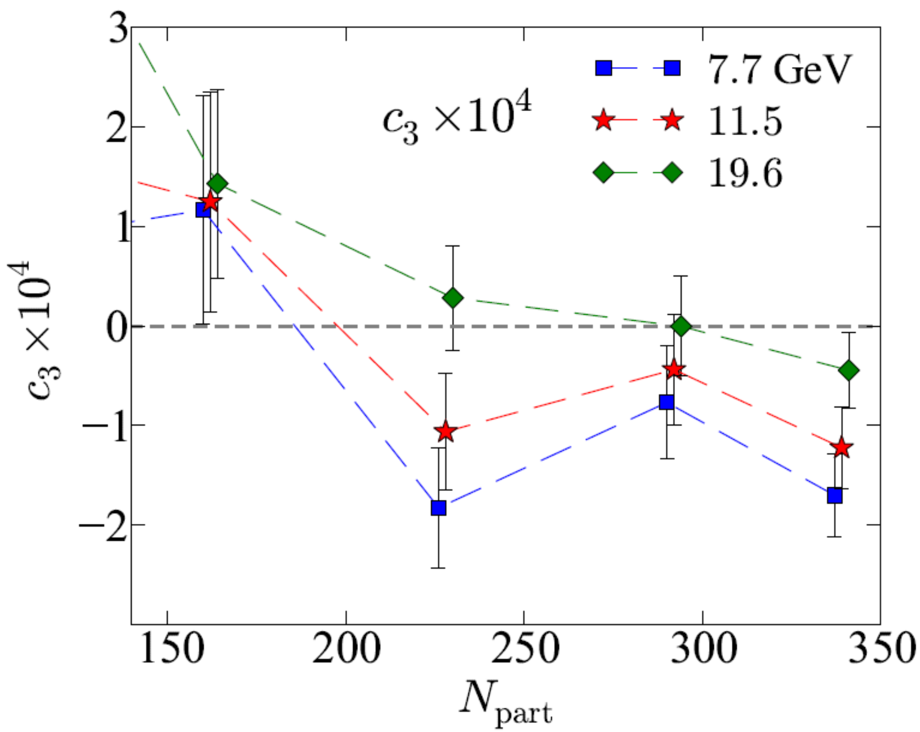
Backup

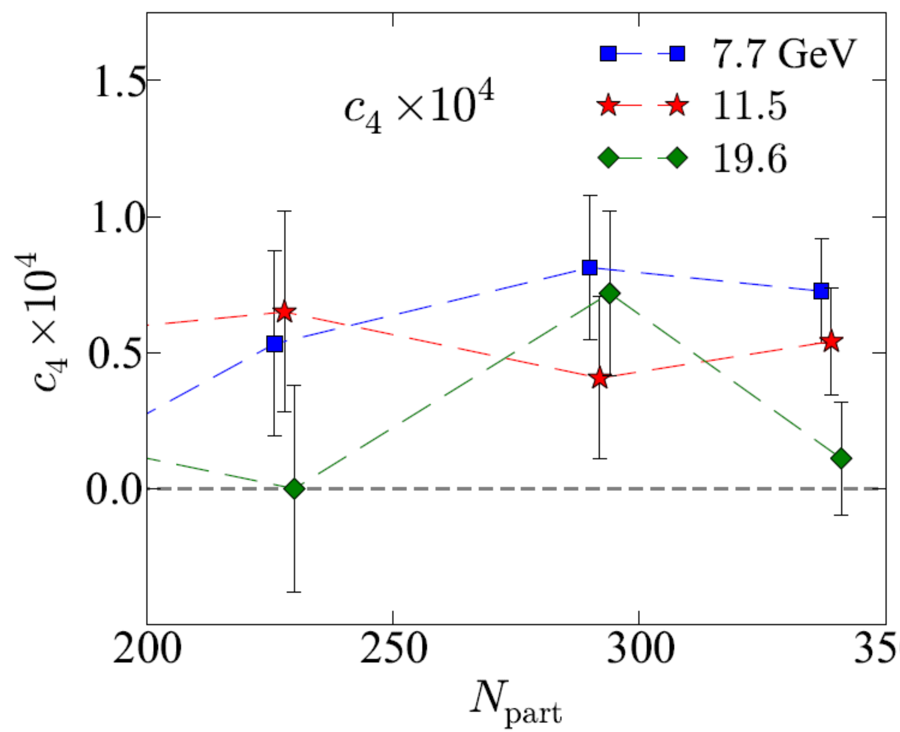
results for c_4

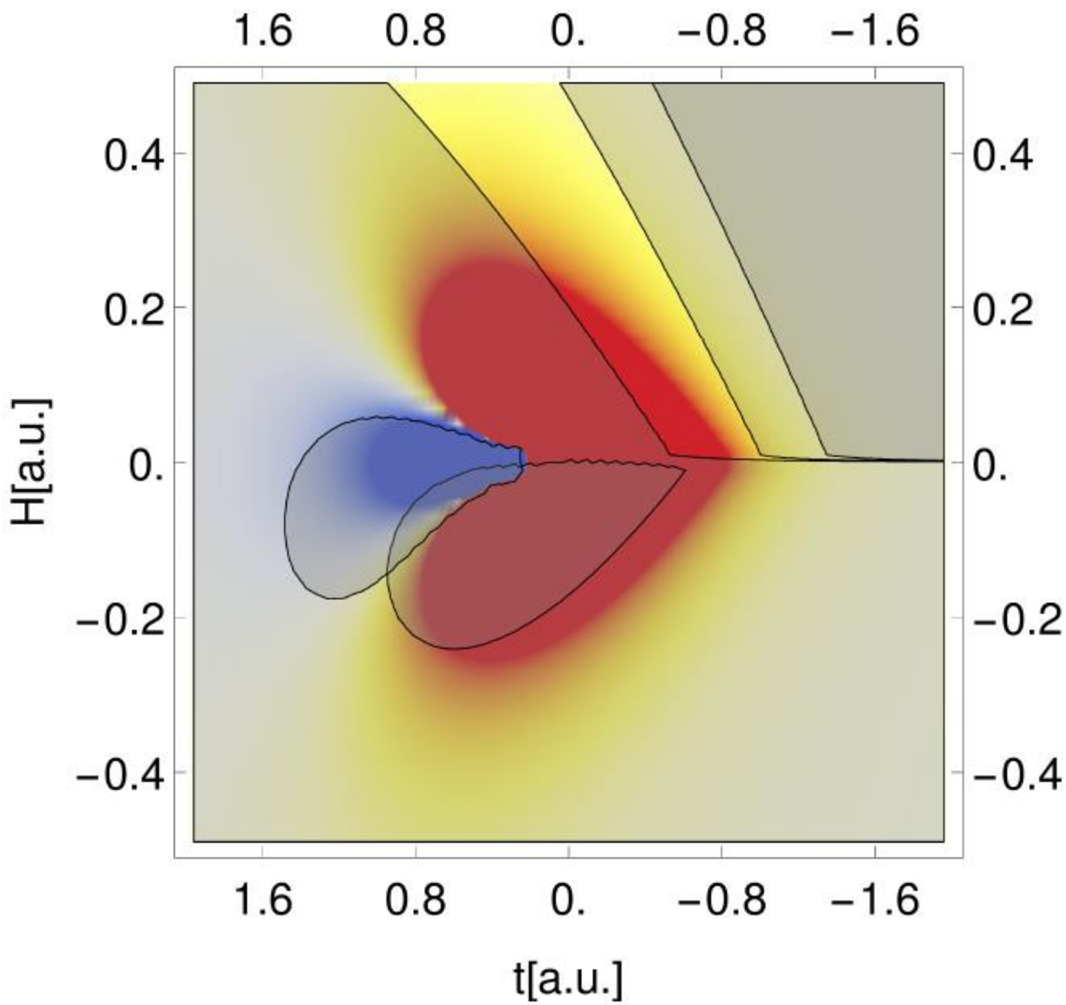




results for central c_3







$$C_2 < 0 \ \& \ C_3 < 0 \ \& \ C_4 > 0$$

We need to do proper unfolding
For example:

$$p(n) = \sum_{N=n}^{\infty} P(N) \frac{N!}{n!(N-n)!} \epsilon^n (1-\epsilon)^{N-n}$$

$$\begin{pmatrix} p(0) \\ p(1) \\ p(2) \\ p(3) \\ p(4) \end{pmatrix} = \begin{pmatrix} 1 & 1-\epsilon & (1-\epsilon)^2 & (1-\epsilon)^3 & (1-\epsilon)^4 \\ 0 & \epsilon & 2\epsilon(1-\epsilon) & 3\epsilon(1-\epsilon)^2 & 4\epsilon(1-\epsilon)^3 \\ 0 & 0 & \epsilon^2 & 3\epsilon^2(1-\epsilon) & 6\epsilon^2(1-\epsilon)^2 \\ 0 & 0 & 0 & \epsilon^3 & 4\epsilon^3(1-\epsilon) \\ 0 & 0 & 0 & 0 & \epsilon^4 \end{pmatrix} \begin{pmatrix} P(0) \\ P(1) \\ P(2) \\ P(3) \\ P(4) \end{pmatrix}$$

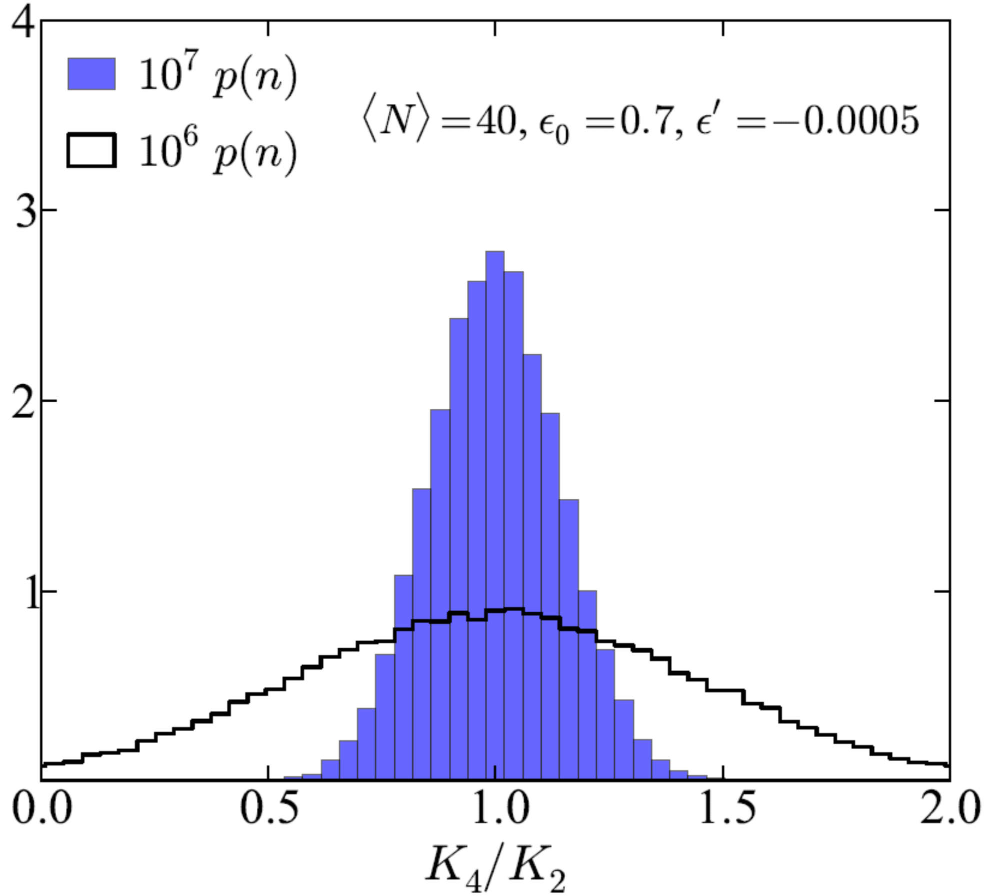
matrix is pseudo-singular

We can easily use $\epsilon(N)$, matrix is much more complicated but it is not a big deal.

In general

$$p(n) = \sum_{N=n} P(N) B(n; N)$$

The method works for $\epsilon(N)$



It works very well, statistical errors are under control