Hadron Structure

James Zanotti
The University of Adelaide

Lattice Summer School, August 6 - 24, 2012, INT, Seattle, USA

Lecture 5

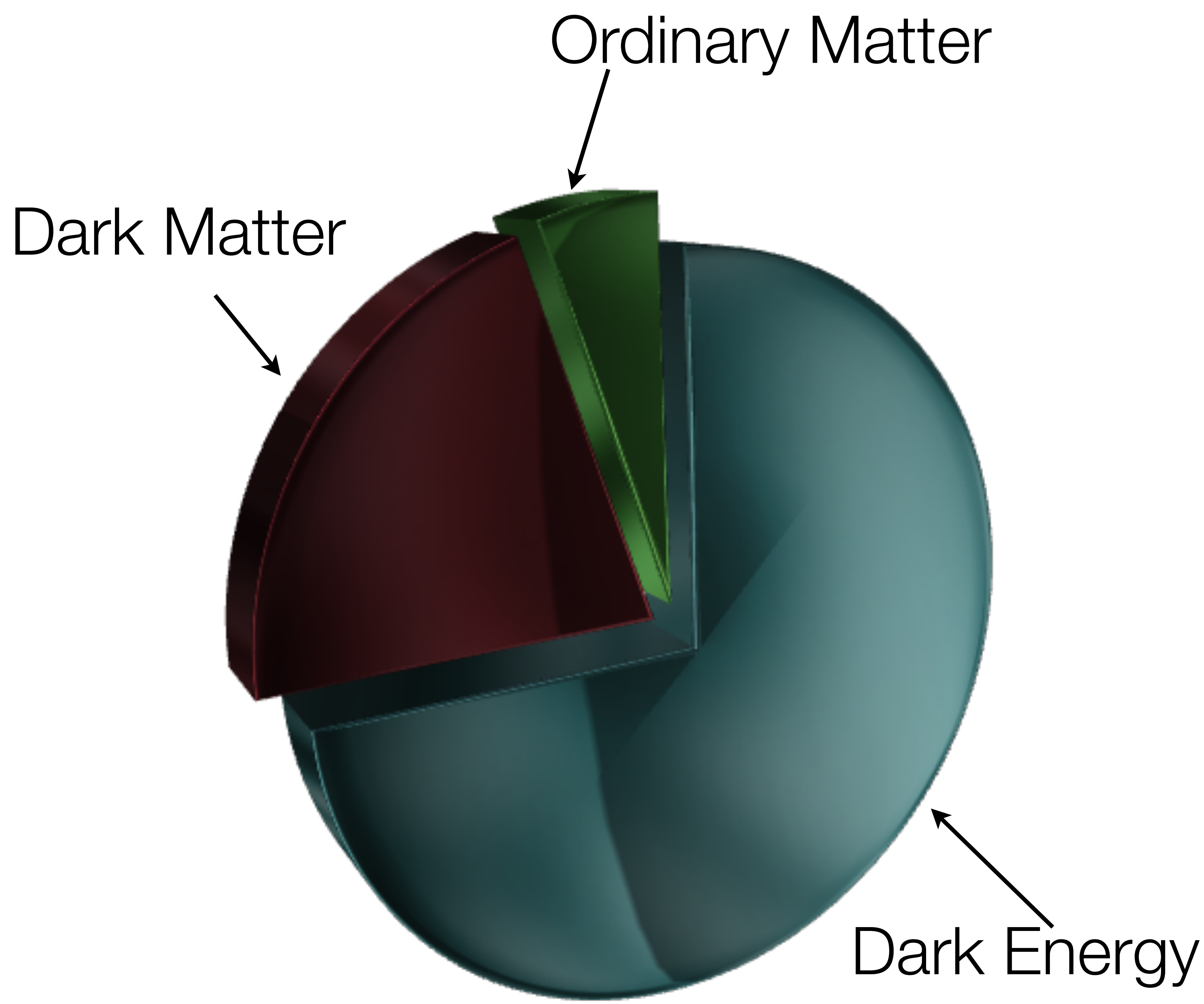
- Hidden flavour - Strangeness in the nucleon
  - Strange quark contribution to nucleon mass (sigma term)
    - Feynman-Hellman
    - Matrix elements
    - Impact for Dark Matter searches
  - Other strangeness contributions
    - Spin
    - Charge and Magnetic form factors
- Semi-Leptonic decays of strange hadrons
  - CKM matrix element $|V_{us}|$

Strangeness Content of the Nucleon & Dark Matter Searches

(See plenary talk by R.Young at Lattice 2012)

Strangeness and Dark Matter

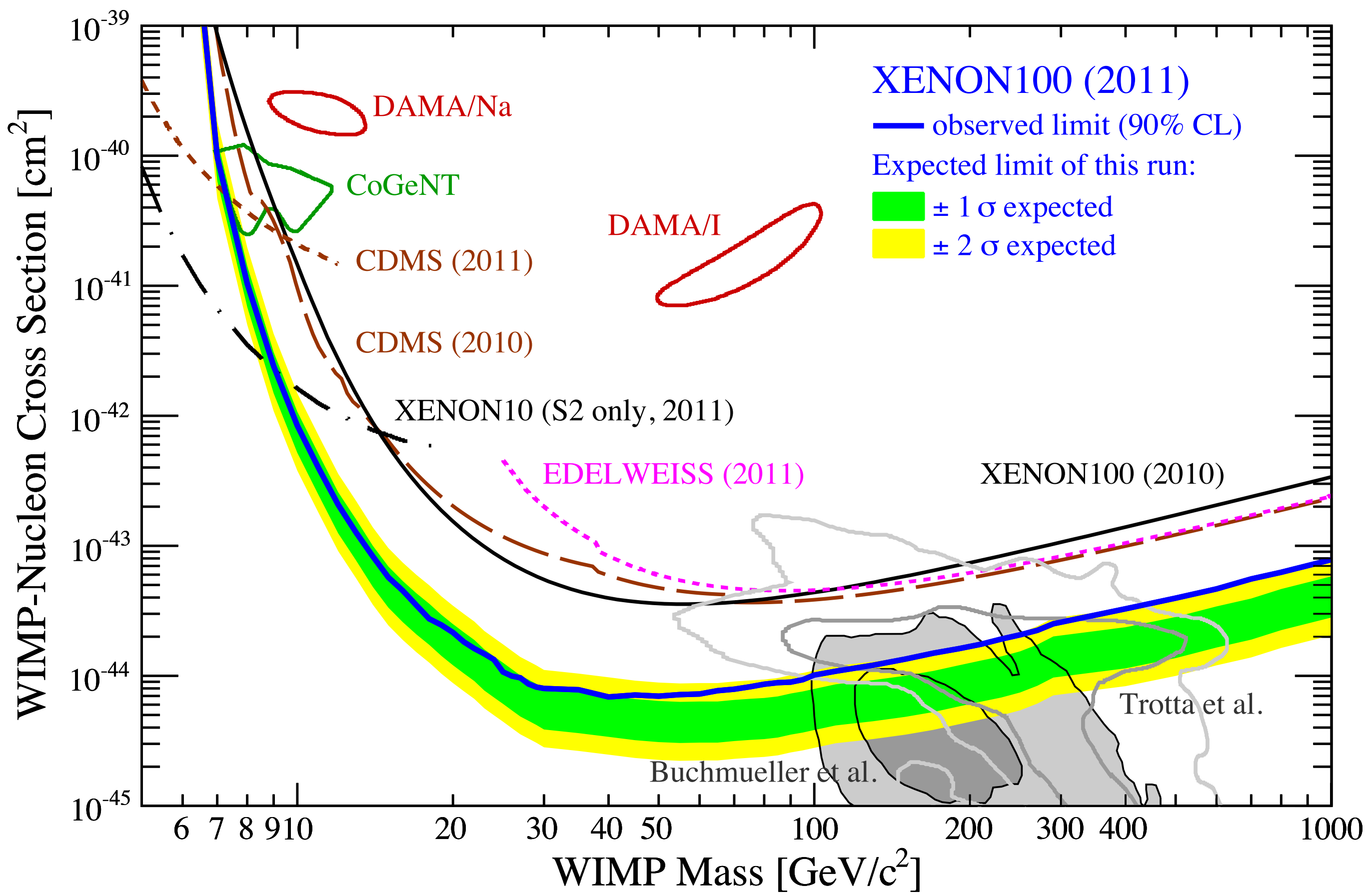
- We have no idea what makes up most of the mass of the universe
  - Strong evidence that Dark Matter is made up of weakly-interacting massive particles:"WIMPs"
  - An example candidate is provided if supersymmetry is not maximally broken in nature
  - Direct detection of such particles extremely challenging



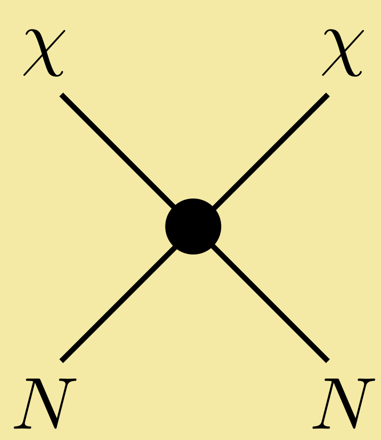
Direct Detection

- Giant underground detectors + a lot of patience
- Cross sections are small, but how small?
- Direct experimental searches depend on WIMP-nucleon cross sections

[XENON100, PRL(2011)]



Spin-independent amplitude



$\sim \bar{\chi}\chi\bar{N}N$

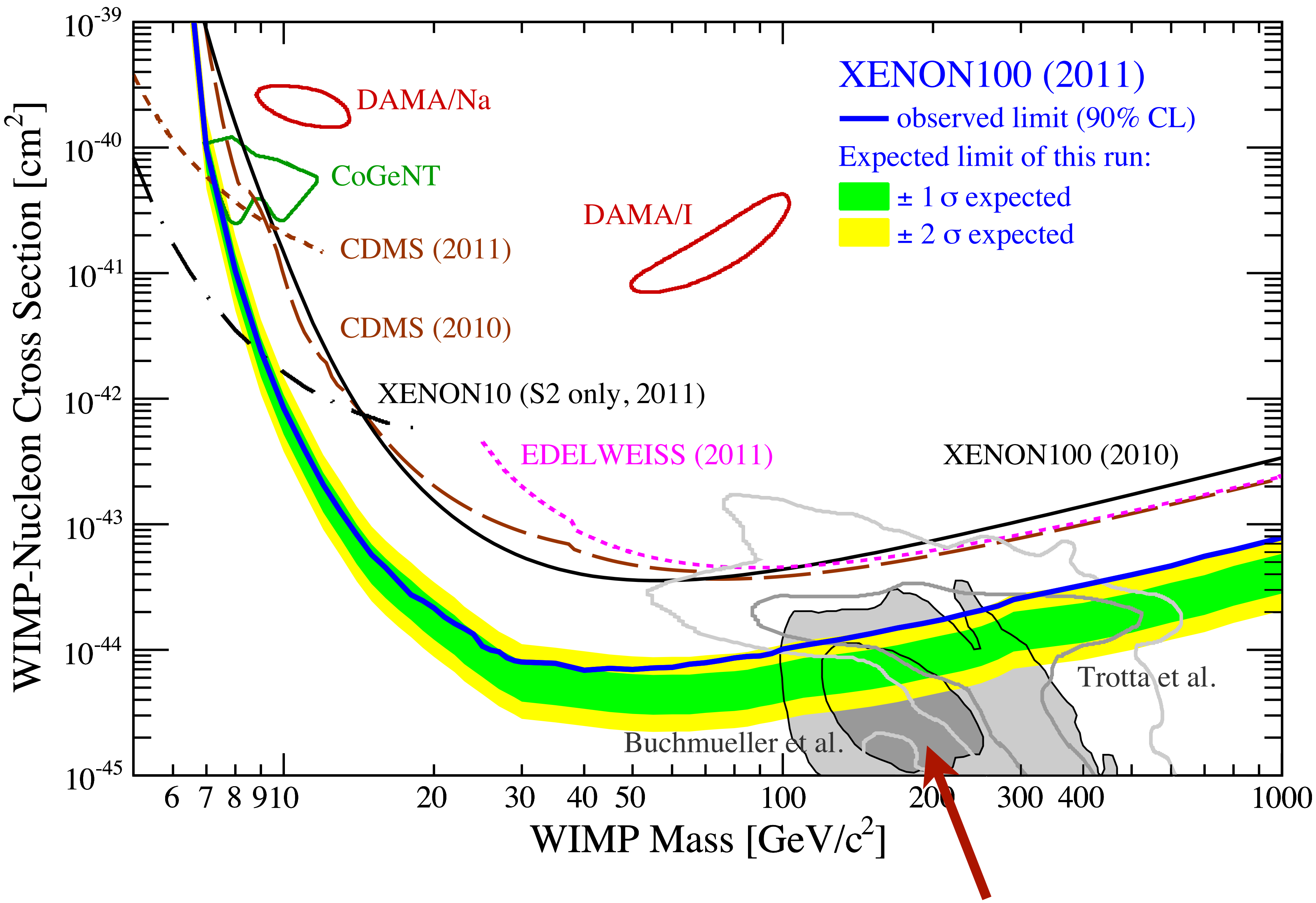
- Scattering amplitude

$$\mathcal{M} \sim \sum_q \underbrace{C_q \langle N | m_q \bar{q} q | N \rangle}$$

Nucleon “sigma” terms

Direct Detection

[XENON100, PRL(2011)]



Expected cross sections for neutralino in CMSSM

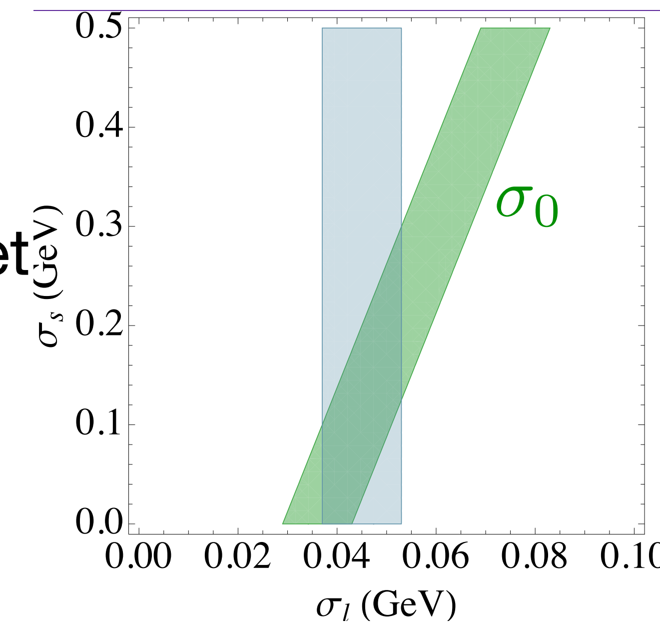
Significant uncertainty coming from nucleon "sigma" terms

$$\sigma_q = m_q \langle N | \bar{q} q | N \rangle$$

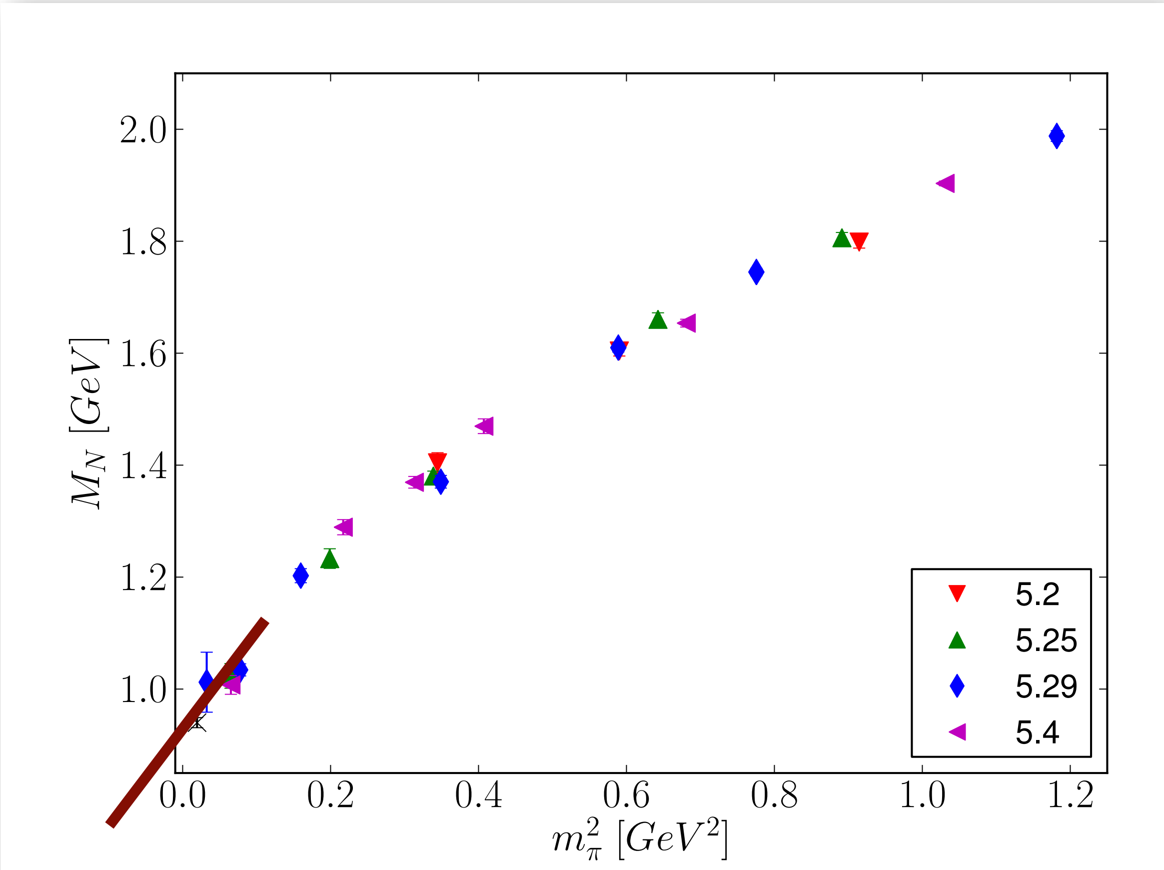
Strangeness and Dark Matter

- $\Sigma_{\pi N} = \sigma_{u+d}$ can be obtained from πN scattering
- σ_s then determined from $\Sigma_{\pi N}$ and an estimate for the non-singlet

$$\sigma_0 \equiv m_l \langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle$$



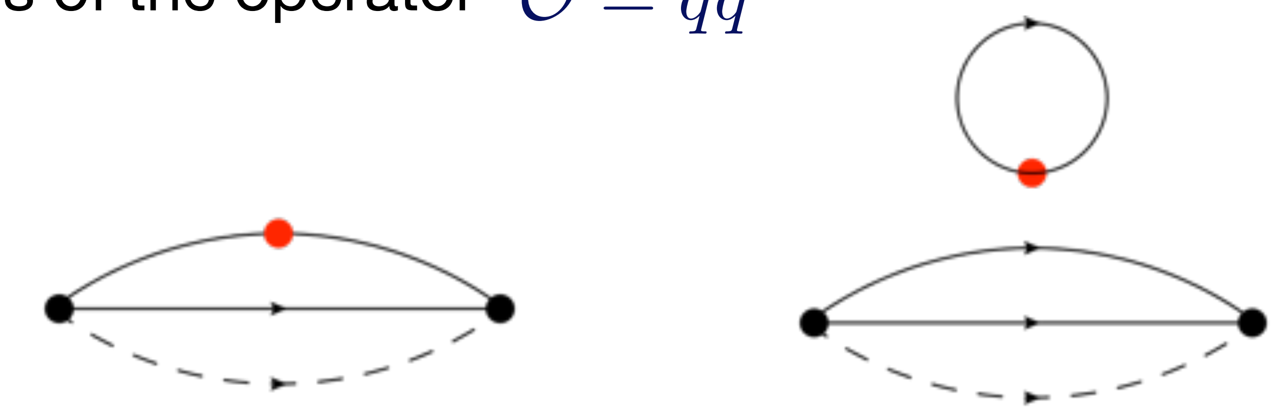
- ⇒ poorly determined $\sigma_s \approx 300 \pm 300$ MeV
- Even with perfect $\Sigma_{\pi N}$ $\Delta\sigma_s = \frac{m_s}{2m_l}\Delta\sigma_0 \sim 90$ MeV
- ⇒ Lattice QCD
- Two options:
  - Determine $\langle N|\bar{q}q|N\rangle$ directly
  - Feynman-Hellmann $\sigma_q = m_q \frac{\partial M_N}{\partial m_q}$



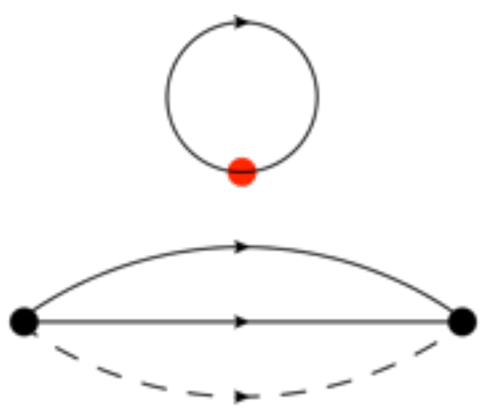
Sigma Terms on the Lattice

- Direct determination proceeds by our established methods of computing lattice three-point functions of the operator $\mathcal{O} = \bar{q}q$

u,d



s



- Disconnected diagram notoriously difficult

  - Scalar current couples to the vacuum ⇒ requires vacuum subtraction

$$R^{\mathrm{dis}}(t_f, t) = \langle \mathrm{Tr}(M^{-1}\mathbb{1})\rangle - \frac{\langle C_2(t_f)\mathrm{Tr}(M^{-1}\mathbb{1})\rangle}{\langle C_2(t_f)\rangle}$$

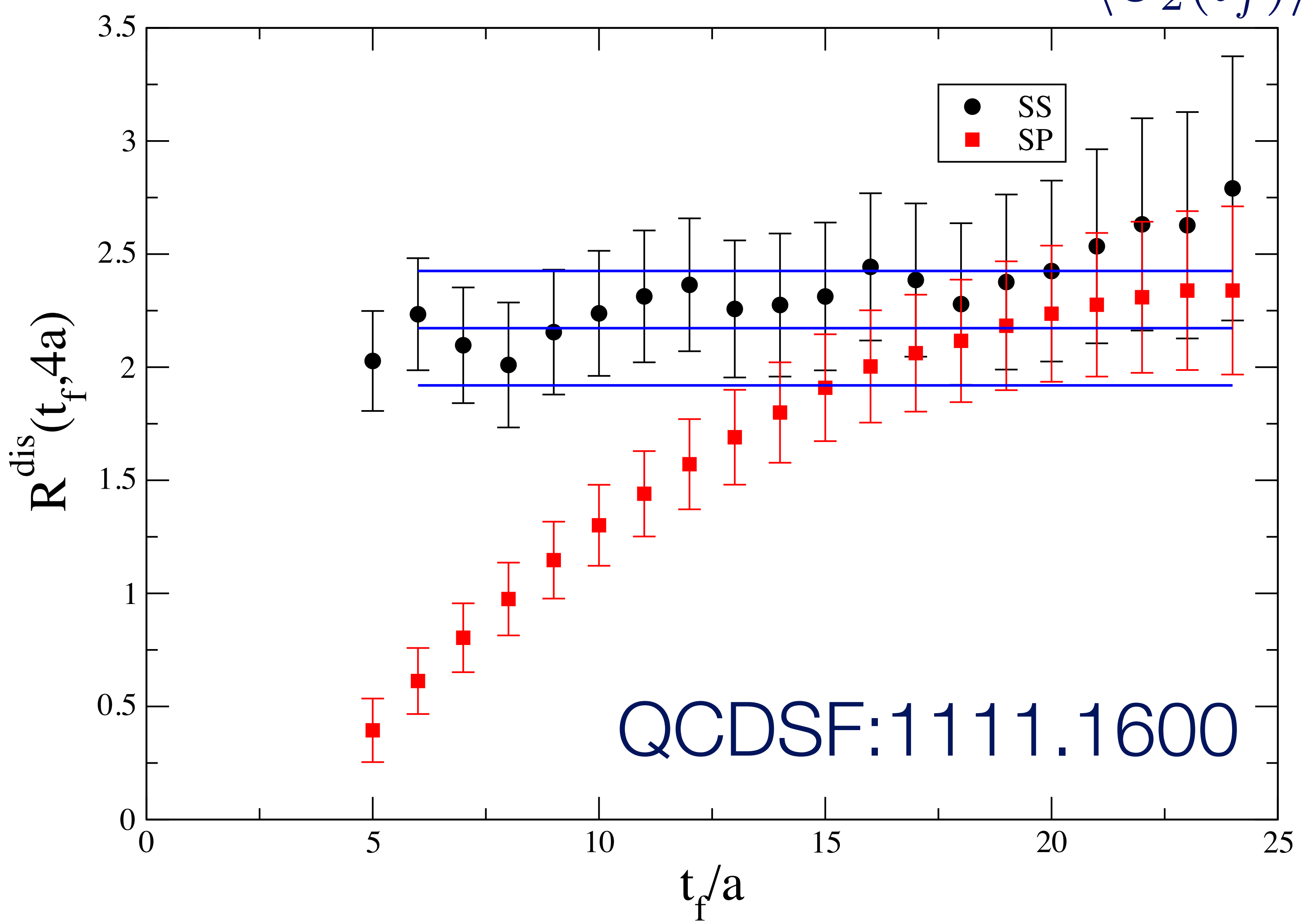
Sigma Terms on the Lattice

- However progress has been made in the computation of the required all-to-all propagators via stochastic noise sources. see e.g.

hep-lat/0505023
0910.3970
1204.0685

- For the disconnected strangeness sigma term

$$R^{\rm dis}(t_f,t) = \langle {\rm Tr}(M^{-1}\mathbb{1})\rangle - \frac{\langle C_2(t_f){\rm Tr}(M^{-1}\mathbb{1})\rangle}{\langle C_2(t_f)\rangle}$$



includes connected piece

$$\sigma_{\pi N} = 38 \pm 12 \text{ MeV}$$

$$\sigma_s = 12^{+23}_{-16} \text{ MeV}$$

Sigma Terms on the Lattice

- An alternative, and to date more popular, method is to use the Feynman-Hellmann relation

  - Differentiate the quark mass dependence

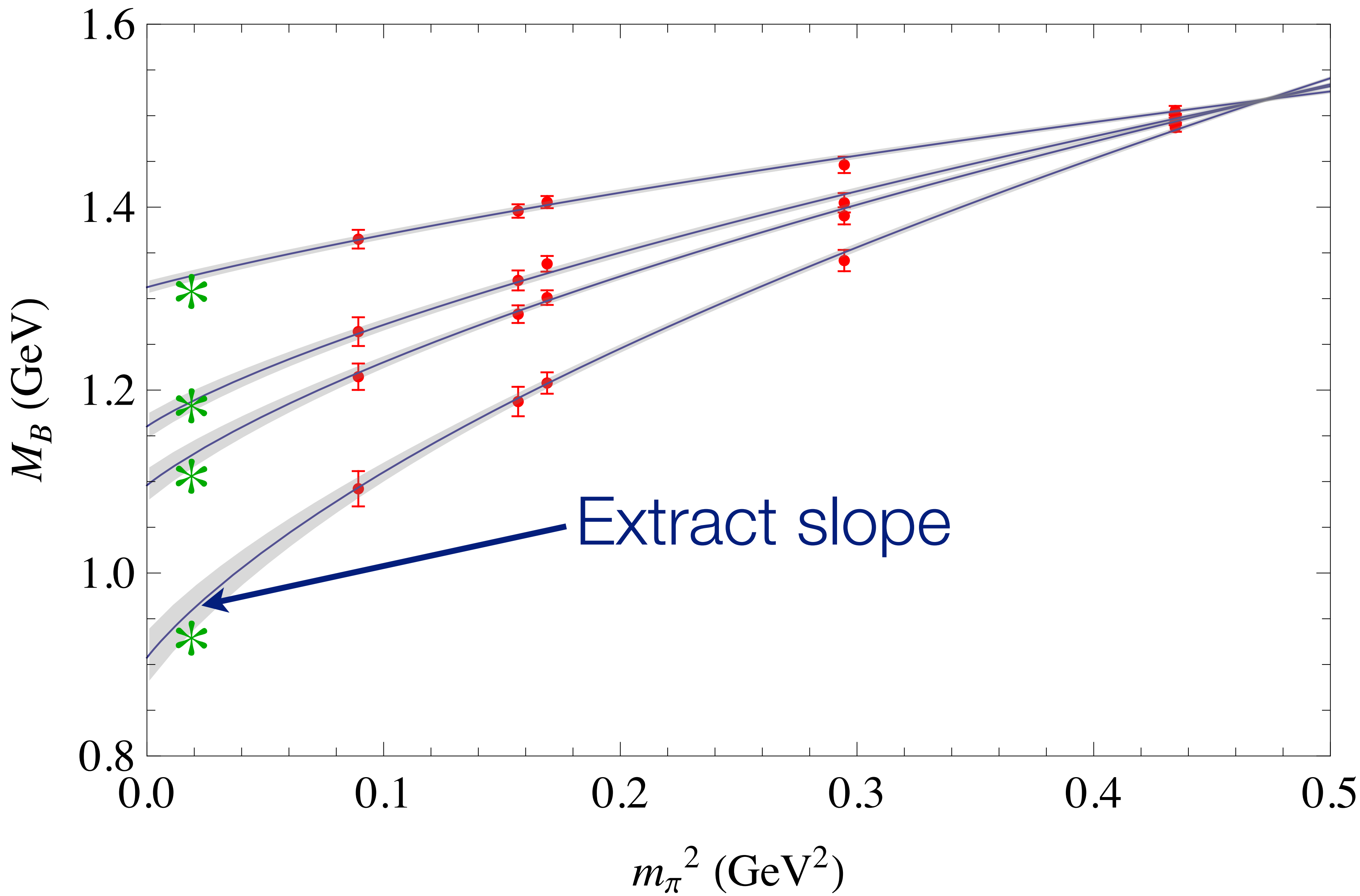
$$\sigma_q = \langle N | m_q \bar{q} q | N \rangle = m_q \frac{\partial M_N}{\partial m_q}$$

  - Requires substantial variation of both light and strange quark masses

  - Depends on the form used to fit the quark mass dependence of the baryon mass (Chiral Perturbation Theory)

Sigma Terms on the Lattice

- Example, Shanahan et al. [1205.5365] fit to PACS-CS data



$$\sigma_{\pi N} = 45 \pm 6 \text{ MeV}$$

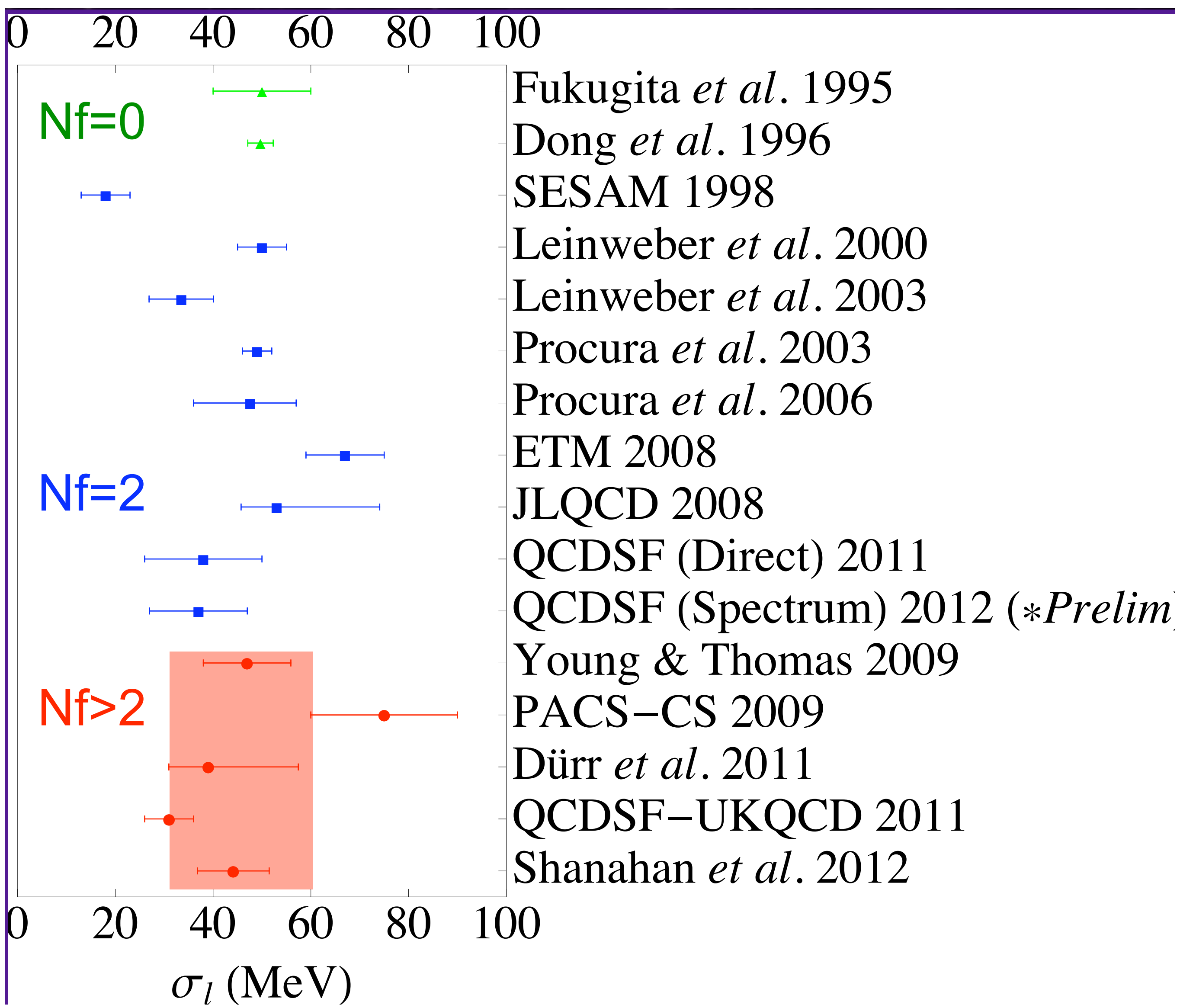
$$\sigma_s = 21 \pm 6 \text{ MeV}$$

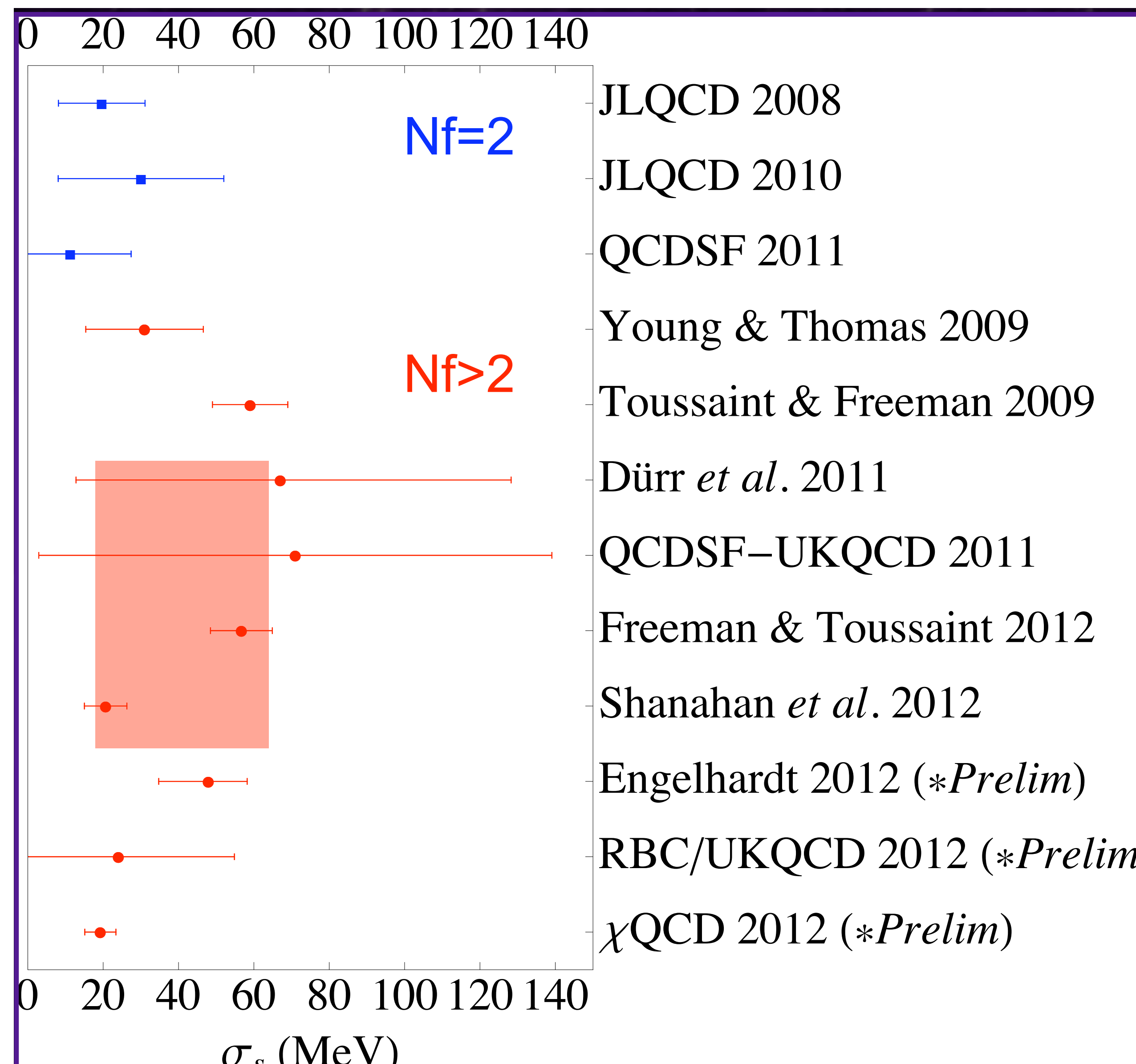
Strangeness and Dark Matter

(Plenary talk by R.Young at Lattice 2012)

Ross Young's Lattice Estimates:

$$30\ \text{MeV} \lesssim \sigma_l \lesssim 60\ \text{MeV} \qquad 20\ \text{MeV} \lesssim \sigma_s \lesssim 60\ \text{MeV}$$





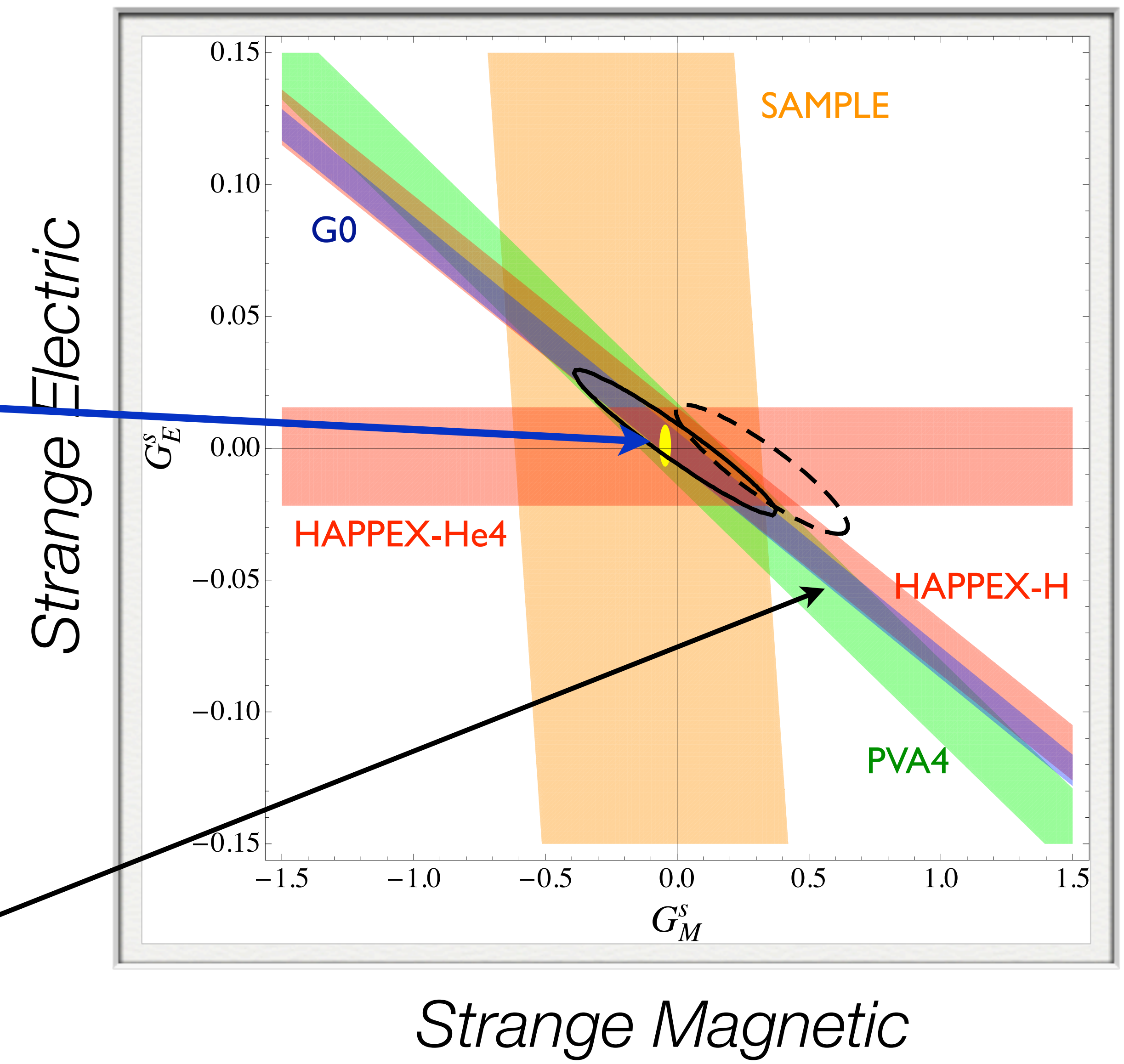
Dramatically improves cross section estimates

Strangeness in the Nucleon

- Other studies of strange quark contributions to nucleon structure

[Not an exhaustive list]

  - $\Delta s = -0.020(10)(4)$QCDSF: 1112.3354,
  - $\Delta s, \langle x \rangle_s, \ J_s$ $\quad \chi$QCD: 1203.6388
  - G_E^s, G_M^s CSSM: hep-lat/0406003, hep-lat/0601025



Latest JLab Experimental bounds

R. Young

Semi-Leptonic Strange Hadron Decays

$$K^+ \to \pi^0 l^+ \nu, \ \Xi^0 \to \Sigma^+ l^- \nu, \ \Sigma^- \to n\, l^- \nu$$

Matter-Antimatter Asymmetry

For every billion ordinary particles annihilating with antimatter in the early Universe, one extra was left "standing."

Matter-Antimatter Asymmetry

Why did matter dominate?

For every billion ordinary particles annihilating with antimatter in the early Universe, one extra was left "standing."

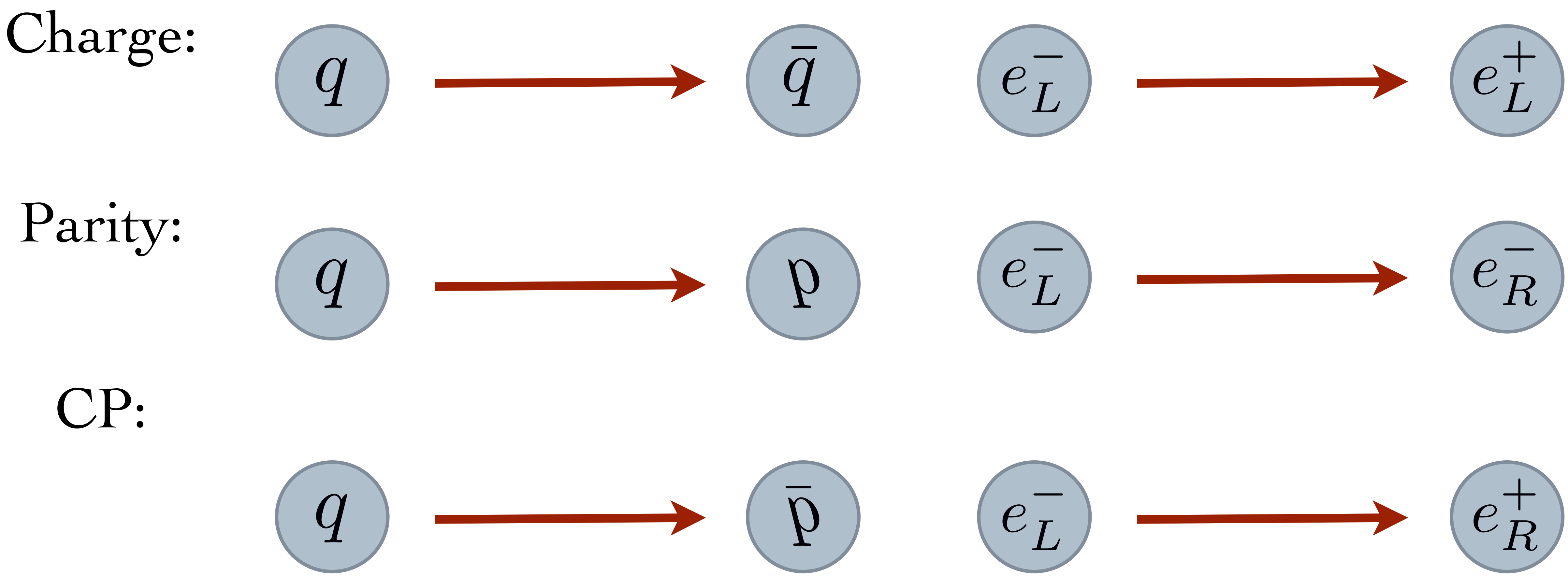
Matter-Antimatter Asymmetry

Why did matter dominate?

CP Violation?

For every billion ordinary particles annihilating with antimatter in the early Universe, one extra was left "standing."

CP Violation



- The Standard Model contains two ways to break CP symmetry
- In the QCD Lagrangian (strong) - not observed
- Via the weak force - observed, but can only account for a small portion of CP-violation

Cabibbo Kobayashi Maskawa Matrix

- Cabibbo (1963) proposed a theory of the weak current in terms of a single mixing angle θ_c to preserve universality of the weak interaction.
  - Explains the difference between the amplitudes of $\Delta S=0$ and $\Delta S=1$ transitions
- Led to a detailed description of semileptonic decays of mesons and baryons
- After the introduction of quarks (1964) the weak current is then written as

$$J_\alpha = \cos\theta_C \bar{u}\gamma_\alpha(1+\gamma_5)d + \sin\theta_C \bar{u}\gamma_\alpha(1+\gamma_5)s$$

- This interaction is described by a unitary 2x2 quark mixing matrix:

$$\begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix}$$

- Has only one free parameter: Cabibbo angle θ_c with $\tan\theta_c = V_{us}/V_{ud}$

CKM Matrix

- A 2x2 matrix can always be reduced to a form with real elements (no phase)
  - Couldn't accommodate experimentally observed CP violation in
    - Neutral Kaon decays (1964) [1980 Nobel Prize]
- Kobayashi & Maskawa (1973) proposed a third generation of quarks since a unitary 3x3 matrix has: [2008 Nobel Prize]
  - 3 real parameters (mixing angles)
  - 1 imaginary (CP-violating) parameter (phase)

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Discovery of b-quark (1976) led to a search for the t-quark later discovered at Fermilab (1995)

CKM Matrix

- The CKM matrix elements are fundamental parameters of the SM, so their precise determination is important for evaluating the solidity of the SM

- The most sensitive test of the unitarity of the CKM matrix is provided by the relation

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta$$

- An important goal of flavour physics is to over constrain the CKM elements

- Processes dominated by loop contributions in the SM are sensitive to new physics, and can be used to extract CKM elements only if the SM is assumed.

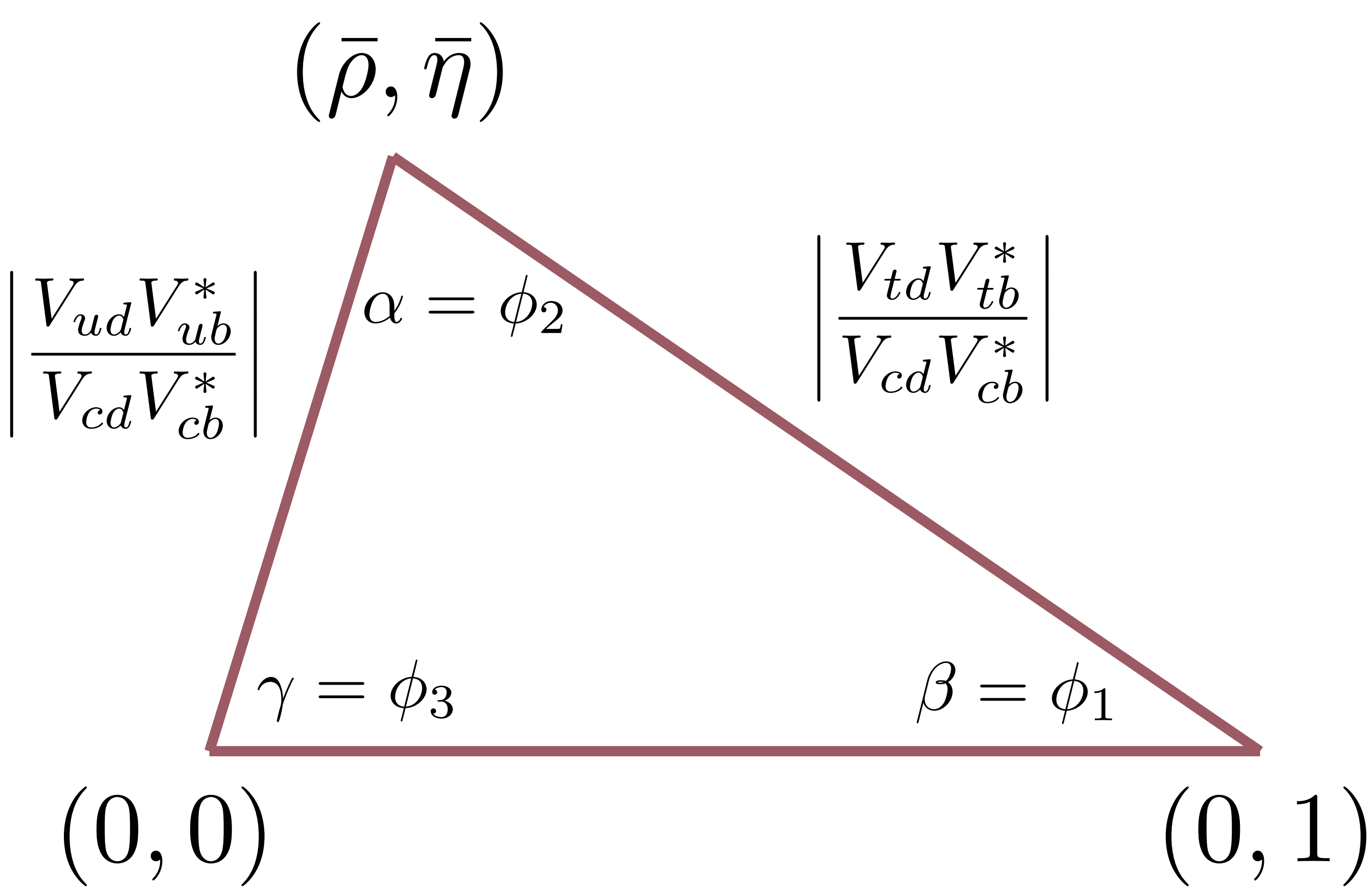
- Search deviations from unitarity $\Longleftrightarrow$ search for physics beyond the SM

Unitarity Triangle

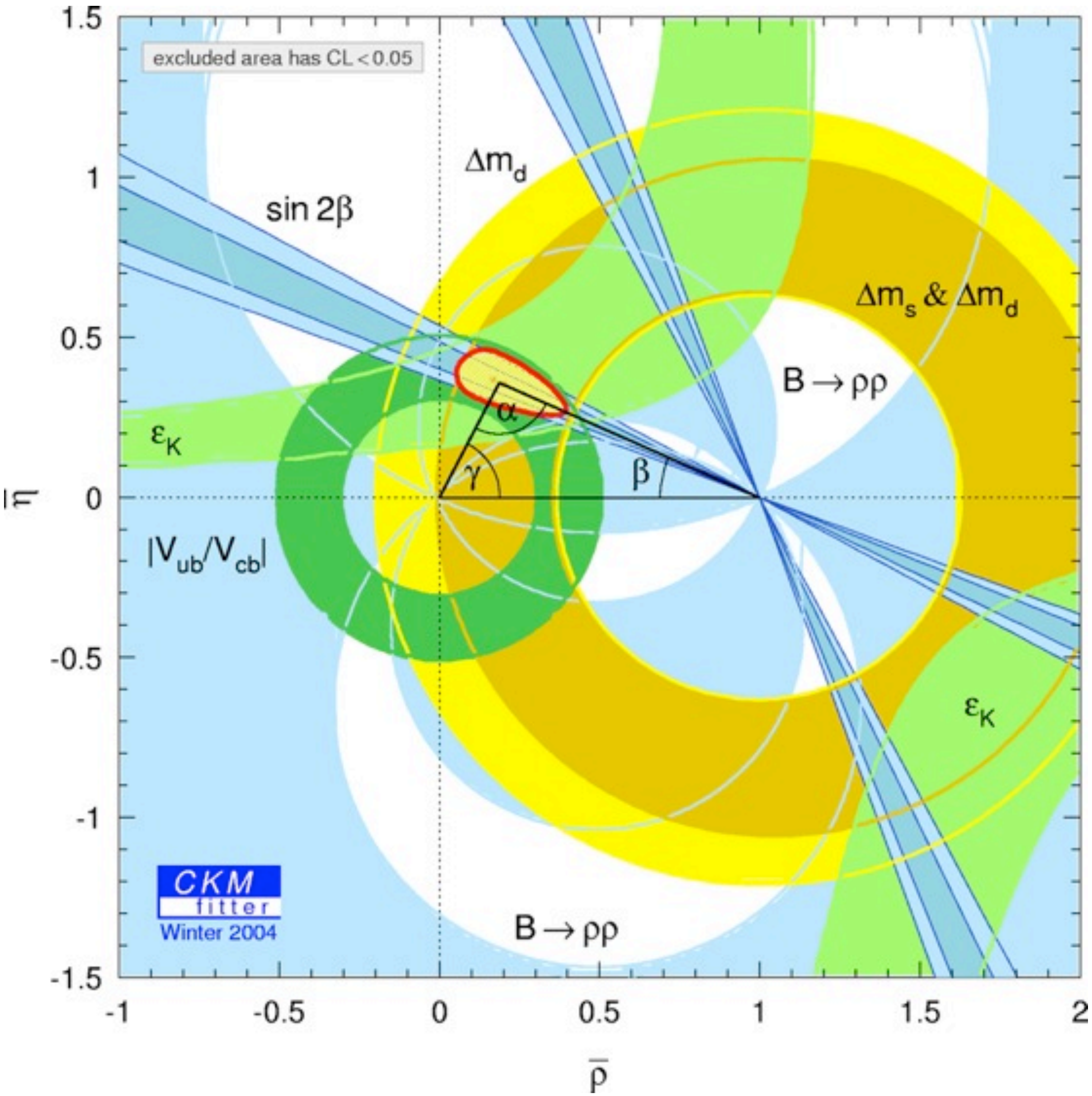
One common parameterisation (Wolfenstein):

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

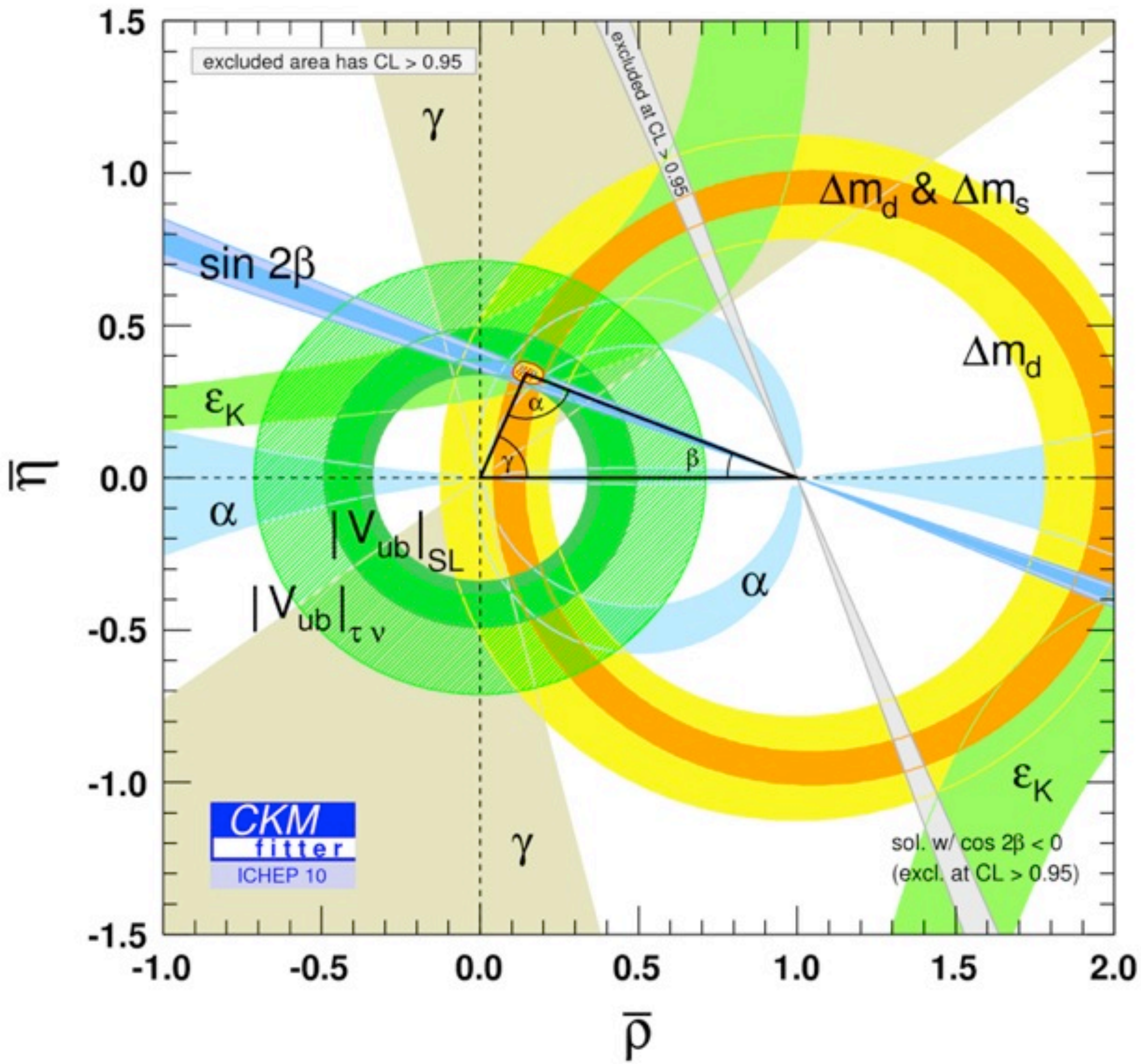
Unitarity: $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$



Unitarity Triangle



Unitarity Triangle



CKM Matrix

PDG (2010):

$$
\begin{aligned}
|V_{ud}| &= 0.97425(22) \\
|V_{us}| &= 0.2252(9) \\
|V_{ub}| &= 3.89(44) \times 10^{-3} \\
|V_{cd}| &= 0.230(11) \\
|V_{cs}| &= 1.023(36) \\
|V_{cb}| &= 40.6(1.3) \times 10^{-3} \\
|V_{td}| &= 8.4(6) \times 10^{-3} \\
|V_{ts}| &= 38.7(2.1) \times 10^{-3} \\
|V_{tb}| &= 0.88(7)
\end{aligned}
$$

CKM Matrix

Lattice Input

This talk

$|V_{us}|$

$K^+ \to \pi^0 l^+ \nu,\ f_\pi / f_K,\ \Xi^0 \to \Sigma^+ l^- \nu,\ \Sigma^- \to n\, l^- \nu$

$|V_{ub}|$

$B \to \pi l \nu$

$|V_{cd}|$

$D \to K l \nu,\ D \to \pi l \nu$

$|V_{cs}|$

$D \to K l \nu,\ D \to \pi l \nu,\ f_{D_s}$

$|V_{td}|$ & $|V_{ts}|$

$f_{B_d}\sqrt{\hat{B}_{B_d}},\ \xi = (f_{B_s}\sqrt{B_{B_s}})/(f_{B_d}\sqrt{B_{B_d}})$

K_{l3}

[Similar for $\Xi^0 \to \Sigma^+ l^- \nu,\ \Sigma^- \to n\, l^- \nu$]

- $K \longrightarrow \pi l \nu\ (K_{l3})$ decay leads to determination of $|V_{us}|$

$$\Gamma_{K\to\pi\ell\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} I S_{EW} [1 + 2\Delta_{SU(2)} + 2\Delta_{EM}] |V_{us}|^2 |f_+(0)|^2$$

- Require precise theoretical determination of $f_+(0)$

- Current conservation $\Longrightarrow$ $f_+(0) = 1\Big|_{su(3)\ flavour\ limit}$

- Ademollo-Gatto Theorem $\Longrightarrow$ second order SU(3) breaking effects in $f_+(0)$

$$f_+(0) = 1 + f_2 + f_4 + \cdots$$

$$\Rightarrow \Delta f = 1 + f_2 - f_+(0)$$

- [Leutwyler & Roos: $f_2 = -0.023$]

Motivation

- Until recently, standard result from Leutwyler & Roos (1984)

$$\Delta f = -0.016(8)$$

- Studied by several lattice groups
- Tension between lattice and ChPT communities
- Situation summarised by FlaviaNet [arXiv:1011.4408]

$$f_+^{K\pi}(0) = 0.956(8)$$

Collaboration	Ref.	N_f	publication status	chiral extrapolation	continuum extrapolation	finite volume errors	$f_+(0)$
RBC/UKQCD 10	[137]	2+1	A	●	■	★	$0.9599(34)(^{+31}_{-47})(14)$
RBC/UKQCD 07	[138]	2+1	A	●	■	★	0.9644(33)(34)(14)
ETM 09A	[139]	2	A	●	●	●	0.9560(57)(62)
QCDSF 07	[140]	2	C	■	■	★	$0.9647(15)_{stat}$
RBC 06	[141]	2	A	■	■	★	0.968(9)(6)
JLQCD 05	[142]	2	C	■	■	★	0.967(6), 0.952(6)

Table 5: Colour code for the data on $f_+(0)$.

Motivation

- Until recently, standard result from Leutwyler & Roos (1984) $\Delta f = -0.016(8)$
- Studied by several lattice groups
- Tension between lattice and ChPT communities
- Situation summarised by FlaviaNet [arXiv:1011.4408] $f_+^{K\pi}(0) = 0.956(8)$

Collaboration	Ref.	N_f	publication status	chiral extrapolation	continuum extrapolation	finite volume errors	$f_+(0)$
RBC/UKQCD 10	[137]	2+1	A	●	■	★	$0.9599(34)(^{+31}_{-47})(14)$
RBC/UKQCD 07	[138]	2+1	A	●	■	★	0.9644(33)(34)(14)
ETM 09A	[139]	2	A	●	●	●	0.9560(57)(62)
QCDSF 07	[140]	2	C	■	■	★	$0.9647(15)_{stat}$
RBC 06	[141]	2	A	■	■	★	0.968(9)(6)
JLQCD 05	[142]	2	C	■	■	★	0.967(6), 0.952(6)

Table 5: Colour code for the data on $f_+(0)$.

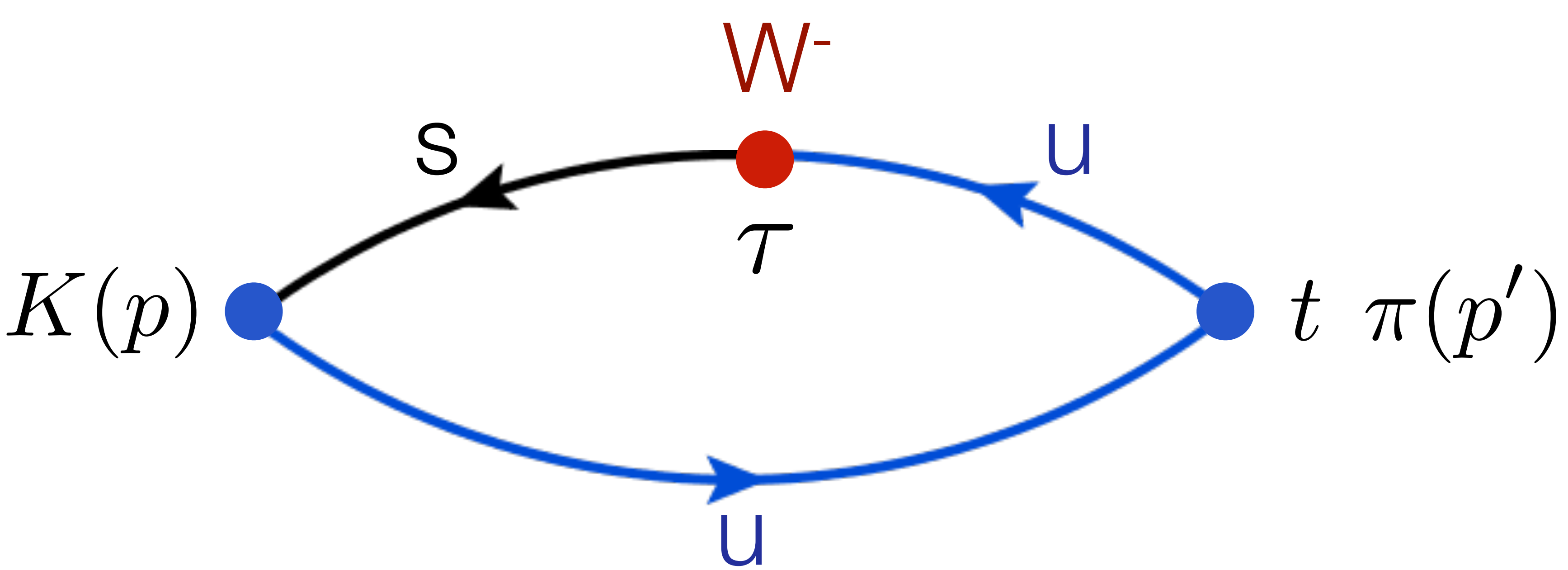
Lattice Techniques

- $K \longrightarrow \pi$ Matrix element

$$\langle \pi(p')|V_\mu|K(p)\rangle = (p_\mu + p'_\mu)f_+(q^2) + (p_\mu - p'_\mu)f_-(q^2), \quad q^2 = (p'-p)^2$$

- Three-point function

$$C_\mu^{PQ}(t', t, \vec{p}', \vec{p}) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}'(\vec{y}-\vec{x})} e^{-i\vec{p}\vec{x}} \langle 0|\mathcal{O}_Q(t')|Q(p')\rangle\langle Q(p')|V_\mu(t)|P(p)\rangle\langle P(p)|\mathcal{O}_P^\dagger(0)|0\rangle$$



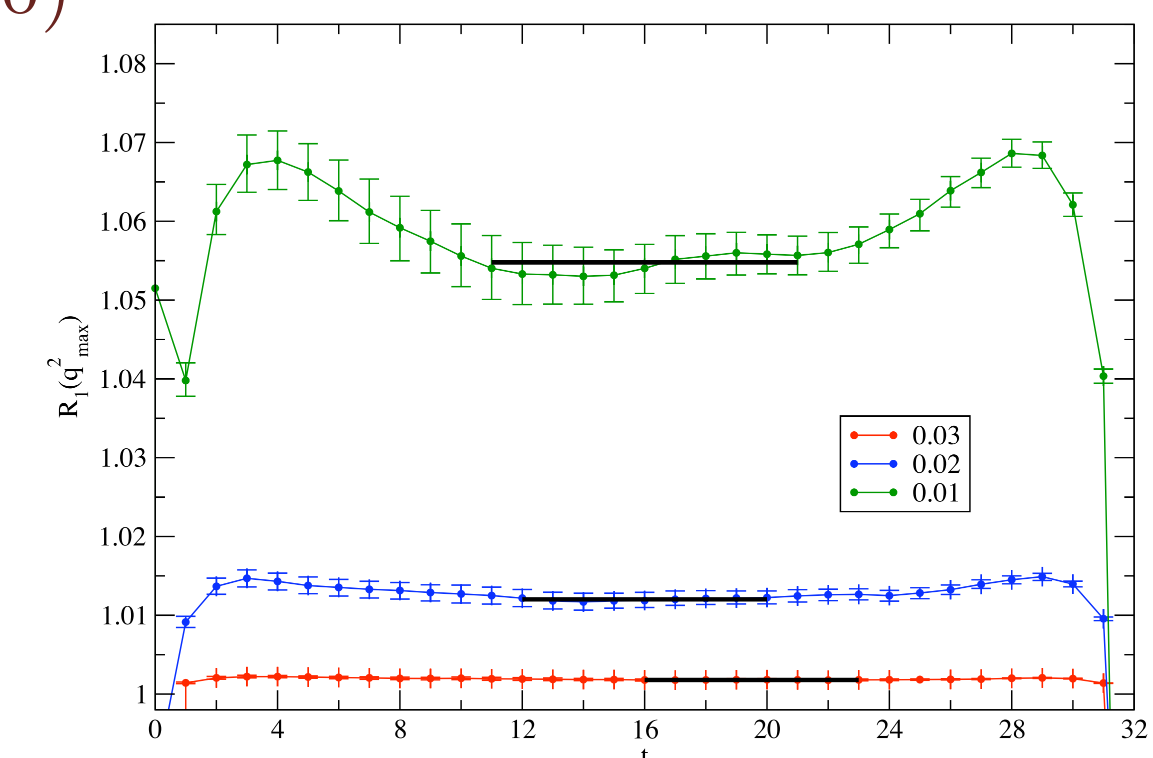
Extraction of Form Factor

- Extract scalar form factor

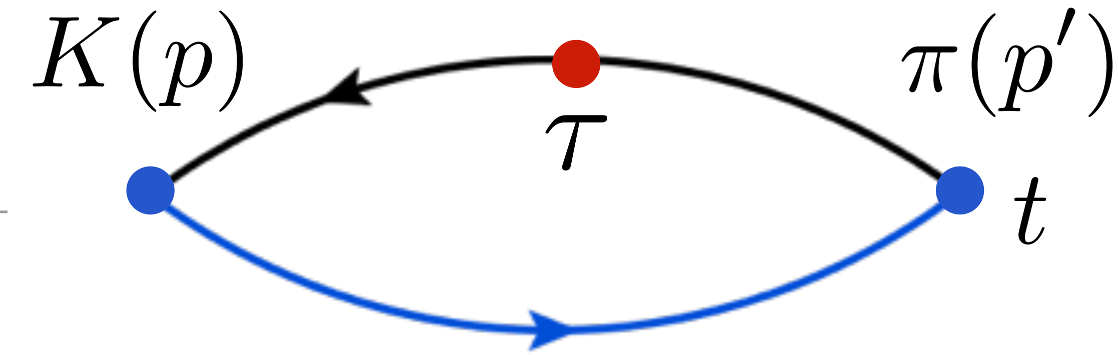
$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m_K^2 - m_\pi^2} f_-(q^2)$$

- at $q^2_{\rm max} = (m_K - m_\pi)^2$ with high precision via

$$R(t',t) = \frac{C_4^{K\pi}(t',t;\vec{0},\vec{0})C_4^{\pi K}(t',t;\vec{0},\vec{0})}{C_4^{KK}(t',t;\vec{0},\vec{0})C_4^{\pi\pi}(t',t;\vec{0},\vec{0})} \longrightarrow \frac{(m_K+m_\pi)^2}{4m_K m_\pi}|f_0(q^2_{\rm max})|^2$$



Extracting Form Factors



More generally at any q

$$\langle \pi(p')|V_\mu|K(p)\rangle = (p_\mu + p'_\mu) f_+(q^2) + (p_\mu - p'_\mu) f_-(q^2), \quad q^2 = (p' - p)^2$$

Construct ratios

$$R_{1,\,P_iP_f}(\vec{p}_i,\vec{p}_f) \;=\; 4\sqrt{E_iE_f}\sqrt{\frac{C_{P_iP_f}(t,\vec{p}_i,\vec{p}_f)\,C_{P_fP_i}(t,\vec{p}_f,\vec{p}_i)}{C_{P_i}(t_{\rm sink},\vec{p}_i)\,C_{P_f}(t_{\rm sink},\vec{p}_f)}},$$

$$R_{3,\,P_iP_f}(\vec{p}_i,\vec{p}_f) \;=\; 4\sqrt{E_iE_f}\,\frac{C_{P_iP_f}(t,\vec{p}_i,\vec{p}_f)}{C_{P_f}(t_{\rm sink},\vec{p}_f)}\sqrt{\frac{C_{P_i}(t_{\rm sink}-t,\vec{p}_i)\,C_{P_f}(t,\vec{p}_f)\,C_{P_f}(t_{\rm sink},\vec{p}_f)}{C_{P_f}(t_{\rm sink}-t,\vec{p}_f)\,C_{P_i}(t,\vec{p}_i)\,C_{P_i}(t_{\rm sink},\vec{p}_i)}}\,.$$

Form system of equations

$$R_{\alpha,K\pi}(\vec{p}_K,\vec{p}_\pi,V_4) = f^+_{K\pi}(q^2)\,(E_K + E_\pi) + f^-_{K\pi}(q^2)\,(E_K - E_\pi)$$

$$R_{\alpha,K\pi}(\vec{p}_K,\vec{p}_\pi,V_i) = f^+_{K\pi}(q^2)\,(\vec{p}_K + \vec{p}_\pi)_i + f^-_{K\pi}(q^2)\,(\vec{p}_K - \vec{p}_\pi)_i$$

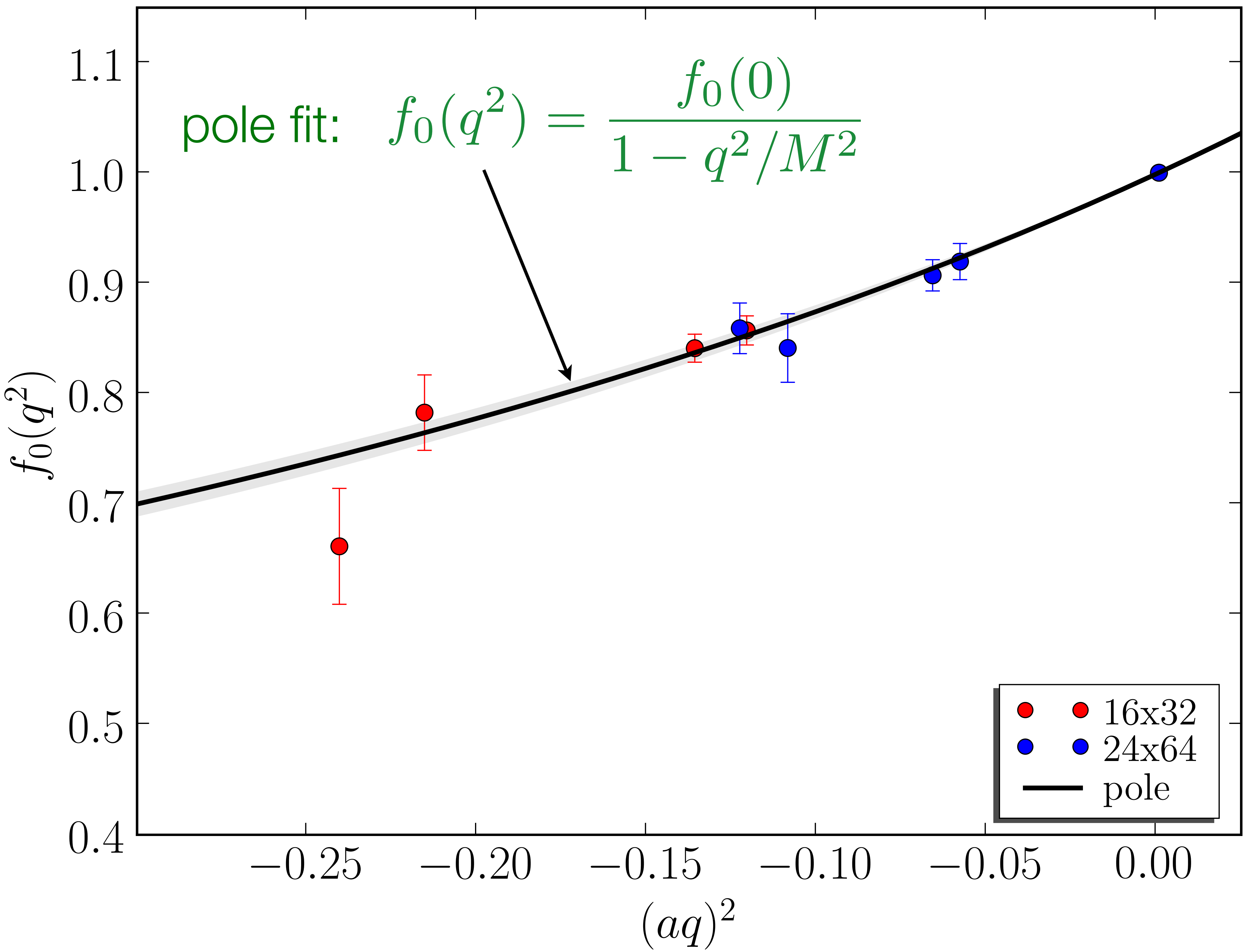
Solve for

$$f^+_{K\pi}(0) \;\&\; f^-_{K\pi}(0)$$

$f_0(q^2)$

RBC/UKQCD: 0710.5136

$m_\pi \approx 670\,\mathrm{MeV}$



Chiral Extrapolation of $f_+(0)$

- $f_+(0) = 1 + f_2 + \Delta f$

$$f_2 = \frac{3}{2} H_{\pi K} + \frac{3}{2} H_{\eta K}$$

- where

$$H_{PQ} = -\frac{1}{64\pi^2 f_\pi^2}\left[M_P^2 + M_Q^2 + \frac{2M_P^2 M_Q^2}{M_P^2 - M_Q^2}\log\left(\frac{M_Q^2}{M_P^2}\right)\right]$$

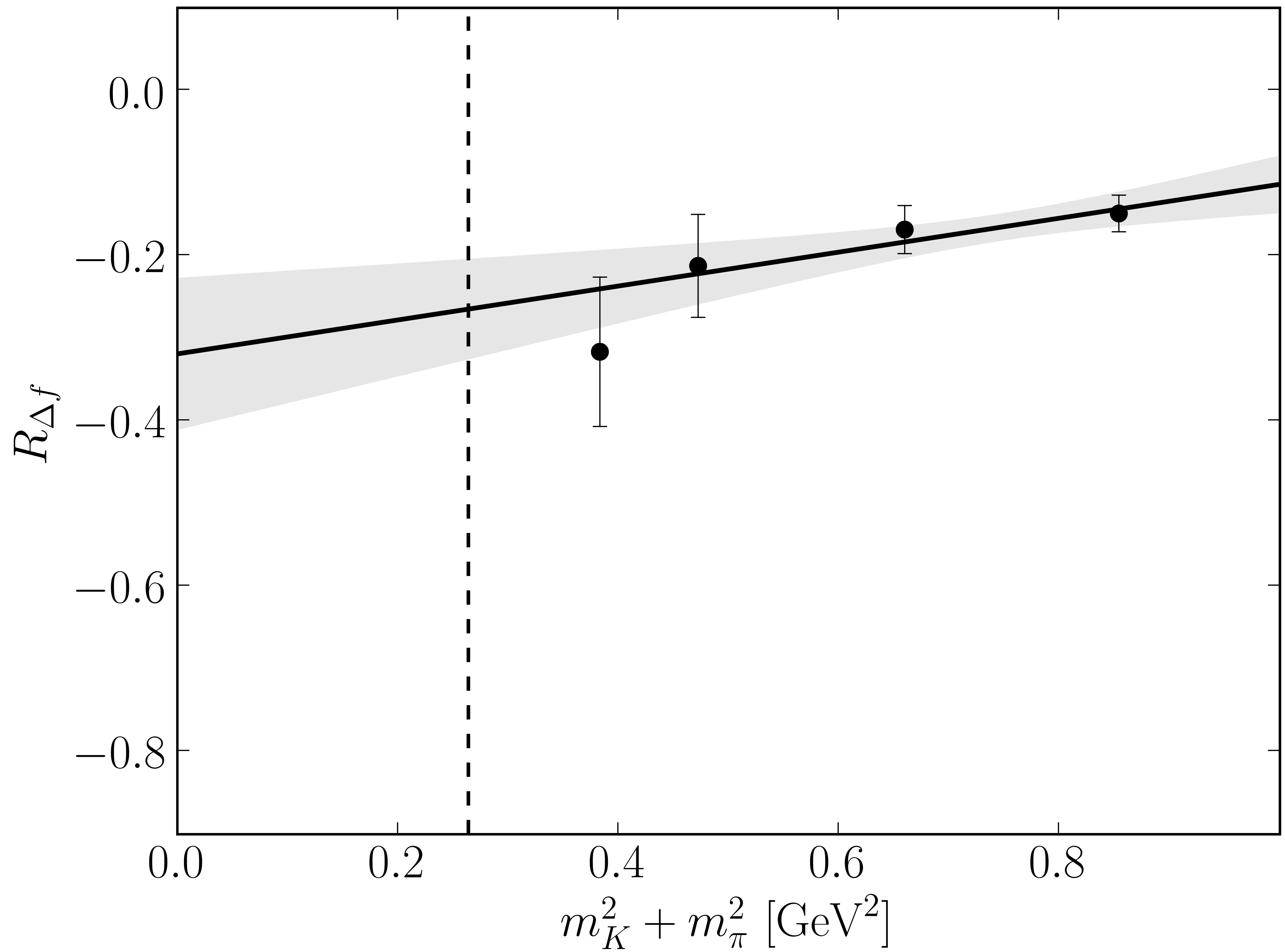
- at the physical masses, $f_2 = -0.023$

$\Delta f \propto (m_s - m_{ud})^2$ → Attempt extrapolation

$$R_{\Delta f} = \frac{\Delta f}{(M_K^2 - M_\pi^2)^2} = a + b(M_K^2 + M_\pi^2)$$

Chiral Extrapolation of $f_+(0)$

$$R_{\Delta f} = \frac{\Delta f}{(M_K^2 - M_\pi^2)^2} = a + b(M_K^2 + M_\pi^2) \qquad \Delta f = -0.0161(46)$$



Simultaneous Fit

[RBC/UKQCD PRL100, 141601 (2008)]

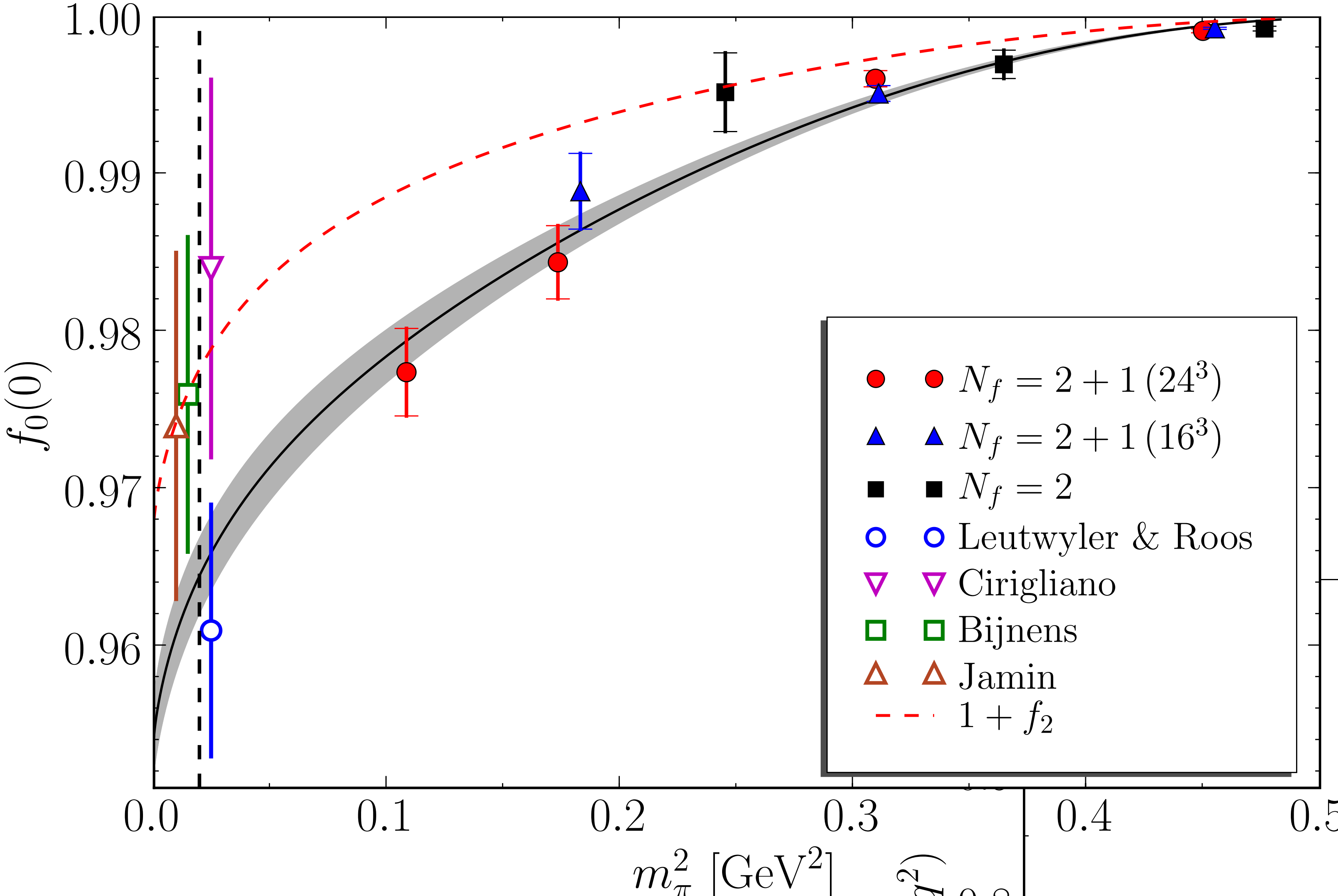
- In an attempt to get as much information as possible out of the lattice data as possible, we attempt to fit the q^2 and the quark mass dependencies simultaneously

$$f_0(q^2, m_\pi^2, m_K^2) = \frac{1 + f_2 + (m_K^2 - m_\pi^2)^2 (A_0 + A_1(m_K^2 + m_\pi^2))}{1 - \frac{q^2}{(M_0 + M_1(m_K^2 + m_\pi^2))^2}}$$

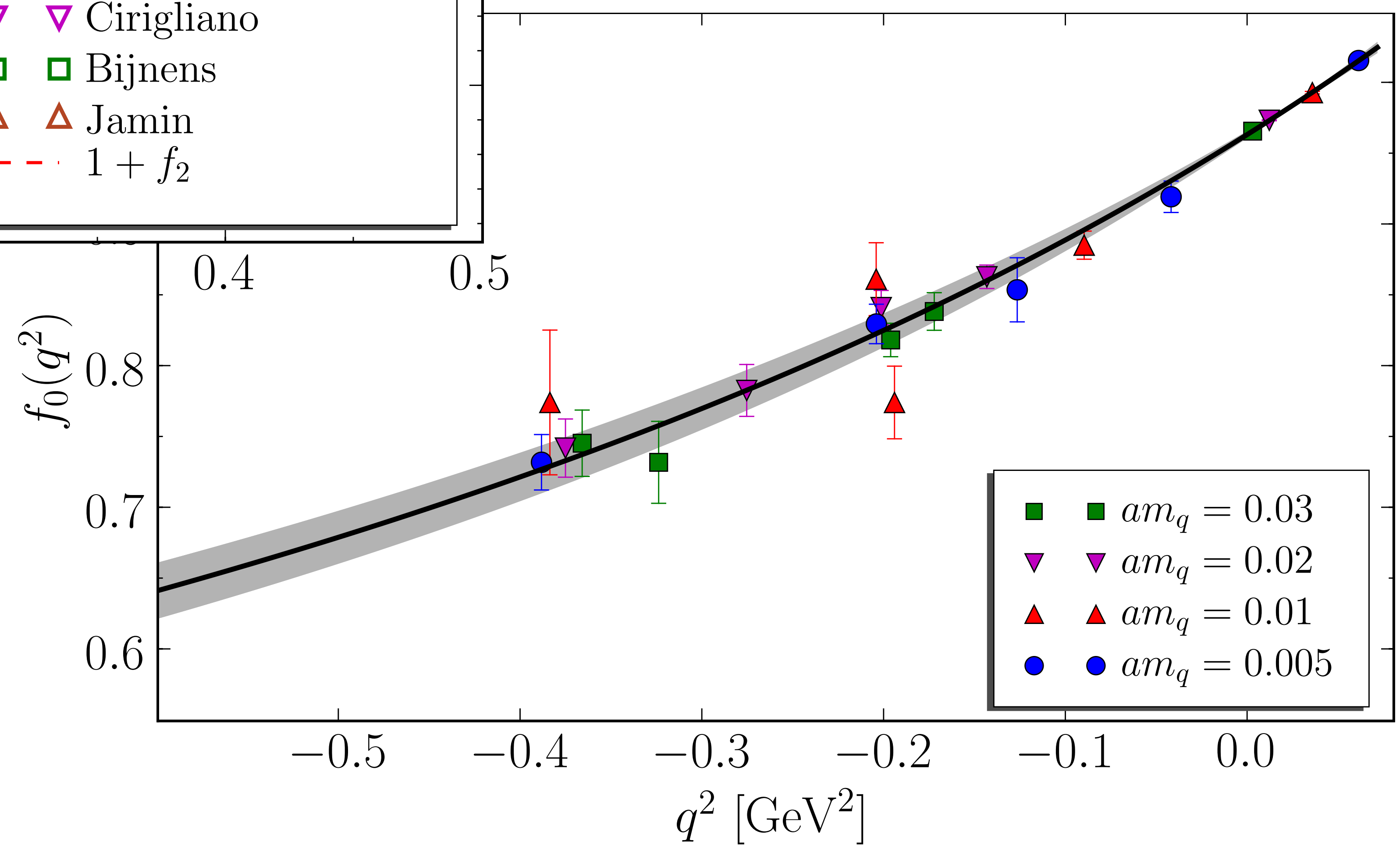
- where A_0, A_1, M_0, and M_1 are fit parameters

- Also construct simultaneous fit based on ansatz quadratic in q^2 and take difference as estimate of systematic error

$$f_0(q^2, m_\pi^2, m_K^2) = \frac{1 + f_2 + (m_K^2 - m_\pi^2)^2 (A_0 + A_1(m_K^2 + m_\pi^2))}{1 - \frac{q^2}{(M_0 + M_1(m_K^2 + m_\pi^2))^2}}$$



$f_+(0) = 0.9644(33)(34)(14)$



PRL100, 141601 (2008)

$|V_{us}|$ [RBC/UKQCD PRL100, 141601 (2008)]

$$\Delta f = -0.0129(33)(34)(14) \Rightarrow f_+^{K\pi}(0) = 0.9644(48)$$

Statistical error after extrapolations

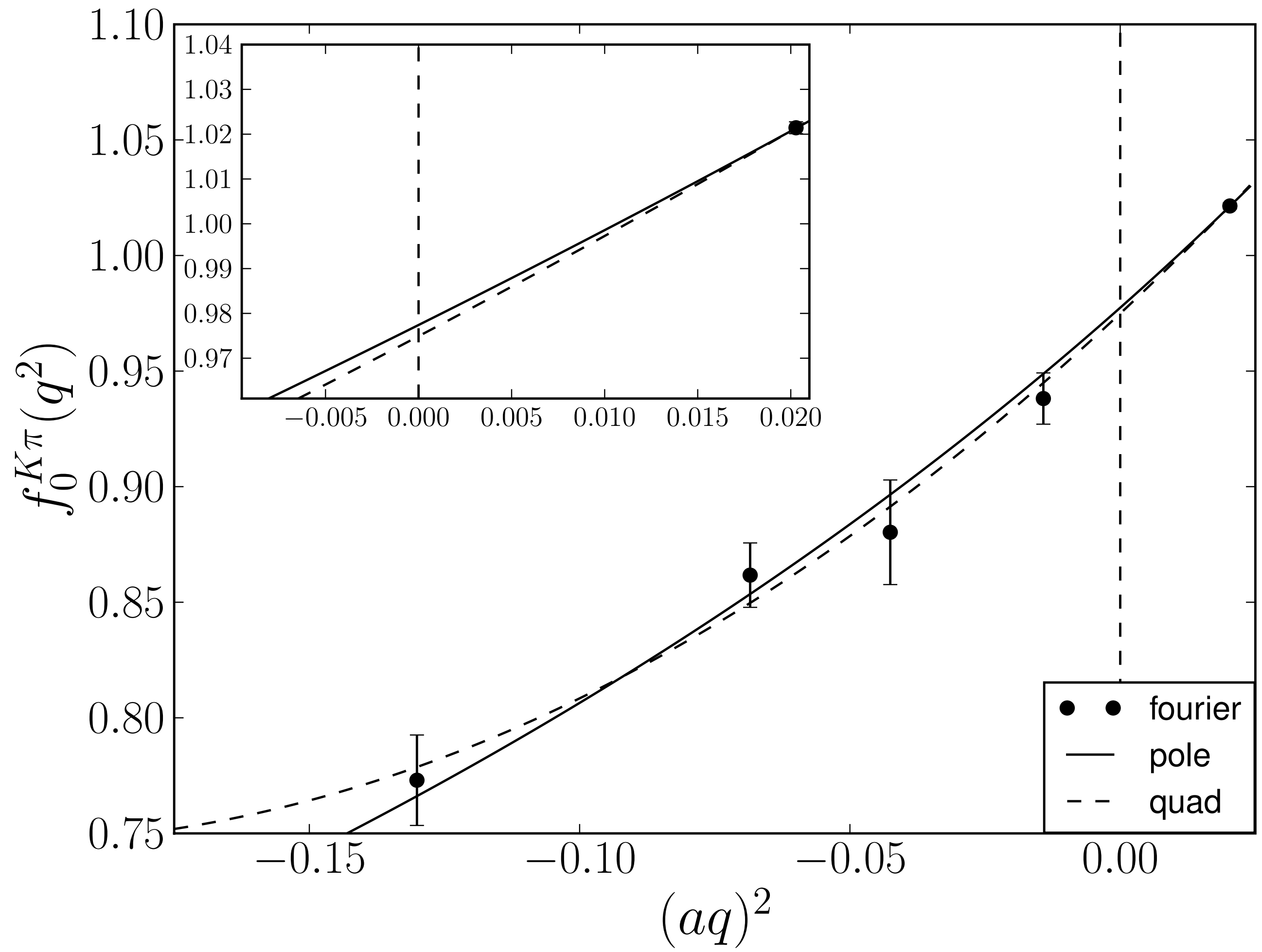
Systematic error from modelling q^2 dependence

Discretisation error

Can we remove this error by using twisted boundary conditions?

$\bar{K}^0 \to \pi^+ \ell \nu_\ell$

Reference point: Boyle et al. [RBC/UKQCD] PRL100, 141601 (2008)



$m_\pi \approx 330\text{MeV}$

$24^3 \times 64$

$a \approx 0.114$ fm

$$f_+^{K\pi}(0)|_{\text{pole}} = 0.9774(35)$$
$$f_+^{K\pi}(0)|_{\text{polynomial}} = 0.9749(59)$$

$\bar{K}^0 \to \pi^+ \ell \nu_\ell$

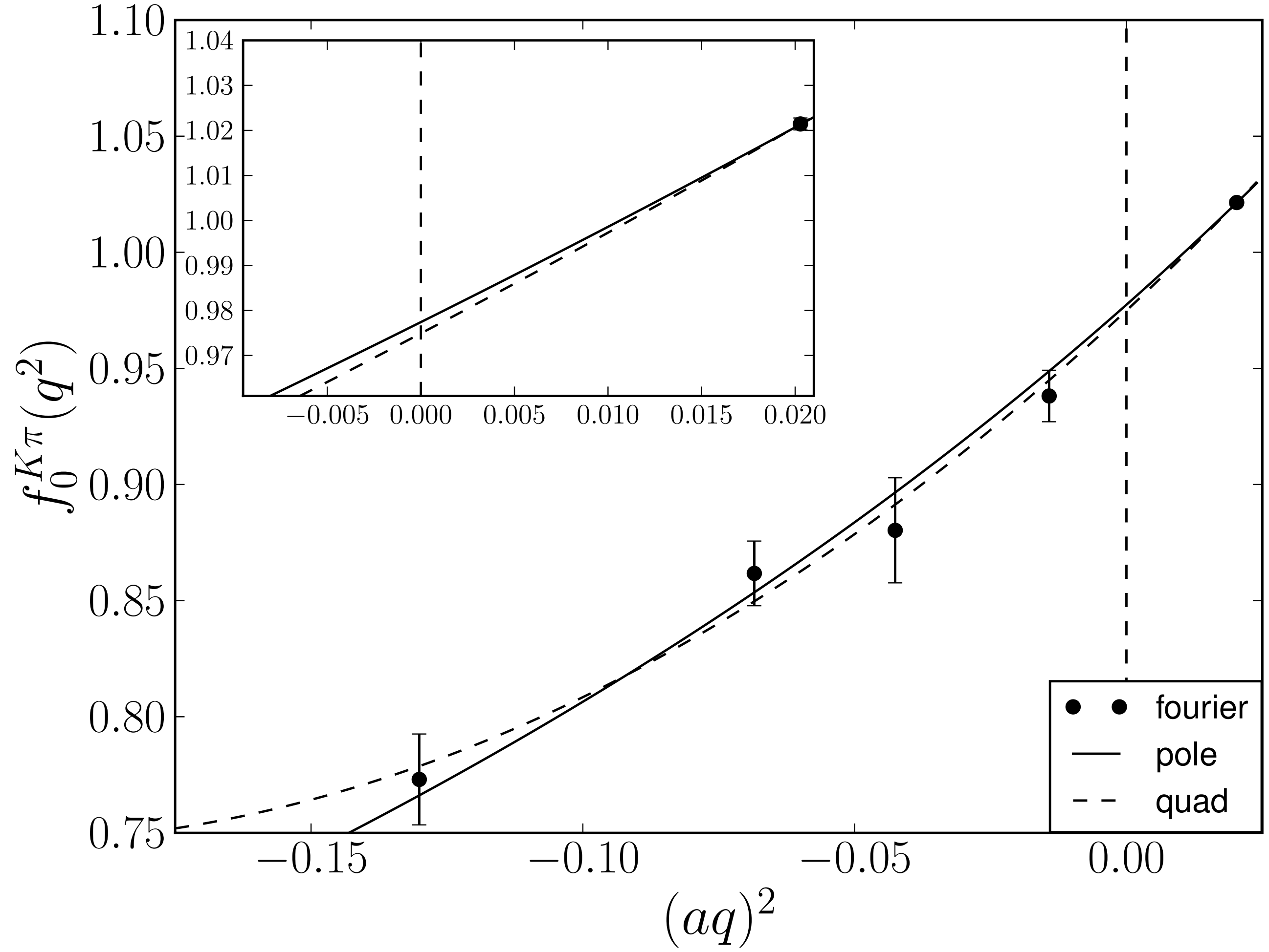
Reference point: Boyle et al. [RBC/UKQCD] PRL100, 141601 (2008)

- Remove source of systematic error by
- using (partially) twisted boundary conditions and tune to q^2=0

$$q^2 = (p_f - p_i)^2 = \left\{ [E_f(\vec{p}_f) - E_i(\vec{p}_i)]^2 - \left[(\vec{p}_{\mathrm{FT},f} + \vec{\theta}_f/L) - (\vec{p}_{\mathrm{FT},i} + \vec{\theta}_i/L) \right]^2 \right\}$$

$$|\vec{\theta}_K| = L\sqrt{(\tfrac{m_K^2+m_\pi^2}{2m_\pi})^2 - m_K^2} \text{ and } \vec{\theta}_\pi = \vec{0}$$

$$|\vec{\theta}_\pi| = L\sqrt{(\tfrac{m_K^2+m_\pi^2}{2m_K})^2 - m_\pi^2} \text{ and } \vec{\theta}_K = \vec{0}\,,$$



$m_\pi \approx 330\mathrm{MeV}$

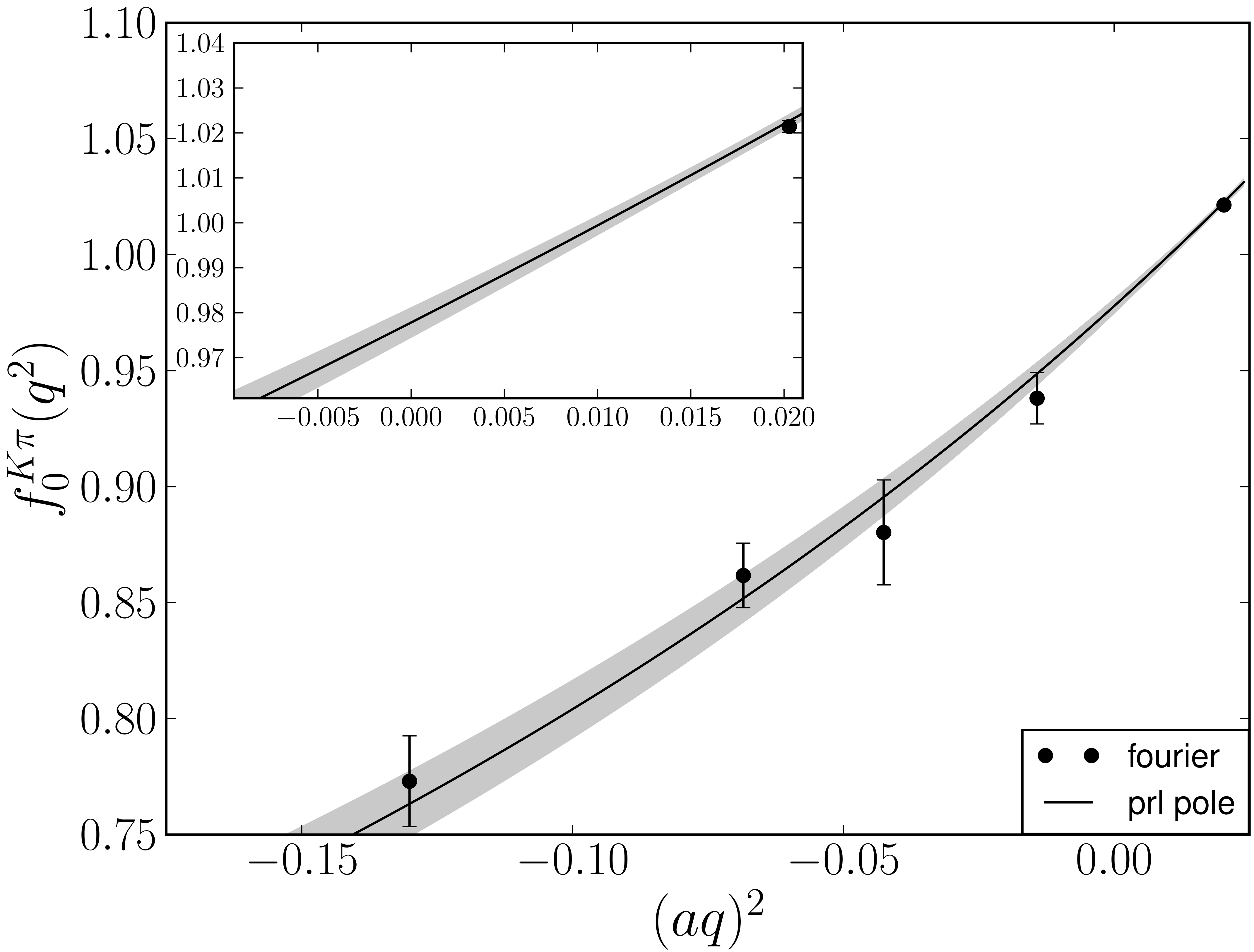
$24^3 \times 64$

$a \approx 0.114$ fm

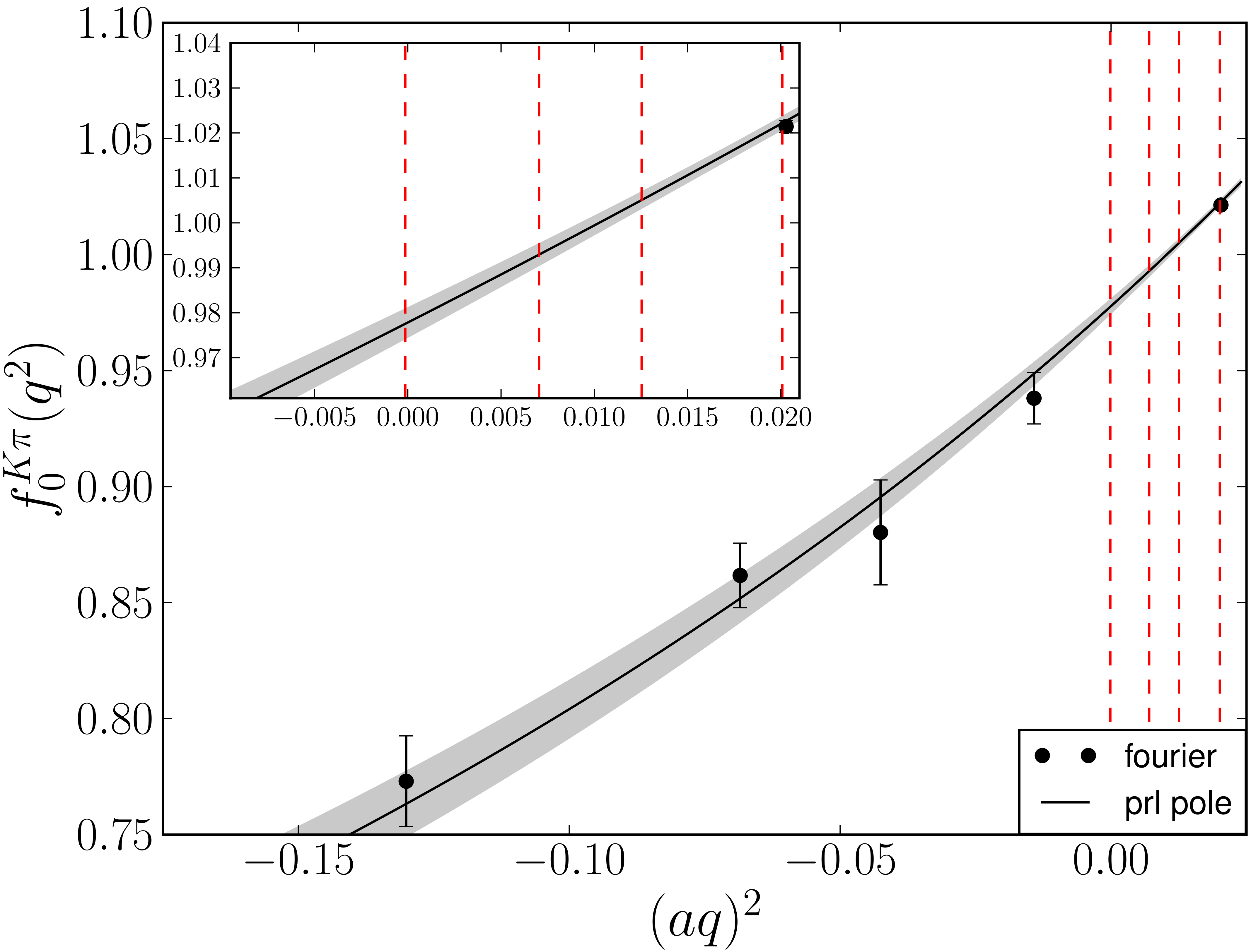
$$f_+^{K\pi}(0)|_{\mathrm{pole}} = 0.9774(35)$$

$$f_+^{K\pi}(0)|_{\mathrm{polynomial}} = 0.9749(59)$$

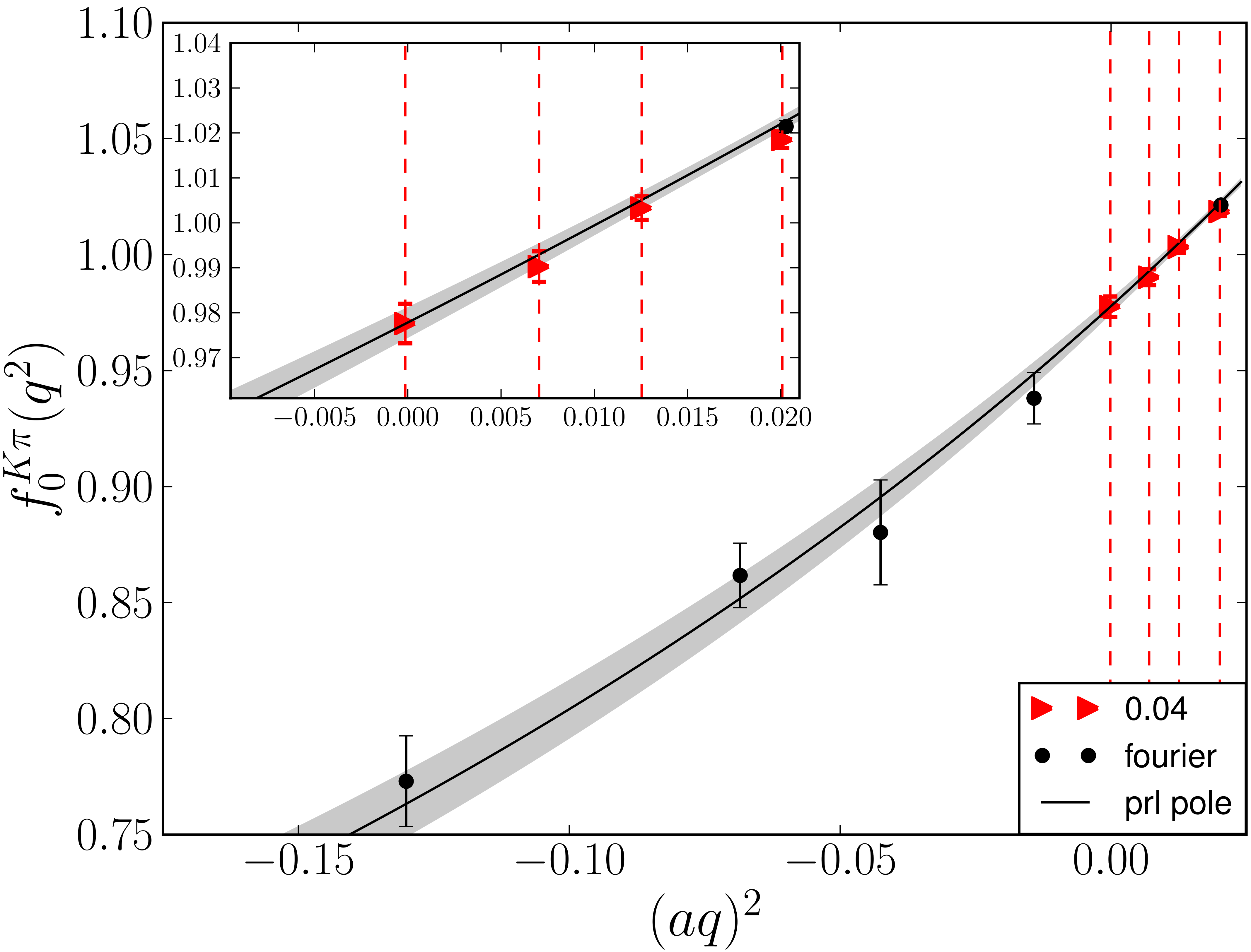
Twisted Results



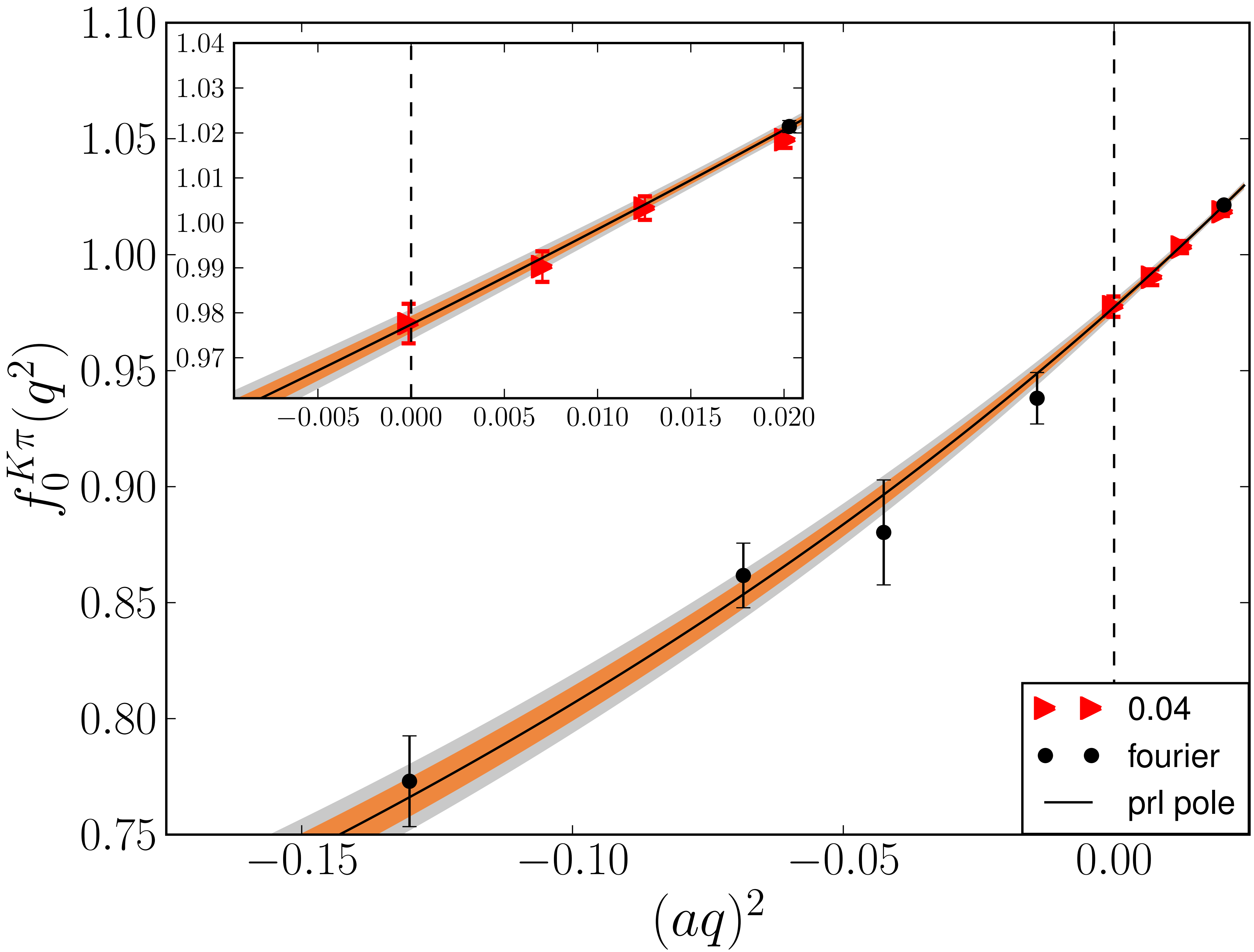
Twisted Results



Twisted Results



Twisted Results



Comparison of determination of $f_+(0)$

$$\mathbf{results\ for\ am_q = 0.005,\ am_s = 0.04}$$

$$\begin{aligned} f_+^{K\pi}(0)\big|_{\rm pole} &= 0.9774(35)\ \ [6]\,, \\ f_+^{K\pi}(0)\big|_{\rm polynomial} &= 0.9749(59)\ \ [6]\,, \\ f_+^{K\pi}(0)\big|_{\rm this\,work} &= 0.9757(44)\,. \end{aligned}$$

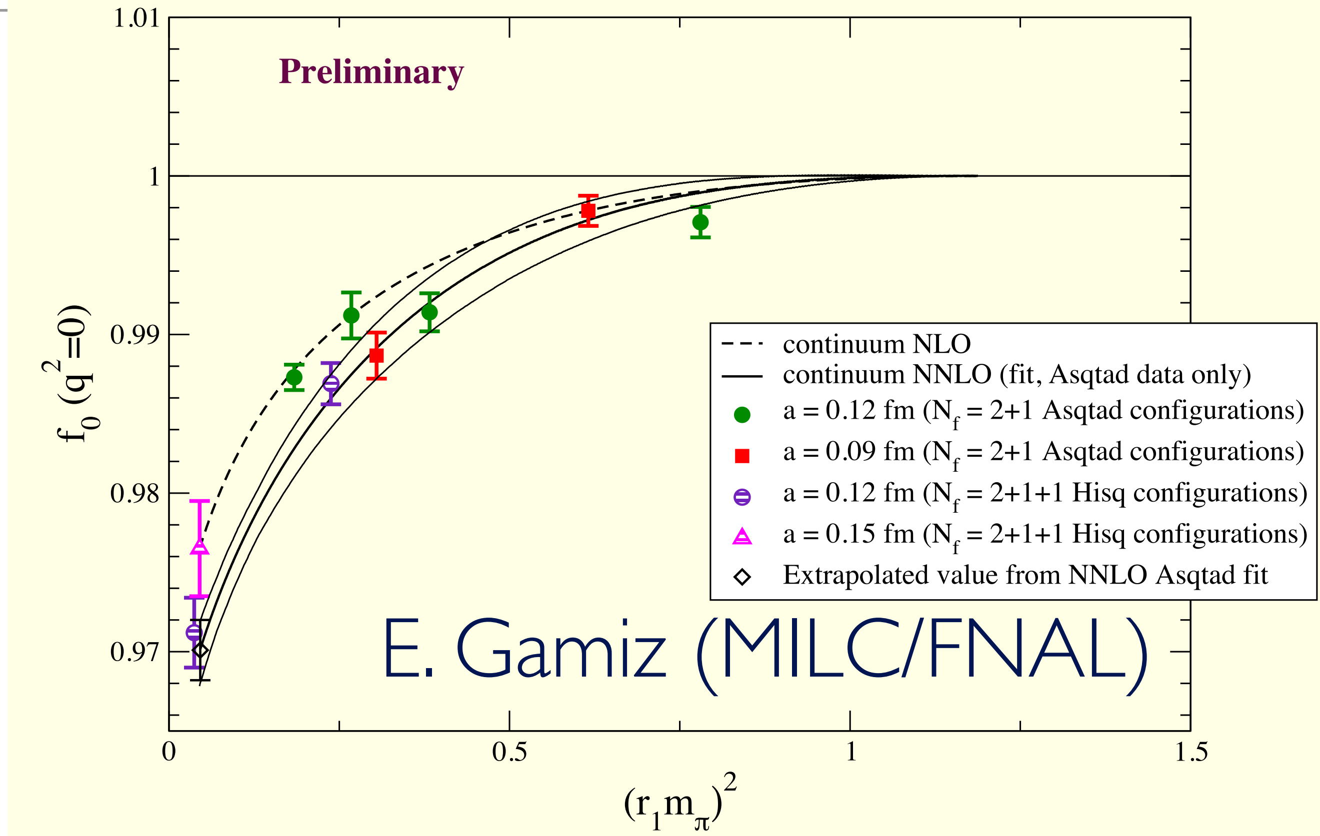
- achieve a result for f+(0) with comparable precision to standard methods

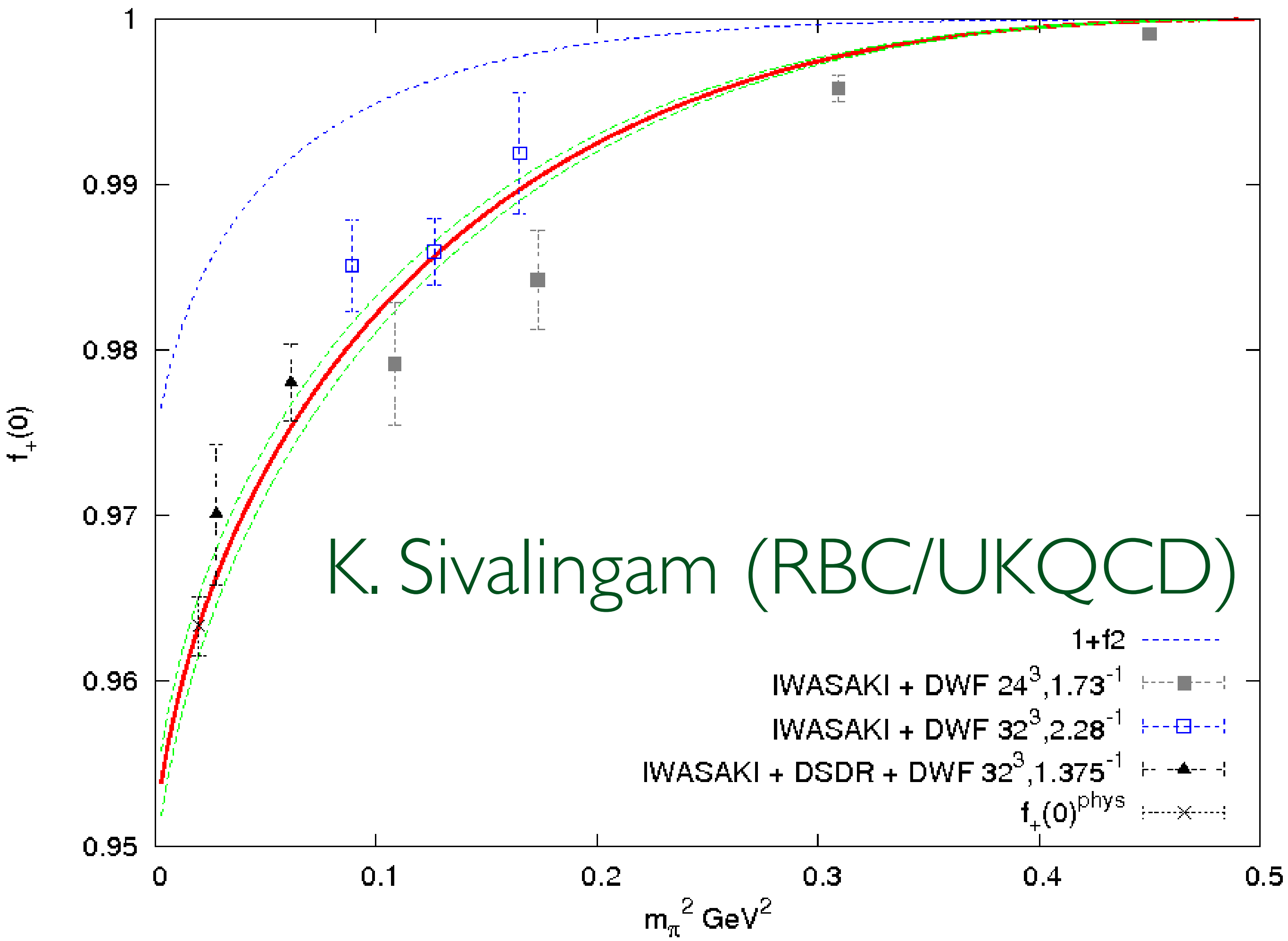
- Removes model dependence of q^2 interpolation

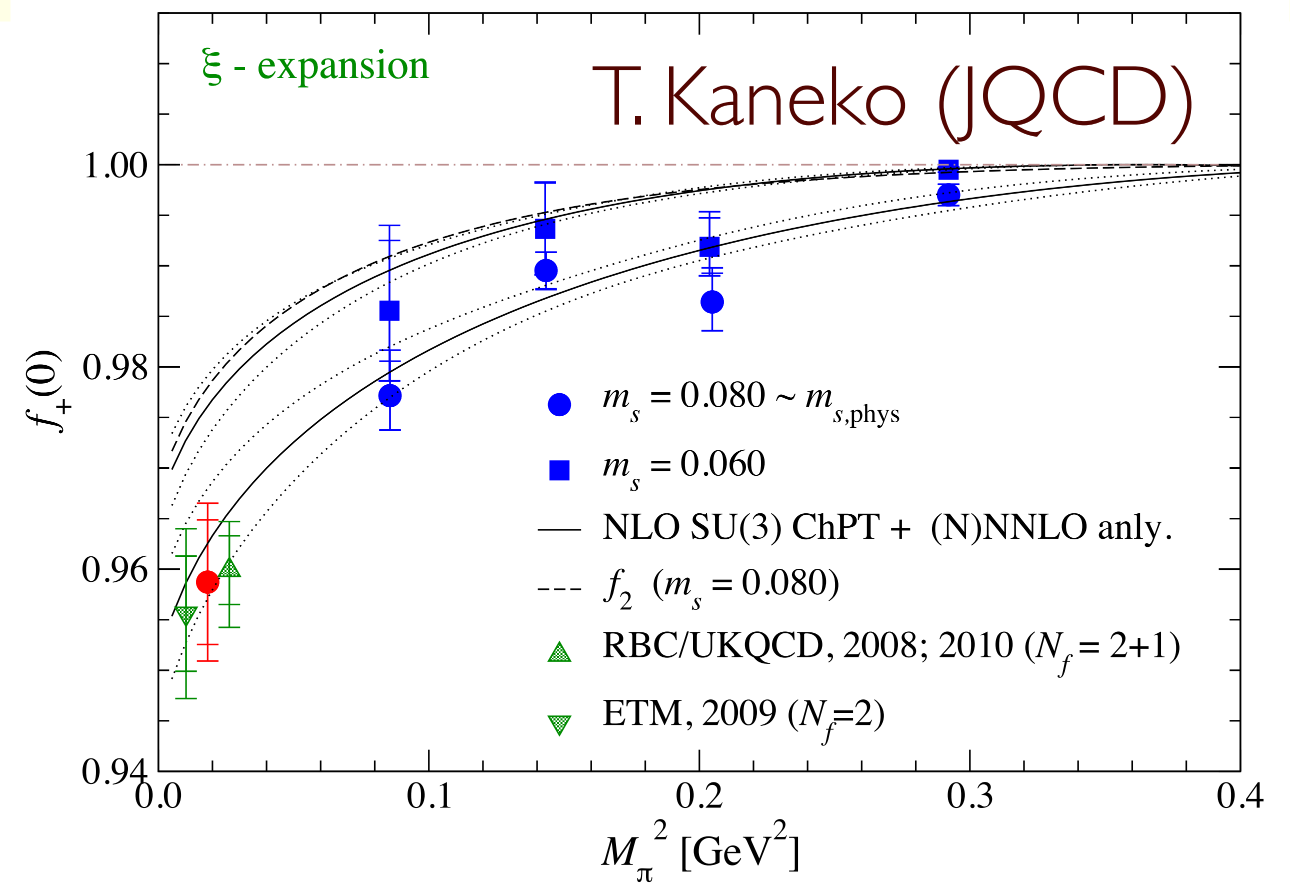
Eur. Phys. J. C69 (2010) 159-167

Recent Progress to Light Quark Masses

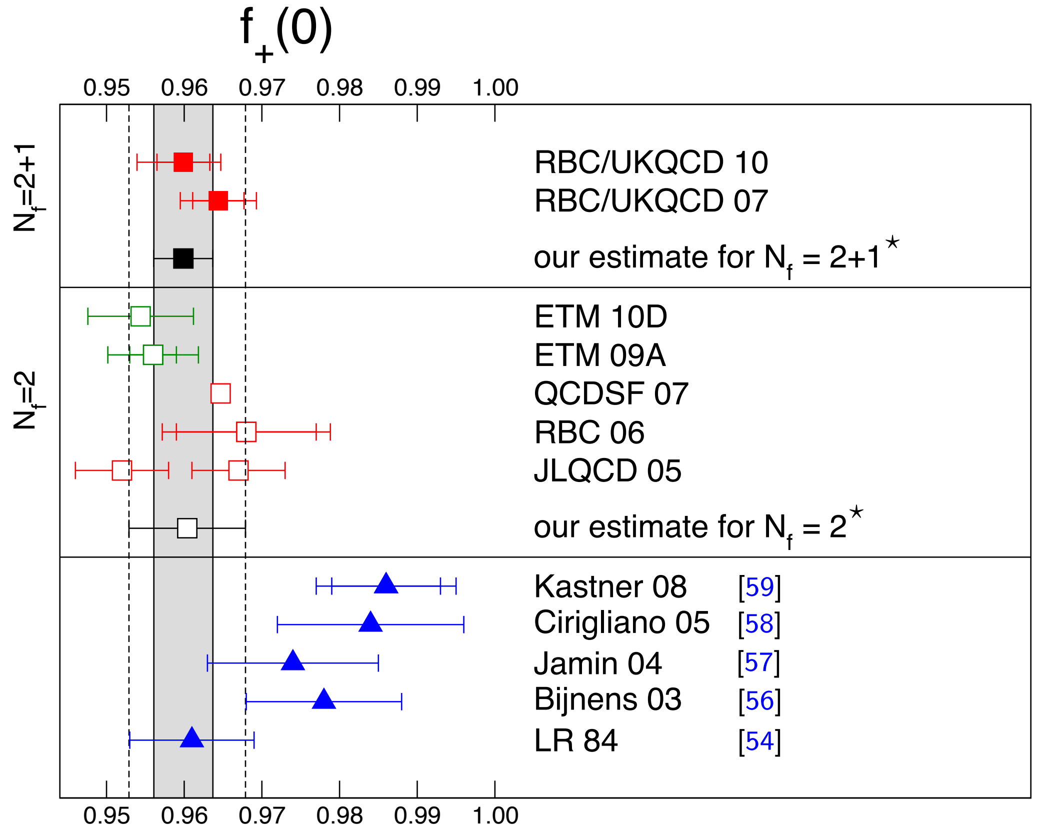
Lattice 2012

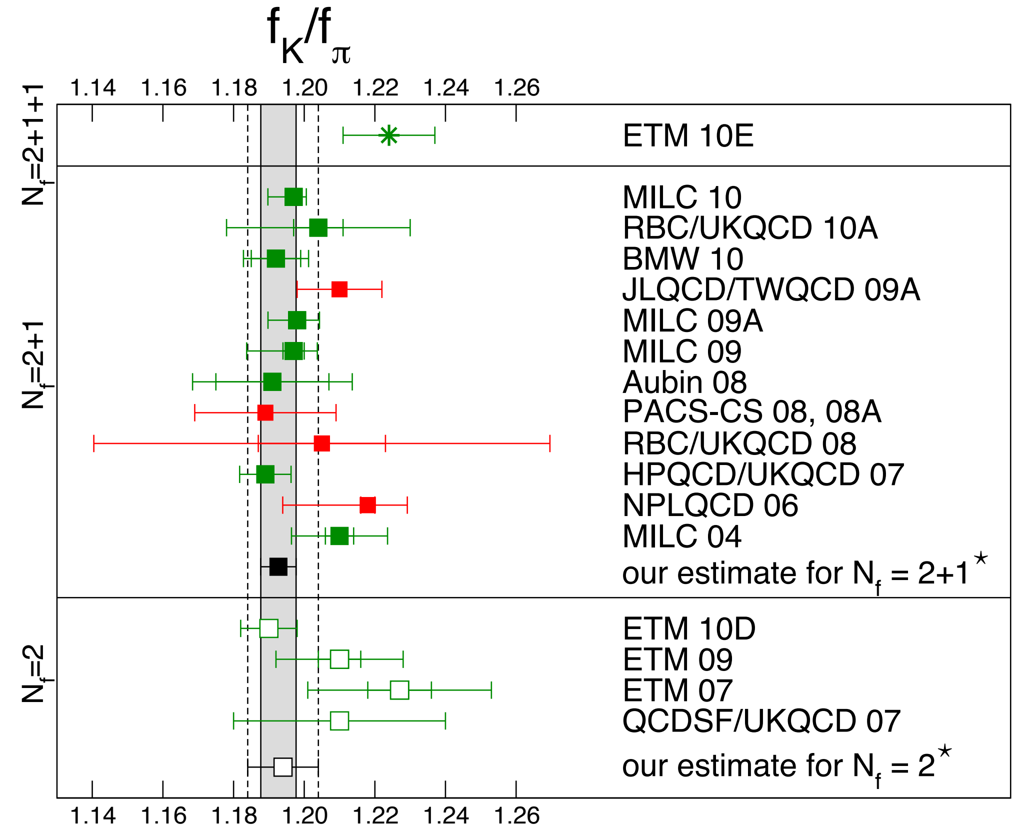




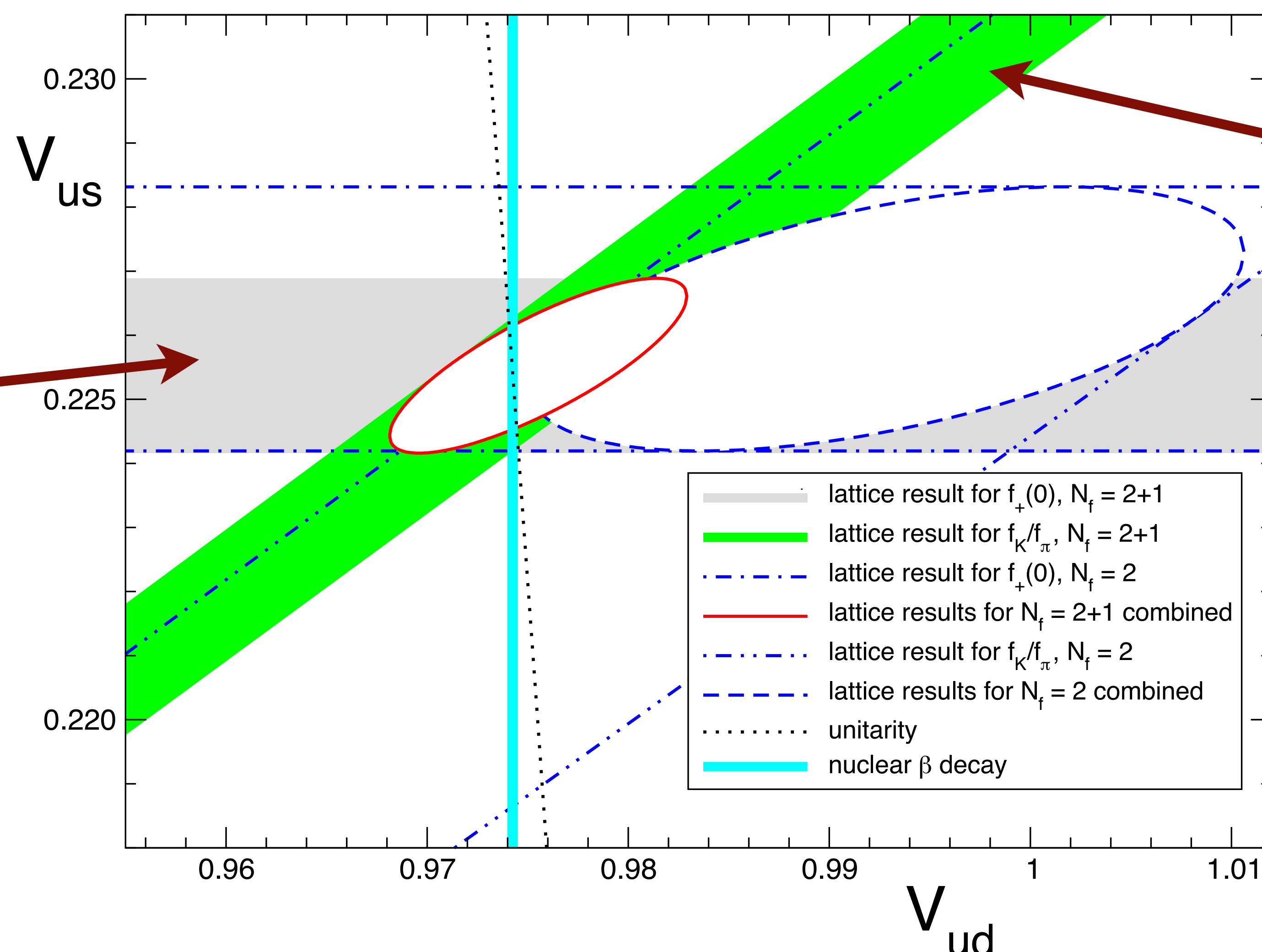


$|V_{us}|$





- Combining lattice results with experimental decay rates, FLAG [1011.4408] finds



$K \to \mu\nu$

f_K/f_π

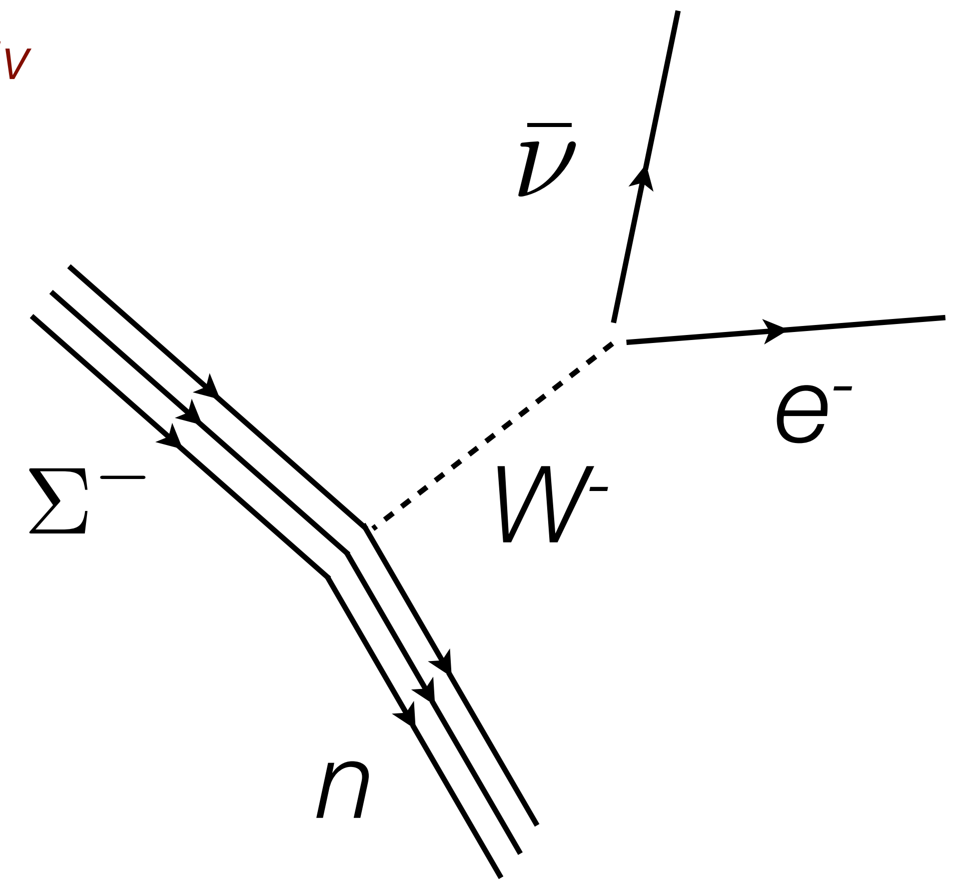
$K \to \pi\ell\nu$

$f_+(q^2=0)$

Hyperon Semi-Leptonic Decays

e.g. $\Sigma^- \to n\ell\nu_\ell$ and $\Xi^0 \to \Sigma^+\ell\nu_\ell$

- Provide an alternative method for determining the CKM matrix element $|V_{us}|$
- The axial semi-leptonic form factor at $q^2=0$ gives g_A/g_V
- $\Xi^0 \to \Sigma^+\ell\nu_\ell$ is analogous to usual β decay
  - expect $g_A/g_V \approx 1.26$



$$\langle B(p',s')|(V_\mu - A_\mu)|b(p,s)\rangle = \bar{u}_B(p',s')\{\gamma_\mu f_1(q^2) + i\frac{\sigma_{\mu\nu}q^\nu}{2M} f_2(q^2) + \frac{q_\mu}{2M} f_3(q^2)$$
$$-[\gamma_\mu\gamma_5 g_1(q^2) + i\frac{\sigma_{\mu\nu}q^\nu}{2M}\gamma_5 g_2(q^2) + \frac{q_\mu}{2M}\gamma_5 g_3(q^2)]\}u_b(p,s)$$

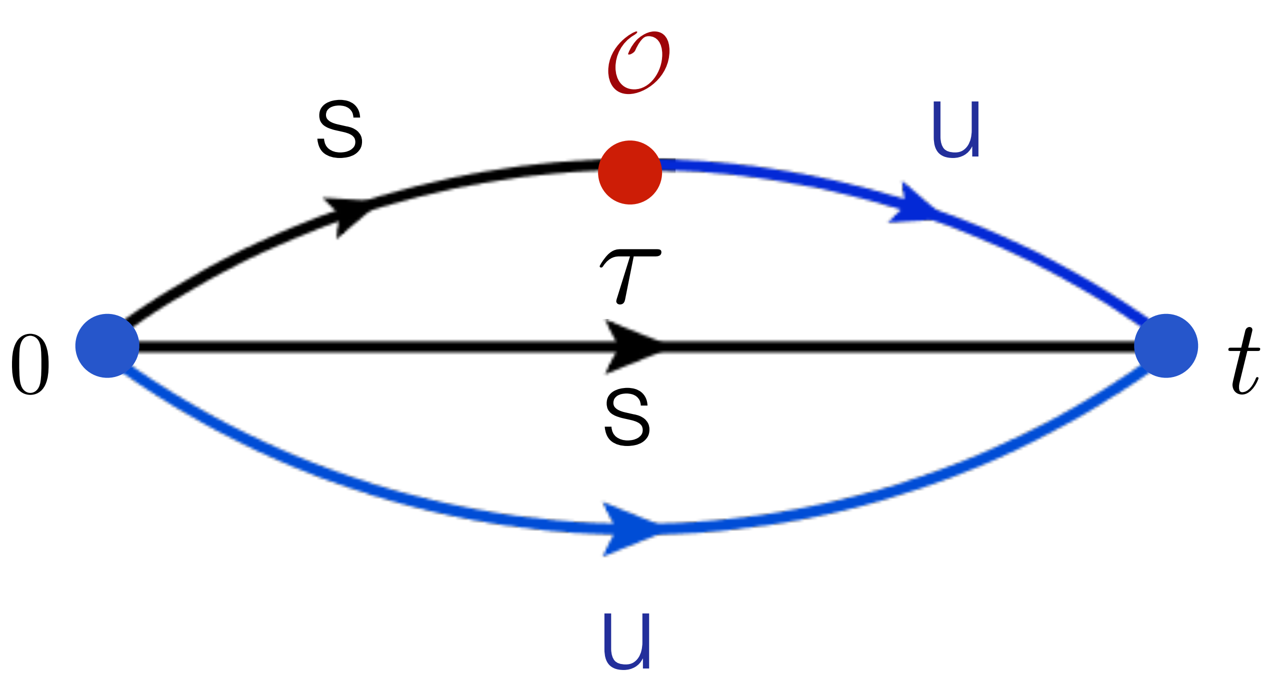
Hyperon Semi-Leptonic Decays

- Experimental decay

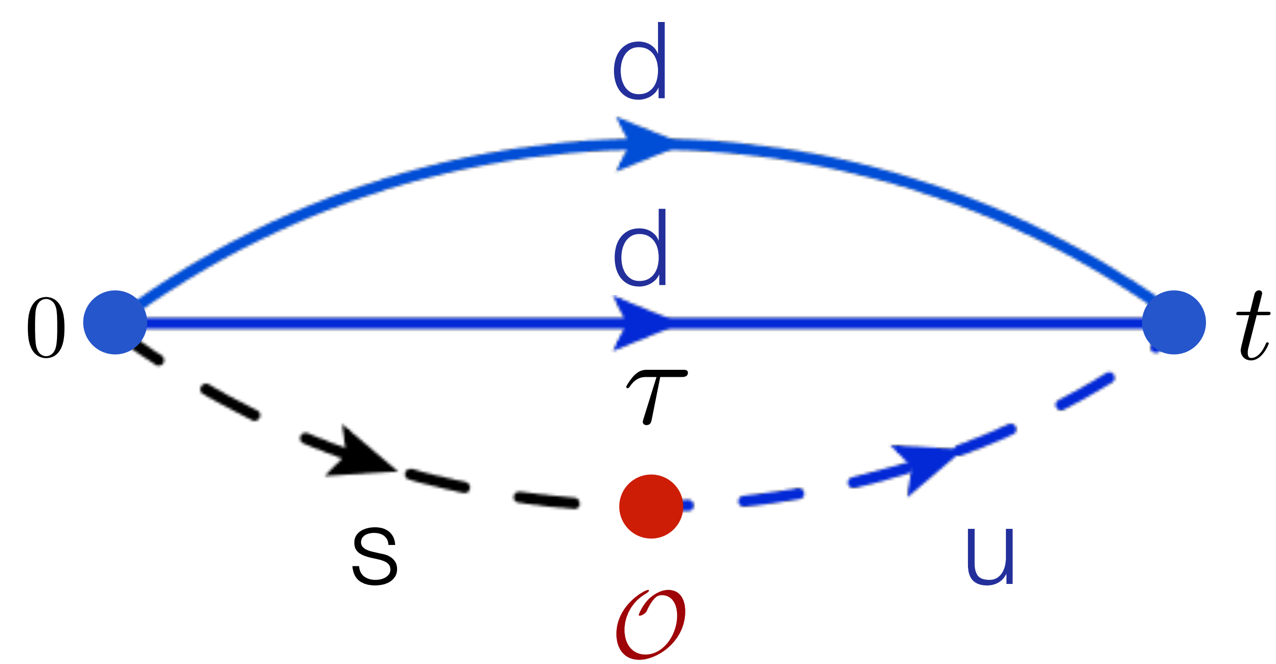
$$\Gamma = \frac{G_F^2}{60\pi^3}(M_B - M_b)^5(1-3\delta)|V_{us}|^2|f_1^{B\to b}(0)|^2\left[1 + 3\left|\frac{g_1^{B\to b}(0)}{f_1^{B\to b}(0)}\right|^2 + \cdots\right]$$

- Lattice 3pt functions

$$\Xi^0 \to \Sigma^+ \ell \nu_\ell$$



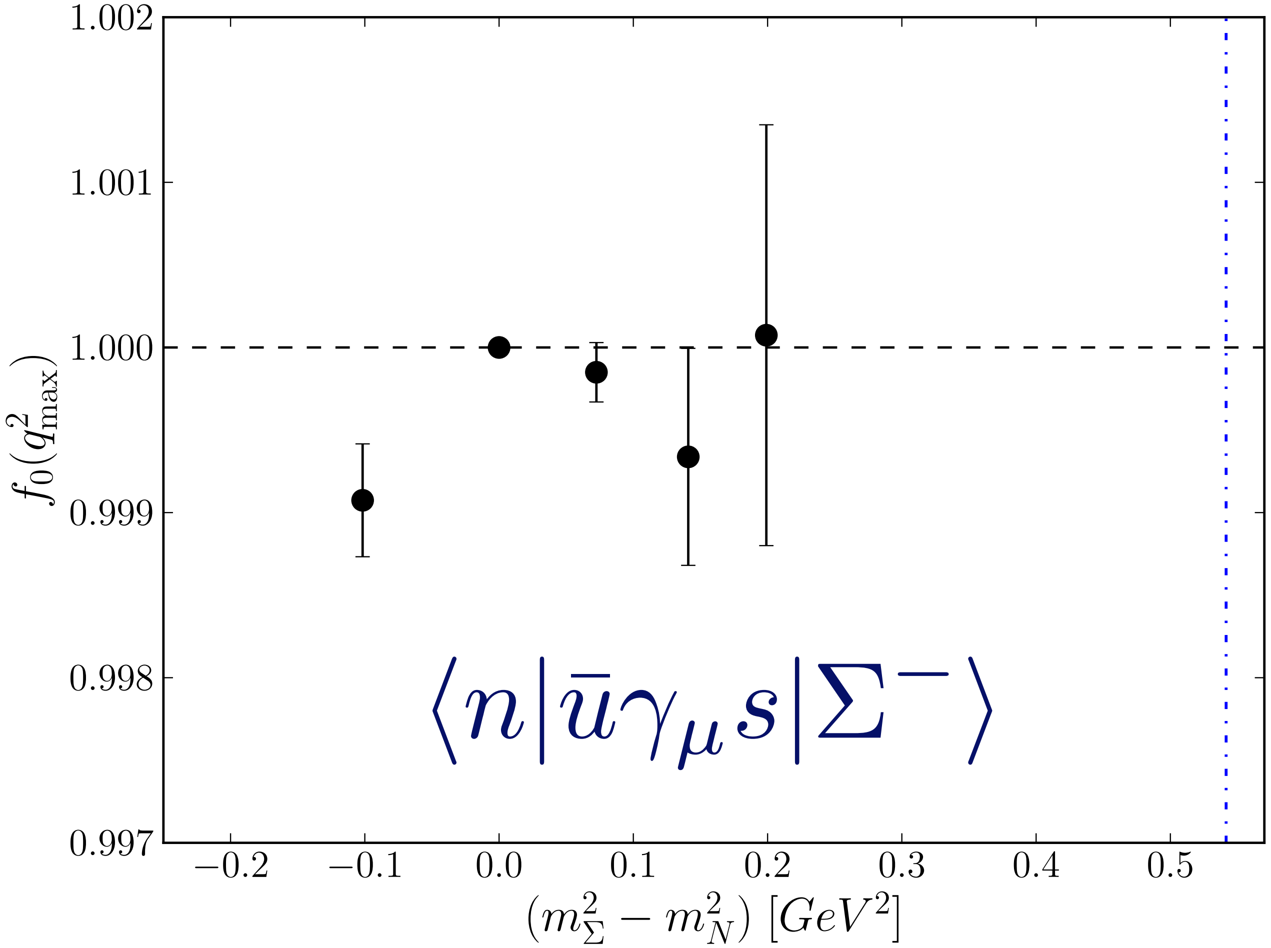
$$\Sigma^- \to n \ell \nu_\ell$$

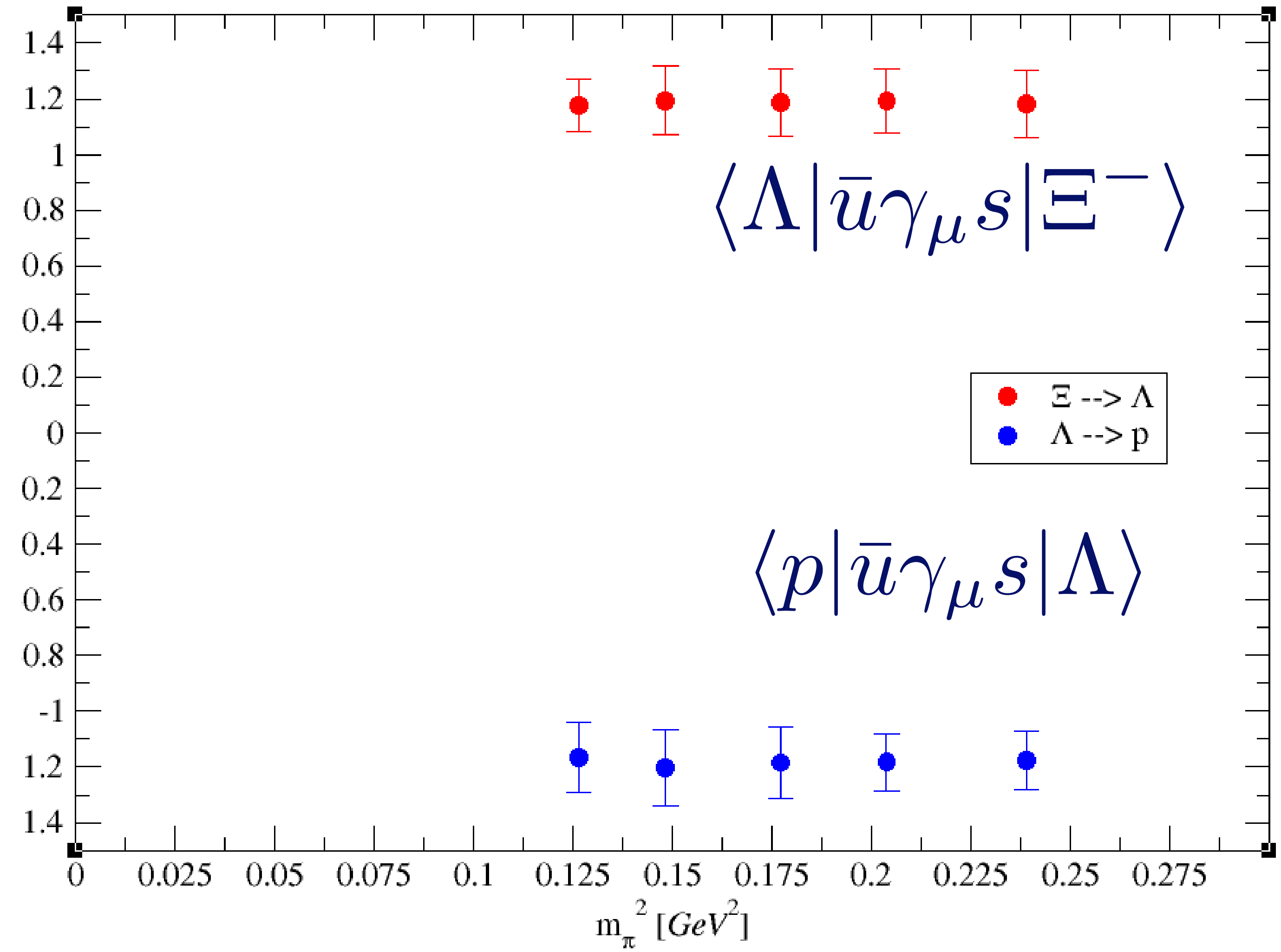


Hyperon semi-leptonic form factor

Weak Vector

[A. Cooke, Lattice 2012]



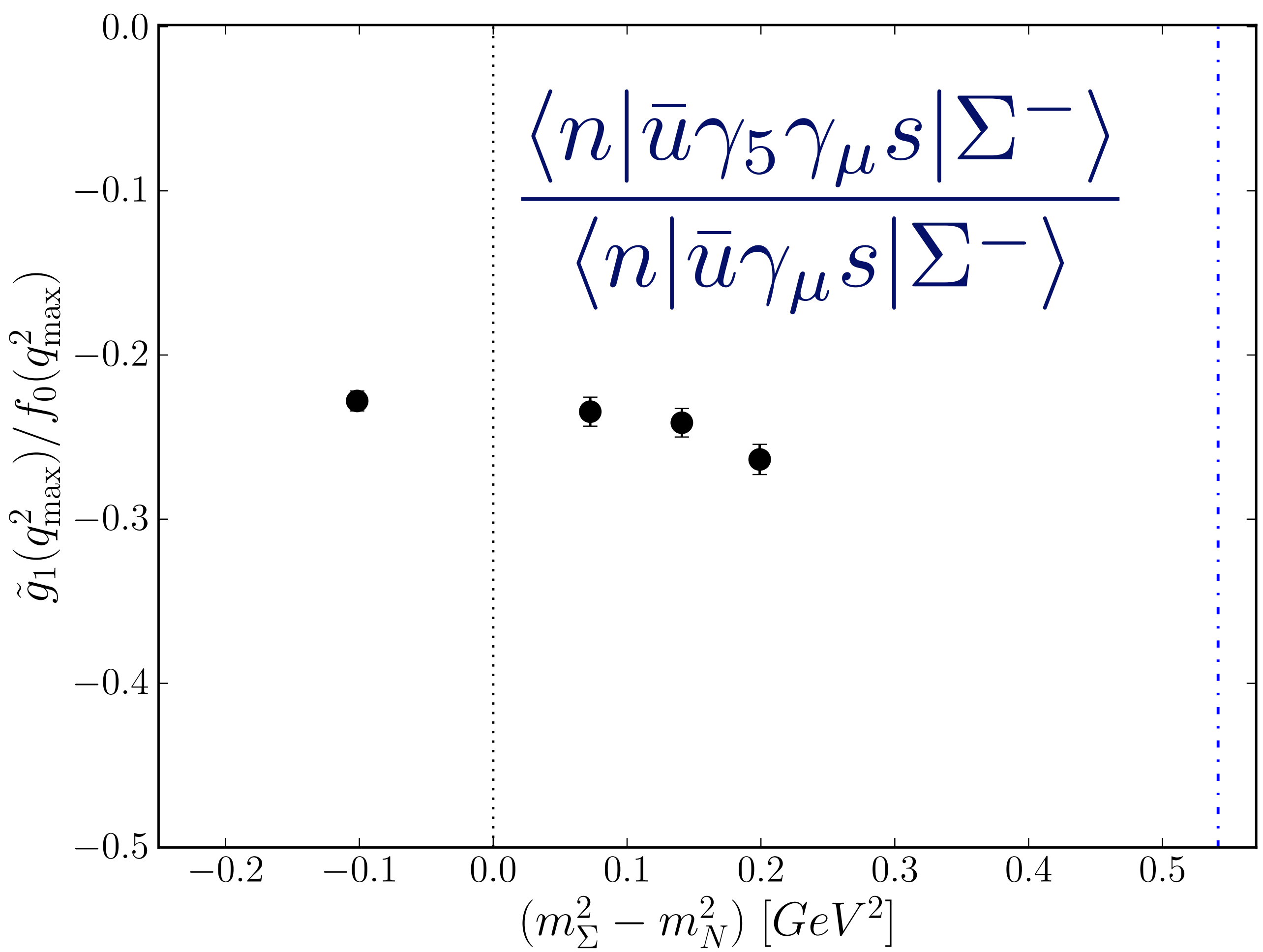


Clebsch-Gordon coefficients (SU(3) limit):

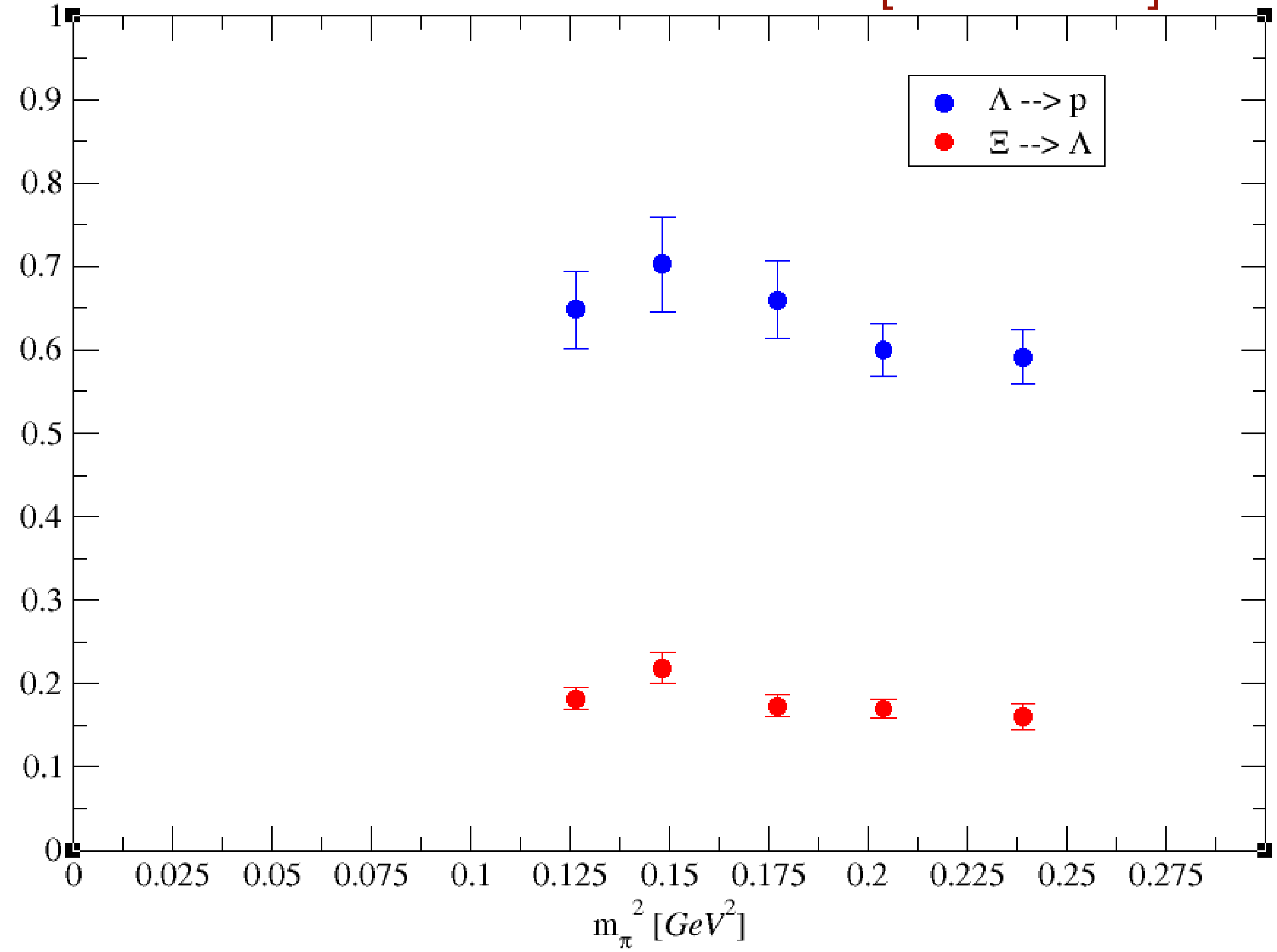
$$1 \qquad \sqrt{\frac{3}{2}} \qquad -\sqrt{\frac{3}{2}}$$

Hyperon semi-leptonic form factor

Weak Axial



[A. Cooke]



SU(3) predictions:

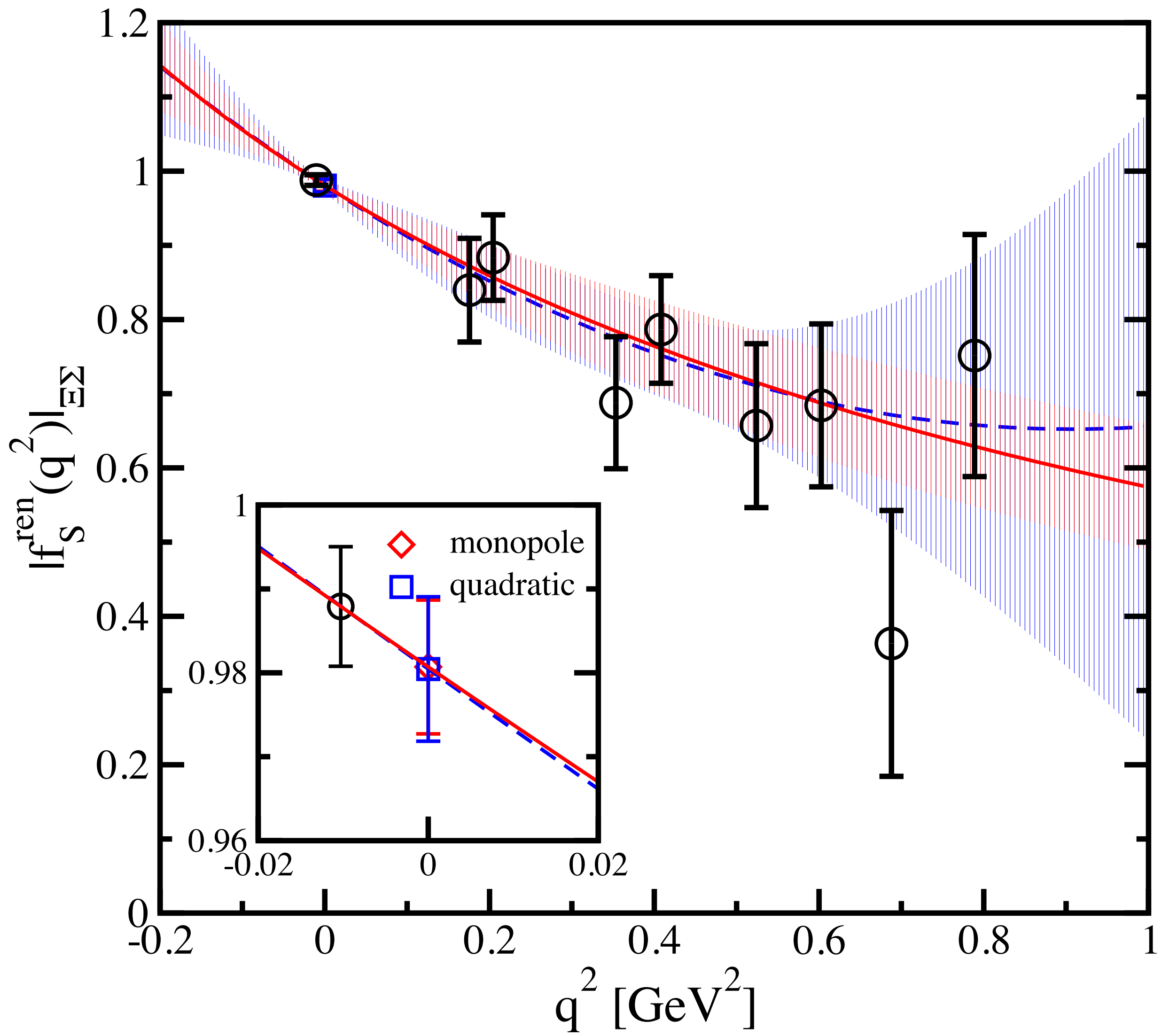
$$\Sigma^- \to n\ (F - D) \qquad \Xi^- \to \Lambda^0 \left(F - \frac{D}{3}\right) \qquad \Lambda^0 \to p \left(F + \frac{D}{3}\right)$$

F≈0.46, D≈0.8

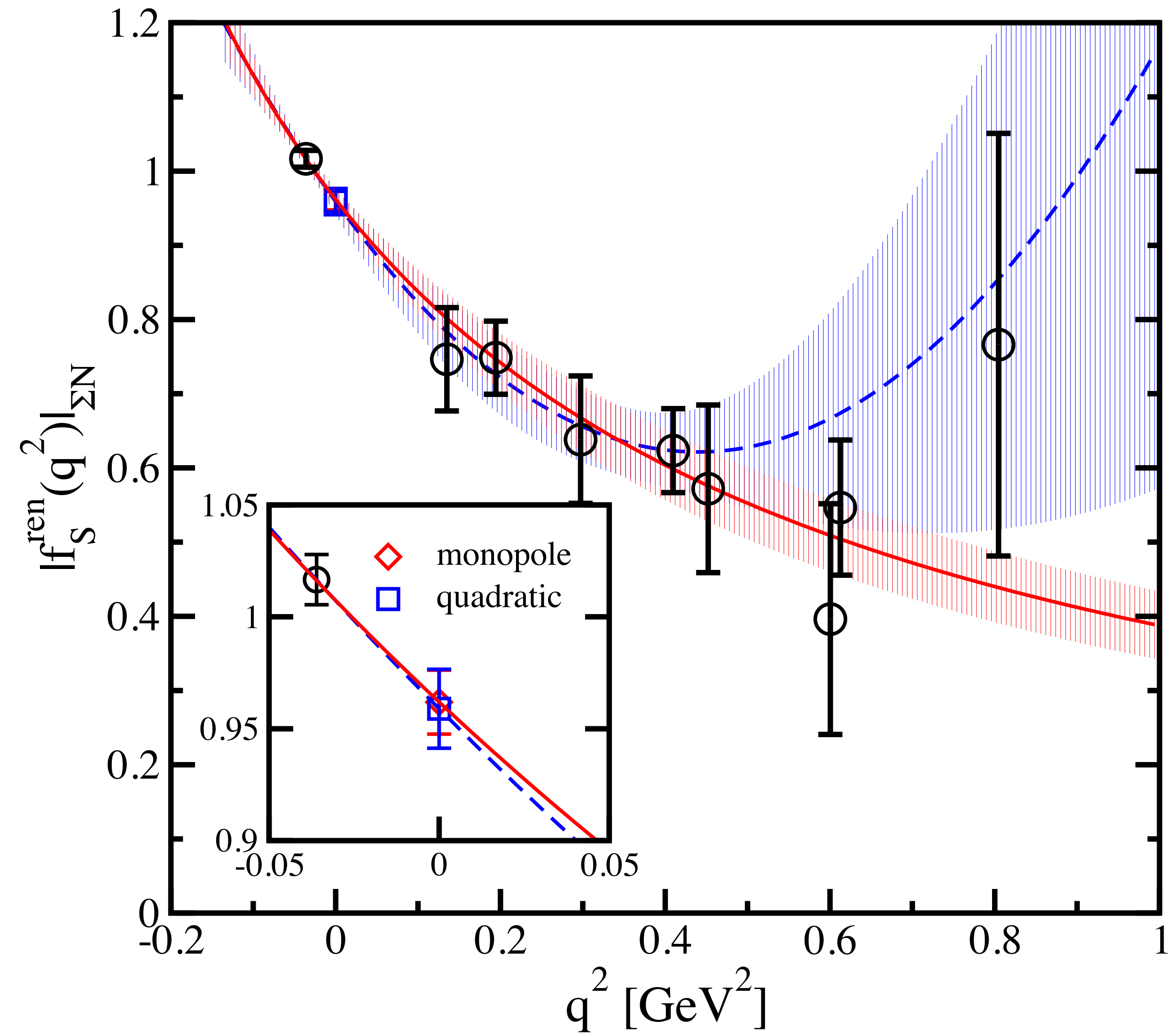
Hyperon semi-leptonic form factor

- Other recent results, S. Sasaki: 1102.4934

$\Xi^0 \to \Sigma^+ \ell \nu_\ell$



$\Sigma^- \to n \ell \nu_\ell$



Summary

Summary

- This week we have looked a variety of aspects of hadron structure and how they can be studied on the Lattice

- From the lattice side, we have learnt about

    - Three point functions

    - Extraction of matrix elements via ratios of 3pt/2pt functions

    - Determination of Form factors, moments of PDFs, GPDs from these matrix elements

Summary

- From a phenomenological point of view, we have studied
  - Elastic scattering ⇒ Form Factors
    - Information on the distribution of charge (quarks) in the transverse plane
  - DIS ⇒ (Moments) of Parton Distribution Functions
    - Distribution of momentum
  - Neutron beta decay ⇒ nucleon axial charge
- Combination of these ideas into a general picture ⇒ Generalised Parton Distribution Functions
    - Transverse densities
    - Spin decomposition

Summary

- Information on hidden flavour, e.g. Strangeness in the nucleon
  - Nucleon sigma terms
  - Implications for Dark Matter searches
- Semileptonic Decays ⇒ $|V_{us}|$