The Renormalization of Effective Field Theory for Nucleon-Nucleon Interactions

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Abstract **–** We review and study the renormalization of chiral nuclear interactions in light of Weinberg's power counting methods. Specifically, we add contact interactions, where needed, to make Weinberg's theory renormalizable at all orders. We then confirm that the theory is still consistent with experimental data at up to next-to-leading order (NLO) for high partial wave interactions. This approach eliminates the need to physically justify vague momentum cutoff ranges, resulting in a more consistent theory, without sacrificing Weinberg's edifice. Since the approach also improves convergence due to large cutoffs, higher order forces, like the three-nucleon (3N) forces may be less important.

I. Introduction

The ultimate goal of nuclear physics is a consistent picture of interactions that can be derived from an understanding of fundamental principles. QCD provides a framework for this understanding, but the details of how nuclear forces arise from QCD are still fuzzy. Due to the strong coupling constant of the QCD lagrangian in the energy regime of nuclear physics, complex quark-gluon interaction pictures cannot be disregarded and hence QCD must be solved non-perturbatively. Lattice simulations provide very promising non-perturbative numerical solution to QCD and have generated good results for meson and baryon properties. Unfortunately, these simulations are currently limited (and computationally expensive) at the scale of Nucleon-Nucleon (NN) interactions (see for example reviews [1,2]).

An effective field theory (EFT) allows us to sidestep the complications of QCD. By producing a general lagrangian, consistent with the symmetries of QCD, we see that, in the low energy regime, all higher momentum interactions (quark-gluon resonance and heavy mesons) can be disregarded or "integrated out". In fact, the only remaining effective degrees of freedom in the EFT lagrangian are nucleons and the Goldstone boson – a result of QCD's spontaneously broken chiral symmetry. Weinberg presented this approach to QCD in 1979 [3] and in doing so, provided a systematic framework for the strong interaction at low energies. EFT creates elegant bridge to first principle (QCD), and also predicts the pion as a primary interaction mechanism under the alias "Goldstone boson"[*]. If it were to match data well EFT would be an attractive and powerful theory for nuclear systems.

II. Power Counting

At the heart of this EFT is a power-counting method that allows us to order the infinite number of terms in the lagrangian by their importance. Key elements of this

[*] The pion is expected to be the primary exchange particle in nuclear physics because it has the smallest mass by a factor of 3.5 and therefore has the longest range, lowest energy interaction of all mesons by a factor of 3.5 (from the uncertainty principle $\Delta l \approx \hbar / mc$). The pion mass ~135 MeV is also in the energy regime of nuclear physics.

ordering are that 1) a finite number of terms contribute at each order and 2) accuracy should always improve by going to higher order. For NN interactions Weinberg suggested such a power-counting with a perturbative expansion in terms of $(q/\mathrm{M})^{\nu}$ where M is the approximate low energy cutoff corresponding to the chiral symmetry breaking limit ~1 GeV (on the order of the nucleon mass M_n) and q is the momentum. His intuition was based on the idea that, since momentum is small in nuclear physics, typically of O(m**.**), higher power momentum terms contribute less to the interaction picture.

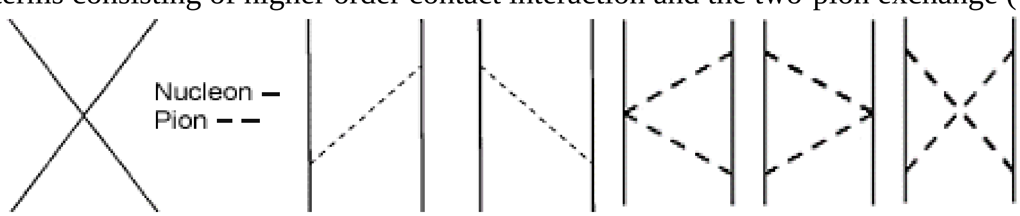
Since terms in the chiral lagrangian refer to specific interaction types we can easily gather a qualitative understanding of the physics occurring in Weinberg's simplified picture (known as chiral perturbation theory). Most obvious are the central term, which represent a power series expansion of local operators describing large momentum (short distance) interactions corresponding to "integrated out" effects such as rho meson exchange and nucleon-internal quark (or pauli exclusion principle) repulsion. This power series extends down all orders and contains unspecified constants that must be fitted to scattering data. All other terms describe pion exchanges and the order has been closely mapped to Feynman exchange diagrams: each pion propagator contributes –2 powers of q, each nucleon contributes –1, each derivative contributes +1, and the four-momentum integration contributes +4. For a diagram with I_n internal nucleon lines, I_p internal pion lines, V_i vertices of type i, and L pion-nucleon loops then:

$$\nu = 4L - I_n - 2I_p + \sum_i V_i d_i$$

or for the (simplified) NN case (f_i are nucleon fields):

$$\nu = 2L + \sum_i V_i (d_i + f_i/2 - 2)$$

At leading order (LO) the simple chiral power-counting scheme predicts one-pion exchange (OPE) and a LO contact interaction (figure 1). Our work also requires NLO terms consisting of higher order contact interaction and the two-pion exchange (TPE).

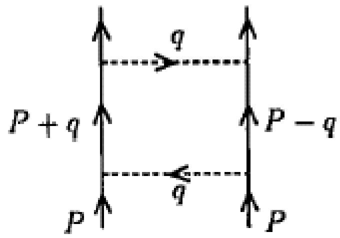


Contact Interaction **LO One Pion Exchange** **NLO Two Pion Exchange**

Figure 1

This simple power counting method justifies the usefulness and accuracy of the perturbative chiral lagrangian for pion-nucleon scattering, however the existence of a bound state for NN systems (the deuteron) would seem to preclude a perturbative treatment in NN systems. The only way to achieve a bound state with the chiral effective lagrangian is through diagrams with a large number of loops, but these have been disregarded as unimportant due to their large order q dependence (1). In 1990 Weinberg suggested [4] that the lack of bound states in a perturbative expansion of the chiral lagrangian was a result of certain loop diagrams being unfairly demoted by the naïve chiral power counting. Namely reducible diagrams – diagrams containing only initial and final particles over some interval– contain infrared divergences in the limit of zero baryon kinetic energy, which increase their importance. The divergence is an artificial result of the limit, but the small recoil energy denominator still makes the loop contribution larger by a factor $M_n/q >> 1$.

Weinberg realized that he could either include these contributions or simply iterate the exchange pictures, thereby implicitly looping the diagrams to arbitrary order. The Lippmann –Schwinger equation is a convenient vehicle for this looping as it describes the propagation of a particle under some interaction picture followed by the propogation of the new state and then a second interaction etc:



$$T = V + VG_0T \equiv V + VG_0V + VG_0VG_0V + ...$$

Figure 2: The reducible one-pion exchange loop diagram, which is iterated to all orders in the Lippmann-Schwinger equation for Weinberg's power counting scheme.

Weinberg's power counting† has been shown [5,6,7] to generate predictions rivaling the accuracy of phenomenological models. Our motivation this summer was, first and foremost, to correct a mathematical inconsistency in Weinberg's power counting which will be discussed in the next section, but we also hoped to further improve accuracy in NN calculations from previous calculations at the same order. It has been shown that certain interaction effects thought to be the result of 3N forces present for N>2 were a result of high sensitivity to NN interaction descriptions [8]. Therefore our secondary goal of better convergence to experiment which may explain away more of the 3N force when applied to N>2 systems, possibly suppressing it to N3LO.

III. Renormalization of Weinberg's Method

There is a remaining problem in Weinberg's construction. – due to strong singularities created by OPE ($1/r^3$) and TPE ($1/r^5$) integration needs to be regularized and renormalized. This short distance, large momentum, artificial singularity, shows up for any finite order but can be regularized by the contact interactions in the lagrangian. This short distance interaction is parameterized by the contact terms‡, however, following Weinberg's power counting strictly, the contact interactions are not always present at low order to renormalize the theory. Since the contact interactions are a simple power series expansion of momentum their order also goes as the angular momentum l . Therefore contact terms are only present at lowest order in S-waves, not for higher partial waves, and the theory becomes non-renormalizable in these waves.

One interpretation of the cutoff dependence is that it provides a measure of error at a given order and that moving to arbitrarily high order should eliminate the singularity

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† Criticisms of Weinberg's power counting have been directed at the analytic non-renormalizability of the infinite set of loop diagrams which theoretically require an infinite set of counter terms to be renormalized (although this problem does not appear in the numerics). Other power counting methods have focused on pionless theory [9] and perturbative pions [10], which fails for higher orders [11] (this motivated our NLO calculations).

‡ The contact interactions act as counter-terms carrying the coupling constants necessary for the renormalization.

anyway. In this case people attempt to find plateaus in cutoff dependence and use physical arguments to explain why the plateaus exist in the range they do. In several partial waves the cutoff dependence is too strong and this approach fails – or gives to large an error margin to be useful.

We propose to artificially promote the contact interactions to leading order for higher partial wave interactions. A similar approach was applied successfully in pionless EFT theory in the 3N system. We need to study whether it is more consistent than Weinberg's power counting. The benefits of this approach are large: it obviates vague physical justifications for specific momentum cutoffs at a given order and it also creates hope for analyzing midrange[§] P,D,F, and G partial waves.

Following a partial wave decomposition of the LO and NLO potentials that follow we can proceed with momentum space calculations of the phase shift which correlates with the interaction cross-section.

$$V^{\mathrm{OPEP}} = -\left(\frac{g_A}{2f_\pi}\right)^2 \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{q^2 + M_\pi^2}, \qquad \text{LO Contact} = C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$V_{\mathrm{NLO}}^{\mathrm{TPEP}} = -\frac{\vec{\tau}_1 \cdot \vec{\tau}_2}{384\pi^2 f_\pi^4} L(\vec{q}) \left\{ 4M_\pi^2 \left(5g_A^4 - 4g_A^2 - 1\right) + q^2\left(23g_A^4 - 10g_A^2 - 1\right) + \frac{48 g_A^4 M_\pi^4}{4M_\pi^2 + q^2} \right\}$$

$$- \frac{3g_A^4}{64\pi^2 f_\pi^4} L(\vec{q}) \left\{ \vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q} - q^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right\}$$

$$\text{NLO Contact} = C_1 \vec{q}^{\,2} + C_2 \vec{k}^2 + (C_3 \vec{q}^{\,2} + C_4 \vec{k}^2) \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$+ iC_5 \frac{\vec{\sigma}_1 + \vec{\sigma}_2}{2} \cdot (\vec{q} \times \vec{k}) + C_6 \vec{q} \cdot \vec{\sigma}_1 \vec{q} \cdot \vec{\sigma}_2 + C_7 \vec{k} \cdot \vec{\sigma}_1 \vec{k} \cdot \vec{\sigma}_2$$

$$\text{With:} \qquad L(q) = \frac{1}{q}\sqrt{4M_\pi^2 + q^2}\, \ln \frac{\sqrt{4M_\pi^2 + q^2} + q}{2M_\pi},$$

where C's are coupling constants to be fitted, $\vec{q} = \vec{p} - \vec{p}'$ and $\vec{k} = (1/2)(\vec{p} + \vec{p}')$, $\sigma_{1,2}$ are pauli spin matrices and $\tau_{1,2}$ are isospin operators, f_π=93 MeV is the pion decay, M_π=138.03 MeV and gA=1.26 is the vertex coupling constant.

Coupling constants are fitted to NN cross-section data at one energy for each state. It is important to note that this model, despite it's reliance on fitting, is far from phenomenological. Only one cross section (phase) is necessary for a good <20% fit to data up to 300 MeV in most cases.

IV. Results and Conclusions

Results are incomplete, but appear promising. Our first small result is a proof that by going from LO to NLO the singularity for the $^3D_2$ channel disappears as evidenced by the simplified cutoff dependence at all energies (figure 3).

[§] At low energies, the $l(l+1)/r$ centrifugal barrier term in the Schrodinger equation means that nucleons do not see any singularity for $l > \sim 5\ \mathrm{fm}^{-1}$. Renormalization is, therefore, not a problem for very high partial waves.

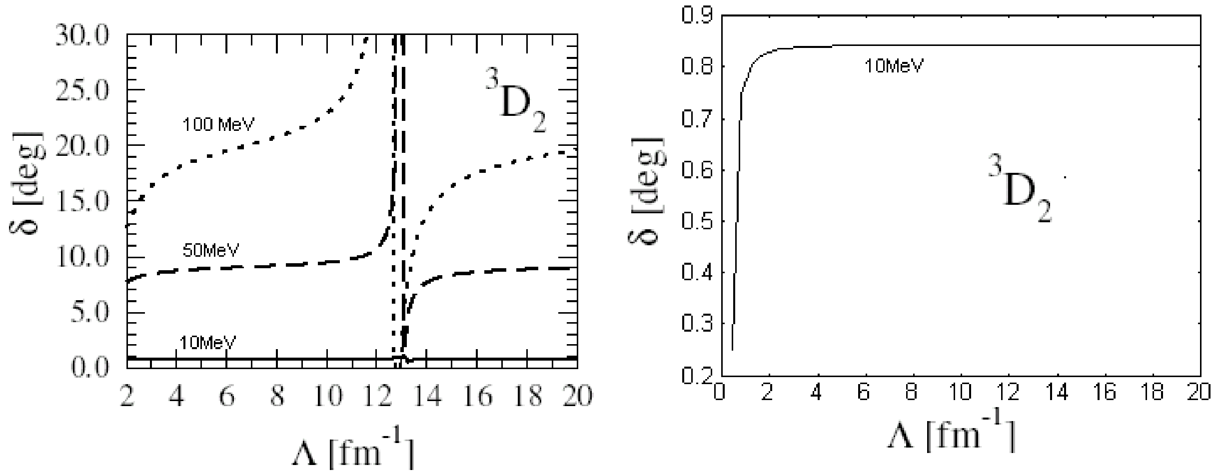

Figure 3: (Left) LO cutoff dependence in $^3D_2$ for different energies. (Right) NLO cutoff dependence in $^3D_2$ for 10 MeV. Note the lack of kinks indicative a singularity at NLO.

We have also found that the 3P0 and 3P1 channels at NLO are an improvement over LO calculations (figure 4) which means that this power-counting is likely to be consistently improved by going to higher orders.

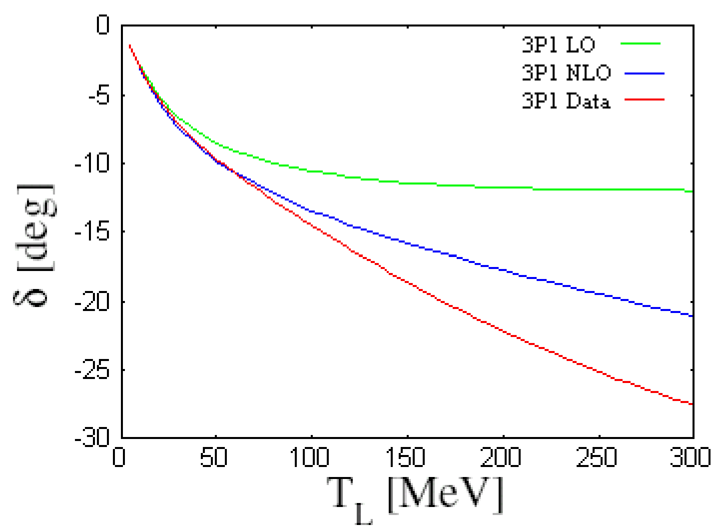

Figure 4: $^3P_1$ phase shift plot at different energies with fitted results traditionally not included by Weinberg's power counting. The NLO results are consistently better than LO results making this altered power counting a promising option.

We would like to complete fitting for the 1S0 channel, which exhibits extreme cutoff dependence (the problem is intractable without some regularization). Unfortunately, the number of parameters required for fitting has slowed progress. Fitting and analysis other NN channels below *l*=6 and eventually an extension to 3N interactions are all part of a large amount of future work which must be done to verify the usefulness of this form of power-counting.

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