Discovering the QCD critical point with net-proton fluctuations

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INT-16-3 – *Exploring the QCD Phase Diagram through Energy Scans*

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Prelude

- **thermal** (fluid dynamical) **fluctuations** important for fluids undergoing phase transitions, nucleation, showing instabilities, ...
- relevant in micro-engineering, molecular biology, combustive ignition, ...
- stochastic correlations between fluctuating quantities
  $\rightarrow$ consider a non-relativistic fluid:
  $\langle \delta v_i(x,t)\delta v_j(x',t)\rangle = \frac{T}{\rho}\delta_{ij}\delta(x-x')$
- (linearized) fluid dynamics propagates these, e.g. as *shear* or *sound* modes

$$\langle \delta v_i^T \delta v_j^T \rangle_{\omega,k} = \frac{2T}{\rho}\left(\delta_{ij} - \hat{k}_i\hat{k}_j\right)\frac{\eta k^2/\rho}{\omega^2 + (\eta k^2/\rho)^2}$$

$$\langle \delta v_i^L \delta v_j^L \rangle_{\omega,k} = \frac{2T}{\rho}\hat{k}_i\hat{k}_j\frac{\omega k^2 \Gamma}{(\omega^2 - c_s^2 k^2)^2 + (\omega k^2 \Gamma)^2}\,;\, \Gamma = \frac{4}{3}\frac{\eta}{\rho} + \ldots$$

Prelude

$\rightarrow$ consider the correlation function

$$G_S^{xyxy} = \langle\{\pi^{xy}, \pi^{xy}\}\rangle_{\omega,k} \simeq \rho^2 \langle\{v_x v_y, v_x v_y\}\rangle_{\omega,k}$$

$\rightarrow$ match this to response function in Kubo-limit ($\omega \rightarrow 0$)

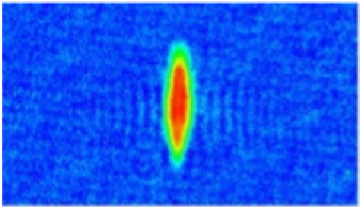
$$G_R^{xyxy} = P + \delta P - i\omega(\eta + \delta\eta) + \omega^2(\eta\tau_\pi + \delta(\eta\tau_\pi))$$

- $\delta P \sim T\Lambda^3$
  cutoff-dependent pressure correction
- $\delta\eta \sim \frac{T\rho\Lambda}{\eta}$
  small $\eta \Rightarrow$ large $\delta\eta$ (there must be a lower bound on η/n induced by fluctuations)
- $\delta(\eta\tau_\pi) \sim \frac{1}{\sqrt{\omega}} \frac{T\rho^{3/2}}{\eta^{3/2}}$
  for $\omega \rightarrow 0$ find $\delta(\eta\tau_\pi) \rightarrow \infty \Rightarrow 2^{\text{nd}}$-order fluid dynamics inconsistent without fluctuations

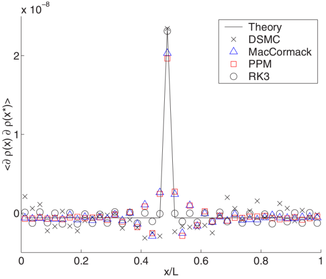
Prelude

Landau-Lifshitz-Navier-Stokes fluid dynamics

$\Rightarrow$ add stochastic flux terms to fluid dynamical equations



anisotropic fluid dynamical expansion of an ultra-cold Fermi gas cloud at unitarity



1d, dilute gas, periodic boundary conditions

- numerics sensitive to discretization of noise $\langle \xi^2 \rangle \propto 1/\Delta V \sim \Lambda^3$
- correct implementation possible; 3$^{\mathrm{rd}}$-order methods seem to work best

J.B. Bell et al., PRE 76 (2007) 016708

Prelude

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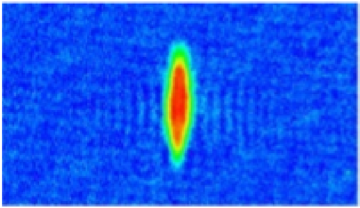

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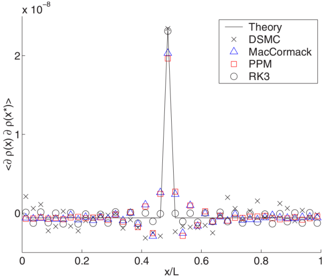

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Prelude

- event-by-event fluctuations of the net-proton number expected to signal a critical point (CP) in the QCD phase diagram
- in td. limit: net-baryon number susceptibility diverges at CP
- associated with fluctuations in the chiral order parameter σ
- higher-order fluctuations, $\langle(\delta\sigma)^n\rangle_c \propto \xi^{m(n)}$, depend stronger on correlation length ξ
- system created in a HIC is short-lived, spatially small, inhomogeneous, highly dynamical
- $\tau \propto \xi^z \Rightarrow$ critical slowing down, memory effects

$\Rightarrow$ Relativistic fluid dynamics including fluctuations/dissipation!

- final stage processes could wash out critical fluctuation signals:
  - isospin redistribution,
  - resonance decays, ...

$\Rightarrow$ Can critical fluctuation signals survive resonance decays?

Y. Hatta and M.A. Stephanov, PRL 91 (2003) 102003; M.A. Stephanov, PRD 65 (2002) 096008; *ibid.* PRL 102 (2009) 032301; B. Berdnikov and K. Rajagopal, PRD 61 (2000) 105017; M. Nahrgang et al., PRC 84 (2011) 024912; S. Mukherjee et al., PRC 92 (2015) 034912; C. Herold et al., PRC 93 (2016) 021902; M. Kitazawa and M. Asakawa, PRC 86 (2012) 024904; M. Nahrgang et al., Eur. Phys. J. C 75 (2015) 573

P A R T – I

Recent progress in simulating relativistic

Fluctuating Dissipative Fluid Dynamics

with Yuri Karpenko

Remarks about the critical mode

- at $\mu \neq 0$, σ mixes with the net-baryon density n (and e, ...)
- in a Ginzburg-Landau formalism:

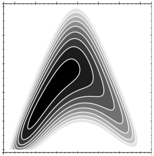
$$V(\sigma, n) = \int d^3x \left(\sum_m [a_m\sigma^m + b_m n^m] + \sum_{m,l} c_{m,l}\sigma^m n^l \right) - h\sigma - jn$$

- $V(\sigma, n)$: flat direction $(a\sigma, -cn)$ with vanishing curvature at CP
- equations of motion (including the symmetries of $V(\sigma, n)$):

$$\partial_t\sigma = -\Gamma(\delta V/\delta\sigma) + \ldots \quad ; \quad \partial_t n = \gamma\nabla^2(\delta V/\delta n) + \ldots$$

- eigenfrequencies and -modes:

$\omega_1 \propto -i\Gamma a$; $(1, 0)$ $\rightarrow$ fast mode
$\omega_2 \propto -i\gamma Dk^2/a$; $(-c, a)$ $\rightarrow$ slow mode



NJL for Z_2-CP

$\Rightarrow$ the diffusive mode becomes the true critical (slow) mode near the CP in the long-time dynamics

H. Fuji, M. Ohtani PRD70 (2004); M. Stephanov, D. Son PRD70 (2004)

Fluid dynamical fluctuations

Conventional fluid dynamics propagates thermal averages of the energy density, pressure, velocities, charge densities, etc.

However, ...

- ... already in equilibrium there are thermal fluctuations!
- ... the fast processes, which lead to local equilibration also lead to noise!

Conventional ideal fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{eq}}$$
$$N^{\mu} = N^{\mu}_{\mathrm{eq}}$$

Y. Minami, T. Kunihiro, PTP122 (2010); P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); J. Kapusta, B. Müller, M. Stephanov PRC85 (2012); J. Kapusta, J. Torres-Rincon PRC86 (2012); C. Chafin, T. Schäfer, PRA87 (2013); P. Romatschke, R. Young, PRA87 (2013); P. Kovtun, G. Moore, P. Romatschke, JHEP1407 (2014); J. Kapusta, C. Young, PRC90 (2014); C. Young, J. Kapusta, C. Gale, S. Jeon, B. Schenke, PRC91 (2015); K. Murase, T. Hirano, 1304.3243; *ibid.* 1601.02260

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Conventional viscous fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{eq}} + \Delta T^{\mu\nu}_{\mathrm{visc}}$$
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Fluctuating viscous fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \Delta T^{\mu\nu}_{\rm visc} + \Xi^{\mu\nu}$$
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- In second-order fluid dynamics there are relaxation equations for $\Xi^{\mu\nu}, \ldots$:

$$u^{\gamma}\partial_{\gamma}\Xi^{\langle\mu\nu\rangle} = -\frac{\Xi^{\mu\nu} - \xi^{\mu\nu}}{\tau_{\pi}}$$

with (white) noise correlators in linear response theory

$$\langle \xi^{\mu\nu}(x)\xi^{\alpha\beta}(x')\rangle = 2T[\eta(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}) + (\zeta - 2/3\eta)\Delta^{\mu\nu}\Delta^{\alpha\beta}]\,\delta^4(x - x')$$

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Fluctuating viscous fluid dynamics: $\Rightarrow$ neglect N^μ for now!

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \Delta T^{\mu\nu}_{\rm visc} + \Xi^{\mu\nu}$$
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Fluid dynamical fluctuations - nonlinearities

- correlation functions in linearized fluid dynamics describe noninteracting modes
- if nonlinearities are included $\rightarrow$ interaction of modes
  $\rightarrow$ modification of correlations
  $\rightarrow$ contributions to transport coefficients, ...
- symmetrized correlator:

$$G_S^{xyxy}(\omega, \mathbf{0}) = \int \mathrm{d}^3x \mathrm{d}t\, e^{i(\omega t - \mathbf{k}\cdot\mathbf{x})} \left\langle \frac{1}{2}\{T^{xy}(t, \mathbf{x}), T^{xy}(0, \mathbf{0})\} \right\rangle$$

- for the shear-shear contribution $\Rightarrow$

$$G^{xyxy}_{R,\mathrm{shear-shear}}(\omega, \mathbf{0}) = -\frac{7T}{90\pi^2}\Lambda^3 - i\omega\frac{7T}{60\pi^2}\frac{\Lambda}{\gamma_\eta} + (i+1)\omega^{3/2}\frac{7T}{90\pi^2}\frac{1}{\gamma_\eta^{3/2}}$$

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cutoff-dependent fluctuation contribution to pressure

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cutoff-dependent fluctuation contribution to pressure

cutoff-dependent correction to η

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cutoff-dependent fluctuation contribution to pressure

cutoff-dependent correction to η

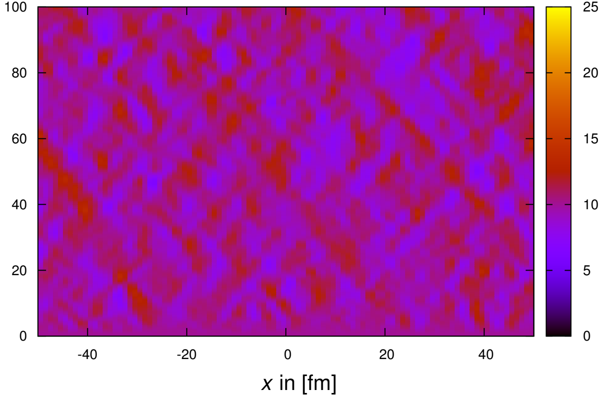
frequency-dependent contribution to η and τ_π

P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); C. Chafin, T. Schäfer, PRA87 (2013); P. Romatschke, R. Young, PRA87 (2013)

Fluid dynamical fluctuations - 1+1d

- Static "box" with periodic boundary conditions in relativistic $1+1$d fluid dynamics
- Initialized at $e_0 = 10$ GeV/fm^3 (without fluctuations nothing would happen)

consider time evolution in [fm] of e in [GeV/fm^3] for $\Delta x = 1.0$ fm



M. Nahrgang, MB, Y. Karpenko, S. Bass, T. Schäfer, work in progress

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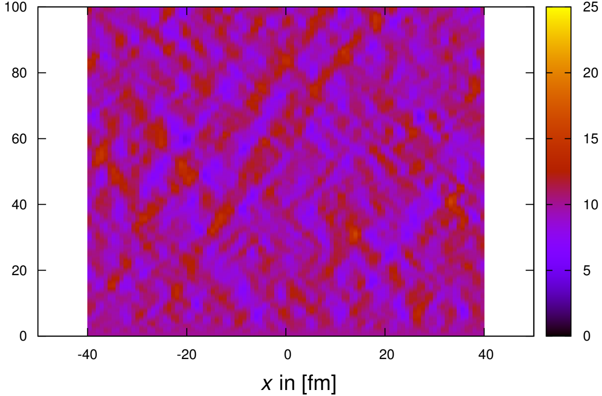
consider time evolution in [fm] of e in [GeV/fm^3] for $\Delta x = 0.8$ fm



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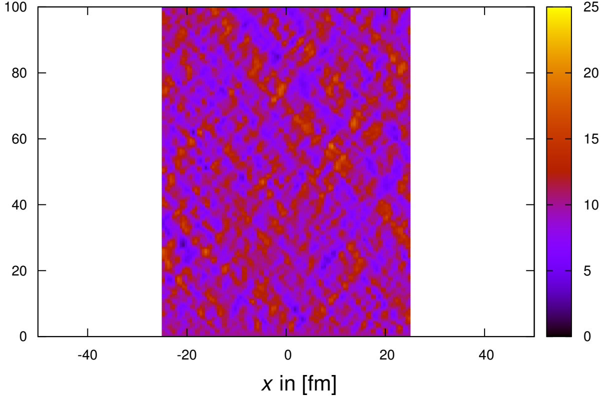
consider time evolution in [fm] of e in [GeV/fm^3] for $\Delta x = 0.5$ fm



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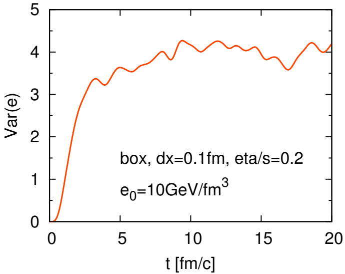
Fluid dynamical fluctuations - 3+1d

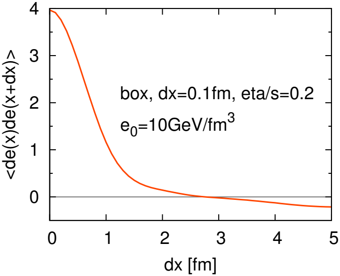
- Static box with periodic boundary conditions in relativistic $3+1$d fluid dynamics

$$\partial_\mu T^{\mu\nu} = \partial_\mu \left(T^{\mu\nu}_{\rm eq} + \Delta T^{\mu\nu}_{\rm visc} + \Xi^{\mu\nu} \right) = 0$$

time evolution of the variance $\langle (\Delta e)^2 \rangle$:



$\langle \delta e(x) \delta e(x+dx) \rangle$ correlation function:



- The variance $\langle (\Delta e)^2 \rangle$ saturates after ~ 5 fm.
- Fluctuations are large in the computational cell of fluid dynamics $\Rightarrow$ noise correlated over ~ 1 fm^3 – reproduced.

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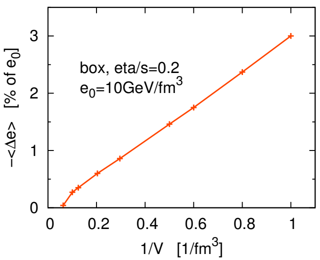
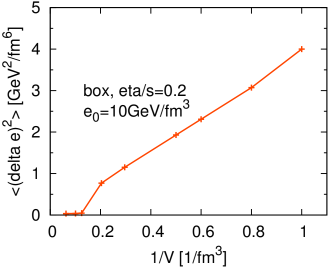
Fluid dynamical fluctuations - 3+1d nonlinearities

- Important check: equilibrium expectations for fluctuations and nonlinear effects!





- Proportionality to $1/V$ reproduced for the correction to the average and the variance of energy density in the local rest frame.
- Implementing fluid dynamical fluctuations is important,

  but requires a sustained and systematic effort!

M. Nahrgang, MB, Y. Karpenko, S. Bass, T. Schäfer, work in progress

What remains to be done!?

- ...apart from the obvious...
- provide realistic initial conditions for HIC
- provide realistic EoS and transport coefficients
- propagate also net-baryon current with fluctuations in fluid dynamics
- evolve σ-field within fluid dynamical background by means of an stochastic Langevin equation or Fokker-Planck equation

$\rightarrow$ Should σ-field fluctuations influence the fluid dynamical evolution?

- make particles from the fluid and evolve them (hadronic stage)

M. Nahrgang et al., PRC 84 (2011) 024912; J. Kapusta and J. Torres-Rincon, PRC 86 (2012) 054911; C. Young, PRC 89 (2014) 024913; S. Mukherjee et al., PRC 92 (2015) 034912; C. Herold et al., PRC 93 (2016) 021902

P A R T – II

Phenomenological study of the

Influence of Resonance Decays on Critical Fluctuations

Critical fluctuations based on Ising-model EoS

$\rightarrow$ need to know equilibrium σ-field fluctuations!

under universality hypothesis:

order parameter magnetization

$$M(r,H) = M_0 R^\beta \theta$$

with parametric representation

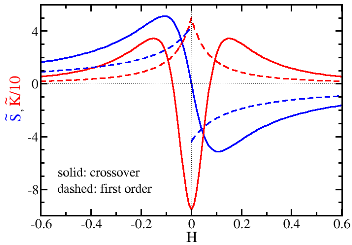
$$r = R(1-\theta^2)\,,\; H = H_0 R^{\beta\delta} \tilde{h}(\theta)$$

σ-field cumulants:

$$\langle (\delta\sigma)^n \rangle_c = \left(\frac{T}{V}\right)^{n-1} \left(\frac{\partial^{n-1} M}{\partial H^{n-1}}\right)_r$$

$\rightarrow$ valid in scaling regime

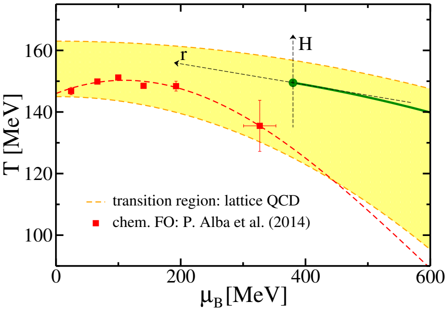
- CP located at $(r,H) = (0,0)$
- for $r > 0$ crossover regime
- for $r < 0$ first-order phase transition at $H = 0$



$\tilde{S} \sim \langle (\delta\sigma)^3 \rangle / \langle (\delta\sigma)^2 \rangle$, $\tilde{K} \sim \langle (\delta\sigma)^4 \rangle_c / \langle (\delta\sigma)^2 \rangle$

P. Schofield, PRL 22 (1969) 606; R. Guida and J. Zinn-Justin, NPB 489 (1997) 626

Mapping to QCD thermodynamics



What is unknown?

- location of CP in (μ_B, T)
- size of critical region
- mapping $r(\mu_B, T)$ and $H(\mu_B, T)$

$\rightarrow$ one preferred direction for r in CP: along first-order transition line

What do we know?

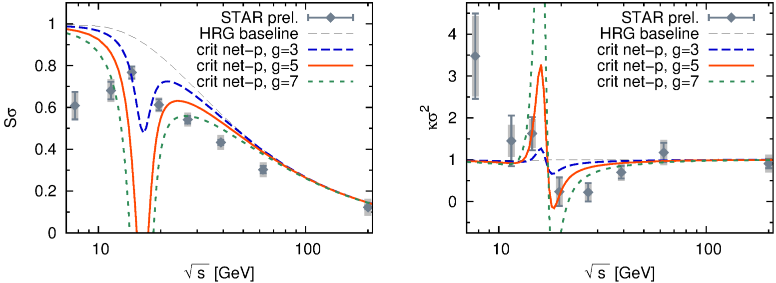
- $T_{c,0} = (154 \pm 9)$ MeV
- chiral crossover curvature $\kappa_c = 0.007 \ldots 0.02$
- chemical freeze-out parameters

Y. Aoki et al., JHEP 06 (2009) 088; A. Bazavov et al., PRD 85 (2012) 054503; O. Kaczmarek et al., PRD 83 (2011) 014504; G. Endrodi et al., JHEP 04 (2011) 001; C. Bonati et al., PRD 92 (2015) 054503; R. Bellwied et al., PLB 751 (2015) 559; P. Cea et al., PRD 93 (2016) 014507; P. Alba et al., PLB 738 (2014) 305; A. Bazavov et al., PRD 93 (2016) 014512

Coupling to observable fluctuations

- effective interaction of particles with σ-field, e.g. $g\bar{p}\sigma p$
- additional critical fluctuation contributions:

$$\delta f_{\mathrm{crit}} = -\frac{g}{T} f^0 (1 \pm f^0) \frac{m}{E} \delta\sigma \quad \text{with} \quad \langle \delta\sigma \rangle = 0$$



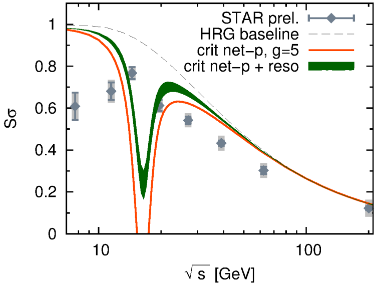
→ critical fluctuations imprinted in net-proton fluctuations

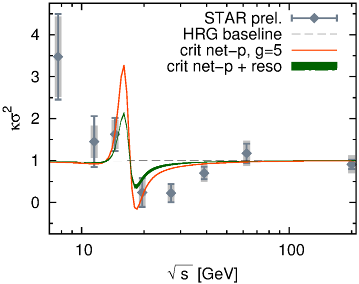
→ magnitude of signal depends on coupling g (and its sign)

M.A. Stephanov et al., PRL 81 (1998) 4816; *ibid.* PRD 60 (1999) 114028; C. Athanasiou et al., PRD 82 (2010) 074008; J. Thäder et al. [STAR Collaboration], 1601.00951

Influence of resonance decays

- resonance decay is a probabilistic process $\rightarrow$ significant contribution to fluctuations
- two limiting cases: no coupling to σ-field vs. chiral model inspired coupling via $g_R = \frac{g}{3}\frac{m_R}{m_p}(3 - |S_R|)$





$\rightarrow$ resonance decays reduce $S\sigma$ by $\sim$ 40% and $\kappa\sigma^2$ by $\sim$ 50%

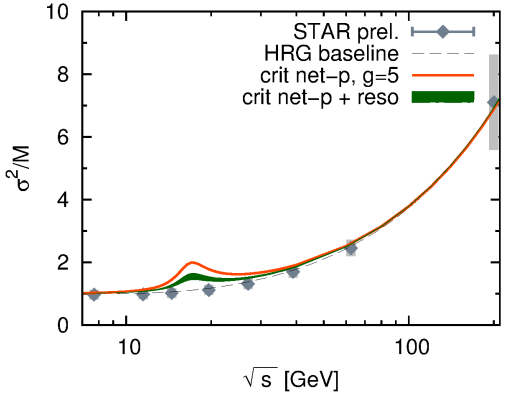
but signal survives!

D. Zschiesche et al., PRC 63 (2001) 025211; V. Dexheimer and S. Schramm, Astroph. J. 683 (2008) 943; J. Thäder et al. [STAR Collaboration], 1601.00951

What about σ^2 / M ?

Absolutely no deviation from the HRG baseline seen in the data.

$\Rightarrow$ provides additional important constraints!



J. Thäder et al. [STAR Collaboration], 1601.00951

Conclusions

- qualitative features of the critical point can be studied with phenomenological models:
  - $\rightarrow$ critical fluctuation signals in net-proton fluctuations are reduced by resonance decays but survive
- for a realistic dynamical treatment need to apply Fluctuating Dissipative Fluid Dynamics:
  - $\rightarrow$ in 1+1d ”box”: evolution of fluctuations clearly visible, volume dependence tested successfully
  - $\rightarrow$ in 3+1d box: expectations for modifications due to nonlinear effects verified, correlations reproduced

$\Rightarrow$ next: study more realistic expansion scenarios!