MaVaNs and Solar Physics: A Preliminary Study

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Abstract

A simple quantum mechanical model of neutrino oscillations fits all experimental data thus far, but the data is sparse enough to allow for many other possibilities. The mass-varying neutrino (MaVaN) model adds one heavy sterile neutrino with a mass proportional to a power of the electron number density. We study oscillations between two active neutrinos after integrating out the sterile neutrino, with the goal of reproducing the standard electron neutrino survival curve for electron neutrinos from the sun. Using a computer scan to find the best-fitting parameters, we achieve this goal, and demonstrate that there are possibilities for new physics in the explanation of neutrino oscillations.

1 Introduction

The discovery of neutrino oscillations by Super-Kamiokande [1] at once established that neutrinos have nonzero masses and excited a search for the theory predicting those masses. The SNO [2], [3] and KamLAND [4] collaborations have further confirmed the nonzero neutrino mass in solar neutrinos. As a result, the solar neutrino oscillation parameters are tightly pinned down, as shown in [5]: $6 \times 10^{-5}\,\mathrm{eV}^2 < \Delta m^2_{21} < 1 \times 10^{-4}\,\mathrm{eV}^2$ and $0.3 < \tan^2(\theta_{12}) < 0.55$ at 2σ. A simple quantum mechanical model with these experimental parameters can reproduce the electron neutrino survival probability curve seen by experiment(see section 2.1 below), but the existing oscillation data is sparse enough to allow for many other possibilities as well. The mass-varying neutrino (MaVaN) model takes as an ansatz one sterile neutrino with a mass that scales as a power of the electron number density. We calculate the survival probability of solar electron neutrinos numerically in the hopes of reproducing the standard curve as a function of energy, as in [6].

The rest of the paper is organized as follows: section 2 outlines background information, including a derivation of the standard quantum mechanical model and reasons for a new approach to neutrino oscillations. Section 3 discusses both the setup and the analysis of the MaVaN proposal, and in section 4 we present our conclusions.

2 Background

2.1 The Standard Neutrino Oscillation Model

The simplest model of neutrino oscillations involves two active neutrinos. There are of course three neutrinos according to the Standard Model of particle physics, but the the dominant oscillations happen between the lightest two neutrinos, so neglecting the third is a good first-order approximation. Oscillations emerge because the flavor eigenstates $|\,\nu_e\rangle$ and $|\,\nu_\mu\rangle$ are different from the mass eigenstates $|\,\nu_1\rangle$ and $|\,\nu_2\rangle$, with masses m_1 and m_2 respectively. We assume that these are labeled such that $m_1 \leq m_2$. In the absence of matter, these two different bases are related by a vacuum mixing angle θ_0, so that going from the first basis to the second means multiplying the latter by a unitary transformation $U(\theta_0)$:

$$\begin{pmatrix} |\,\nu_e\rangle \\ |\,\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos(\theta_0) & \sin(\theta_0) \\ -\sin(\theta_0) & \cos(\theta_0) \end{pmatrix} \begin{pmatrix} |\,\nu_1\rangle \\ |\,\nu_2\rangle \end{pmatrix} \tag{1}$$

In the mass basis, the mass matrix for the system is

$$M_{\text{mass}} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \tag{2}$$

and so in the flavor basis it is $M_{\text{flavor}} = U(\theta_0) M_{\text{mass}} U^\dagger(\theta_0)$ where $U(\theta_0)$ is as above. Using the relativistic approximation with

$$E = \sqrt{p^2 + m^2} \approx p + \frac{m^2}{2p} \tag{3}$$

we have a kinetic contribution to the Hamiltonian of

$$\begin{aligned} T &\approx p\,\mathbb{1} + \frac{1}{4p} M_{\text{flavor}}^2 & (4)\\ &= p\,\mathbb{1} + \frac{1}{4p} \begin{pmatrix} \cos^2(\theta_0) m_1^2 + \sin^2(\theta_0) m_2^2 & \sin(\theta_0)\cos(\theta_0)\left(m_2^2 - m_1^2\right) \\ \sin(\theta_0)\cos(\theta_0)\left(m_2^2 - m_1^2\right) & \sin^2(\theta_0) m_1^2 + \cos^2(\theta_0) m_2^2 \end{pmatrix} & (5)\\ &= \frac{1}{4p}\left(m_2^2 - m_1^2\right) \begin{pmatrix} -\cos(2\theta_0) & \sin(2\theta_0) \\ \sin(2\theta_0) & \cos(2\theta_0) \end{pmatrix} + \frac{1}{4p}\left(4p^2 + m_2^2 + m_1^2\right)\mathbb{1} & (6) \end{aligned}$$

The term proportional to the identity will add the same amount to the energy of each eigenstate. Since it will be the difference in energies that gives rise to oscillations, the term proportional to the identity will have no oscillatory effect, and we will neglect it from now on. Introducing the standard notation $\Delta m^2 = m_2^2 - m_1^2$ and approximating $\frac{1}{p} \approx \frac{1}{E}$ to first order, we have the following kinetic term:

$$T = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos(2\theta_0) & \sin(2\theta_0) \\ \sin(2\theta_0) & \cos(2\theta_0) \end{pmatrix} \tag{7}$$

There is a potential term in the Hamiltonian that we have thus far neglected, as Wolfenstein first noticed in 1978 [7]. When neutrinos pass through matter, which is almost completely first generation leptons and quarks, weak current interactions single out the electron neutrino component. This contributes a potential term

$$V_{\text{MSW}} = \sqrt{2} G_F n_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \frac{\sqrt{2}}{2} G_F n_e \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{\sqrt{2}}{2} G_F n_e\, \mathbb{1} \tag{8}$$

where n_e is the electron density. If we write

$$\Omega = \frac{2\sqrt{2} G_F n_e E}{\Delta m^2} \tag{9}$$

and drop the term proportional to the identity, the Hamiltonian can then be written

$$H = \frac{\Delta m^2}{4} \begin{pmatrix} -\left(\cos(2\theta_0) - \Omega\right) & \sin(2\theta_0) \\ \sin(2\theta_0) & \cos(2\theta_0) - \Omega \end{pmatrix} \tag{10}$$

From this we see that the mixing angle in matter θ_m is given by

$$\sin^2(2\theta_m) = \frac{\sin^2(2\theta_0)}{\sin^2(2\theta_0) + (\cos(2\theta_0) - \Omega)^2} \tag{11}$$

The energies (or more accurately, the energy splittings, since we have dropped parts of the Hamiltonian proportional to the identity) are $\pm\xi$, where

$$\xi = \frac{\Delta m^2}{4E}\sqrt{\Omega^2 - 2\Omega\cos(2\theta_0) + 1} \tag{12}$$

We now derive the survival probability for solar electron neutrinos. From above, flavor and mass eigenstates in the sun are related by

$$\begin{pmatrix} |\,\nu_e\rangle \\ |\,\nu_\mu\rangle \end{pmatrix} = U(\theta_m)\begin{pmatrix} |\,\nu_1\rangle \\ |\,\nu_2\rangle \end{pmatrix} \tag{13}$$

The mass eigenstates evolve in time as usual, i.e.

$$|\,\nu_j(t)\rangle = |\,\nu_j(0)\rangle \exp\left(-i\int_0^t E_j(t')\,\mathrm{d}t'\right) \tag{14}$$

Differentiating the relation between the sets of eigenstates, we get

$$\frac{\partial}{\partial t}\begin{pmatrix} |\,\nu_e\rangle \\ |\,\nu_\mu\rangle \end{pmatrix} = \frac{\partial U(\theta)}{\partial t}\begin{pmatrix} |\,\nu_1\rangle \\ |\,\nu_2\rangle \end{pmatrix} + U(\theta)\frac{\partial}{\partial t}\begin{pmatrix} |\,\nu_1\rangle \\ |\,\nu_2\rangle \end{pmatrix} \tag{15}$$

and the Schrödinger equation tells us that

$$i\frac{\partial}{\partial t}\begin{pmatrix} |\,\nu_1\rangle \\ |\,\nu_2\rangle \end{pmatrix} = U(\theta)\begin{pmatrix} -\xi & 0 \\ 0 & \xi \end{pmatrix} U^\dagger(\theta)\begin{pmatrix} |\,\nu_e\rangle \\ |\,\nu_\mu\rangle \end{pmatrix} \tag{16}$$

Combining these two gives

$$i\frac{\partial}{\partial t}\begin{pmatrix} |\,\nu_1\rangle \\ |\,\nu_2\rangle \end{pmatrix} = \begin{pmatrix} -\xi & -i\frac{\partial\theta}{\partial t} \\ i\frac{\partial\theta}{\partial t} & \xi \end{pmatrix}\begin{pmatrix} |\,\nu_1\rangle \\ |\,\nu_2\rangle \end{pmatrix} \tag{17}$$

The adiabatic approximation is valid in the region where the off-diagonal terms are much smaller than the difference in diagonal terms, i.e.

$$\frac{\partial\theta}{\partial t} \ll 2\xi \qquad \text{or equivalently} \qquad \frac{1}{n_e}\frac{\partial n_e}{\partial t} \ll \frac{\Delta m^2}{2E}\frac{\sin^2(2\theta)}{\cos(2\theta)} \tag{18}$$

In this approximation the different mass eigenstates do not mix: $\langle\nu_i(t)\,|\,\nu_j(t)\rangle = \delta_{ij}$. The probability that an electron neutrino generated in the sun at $t = 0$ is measured as an electron neutrino a time T later is

$$P_{\nu_e\to\nu_e} = |\langle\nu_e(T)\,|\,\nu_e(0)\rangle|^2 \tag{19}$$

$$= \left|\sum_{j,k}\langle\nu_e(T)\,|\,\nu_j(T)\rangle\,\langle\nu_j(T)\,|\,\nu_k(0)\rangle\,\langle\nu_k(0)\,|\,\nu_e(0)\rangle\right|^2 \tag{20}$$

$$= \left|\sum_{j,k}U^*_{ej}(\theta_m)U_{ek}(\theta_0)\delta_{jk}\exp\left(-i\int_0^T E_j(t')\,\mathrm{d}t'\right)\right|^2 \tag{21}$$

$$= \cos^2(\theta_m)\cos^2(\theta_0) + \sin^2(\theta_m)\sin^2(\theta_0) + \frac{1}{2}\sin(2\theta_m)\sin(2\theta_0)\cos\left(\int_0^T (E_2(t') - E_1(t'))\,\mathrm{d}t'\right) \tag{22}$$

The last term oscillates very rapidly for solar neutrinos, and so averages out to zero. With a little trigonometry, we can write the remaining terms as

$$P_{\nu_e \to \nu_e} = \frac{1}{2} + \frac{1}{2}\cos(2\theta_m)\cos(2\theta_0) \tag{23}$$

The current best values of the mass squared difference and the vacuum mixing angle are $\Delta m^2 = 8.3 \times 10^{-5}\,\mathrm{eV}^2$ and $\tan^2(\theta) = 0.36$ (see [8]). Plugging these in, along with Bahcall's value for the solar electron density [9], we can plot the survival probability for electron neutrinos as a function of energy. Electron

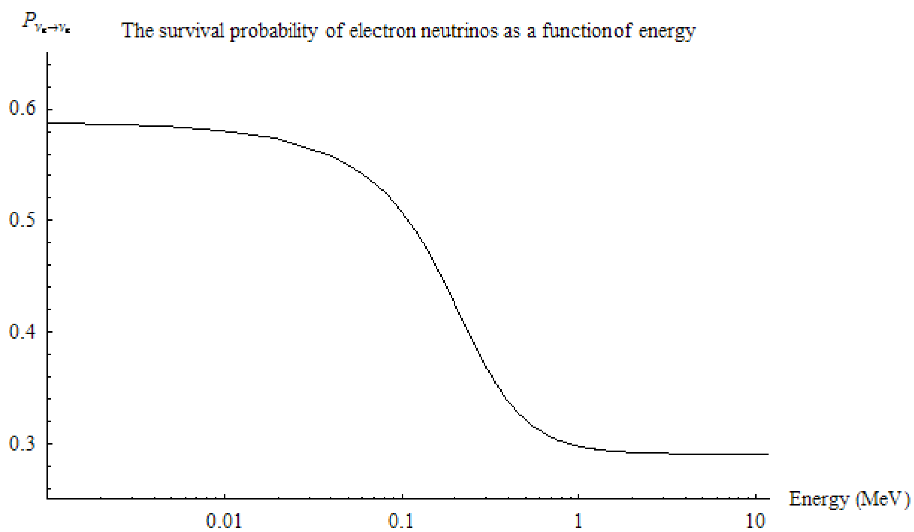

Figure 1: The standard adiabatic electron neutrino survival probability as a function of energy

neutrinos with larger energies are less probable to remain electron neutrinos by the time they reach the detector on Earth. If instead of assuming adiabaticity we make the Landau-Zener approximation, the probability begins to rise again for large energies (asymptoting to the same probability value as at zero energy) [10], [11]. However, we will be assuming that all processes are adiabatic in developing the MaVaN theory, so we will not need this addition.

2.2 Reasons for a New Approach

While the approach outlined above does fit all the experimental data, there is so little data available that this is hardly telling. There is substantial room for effects due to new physics, and at this early stage many of these effects cannot be ruled out.

3 Methods

3.1 Setup

We consider a universe with two active neutrinos and one heavy sterile neutrino. We take as an ansatz that the mass of the sterile neutrino varies as a function of the electron number density according to

$$m_{\text{sterile}} = K n_e^r \tag{24}$$

for some constants K and r. In this system, the initial mass matrix in the flavor basis is

$$M_3 = \begin{pmatrix} 0 & 0 & m_D \sin(\theta) \\ 0 & 0 & m_D \cos(\theta) \\ m_D \sin(\theta) & m_D \cos(\theta) & K n_e^r \end{pmatrix} \tag{25}$$

where m_D is an unspecified Dirac mass, and θ is the vacuum mixing angle. On the assumption that the sterile neutrino is very heavy, we integrate it out, and the square of our mass matrix becomes

$$M_2^2 = C^2 n_e^{-2r} \begin{pmatrix} \sin^2(\theta) & \sin(\theta)\cos(\theta) \\ \sin(\theta)\cos(\theta) & \cos^2(\theta) \end{pmatrix} \tag{26}$$

where $C = \frac{m_D^2}{K}$ is the relevant combination of free parameters. Neglecting the terms proportional to the identity, the Hamiltonian for this system is given by $H = \frac{1}{2E} M_2^2 + V$ where V is the MSW potential:

$$H = \frac{1}{2E} C^2 n_e^{-2r} \begin{pmatrix} \frac{2\sqrt{2} G_F n_e^{2r+1} E}{C^2} + \sin^2(\theta) & \sin(\theta)\cos(\theta) \\ \sin(\theta)\cos(\theta) & \cos^2(\theta) \end{pmatrix} \tag{27}$$

3.2 Analysis

We consider two points, one at the Sun and one on Earth. For these two points, we plug in the known values of the parameters in the Hamiltonian, such as electron density. With this we have the MaVaN Hamiltonian at the two points under consideration. There are two parameters whose values we do not know – the ratio C and the power dependence r (we have assumed that standard parameters like θ take on their currently accepted measured values). The MaVaN theory does not place constraints on C or r; however, since the goal of the paper was to demonstrate that an alternate theory could reproduce the experimental neutrino oscillation results, finding an appropriate pair is enough. That said, we know from working with neutrino oscillations that masses tend to be on the order of 10^{-2} eV, and we use this to limit the parameter search. Plotting the electron survival probability as a function of energy with various candidate search results, the results appear very promising. The value of r mainly influences the steepness of the descent, and weakly influences the energy value where

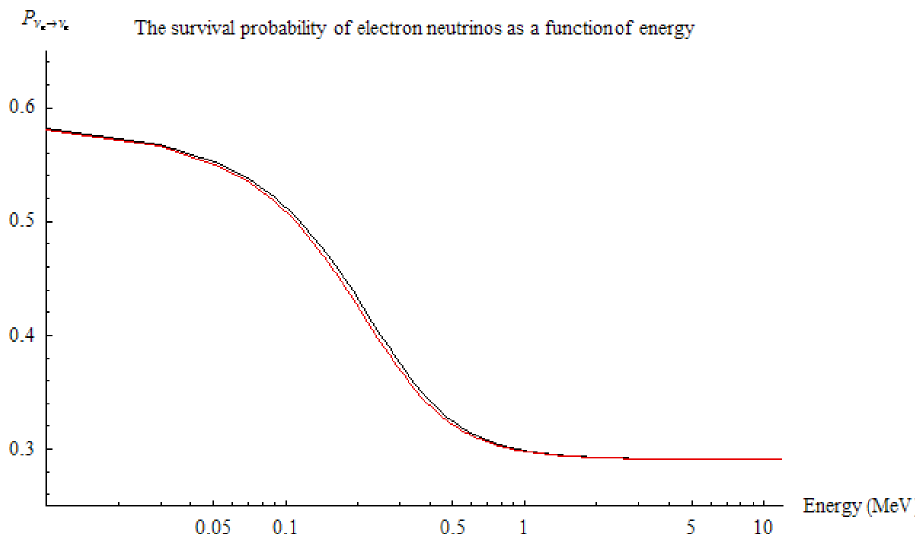

Figure 2: The standard and MaVaN adiabatic electron neutrino survival probability as functions of energy. The MaVaN curve, which is the upper curve, can be fine tuned to lie directly on top of the standard curve.

it takes place. The value of C primarily affects this latter quantity, and so using both we can tune the curve to closely replicate the standard curve. An example with $r = .58$ is shown in figure 2. The curve predicted by the MaVaN theory is the upper curve. Further fine tuning can put the MaVaN curve directly on top of the standard curve. It can also make the transition steeper, shallower, begin later, begin earlier, and a host of other possibilities. Most importantly, though, the graph contains the essential features of the standard electron neutrino survival curve, which was what we were attempting to reproduce.

4 Conclusion

The simplest explanation of neutrino oscillations agrees with current experimental data, but measured data is so scarce that this is hardly a stringent test. The mass-varying neutrino (MaVaN) model takes as an ansatz one sterile neutrino with a mass that scales as a power of the electron density. We have demonstrated that this theory can calculate the survival probability of solar electron neutrinos in agreement with the standard curve. While the standard explanation of neutrino oscillations may be satisfactory, this result means that alternate explanations for neutrino oscillations cannot be ruled out.

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