# Gluon Spin, Canonical Momentum, and Gauge Symmetry

Xiangdong Ji,[1,2,3] Yang Xu,[2,3] and Yong Zhao[3]

[1] *INPAC, Department of Physics, and Shanghai Key Lab for Particle Physics and Cosmology, Shanghai Jiao Tong University, Shanghai, 200240, P. R. China*

[2] *Center for High-Energy Physics, Peking University, Beijing, 100080, P. R. China*

[3] *Maryland Center for Fundamental Physics, University of Maryland, College Park, Maryland 20742, USA*

(Dated: April 30, 2012)

## Abstract

It is well known that in gauge theories, the spin (and orbital angular momentum) of gauge particles is not gauge-invariant, although the helicity is; neither are the canonical momentum and canonical angular momentum of charged particles. However, the simple appeal of these concepts has motivated repeated attempts to resurrect them as physical descriptions of gauge systems. In particular, measurability of the gluon-spin-contribution to the proton helicity in polarized proton scattering has generated much theoretical efforts in generalizing it and others as gauge-invariant quantities. In this work, we analyze constraints of gauge symmetry, the significance of gluon spin in the light-cone gauge, and what-is-possible-and-natural in QCD parton physics, emphasizing experimental observability and physical interpretation in the structure of bound states. We also comment on the measurability of the orbital angular momentum of the Laguerre-Gaussian laser modes in optics.

## I. INTRODUCTION

Feynman's parton is a powerful idea in describing high-energy collisions of hadrons [1]. The parton concept grows out of a simple model for hadrons in the infinite momentum frame (IMF), i.e., observers or systems traveling at the speed of light, in which the interactions are slowed down by Lorentz dilation and the hadrons appear to be simply a collection of non-interacting partons. When quantum chromodynamics (QCD) was proposed as the fundamental quantum theory to describe the physics of strong interactions, the parton picture survived with two minor modifications: First, it emerges in a special choice of gauge, the light-cone gauge $A^+ = 0$ (where $+$ means a combination $(A^0 + A^3)/\sqrt{2}$ of $A^\mu$, $\mu = 0, 1, 2, 3$, if the hadron is moving in the 3-direction) because the quarks do interact with each other through an SU(3) color gauge field, even in IMF. In this choice of gauge, the interaction effects are nominally gone. Second, partons are differently at different (momentum) resolution scales again due to color interactions; however, the scale evolution can be computed in QCD perturbation theory and is believed under control [2]. Of course, QCD also permits studying effects beyond the simple parton model by including parton correlations, transverse momentum, off-shellnesss, etc. These effects cannot be studied model-independently, and one must invoke high-twist factorization and expansions [3, 4].

Because the parton concept has been so appealing, it often motivates attempts to do away with the gauge symmetry of QCD and search for "physical observables" which are intuitive in non-interacting theory but "unnatural" due to gauge symmetry constraints. These attempts occasionally have met remarkable successes. For example, although the gluon spin is not a gauge-invariant concept, the gluons in IMF have momentum predominantly along the 3-direction and appear to be on-shell. A free gluon is defined to have helicity either $+1$ or $-1$. It is then a simple matter to define the gluon helicity density $\Delta g(x)$ as the density difference of two different helicities. The total gluon polarization $\Delta G = \int \Delta g(x)$ is a measurable quantity through hard scattering, and therefore must be gauge-invariant. On the other hand, in the rest frame of the nucleon, the transverse component of the gluon momentum is by no means negligible, and the projection of the gluon spin along the 3-direction is no longer gauge invariant. Moreover, as a part of the bound state, the gluons are necessarily off-shell. An off-shell gluon, as in the gluon propagator, is hard to define gauge-invariantly. Therefore, $\Delta g(x)$ is no longer simple to interpret as the bound state property of the nucleon, although it is a matrix element of a gauge-invariant operator, non-local and complicated in general framework [5]. Nonetheless, in the light-cone gauge, it reduces to the simple gluon spin operator along the 3-direction, reminiscent of the IMF physics.

The example of the IMF gluon helicity has motivated myriad attempts to define parton canonical momentum and canonical orbital angular momentum (OAM). In particular, there appears a recent attempt to completely reinvent the concept of gauge invariance by proposing decomposing the gauge potential into the so-called physical and pure-gauge parts, and defining various "physical observables" with the physical part [6, 7]. The standard textbook result that gluons carry about one half of the nucleon momentum has been challenged by the new definition of the parton canonical momentum. There have been a number of follow-up studies in the literature [8–12].

In a different context, there have been many studies about the parton transverse-momentum-dependent (TMD) factorization "theorems" in the literature. While in some cases, these theorems can be argued or even proved and the relevant distributions are gauge-invariant [13]. In many other cases, such factorizations are simply model assumptions, and do not meet the requirement of gauge symmetry. In particular, when the parton transverse momentum is fully taken into account, the twist-expansion is often lost in the sense that all twists are now involved. There seems no systematic expansion since there are no arguments why extra gluons with transverse polarizations shall not be included in the calculations [14].

In this paper, we analyze the constraints of gauge symmetry on bound state properties and parton physics. In Sec. II, we make some general remarks about gauge symmetry, in particular, how to maintain gauge symmetry in a fixed-gauge calculation. We emphasize that the wave function is a gauge-dependent concept and a gauge symmetric matrix element emerges only when the operator is manifestly gauge symmetric, modulo the so-called BRST exact operators and equations-of-motion operators in the covariant gauge. In Sec. III, we analyze the gauge symmetry in parton physics and in particular in the case of the gluon polarization. We point out that while gluons appear as on-shell radiation in IMF, they are off-shell and have transverse-momentum as a part of the bound state. We introduce the concept of gauge-invariant extension (GIE) to make connection between parton physics and gauge symmetry. In Sec. IV, we discuss GIE of the Coulomb gauge in connection with the work of Chen et al. We comment that result of the Wakamatsu's calculation of the scale evolution for the gluon spin is in fact a light-cone gauge result [9]. We will also discuss the spin and OAM in the case of photon radiation fields in optics. We conclude the paper in Sec. V.

## II. GENERAL REMARKS ABOUT GAUGE SYMMETRY

In this section, we are going to make some general remarks about gauge symmetry. We emphasize that wave functions in non-relativistic quantum mechanics are gauge dependent, and the mechanical momentum of the charged particles corresponds to the covariant derivative in quantum theory. We argue that in atomic systems, because of the non-relativistic approximation, the partial derivative is actually gauge-invariant to leading order in $v/c$ expansion. We then consider the gauge symmetry in quantum field theories, reminding the reader why gauge choice is a necessity and how a gauge-invariant result emerges from a fixed-gauge calculation. We also comment on the relation between gauge symmetry and Lorentz symmetry. These remarks are by no means novel, and many are present in textbooks or papers. However, they lay important general framework and will be useful for the discussions in the remainder of the paper.

### A. Gauge symmetry in non-relativistic quantum mechanics

Let's first consider gauge symmetry in non-relativistic quantum theory with electromagnetic interactions. For a charged particle moving in an external electromagnetic field, the Hamiltonian is,

$$H = \frac{(\vec{P} - e\vec{A})^2}{2m} + e\phi \ , \tag{1}$$

where $\vec{P}$ is the canonical momentum and we have customarily called $A^0 \equiv \phi$. It has the following eigenvalue problem,

$$H\psi_n(\vec{r}) = E_n\psi_n(\vec{r}) \ , \tag{2}$$

with energy eigen-values $E_n$ and energy eigenfunctions $\psi_n(\vec{r})$.

Under a time-independent gauge transformation,

$$A^\mu(\vec{r}) \to A^{\mu\prime}(\vec{r}) = A^\mu(\vec{r}) + \partial^\mu \chi(\vec{r}) \ , \tag{3}$$

we obtain a new Hamiltonian,

$$H' = \frac{(\vec{P} - e\vec{A}')^2}{2m} + e\phi \ , \tag{4}$$

which is manifestly different. However, it has the same quantum mechanical eigen-energy,

$$H'\psi_n'(\vec{r}) = E_n \psi_n'(\vec{r}) \ , \tag{5}$$

where $\psi_n' = e^{ie\chi(\vec{r})}\psi_n(\vec{r})$ is the eigenfunction in the new gauge. Thus, while the charged particle probability density

$$\rho_n(\vec{r}) = \psi_n^*(\vec{r})\psi_n(\vec{r}) \ , \tag{6}$$

is gauge invariant, the charged particle canonical momentum $\vec{P}$ (which quantizes as a partial derivative) expectation value is not

$$\left\langle \psi_m' \left| \vec{P} \right| \psi_n' \right\rangle = \left\langle \psi_m \left| \vec{P} \right| \psi_n \right\rangle + e \left\langle \psi_m \left| \nabla \chi(\vec{r}) \right| \psi_n \right\rangle \ . \tag{7}$$

Thus the proof that the canonical momentum matrix element is gauge-invariant in Ref. [11] is likely incorrect. Actually, in Ref. [11] the gauge transformation leaves the physical states invariant, but that is not true according to Eq. (5): wavefunction is a not a gauge-independent quantity. It is the matrix element of the *mechanical momentum* $\vec{p} = \vec{P} - e\vec{A}$ that is gauge invariant,

$$\left\langle \psi_m' \left| \vec{P} - e\vec{A}' \right| \psi_n' \right\rangle = \left\langle \psi_m \left| \vec{P} - e\vec{A} \right| \psi_n \right\rangle \ , \tag{8}$$

which corresponds to the covariant derivative in quantization, $\vec{p} = \vec{P} - e\vec{A} \equiv -i\vec{D}$.

In Feynman's lectures on physics, he gave a simple example to demonstrate that it is the covariant derivative that corresponds to the mechanical momentum in quantum mechanics [15]: Consider a charged particle near a solenoid, which has a quantum mechanical wave function $\psi(\vec{r}, t)$. The solenoid does not have any current in the beginning. At some point in time, a current passes through the solenoid and a stable magnetic field is established. In the process, the particle gets a momentum kick because a changing magnetic field induces an electric field which exerts a force on the charged particle. In quantum mechanical calculation, this momentum kick cannot be obtained from the partial derivative acting on the wave function because the latter is continuous in time. This momentum kick comes from the establishment of the vector potential $\vec{A}$ in the system.

### B. Atom systems and Coulomb gauge

Atoms are to a very good approximation non-relativistic systems. In such systems, the charge effects are much larger than current interactions, and the dominant interaction is the static Coulomb force. Let us establish a power counting in terms of the velocity $v$ and electromagnetic coupling $e$ [16]. The fine structure constant $e^2/4\pi = \alpha_{\rm em}$ is a small quantity, and so is the typical velocity of the charged particle $v/c \sim \alpha_{\rm em}$. We will show through power counting that the canonical momentum and canonical OAM are approximately gauge-invariant.

In non-relativistic systems,

$$B/E \sim v/c \ . \tag{9}$$

We are going to use this to establish a power counting for the four-vector potentials. According to the definition

$$\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \nabla \times \vec{A} \ , \tag{10}$$

Since the electrostatic interaction $e\phi$ is order of $\alpha_{\rm em}^2$ (recall the Bohr energy), we have

$$\phi \sim e\alpha_{\rm em}; \qquad E \sim ev\alpha_{\rm em} \ . \tag{11}$$

In principle, we can choose $\vec{A}$ to be of the same order of magnitude as the above. However, since $\vec{B}$ is suppressed by a factor of $v/c$ relative to $\vec{E}$, the transverse part of $\vec{A}$ must be suppressed by the same factor. To have a natural power counting, one shall work only in the class of gauges in which the longitudinal part of $\vec{A}$ is similarly suppressed. One gauge that satisfies this condition is the Coulomb gauge,

$$\nabla \cdot \vec{A} = 0 \ . \tag{12}$$

which eliminates the longitudinal part of the gauge potential entirely. Thus the Coulomb gauge is a natural choice for non-relativistic systems.

According to the above, the counting for the vector potential in Coulomb gauge is

$$\vec{A} \sim ev\alpha_{\rm em} \ . \tag{13}$$

Thus $e\vec{A}$ in Coulomb gauge is $\alpha_{\rm em}^2$ suppressed related to the canonical momentum $\vec{P} = -i\nabla \sim v$. Thus,

$$\vec{p} = \vec{P} + \mathcal{O}(v\alpha_{\rm em}^2) \ , \tag{14}$$

and the canonical momentum is gauge-invariant to the relative order $\alpha_{\rm em}^2$. This explains why the canonical momentum and canonical OAM are useful concepts in atomic systems.

Of course, one can choose to work in a gauge in which $e\vec{A}$ is not suppressed at all relative to $\vec{P}$. Then the atomic wave functions will be different, and the physical momentum and OAM must follow from the mechanical momentum and mechanical OAM.

In the QCD bound states, such as the nucleon, the above power counting fails. In particular, the color magnetic field $\vec{B}^a$ is not suppressed relative to the color electric field $\vec{E}^a$. Moreover, the quark motion is entirely relativistic. Therefore, the $e\vec{A}$ and $\vec{P}$ have the same order of magnitude. The Coulomb gauge is no longer special to keep the power counting natural. In the factorization theorems involving parton transverse momentum, it is often assumed that parton canonical momentum is much larger than the transverse gauge potential in light-cone gauge. There is no known physical basis for this assumption.

### C. Quantization of QED and gauge symmetry

Here we consider gauge-symmetry constraints in quantum field theories. For simplicity, we take the U(1) gauge theory with a spin-1/2 fermion, such as quantum electrodynamics

(QED). The Lagrangian density is

$$\mathcal{L} = \overline{\psi}(i\gamma_\mu D^\mu - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \ , \tag{15}$$

where $\psi$ is the electron's Dirac field and $A^\mu$ is the $U(1)$ gauge potential.

$A^\mu$ has two physical degrees of freedom; it also mediates static electric and magnetic interactions among charged particles. However, $A^\mu$ does have spurious degrees of freedom that must be constrained in order to have a finite photon propagator. Thus, quantization of a gauge theory requires gauge fixing, which ironically breaks the gauge symmetry. Although the symmetry is no longer manifest after quantization, the physical observables in terms of the physical degrees of freedom shall be independent of the spurious gauge dynamics. This can be verified either through residual gauge symmetry after gauge fixing or through comparing calculations in different gauges.

Different gauge fixings often lead to different Hilbert spaces, and the physical states are gauge dependent, just as in the case of non-relativistic system described in the previous subsection. For instance, in the Coulomb gauge, all quanta in the Hilbert space are physical degrees of freedoms, whereas in the Weyl gauge $A^0 = 0$, the longitudinal polarization of the photon is allowed, and the physical states are chosen through Coulomb's law constraint $(\nabla \cdot \vec{E} - \rho)|\Psi_{\rm phys}\rangle = 0$. In the covariant Lorentz gauge, all four polarizations of the photons are allowed and the physical Hilbert space is constrained by Lorentz gauge condition $\partial_\mu A^\mu|\psi_{\rm phys}\rangle = 0$. However, due to the residual gauge freedoms, the physical states is still not unique. It can contain non-physical quanta.

Despite the fact that all Green's functions and wavefunctions are gauge-dependent, the poles of the Green's functions and scattering S-matrix are gauge invariant. Usually the proof is done using residual gauge symmetry. Here we are concerned with the physical matrix elements in an asymptotic states $\langle\psi|O|\phi\rangle$. In order for them to be gauge invariant, we have to study general gauge symmetry transformation. Let's assume a super-operator $G$ which brings the physical states from one gauge to another,

$$|\psi\rangle' = G|\psi\rangle \ . \tag{16}$$

Then the matrix element is invariant under gauge transformation,

$$\langle\psi'|O'|\phi'\rangle = \langle\psi|O|\phi\rangle \ , \tag{17}$$

only if $O' = GOG^{-1}$ is gauge invariant, that is, has the same dependence on the gauge potential as in $O$. Clearly the gauge transformations here cannot be a classical one, it must be quantum operators.

The requirement for matrix elements to be gauge-invariant is much simpler to see in path-integral formulation [17]. The sufficient condition again is that the operator has to be manifestly gauge-invariant.

Of course, it is not necessary to have gauge-invariant operators in order for its physical matrix elements to be gauge-invariant. In covariant quantization, it is known that the equations-of-motion operators and the so-called BRST-exact operator do vanish in the physical states [18]. For the example of energy-momentum tensor, see [19]. It is also taken as true that the gauge-variant total derivative operators do vanish in well-localized states. However, in the plane wave states, the total derivative operators do contribute to the non-forward matrix elements. Thus one has to be careful about the total derivative operators particularly when they are gauge dependent.

The matrix element of a gauge-dependent operator usually vanishes when averaging over all gauges, just like the velocity vector vanishes when summing over contributions from all frames.

### D. Lorentz symmetry

According to Einstein's theory of special relativity, all physical quantities shall be Lorentz tensors, i.e., when coordinates transform, they behave either as scalars, four-vectors, or high-order tensors. Therefore, when making a frame transformation, one easily obtains the physical quantities in one frame from the others. In gauge theories, because not all four-components of $A^\mu$ are physical, calculations are found not to be manifestly Lorentz covariant. For example, the gauge choices often break Lorentz symmetry when the gauge conditions are frame-dependent. However, for local gauge-invariant quantities, Lorentz symmetry is manifestly kept in calculations [20]. For non-local operators, one has to introduce gauge links which often involve geometrical lines. The transformation of such a line is usually complicated, and Lorentz symmetry transformation of non-local operators often becomes intractable. In this case, one considers the subclass of Lorentz transformation which leaves the line invariant.

In the case of non-covariant gauges, calculations in different frame often correspond to different gauge choices. Therefore, gauge symmetry of the calculations is a necessary condition to maintain Lorentz symmetry, or vice versa. We note that in quantum theory, the wave function of a system is not a Lorentz invariant quantity, it changes with the coordinates. Such a change is not just a kinematic transformation, it involves Lorentz boost which is usually interaction-dependent. Thus boosting a wave function is a highly nontrivial matter in a relativistic interacting theory.

## III. EXTENDED GAUGE INVARIANCE IN PARTON PHYSICS

In high-energy scattering in QCD, partons are often used to describe the scattering processes involving large momentum transfers [1]. Total scattering amplitudes usually contain two factorized parts, described as factorization theorems [2]. One part contains the product of parton densities and/or fragmentation functions and/or soft functions. As far as the hard scattering is concerned, the hadrons can be viewed as beams of non-interacting partons, and the parton densities simply describe the properties of the beams. The second part is the parton scattering cross section, which involves scattering of on-shell colored particles. Feynman's parton language greatly simplifies the description of high-energy scattering. The prove of the factorization theorems is to show that the cross section has a systematic expansion in terms of the inverse powers of large momenta and that the leading factorized parts are naturally gauge-invariant.

### A. Gauge symmetry of parton density and physics in light-cone gauge

One of the keys to the parton language description of high-energy scattering is gauge symmetry and light-cone gauge, $A^+ = 0$. [Here we use the light-cone coordinates $A^\pm = (A^0 \pm A^3)/\sqrt{2}$, $A_\perp = (A^1, A^2)$, and it is well known that the light-cone coordinates are equivalent to the use of IMF.] The parton density can be described as the matrix element of gauge-invariant operators in hadron states, and since the cross section is gauge-invariant by itself, this means that the on-shell parton scattering cross section is also gauge-invariant and can be calculated independently in any gauge. This can also be seen from that the parton cross sections involve only on-shell parton scattering. The gauge-invariance, however, masks the physical property of the parton density operator. For example, the quark longitudinal-momentum density distribution

$$q(x) = \frac{1}{2P^+} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\overline{\Psi}(\lambda n)\gamma^+ \Psi(0)|PS\rangle \ , \tag{18}$$

where $n$ is a light-like four-vector $n^2 = 0$ and the nucleon momentum is $P^\mu = (P^0, 0, 0, P^3)$, with $P \cdot n = 1$. $\Psi(\xi)$ is a gauge-invariant quark field defined through multiplication of a light-cone gauge link

$$\Psi(\xi) = \exp\left(-ig\int_0^\infty n \cdot A(\xi + \lambda n) d\lambda\right) \psi(\xi) \ . \tag{19}$$

This gauge link ensures that whenever a partial derivative or canonical momentum of colored quarks appears, the gauge potential $A^\mu$ must be present simultaneously to make it a covariant derivative (kinetic momentum), $D^\mu = \partial^\mu + igA^\mu$. Indeed, taking its moments,

$$\int x^{n-1} q(x) dx \quad \sim \quad n_{\mu_1} ... n_{\mu_n} \langle P|\overline{\psi}(0)\gamma^{\mu_1} iD^{\mu_2} ... iD^{\mu_n} \psi(0)|P\rangle \ , \tag{20}$$

we see that the parton momentum distribution refers to the gauge-invariant kinetic momentum! The kinetic momentum structure is clearly seen through Feynman diagrams in Fig. 1: Gauge symmetry requires that a parton with kinetic momentum $k^+ = xP^+$ includes the sum of all diagrams with towers of longitudinal gluon $A^+$ insertions.

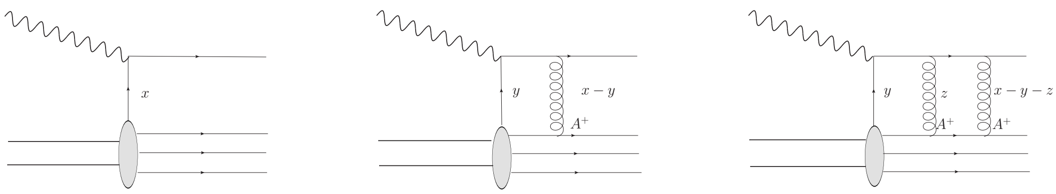


FIG. 1: Deep-inelastic scattering process in which the gauge invariance involving the longitudinal quark kinetic momentum $xP^+$ is achieved through insertions of gluons with longitudinal polarization $A^+$.

Simple parton physics emerges in the light-cone gauge $A^+ = 0$, where the light-cone gauge link disappears, all the covariant derivatives in Eq. (20) become partial ones, i.e., the canonical momentum and the simple quark field become physical. One can use the light-cone quantization to write

$$\psi_+(\xi) = \int \frac{dk^+ d^2k_\perp}{2k^+(2\pi)^3} \left[d^\dagger(k^+, k_\perp)v(k^+, k_\perp)e^{i(k^+\xi^- - \vec{k}_\perp \cdot \vec{\xi}_\perp)} + b(k^+, k_\perp)u(k^+, k_\perp)e^{-i(k^+\xi^- - \vec{k}_\perp \cdot \vec{\xi}_\perp)}\right] \tag{21}$$

Inserting the above expression into the parton density, one finds for $x > 0$,

$$q(x) = \int \frac{d^2k_\perp}{2k^+(2\pi)^3} \langle P|b^\dagger(xP^+, k_\perp)b(xP^+, k_\perp)|P\rangle \ , \tag{22}$$

which is a genuine number-density distribution.

While the parton language simplifies the description of a scattering process considerably, one does not have to choose the light-cone gauge $A^+ = 0$. There shall always exist a natural gauge-invariant formalism which embodies the factorization, such as the presence of the matrix elements of the twist-two gauge-invariant operators.

In recent years, there have been many discussions on the so-called $k_T$-dependent factorizations. The motivation is to take into account the effects of parton transverse momentum in the scattering processes. In some cases where there is a systematic power counting for the relevant physical observable, the factorization is gauge-invariant [13]. However, in many other cases, the factorization is basically a model assumption which cannot be justified because other contributions of the same order have been neglected and the result is not gauge invariant [14].

### B. Gluon spin

It is well-known that in gauge theories, the spin operator of a gauge particle, defined as

$$\vec{S}_g = \int d^3\xi \vec{E} \times \vec{A} \ , \tag{23}$$

is not a gauge invariant quantity [21], because of the manifest dependence on the gauge potential $\vec{A}$. This applies for photons in QED and gluons in QCD. However, its helicity, i.e., the projection of $\vec{S}_g$ along the direction of momentum, is gauge-invariant. This can be easily seen from vanishing of the gauge-dependent term $(\vec{\nabla} \times \vec{\nabla}\chi) \cdot \vec{E}$. Helicity is also a projection of the total angular momentum along the direction of motion, as obviously the orbital part does not contribute. The helicity of a vector gauge particle is defined to be $+1$ or $-1$, corresponding to right-handed or left-handed polarization, respectively.

While helicity is a useful gauge-invariant concept, it shall not be confused with spin projection. For example, the spin vector $\vec{S}_g$ calculated along the $\perp$-direction for a photon moving in the 3-direction definitely depends on the choice of the wave function or gauge. Many concepts developed in radiation physics use helicity as a starting point. For example, the well-known electromagnetic multipoles are constructed with Wigner rotation matrices applying to the plane-wave states with good helicities [21].

A nucleon with large 3-momentum $P_3 \to \infty$ are made of off-shell (virtual) constituents with large 3-momenta, $xP_3$, as well. In high-energy scattering, only this 3-momentum is important and the transverse component and off-shellness can be neglected, because they produce subleading effects in the parton scattering cross section. Therefore, as far as the hard scattering process is concerned, the nucleon appears to be made of a beams of massless *on-shell* partons (quarks and gluons) moving in the same direction (called equivalent-photon or Weizsäcker-Williams approximation in QED [22]). For such a beam of gluons, the helicity is a good quantum number. The total beam helicity, the difference between the number of particles with positive and negative helicity, is also a physical observable. Thus, we have the gauge -invariant gluon helicity distribution $\Delta g(x)$,

$$\Delta g(x) = g_+(x) - g_-(x) \ , \tag{24}$$

where $g_\pm(x)$ is the density of gluons with momentum $x$ and helicity $\hat{P} \cdot \vec{S}_g = \pm 1$.

However, in the bound state nucleon, the gluons are virtual quanta which not only involve transverse polarization but also longitudinal one. They are off-shell particles whose wave function cannot be defined in a gauge-invariant way. Thus, the gluon helicity distribution formally is a matrix element of a complicated non-local operator,

$$\Delta g(x) = \frac{i}{2x(P^+)^2} \int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle PS| F^{+\alpha}(\lambda n) L_{[\lambda n^-,0]} \tilde{F}^{+}{}_{\alpha}(0)|PS\rangle \ . \tag{25}$$

where $\tilde{F}^{\alpha\beta} = 1/2\epsilon^{\alpha\beta\mu\nu}F_{\mu\nu}$ and $L_{[\lambda n^-,0]}$ is a light-cone gauge link. The need for gauge invariance dictates that it must be the gluon field strength $F^{\mu\nu}$ that appears and the gauge link ensures cancelation of the gauge transformation factors $U$ from two different spacetime points. The QCD expression only ensures that in IMF and light-cone gauge, it reduces to the simple gluon helicity counting,

$$\Delta g(x) = \int \frac{d^2k_\perp}{2k^+(2\pi)^2} \langle P| a^\dagger_{+1}(xP^+, k_\perp) a_{+1}(xP^+, k_\perp) - a^\dagger_{-1}(xP^+, k_\perp) a_{-1}(xP^+, k_\perp)|P\rangle \tag{26}$$

In light-cone quantization with $A^+ = 0$, an on-shell gluon has two physical polarizations, described by the transverse components of the gauge potential $A_\perp$. The matrix element is, however, independent of the momentum in the 3-direction and can be taken in the rest frame of the nucleon. However, in such a frame, the gluon parton's transverse momentum definitely cannot be neglected, neither does its off-shellness. For these gluons, their spin projection in the 3-direction is no longer gauge-invariant; and one cannot define the gauge-invariant gluon helicity in a simple fashion. Therefore, $\Delta g(x)$ does not have a straightforward physical interpretation as the property of the bound state.

Indeed, when integrating over all $x$, one gets the total parton helicity operator in a longitudinally polarized nucleon

$$S_g^{\rm inv} = \frac{i}{2} \int \frac{dx}{xP^+} \int d^3\xi e^{ix\xi^- P^+} F^{+\alpha}(\xi^-) L_{[\xi^-,0]} \tilde{F}^{+}{}_{\alpha}(0) \ , \tag{27}$$

which is a non-local operator, and again has no clear gauge-invariant physical interpretation. $S_g$ has a simple interpretation only in the light-cone gauge: it reduces to the the gluon spin,

$$S_g^{\rm inv}|_{A^+=0} = \int d^3\xi (\vec{E} \times \vec{A})^3 \ . \tag{28}$$

Of course, as we have discussed in Sec. II, the same operator defined in other gauges has entirely a different bound-state matrix element.

The above discussion makes clear that while gluon spin is not a gauge-invariant quantity, but in the light-cone gauge and frame, the gluons in a bound state may be regarded as on-shell and the gluon helicity becomes a useful concept and can be measured experimentally [22]. Leaving this specific gauge and frame, this quantity involves longitudinally polarized gluons and no longer has a clear physical interpretation despite that it can be defined and calculated gauge-invariantly. This motivates going beyond the traditional sense of gauge symmetry by introducing the *gauge-invariant extension* (GIE) of some apparently gauge-dependent quantities. A GIE of a gauge-variant quantity is to generalize the fixed gauge result extrapolated to any other gauge. There are an infinite number of GIE for a gauge-dependent quantity depending on the gauges in which one decides to generalize. The gluon spin operator in the light-cone gauge has its GIE in Eq. (27) [5].

### C. GIE of parton observables

Parton physics emerges in the set-up of IMF and light-cone gauge. In a fixed gauge, not all results are naturally gauge-invariant, as gauge fixing often masks the gauge symmetry properties of both gauge-invariant as well as non-invariant quantities. This is similar to calculations in a fixed system of coordinates where physical quantities often do not have apparent properties of the underlying tensors.

One can make GIE for any partonic operators using the same approach as in the case of the gluon spin, so that in any other gauges, GIE partonic operators produce the same matrix elements as in the light-cone gauge.

For example, a GIE of the partial derivative in light-cone gauge is

$$i\partial_\xi^\perp = iD_\xi^\perp + \int^{\xi^-} d\eta^- L_{[\xi^-,\eta^-]} gF^{+\perp}(\eta^-,\xi_\perp) L_{[\eta^-,\xi^-]} \ . \tag{29}$$

In this way, the partial derivative in the light-cone gauge becomes gauge-invariant, and therefore one can talk about "gauge-invariant" canonical momentum and orbital angular momentum. In fact, the canonical version of the spin sum rule discussed in Ref. [23, 24] and its scale evolution [17] can be understood as "gauge-invariant" in this sense [25, 26].

In the same sense, one may regard all $k_T$ dependent factorization result as gauge invariant.

### D. Problems with gauge-invariant extension

Clearly, GIE of an intrinsically gauge-noninvariant quantity is not naturally gauge invariant. As such, its usage has a number of problems:

First, the operators resulted from GIE are in general non-local. Non-local operators do not usually have a clear physical meaning in general gauges, although they usually do in the fixed gauge. It is also difficult to perform calculations which are most practically done in the fixed gauge. In fact, all GIE of partonic operators cannot be calculated in lattice QCD because they involve intrinsically the real time which becomes imaginary upon the Wick rotation. This, for example, is the case for the total gluon helicity $\Delta G = \int dx \Delta g(x)$.

Second, GIE operators do not transform simply under Lorentz transformations. While local gauge-invariant operators often have simple classifications in terms of representations of the Lorentz group, non-local operators involve geometrical lines as gauge links which do not have simple Lorentz transformation properties. Many gauge conditions are not Lorentz-covariant.

Third, there are (infinite) many non-local operators that have the same quantum numbers. For instance, one can merely change the path of a gauge link to get a new operator. All of these operators can mix under scale evolution. Therefore scale-mixing calculations become intractable [27].

Finally, GIE operators are in general unmeasurable. So far, the only example is offered in high-energy scattering in which certain partonic GIE operators may be measured. For GIE operators in Coulomb gauge, to which we will turn to in the next section, there is no known physical measurement for any of them.

For the transverse-momentum dependent factorization, many of the so-called factorization formulae are intrinsically gauge dependent. There are contributions of the same order considered that have been neglected, for instance, contributions from additional transversely polarized gluons. Here gauge dependence is a signal that there are legitimate contributions that have been left out. Thus one makes little gain by claiming the result is gauge invariant through GIE.

## IV. GAUGE-INVARIANT EXTENSION OF COULOMB GAUGE

Coulomb gauge is a natural gauge choice in atomic physics, which as we discussed in Sec. II preserves the power counting in non-relativistic effective theory. In this section, one considers GIE of the Coulomb gauge, i.e. extending a Coulomb gauge result gauge-invariantly to other gauges. For a natural gauge-invariant quantity, this of course does not generate anything new. However, if one believes that the Coulomb gauge potential $A_c^\mu$ is special, any operator made from it can be made gauge-invariant through GIE. As we shall see, this formalism is just what Chen et al has been proposed as a new "gauge-invariant" formulation [6, 7]. Being intrinsically non-relativistic, however, Coulomb gauge is not a natural gauge choice for describing the nucleon structure as quarks and gluons are relativistic particles.

In this section, we will first define the GIE of the Coulomb gauge. Then we argue that Chen et al's proposal is just the same. We show that the so-called gauge-invariant anomalous dimension of the gluon spin operator calculated in Ref. [9] is in fact corresponding to GIE of the gluon spin in the light-cone gauge. Finally, the last subsection, we discuss why the spin and orbital angular momentum separation seems possible in optics.

### A. Coulomb gauge extension

Let us consider how to make GIE of the Coulomb gauge. For simplicity, we consider QED only and use a subscript $c$ to denote quantities in this gauge. The gauge condition is

$$\vec{\nabla} \cdot \vec{A}_c = 0 \ , \tag{30}$$

and $A_c^0$ becomes a non-dynamical variable satisfying constraint

$$\nabla^2 A_c^0 = -\vec{\nabla} \cdot \vec{E} = -\rho \ . \tag{31}$$

Let us consider any Coulomb-gauge operator $O_c$ as a function of $A_c^\mu$. Its matrix element evaluated in the Coulomb gauge wave functions are $\langle \psi_c | O_c | \phi_c \rangle$. Any such operator has a GIE $\mathbf{O} \equiv O_c(A_c^\mu)$ by replacing every $A_c^\mu$ in it by

$$A_c^\mu = \frac{1}{\nabla^2} \partial^i F_{i\mu} \ . \tag{32}$$

which depends on the gauge-invariant field $F_{\mu\nu}$. $\mathbf{O}$ is now manifestly gauge-invariant as $A_\mu$ only appears in the field strength tensor. If we now specialize $\mathbf{O}$ in any gauge by letting

$A^\mu$ satisfying a new gauge condition $g$, calling it $\mathbf{O}_g$, its matrix element is then the same as that in Coulomb gauge.

$$\langle\psi_g|\mathbf{O}_g|\phi_g\rangle \equiv \langle\psi_c|O_c|\phi_c\rangle \ . \tag{33}$$

Note that the states $|\psi_g\rangle$ and $|\phi_g\rangle$ are correspondingly obtained in the same gauge $g$. In this sense, any operator in Coulomb gauge $O_c$ is "gauge-invariant" through $\mathbf{O}$, although it is generally non-local in terms of the non-gauge-fixed potential $A_\mu$.

### B. Chen et al's proposal

Chen et al. claim to have found a way to describe the spin of a gauge particle through a new formulation of gauge symmetry [6]. The first step in their theory is to decompose the gauge potential $A^\mu$ into two parts.

$$A^\mu = A^\mu_{\rm pure} + A^\mu_{\rm phys} \ . \tag{34}$$

The part $A^\mu_{\rm pure}$ carries all the gauge degrees of freedom and hence has no physical consequence. The other part is called "physical" because it is invariant under gauge transformations. In their choice, they demand $\vec{A}_{\rm phys}$ to satisfy the divergence-free condition

$$\vec{\nabla}\cdot\vec{A}_{\rm phys} = 0 \ , \tag{35}$$

whereas $\vec{A}_{\rm pure}$ to satisfy the curl-free condition

$$\vec{\nabla}\times\vec{A}_{\rm pure} = 0 \ . \tag{36}$$

Under gauge transformation, one has

$$\vec{A}_{\rm pure} \to \vec{A}'_{\rm pure} = \vec{A}_{\rm pure} + \nabla\chi; \quad \vec{A}_{\rm phys} \to \vec{A}'_{\rm phys} = \vec{A}_{\rm phys} \ . \tag{37}$$

Therefore, one can construct gauge-invariant observables using $\vec{A}_{\rm phys}$, including for example, the gluon spin density, $\vec{E}\times\vec{A}_{\rm phys}$.

It is easy to see that this proposal is the same as the GIE of the Coulomb gauge. If one is to make a calculation, the simplest way is to choose,

$$\vec{A}_{\rm pure} = \vec{\nabla}\frac{1}{\nabla^2}\vec{\nabla}\cdot\vec{A} = 0 \ . \tag{38}$$

which is the same as the Coulomb gauge condition, $\vec{\nabla}\cdot\vec{A} = 0$. Then, $\vec{A}_{\rm phys}$ is the physical part of $A^\mu$ in the Coulomb gauge which also satisfies the condition $\vec{\nabla}\cdot\vec{A}_{\rm phys} = 0$. The QED lagrangian contains only $\vec{A}_{\rm phys}$, and all calculations proceed in the exactly the same way as in the Coulomb gauge, and all results are naturally the same as those in that gauge. In particular, the matrix element $\vec{E}\times\vec{A}_{\rm phys}$ is the same as $\vec{E}\times\vec{A}$ calculated in Coulomb gauge.

The procedure that Chen et al. proposed is to ensure calculations in any gauge involving $\vec{A}_{\rm phys} = \vec{A} - \vec{\nabla}\frac{1}{\vec{\nabla}^2}\vec{\nabla}\cdot\vec{A}$ yield the same result, as that in the Coulomb gauge. This is the significance of "gauge invariance" associated with the matrix element such as $\vec{E}\times\vec{A}_{\rm phys}$. In fact, the propagator $\langle 0|TA^\mu_{\rm phys}A^\nu_{\rm phys}|0\rangle$, considered as gauge-invariant and physical, computed in any other gauge will yield the Coulomb gauge result,

$$\langle 0|TA^\mu_{\rm phys}A^\nu_{\rm phys}|0\rangle = \frac{i}{k^2+i\epsilon}\left(-g^{\mu\nu} - \frac{k^\mu k^\nu}{((k\cdot\eta)^2-k^2)} + \frac{(k\cdot\eta)(k^\mu\eta^\nu + k^\nu\eta^\mu)}{((k\cdot\eta)^2-k^2)}\right) \ , \tag{39}$$

where $\eta^\mu$ is a temporal vector $\eta = (1, 0, 0, 0)$. This confirms the conclusion that Chen et al.'s approach is the same as GIE of the Coulomb gauge.

However, the above approach is not really gauge-invariant in the textbook sense [28, 29]. While the Coulomb gauge has been very useful for atomic physics, there is no natural place for it in QCD.

### C. Gauge-invariance of the gluon spin operator?

Following the work by Chen et al., Wakamatsu [8] proposed his gauge- and frame-independent decomposition of the nucleon spin, and showed that the evolution of the gluon spin in this decomposition is gauge invariant [9]. What actually Wakamatsu did was instead of making GIE of the Coulomb gauge, he made GIE of the light-cone gauge. Therefore, his anomalous dimension result has to coincide with the light-cone gauge calculation of the gluon spin operator, and this is exactly what he found. Thus his calculation does not at all support that the gluon spin is gauge invariant.

The first step in Wakamatsu's work is also to decompose the gauge field $A^\mu$ into $A^\mu_{\rm pure}$ and $A^\mu_{\rm phys}$. In the paper [9], $A^\mu_{\rm phys}$ is defined by imposing the light-cone condition $n \cdot A_{phys} = 0$, where $n^2 = 0$. The propagator $\langle 0|T[A^\mu_{\rm phys} A^\nu_{\rm phys}]|0\rangle$ computed in any gauge just yields the light-cone result:

$$\langle 0|T[A^{a,\mu}_{\rm phys}(x) A^{b,\nu}_{\rm phys}(y)]|0\rangle \;=\; \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot(x-y)} \left(\frac{-i\delta^{ab}}{k^2 + i\epsilon}\right) \left(g^{\mu\nu} - \frac{n^\mu k^\nu + n^\nu k^\mu}{n\cdot k}\right) \tag{40}$$

The gluon spin operator is constructed from $A^\mu_{\rm phys}$:

$$M^{+12}_{\rm g-spin} \;=\; 2{\rm Tr}[F^{+1} A^2_{\rm phys} - F^{+2} A^1_{\rm phys}] \; . \tag{41}$$

This of course is the same spin operator as in the light-cone gauge, which can be make gauge invariant by the procedure discussed in the previous section. Once this operator is generalized to the other gauges, Wakamatsu calculated the one-loop anomalous dimension in Feynman gauge, which turned out to be the same as that obtained within the light-cone gauge by Hoodbhoy et. al [17]. This result is hardly novel from the GIE point of view.

The GIE of the gluon spin operator from the light-cone gauge is realized by choosing

$$A^\mu_{\rm phys}(\xi) \;=\; \int^{\xi^-} d\eta^- \; L_{[\xi^-,\eta^-]} \; F^{+\mu}(\eta^-, \xi_\perp) \; L_{[\eta^-,\xi^-]} \; , \tag{42}$$

where $L$ are light-cone gauge links. In this sense, the gluon spin operator constructed in Eq. (41) is gauge invariant, and its matrix elements in any other gauge should be just the same as they are in the light-cone gauge. This is also true for its anomalous dimension. We have verified our conclusion by explicit calculations in several choices of gauges. At the lowest order of $A^\mu_{\rm phys}$, we have calculated $\Delta\gamma^{(0)}_{Gq}$ at one-loop level in general covariant and non-covariant gauges, and all the additional integrals with respect to the Feynman gauge result canceled exactly among themselves. Therefore, Wakamatsu's result in Feynman gauge cannot be any thing other than that in the light-cone gauge.

### D. Photon spin and orbital angular momentum in atomic physics

It has often been claimed that the photon spin and orbital angular momenta in QED can be separately defined and measured, and therefore they must be separately gauge invariant. This has served as another important motivation to look for gauge-invariant definition of the gluon spin. In this subsection, we discuss the examples in optics, pointing out that this is possible only for optical modes with fixed frequency.

We first recall a bit of history about photon's angular momentum. For a circularly polarized plane wave, R. Beth [30] was the first one who measured the spin angular momentum of a circularly polarized light by measuring the torque exerted on the quartz wave plate it passed through. As we explained above, this is just the helicity and is gauge invariant in theory. In 1992, L. Allen et al. pointed out that Laguerre-Gaussian laser modes also have a well-defined orbital angular momentum [31]. Based on this, several experiments [32–34] have been set up to observe and measure the orbital angular momentum of a Laguerre-Gaussian photon.

For radiation field with $e^{-i\omega t}$ time-dependence, using the Maxwell equation

$$\vec{B} = -\frac{i}{\omega}\nabla\times\vec{E} \ , \tag{43}$$

one always has the gauge invariant decomposition

$$\begin{aligned}\vec{J} &= \frac{1}{2}\int d^3\xi \ [\vec{\xi}\times(\vec{E}^*\times\vec{B}+\vec{E}\times\vec{B}^*)] \\ &= -\frac{i\epsilon_0}{\omega}\int d^3\xi[\vec{E}_i^*(\vec{\xi}\times\nabla)\vec{E}_i+\vec{E}^*\times\vec{E}] \ .\end{aligned} \tag{44}$$

The two terms may be identified as the orbital and spin angular momentum respectively and are separately gauge invariant. In particular, this is true for photon electric and magnetic multipoles which are often used in transitions between atomic or nuclear states. The photon orbital angular momentum can be defined without referencing to the gauge potential at all [22].

It can be easily checked from the above equation that the spin equals $\mp 1$ for left-handed or right-handed circularly polarized light, and 0 for linearly polarized light. In paraxial approximation, a Laguerre-Gaussian mode with azimuthal angular dependence of $\exp(il\phi)$ is an eigen-mode of the operator $L_z = -i\partial/\partial\phi$, and carries orbital angular momentum $l\hbar$. It is remarkable that the experimenters are able to find way to detect the effects of the orbital angular momentum alone [32–34].

Clearly, the above procedure only applies for a specific type of radiation field. In the case of the QCD, the gluons in the nucleon cannot be said to be this type. In particular, they are off-shell and do not satisfy the on-shell equations of motion. The best one can do is to go the IMF where the gluons appear as on-shell radiation, then the gluon helicity and orbital angular momentum could be defined and measured "naturally" in the light-cone gauge. Thus we are back to the discussions made in the previous section.

## V. CONCLUSION

This paper examines the constraints of gauge invariance on physical observables in a gauge theory. We advocate the standard textbook view that the naturally-gauge-invariant (local) operators must be manifestly invariant under gauge transformations. Thus, the spin of gauge particles, canonical momentum, and canonical OAM are not strictly-speaking gauge-invariant. Only in special context, it makes sense to talk about them. These special cases arises 1) in non-relativistic quantum systems such as atoms where the Coulomb gauge is a natural choice, 2) in radiation fields where the time variation is in harmonic oscillation (such as the electromagnetic multipoles and Laguerre-Gaussian laser modes), and 3) in high-energy hadron scattering in which the gauge partons appear to have two physical polarizations and be on-shell. In the late case, the simple physics in the IMF and light-cone gauge cannot be extended to that of the bound state because the same quantity does not have a simple gauge-invariant interpretation in the rest frame of the nucleon. In fact, there is no gauge-invariant notion of the gluon spin contribution to the spin of the nucleon. The IMF frame result can only be translated into the physics of bound state only in light-cone gauge.

Of course, high-energy scattering has provided great tools to learn about parton physics. Thus, it is possible to probe, for example, the canonical momentum and OAM of the partons in the IMF and light-cone gauge [25, 26]. However, they cannot be taken as gauge-invariant properties of bound states, contrary to many assertions made in the literature so far. Also many of the TMD factorization theorems are practically model assumptions which must be taken with caution [14].

Thus, the efforts of trying to generalize the gauge symmetry from a fixed gauge calculation is of limited use. In particular, generalizing the Coulomb gauge results as “gauge-invariant” in the case of QCD is neither physically useful nor experimentally relevant [28, 29].

This work was partially supported by the U. S. Department of Energy via grants DE-FG02-93ER-40762 and a grant (No. 11DZ2260700) from the Office of Science and Technology in Shanghai Municipal Government.

---

[1] R. P. Feynman, Phys. Rev. Lett. **23**, 1415 (1969).
[2] G. F. Sterman, “An Introduction to quantum field theory,” Cambridge, UK: Univ. Pr. (1993).
[3] R. K. Ellis, W. Furmanski and R. Petronzio, Nucl. Phys. B **207**, 1 (1982).
[4] R. K. Ellis, W. Furmanski and R. Petronzio, Nucl. Phys. B **212**, 29 (1983).
[5] A. V. Manohar, Phys. Rev. Lett. **66**, 289 (1991).
[6] X. -S. Chen, X. -F. Lu, W. -M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. **100**, 232002 (2008) [arXiv:0806.3166 [hep-ph]].
[7] X. -S. Chen, W. -M. Sun, X. -F. Lu, F. Wang and T. Goldman, Phys. Rev. Lett. **103**, 062001 (2009) [arXiv:0904.0321 [hep-ph]].
[8] M. Wakamatsu, Phys. Rev. D **83**, 014012 (2011) [arXiv:1007.5355 [hep-ph]].
[9] M. Wakamatsu, Phys. Rev. D **84**, 037501 (2011) [arXiv:1104.1465 [hep-ph]].
[10] Y. Hatta, Phys. Rev. D **84**, 041701 (2011) [arXiv:1101.5989 [hep-ph]].
[11] E. Leader, Phys. Rev. D **83**, 096012 (2011) [Erratum-ibid. D **85**, 039905 (2012)] [arXiv:1101.5956 [hep-ph]].
[12] B. -H. Zhou and Y. -C. Huang, Phys. Rev. D **84**, 047701 (2011).
[13] See, for example, J. C. Collins, D. E. Soper and G. F. Sterman, Nucl. Phys. B **250**, 199 (1985); X. -d. Ji, J. -p. Ma and F. Yuan, Phys. Rev. D **71**, 034005 (2005) [hep-ph/0404183].
[14] See, example, H. -n. Li and G. F. Sterman, Nucl. Phys. B **381**, 129 (1992); F. Feng, J. P. Ma and Q. Wang, Phys. Lett. B **674**, 176 (2009) [arXiv:0807.0296 [hep-ph]].

[15] R. P. Feynman, R. B. Leighton, M. Sands, "Feynman Lectures on Physics," Vol. III, Addison Wesley Longman (January 1970).

[16] G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995) [Erratum-ibid. D **55**, 5853 (1997)] [hep-ph/9407339].

[17] P. Hoodbhoy and X. -D. Ji, Phys. Rev. D **60**, 114042 (1999) [hep-ph/9908275].

[18] J. C. Collins, "Renormalization. An Introduction To Renormalization, The Renormalization Group, And The Operator Product Expansion," Cambridge, Uk: Univ. Pr. ( 1984).

[19] X. -D. Ji, Phys. Rev. D **52**, 271 (1995) [hep-ph/9502213].

[20] X. Ji, Phys. Rev. D **58**, 056003 (1998).

[21] V. B. Berestetskii, L. P. Pitaevskii, and E.M. Lifshitz, "Quantum Electrodynamics," Second Edition: Volume 4 in Courses in Theoretical Physics, Butterworth-Heineman, 1982.

[22] J. D. Jackson, "Classical Electrodynamics," Third Edition, John Wiley & Sons, Inc (1999).

[23] R. L. Jaffe and A. Manohar, Nucl. Phys. B **337**, 509 (1990).

[24] S. Bashinsky and R. L. Jaffe, Nucl. Phys. B **536**, 303 (1998); P. Hagler and A. Schafer, Phys. Lett. B **430**, 179 (1998).

[25] X. Ji, X. Xiong and F. Yuan, arXiv:1202.2843 [hep-ph].

[26] Y. Hatta, arXiv:1111.3547 [hep-ph].

[27] P. Hoodbhoy, X. Ji and W. Lu, Phys. Rev. D **59**, 014013 (1999).

[28] X. Ji, Phys. Rev. Lett. **104**, 039101 (2010).

[29] X. Ji, Phys. Rev. Lett. **106**, 259101 (2011) [arXiv:0910.5022 [hep-ph]].

[30] R. A. Beth, Phys. Rev. **50**, 115(1936).

[31] L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, Phys. Rev. A **45**, 8185(1992).

[32] A. T. O'Neil, I. MacVicar, L. Allen, and M. J. Padgett, Phys. Rev. Lett. ,**88**, 053601(2002).

[33] M. F. Anderson, C. Ryu, Pierre Cladé, Vasant Natarjan, A. Vaziri, K. Helmerson, and W. D. Phillips, Phys. Rev. Lett. **97**, 170406(2006).

[34] J. Leach, M. J. Padgett, S. M. Barnett, S. Franke-Arnold, and J. Courtial, Phys. Rev. Lett. **88**, 257901(2002).