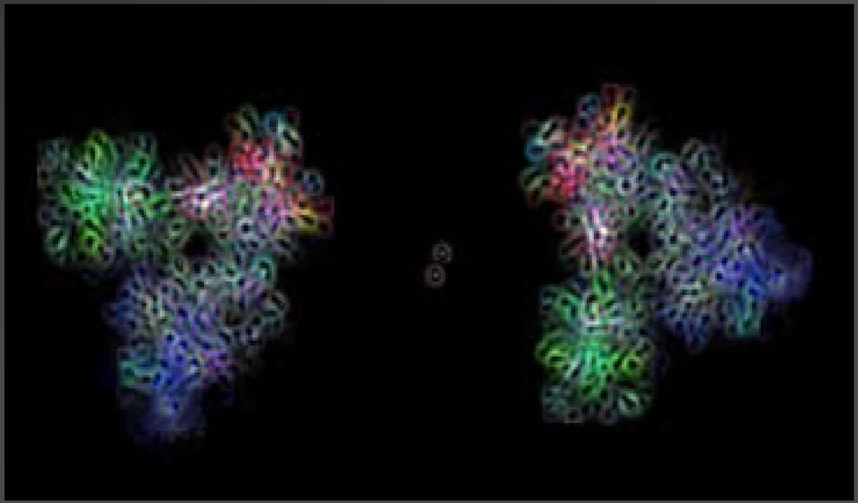
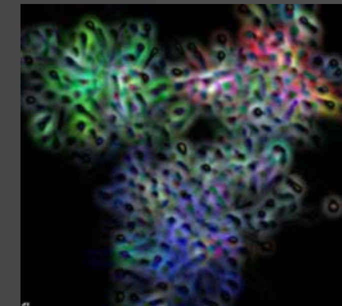
The Renormalization of Effective Field Theory for Nucleon-Nucleon Interactions

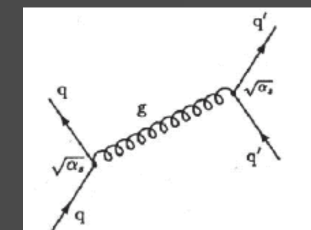


Presented by Sam Leitner
Wesleyan University
Advisor: Andreas Nogga

Quantum Chromo Dynamics (QCD)



- Gluons: exchange particles for 3 quarks
  - Strongly self interacting, produce own field

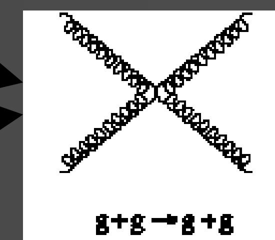


$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}_e\gamma^\mu[\partial_\mu + ieA_\mu]\psi_e - m_e\bar{\psi}_e\psi_e$$

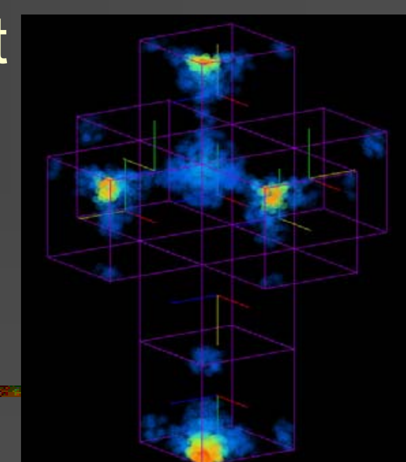
$$\mathcal{L}_{QCD} = -\frac{1}{4}F^\alpha_{\mu\nu}F^{\mu\nu}_\alpha - \sum_n \bar{\psi}_n\gamma^\mu[\partial_\mu - igA^\alpha_\mu t_\alpha]\psi_n - \sum_n m_n\bar{\psi}_n\psi_n$$

$$\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F^\alpha_{\mu\nu} = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\nu + C^\alpha_{B\gamma}A^B_\mu A^\gamma_\nu$$



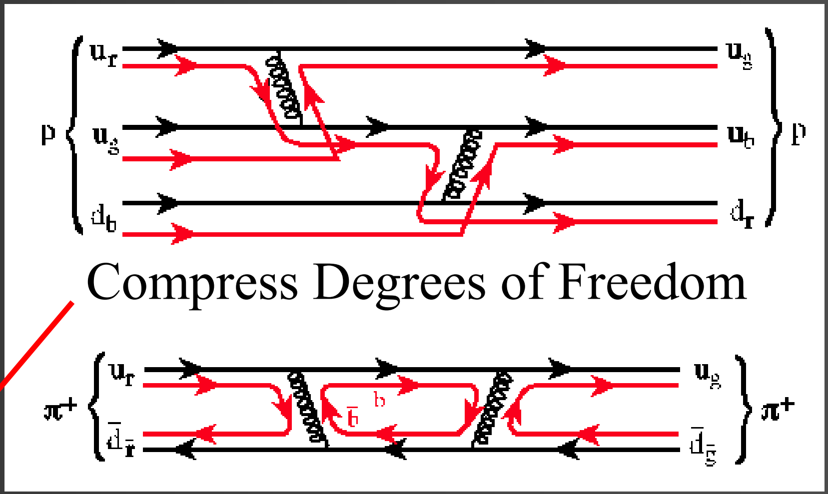
- Large Coupling Constant
  - Non-perturbative - complex pictures important
- ->Interactions analytically intractable turn to numerics … QCD lattice simulations:
  - Computationally expensive (4d =time+space)
  - Fundamentally limited = “Sine Problem”



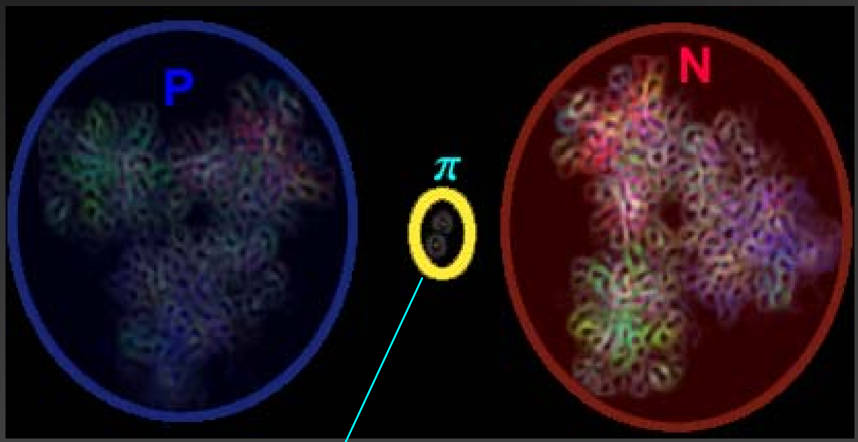
What now?

- Lattice simulations are difficult or impossible for Nucleon-Nucleon scales.
- QCD perturbation theory is useless for the small energy scales of nuclear systems.
- Towards something less fundamental, but more useful …

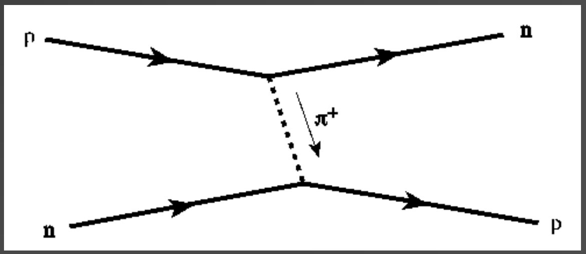
An Effective Theory

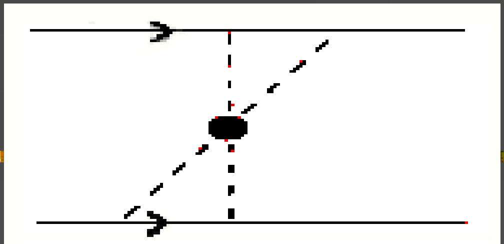


NEW "Fundamental" Particles



What we care about now = Feynman diagrams of pion exchange:





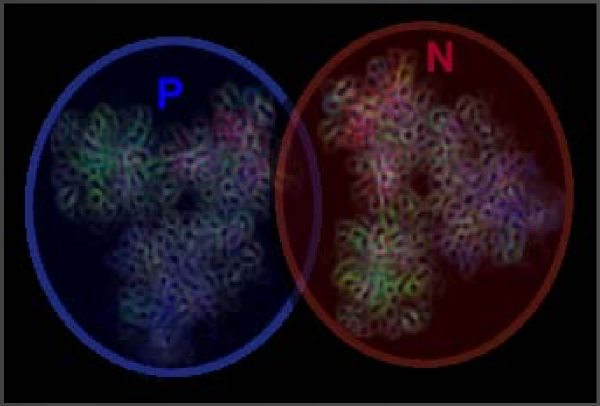
Exchange particle? Pion is in the correct energy scale (50-150MeV)

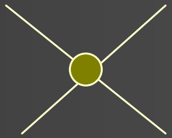
Particle and Symbol	Makeup	Rest Mass (MeV)
Pion: π^+	ud	139.6
Pion: π^0	(uu+dd)/($2^{1/2}$)	135
Kaon: K^+	us	493.7
Kaon: K_S^0	*	497.7
Kaon: K_L^0	*	497.7
Eta: η^0	(uu+dd-2ss)/($6^{1/2}$)	548.8
Rho: ρ^+	ud	770

Interaction Diagrams

- Short Distance = “Contact Interaction”
  - Phenomenological understanding only - modeled as a power series expansion (here in momentum).

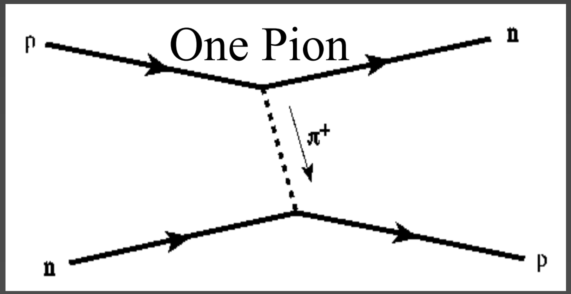
(p and p’ are the momentum of each nucleon)

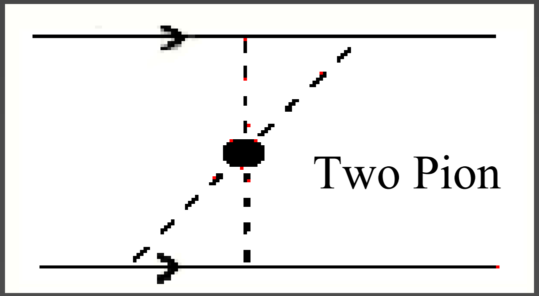




$$= \sum_{i=0}^{\infty} c_i q^{2i}$$

- Long Range = Pion Exchange





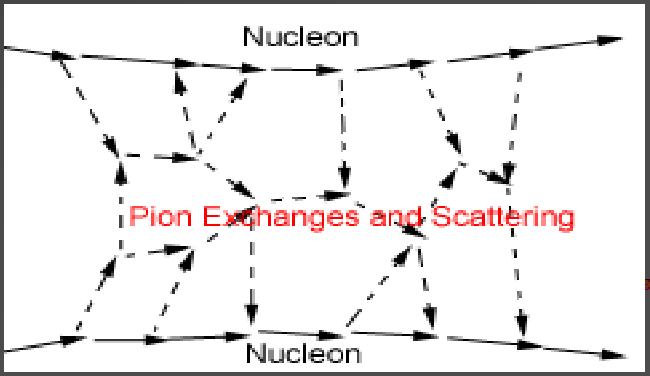


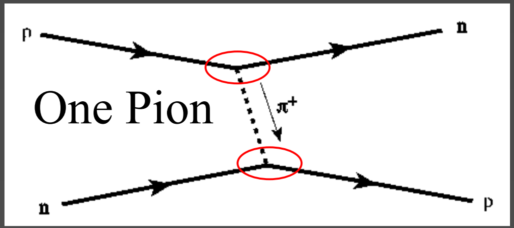
Diagram Ordering & Power Counting

- Infinite number of possible exchange diagrams.
- Approximate by looking at most important.
- Chiral Perturbation Theory - Important diagrams have lower order momentum dependence
  - Momentum is small (higher power means smaller number)
  - Count power of the momentum dependence ="power counting"
- New order of momentum dependence for each vertex.

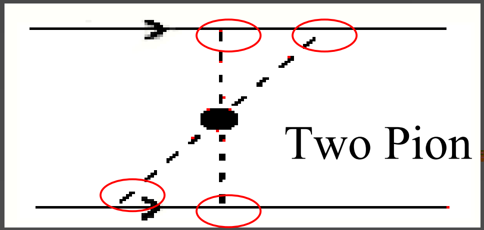
Two Vertices=q^2

Four Vertices=q^4

Eg:



Leading Order Term

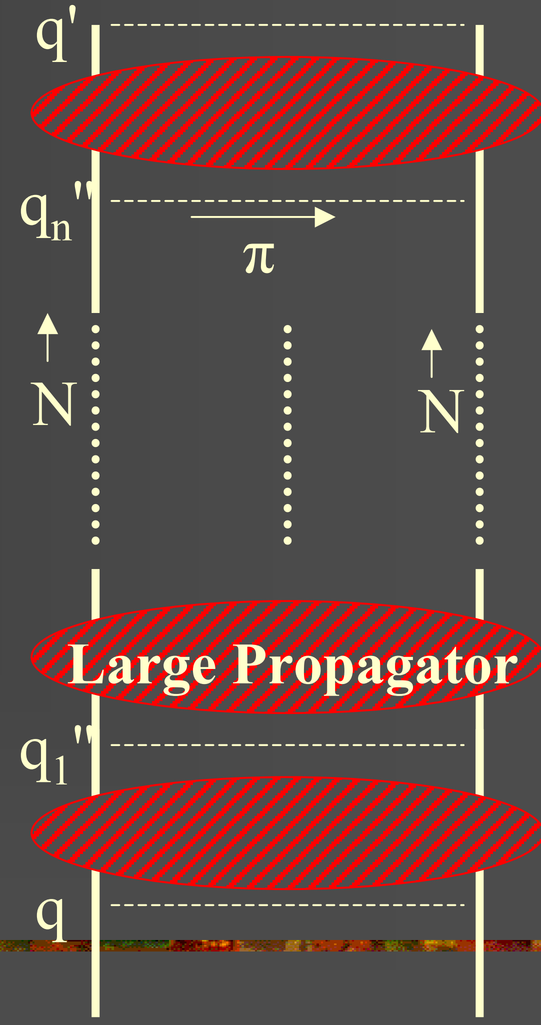


Next to Leading Order Term

Weinberg - Infrared Enhancement

- For small momentum (low energy = infrared) the free nucleon motion between successive pion exchanges is important.
- Weinberg - need to iterate the interaction to include what happens in between
  - Iterated Interaction = Interaction -> Propagation->Interaction….
    - Reformulation of the Schrodinger eq:

$$T=V+VG_0T$$

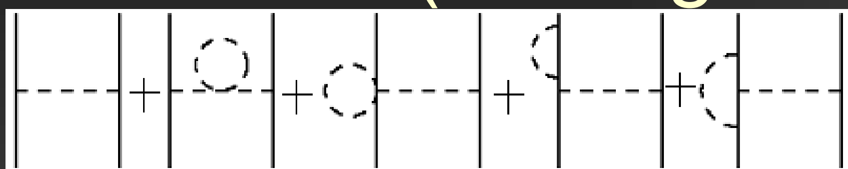


- Iterated interaction -> Potential!

Iterated Pion Exchange

- One Pion (leading order)

scary but useable potentials!

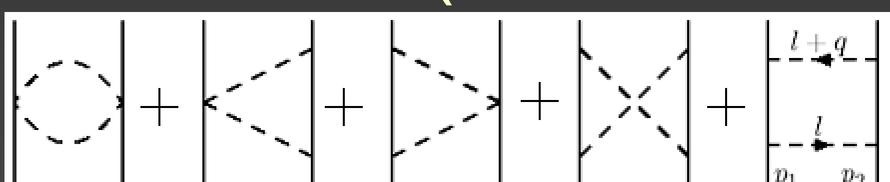


$$=$$

Orbital and Spin momentum coupling

$$V(\vec{p},\vec{p}\,') = -\frac{1}{(2\pi)^3}\left(\frac{g_A}{2f_\pi}\right)^2 \tau_1\cdot\tau_2\,\frac{\vec{q}\cdot\sigma_1\,\vec{q}\cdot\sigma_2}{(\vec{q})^2+m_\pi^2}$$

- Two Pion (next to leading order)



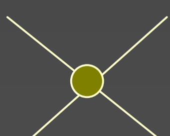
$$=$$

$$V_{2\pi} = -\frac{1}{(2\pi)^3}\cdot\frac{t_1\cdot t_2}{384\pi^2 f_\pi{}^4}\cdot L(q)\cdot\left[\begin{array}{l}4m_\pi{}^2\cdot(5\cdot gA^4-4\cdot gA^2-1)\\ +q^2(23\cdot gA^4-10\cdot gA^2-1)\\ +\dfrac{48\cdot gA^4\cdot m_\pi{}^4}{4m_\pi{}^2+q^2}\end{array}\right]$$

$$-\frac{1}{(2\pi)^3}\cdot\frac{3\cdot gA^4}{384\pi^2 f_\pi{}^4}\cdot L(q)\cdot\left[(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})-q^2\vec{\sigma}_1\cdot\vec{\sigma}_2\right]$$

$$L(q) = \frac{1}{q}\sqrt{4m_\pi{}^2+q^2}\,\ln\left(\frac{\sqrt{4m_\pi{}^2+q^2}+q}{2m_\pi}\right)$$

- Contact Interaction*



$$= \sum_{i=0}^{\infty} c_i q^{2(i+l)}$$

*contact order is angular momentum dependent (ℓ) ~ $(q')^{\ell+2i}$

Summary

- Effective theory of nucleons exchanging pions (ignore quarks and gluons).
- Power counting of momentum to determine importance of different exchange pictures.
- Iteration of each exchange picture to create a potential.
- Now calculate by integrating the potentials using a reformulation of the Schrodinger eq.

Calculations - Renormalization

- Renormalization of one pion $1/r^3$ and two pion $1/r^5$
  - Want to perform integral to infinity - unphysical and divergent. Instead create a high momentum (short distance) cutoff:

$$I(\alpha,\beta)=\int_0^{\infty}F(k,\alpha,\beta)dk \rightarrow I(\alpha,\beta,\Lambda)=\int_0^{\Lambda}F(k,\alpha,\beta)dk$$

  - Absorb functional form of cutoff dependence into physical constants… get a consistent answer:

$$I(\alpha,\beta,\Lambda)\rightarrow I(\alpha(\Lambda),\beta(\Lambda))=\int_0^{\Lambda}F(k,\alpha,\beta)dk$$

  - Match data at a point to fix constants at a cutoff.

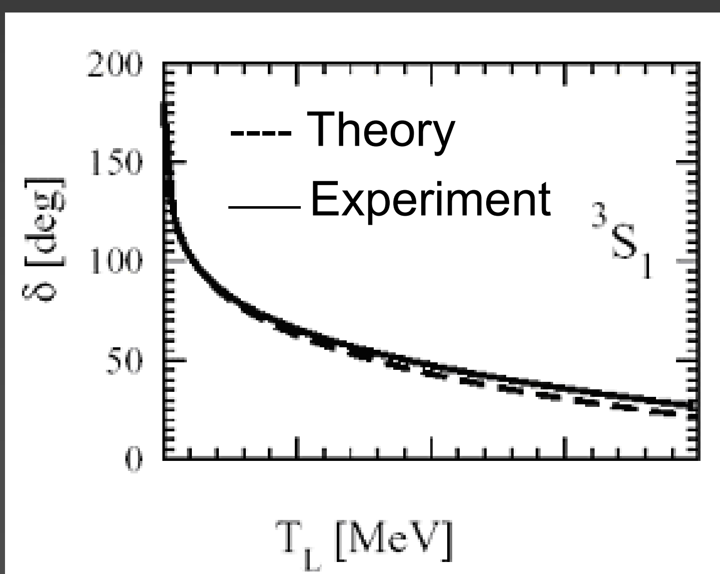
The Problem

- Total Potential: $Potential = V_{1\pi} + V_{2\pi} + ... + \sum_{i=0}^{\infty} c_i q^{2(i+l)}$
- Leading order for ●=0 $L.O. \;\; Term = V_{1\pi} + c_0 + c_1 q^2$
  - But leading order for ●=1: $L.O. \;\; Term = V_{1\pi} + c_0 q^2$
    - Recall contact interaction momentum order ~ 2(i+●)
- Not enough constants to renormalize for higher angular momentum
  - ●=1 already lost c_1 in leading order
- Too few constants to renormalize at given order
- Are Weinberg’s method and power counting inconsistent?

Inconsistent? Yes. Make the Best of it

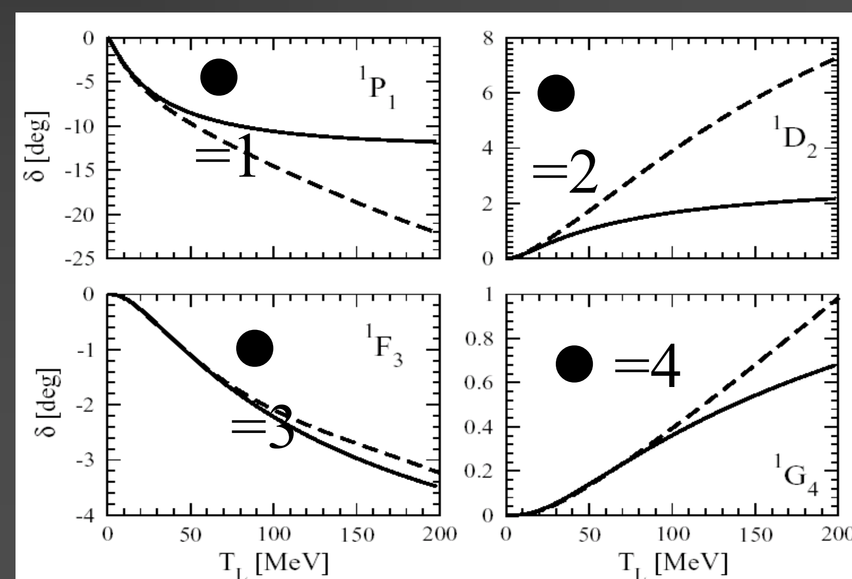
(1)
Angular Momentum ● = 0

Good agreement with data so use it.

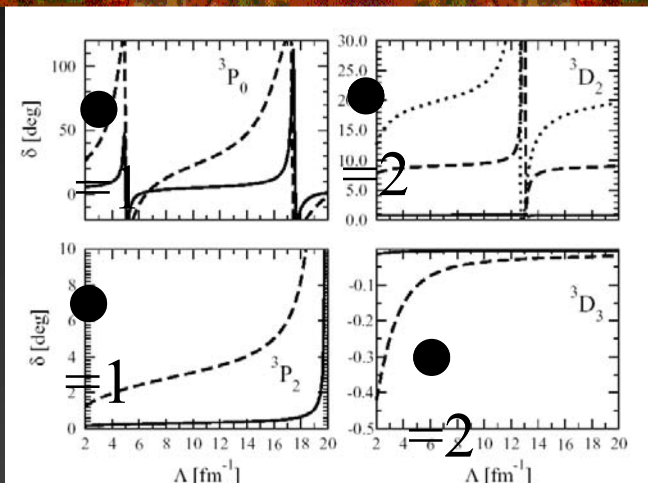


(2)
Angular Momentum ● > 0

Find a cutoff range with little variability and fit experiment (that's the best Weinberg power counting gives - any error is from only including low order in q)



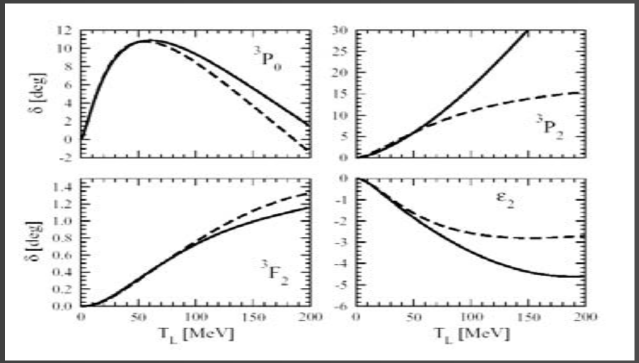
Or Make it Better



Phase vs. Cutoff for several energies and momentum

- Sometimes for ● > 0 cutoff dependence is too strong to pretend we have renormalized equation even at low energies (further from the cutoff).
  - Note $^3P_0$ especially
- Fix = artificially boost the importance of "lower order" contact interactions by bringing them up to the un-renormalized order
  - They once again provide constants for renormalization
- For ● =1 $L.O.\ \ Term = V_{1\pi} + c_0 q^2 \Rightarrow L.O.\ \ Term = V_{1\pi} + c_0 q^2 + c_1 q^4$
  - Despite the q^4 strictly being of higher order

Results/Conclusions



- One pion results look good

even for high ● ->

- Future Work
  - Insert two pion exchange and check accuracy
    - Check whether cutoff dependence can be absorbed into a single contact interaction (single constant)
  - Check high and low partial waves against experiment
  - Extend results to a 3 nucleon system (which is at leading order basically the same)

Acknowledgements

Many Thanks To:

Fellow REUers

NSF and UW REU Program

Andreas Nogga