An investigation of the combinatorial background to resonant mass peaks in multi-jet high energy events

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I. INTRODUCTION

High energy particle colliders are central in the continuing search for the fundamental laws of nature, in particular for new physics beyond the Standard Model. Interpreting the data from the events at such machines, including the Large Hadron Collider currently under construction, is generally an involved and complicated endeavor. It is always important to know what particles are being produced in these collisions, and one common method to find out is to construct the invariant-mass spectrum of jet pairs. If many events involve a particle (for example, a W boson) with a given mass m_0 that decays to two jets, then the expectation is that such a spectrum will have a peak at m_0. However, since there is generally little or no information indicating which two jets came from the decay of a W, a spectrum must be constructed from all possible pairs of jets from each event. Since the number of wrong choices can greatly outnumber the single correct pairing, the expected peak may be totally obscured by the background distribution from those incorrect choices, rendering it difficult or impossible to determine if there actually is some particle of this type being produced.

In this paper, we investigate this invariant-mass background, how it changes with the ratio between m_0 and the scale of the momentum of the jets, b. First we cover some elementary ideas of particle phenomenology with the aim of making this paper accessible to anyone with background in special relativity and some modern physics. Next we describe our work and results relating to the invariant-mass distributions of incorrectly chosen jet pairs. In section three we discuss some insights that basic combinatorics gives into how we should use any additional data about specific jets in searching for an invariant-mass peak. Finally, we suggest paths for future work and summarize our results.

II. SOME ELEMENTARY HIGH ENERGY PHYSICS

The primary method of investigating the fundamental laws of physics is accelerating two particles—such as electron plus positron or proton plus proton—to greater than 99% the speed of light and colliding them with each other. From this collision comes other particles that fly away from the interaction point at relativistic velocities while transmuting and decaying into a menagerie of other particles, which are then detected. The high energies of the colliding particles are required to probe the short distances over which the fundamental forces act and to produce the heavy particles that are important to the theories of those forces. The data that we collect from detectors built around the intersection point consists of particle trails in tracking chambers, measures of the particles' energies, and other information such as electric charge and whether the particles in the final state interact via the strong force or only the weak force. For our purposes we may simply consider that we have measured the relativistic 4-vector of each particle that is in the final state of the interaction.

There are great many such particles produced in a typical collision, so it is useful to simplify the problem of analyzing the data. A ubiquitous method for doing so is systematically grouping the particles together into constructs called jets. Due to the strong forces and short lifetimes of the quarks and gluons produced in the collisions, we do not measure them directly but rather observe a whole shower of particles for each quark. Since the initial quarks are traveling at high speed, the particles that they produce travel in a cone centered in the direction of the original quark's velocity. These clusters of particles, traveling in roughly the same direction and with a common vertex at or near the original collision point, are what we call jets. Operationally, they are defined by an algorithm that searches for clusters of energy deposited in the detector within a certain spatial extent. Each jet, we assume, has approximately the same energy and momentum as its primordial quark. The analysis of this paper is done at the level of these jets.

Now that we have these jets, how do we deduce from them what is happening in the events? For concreteness, suppose we are looking for the W boson, one of the mediators of the weak interaction. This will be one method of looking at the data from the LHC, for instance. At the energies there, higher than any yet obtained in colliders, new particles will be produced, and their decays—whether into Ws or other particles—are clues into their nature and the new physics we hope to learn. W bosons have about a 70% probability of decaying into two quarks, which we detect as two jets (Eidelman *et al.*, 2004). We determine that two jets came from the decay of a W using the concept of invariant-mass. An invariant-mass m is defined for any two 4-vectors:

$m^2 = (p_{1\mu} + p_{2\mu})^2 = (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2$, where we use the usual choice of units to set $\hbar = c = 1$. If two bodies came from the decay of one particle with a rest mass of m_0, the invariant-mass of the two bodies is equal to m_0. This can be seen best in the center of momentum frame, where the last term equals zero and so m = total CM energy after the decay = total CM energy before the decay = m_0. For our purposes, the invariant-mass of two objects that did not come from the decay of one particle is just some mass scale without any important meaning.

With this tool in hand, we now see that if we have a number of jets in an event and want to know if a W produced two of them, we can just find the invariant-mass of each pair and check it against the known mass of the W, m_W. Of course it's not that simple. Since our detectors only have a certain resolution and the W mass actually has a non-zero width, the invariant-mass of the correct pair will only approximately be equal to m_W. Furthermore, if the other jets in the event have energies on the same order as the W mass, than the invariant-mass of the incorrect pairs could, by chance, lie near m_W. When we create a histogram of invariant-masses for many events, we might expect the (narrow) peak around m_W to stand out from the (presumably broad) distribution from the random pairs, but the key point is that there are many more incorrect pairs than correct ones. For example, a typical event might have six jets, two of which came from a W. Then there would be $\binom{6}{2} = 15$ different pairs, 14 of them wrong (background) and only one right (signal). The signal would be an order of magnitude less than the background, and we would probably not be able to see it.

III. DERIVING THE INVARIANT-MASS SPECTRA FOR INCORRECT JET PAIRS

A better understanding of this background invariant-mass spectrum could aid in picking out the signal, or at least let us know under what circumstances we can or cannot hope to see it. Our first goal, then, is to investigate the shape of the probability density distribution of invariant-mass for jet pairs given certain assumptions about the jets. The starting point is the probability distribution for the momentum of jets that are not from the decay of a W or other such particle. Throughout we assume massless jets to facilitate calculations. Jets are generally composed of many particles, each having a much greater kinetic energy than rest energy and hence massless to good approximation. The composite of several massless particles is massless when its components are collimated, that is, all traveling in the same direction; this is approximately true of many jets. These jet momentum distributions are empirical; the particular ones used here are especially simple, again to make the calculations tractable. From elementary probability theory, if some random variable f depends on random variables x and y, $f = F(x, y)$, then the distribution of f in terms of the distributions of x and y is $P(f) = \iint P_x(x) P_y(y)\, \delta(F(x, y) - f)\, dxdy$ (Riley *et al.*, 2002). Thus the distribution of the invariant-mass $\widetilde{m}$ of two jets is given by

$$P(\widetilde{m}) = 2\widetilde{m}\, P_{\widetilde{m}^2}(\widetilde{m}^2) = 2\widetilde{m} \iint P_1(\mathbf{p}_1) P_2(\mathbf{p}_2)\, \delta(m^2(\mathbf{p}_1, \mathbf{p}_2) - \widetilde{m}^2)\, \frac{d^3p_1}{E_1} \frac{d^3p_2}{E_2}, \tag{1}$$

where $\frac{d^3p}{E}$ is the infinitesimal for invariant phase space. In general the distributions of the two jets need not be the same. The normalizations of these probability distributions are $\int P_{1,2}(\mathbf{p})\, \frac{d^3p}{E} = 1$ and $\int P(\widetilde{m})\, d\widetilde{m} = 1$. There are three distinct kinds of incorrect pairs that we must consider: two random jets, one random jet with one from a W, and one jet from a W with another from a different W. (The fourth possibility, two jets from the same W, is of course the signal.)

There are several momentum distributions that might plausibly be used in actually performing this calculation. The most realistic for hadron colliders, such as the LHC, uses pseudo-rapidity coordinates. Events at hadron colliders have, on the average, a kind of cylindrical symmetry. Letting the z-axis lie along the beam line, the jets' momenta are evenly distributed around the usual φ angle. The magnitude of the momentum in the x-y plane, the transverse momentum, typically enters the probability density as an inverse power α between roughly 5 and 6: $p_T^{-\alpha}$. The pseudo-rapidity coordinate η is a measure of the particle's velocity along the beam line. It is defined $\eta = \ln(\cot \frac{\theta}{2})$ and for highly relativistic particles is a good approximation to the rapidity $y = \tanh^{-1}(v_z)$, which is the third coordinate in this system. The advantage of rapidity, or in this common approximation, pseudo-rapidity, is that its transformation rule for boosts in the z direction is simple addition and subtraction (Rolnick, 1994). This feature results in the simple distribution of jets in rapidity: constant out to the limits of the detector, which will typically cover a range of approximately $-4 \leq \eta \leq 4$. To derive the distributions of jets from the decay of Ws we use the joint distribution $P_1(\mathbf{p}_1) P_1(\mathbf{p}_2)\, \delta(m_{12}^2 - m_W^2)$, where P_1 is the random jet distribution, to describe both jets and integrate over one of the jets to find the marginal distribution of the other.

The integrals involved in finding $P(\widetilde{m})$ given the above distributions for random jets and jets originating in Ws were not successfully evaluated. An exponential rather than power-law falloff in p_T was used, in hope of easing the computations, giving the normalized distribution $P_1(\mathbf{p}) = \frac{1}{4\pi\eta_0 b^2}\, e^{-p_T/b}$ for $|\eta| \leq \eta_0$, where b sets the scale and has units of momentum (= units of energy). The six-dimensional integral for two random jets was successfully reduced to a two-dimensional integral analytically, and the remaining integrals were evaluated numerically. The evaluation of this integral is described in detail below as an typical example of these types of integrals. However, the nine-dimensional integral for one random jet and one jet from a W was not successfully evaluated, even numerically (the twelve-dimensional integral for the double W case was not attempted). Numerical integration was done using Mathematica's `NIntegrate` command and (with somewhat more success) the `Vegas` routine of the Cuba library (Hahn, 2005). Since we cannot compare the three different sources of background given this momentum distribution, we also examine a simpler case.

The general equation (1) becomes, for the distribution and coordinate system described above,

$$P(\widetilde{m}^2) = \left(\frac{1}{4\pi\eta_0 b^2}\right)^2 \iiint \iiint e^{-(p_{T_1}+p_{T_2})/b}\, p_{T_1}\, p_{T_2}\, \delta\big(2p_{T_1}\, p_{T_2}(\cosh(\eta_1-\eta_2)-\cos(\varphi_1-\varphi_2))-\widetilde{m}^2\big)$$
$$dp_{T_1} d\eta_1 d\varphi_1\; dp_{T_2} d\eta_2 d\varphi_2\,, \tag{2}$$

where the limits are $0 \leq p_{T_1}, p_{T_2} \leq \infty$; $-\eta_0 \leq \eta_1, \eta_2 \leq +\eta_0$; and $0 \leq \varphi_1, \varphi_2 \leq 2\pi$. The first two steps in attacking this integral are changing variables to the dimensionless quantities $q = p_T/b$ and $\tilde{s} = \widetilde{m}/b$ and evaluating one of the φ integrals. Since the integrand depends only on the cosine of $\Delta\varphi = \varphi_1 - \varphi_2$, we may change variables to $\Delta\varphi$ and $\varphi = \varphi_1$ and evaluate the trivial φ integral to obtain 2π. These manipulations to (2) yield

$$P(\widetilde{m}^2) = 2\pi b^2 \left(\frac{1}{4\pi\eta_0 b^2}\right)^2 \iiiint\!\!\int e^{-(q_1+q_2)}\, q_1 q_2\, \delta\big(2q_1 q_2(\cosh(\eta_1-\eta_2)-\cos\Delta\varphi)-\tilde{s}^2\big) \tag{3}$$
$$dq_1 d\eta_1 dq_2 d\eta_2 d\Delta\varphi\,,$$

where the change of variables produces a $1/b^2$ from the delta function and a b^4 from the other p_Ts, leaving just b^2. Another of the six integrals may be evaluated easily by a change of variables, this time in η. Changing variables from η_1 and η_2 to $\Delta\eta = \eta_1 - \eta_2$ and $\eta_+ = \eta_1 + \eta_2$ introduces a factor of $1/2$ from the Jacobian and allows us the evaluate the η_+ integral. The natural limits of $\Delta\eta$ are $-2\eta_0$ to $+2\eta_0$, but since the integrand is even in $\Delta\eta$, we can put in a factor of two and use $0 \leq \Delta\eta \leq 2\eta_0$. Since η_+ does not appear in the integrand, we can integrate over its range $\Delta\eta - 2\eta_0 \leq \eta_+ \leq 2\eta_0 - \Delta\eta$, giving a factor of $2(2\eta_0 - \Delta\eta)$. Doing this, we find

$$P(\widetilde{m}^2) = 4\pi b^2 \left(\frac{1}{4\pi\eta_0 b^2}\right)^2 \iiiint e^{-(q_1+q_2)}\, q_1 q_2\, (2\eta_0-\Delta\eta)\, \delta\big(2q_1 q_2(\cosh\Delta\eta-\cos\Delta\varphi)-\tilde{s}^2\big) \tag{4}$$
$$dq_1\, dq_2\, d\Delta\eta\, d\Delta\varphi\,.$$

At last it is time to use the delta function. At first glance, there is little to suggest which of the four integrals it would be best to evaluate using the delta function, but the hard-won experience of the author dictates that the $\Delta\eta$ integral be eliminated at this point (this has the advantage of keeping the nice symmetric exponentials simple). This takes $\Delta\eta \to \pm\cosh^{-1}(\frac{\tilde{s}^2}{2q_1 q_2} + \cos\Delta\varphi)$ and introduces a factor of one over the Jacobian, $J = 2q_1 q_2 \sinh\Delta\eta$. Since everything is even in $\Delta\eta$, we may take just the positive root so long as we include a factor of two in front. Care also must be taken of the limits of integration. Two theta functions are necessary since the argument of the delta function only has a zero for certain ranges of the remaining variables, one to guarantee that the argument of the $\cosh^{-1}$ is greater than one, and another reflecting the bound $\Delta\eta \leq 2\eta_0$. All of this becomes

$$P(\widetilde{m}^2) = 4\pi b^2 \left(\frac{1}{4\pi\eta_0 b^2}\right)^2 \int_0^\infty dq_1 \int_0^\infty dq_2 \int_0^{2\pi} d\Delta\varphi \, \frac{e^{-(q_1+q_2)}\,(2\eta_0 - \cosh^{-1}(\frac{\tilde{s}^2}{2q_1q_2} + \cos\Delta\varphi))}{\sqrt{(\frac{\tilde{s}^2}{2q_1q_2} + \cos\Delta\varphi)^2 - 1}} \tag{5}$$
$$\theta\Big(\frac{\tilde{s}^2}{2q_1q_2} + \cos\Delta\varphi - 1\Big)\,\theta\Big(\cosh 2\eta_0 - \frac{\tilde{s}^2}{2q_1q_2} - \cos\Delta\varphi\Big).$$

The next step comes from noticing that the limits of integration, complicated as they are written now, are simple in terms of the product $\rho = q_1 q_2$. Since the integrand depends on ρ and $\sigma = q_1 + q_2$, we now change to those two variables. With the Jacobian $1/\sqrt{\sigma^2 - 4\rho}$, we have

$$P(\widetilde{m}^2) = 4\pi b^2 \left(\frac{1}{4\pi\eta_0 b^2}\right)^2 \int_0^{2\pi} d\Delta\varphi \int_{\rho_{\min}}^{\rho_{\max}} d\rho \int_{2\sqrt{\rho}}^\infty d\sigma \, \frac{e^{-\sigma}\,(2\eta_0 - \cosh^{-1}(\frac{\tilde{s}^2}{2\rho} + \cos\Delta\varphi))}{\sqrt{(\sigma^2 - 4\rho)(\frac{\tilde{s}^2}{2\rho} + \cos\Delta\varphi)^2 - 1)}}, \tag{6}$$

where $\rho_{\min} = \frac{\tilde{s}^2/2}{\cosh 2\eta_0 - \cos\Delta\varphi}$ and $\rho_{\max} = \frac{\tilde{s}^2/2}{1 - \cos\Delta\varphi}$. The σ integral is our next target, and the change of variables $\sigma \to \bar{\sigma} = \frac{\sigma}{2\sqrt{\rho}}$ reveals it to be nothing but the defining integral of the K_0 Bessel function:

$$\int_{2\sqrt{\rho}}^\infty \frac{e^{-\sigma}}{\sqrt{\sigma^2 - 4\rho}}\, d\sigma = \int_1^\infty \frac{e^{-\bar{\sigma}(2\sqrt{\rho})}}{\sqrt{\bar{\sigma}^2 - 1}}\, d\bar{\sigma} = K_0(2\sqrt{\rho}).$$

Finally, we are left with a two dimensional integral:

$$P(\widetilde{m}^2) = 4\pi b^2 \left(\frac{1}{4\pi\eta_0 b^2}\right)^2 \int_0^{2\pi} d\Delta\varphi \int_{\rho_{\min}}^{\rho_{\max}} d\rho \, K_0(2\sqrt{\rho}) \, \frac{2\eta_0 - \cosh^{-1}(\frac{\tilde{s}^2}{2\rho} + \cos\Delta\varphi)}{\sqrt{(\frac{\tilde{s}^2}{2\rho} + \cos\Delta\varphi)^2 - 1}}. \tag{7}$$

Attempts to continue to evaluate this integral analytically have not met with success, so the final integrals must be evaluated numerically with a computer program such as Mathematica, which was done successfully. This sample integral is typical of the ones that were evaluated to find the invariant-mass distributions for other jet momentum distributions.

Instead of even distributions in rapidity, we next use spherically symmetric distributions with exponential falloff in $|\mathbf{p}|$. This simplifies matters considerably, as there is no preferred direction such as the z-axis was in the rapidity case. The only angle the integrals care about is the one between the two jets, making it feasible to proceed further analytically (in particular, the expression for invariant-mass squared in the delta function is much simpler). Here we are able to evaluate the two lower-dimensional integrals entirely analytically, and the last can be done be numerically without difficulty. It is in comparing the distributions of the three sources of background that we see that when the resonant mass m_0 and the jet scale b are roughly the same ($1 \lesssim \frac{m_0}{b} \lesssim 3$), the three distributions are all similar in shape and magnitude, as seen in Fig. 1. This suggests that we group the three sources of background together and examine the amount of background as a whole.

IV. INCREASING THE SIGNAL-TO-BACKGROUND RATIO

In this final section we consider the total integrated background and compare how its magnitude relative to the signal varies with the total number of jets in each event and with the use of a tagging algorithm (with efficiency and false positive rates) that can be used to reject jets that do not come from a W. Methods that attempt to tag jets as not from a W commonly use the detailed information about the constitutions of each jet. One example is b-tagging: the b mesons that come from b quarks have lifetimes such that they may fly several millimeters before decaying, so if the constituents of a jet include one or more tracks indicating some vertex spatially separated from the vertex of all the other tracks in the event, that jet may be tagged with a certain confidence as originating in a b quark. Since Ws only very rarely produce a b quark in their decays, b tagged jets may be removed from consideration when searching for Ws. However, the rate at which jets from b quarks are successfully tagged is commonly low, around 50%, and the rate at which non-b jets are tagged is greater than zero, perhaps 15% (the exact numbers depend on the details of the detector and the event and the precise algorithm used in tagging). Thus, in looking at events that have b-tagged jets, we are compromising our data in two ways. We are using a smaller subset of events, since not all events will get b-tags, and we are throwing away a few correct jets in the events we are using. The first issue is one of the total quantity of data collected: do we have so few total events that if we look just for those with the maximum possible number of tags we will be left with a sample too small to give any statistically significant results? The second is a question of combinatorics: how does the signal-to-background ratio change with the number of tags in each event? Due to the constraints of time on our research, this paper will cover only the second question.

The signal-to-background ratio for n-jet events with k tags where a fraction f_k of the events have two jets from a W, neither of which is among the tagged jets, is given by

$$\frac{S}{B} = \frac{f_k}{\binom{n-k}{2} - f_k} \,. \tag{8}$$

By rejecting from consideration those jets that are tagged, we are reducing $n - k$ and therefore $\binom{n-k}{2} = (n-k)(n-k-1)/2$, while also reducing f due to false positive tags of jets that actually are from a W. It is easy to show that if $k \leq n-2$ jets are removed from each event purely at random then S/B remains unchanged. We therefore expect that any tagging method even slightly better than random will improve S/B. We choose to examine in detail the case $n = 4$, for which useful events could have zero, one, or two tags. The only thing needed to find S/B in these three cases is the formula for f in terms of the tagging efficiency ϵ and the false positive rate ρ. To illustrate the routine for calculating f in general, we show the calculation in detail for a slightly simpler case: 2 jets events where one is from a W (jet A) and one is not (jet B), there is one tag, and f is the fraction of events with one tag where the non-W jet was the one tagged. There are two ways for a given event to be placed in the one-tag group: A could be tagged and B not, with probability $\epsilon(1-\rho)$; or B could be tagged and A not, with probability $(1-\epsilon)\rho$. The ratio of the correctly one-tagged events to all the one-tagged events is $f = \frac{\epsilon(1-\rho)}{\epsilon(1-\rho)+(1-\epsilon)\rho}$. Using this logic we find in the $n = 4$ case, after algebraic simplifications,

$$f_0 = 1$$

$$f_1 = \frac{\epsilon(1-\rho)}{\epsilon(1-\rho)+(1-\epsilon)\rho}$$

$$f_2 = \frac{\epsilon^2(1-\rho)^2}{\epsilon^2(1-\rho)^2 + 4\epsilon(1-\epsilon)\rho(1-\rho) + (1-\epsilon)^2\rho^2}$$

To see the effect on S/B of using events with tags, we plug these formulae into (8) and and plot S/B versus ϵ for a given ρ in Fig. 2. As the figure shows, S/B in the $k = 2$ case is much greater than in the other cases. It is also instructive to calculate how much the use of b-tagging would improve S/B in a somewhat realistic case. Given the b-tagging efficiency and false positive rates above, 50% and 15%, we also must have information about how actually being from a b quark is correlated with not being from a W. The fraction of quarks produced by W decay that are b quarks is very small, let us say 1%. The fraction of jets not from a W that are b quark jets depends on the specifics of the event, but 20% is a reasonable number. These numbers combine to give us our ultimate efficiency and false positive rates in the following way:

$$\epsilon = 0.2 \cdot 0.5 + 0.01 \cdot 0.15 \approx 0.1$$

$$\rho = 0.2 \cdot 0.15 + 0.01 \cdot 0.5 \approx 0.03$$

Plugging these numbers into the formulae for S/B in the $n = 4$ case, we find that S/B increases from 0.20 to 0.35 to 0.84 as the number of jets tagged goes from 0 to 2. In the $n = 5$ case, S/B goes as 0.07, 0.16, 0.27, 0.52 as the number tagged goes from 0 to 3.

From this we conclude that great improvements can be made to the signal-to-background ratio by utilizing a tagging method to the fullest extent possible, even when that method is quite inefficient. Qualitatively, this may be understood as the quadratic dependence of $\binom{n}{2}$ on n overwhelming the weaker dependence of f on ϵ and ρ. The gives new force to what in some sense we already knew: the sheer number of combinatorial possibilities is the source of the problem, and anything that can be used to cut down the number of jets should be used.

V. CONCLUSION AND POSSIBLE FUTURE WORK

The combinatorial background to invariant-mass distributions at particle colliders was investigated by means of evaluating high-dimensional integrals both analytically and numerically for several distributions of jet momentum. It was found that for certain scales the three components of the background had similar distributions. This suggested an analysis that grouped the different parts of the background together and focused on the signal-to-background ratio. This analysis offers a simple rule of thumb to make the signal stand out from the background: use all available information to reduce the number of jets under consideration in each event as much as possible.

Probably the biggest gap in this research as it stands is the effect of following the above rule on the sample size under consideration. Given finite data, it must be the case that there is some optimum level beyond which making still more cuts will reduce the sample under consideration so much as to outweigh the benefit of a cleaner signal. This, and the predictions about S/B, could be tested against simulated data. Another obvious way to extend this work would be to push forward with the numerical integration necessary to find the invariant-mass distributions for more realistic momentum distributions.

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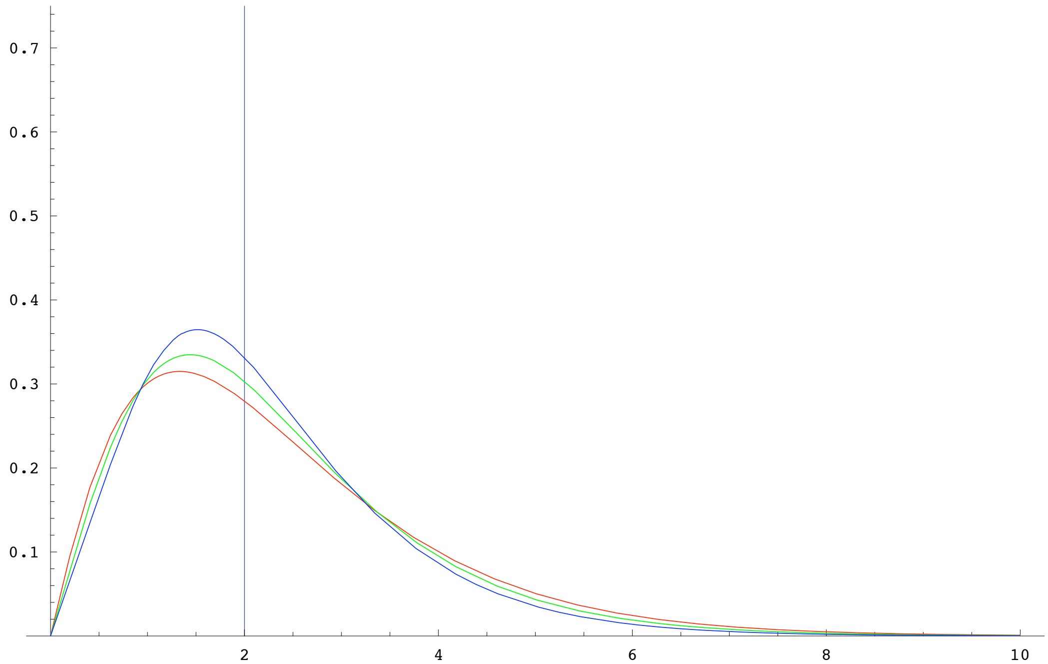

FIG. 1 The invariant mass distributions for the three different types of pairs. This is for the spherical case with an exponential falloff in $|\mathbf{p}|$, $b = 1$, and $m_0 = 2$. Running from most to least in the vicinity of $m = 2$ are the distributions for jets from two Ws, one random and one W, and two random.

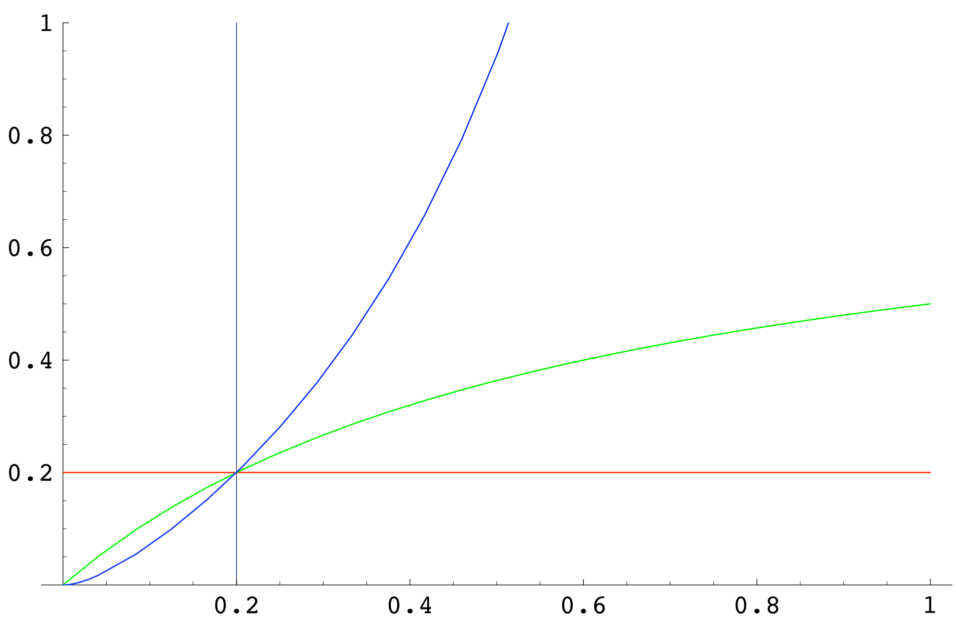

FIG. 2 S/B plotted against ϵ for $\rho = 0.2$ in the case $n = 4$. The flat line is for events with no tags, the concave down line for one tag events, and the concave up line for two tag events ($k = 0$, 1, or 2, respectively).