Neutrino Oscillations and the MaVaN Model

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Outline

- Massive neutrinos and the Standard Model
- Neutrino oscillations
  - The standard picture
  - The MaVaN picture
- Outlook

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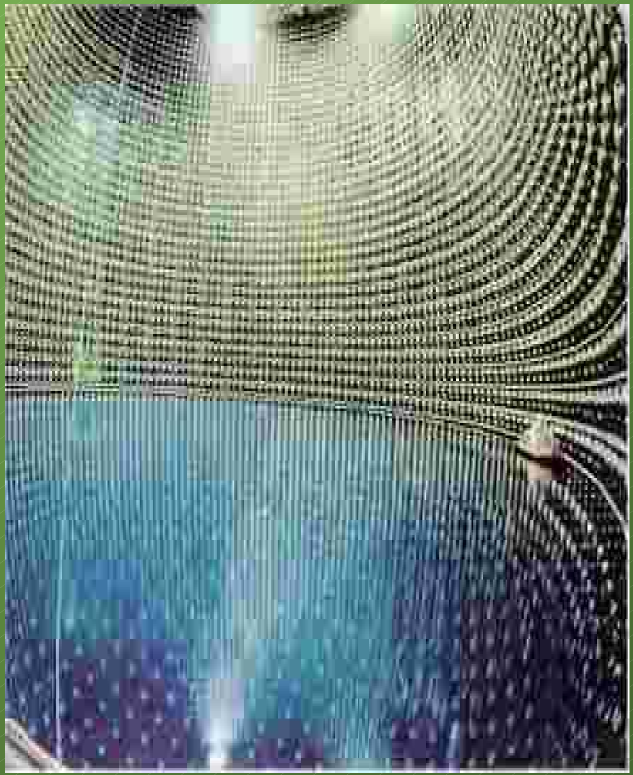
The Standard Model

- According to the Standard Model, all three neutrino masses are zero
- Nonzero masses can work, but require an extension of the model
- Difficult to measure masses because neutrinos rarely interact
  - A neutrino of moderate energy can penetrate many light years of lead!
  - They're passing through us *right now*

Super-Kamiokande

- 50,000 ton underground water tank
- Photomultiplier tubes see results of electron neutrino interactions
  - But not the actual neutrinos
- Fewer results than expected
  - Resolution: neutrino oscillations, which can only happen with nonzero mass



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Neutrino Oscillations

- Consider only electron and muon neutrinos
  - "Flavor eigenstates" $|\nu_e\rangle$, $|\nu_\mu\rangle$
- Different from the eigenstates of the Hamiltonian
  - "Mass eigenstates" $|\nu_1\rangle$, $|\nu_2\rangle$
- Related by a (vacuum) mixing angle θ_0
  - To get from one basis to the other, multiply by a unitary transformation $U(\theta_0)$:

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos(\theta_0) & \sin(\theta_0) \\ -\sin(\theta_0) & \cos(\theta_0) \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

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A Quick Derivation (1 of 4)

- In the mass eigenstate basis, the mass matrix is

$$M_{\text{mass}} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

- In the flavor eigenstate basis, it is then

$$\begin{aligned} M_{\text{flavor}} &= U(\theta_0) M_{\text{mass}} U^\dagger(\theta_0) \\ &= \left(m_2^2 - m_1^2\right) \begin{pmatrix} -\cos(2\theta_0) & \sin(2\theta_0) \\ \sin(2\theta_0) & \cos(2\theta_0) \end{pmatrix} + \frac{1}{2}\left(m_2^2 + m_1^2\right) \mathbb{1} \end{aligned}$$

A Quick Derivation (2 of 4)

- In the non-relativistic limit, $\sqrt{p^2+m^2} \approx p + \frac{m^2}{2p}$
- The kinetic energy is then

$$
\begin{aligned}
T &= p\mathbb{1} + \frac{1}{2p} M^2_{\text{flavor}} \\
&= \frac{1}{4p}\left(m_2^2 - m_1^2\right)\begin{pmatrix} -\cos(2\theta_0) & \sin(2\theta_0) \\ \sin(2\theta_0) & \cos(2\theta_0) \end{pmatrix} + \frac{1}{4p}\left(4p^2 + m_2^2 + m_1^2\right)\mathbb{1}
\end{aligned}
$$

- Wolfenstein (1978) derives potential term (MSW effect)
  - Matter almost entirely first-generation leptons and quarks
  - Weak charged current interactions single out the electron neutrino component

$$
V_{\text{MSW}} = \sqrt{2} G_F n_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}
$$

A Quick Derivation (3 of 4)

- Define $\delta m^2 = m_2^2 - m_1^2$
- Drop terms proportional to the identity
- Define $\Omega = \frac{2\sqrt{2}G_F n_e E}{\delta m^2}$

$$H_{\text{eff}} = \frac{\delta m^2}{4}\begin{pmatrix} -(\cos(2\theta_0) - \Omega) & \sin(2\theta_0) \\ \sin(2\theta_0) & \cos(2\theta_0) - \Omega \end{pmatrix}$$

- After some algebra and trigonometry:

$$\sin^2(2\theta_m) = \frac{\sin^2(2\theta_0)}{\sin^2(2\theta_0) + (\cos(2\theta_0) - \Omega)^2}$$

- Mixing angle changes in matter!

A Quick Derivation (4 of 4)

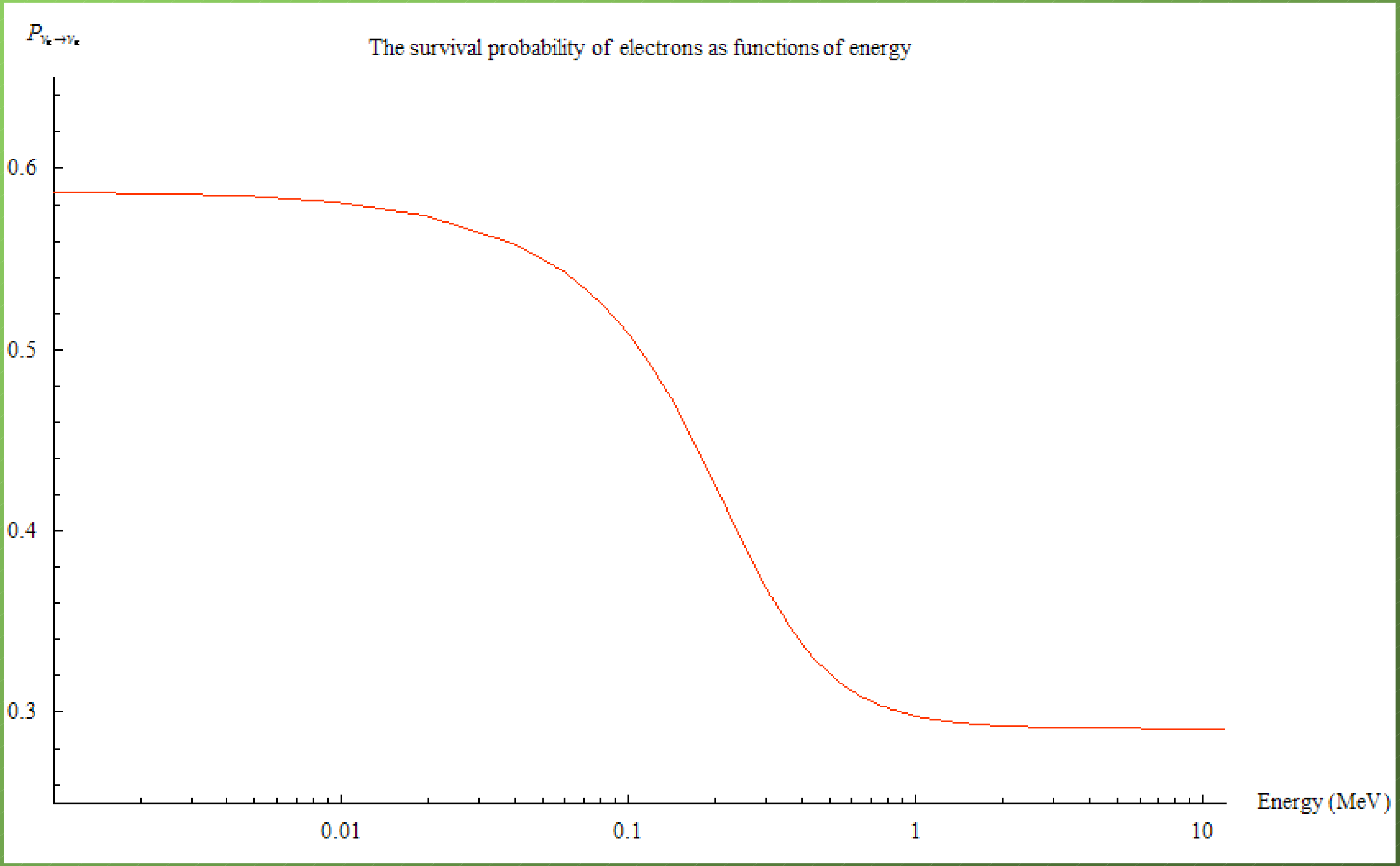
- From QM,

$$\left|\nu_j(t)\right\rangle = \left|\nu_j(0)\right\rangle \exp\left(-\mathrm{i}\int_0^t E_j(t')\,\mathrm{d}t'\right)$$

- Use the adiabatic approximation (essentially assuming density varies slowly):

$$\begin{aligned} P_{\nu_e \to \nu_e} &= |\langle \nu_e(T) \mid \nu_e(0)\rangle|^2 \\ &= \cos^2(\theta_m)\cos^2(\theta_0) + \sin^2(\theta_m)\sin^2(\theta_0) + \\ &\quad \frac{1}{2}\sin(2\theta_m)\sin(2\theta_0)\cos\left(\int_0^T (E_2(t') - E_1(t'))\,\mathrm{d}t'\right) \\ &\to \frac{1}{2} + \frac{1}{2}\cos(2\theta_m)\cos(2\theta_0) \end{aligned}$$

No More Derivations!



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Mass Varying Neutrinos (MaVaNs)

- Add a new scalar field, the “acceleron” A
- Postulate a heavy sterile (dark) neutrino
  - Mass dependent on expectation value **h**A**i**, which is a function of n_e
- Physical justification: measured dark energy density and neutrino energy density are similar
  - MaVaNs can help explain this without fine tuning
  - Many more cosmological justifications - see hep-ph/0309800
- But do they agree with experimental results?

My Last Few Weeks

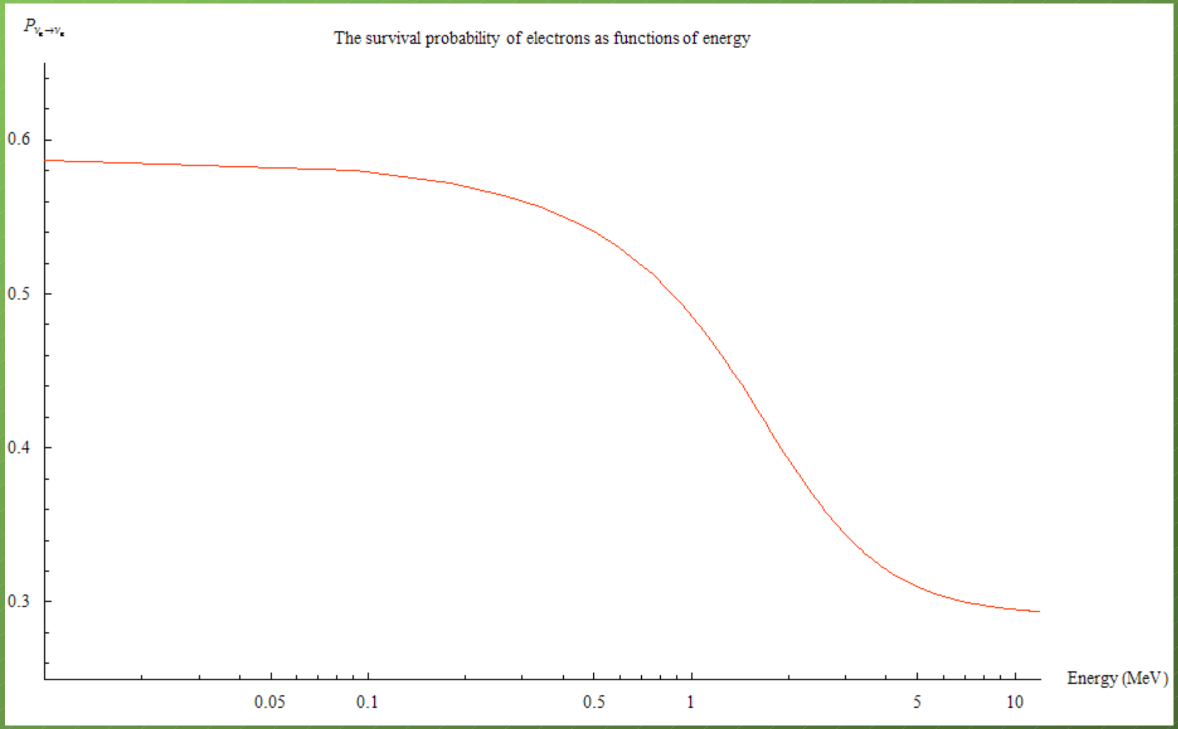
- Simplify model by integrating out heavy sterile neutrino
  - Assume $M_{\mathrm{sterile}} = K n_e^r$
- New Hamiltonian is

$$H_{\mathrm{MaVaN}} = \frac{1}{2E}\frac{m_D^4}{K^2} n_e^{-2r} \begin{pmatrix} \frac{2\sqrt{2}K^2 G_F n_e^{2r+1} E}{m_D^4} + \sin^2(\theta) & \sin(\theta)\cos(\theta) \\ \sin(\theta)\cos(\theta) & \cos^2(\theta) \end{pmatrix}$$

- Radically different electron density dependence

Reproducing Measurements

- Theory of neutrino masses has a new basis
  - Must reproduce experimental results to be useful
- We can!



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Outlook

- This is a very positive result -- no a priori reason that MaVaNs should reproduce experimental results
- Experiments like KamLAND provide further constraints to test
- Lots to explore!

Thanks

- To my two great advisors, Ann Nelson and Neal Weiner
- To my fellow REU students
- To UW Physics
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