Comments on the Draft "Gauge Symmetry, Gluon Spin and Canonical Momentum", and on some of the discussion thereof

by **Ji, Xu and Zhao**

Elliot Leader: 22nd May 2012

In general I find some parts of this draft very enlightening. However some of the statements require modification, in my opinion.

II Section A: It is stated that my proof of the gauge invariance of the matrix elements of the canonical momentum "must be incorrect" and it is suggested that this is because I have not taken into account the change in the physical states. This argument is not correct, as can be seen from a more familiar example.
Suppose we wish to show that S-matrix elements are Lorentz invariant. Let $P = \Lambda p$, $P' = \Lambda p'$. We wish to show that

$$\langle P' \,|\, S \,|\, P \rangle = \langle p' \,|\, S \,|\, p \rangle \tag{1}$$

Now

$$|\, P \rangle = U(\Lambda) |\, p \rangle \qquad \text{etc} \tag{2}$$

so that we have to show that

$$\langle p' \,|\, U^\dagger \, S \, U |\, p \rangle = \langle p' \,|\, S \,|\, p \rangle \tag{3}$$

We can do this in two ways:

(1) Show that

$$U^\dagger \, S \, U = S \tag{4}$$

or

(2) If

$$U^\dagger \, S \, U = S + \delta \, S \tag{5}$$

show that

$$\langle p' \,|\, \delta \, S \,|\, p \rangle = 0. \tag{6}$$

My proof followed exactly this second method.

So why the claim that my result must be incorrect? That comes from a calculation by Hoodbhoy, Ji and Lu showing that the matrix elements are different in covariant and axial gauge. The resolution to this lies in the traditional careless confusion of gauge transformations in classical and quantum theories. Namely I considered c-number gauge transformations and these cannot transform a **quantum** theory from covariant to axial gauge. I don't know the answer for the axial gauge, but for the case of the Coulomb gauge and covariant gauges it is known that one can find an operator which connects these gauges in the , **quantized theory**, but it is **NOT** a gauge transformation. I am grateful to Wakamatsu for drawing my attention to this result in the textbook "Photons and Atoms" by Cohen-Tannoudji, Dupont-Roc and Grynberg.

I am writing a short paper to try to clarify this issue, which is rarely or never explained in textbooks on Field Theory.

II Section C: Eqs. (16) and (17) are misleading, since (17) is automatically satisfied if

$$O' = GOG^{-1}. \tag{7}$$

The example of Lorentz invariance above is the correct analogue for discussing gauge invariance.

Also the last line on page 5 contradicts the assertion in the line 7 above it, beginning "Of course..."

III Section A: After Eq. (23) it is really the gluon **helicity**that is given by (23).

IV Section C:

The Ji-Wakamatsu discussion

I think that Ji's reply to Wakamatsu i.e. that Wakamatsu has introduced additional degrees of freedom by introducing physical and pure fields, is a little misleading, since that is only because Wakamatsu has not specified how to obtain his physical and pure fields from A_μ. Hatta, for example, gives explicit expressions for his physical and pure fields in terms of the original A_μ, so no new degrees of freedom are introduced.
So I don't think that is a problem. But, of course, you are now defining **densities**, which intuitively you would expect to be local quantities, in terms of non-local fields. This I do not like at all.

Also, it should be realized that once you give a precise expression for these fields you **lose frame independence**.

Gauge invariance of the gluon spin operator

I like very much the discussion about Chen et al and the Coulomb gauge. Could not the discussion here, of the axial gauge, be made much simpler by analogously choosing a gauge where $A_{pure} = 0$. Such a choice should not interfere with the condition $n \cdot A_{phys} = 0$ since the transformation simply mixes up the colour labels on A_{phys} and does not affect the vector components?