Geraldine Cochran
August 11, 2004

Introduction

Previous research has been done on student understanding of kinematics. In the paper, *Investigation of student understanding of the concept of acceleration in one dimension,* Dr. David E. Trowbridge and Dr. Lillian C. McDermott assessed student understanding of acceleration by administering individual demonstration interviews and supplemented that information with analysis of course examinations.[1] I found very interesting their work with Acceleration Comparison Task 2. A replication of Acceleration Comparison Task 2 is provided in the Appendix on page 13. According to the paper, this task was administered "in order to examine quantitative understanding of the concept of acceleration." A student was said to have successfully completed the task if he or she were able to answer the question correctly using a valid procedure. To complete this task, students had to differentiate between instantaneous velocity and average velocity in order to calculate the change in velocity needed to find acceleration. The results showed that students had difficulty in doing so before instruction. What is more, the conceptual difficulties that resulted in unsuccessful completion of the task were evident after instruction as well.

My goal for this project was to evaluate whether or not inservice teachers in the National Science Foundation's Summer Institute taught by the Physics Education Group displayed a robust understanding of average velocity and instantaneous velocity. In particular, I was interested in determining whether or not these teachers were capable of using their understanding of these concepts to determine acceleration after instruction.

---

[1] *Investigation of student understanding of the concept of acceleration in one dimension.* Dr. David E. Trowbridge and Dr. Lillian C. McDermott, Am. J. Phys. **49**, 242 (1981)

Background

The Summer Institute is a six-week course for inservice teachers designed to strengthen their understanding of physics and to introduce them to inquiry-based learning. The curriculum for the course came from revisions of *Physics by Inquiry*, one of the instructional materials developed by the University of Washington Physics Education Group.[2] The topics covered by various teachers included: properties of matter, electric circuits, waves and physical optics, kinematics and dynamics, and observational astronomy.

The participants in the section I worked with were high school and middle school teachers, as well as a few returning elementary school teachers. Their backgrounds varied greatly. The high school teachers all had kinematics and dynamics in college. Many taught this material. One of the reasons that I was particularly interested in their understanding of instantaneous velocity, average velocity, and acceleration is because the kinematics instructors emphasized the importance of operationally defining and interpreting these concepts. Furthermore, due to time constraints, the kinematics curriculum was abbreviated. The teachers covered kinematics, dynamics, and electric circuits all in six weeks. Thus, an evaluation of their understanding of the kinematical concepts will shed light on whether or not it is beneficial to cut part of the kinematics curriculum and if more time should or should not be spent on this material.

Procedure

As was mentioned in the introduction, my goal for this project is to evaluate whether or not teachers in the Summer Institute displayed a robust understanding of

[2] *Physics by Inquiry*. L. C. McDermott and the Physics Education Group (John Wiley & Sons, Inc., New York, NY, 1997).

average velocity and instantaneous velocity. Thus, I began by analyzing the responses to a module pretest question given to the teachers before any instruction. I analyzed their responses to this question in an endeavor to determine whether or not the teachers entered the course with an understanding of average or instantaneous velocity. I then analyzed the responses to the pretest for section 10, entitled "the concept of acceleration," of the revised *Physics by Inquiry* curriculum. At this point, the teachers had covered average velocity and instantaneous velocity and had been introduced to acceleration. This question gave me an idea as to whether or not the teachers were able to determine acceleration using their knowledge of acceleration, as well as average and instantaneous velocity. I then examined the responses to a question included on their mid-term exam. At this point, the teachers had revisited the concept of acceleration and some had completed homework and a practice problem on acceleration. Analysis of this question helped me determine whether or not the teachers had an understanding of the operational definitions of average and instantaneous velocity, as well as a physically significant interpretation of average velocity. Finally, during the last three days of the course, I conducted interviews with twenty-one of the teachers, all of whom had completed the kinematics curriculum and were working on the dynamics curriculum. These interviews indicated whether or not the teachers were able to articulate their understanding of the kinematical concepts and whether or not they were able to use their understanding of these concepts to design a valid method to be used in solving a motion problem involving a real object.

Results

As was mentioned in the previous section, I began by analyzing the student responses to a module pretest question given to the teachers before any instruction. A replication of this pretest question appears in the Appendix on page 14. Only 22% of the teachers answered all four portions of this question correctly. Thus, the majority of the teachers did not enter the course with an understanding of instantaneous velocity and average velocity.

I then considered the responses to the pretest for section 10. A replication of this pretest appears in the Appendix on page 15. To reiterate, at this point the teachers had covered the concepts of average velocity and instantaneous velocity and had already had a brief introduction to the concept of acceleration. Only 10 of the 23 teachers answered this question correctly. Three of the teachers answered incorrectly because they failed to discriminate between instantaneous and average velocity. Thus, at this point less than half of the teachers were able to use their knowledge of instantaneous and average velocity to calculate acceleration.

I then examined the answers given on a question included on the mid-term exam. A replication is given in the Appendix on page 14. Before taking the mid-term, the teachers revisited the concept of acceleration and some of them completed homework and a practice problem on acceleration. In doing so, these teachers also reviewed the concepts of instantaneous and average velocity. According to the exam results for part A, 24 of the 27 teachers correctly determined the instantaneous speed of the cart and correctly explained their reasoning. Regarding part B, 24 of the teachers realized that they could not determine the instantaneous velocity of the cart from the information given. In part C, all of the teachers correctly calculated the average velocity, but only 18 were able to give an interpretation of this number in part D. Thus, the majority of the teachers now had an understanding of the operational definitions of instantaneous and average velocity, in the sense that they were able to calculate them for specific cases. However, only about two-thirds of the teachers were able to give a physically meaningful interpretation average velocity.

Finally, I conducted interviews with 21 of the teachers. During the interviews, they were asked to describe the method they would use in finding the acceleration in a scenario similar to the one given in Acceleration Comparison Task 2. A replication of interview question 1 is provided in the Appendix on page 15. The teachers were asked to define acceleration, instantaneous velocity, and average velocity. They were also asked to interpret and differentiate between instantaneous velocity and average velocity. Finally, teachers were asked to determine what kind of velocity they referred to in their definition of acceleration. These interviews were quite informative, and the results are presented below in sections. I have included excerpts from various interviews in an endeavor to illustrate cases in which the teachers were successfully able to answer the questions, as well as cases that provide insight into student difficulties or misunderstandings. The results section will be followed by a section presenting uncommon difficulties that I found interesting.

Acceleration.

To begin with, when asked to define acceleration, all of the teachers were able to do so successfully. Moreover, 17 of these 21 teachers were able to describe a valid method to determine the acceleration for the scenario given during the interview. In order to complete this task, the teachers had to differentiate between instantaneous and average velocity. They also had to decide what information was necessary to complete the task and what was not. One student reasoned through how to use the information by referring to her operational definitions. Note her reasoning:

> And so what am I going to do with you [the length of ramp 2]? You've got 3 meters covered in 3 seconds . . . The change in time was 3 seconds and you know that you went 3 meters. Does that matter? . . . It's your final velocity minus your initial velocity. And we already know that . . .

This teacher was eventually able to determine the method to use for solving this problem. Referring back to her definition of acceleration, she was able to conclude that the fact that the ball covered 3 meters in 3 seconds on the second ramp was extraneous information.

However, four of the teachers were unable to complete this task. Each of them had different reasoning. Thus, I will present all four cases. In case one, the teacher felt that she didn't have enough information. She wasn't sure exactly what she needed, but mentioned that she needed to know how to break up the entire motion of the ball into sections. In case two, the teacher mentioned using graphs and derived kinematics equations to determine the acceleration, but stated that those were the only ways that it could be done. In case three, the teacher used the average velocity of the ball on ramp 3. He states:

> It's going to be—change in distance. It's going to be 3 meters. Change in time, it's going to be 3 seconds. So, that's going to give us basically a meter per second. That's going to be velocity. I want to say it's going to be 1 meter per second per second.

After being asked if his method for determining the acceleration was consistent with his definition, he attempts to determine the acceleration again. He says:

> Even though it was in constant motion prior to that [entering ramp 2] you weren't seeing any acceleration . . .So, it has a starting velocity of zero at that point in terms of the acceleration. So, it's going to be 0 meters per second. 3 meters per second minus 0 meters per second over a 3 second time frame. . . That [the acceleration] would be 1 meter per second per second.

Here he has reasoned that, because the time frame at the top of the ramp is zero seconds, the initial velocity is zero meters per second and that, because the time frame at the bottom of the ramp is 3 seconds, the final velocity is 3 meters per second.

In case 4, the teacher successfully finds the initial velocity by finding the average velocity along ramp 1. However, she finds the final velocity by taking the average velocity along ramp 2, the sloping ramp. Using these two velocities and the time spent along ramp 2, she calculates the acceleration and thus obtains an incorrect value.

From this question, we see that the majority of the teachers were able to describe a valid method to be used in determining the acceleration by differentiating between instantaneous velocity and average velocity. In cases 3 and 4, an inability to differentiate between instantaneous and average velocity resulted in failure to complete the task.

Instantaneous velocity.

All of the teachers gave a similar interpretation for instantaneous velocity that clearly distinguished it from average velocity. They mentioned that instantaneous velocity is the velocity at an instant in time. However, it was not the interpretation that the kinematics instructors tried to emphasize. They stressed that instantaneous velocity provides information about how far the object would travel if it continued to move with uniform motion at that same velocity and if the motion continued for an entire second.

This may be a result of more time spent on how to calculate instantaneous velocity than its interpretation. Nonetheless, regarding the definition of instantaneous velocity, 17 of the teachers were able to define or to explain how to calculate instantaneous velocity. Two of the teachers incorrectly defined instantaneous velocity. The first teacher says, "instantaneous tells you the change in x [position] over change in t [time] at a certain little particular moment. So, it's like x over t in a way." However, she soon stated that her answer didn't make any sense, but she could not explain how she could calculate instantaneous velocity. The second teacher's method for calculating instantaneous velocity will be discussed in the difficulties section.

Average velocity.

Nineteen of the teachers gave a correct interpretation of average velocity. One teacher interpreted the average velocity incorrectly. The other student did not give an interpretation. Sixteen of the teachers defined or described how to calculate average velocity. The majority of the teachers mentioned calculating the displacement and dividing it by the time it took to cover the distance. However, five of the teachers employed a method that involved adding up instantaneous velocities and dividing by a number. There were three different methods mentioned by the teachers for calculating the average velocity. The following quotations will illustrate two of those methods.

> Method 1. "Obviously there is the definition of average which is like the beginning plus the end divided by two."
>
> Method 2. "I guess what you could do is add up all the velocities and divide them just like you would do an average . . . "

Method one works only if the acceleration is constant. Thus, this answer was considered incorrect since the teachers were asked for a method that works in any physical situation. Two of the teachers applied method one and two of the teachers applied method two. However, when given the scenario of a cart traveling 1 meter in 3 seconds, all four of those teachers were able to determine average velocity using the method of displacement over time. The fifth teacher employed a unique method discussed in the misunderstandings section. This teacher was not able to determine the average velocity in the cart scenario. Interestingly, all but one of these five teachers skipped sections 12-14 of the curriculum. Teachers were asked to skip these sections, depending on their progress with the material, in an endeavor to condense the kinematics curriculum. It appears that section 13, entitled calculating averages, would have been beneficial to these teachers and may have prevented them from making this error. As far as differentiating between instantaneous velocity and average velocity, all of the teachers were able to do so.

Revisiting Acceleration.

All of the teachers were again asked to define acceleration. As before, all did so successfully. They were then asked to identify what kind of velocity they were referring to in their definition of acceleration. Only 15 of the teachers answered this correctly. Of those answering correctly, 7 explained their reasoning by referring back to the method they described for finding acceleration during the first question of the interview. One such student stated:

> I was saying that the first section of the ramp . . . had uniform motion so that [at] any point along the ramp the instantaneous velocity and the average velocity [are] . . the same. And . . . ramp [3] was also like that. . .

> . and we would be comparing the change in instantaneous velocities over that time interval.

Six of the teachers stated that they use average velocity. The reasoning for this varied. In one case, the student stated: "I would say it's average velocity because it's describing entire time increments. Whereas instantaneous is only describing small increments." Another student stated:

> I think it's the average velocity because . . if you're doing instantaneous velocity you have instantaneous acceleration. . . . because you're talking about a particular point. And typically when we talk about acceleration it's over a duration."

However, of these 6 teachers who answered incorrectly, 2 of them correctly stated that they were referring to instantaneous velocities when asked if their definition was consistent with the method they described in question one.

Difficulties

The dialogue during the interviews uncovered a few difficulties teachers were having even after instruction. All of these will be briefly described and then illustrated through excerpts from the interviews. To begin with, one of the teachers had trouble explaining how to calculate instantaneous velocity. Note the dialogue below:

> Student— . . . . It would be a velocity time graph. If I had that I could do a tangent line to find the instantaneous velocity at a point.
>
> Interviewer—On a velocity time graph you would do a tangent line to what?
>
> Student—To . . the point of interest on that graph.
>
> Interviewer—(Draws a velocity versus time graph). . . .Let's say this corresponds to two seconds. How would I . . .
>
> Student—If that's my point of interest on the graph, to find the

> instantaneous velocity of that I would draw the tangent line at that point [two seconds]. [I could] figure out the velocities of each of these [two points on the tangent line] . . . and then I can basically just find the average of those two.

This student used a similar method when given another scenario. He states that you can find the instantaneous velocity at a point by finding the instantaneous velocities at two points surrounding that point and averaging them. This is correct only if acceleration is constant and one is finding the instantaneous velocity at the midpoint in time. This student also showed an inability to determine average velocity. Given a scenario of a cart with non-uniform motion traveling a distance of one meter in 3 seconds, the student states that he cannot determine the average velocity if the cart stops during any part of that motion. He states:

> I could do average velocity within the points where there is motion, but at the point where it's at rest there is no velocity at that point. So, I could do from where it started to that rest point. When it's resting it's going to be zero velocity. So, now I can do it from when it starts up again to where it finishes.

One student repeatedly stated that average velocity is related to the final velocity minus the initial velocity. In explaining acceleration, she states:

> I just know that it's just the change in velocity over time. So, I don't know if that means your average velocity. I'm assuming it does because average velocity is basically your final velocity minus initial velocity and then divide it by the time interval.

These difficulties were not common, but rather unique to the individual teachers who stated them. However, they were quite interesting and gave evidence of how the teachers incorrectly modeled various kinematical concepts.

Conclusion

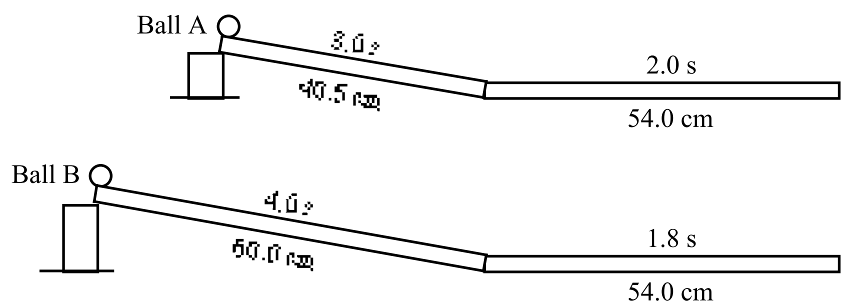
In conclusion, student understanding of the concepts of average velocity, instantaneous velocity, and acceleration increased as a result of instruction using *Physics by Inquiry*. The number of teachers able to use these concepts to determine the acceleration given the motion of real objects also increased. However, there still remained a few who had difficulty understanding the concepts, which led to an inability to make calculations and provide interpretations. The interviews in particular uncovered that a small percentage of teachers were unable to articulate their understandings of these concepts even after instruction. Standards for the teachers are very high due to the fact that they are or will be teaching these concepts to many students. Thus, instructors of the National Science Foundation's Summer Institute would like to see all of the teachers demonstrate an in-depth and functional understanding of the concepts covered. The results presented in this paper have implications for future instruction. Indeed, such data will help the institute instructors determine both the extent to which the kinematics curriculum may be abridged and the amount of time required for teachers to develop a robust understanding of this material. A comparison of this data to a larger sample size would help to better clarify the results. Furthermore, more thorough interviews would help to uncover or shed more light on misunderstandings and/or student difficulties.

Appendix

Note that the questions in the appendix are not replications of the actual questions.

In some cases, the wording has been paraphrased and the diagrams are not exact.

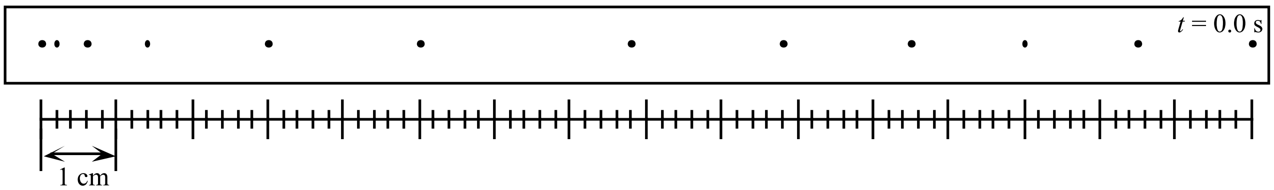
Acceleration Comparison Task 2



Given that the two balls start from rest, use the information provided to determine which ball has the greater acceleration on the sloping section. Note: No credit will be given for using the formula $\Delta s = 1/2\ a\ \Delta t^2$.

Exam Question 1

The small piece of ticker tape shown below was attached to a small cart moving along a straight path. As the cart moved, the tape was pulled through a ticker-tape timer that made a dot on the tape every 0.1 seconds. The dot representing the beginning of the motion (t = 0.0 s) is indicated on the tape.



A. What was the speed of the cart at t = 0.2 s? Explain your reasoning. If it is not possible to determine the speed of the cart at t = 0.2 s, explain why not.

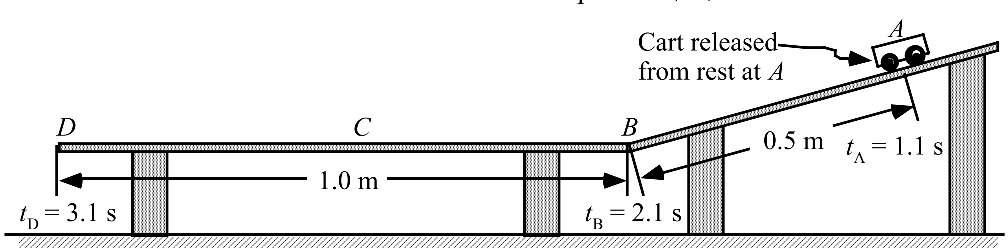
B. What was the speed of the cart at t = 0.9 s? Explain your reasoning. If it is not possible to determine the speed of the cart at t = 0.9 s, explain why not.

C. What was the absolute value of the average velocity of the cart over the entire interval shown on the tape? Explain your reasoning.

D. Give an interpretation of the number you obtained in part C.

Module Pretest Question 3

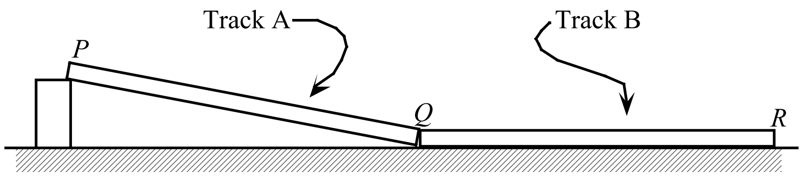
A cart is released from rest and speeds up as it moves down an incline. The cart then moves with constant speed on a 1.0 m segment of track. A diagram of the apparatus is reproduced below. Also shown are the times when the cart was located at points *A, B,* and *D.*



Use this information to find the speed of the cart, if possible, at each of the points *A–D.* If you do not have enough information to find the speed at one or more of the points, state what information you would need.

Question from Pretest for Section 10

A group of students performs an experiment using a ball and straight aluminum tracks, A and B, as shown. Each track is 2 m long. Track B has been arranged so that the ball will roll along it with uniform motion.

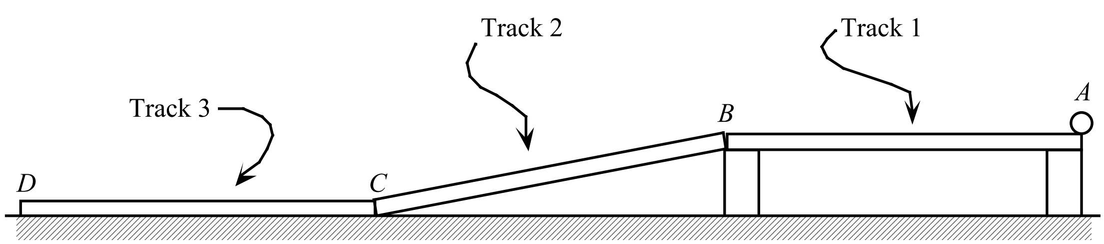


The students release the ball from rest at point *P* and examine its subsequent motion. Suppose the students use 3 clocks that run synchronously (*i.e.,* they are all started at the same instant):

- Clock 1 is stopped at the instant that the ball is released from rest at point *P*.
- Clock 2 is stopped when the ball reaches point *Q*.
- Clock 3 is stopped when the ball reaches point *R*.

At the end of the experiment, clock 1 reads 1.2 s, clock 2 reads 4.4 s, and clock 3 reads 6.0 s. (Ignore uncertainties in the time measurements made by the students.)

Using their data, is it possible to determine the absolute value of the acceleration of the ball on track A? If so, determine this value and explain your reasoning. If it is not possible, explain why not and describe any additional information you would need.



Interview Question 1

Given that the ball starts from rest at the beginnning of track one and rolls along track 1 with uniform motion, down track 2 with constant acceleration, and onto track 3 with uniform motion, describe a method that can be used to determine the acceleration along track 2 if the times A-D are known and the length of the tracks are known.