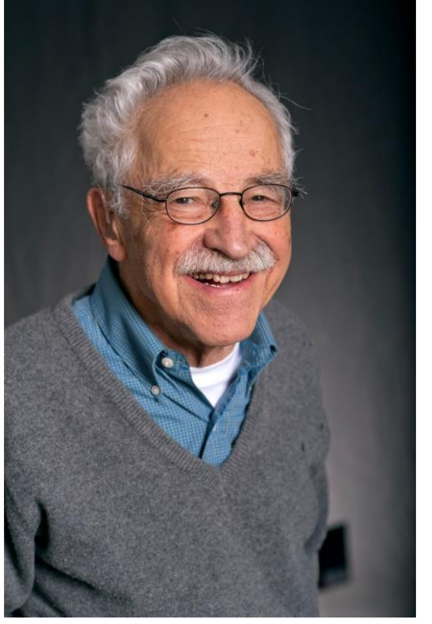
Dedicated to a friend
and mentor (1981-1983)
who taught me to know
and appreciate the power
of symmetries



Five-Dimensional Physics and Soft QCD

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1. Holography…the ADS/CFT approach…successes
2. Hadron Polarizability – Why it's Important
3. Why ADS/CFT fails on Polarizabilities
4. Attempting A Rescue Operation

Augmenting the Gauge-Gravity Correspondence to include Hadron Polarizabilities

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Abstract

ADS/CFT models have achieved considerable success in describing masses and form factors of hadrons but hadronic electric and magnetic polarizabilities vanish if the minimal electromagnetic vertex is used. This contradicts both intuition and experiment. However, if effective vertices are used, and axial and vector mesons are allowed to propagate as intermediate states, then the static polarizabilities can in principle be computed from ADS/CFT.

PACS numbers: 11.25.Tq, 12.38.-t, 13.60.Fz, 24.85.+p

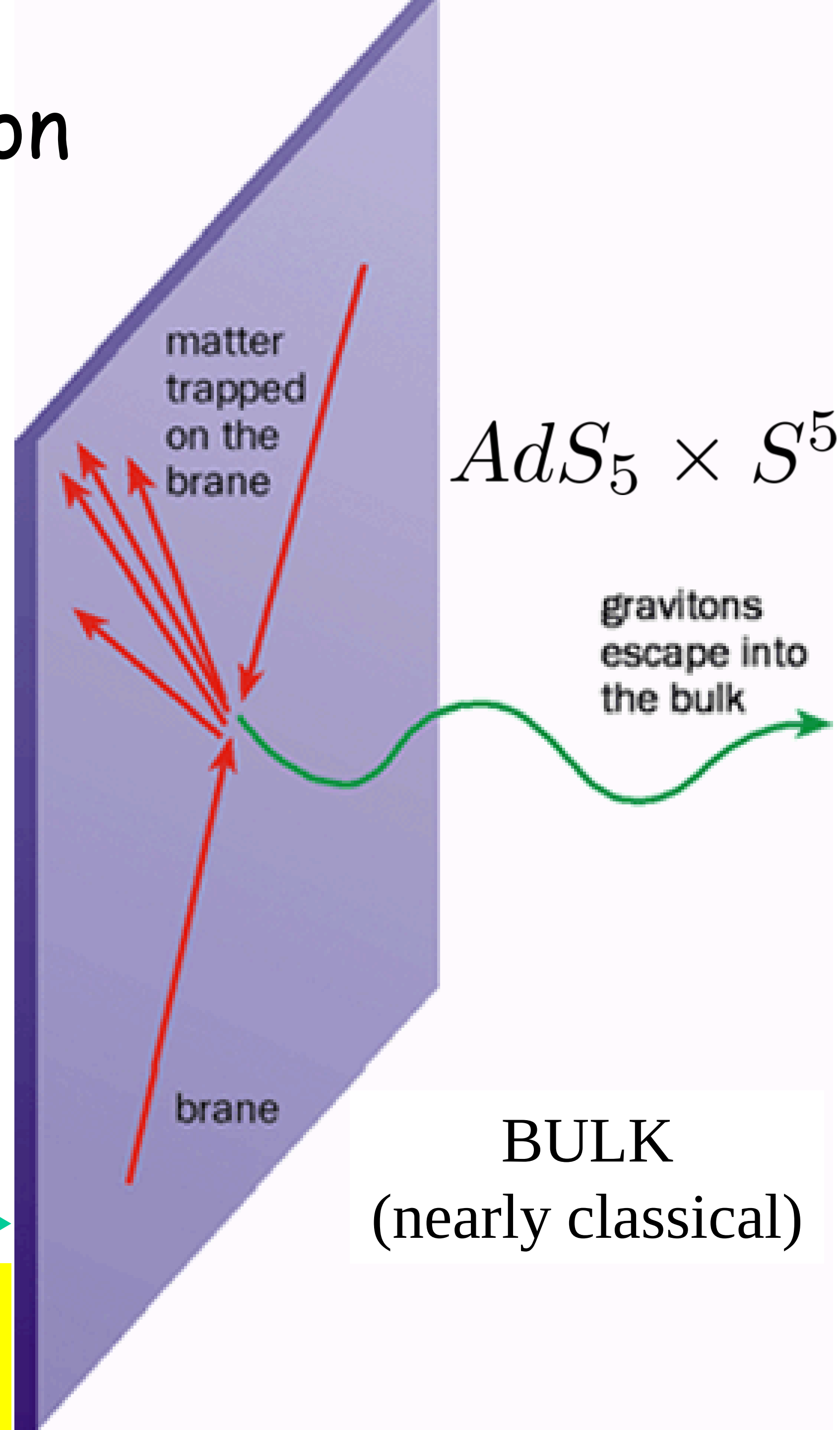
The Maldacena Revolution is Symmetry Based

– We live in a 3+1 dimensional subspace: the **3-brane**.

– The brane is embedded in a 4+1 dim space: the **bulk**.

– Bulk isometries match QCD symmetries on the brane

Poincare + Conformal
Quantum Field Theory



QFT (4-D) = Classical Theory (5-D)

Doing QCD without quarks and gluons

Source currents: the boundary theory has no quarks or gluons but we can still insist upon correct global symmetries:

$$Boundary\ Symmetry \leftrightarrow Bulk\ Symmetry$$

$$J_L^{a\mu} = \bar{q}_L \gamma^\mu t^a q_L \leftrightarrow A_L^{a\mu}$$

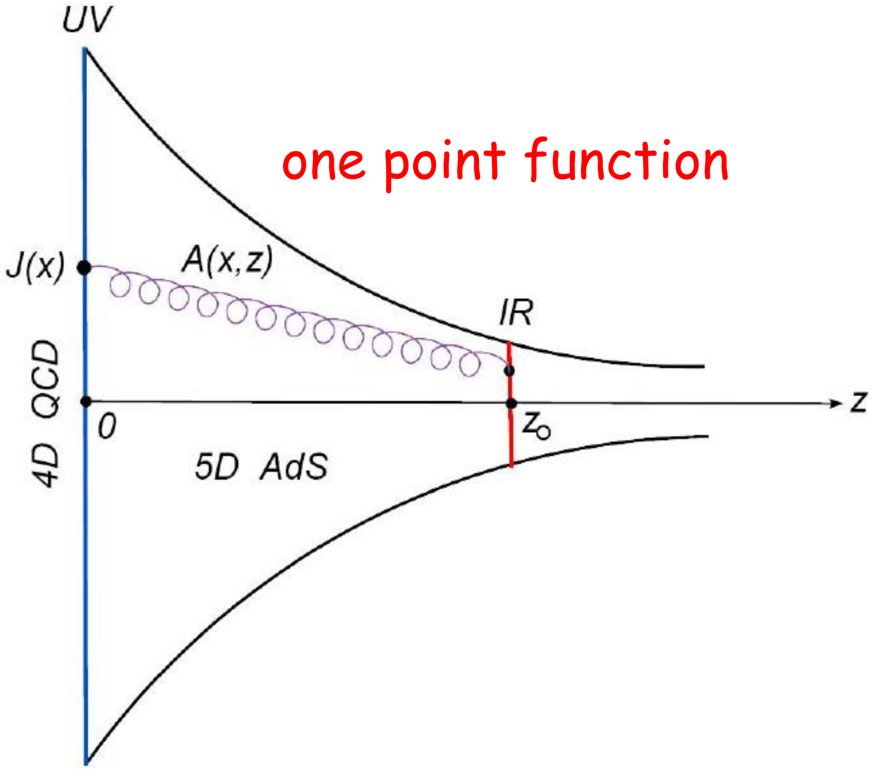
$$J_R^{a\mu} = \bar{q}_R \gamma^\mu t^a q_R \leftrightarrow A_R^{a\mu}$$

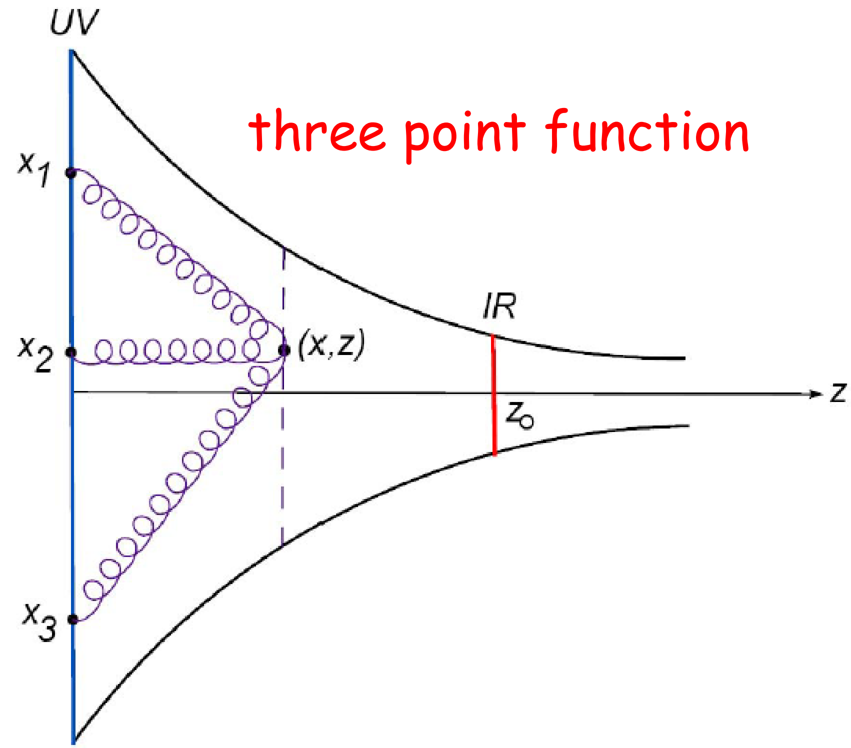
- ***Maldacena Duality:*** $\mathbf{Z}_{CFT} = \mathbf{Z}_{Bulk}$

hard to calculate

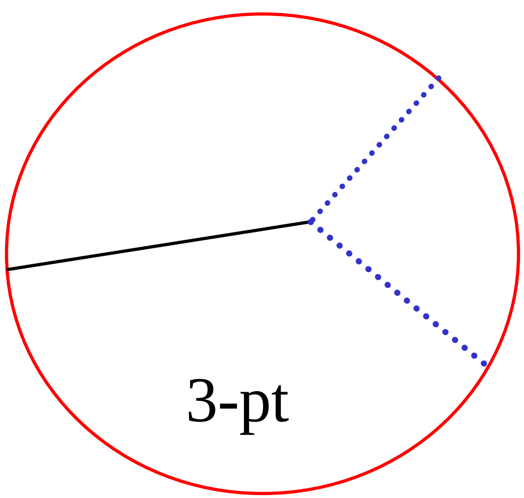
easy to calculate

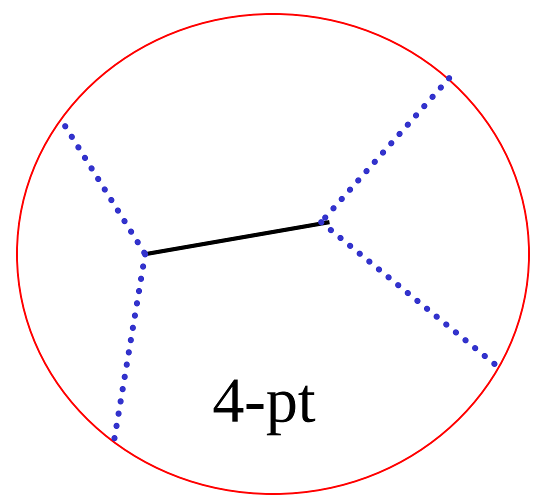
$$\langle O(x)O(y)O(z)O(w)\rangle_c = \frac{\delta^4}{\delta\phi_0(x)\,\delta\phi_0(y)\,\delta\phi_0(z)\,\delta\phi_0(w)} \mathrm{Z}_{\mathrm{Bulk}}\left[\phi_0\right]$$





Disturbing the 5-D bulk buys you knowledge of our 4-D world





Some Impressive Successes of ADS/CFT
(Brodsky, de Teramond, Lorce)

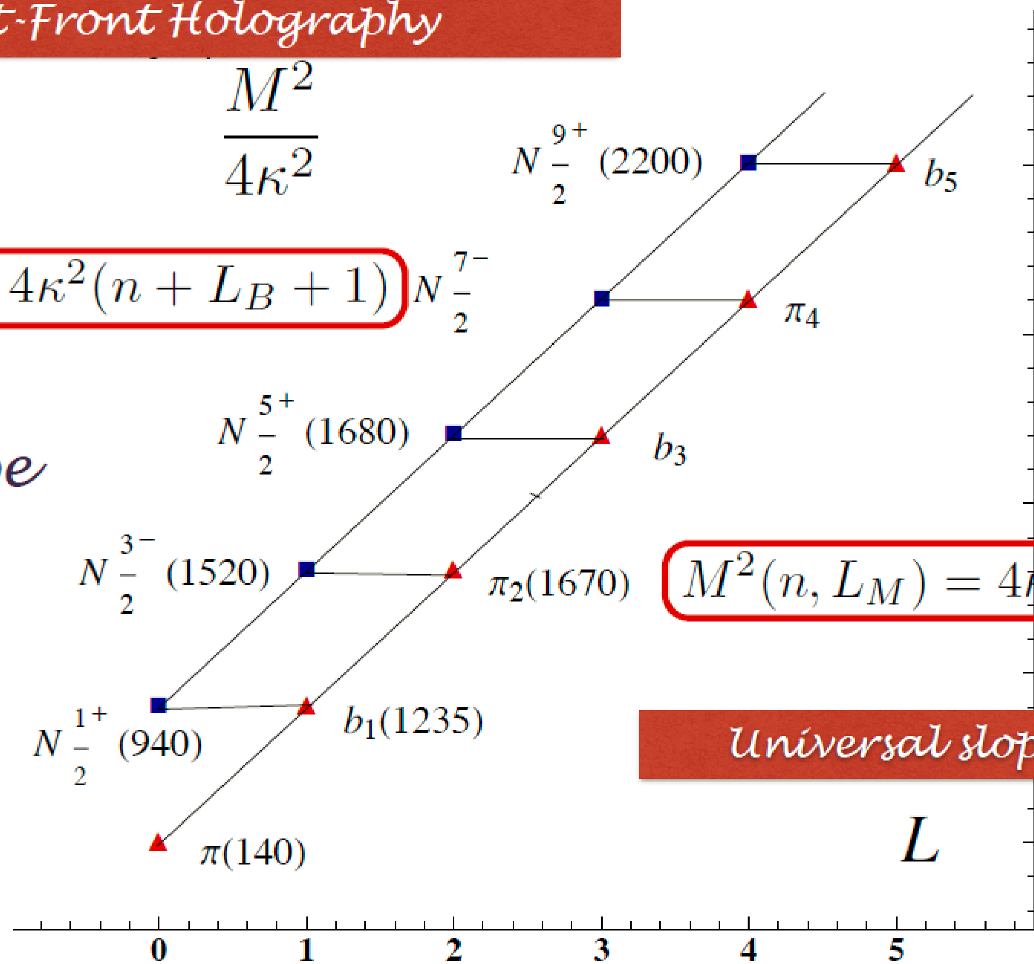
Superconformal Quantum Mechanics
Light-Front Holography

de Tèramond, Dosch, Lorcè, sjb

$$\frac{M^2}{4\kappa^2}$$

$$\boxed{M^2(n, L_B) = 4\kappa^2(n + L_B + 1)}$$

Same slope



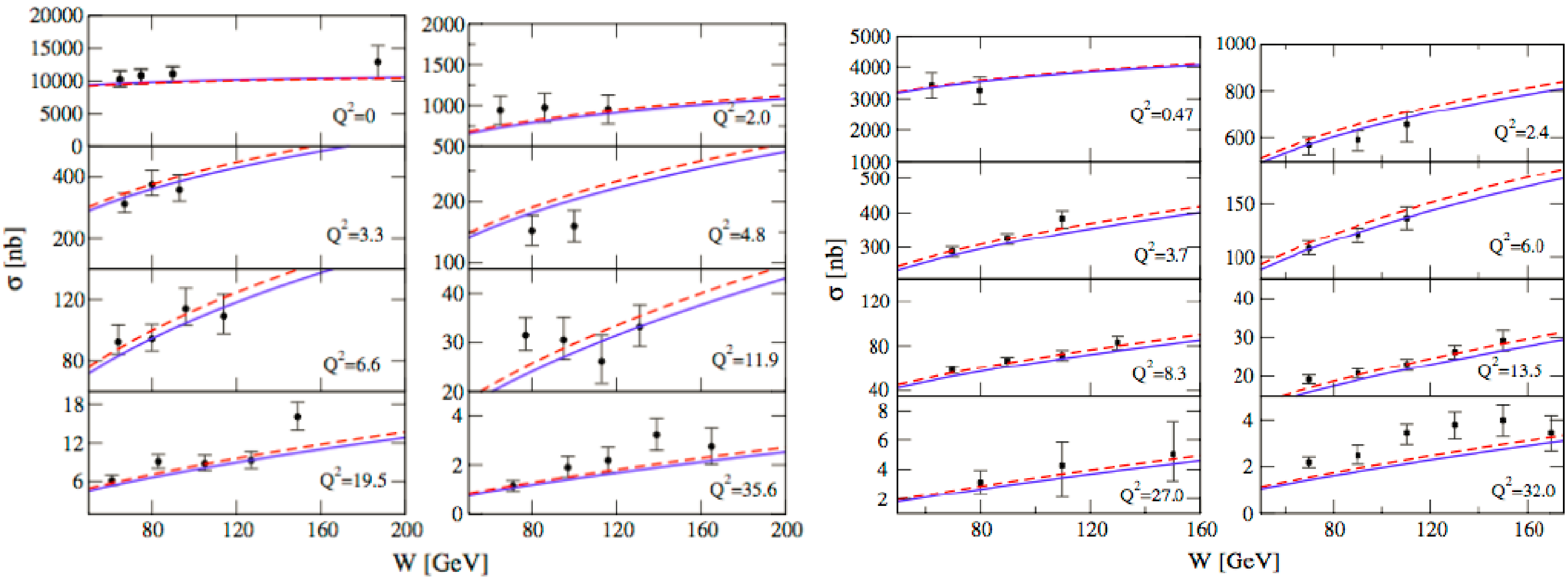
$$\boxed{M^2(n, L_M) = 4\kappa^2(n + L_M)}$$

Universal slopes in n, L

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for L_M=L_B+1

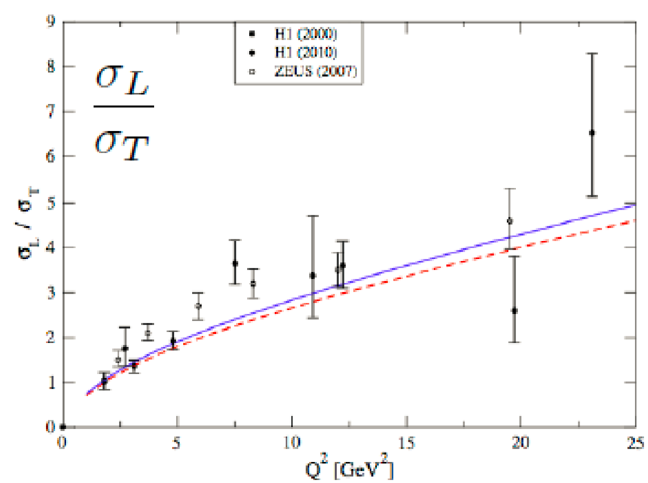
AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



(a) H1 (b) ZEUS

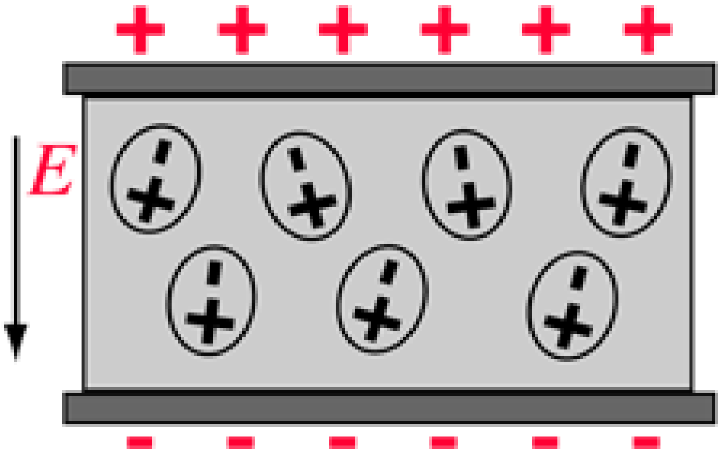
J. R. Forshaw, R. Sandapen

$$\gamma^* p \to \rho^0 p'$$



$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1}}$$

Polarizability



$$\mathcal{E} = -\frac{1}{2}(\alpha_E \vec{E}^2 + \beta_M \vec{B}^2)$$

Measuring α_E, β_M for charged hadrons problematic:

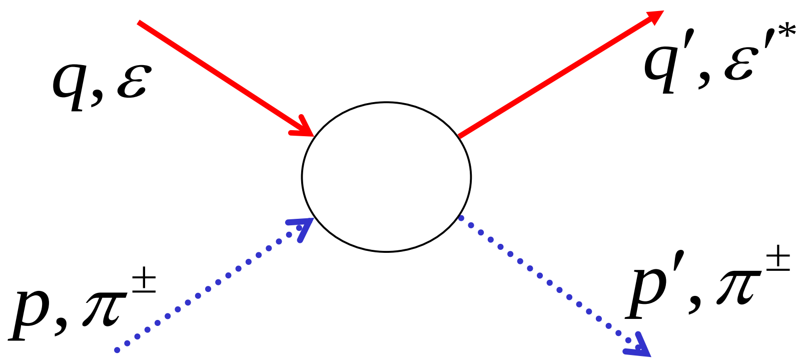
1. Very high $\vec{E}, \vec{B}$ fields needed

2. The hadron can't be nailed down

Solution: Use Compton Scattering + Low Energy Theorem

(*Low, Gell-Mann, Goldberger*)

Compton scattering from a (spinless) pion target



$$f = -\frac{e^2}{m}\varepsilon'^* \cdot \varepsilon + \alpha_E \omega\omega' \varepsilon'^* \cdot \varepsilon + \beta_M \omega\omega' (\varepsilon'^* \times \hat{\mathbf{q}}') \cdot (\varepsilon \times \hat{\mathbf{q}}) + O(\omega^3)$$

Low Energy Theorem (Low, Gellmann, Goldberger, from 1950’s)

$$\begin{aligned} T &= T_{Born} + \frac{1}{2} b_1(0) f^{\mu\nu} f'_{\mu\nu} + b_2(0) P_\mu f^{\mu\nu} P^\rho f'_{\rho\nu}, \\ P_\mu &= p_\mu + p'_\mu, \\ f_{\mu\nu} &= -i(q_\mu \varepsilon_\nu - q_\nu \varepsilon_\mu), \\ f'_{\mu\nu} &= i(q'_\mu \varepsilon'^*_\nu - q'_\nu \varepsilon'^*_\mu). \end{aligned}$$

L’vov, Scherer, Pasquini, Unkmeir, Drechsel (2001)

$$\boxed{\alpha_E = -\frac{1}{2m} b_1(0) - \frac{m}{2} b_2(0) \qquad \beta_M = \frac{1}{2m} b_1(0)}$$

An ADS calculation of Compton scattering by Marquet, Roiesnelb, Wallon (2010)

Scalar + Vector fields in 5-D

$$S_0 = \int d^4x dz \sqrt{-g} \left(-\frac{1}{4} F_{mn} F^{mn} + e^{-\chi} D^m \Phi^* D_m \Phi + e^{-\chi} \mu_S^2 \Phi^* \Phi \right)$$

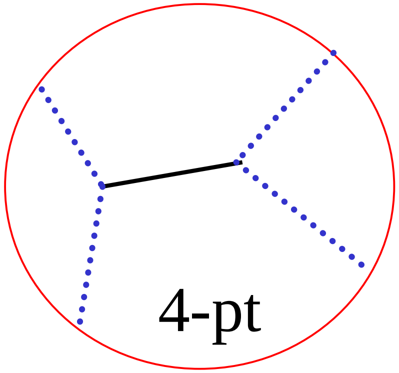
$$D_m \Phi = \partial_m \Phi - ieA_m \Phi$$

dilaton

EOM reduces to Sturm-Liouville eigenvalue problem:

$$H_S \Phi_n = -m_n^2 \Phi_n \qquad H_S = \frac{z^3}{e^{-\chi}} \partial_z \left(\frac{e^{-\chi}}{z^3} \partial_z \right) - \left(\frac{\mu_S^2 R^2}{z^2} \right)$$

Normalization: $\displaystyle\int_0^\infty dz \frac{e^{-\chi(z)}}{z^3} \Phi_n^*(z) \Phi_m(z) = \delta_{mn}$

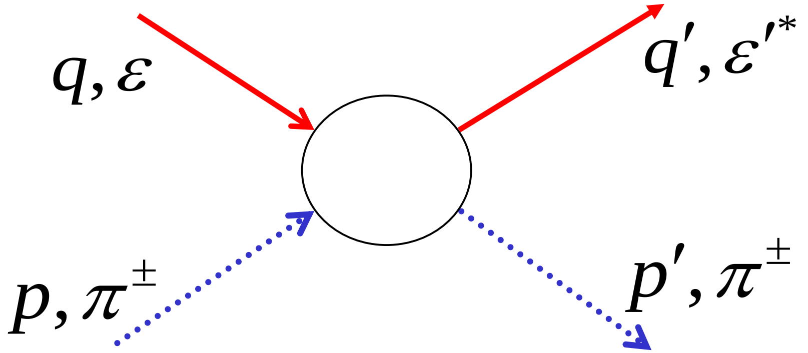


bulk-to-bulk propagator for 4-pt calculations:

$$\hat{G}(z, z', k) = -\sum_{n=0}^{\infty} \frac{\Phi_n^*(z) \Phi_n(z')}{k^2 + m_n^2 - i\varepsilon}$$

Apply to Compton Scattering

Marquet, Roiesnelb, Wallon (2010)



$$T = (2\pi)^4 \delta^4(p + q - p' - q')\mathcal{M}_{Born}$$

$$\mathcal{M}_{Born} = e^2 \varepsilon'^{*\mu} \left(2\eta_{\mu\nu} - \frac{(2p+q)_\mu (2p'+q')_\nu}{s+m^2} - \frac{(2p'-q)_\mu (2p-q')_\nu}{u+m^2} \right) \varepsilon^\nu$$

$$\text{Expand out to } O(\omega^2) \Longrightarrow b_1(0) = b_2(0) = 0$$

$$\alpha_E = \beta_M = 0.$$

$$\text{Experiment: } \alpha_E - \beta_M = 13^{+2.6}_{-1.9}, \; \alpha_E + \beta_M = 0.18^{+0.11}_{-0.02}$$

Conclude: canonical ADS/CFT does not **sufficiently** recognize the compositeness of extended objects (pions)

Augmenting Canonical ADS/CFT to include polarizabilities

Modify the ADS/CFT action – add axial-vector mesons

$$S_a = -\int d^4x dz \sqrt{-g} e^{-\chi} \left(\frac{1}{2} a^*_{mn} a^{mn} + \mu_A^2 a^*_m a^m + \frac{1}{2} e g_A F^{mn} (a^*_{mn} \Phi + a_{mn} \Phi^*) \right)$$

$$a_{mn} = \partial_m a_n - \partial_n a_m \qquad \text{Gauge choice: } a_5(x,z) = 0$$

$$a_\mu(x,z) \text{ sourced by } A_\mu(x) = \bar{q} \gamma_5 \gamma_\mu q$$

Green's function $\hat{G}_{\mu\nu}(x,z,x',z')$

$$\left[\frac{z}{e^{-\chi}} \partial_z \left(\frac{e^{-\chi}}{z} \partial_z \right) + \eta^{\mu\nu} \partial_\mu \partial_\nu - \left(\frac{\mu_A^2 R^2}{z^2} \right) \right] \hat{G}_{\mu\nu} = \frac{g_{\mu\nu}}{\sqrt{-g} e^{-\chi}} \delta^4(x - x') \delta(z - z').$$

Axial propagator: $h_A(z, z', k) = -\sum_{n=0}^{\infty} \frac{\Psi^*_{An}(z) \Psi_{An}(z')}{k^2 + M^2_{An} - i\varepsilon},$

Normalization: $\int_0^\infty dz \frac{e^{-\chi(z)}}{z} \Psi^*_{An}(z) \Psi_{Am}(z) = \delta_{mn}$

Calculate scattering amplitude after adding axial-vector mesons

$$\mathcal{M}_A = \frac{1}{4} e^2 g_A^2 \underbrace{P_\mu f^{\mu\nu}(q) P^\rho f_{\rho\nu}(q')}_{\text{correct structure for } b_2(0)} \int \frac{dz}{z^5}\frac{dz'}{z'^5} e^{-\chi(z)} e^{-\chi(z')} (z^4 + z'^4) \Phi_0(z) h_A(z,z',k) \Phi_0^*(z')$$

$$\boxed{b_2(0) \quad = \quad e^2 g_A^2 \sum_{n=0}^{\infty} \frac{C_n D_n}{-m_0^2 + M_{An}^2},}$$

$$C_n = \int_0^\infty \frac{dz}{z} e^{-\chi(z)} \Phi_0(z) \Psi^*_{An}(z),$$

$$D_n = \int_0^\infty \frac{dz}{z^5} e^{-\chi(z)} \Psi_{An}(z) \Phi_0^*(z).$$

$$\Phi_n(z) \sim z^a,\ a = \tfrac{1}{2} + \sqrt{4 + \mu_S^2 R^2}$$
$$\Psi_n(z) \sim z^b,\ \ b = \tfrac{1}{2} + \sqrt{1 + \mu_A^2 R^2}$$

What about $b_1(0)$?

Add in vector mesons (while preserving discrete symmetries)

$$S_{\mathrm{v}} = -\int d^4x dz \sqrt{-g} e^{-\chi} \left(\frac{1}{2} \mathrm{v}^*_{mn} \mathrm{v}^{mn} + \mu_{\mathrm{v}}^2 \mathrm{v}^*_m \mathrm{v}^m + \frac{1}{4} e g_{\mathrm{v}} \tilde{F}^{mn} (\mathrm{v}^*_{mn} \Phi + \mathrm{v}_{mn} \Phi^*) \right)$$

$$\mathrm{v}_{mn} = \partial_m \mathrm{v}_n - \partial_n \mathrm{v}_m, \qquad \tilde{F}^{mn} = \varepsilon^{mnpqz} F_{pq}$$

$SU(2)_L \times SU(2)_R$ a symmetry? NO: $m_{A_1}^2 \approx 2m_\rho^2$

$$\mathcal{M}_V = e^2 g_A^2 m^2 \underbrace{\tilde{f}^{\mu\nu}(q) \tilde{f}_{\mu\nu}(q')}_{-4 f^{\mu\nu}(q) f_{\mu\nu}(q')} \int \frac{dz}{z^5} \frac{dz'}{z'^5} e^{-\chi(z)} e^{-\chi(z')} (z^4 + z'^4) \Phi_0(z) h_V(z, z', k) \Phi_0^*(z')$$

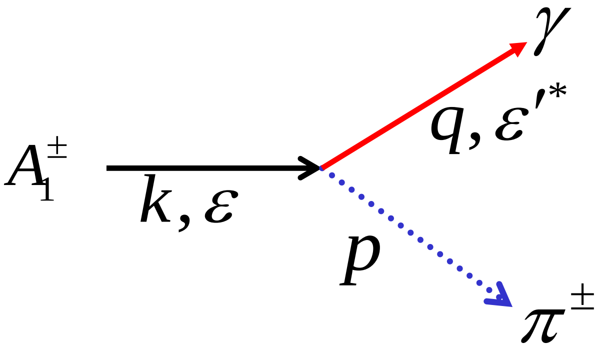
is the correct structure for $b_1(0)$

$$\boxed{b_1(0) = -e^2 m^2 g_V^2 \sum_{n=0}^{\infty} \frac{E_n F_n}{-m_0^2 + M_{Vn}^2}}$$

$$E_n = \int_0^\infty \frac{dz}{z} e^{-\chi(z)} \Phi_0(z) \Psi_{Vn}^*(z) \qquad F_n = \int_0^\infty \frac{dz}{z^5} e^{-\chi(z)} \Psi_{Vn}(z) \Phi_0^*(z)$$

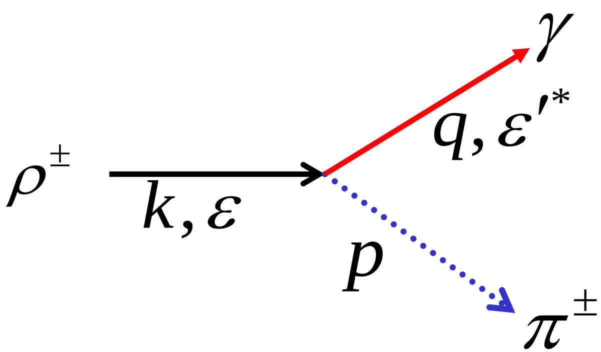
How to get a handle on g_A^2 and g_V^2 ?

$$\mathcal{M}_{A_1\to\gamma\pi} = (2\pi)^4\delta^4(k-q-p)\times e g_A \times (\varepsilon'^{*}\cdot\varepsilon q\cdot k - \varepsilon\cdot q\varepsilon'^{*}\cdot k)C_0$$



$$C_0 = \int_0^\infty \frac{dz}{z} e^{-\chi(z)}\Phi_0(z)\Psi^*_{A0}(z)$$

$$\mathcal{M}_{\rho\to\gamma\pi} = (2\pi)^4\delta^4(k-q-p)\times e g_V \times \varepsilon^{\mu\nu\rho\sigma}k_\mu\varepsilon_\nu q_\rho\varepsilon'^{*}_\sigma E_0$$



$$E_0 = \int_0^\infty \frac{dz}{z} e^{-\chi(z)}\Phi_0(z)\Psi^*_{V0}(z)$$

Conclude: we now have a framework in which to compute hadron polarizabilities

Opinion about ADS/CFT

- ADS/CFT is impressive and offers a path to strong interactions.
- Since no QCD dual is available we must keep supplementing.
- Until that time the best strategy is to aim for models that impose the right symmetries. Although crude, they are quite successful in reproducing interesting phenomenology (Brodsky, et. al. and their ADS-supplemented light-cone models).