Capacitance Position Sensor for LISA Noise Measurement

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Abstract

This paper discusses two techniques investigated for use in a capacitance-based position sensor for the Eöt-Wash LISA noise measurement. The advantages and disadvantages of both techniques are reviewed and some elementary mathematical analysis is presented. A prototype implementation based on the more successful of the two techniques has been constructed.

1 Background

The Laser Interferometer Space Antenna (LISA) satellite system is being designed to detect gravitational waves by precise monitoring of free-floating proof-masses. The task of the Eöt-Wash LISA noise measurement is to search for any forces which may exist between the spacecraft and the proof-masses. This search is accomplished though the use of a torsion pendulum setup which measures the angular oscillations of a pendulum as a function of its distance to a movable plate. In the analysis of the angular position data, it is important to be able to isolate components correlated to translational motion of the pendulum. For this purpose, a sensor capable of unobtrusively sensing the pendulum's translational position is necessary.

2 Sensor Requirements

To be useful for the purpose of enabling identification of translation correlated observables, the sensor is required to have stable resolution at least at the level of one micron. Additionally, the sensor must not introduce any significant forces which act on the pendulum. It is also desirable that the sensor system be relatively easy to incorporate into the existing experimental setup.

3 Basic Principles

The concept investigated for sensing the position of the pendulum is to use the pendulum, or some other electrode attached to the pendulum's supporting torsion fiber, as one plate of a parallel plate capacitor.

The magnitude of the capacitance of a parallel plate capacitor is:

$$C = \frac{\epsilon_0 A}{d}$$

where A is the area shared between the plates, d is the distance between the plates and ϵ_0 is the permittivity of free space. Thus, if the area between the plates of the capacitor is held constant, the distance between the plates can be determined by measuring the capacitance.

4 Measurement Circuits

Two different capacitance measurement circuit types were investigated for use in the position sensor. The first type investigated used an "active" design which produced a direct measure of the capacitance [1]. The second type uses a "passive" design which produces a differential measurement of the difference in magnitude between two capacitors [2].

4.1 Direct Measurement

Using the direct measurement technique the pendulum and the movable plate would form the capacitor used to measure the position of the pendulum. Figure 1 shows the basic circuit layout. The circuit works by measuring the current induced by rapidly charging and discharging the capacitor.

Each clock cycle of the circuit can be divided into two phases of equal duration:

1. C_x is charged to V_c
2. C_x is discharged

The charge transfered during each phase is:

$$Q = V_c C_x$$

Thus, the current is:

$$I = fQ = fV_c C_x$$

This gives:

$$V_1 = -fV_c C_x R_f$$

$$V_2 = fV_c C_x R_f$$

$$V_3 = -2fV_c C_x R_f$$

where V_3 is considered to be the measured circuit output. It should be noted that the negative signs in the voltages come because the operational amplifiers labeled 1 and 2 form inverting amplifiers.

The direct measurement technique possesses a number of advantages relating to its use in the current setup. Primarily, its implementation would require minimal change to the existing experimental equipment because leads from the pendulum, the movable plate and the other electrodes already exist outside of the vacuum volume. Additionally, it is able to operate with V_c at fractions of a millivolt in addition to keeping the pendulum at a virtual ground thus minimizing any additional forces acting on the pendulum. The circuit also features a total cost which ranks as relatively inexpensive.

Unfortunately, the direct measurement technique also sports a number of important disadvantages. For low values of V_c, non-ideal behavior of the operational amplifiers and other factors introduce large offsets in the output voltage which complicate calibration. Most importantly, when our implementation was connected to the experimental setup its sensitivity was much lower than expected.

4.2 Differential Measurement

The second option investigated for the purpose of measuring the pendulum's position using capacitance was a differential measurement using a LC bridge circuit (Figure 2).

The circuit works by measuring the current flowing across the middle of the bridge circuit (the equalizing or balancing current). The current is measured with a lock-in amplifier so that only changes at the driving frequency are measured; thereby eliminating most noise. As the phase of the signal into capacitor C_2 is 180° out of phase compared to the signal into C_1, both

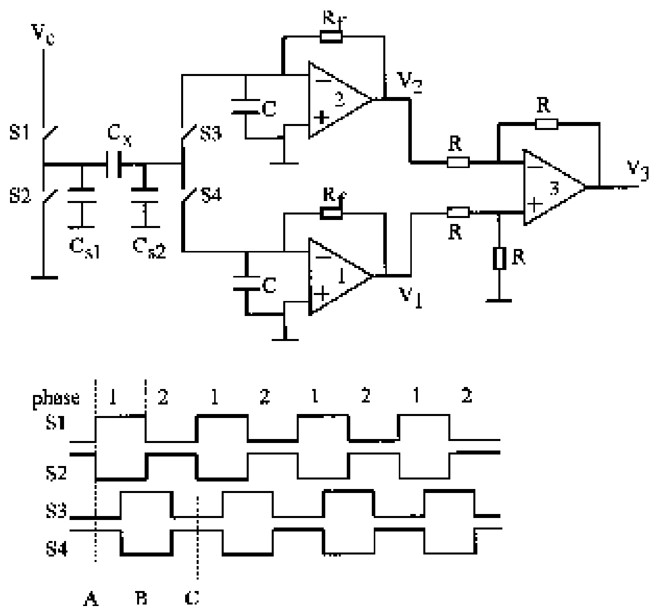

Figure 1: Direct Capacitance Measurement Circuit

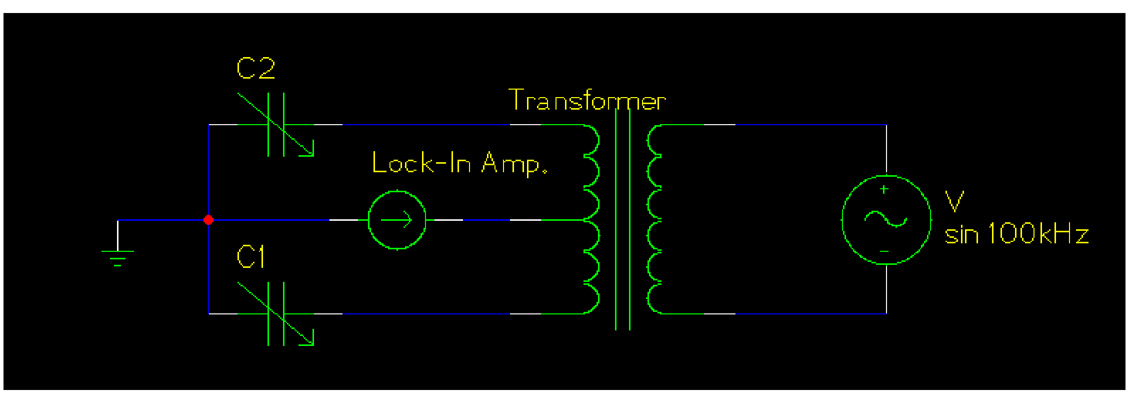

Figure 2: Differential Capacitance Measurement Circuit

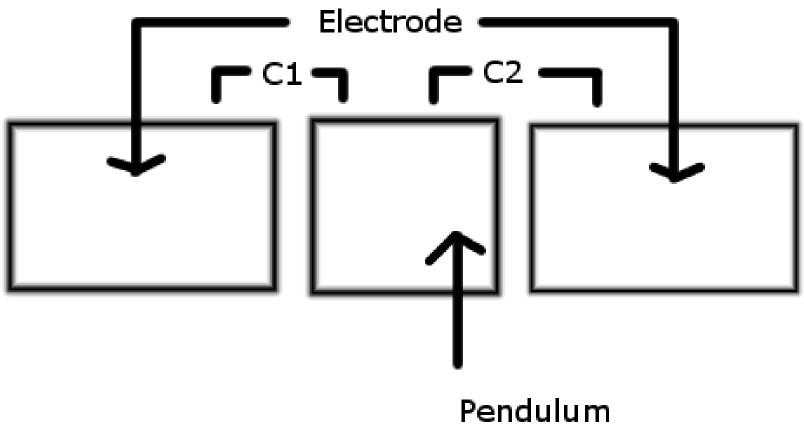

Figure 3: Differential Capacitance Schematic

signals effectively cancel when the bridge is balanced (when $C_1 = C_2$). Thus, by measuring both the amplitude and the phase of the current, it is possible to determine not only the magnitude of the capacitance difference, but the sign of the difference as well.

Calculation of the current is simply a matter of defining voltage loops around connected paths in the circuit and equating them to zero.

$$V - Z_{C_1} I_1 - (I_1 - I_2)R = 0$$
$$V - Z_{C_2} I_2 - (I_2 - I_1)R = 0$$

where:

$$Z_{C_1} = \frac{1}{\imath\omega C_1}$$
$$Z_{C_2} = \frac{1}{\imath\omega C_2}$$

As seen in figure 3, the two capacitors on each axis are not independent. The middle electrode, which is attached to the torsion fiber holding the pendulum, has some equilibrium distance from the two side electrodes called X. As the pendulum swings, the gap between the middle electrode and one of the side electrodes will decrease by some distance x causing the gap between the middle electrode and the other side electrode to grow by x. Thus, the values for C_1 and C_2 can be given as:

$$C_1 = \frac{\epsilon_0 A}{X + x}$$
$$C_2 = \frac{\epsilon_0 A}{X - x}$$

Solving yields:

$$Re(I_1 - I_2) = -\frac{V\omega(C_1 - C_2)}{1 + (R\omega(C_1 + C_2))^2}$$

The current $I_1 - I_2$ is determined by measuring the voltage drop across the resistor R. Although the full expression for $Re(I_1 - I_2)$ in terms of A, X and x is quite complex, assuming that x is small compared to X and that changes in ω are small allows the use of a series expansion to simplify the analysis. With respect to x and ω, the real part of the current difference is:

$$Re(I_1 - I_2) = \frac{2\epsilon_0 AV\omega x}{X^2} + \mathcal{O}(x^3, \omega^3)$$

Conveniently, for a constant ω and small values of x, the measured current varies linearly with the displacement of the middle electrode.

Our current prototype uses end electrodes with face dimensions of 0.38" wide by 0.76" tall. The middle electrode, which is grounded, has face dimensions of 0.5" wide by 0.76" tall. The equilibrium electrode separation distance is 0.018". Thus, the equilibrium capacitance is approximately 4 pF. A 10 Ω resistor is used to measure the equalization current $I_1 - I_2$. The transformer has a 50 winding primary coil with an inductance of 3 mH and two connected 10 winding secondary coils each with an inductance of 141 μH. The transformer is placed far away from the electrode assembly and wrapped with grounded aluminum foil as a shield from electric coupling. The wires forming the input to the transformer are twisted together and kept as far as possible from the wires connecting to the secondary windings. Additionally, the wire from the middle point of the secondary windings is twisted together with a grounded wire. A 100 Ω resistor is in series with the primary coil to match the output impedance of the signal generator. The signal generator produces a 30 kHz sine wave with an amplitude of 1.0 V RMS.

The lock-in amplifier can be configured to compute the real part of the voltage measurement from which the real part of the current difference given by the equation above can be determined. When proper component shielding is employed to eliminate measurement offsets due to electro-magnetic couplings, calibration data collected using our implementation very closely matches the expected results (Figure 4).

The differential measurement technique has several advantages which make it well qualified for use as a position sensor for the pendulum. Primarily, it has very good sensitivity. Additionally, apart from the lock-in amplifier, it consists of only a small number of passive components which can be put in the temperature controlled vacuum container to make the output very stable. This technique also results in an output almost completely linear with respect to the electrode separation distance which makes interpretation straightforward.

This technique, however, does not come without its downsides. Non-ideal component behavior leads to resonance peaks in the circuit's frequency response. Although the circuit will be run at a fixed frequency, understanding these resonances is important in choosing an optimal operating frequency. Also, our implementation requires two lock-in amplifiers, one for each axis, and these are expensive pieces of equipment.

Currently, we are working on understanding the nature of the resonances which exist within the circuit and the optimal shielding and geometric layout to minimize unwanted electromagnetic couplings.

5 Conclusion

Multiple techniques are available for measuring small capacitances. Two of the most common were investigated for use as a position sensor for the torsion pendulum in the LISA noise measurement experiment. The differential measurement technique has proved the most promising and we are actively working on improving a prototype implementation before its inclusion into

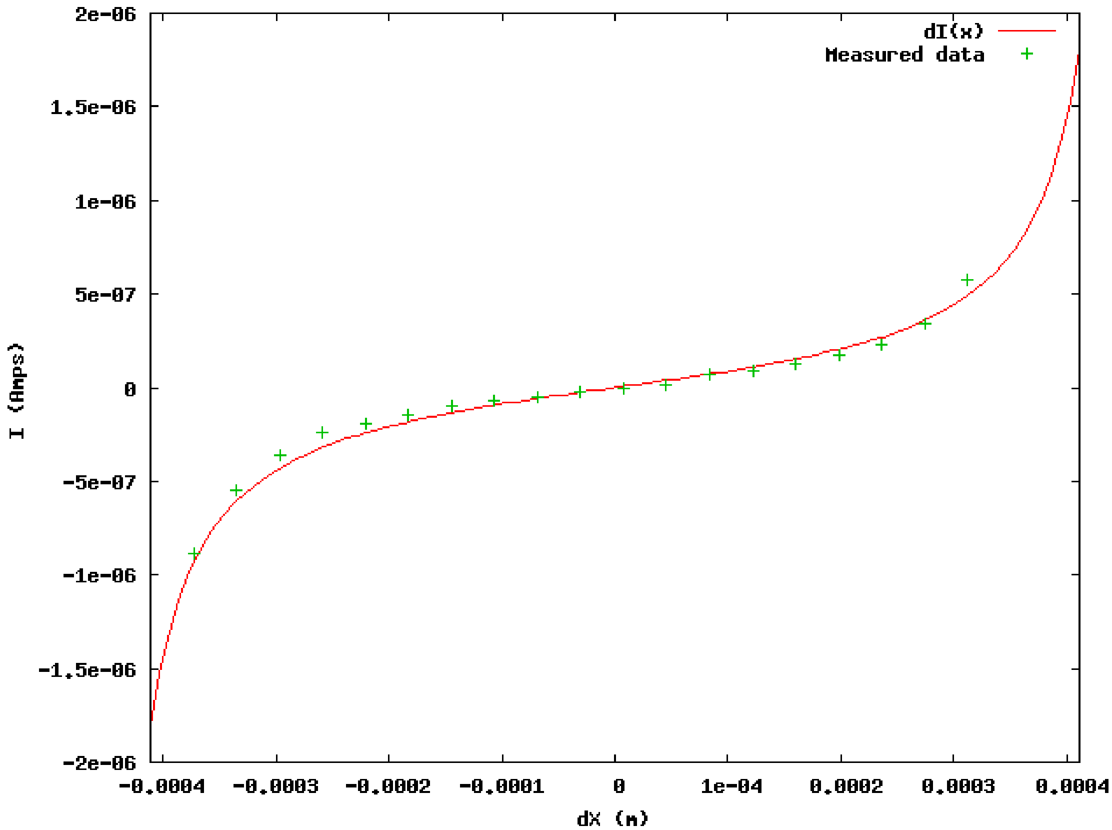

Figure 4: Expected and measured equalization current in the differential circuit

the actual experimental apparatus.

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References

[1] San Xu Yang and Wu Qiang Yang. A portable stray-immune capacitance meter. *Review of Scientific Instruments*, 73(4):1958–1961, April 2002.

[2] R. V. Jones and J. C. S. Richards. The design and some applications of sensitive capacitance micrometers. *Journal of Physics E: Scientific Instruments*, 6(7):589–600, July 1973.