Baryon quadrupole and octupole moments

Alfons Buchmann
University of Tübingen

1. How I came to know Prof. Ernest Henley
2. Collaboration with E. M. Henley
3. Baryon quadrupole moments
4. Baryon octupole moments
5. Summary

INT Symposium
Symmetries in Subatomic Physics
In memory of E. M. Henley
University of Washington, Seattle 2018

1. How I came to know Prof. Ernest Henley

The University of Washington

To all to whom these Letters shall come, Greeting:

The Regents of the University on recommendation of the University Faculty and by virtue of the Authority vested in Them by Law have this day admitted

Alfons Buchmann

to the degree of

Bachelor of Science

and have granted all the Rights, Privileges and Honors thereto pertaining

Given at Seattle, in the State of Washington, this twenty-first Day of August, One Thousand Nine Hundred and Eighty-one, in the One Hundred and twenty-first Year of the University.

President of the University

President of the Board of Regents

Ernest M. Henley
Dean

21 August 1981

E. M. Henley, Humboldt Fellow (1984)

Ernest Henley was the recipient of the **Humboldt prize** in 1984.

In 1984 Prof. Henley made a research visit to the

Johannes Gutenberg University, Mainz.

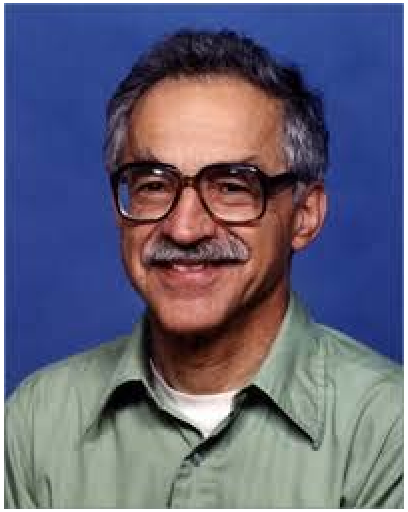
He gave a series of lectures on

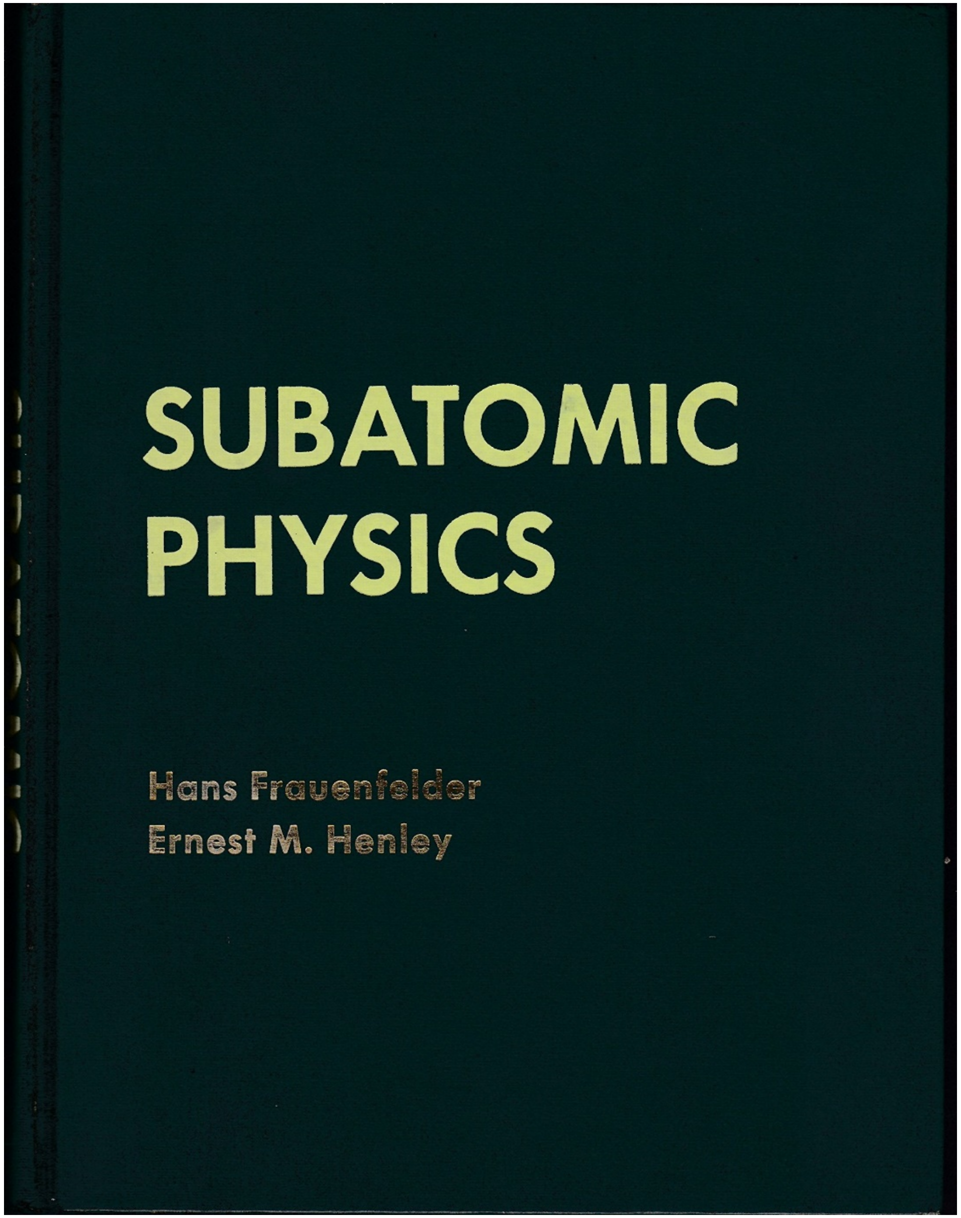
„Symmetries of Electroweak Interactions“
at the
Students‘ Workshop on Electromagnetic Interactions
in Bosen (near Mainz) in 1984.

Students‘ workshop on electromagnetic interactions (Bosen 1984)

Walk with Prof. Henley around the lake in Bosen

E. M. Henley and Subatomic Physics





My favorite textbook

E. M. Henley and the BMW

At the time (1984), the BMW company provided Humboldt prize winners with a BMW car for their stay in Germany free of charge.

Later they could take this car home at a reduced price.

Ernest Henley was not too fond of this arrangement and said:

„I would have prefered a bicycle.“

Story related to me by Dr. Lothar Tiator, University of Mainz
Coorganizer of the Students‘ Workshop in Bosen, Germany

E. M. Henley in Tübingen

As a Humboldt fellow, Prof. Ernest Henley repeatedly visited the

University of Tübingen,

where I worked as a postdoc (1993-1999).

During his visits in 1998 and 1999 Ernest and I discussed

results on charge radii and quadrupole moments

derived from a constituent quark model with exchange currents.

We thought about whether these results could be generalized.

2. Collaboration with Prof. Ernest Henley

Beginning of collaboration with E.M. Henley

Date: Thu, 29 Jul 1999 11:59:09 -0700
From: HENLEY@nucthy.phys.washington.edu
To: alfons.buchmann@uni-tuebingen.de
Subject: saga

Dear Alfons, **July 29**

I looked at the various Morpurgo articles and now understand things a lot better.
……
I thought that you might be able to generalize his idea to meson-baryon couplings…

Do you want to look at the inherent Q? The easiest way to do so might be to factor out the Clebsch-Gordon coefficient (which vanishes for j=1/2).

If I can get you here for a month or more, we might put something together with all this "stuff.„

Best regards **Ernest**

Collaboration with E. M. Henley (1999-2013)

A. B. and E. M. Henley, Pion-baryon couplings,
Phys. Lett. B 484 (2000) 255

A. B. and E. M. Henley, Intrinsic quadrupole moment of the nucleon,
Phys. Rev. C 63 (2001) 015202

A. B. and E. M. Henley, Quadrupole moments of baryons,
Phys. Rev. D 65 (2002) 073017

A. B. and E. M. Henley, Baryon octupole moments,
Eur. Phys. J. 35 (2008) 267

A. B. and E. M. Henley, Spin of ground state baryons,
Phys. Rev. D 83 (2011) 096011

A. B. and E. M. Henley, Three-quark currents and baryon spin,
Eur. Phys. J. 55 (2014) 749

3. Baryon quadrupole moments

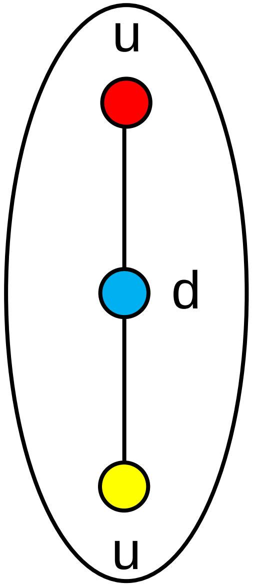
Purpose of this talk

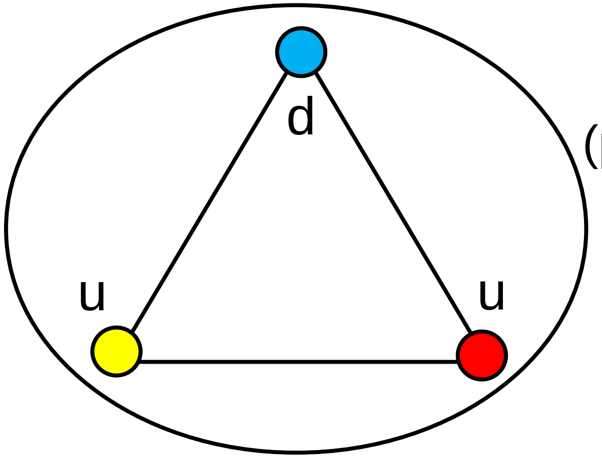
Review the work with E.M. Henley

adressing the question:

What is the shape of the proton and its first excited state $\Delta^+(1232)$?

prolate
(cigar shaped)
p





oblate
(pancake shaped)
Δ^+

Electromagnetic multipole moments

Electromagnetic multipole moments ($\mathbf{2}^{J}$ $-$poles) are directly connected with the **charge and current distributions** in baryons.

In particular, **the sign and size** of the $J = 2$ and $J = 3$

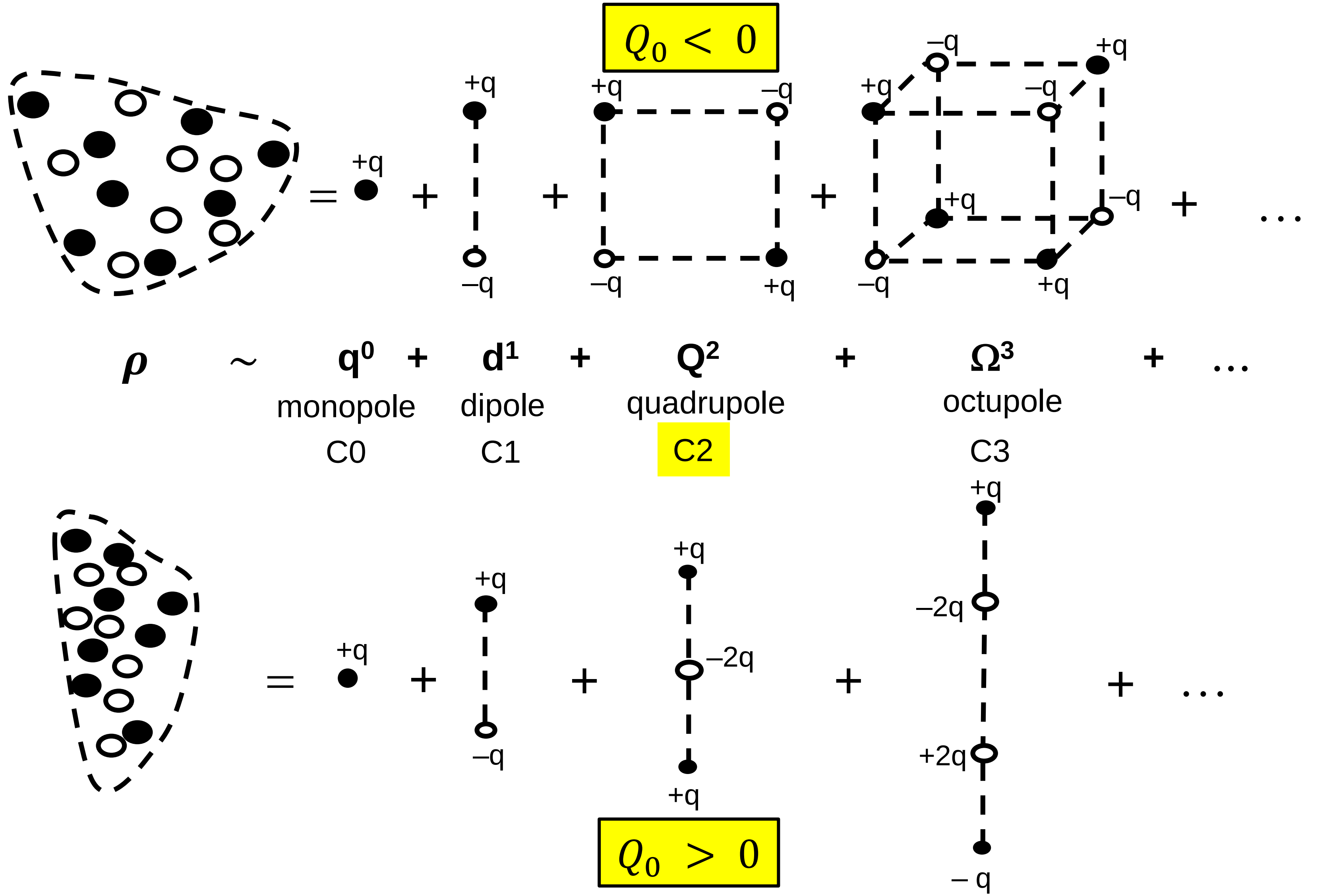
- **charge quadrupole (C2) moments**
- **magnetic octupole (M3) moments**

provide information on the

geometric shape of baryons

not available from the corresponding leading multipole moments C0 and M1.

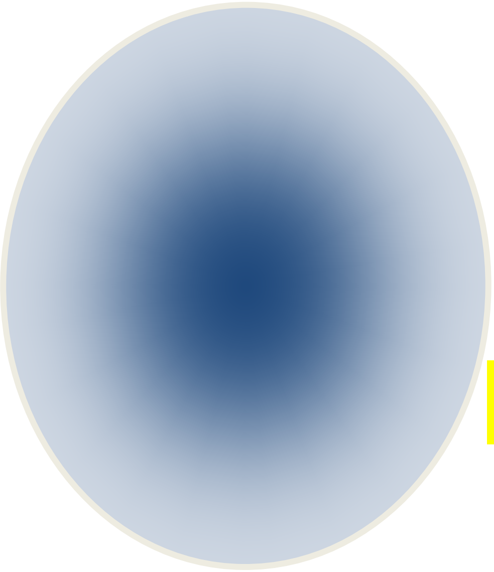
Classical charge multipole moments



In quantum mechanics only **even** charge multipoles e.g. C0,C2,… are allowed.

Geometric shape of proton charge distribution

nonspherical
charge distribution of proton



$$\rho^p(\vec{r}) = \rho^p(r, \theta, \varphi)$$

angular charge distribution

→ shape information

How can one get information

on the angular dependence of the proton charge distribution?

Angular momentum selection rules

$$J_i + J_{op} \rightarrow J_f$$

Here: $J_{op} = 2$

Nucleon→ Nucleon $J_i = J_f = \frac{1}{2}$

$$\left\langle \frac{1}{2} \middle| Q^{[2]} \middle| \frac{1}{2} \right\rangle = Q_N \equiv 0$$

elastic electron scattering

$$1/2 + 2 \nrightarrow 1/2$$

no spectroscopic quadrupole moment

Nucleon $J_i = \frac{1}{2} \rightarrow$ Delta $J_f = \frac{3}{2}$

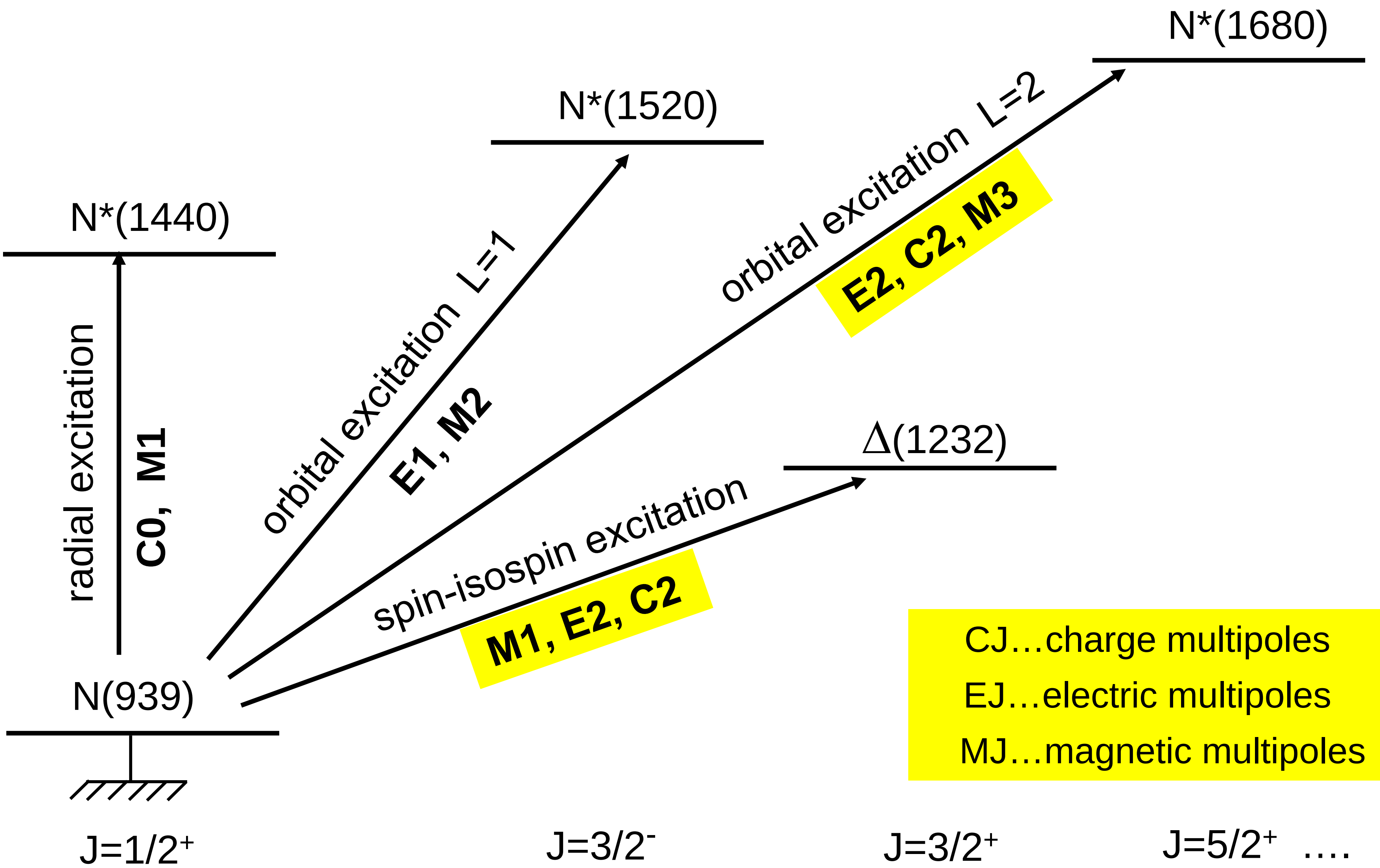
$$\left\langle \frac{3}{2} \middle| Q^{[2]} \middle| \frac{1}{2} \right\rangle = Q_{N \rightarrow \Delta}$$

inelastic electron scattering

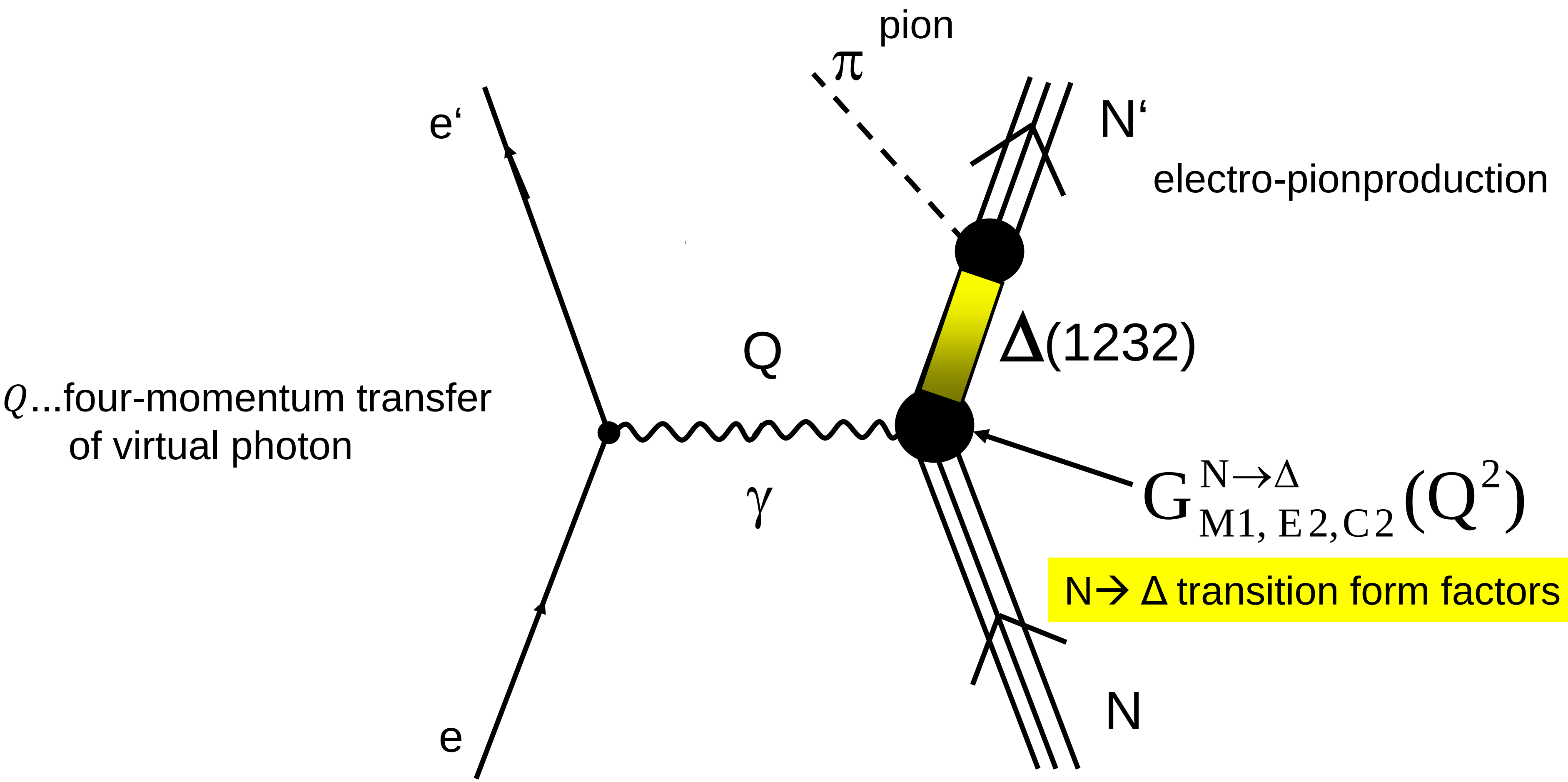
$$1/2 + 2 \rightarrow 3/2$$

spectroscopic quadrupole moment exists

Nucleon excitation spectrum



Inelastic electron-nucleon scattering



$$Q_{p\to\Delta^+} := G_{C2}^{p\to\Delta^+}(Q^2 = 0)$$

transition quadrupole moment

spectroscopic
transition quadrupole moment
$\boldsymbol{Q_{N\to\Delta}(exp) = -0.0846(33)}$ **fm²**

3.1 Spectroscopic vs. intrinsic quadrupole moments

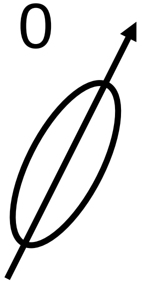
Intrinsic quadrupole moment Q_0

To learn about the shape of a particle one has to determine its **intrinsic quadrupole moment**, defined with respect to a **body-fixed frame**

$$Q_0 = \int \rho(\vec{r})\,(3z^2 - r^2)\,d^3\vec{r}$$

If ρ concentrated along z-axis, $3z^2$-term dominates $\rightarrow Q_0 > 0$

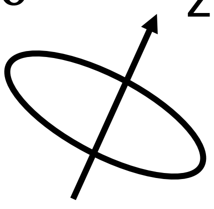
z



prolate

If ρ concentrated in x-y plane, r^2 -term dominates $\rightarrow Q_0 < 0$

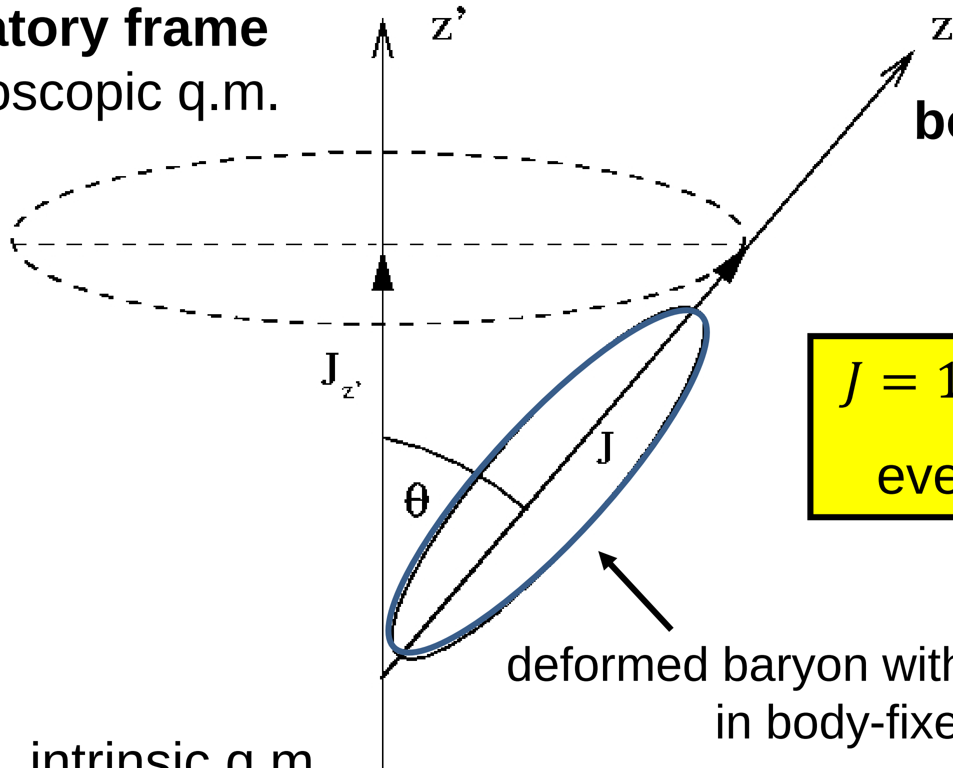
z



oblate

Intrinsic vs. spectroscopic quadrupole moment

laboratory frame
spectroscopic q.m.



body-fixed frame
intrinsic q.m.

$$\cos\theta = \frac{J_z}{\sqrt{J^2}}$$

$J = 1/2 \;\rightarrow\; Q = 0$

even if $Q_0 \neq 0$.

deformed baryon with $J = 0$ or $J = 1/2$
in body-fixed frame

spectroscopic q.m. intrinsic q.m.

$$Q = P_2(\cos\theta)\, Q_0 = \frac{1}{2}(3\cos^2\theta - 1)\, Q_0 = \left(\frac{3\,J_z^2 - J(J+1)}{2J(J+1)}\right) Q_0$$

Legendre function P_2 prevents Q_0 from being seen in lab., if $J = 0$ or $J = 1/2$.

Do intrinsic quadrupole moments exist?

Critique

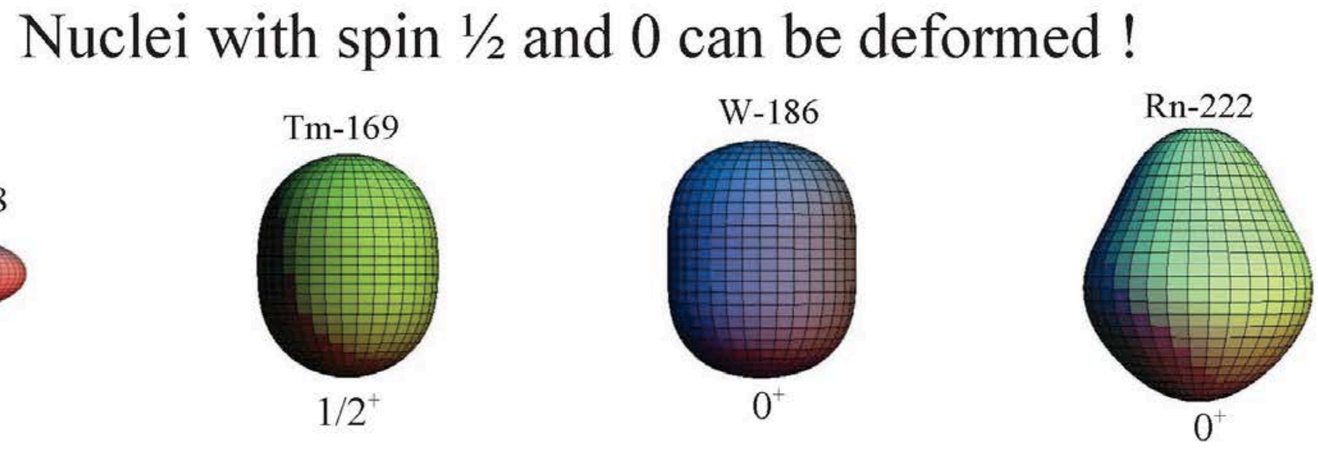
”Intrinsic quadrupole moments cannot be directly measured.
Therefore, they do not exist.“

Heisenberg: ”An elephant in an S-state remains an elephant.“

The nuclear collective model of deformed nuclei (Bohr & Mottelson) shows:

- Nuclei with spins $J = 0$ and $J = \frac{1}{2}$ can have **intrinsic** quadrupole moments even though their **spectroscopic** quadrupole moments are zero.
- The sign of the **intrinsic quadrupole moment** determines the geometric shape of the nucleus, i.e. whether it is prolate or oblate.

Intrinsic quadrupole moment of J=0 & J=1/2 nuclei



Due to the work of A. Bohr and B. Mottelson and others, the notion of intrinsic quadrupole moment of spin 0 and spin ½ nuclei is now generally accepted.

Obtaining information on intrinsic quadrupole moments Q_0 of nuclei from experimental data is possible!

Status prior to our investigation

Model		Q
Quark model with pion cloud (Vento & Rho, PLB 102 (1981) 97	oblate	$Q = ?$
Quark-diquark model (Migdal, PLB 228 (1989) 167	prolate	$Q = +0.17\ \mathrm{fm}^2$
Chiral bag model (Ma & Wambach, PLB 132 (1983) 1	oblate	$Q = ?$
Isobar model (Giannini et al., PLB 88 (1979) 13	oblate	$Q = -0.18\ \mathrm{fm}^2$
Skyrme model (Rahimov et al., PLB 378 (1996) 12	prolate	$Q = +0.02\ \mathrm{fm}^2$

None of this models features a clear distinction between Q_0 and Q.

Model calculations of Q_0

We have calculated the intrinsic quadrupole moment Q_0

in three rather different nucleon models

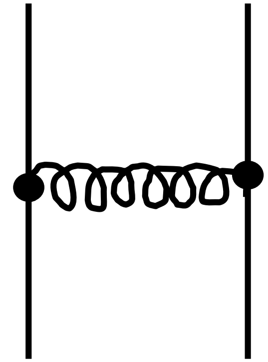
- quark model
- pion-nucleon model
- collective model

All three models lead to the same signs for
the intrinsic quadrupole moments $Q_0(p)$ and $Q_0(\Delta^+)$.

3.2 Quark model

One-gluon exchange potential

quark hyperfine interaction
causes N-Δ mass splitting

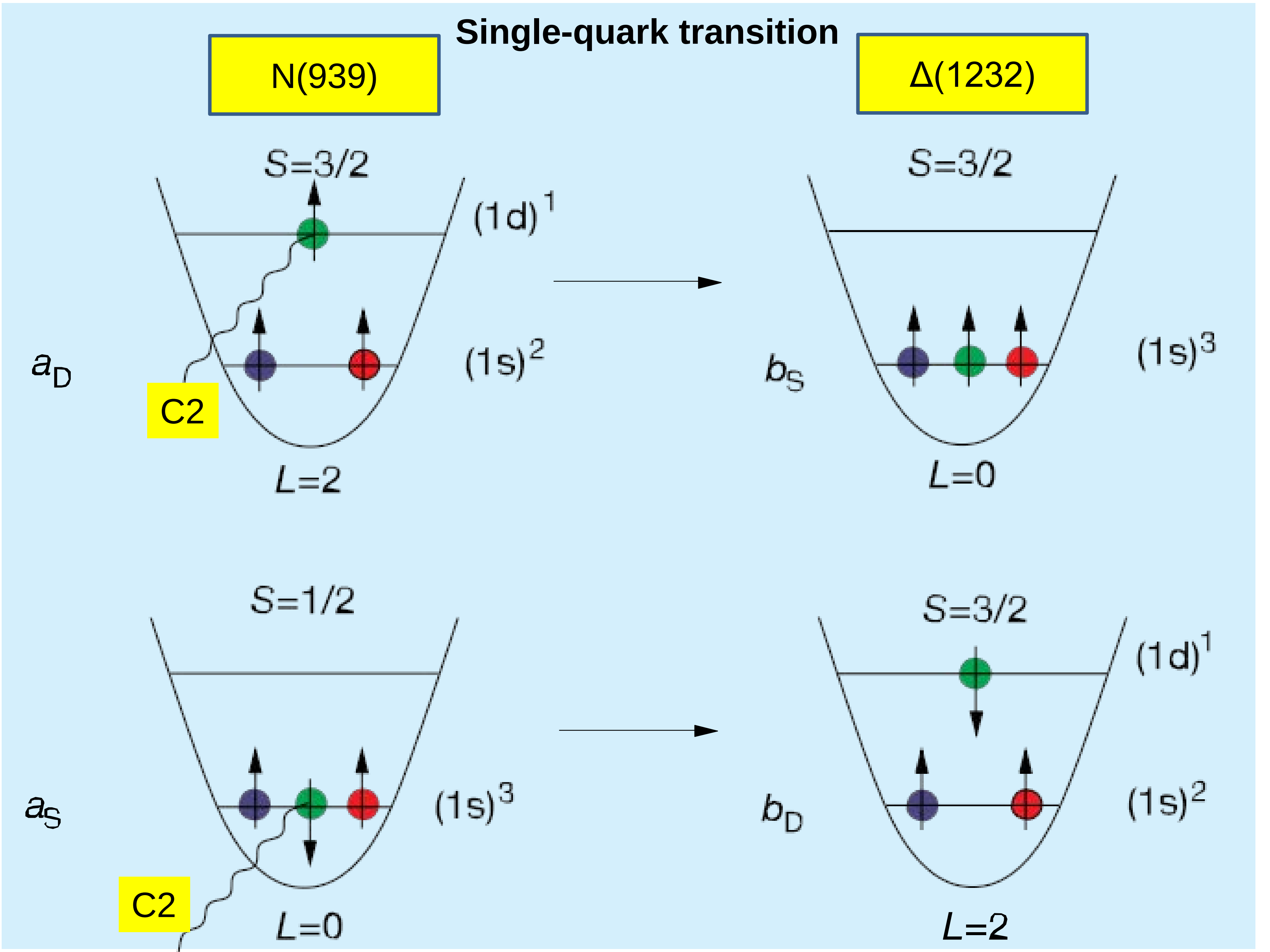


quark tensor force
causes D state admixture in
N and Δ wave functions

$$V^{gluon} = \alpha_S \left\{ \frac{1}{r} - \frac{\pi}{m_q^2}\left(1 + \frac{2}{3}\vec{\sigma}_i \cdot \vec{\sigma}_j\right)\delta(\vec{r}) - \frac{1}{4m_q^2}\frac{1}{r^3}\left(3\,\vec{\sigma}_i \cdot \hat{r}\,\vec{\sigma}_j \cdot \hat{r} - \vec{\sigma}_i \cdot \vec{\sigma}_j\right) + \cdots \right\}$$

central   tensor

Analogous to Fermi-Breit interaction in QED

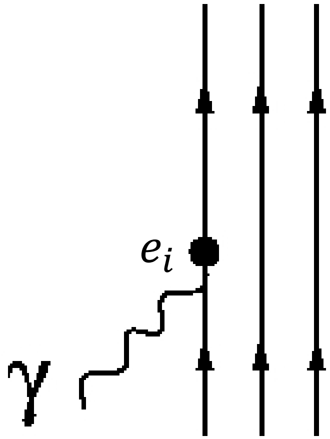

Single-quark transition
N(939)
Δ(1232)
S=3/2
(1d)^1
(1s)^2
a_D
C2
L=2
S=3/2
(1s)^3
b_S
L=0
S=1/2
(1s)^3
a_S
C2
L=0
S=3/2
(1d)^1
(1s)^2
b_D
L=2

Single quark charge density

$$\rho_{[1]} = \sum_{i=1}^{3} e_i \, e^{i\vec{q}\cdot\vec{r}_i}$$

e_i ⋯ quark charge

$\vec{q}$ ⋯ photon momentum



$$Q_{p\to\Delta^+} = b^2 \frac{4}{\sqrt{30}}(a_S b_D - a_D b_S)$$

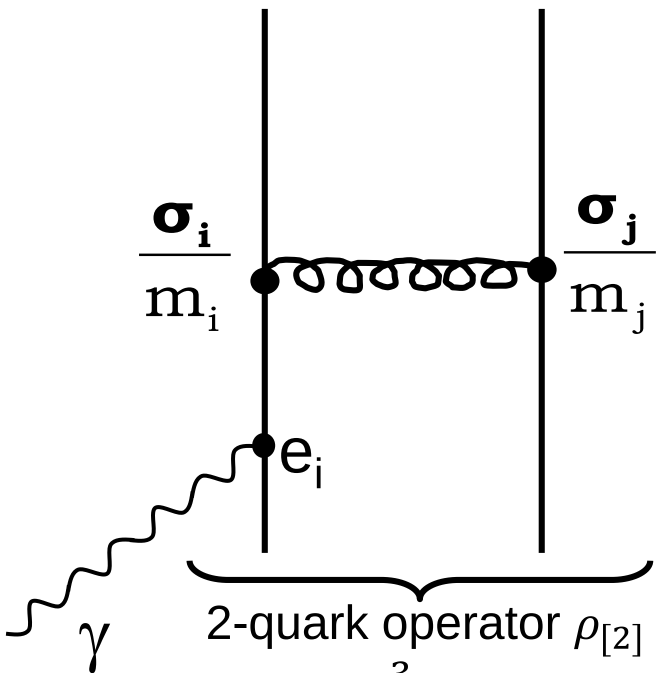
With typical D-state probabilities in the N(939) and Δ(1232) of

$$\boldsymbol{P_D(N) \approx P_D(\Delta) \approx 0.2\%,}$$

$Q_{p\to\Delta^+}$ is **too small** to account for the experimental $N \to \Delta$ quadrupole moment.

Which degrees of freedom are missing?

Two-quark charge density



$e_i \cdots$ quark charge
$\sigma_i \cdots$ quark spin
$m_i \cdots$quark mass
$\vec{q} \cdots$ photon momentum

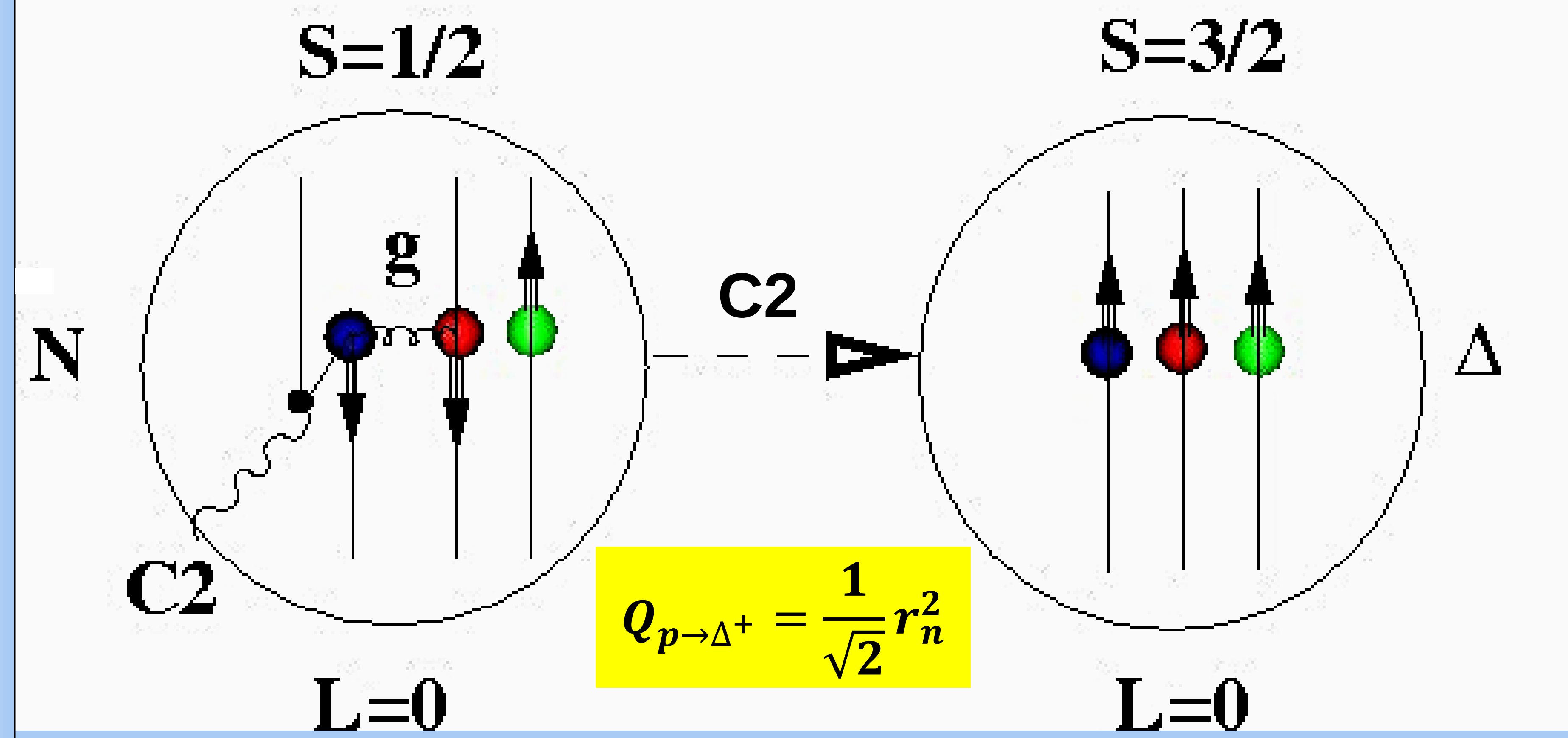
$$e_i = \frac{1}{2}\left(\lambda_{3i} + \frac{1}{\sqrt{3}}\lambda_{8i}\right)$$

$$\rho_{[2]} \sim -\, i\,\frac{\alpha_s}{16\, m_q^3}\sum_{i<j}^{3}\left(e_i\, e^{i\vec{q}\cdot\vec{r}_i}\ \vec{\sigma}_i \times \vec{q}\ \ \vec{\sigma}_j \times \vec{r} + (i \leftrightarrow j)\right)\frac{1}{r^3}$$

After angular momentum recoupling we can rewrite this as

$$\rho_{[2]} = -B\sum_{i\neq j}^{3} e_i \left[\, \underbrace{2\,\vec{\sigma}_i\cdot\vec{\sigma}_j}_{\substack{\text{scalar}\\ (\text{J}=0)}} - \underbrace{\left(3\,\sigma_{iz}\,\sigma_{jz} - \vec{\sigma}_i\cdot\vec{\sigma}_j\right)}_{\substack{\text{tensor}\\ (\text{J}=2)}}\right]$$

Double spin flip mechanism



$$Q_{p\to\Delta^+} = \frac{1}{\sqrt{2}} r_n^2$$

Simultaneously flipping the spin of two quarks via one-gluon exchange current

A. J. Buchmann, E. Hernandez, A. Faessler, Phys. Rev. C55 (1997) 448

Neutron charge radius & N→ Δ quadrupole moment

$$\rho_{[2]} = -B \sum_{i \neq j}^{3} e_i \left[2 \underbrace{\vec{\sigma}_i \cdot \vec{\sigma}_j}_{\substack{\text{scalar} \\ (\text{J}=0)}} - \underbrace{\left(3\, \sigma_{iz}\, \sigma_{jz} - \vec{\sigma}_i \cdot \vec{\sigma}_j \right)}_{\substack{\text{tensor} \\ (\text{J}=2)}} \right]$$

two-body charge density

neutron charge radius

$$r_n^2 = \left\langle 56_n \middle| \rho_{[2]}^{J=0} \middle| 56_n \right\rangle = 4\, B$$

N→Δ transition
quadrupole moment

$$Q_{p\to\Delta^+} = \left\langle 56_{\Delta^+} \middle| \rho_{[2]}^{J=2} \middle| 56_p \right\rangle = 2\sqrt{2}\, B$$

more general derivation
independent of qq interaction

$$Q_{p\to\Delta^+} = \frac{1}{\sqrt{2}} r_n^2$$

Neutron charge radius r_n^2 and $Q_{p\to\Delta^+}$ are dominated by the same degrees of freedom

A. J. Buchmann and E. M. Henley, PRC 63 (2001) 015202

Comparison with experiment

Theory (quark model with two-body e.m. currents)

$$Q_{p\to\Delta^+} := G_{C2}^{p\to\Delta^+}(Q^2=0) = \frac{1}{\sqrt{2}}\, r_n^2$$

AJB and E. M. Henley,

PRC 63 (2001) 015202

With the experimental r_n^2 our theory gives

$$Q_{p\to\Delta^+} = \frac{1}{\sqrt{2}}\, r_n^2 = -0.0811(33)\ \mathrm{fm}^2$$

Experimental results

$Q_{N\to\Delta} = -0.108(9)\ \mathrm{fm}^2$ LEGS: Blanpied et al., Phys. Rev. C 64 (2001) 025203

$Q_{N\to\Delta} = -0.0846(33)\ \mathrm{fm}^2$ MAID: Tiator et al., Eur. Phys. J. A 17 (2003) 357

Spectroscopic quadrupole moment of the proton

Clebsch-Gordan coefficients

$$|p\rangle = \frac{1}{\sqrt{2}}\left\{\underbrace{\frac{1}{\sqrt{6}}|(2uud - udu - duu)\rangle}_{\text{mixed sym.}}\underbrace{\frac{1}{\sqrt{6}}|(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)\rangle}_{\text{mixed sym.}}\right.$$

$$\left. + \underbrace{\frac{1}{\sqrt{2}}|(udu - duu)\rangle}_{\text{mixed antisym.}}\ \underbrace{\frac{1}{\sqrt{2}}|(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)\rangle}_{\text{mixed antisym.}}\right\}$$

$$(3\,\sigma_{iz}\,\sigma_{jz} - \vec{\sigma}_i\cdot\vec{\sigma}_j)\frac{1}{\sqrt{6}}|(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)\rangle = \frac{4}{\sqrt{6}}|(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)\rangle$$

$$\boxed{Q_p = \langle p|Q_{[2]}|p\rangle = B\,(2-1-1) = 0}$$

The spectroscopic quadrupole moment of the proton is zero!

Intrinsic quadrupole moment of the proton

Renormalize all Clebsch-Gordan coefficients in the spin part to 1.

$$|\tilde{p}\rangle = \frac{1}{\sqrt{2}}\left\{\left[\frac{1}{\sqrt{6}}|(2uud - udu - duu)\rangle + \frac{1}{\sqrt{2}}|(udu - duu)\rangle\right] \times \frac{1}{\sqrt{3}}|(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)\rangle\right\}$$

Clebsch-Gordan coefficients renormalized to 1

This amounts to undoing the averaging over the all spin directions.

$$Q_0^p := \langle\tilde{p}|Q_{[2]}|\tilde{p}\rangle = 2B\left(\frac{2}{3} - \frac{8}{3}\right) = -4B = -r_n^2 > 0$$

prolate

Analogously we obtain for the $\Delta(1232)$

$$Q_0^{\Delta^+} = Q^{\Delta^+} = r_n^2$$

$$Q_0^{\Delta^+} = -Q_0^p < 0$$

oblate

Interpretation in quark model

Two-quark spin-spin operators are **repulsive for quark pairs with spin 1**.

In the neutron, both down quarks are in a spin 1 state, and are repelled more strongly than an up-down pair.

→ elongated (prolate) charge distribution

→ negative neutron charge radius r_n^2

→ positive intrinsic quadrupole mom. Q_0

In the Δ^0, all quark pairs have spin 1. Equal distance between down-down and up-down pairs.

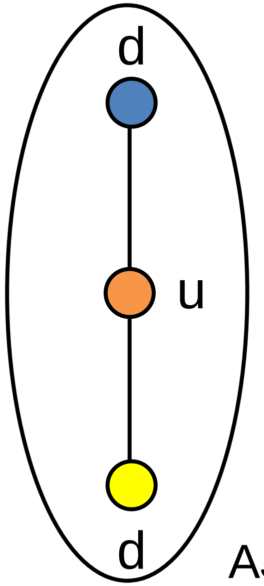
→ planar (oblate) charge distribution

→ vanishing $\mathbf{\Delta^0}$ charge radius $r_{\Delta^0}^2$

→ negative intrinsic quadrupole mom. Q_0

n

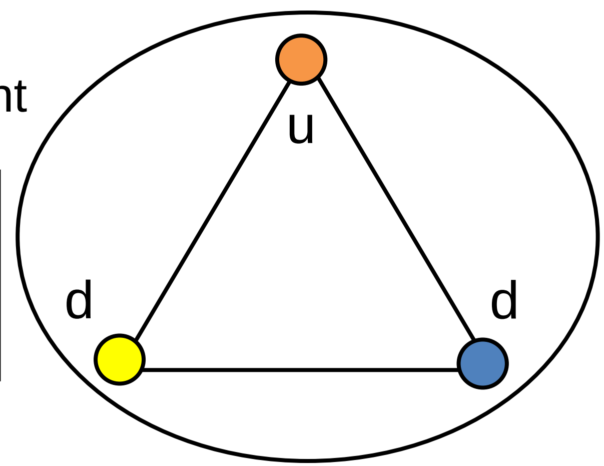
J=1/2



intrinsic quadrupole moment

$$Q_0(n) = -r_n{}^2 > 0$$

$$Q_0(\Delta^0) = r_n{}^2 < 0$$



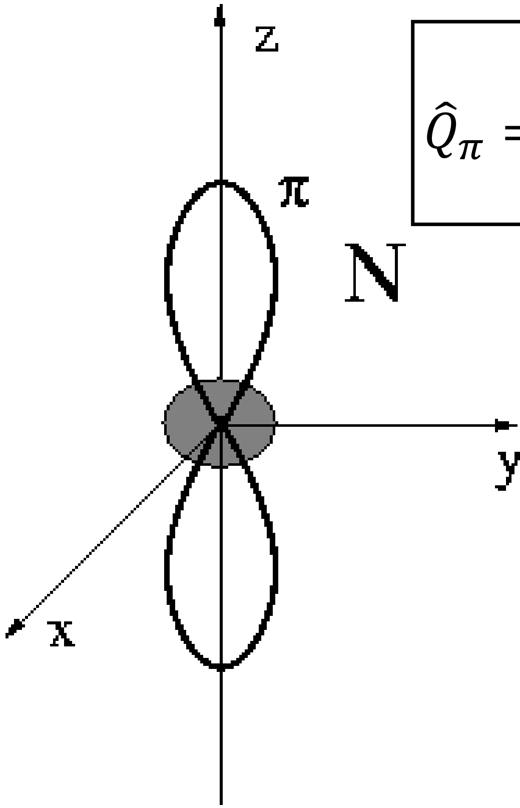
Δ^0

J=3/2

AJB and E. M. Henley, PRC 63 (2001) 015202

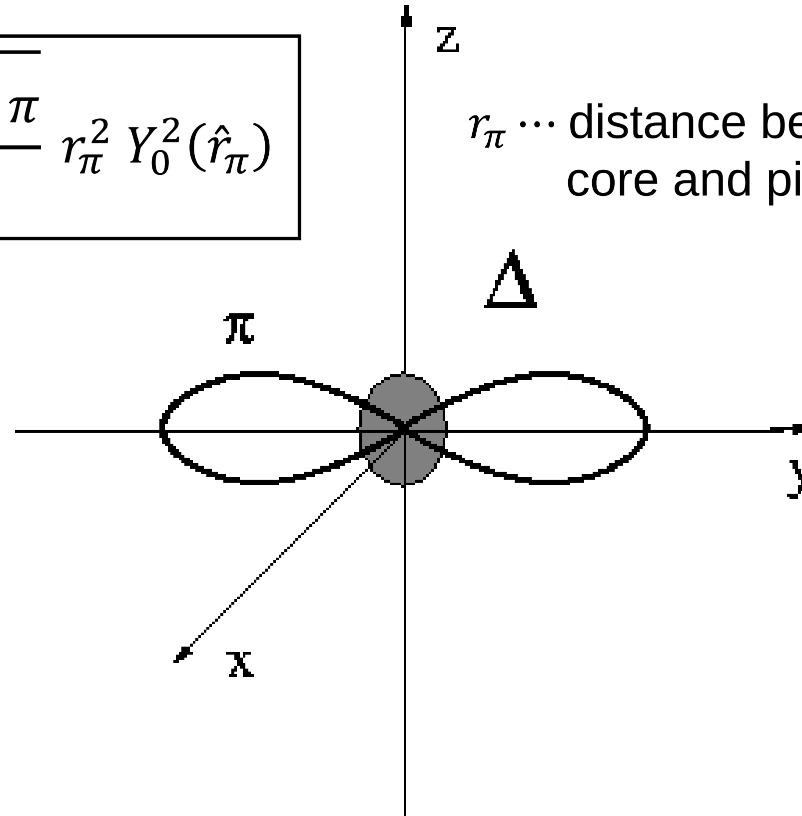
3.3 Pion-nucleon and collective model

Interpretation in pion-nucleon model



$$\hat{Q}_\pi = e_\pi \sqrt{\frac{16\,\pi}{5}}\; r_\pi^2 \; Y_0^2(\hat{r}_\pi)$$

$r_\pi \cdots$ distance between core and pion



$Y_0^1(\hat{r}_\pi)$ pion preferentially emitted along spin direction z

$$Q_0^p = -r_n^2 > 0$$

prolate

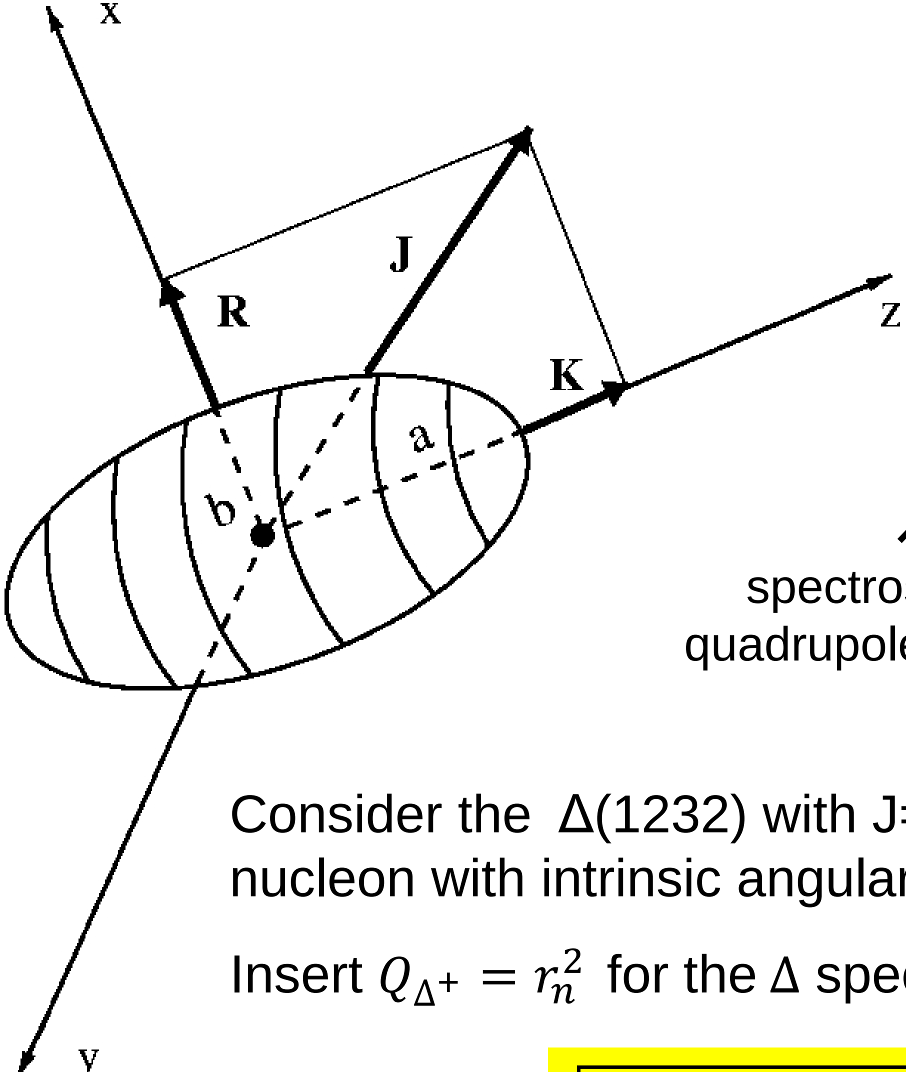
$Y_1^1(\hat{r}_\pi)$ pion preferentially emitted in equatorial x-y plane

$$Q_0^{\Delta^+} = r_n^2 < \; 0$$

oblate

More details in: A. J. Buchmann and E. M. Henley, PRC 63 (2001) 015202

Interpretation in collective model



J… total angular momentum
K… projection of J onto z-axis
R… collective orbital angular momentum

$$Q = \frac{3\,K^2 - J\,(J+1)}{(J+1)(2J+3)}\;Q_0$$

spectroscopic quadrupole moment

intrinsic quadrupole moment

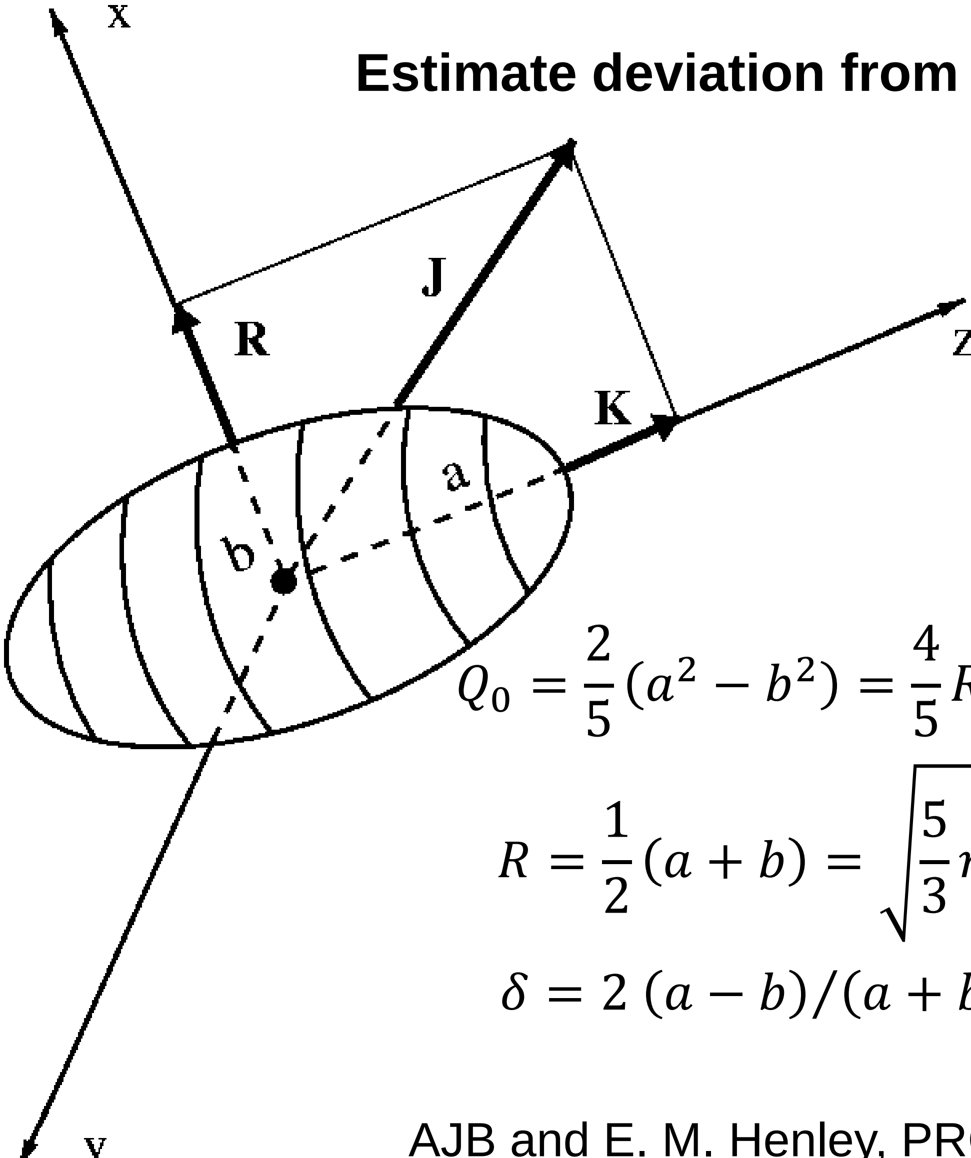
Consider the Δ(1232) with J=3/2 as a collective rotation of the nucleon with intrinsic angular momentum K=1/2.

Insert $Q_{\Delta^+} = r_n^2$ for the Δ spectroscopic quadrupole moment

$$Q_0^p = -5\,r_n^2 > 0$$

overestimate due to crudeness of the model

Interpretation in collective model



Estimate deviation from spherical symmetry

$$Q_0 = \frac{2}{5}(a^2 - b^2) = \frac{4}{5}R^2\,\delta$$

$$R = \frac{1}{2}(a+b) = \sqrt{\frac{5}{3}r_p^2}$$

$$\delta = 2\,(a-b)/(a+b)$$

For given

$r_p = 0.842$ fm

$Q_0 = -r_n^2 = 0.115$ fm²,

calculate half axes $\boldsymbol{a}$ and $\boldsymbol{b}$.

$a/b = 1.13$

big!

compare with deuteron

$a/b = 1.05$

AJB and E. M. Henley, PRC 63 (2001) 015202

3.4 Broken SU(6) spin-flavor symmetry

SU(6) spin-flavor symmetry analysis

SU(6) spin-flavor symmetry combines SU(3) multiplets

with

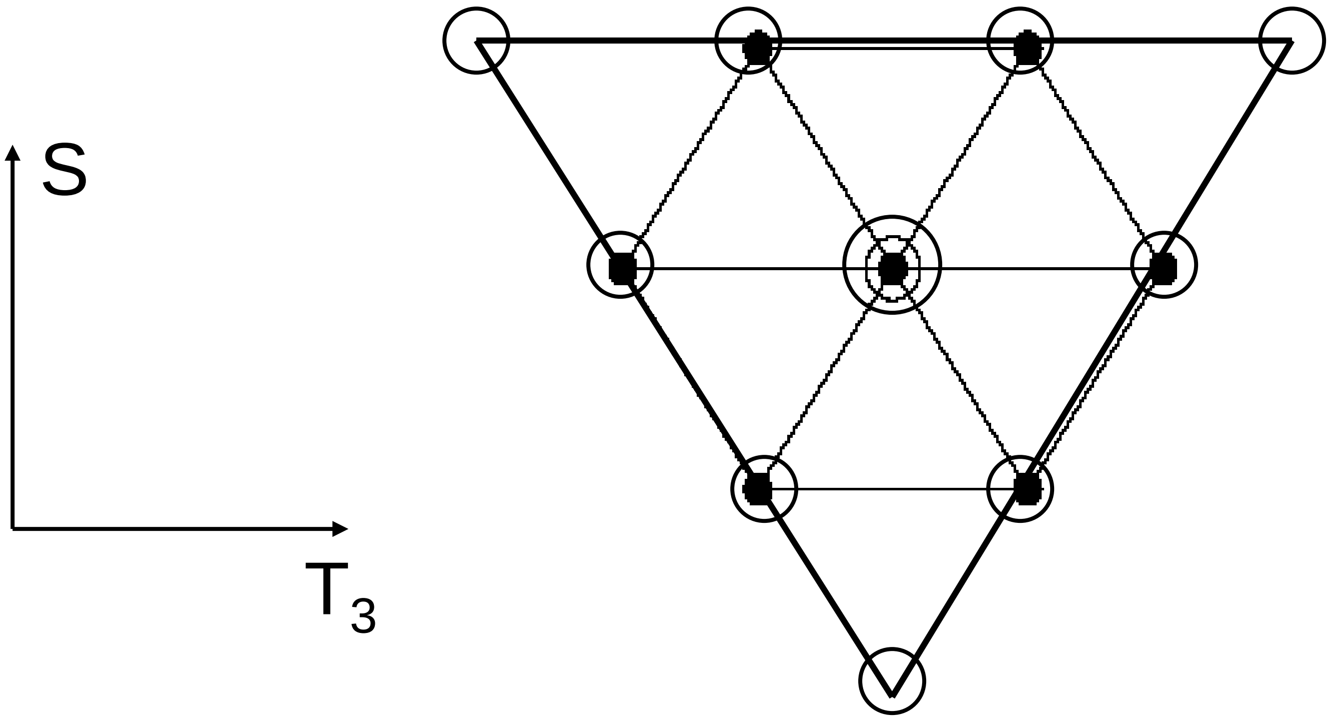
different **spin** and **flavor**

to

SU(6) spin-flavor supermultiplets.

Gürsey, Radicati, Sakita (1964)

SU(6) spin-flavor supermultiplet



baryon
supermultiplet
56 dimensional

$$56 = (8\,,2)\ \ +\ (10\,,4)$$

flavor spin flavor spin

dimensionality in spin space $2J + 1$

General spin-flavor operator $\hat{O}$

$$\hat{O} = A\,\hat{O}_{[1]} + B\,\hat{O}_{[2]} + C\,\hat{O}_{[3]}$$

one-body two-body three-body

Constants A, B, C are determined from experiment.

$\hat{O}_{[k]}$ … allowed operators in spin-flavor space $k = 1,2,3$

How are the spin-flavor operators constructed?

Operators are built from the 35 generators of the SU(6) group:

$$\underbrace{\sigma_i}_{3}, \quad \underbrace{\lambda_j}_{8}, \quad \underbrace{\sigma_i\,\lambda_j}_{24}$$

$\sigma_i \cdots$ spin operator

$\lambda_j \cdots$ flavor operator

Ex.: Charge operator $Q = \frac{1}{2}\left(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right)$ Gell-Mann Nishijima relation

transforms as a flavor 8 rep.

SU(6) spin-flavor symmetry and its breaking

Due to the SU(6) symmetry, there is a connection between observables of different tensor rank J

e.g. between charge radii ($J = 0$) and quadrupole moments ($J = 2$).

Example: Multipole expansion of $\rho_{[2]}$ in spin-flavor space

$$\rho_{[2]} = -B \sum_{i \neq j}^{3} e_i \left[2 \underbrace{\vec{\sigma}_i \cdot \vec{\sigma}_j}_{\substack{\text{scalar} \\ (\mathrm{J}=0)}} - \underbrace{\left(3\, \sigma_{iz}\, \sigma_{jz} - \vec{\sigma}_i \cdot \vec{\sigma}_j \right)}_{\substack{\text{tensor} \\ (\mathrm{J}=2)}} \right]$$

The prefactors of the spin scalar $(+2)$ and spin tensor (-1) terms are determined by the SU(6) group algebra.

Spin-flavor selection rule for the 56-plet

Ground state baryon observable $\hat{O}^{[R]}$ must satisfy the following selection rule

$$\langle \hat{O} \rangle = \langle \overline{56} | \hat{O}^{[R]} | 56 \rangle$$

Selection rule

$$\langle \hat{\boldsymbol{O}} \rangle \neq \mathbf{0}$$

only if $\hat{O}^{[R]}$ transforms according to one of the irreducible representations R contained in the product $\overline{56} \times 56$

$$\overline{56} \times 56 = 1 + 35 + 405 + 2695$$

0-body 1-body 2-body 3-body

SU(3) x SU(2) decomposition of 35-plet

Decompose one-body SU(6) tensor **35** into SU(3) and SU(2) subtensors

$$35 = (8,1) + (8,3) + (1,3)$$

scalar $J = 0$

vector $J = 1$

First entry: dimension of SU(3) flavor operator

Second entry: dimension of SU(2) spin operator $(2J + 1)$

Example: Charge multipole operator transforms as **flavor 8**.

Charge multipoles transform as **J=even** tensors in spin space.

The **35** dim. irrep **does not contain a 5-dim. rep. (8,5) in spin space necessary for a rank J = 2 quadrupole moment operator.**

Therefore, first order SU(6) symmetry breaking one-quark operators cannot produce nonvanishing quadrupole moments.

SU(3) x SU(2) decomposition of 405-plet

Decompose two-body SU(6) tensor 405 into SU(3) and SU(2) subtensors

$$405 = (1,1) + \boxed{(8,1)} + (27,1) \qquad \text{scalar } J = 0$$

$$+\, 2(8,3) + (10,3) + (\overline{10},3) + (27,3) \qquad \text{vector } J = 1$$

$$+\, (1,5) + \boxed{(8,5)} + (27,5) \qquad \text{tensor } J = 2$$

First entry: dimension of SU(3) flavor operator
Second entry: dimension of SU(2) spin operator $(2J + 1)$

Example: Charge multipole operator transforms as **flavor 8**

and as a **spin tensor** with rank **J=even**.

The spin scalar **(8,1)** and spin tensor **(8,5)** are the only components of the SU(6) tensor **405** that contribute to the two-body charge density $\rho_{[2]}$.

SU(6) Wigner-Eckart Theorem

$$\langle \hat{O} \rangle = \left\langle \overline{56_f} \middle| \rho^{[405]}_{[2]\,\mu} \middle| 56_i \right\rangle = \left\langle \overline{56} \middle\| \rho^{[405]}_{[2]} \middle\| 56 \right\rangle \cdot (\mathrm{SU(6)\ CG\ coefficient})$$

reduced matrix element has the same value for the entire 56 multiplet

provides relations between different components μ of the 405 tensor and of different members of the 56 multiplet

- Explains the prefactors (-2) for the spin scalar (charge radii) and (-1) for the spin tensor (quadrupole moment) operators
- Explains why the matrix elements of the spin scalar and the spin tensor operators taken between the 56 dimensional representation are related.

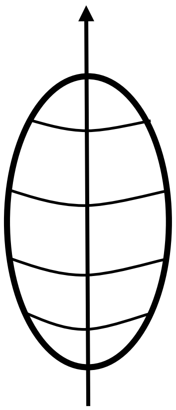
$$\rho^{[405]}_{[2],(8,1)} \sim -B \sum_{i \neq j}^{3} e_i\, \vec{\sigma}_i \cdot \vec{\sigma}_j \qquad \rho^{[405]}_{[2],(8,5)} \sim -B \sum_{i \neq j}^{3} e_i (3\, \sigma_{iz}\, \sigma_{jz} - \vec{\sigma}_i \cdot \vec{\sigma}_j)$$

AJB, Proceedings of NSTAR 2009 Workshop , Chin. Phys. C 33, 1257 (2009)

4. Baryon octupole moments

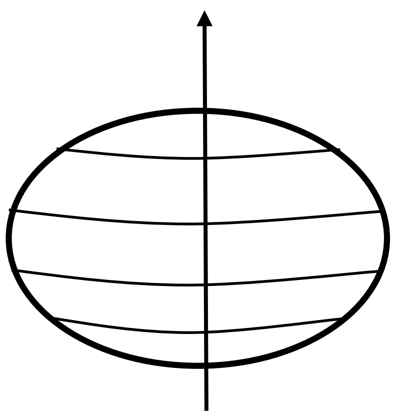
Magnetic octupole moment Ω

The magnetic octupole moment Ω measures the deviation of the magnetic moment distribution from spherical symmetry.



$\Omega > 0$ magnetic moment density is prolate

$$\Omega := \frac{3}{8} \int d^3\vec{r} \, \left(\vec{r} \times \vec{J}(\vec{r}) \right)_z (3z^2 - r^2)$$



$\Omega < 0$ magnetic moment density is oblate

SU(3) x SU(2) decomposition of the 2695-plet

$$2695 = (1,7) + (1,3)$$
$$+ (8,7) + 2(8,5) + 2(8,3) + (8,1)$$
$$+ (10,5) + (\overline{10},5) + (10,3) + (\overline{10},3) + (10,1 + (\overline{10},1)$$
$$+ (27,7) + 2(27,5) + 3(27,3) + (27,1)$$
$$+ (35,5) + (\overline{35},5) + (35,3) + (\overline{35},3)$$
$$+ (64,7) + (64,5) + (64,3) + (64,1).$$

- **no one-quark and no two-quark octupole moment operator**
- **unique three-quark octupole moment operator $(8,7)$**

$$\Omega = C \sum_{i \neq j \neq k}^{3} e_i \left(3\, \sigma_{iz}\, \sigma_{jz} - \vec{\sigma}_i \cdot \vec{\sigma}_j\right) \vec{\sigma}_k$$

three-body op. $O\left(\frac{1}{N_c^2}\right)$

A.J.B. and E.M. Henley, EPJA 35, 267 (2008)

Magnetic octupole moments Ω

Baryon	$\Omega(r=1)$	$\Omega(r\neq 1)$
Δ^-	$-4C$	$-4C$
Δ^0	0	0
Δ^+	$4C$	$4C$
Δ^{++}	$8C$	$8C$
Σ^{*-}	$-4C$	$-4C(1+\mathrm{r}+\mathrm{r}^2)/3$
Σ^{*0}	0	$2C(1+\mathrm{r}-2\mathrm{r}^2)/3$
Σ^{*+}	$4C$	$4C(1+2\mathrm{r}-\mathrm{r}^2)/3$
Ξ^{*-}	$-4C$	$-4C(\mathrm{r}+\mathrm{r}^2+\mathrm{r}^3)/3$
Ξ^{*0}	0	$2C(2\mathrm{r}-\mathrm{r}^2-\mathrm{r}^3)/3$
Ω^-	$-4C$	$-4C\mathrm{r}^3$

$$r = \frac{m_u}{m_s}$$

SU(3) breaking parameter

A.J.B. and E.M. Henley, EPJA 35, 267 (2008)

Magnetic octupole moment $\boldsymbol{\Omega}$ in pion model

$$\hat{\Omega}_\pi = \sqrt{\frac{16\,\pi}{5}}\; r_\pi^2\, Y_0^2(\hat{r}_\pi)\, \tau_z^N\;\; \sigma_z^N$$

$$\left\langle \Delta^+ \uparrow J_z = \frac{3}{2} \right|\; \hat{\Omega}_\pi \left| \Delta^+ \uparrow J_z = \frac{3}{2} \right\rangle = -\frac{2}{15} \beta'^2\;\; r_\pi^2\, \mu_N = r_n^2\, \mu_N < 0$$

$$\Omega_{\Delta^+} \;=\; -0.012\ \mathrm{fm}^3$$

$\mu_N = \frac{1}{2M_N} \cdots$ nuclear magneton

$r_\pi \cdots$ distance between core and pion

$\beta' \cdots$ nucleon core pion admixture

$\Omega_{\Delta^+} < 0\;\; \rightarrow$ Δ magnetic moment distribution is oblate

A.J.B. and E.M. Henley, EPJA 35, 267 (2008)

Determine constant C in pion model

$$\Omega(\Delta^{+}) = 4C = -0.012\ \mathrm{fm}^3$$

$$\rightarrow C = -0.003$$

$$\Omega(\Omega^{-}) = -4C\, r^3 = 0.0026\ \mathrm{fm}^3$$

Comparison with other models

$\Omega(\Delta^{+}) = -0.0035\ \mathrm{fm}^3$ (Ramalho, Peña, Gross, PLB 678, 355 (2009))

$\Omega(\Omega^{-}) = 0.016\ \mathrm{fm}^3$ (Aliev, Aziz, Savici, PLB 681, 240 (2009))

Models agree in sign but not in magnitude.

Magnetic octupole moment relations

The 10 diagonal decuplet baryons are described by just one parameter C.

→

There are 9 relations between them.

6 of these do not depend on the flavor-symmetry breaking parameter r.

$$\Omega_{\Delta^+} = -\Omega_{\Delta^-}$$

$$\Omega_{\Delta^0} = 0$$

$$\Omega_{\Delta^{++}} = -2\,\Omega_{\Delta^-}$$

$$\Omega_{\Sigma^{*+}} - \Omega_{\Sigma^{*-}} = 2\,\Omega_{\Sigma^{*0}}$$

$$3\,(\Omega_{\Xi^{*-}} - \Omega_{\Sigma^{*-}}) = \Omega_{\Omega^-} - \Omega_{\Delta^-}$$

$$\Omega_{\Xi^{*0}} + 2\,\Omega_{\Xi^{*-}} = \Omega_{\Sigma^{*-}} - \Omega_{\Sigma^{*+}}$$

A.J.B. and E.M. Henley, Eur. Phys. J. A 35, 267 (2008)

5. Summary

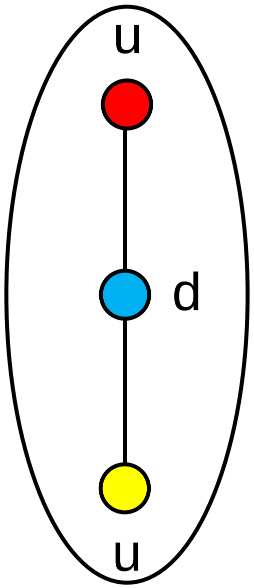
Results obtained with E. M. Henley

Three very different nucleon models agree concerning the sign

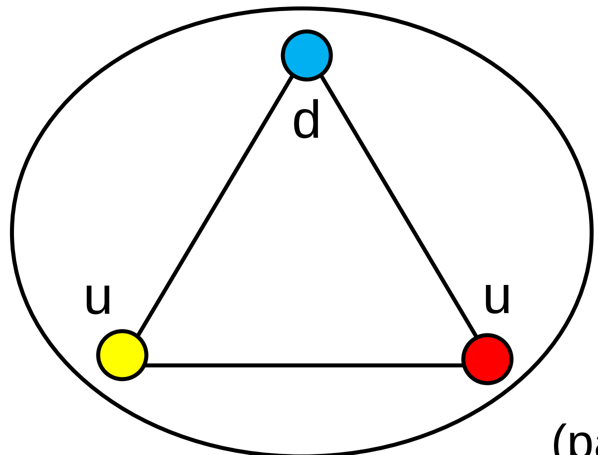
of the intrinsic quadrupole moments Q_0

of the proton and its first excited state Δ^+(1232).

p



prolate
(cigar-shaped)



Δ^+

oblate
(pancake shaped)

$$Q_0(p) = -r_n^2 > 0$$

$$Q_0(\Delta^+) = r_n^2 < 0$$

$r_n^2 = -0.1149(35)\ \mathrm{fm}^2$ experimental neutron charge radius

Magnetic octupole moment Ω of the Δ

Spectroscopic octupole moment of the Δ^+

$$\Omega_{\Delta^+} = -0.012\ \mathrm{fm}^3$$

Intrinsic octupole moment of the proton and Δ^+

$$\Omega_0(p) = -r_n^2\ \mu_N > 0$$

$$\Omega_0(\Delta^+) = r_n^2\ \mu_N < 0$$

$\Omega_0(p) > 0 \rightarrow$ proton magnetic moment distribution is prolate

$\Omega_0(\Delta^+) < 0 \rightarrow \Delta^+$ magnetic moment distribution is oblate

Best chance to get information on **Ω** is from exp. N→N*(1680) M3 transition amplitude.

Implications for other observables

The intrinsic deformation of the nucleon may be seen in other observables, e.g.

- **spin structure of the nucleon**

  AJB. and E. M. Henley, Spin of ground state baryons,
  Phys. Rev. D 83, 096011 (2011)

  AJB. and E. M. Henley, Three-quark currents and baryon spin,
  Eur. Phys. J. 55, 749 (2014)

- **πNΔ and πΔ Δ couplings**

  AJB and E. M. Henley, Pion-baryon couplings,
  Phys. Lett. B 484, 255 (2000)

  AJB and S. Moszkowski, Phys. Rev. C 87, 028203 (2013)

- **atomic hydrogen spectroscopy**

  AJB, Nucleon deformation and atomic spectroscopy,
  Can. J. Phys. 83, 455 (2005)

  AJB, Nonspherical proton shape and hydrogen hyperfine splitting,
  Can. J. Phys. 87, 773 (2009)

Prof. Ernest Henley was a man of the highest scientific and personal integrity, which was reflected in his humanity, empathy, and modesty.

Back up material

Form factor relations

$\mathrm{N} \rightarrow \Delta$ form factor relations

$$G_{M1}^{p\rightarrow\Delta^+}(Q^2) = -\sqrt{2}\, G_M^n(Q^2)$$

$$\mu_{p\rightarrow\Delta^+} = -\sqrt{2}\, \mu_n$$

magnetic form factors
Beg, Lee, Pais, 1964

$$G_{C2}^{p\rightarrow\Delta^+}(Q^2) = -\frac{3\sqrt{2}}{Q^2}\, G_C^n(Q^2)$$

$$Q_{p\rightarrow\Delta^+} = \frac{1}{\sqrt{2}}\, r_n^2$$

charge form factors
AJB, Phys. Rev. Lett. 93 (2004) 212301

Definition of C2/M1 ratio

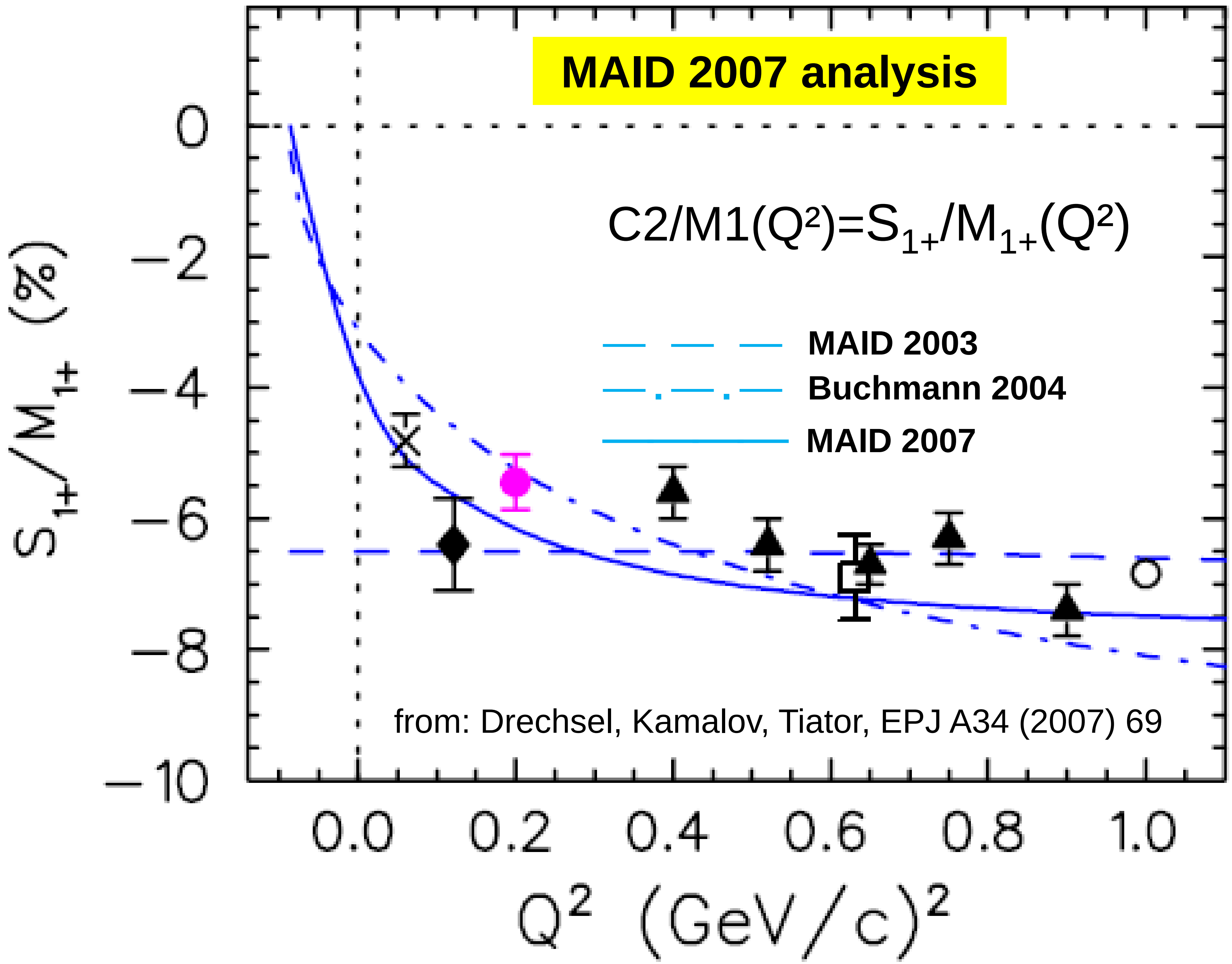
$$\frac{C2}{M1}(Q^2) := \frac{|\vec{q}|M_N}{6}\,\frac{G_{C2}^{p\to\Delta^+}(Q^2)}{G_{M1}^{p\to\Delta^+}(Q^2)}$$

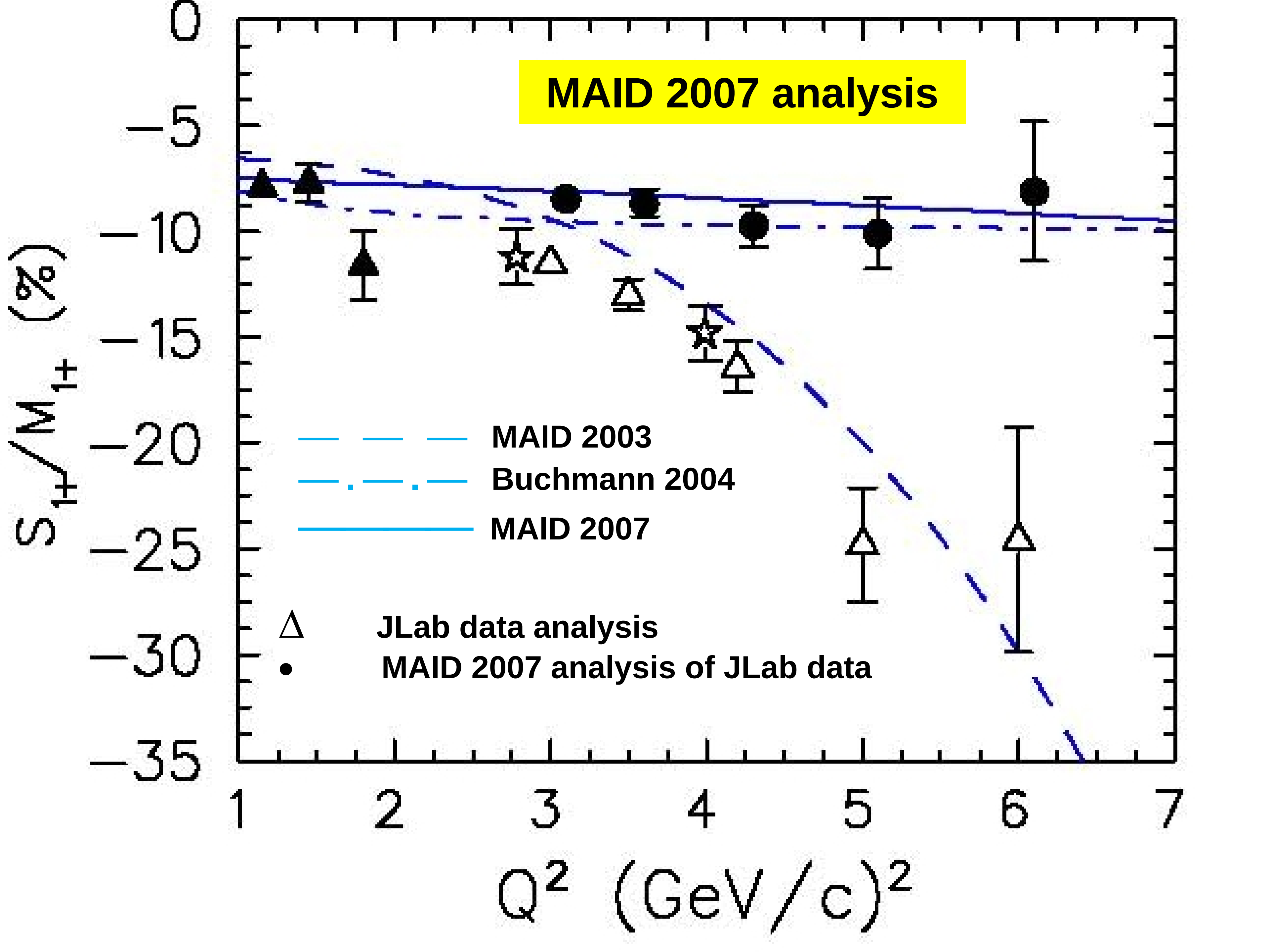
Insert form factor relations

$$\frac{C2}{M1}(Q^2) = \frac{|\vec{q}|M_N}{2Q^2}\,\frac{G_C^n(Q^2)}{G_M^n(Q^2)}$$

C2/M1 ratio expressed via neutron elastic form factors

A. J. Buchmann, Phys. Rev. Lett. 93 (2004) 212301

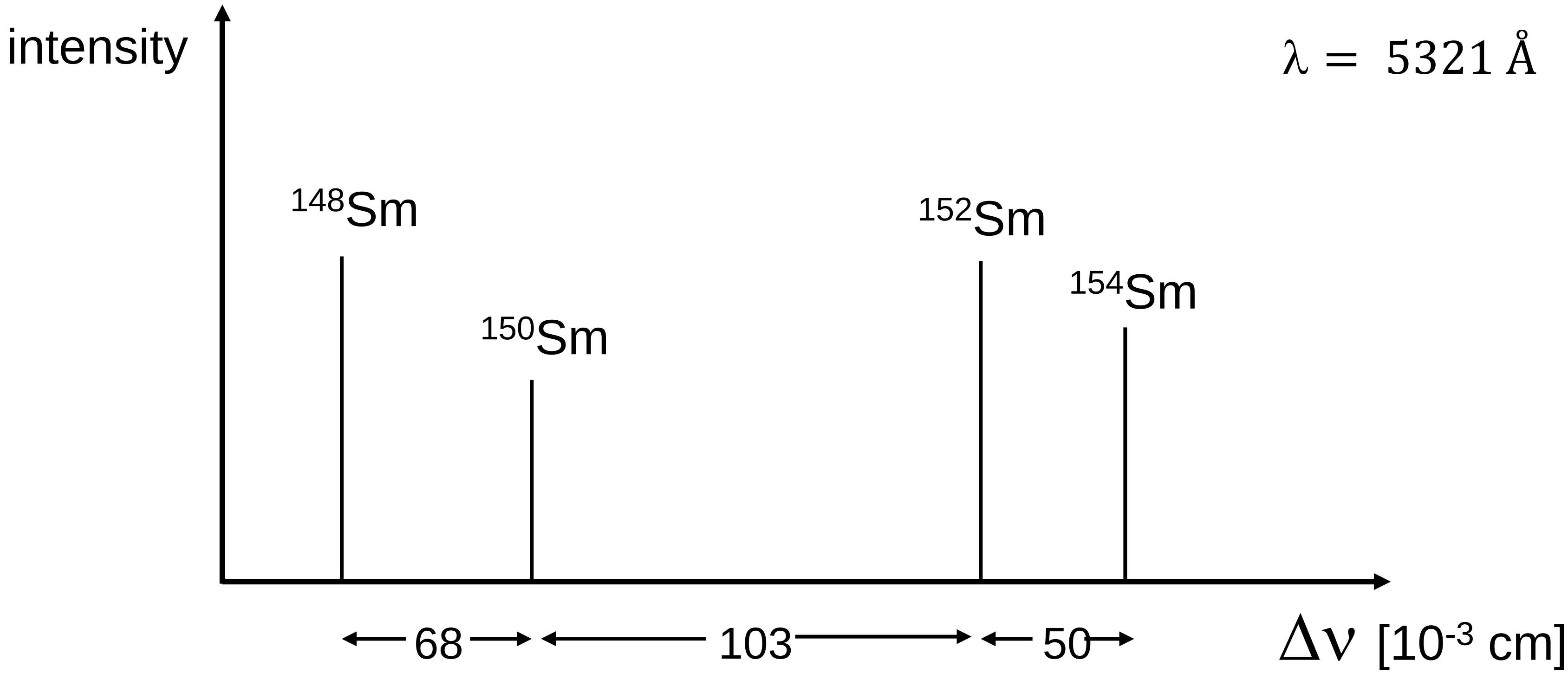

MAID 2007 analysis
C2/M1(Q²)=S1+/M1+(Q²)
MAID 2003
Buchmann 2004
MAID 2007
from: Drechsel, Kamalov, Tiator, EPJ A34 (2007) 69
S1+/M1+ (%)
0
−2
−4
−6
−8
−10
0.0
0.2
0.4
0.6
0.8
1.0
Q² (GeV/c)²


MAID 2007 analysis
MAID 2003
Buchmann 2004
MAID 2007
Δ JLab data analysis
• MAID 2007 analysis of JLab data
S_{1+}/M_{1+} (%)
Q^2 (GeV/c)^2
0
−5
−10
−15
−20
−25
−30
−35
1
2
3
4
5
6
7

Historical notes on the discovery of intrinsic quadrupole moments

Discovery of intrinsic quadrupole moments

In 1934 Schüler and Schmidt found an anomously large isotope shift between the lines of ^{150}Sm and ^{152}Sm isotopes with nuclear spin I=0.



- shift between ^{150}Sm and ^{152}Sm is twice as large as between ^{152}Sm and ^{154}Sm
- cannot be explained by a mass or volume effect

Explanation by Brix and Kopfermann (1947)

Idea

The anomalously large isotope shift in Sm isotopes is connected with the **big jump of spectroscopic quadrupole moments in Eu isotopes**

$$Q(^{151}\text{Eu}) = 150\ \text{fm}^2$$

$$Q(^{153}\text{Eu}) = 320\ \text{fm}^2$$

The huge quadrupole moments of ^{151}Eu and ^{153}Eu **cannot be generated by the addition of a single proton**.

$$^{151}\text{Eu}(\text{I} = 5/2) = {}^{150}\text{Sm}(\text{I} = 0) + \text{p}$$

$$^{153}\text{Eu}(\text{I} = 5/2) = {}^{152}\text{Sm}(\text{I} = 0) + \text{p}$$

They must have been already present in the $I = 0$ nuclei ^{150}Sm and ^{152}Sm.

Conclusion

The ^{150}Sm and ^{152}Sm nuclei with nuclear spin $I = 0$ have the same **intrinsic deformation** as the ^{151}Eu and ^{153}Eu nuclei with spin $I = 5/2$.

intrinsic quadrupole moment: $Q_0(\text{Eu}) \approx Q_0(\text{Sm})$

Effect of nuclear deformation on isostope shift

$$\delta E_{IS} = \frac{2\pi}{3} Z e^2 \left|\Psi_e(0)\right|^2 \delta\langle r^2 \rangle$$

$$\left|\Psi_e(0)\right|^2 = \frac{1}{\pi}\left(\frac{Z}{n a_0}\right)^3$$

Ψ_e ... electron wave function at r=0

a_0 ... Bohr radius

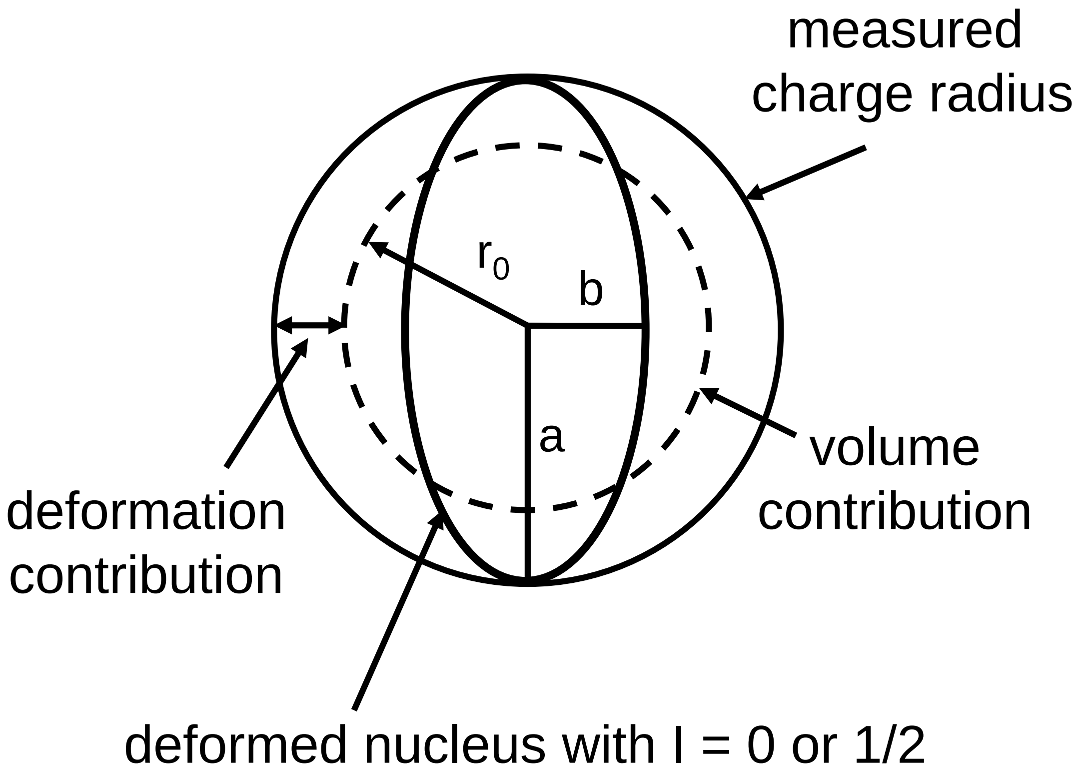
$$\underbrace{\delta\langle r^2 \rangle}_{\text{measured}} = \underbrace{\delta\langle r^2 \rangle_{vol}}_{\sim A^{-1/3}\ \text{calculated}} + \underbrace{\delta\langle r^2 \rangle_{def}}_{\sim \delta Q_0\ \text{extracted}}$$

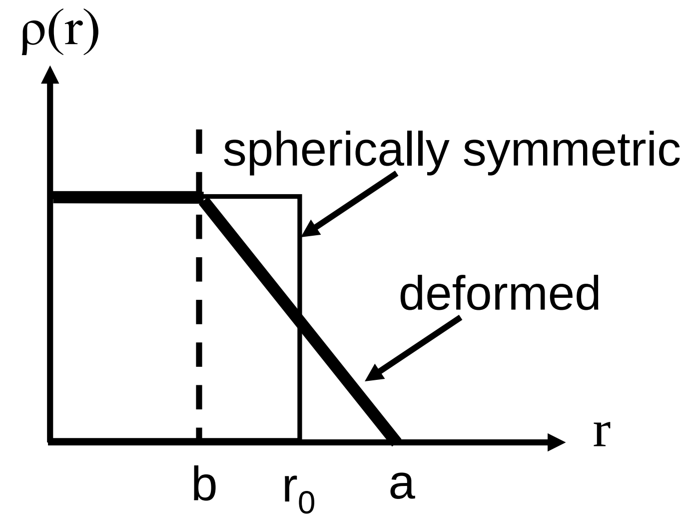
$r_{vol}^2 \sim A^{2/3}$

The Sm isotope shift is increased due to large change in nuclear deformation.

The large δE_{IS} between Sm isotopes is explained using δQ_0 of Eu isotopes.

Effect of nonsphericity on nuclear charge radius





$$r^2 = \int dr\, r^2 \rho(r)$$

Effect of deformation

Larger charge radius but the same area (total charge).

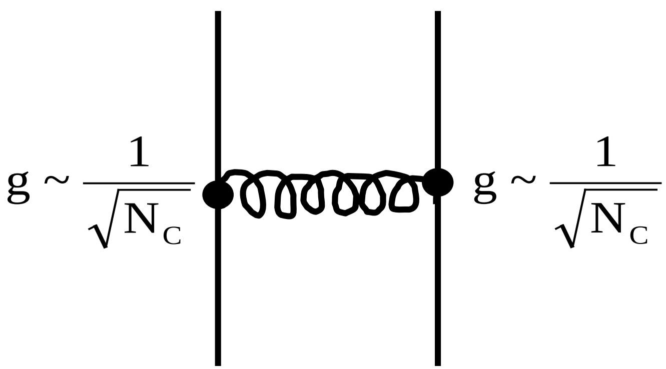
N→ Δ quadrupole moment in $1/N_C$

$1/N_C$ expansion of QCD processes

N_C ... number of colors

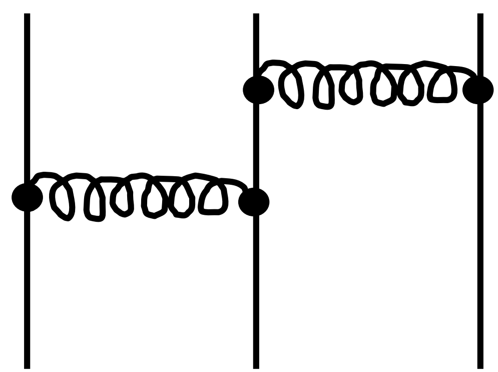
strong coupling $$\alpha_S(Q^2) = \frac{g^2(Q^2)}{4\pi} = \frac{12\pi}{(11\,\boldsymbol{N_c} - 2\,N_f)\ln\left(\frac{Q^2}{\Lambda^2}\right)} \sim \frac{\mathbf{1}}{\boldsymbol{N_c}}$$

two-quark operator



$$O\left(\frac{1}{N_c^1}\right)$$

three-quark operator



$$O\left(\frac{1}{N_c^2}\right)$$

. . .

N→ Δ quadrupole moment in $1/N_C$

$$Q_{p\to\Delta^+} = \left(\frac{B}{N_C} - 2\,\frac{C}{N_C^2}\right)\sqrt{\frac{(N_C+5)(N_C-1)}{2}}$$

including 3-body operators (C terms)

$$r_n^2 = \left(\frac{B}{N_C} - 2\,\frac{C}{N_C^2}\right)\frac{(N_C+5)(N_C+3)}{N_C}$$

$$Q_{p\to\Delta^+} = \frac{1}{\sqrt{2}} r_n^2 \underbrace{\frac{N_C}{N_C+3}\sqrt{\frac{N_C+5}{N_c-1}}}_{\substack{1 \\ \text{for } N_C=3 \text{ and } N_C=\infty}}$$

indicative of a more general validity

AJB, J. Hester, R. Lebed, PRD 66, 056002 (2002)