Muon Followers in the Sudbury Neutrino Observatory

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The Sudbury Neutrino Observatory (SNO) is a Cerenkov detector composed of a spherical acrylic vessel filled with 1,000 metric tonness of ultra pure heavy water. It measures 12 m in diameter and is located about 2 km deep within a Canadian nickel mine. Even at this depth, cosmic rays penetrate the detector at a rate of roughly three per hour. Muons passing through the detector will interact with the heavy water and release spallation neutrons, also called muon followers. They can be measured in the detector via their capture on deuterium (for SNO's Pure D_2O phase) or chlorine (for SNO's Salt phase). Our analysis focused on the differences between spallation neutrons and neutrons that are born thermal in both the Pure D_2O and Salt phases. This paper will provide an overview of the capture efficiency for both low energy neutrons and high energy neutrons as a function of radius. From this efficiency study, the rate of neutrons released as each muon passes through can be measured. Understanding and measuring the neutrons produced by muons will help in the optimization of future sensitive underground experiments.

I. INTRODUCTION

The Sudbury Neutrino Observatory (SNO) is perhaps best known for resolving the solar neutrino problem in 2002 by determining that electron neutrinos indeed change flavors between the time they are produced in the Sun and the time they are detected on Earth. The detector, which measures approximately 10 stories, was designed for the purpose of studying neutrinos produced by the sun. It consists of 9,456 photomultiplier tubes (PMTs) surrounding an acrylic vessel (AV), which is filled with 1,000 tonnes of ultra-pure heavy water. The AV and the PMTs are suspended in ordinary water to shield the heavy water from radioactivity [1].

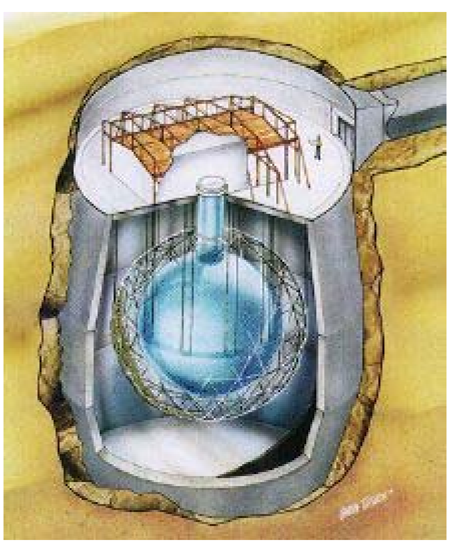

FIG 1: Sketch of SNO courtesy of http://www.sno.phy.queensu.ca/

In addressing the solar neutrino problem, SNO measures both the solar ^8B electron neutrino flux and the flux of all active neutrinos coming the Sun. It does so through three reactions:

$$\nu_e + d \rightarrow e^- + p + p \ (CC)$$

$$\nu_x + d \rightarrow \nu_x + p + n \ (NC)$$

$$\nu_x + e^- \rightarrow \upsilon_x + e^- \ (ES)$$

with ν_x being any flavor neutrino. In each reaction, a neutrino is detected through its characteristic ring of Cerenkov light emitted by a high-speed electron. In the first reaction, the charged current reaction, an electron neutrino, ν_e, interacts with a deuterium nucleus, *d*. The electron produced, e^-, will then pass through the water and release Cerenkov light, which is then detected by the PMTs. In the second reaction, the neutral current reaction, any flavor of neutrino, ν_x, interacts with a deuterium nucleus. The neutron that is produced, *n*, may then capture on another deuterium nucleus, producing a gamma ray, which in turn Compton scatters free atomic electrons whose Cerenkov light is detected. Lastly, the elastic scattering reaction is the elastic scattering of a neutrino and an atomic electron [2].

SNO operates in three phases. The first phase, the Pure D_2O phase, uses only deuterium to measure the NC reaction rate in SNO. This phase of the experiment was completed in June 2001. In order to increase the efficiency of measuring the NC reactions occurring in SNO, table salt, NaCl, was dissolved in the D_2O in the second phase, the Salt phase. The ^{35}Cl nucleus is able to capture neutrons about 100 times greater than deuterium nuclei. Again, once the neutron is captured, gamma rays are released in the de-excitation of ^{36}Cl, producing a detectable signal. This phase of SNO was completed in September 2003. The SNO experiment is currently in its third phase, the NCD phase, in which ^3He proportional counters, called neutral-current detectors (NCDs) are inserted into the D_2O volume and detect neutrons without the production of Cerenkov light. This allows event-by-event separation of CC and NC reactions [3].

II. MUON FOLLOWERS

In our study, we examined neutrons produced from muon spallation, in which neutrons are produced from the interaction of muons passing through the detector and the heavy water, as described in the following spallation reactions:

$$\mu + d \rightarrow \mu + n + X$$

$$\mu + {}^{16}O \rightarrow \mu + n + X\ ,$$

where X is the recoil nucleus and n is the spallation neutron, also called muon follower[1].

Studying muon followers is important since they remain one of the dominant backgrounds for many sensitive underground experiments. Understanding and measuring the neutrons produced by muons will help in optimizing future sensitive underground experiments.

Muon followers can be measured in the detector via their capture on deuterium (for SNO's Pure D_2O phase) or chlorine (for SNO's Salt phase), as described by the following reactions:

$n + d \rightarrow t + \gamma$ (neutron capture on deuterium)

$n + {}^{35}Cl \rightarrow {}^{36}Cl + \sum \gamma$ (neutron capture on chlorine)

We will present analysis focused on the differences between spallation neutrons and neutrons that are born thermal in both the Pure D_2O and Salt phases. This paper will use the term spallation neutrons interchangeably with "high energy" neutrons, which may have energies as high as 2 GeV. Neutrons that are born near thermal energies may be referred to as "low energy" neutrons, with energies ranging from 0.01 eV to 1 eV. Lastly, we will present an overview of the capture efficiency for both low and high energy neutrons as a function of radius, where the radius is measured from the center of the AV. This will enable us to measure the rate of neutrons released as each muon passes through.

The rate of neutrons produced from muons passing through the detector is given by the expression:

$$N_n(b_\mu) = F_\mu \cdot \left(\sum \frac{\sigma_i M_i(b_\mu)\varepsilon_{n,i}(b_\mu)}{A_i}\right) \cdot N_a \cdot t \cdot \varepsilon_\mu(b_\mu) + B_n$$

where F_μ is the muon flux, N_a is Avogadro's number, t is the sampling time, ε_μ is the muon detection efficiency, B_n is the muon follower background, σ is the muon nuclear cross-section, M is the mass of the target, A is the atomic number of the target, and b_μ is the maximum impact parameter for an incoming muon [4]. This equation can be simplified to the following:

$$N_n(b_\mu) = F_\mu \cdot \omega(b_\mu) \cdot \varepsilon_n(b_\mu) \cdot M \cdot (1 + \chi(b_\mu)) \cdot t \cdot \varepsilon_\mu(b_\mu) + B_n$$

where $\chi(b_\mu)$ is the correction term due to high energy neutrons and $\omega(b_\mu)$ is the neutron production rate from muons. It has units of $(n/\mu)cm^2g^{-1}$[7].

If one takes into account the muon rate, the muon flux and efficiency terms cancel out, leaving:

$$N_\mu(b_\mu) = F_\mu \cdot A_{eff}(b_\mu) \cdot t \cdot \varepsilon_\mu(b_\mu) + B_\mu\ ,$$

where where $A_{eff}(b_\mu)$[2] is the effective area of the detector, and B_μ is the instrumental background muon events [4].

With some simplification, it can be shown that

$$\omega(b_\mu) = \frac{(N_n - B_n)}{(N_\mu - B_\mu)} \cdot \frac{1}{1 + \chi(b_\mu)} \cdot \frac{A_{eff}(b_\mu)}{\varepsilon_n(b_\mu) \cdot M_{D_2O}}\ .$$

The correction term due to high energy neutrons, $\chi(b_\mu)$, can be rewritten as

$$\chi(b_\mu) = \left(\frac{\varepsilon_{H_2O}}{\varepsilon_{D_2O}}\right) \cdot \frac{\rho(H_2O)}{\rho(D_2O)} \cdot \frac{A_{D_2O}}{A_{H_2O}} \cdot \left(\left(\frac{R_{H_2O}}{R_{D_2O}}\right)^3 - 1\right)$$

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[1] One or more muon followers may be produced in a single spallation reaction.

[2] A_{eff} was previously determined to be ≈ 175.5 +/- 5.4 m^2 for a b_μ value of 750 cm [7].

where ε is the capture efficiency, ρ is the mass density, and A is the atomic number.

Thus, part of our study essentially focuses on helping to determine the $\chi(b_\mu)$ term.

III. MUON ENERGY SPECTRUM

The first task in our study was modeling a realistic muon energy spectrum, since the spallation neutron energy depends on the muon energy. Using a data file from a model of muons penetrating the SNO detector, we obtained a collection of over 100,000 muons at SNO's depth with their respective energies and angular distributions. The histogram of this data shows the muon flux, $F_\mu(E_\mu)$, below.

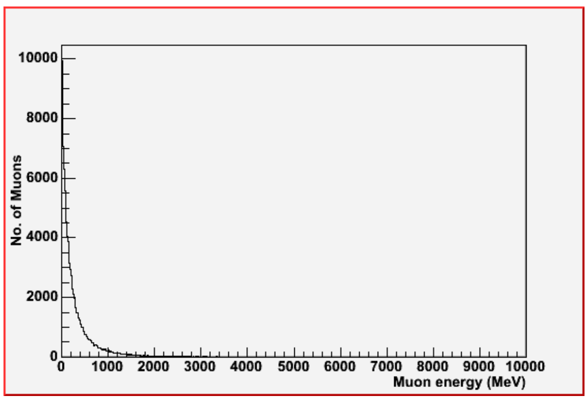

FIG 2: Muon energy spectrum

I developed a FORTRAN program (Monte Carlo routine) that returns, for each call, an energy, E_μ, that, when sampled many times, returns a distribution similar to the original ASCII file shown above. As one can see, the muon energies are quite broad, so the FORTRAN routine was rather slow. Since we did not want to read in the ASCII file each time I ran the program, we decided that a parameterization of the muon energy spectrum (that is, a fit to the distribution) would be better. Below is a log plot showing the muon energy distribution as well as the fit to the distribution (thick black line).

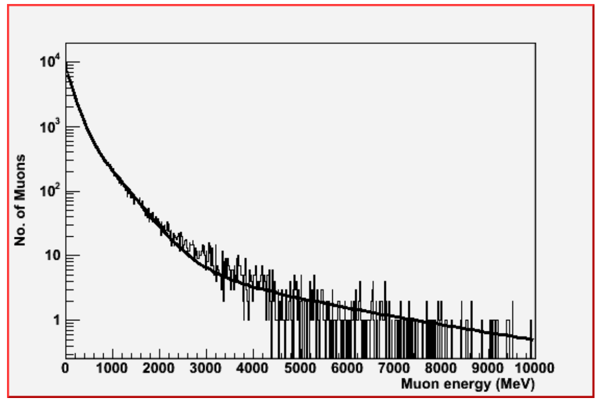

FIG 3: Muon energy spectrum with fit

The fit was done through the analysis program, ROOT[3]. The fitting function,

$$F(x) = ae^{-bx} / x + ce^{-dx} + fxe^{-gx}$$

has the following parameters:

parameter	value
a	2.41×10^{4}
b	1.58×10^{-4}
c	7.69×10^{3}
d	2.82×10^{-3}
f	-1.90×10^{1}
g	4.24×10^{-3}

FIG 4: Muon energy spectrum fit parameter values

and provides a fit with a chi square/degree of freedom of 1.0908.

After obtaining the fitting function, I was able to rewrite my FORTRAN routine to produce the proper muon energy distribution when called. In the next step, I added some neutron code that converts each muon energy into a potential neutron energy (E_n). This step essentially entails implementing the following formula:

$$\frac{dN}{dE_n} = A\left(\frac{e^{-7E_n}}{E_n} + B(E_\mu)e^{-2E_n}\right)$$

where A is a normalization factor, and

$$B(E_\mu) = 0.52 - 0.58e^{-0.0099E_\mu} \quad [5].$$

[3] Prior to this study, I had never used ROOT or C++. See reference [6] for a good C++ book.

Here is a plot of the neutron energy spectrum, which is in good agreement with the data presented in "Predicting Neutron Production from Cosmic-ray Muons" by Y-F. Wang et al. [5]. The main difference is that our plot is a spectrum of neutron energies created by a spectrum of muon energies, as opposed to a fixed muon energy.

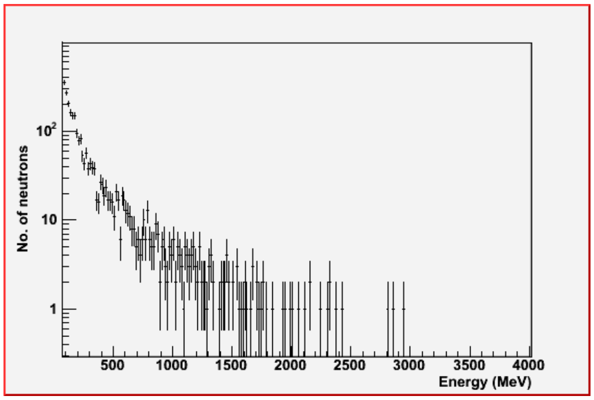

FIG 5: Neutron energy spectrum

IV. NEUTRON CAPTURE EFFICIENCY

The Monte Carlo routine was now able to generate neutrons with the proper energies. With this, I generated neutrons across the entire sphere, up to 800 cm in radius. In the next part of our analysis, I compared a set of neutrons with low energies and spallation energies for both the Pure D_2O and Salt phases. Each of the four Monte Carlos were run with 50,000 events, with the data written to a root file. The simulations were conducted by running my routine along with SNOMAN, the SNO Monte carlo and ANalysis program, the primary simulation and data analysis program used in the SNO experiment. (It simulates event by event in the SNO detector). The Monte Carlo events were then needed to be selected based on reconstructed variables in order to compare what I produced versus what I actually saw[4]. The selection criteria used are as follows:

a) 4.0 < Kinetic Energy (K.E. = Energy – 0.511 MeV) < 20 MeV (the energy selection)

b) 0 < R < 600 cm (the radius selection)

c) itr > 0.55 (goodness of fit variable selection)[5]

In the various cases, we wanted to understand the efficiency of various samples, which is defined as

ε = Number that pass cuts/ Number Generated

I decided to study efficiency as a function of generated radius. That is, I looked at the number of events generated in a sub-volume dV a distance r away from the center and then examined how many events of the ones generated within this sub-volume actually passed all my selection criteria. In order to do this, I had to learn how to draw curves, extract information from the Monte Carlo, and store information into files. Below are some plots of this efficiency[6] as a function of generated radius.

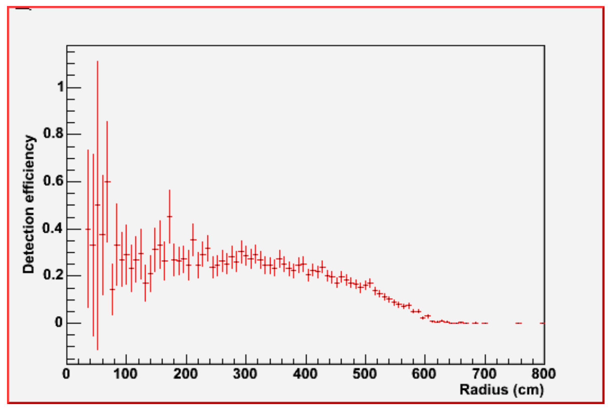

FIG 6: Efficiency curve for spallation neutrons in the pure D_2O phase.

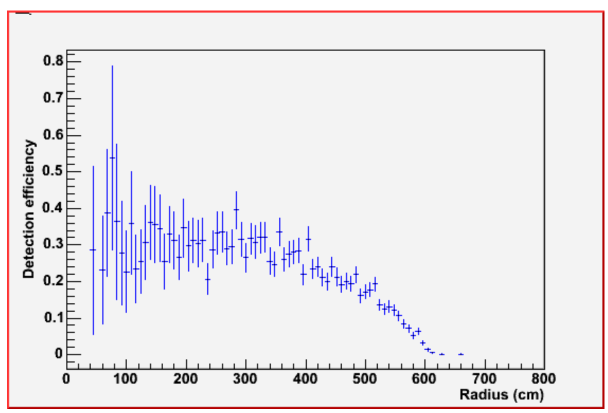

FIG 7: Efficiency curve for thermal neutrons in the pure D_2O phase.

[4] Generated variables are the actual values an event has. Since in actual data taking we do not have access to generated values, we work with reconstructed variables, which are the values we think the event has.

[5] The itr number helps to determine if an event is a physical event or just instrumental background.
[6] The relative efficiencies are valid but the absolute efficiencies are still under investigation.

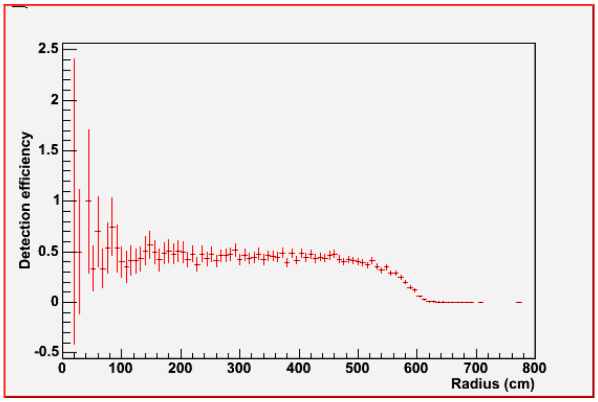

FIG 8: Efficiency curve for spallation neutrons in the salt phase.

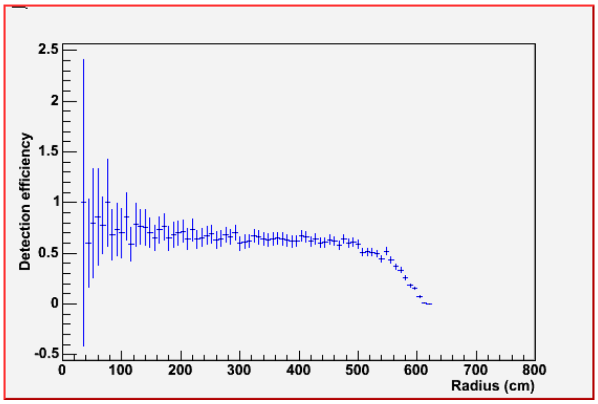

FIG 9: Efficiency curve for thermal neutrons in the Salt phase.

We observe that, as expected, the capture efficiency as a function of generated radius for low energy neutrons stops abruptly at about the 600 cm mark, where the AV ends. However, the spallation neutrons are still able to be captured by the deuterium even when they start outside the AV. Being much higher in energy, some of the spallation neutrons are able to penetrate the H_2O and the AV, into the heavy water. Below is a table showing the percent of neutrons detected when they start out between 600 cm and 800 cm.

neutron type	% detected
spallation, D2O	0.1299
low energy, D2O	0.04416
spallation, Salt	0.2532
low energy, Salt	0.1644

FIG 10: Percent of neutrons detected when they start out between 600 cm and 800 cm

Next, in order to parameterize the efficiency curves for both low and high energy neutrons as a function of generated radius, we needed an analytic model for the efficiency for a source at position, say s. The following equation does just that:

$$\in = \frac{\Sigma_D}{\Sigma} x\{1 - F_{escape}\}$$

where

$$F^{PtSrc}_{Escape} = \frac{R}{s} \frac{\sinh(\frac{s}{l})}{\sinh(\frac{R_e}{l})} \left[\cosh(\frac{R_e - R}{l}) + \frac{l}{R} \sinh(\frac{R_e - R}{l}) \right]$$

with $l = \sqrt{\frac{D}{\Sigma}}$ [7].

The parameter l is called the diffusion length, and depends upon Σ and Σ_D, which are the macroscopic cross-section for SNO heavy water and for deuterium, respectively. The other neutron transport parameter, D, is called the diffusion constant. It describes the mean free path of neutrons in the SNO heavy water. The two constants in the equation, R and R_e, are unique to the SNO spherical vessel, in which R = 600 cm and R_e = 616 cm. Lastly, the function F_{escape} represents the neutrons lost from the heavy and light water [5].

By using ROOT, I was able to parameterize the efficiency curves with the equation above. I did this holding R fixed and then letting R be a free parameter[7]. I obtained the following parameter values:

Neutron type	Parameter	value	χ^2/ndf
Spallation, D2O	$\frac{\Sigma_D}{\Sigma}$ L	0.355128 153.09	0.4842
thermal, D2O	$\frac{\Sigma_D}{\Sigma}$ L	0.363989 135.96	0.711
spallation, Salt	$\frac{\Sigma_D}{\Sigma}$ L	0.463736 50.8892	0.3797
thermal, Salt	$\frac{\Sigma_D}{\Sigma}$ L	0.670514 -57.3297	0.4245

FIG 11: Parameter values for R fixed.

[7] The curves were fitted from 0 to 600 cm.

neutron type	Parameter	Value	χ^2/ndf
spallation, D2O	$\frac{\Sigma_D}{\Sigma}$	0.322366	0.4485
	L	130.101	
	R	652.906	
thermal, D2O	$\frac{\Sigma_D}{\Sigma}$	0.347964	0.7079
	L	124.802	
	R	585.625	
spallation, Salt	$\frac{\Sigma_D}{\Sigma}$	0.459867	0.3675
	L	48.8738	
	R	593.317	
thermal, Salt	$\frac{\Sigma_D}{\Sigma}$	0.659948	0.3721
	L	-50.5529	
	R	590.287	

FIG 12: Parameter values for R not fixed.

In the last part of our analysis, we decided to conduct a test to see if the program was running properly. I did this by changing the neutron source from being distributed throughout the spherical volume to a point source located at the center. I ran these simulations each with 10,000 events. We wanted to know how far a neutron, starting at the center, travels before it gets captured. To do this, I plotted the reconstructed radius for events that passed all of my cuts. I did this for both high and low energy neutrons for the Pure D_2O and Salt phases.

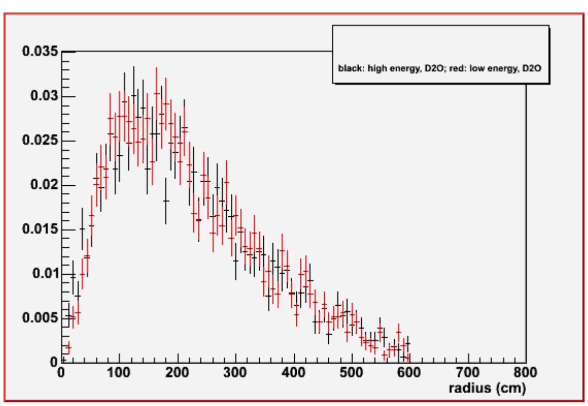

FIG 13: (scaled) Capture efficiency as a function of reconstructed radius

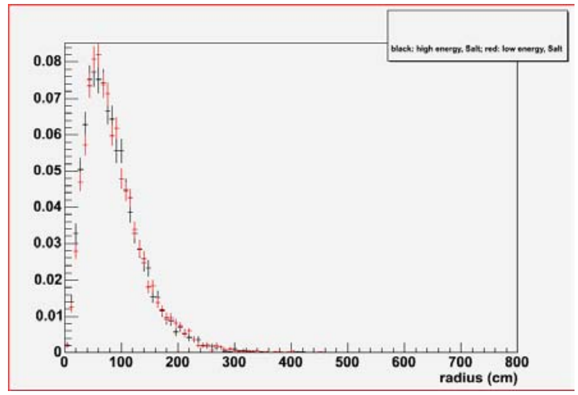

FIG 14: (scaled) Capture efficiency as a function of reconstructed radius

The above plots indeed reflect what we expect. With the source of neutrons being at the center, i.e. at 0 cm, we see that the distance a neutron travels before being captured is much shorter in the Salt phase as compared to the D_2O phase. This is expected since the chlorine added in the Salt phase has a much higher cross-section than deuterium.

V. SUMMARY

I have written a routine that produces muons and muon followers with the proper energy distributions. The capture efficiencies for spallation neutrons and neutrons that are born thermal have been produced and parameterized. They have been compared for both the Pure D_2O and Salt phases of SNO using an isotropic volume neutron source. The relative efficiencies have been tested using a point neutron source at the center. Further investigation will be conducted on the absolute efficiencies.

VI. ACKNOWLEDGEMENTS

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VII. REFERENCES

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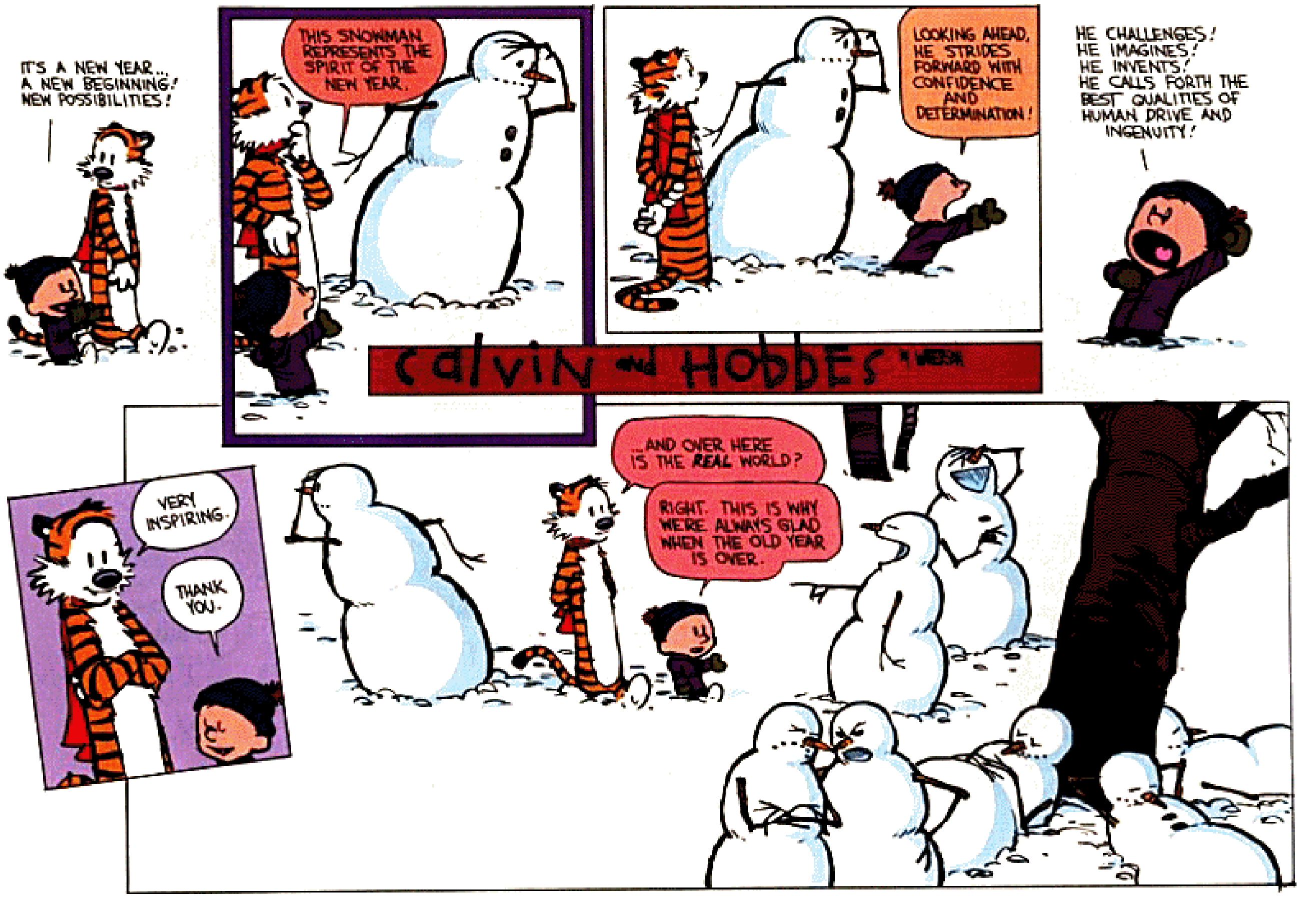
VIII. FINAL (AND FUNNY) NOTE



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