Hadron Structure

James Zanotti
The University of Adelaide

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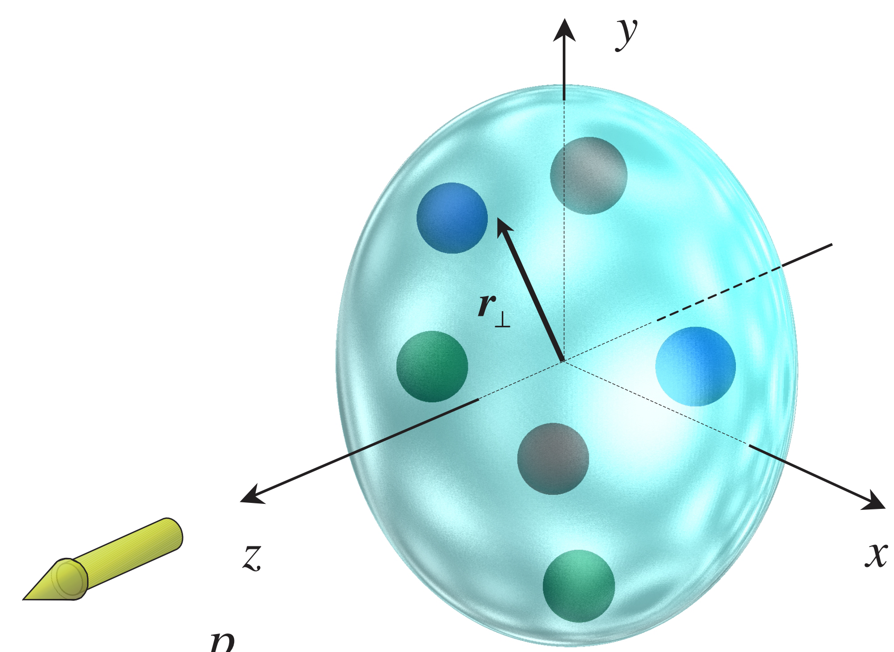
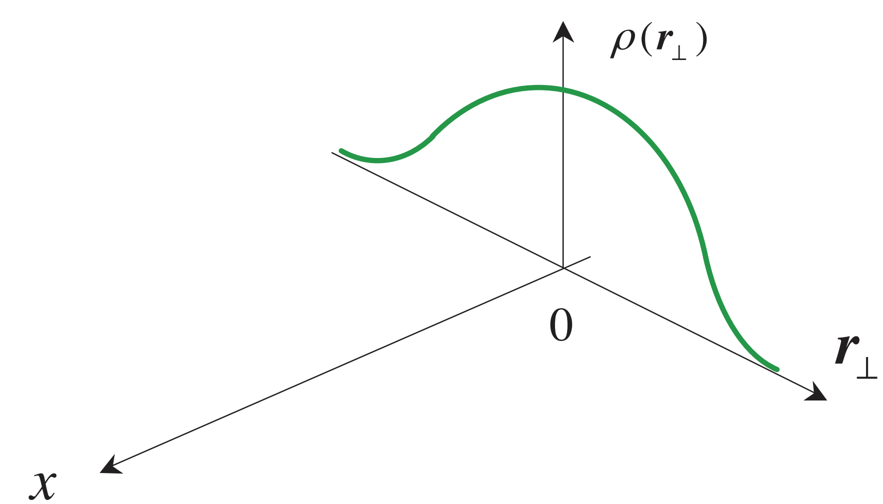
Lecture 4

- Generalised Parton Distributions
    - Definition
    - Relevant limiting cases
    - Impact Parameter GPDs
    - Nucleon Spin
    - Transverse Spin Densities
- Transverse Momentum Dependent Parton Distribution Functions

Generalised Parton Distributions

- So far we have studied
    - Form Factors
        - Information on the distribution of charge (quarks) in the transverse plane





Generalised Parton Distributions

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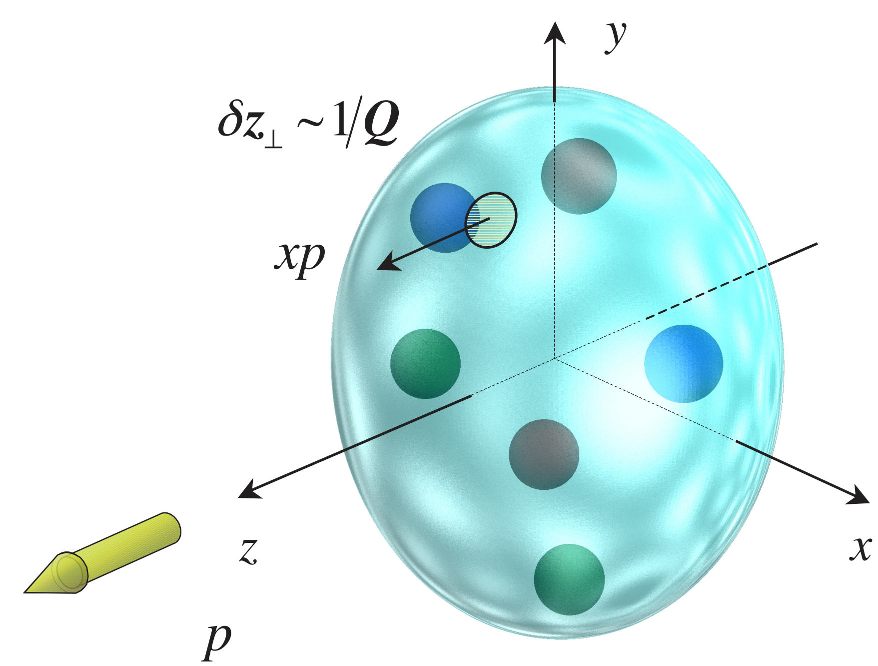
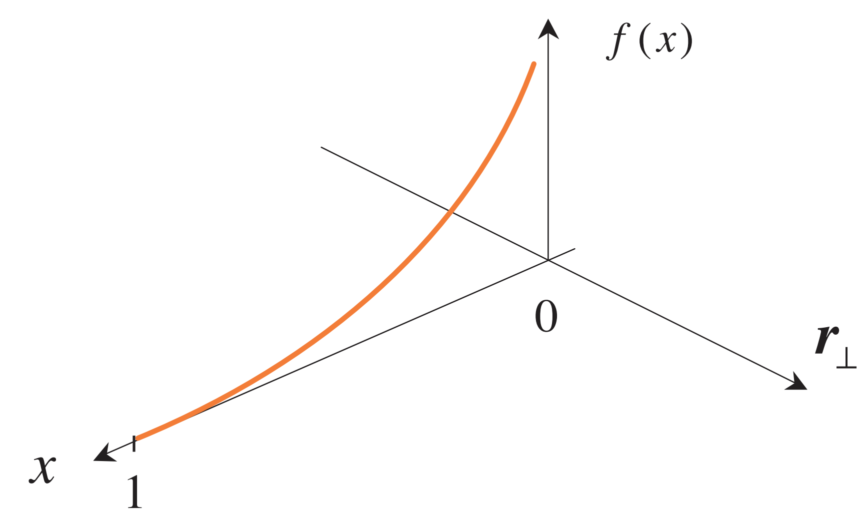
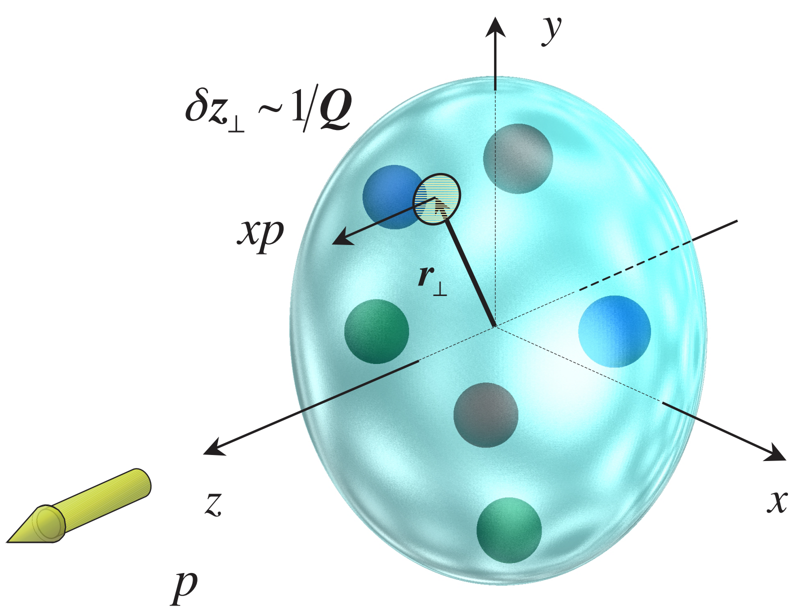
  - (Moments) of Parton Distribution Functions
    - Distribution of momentum

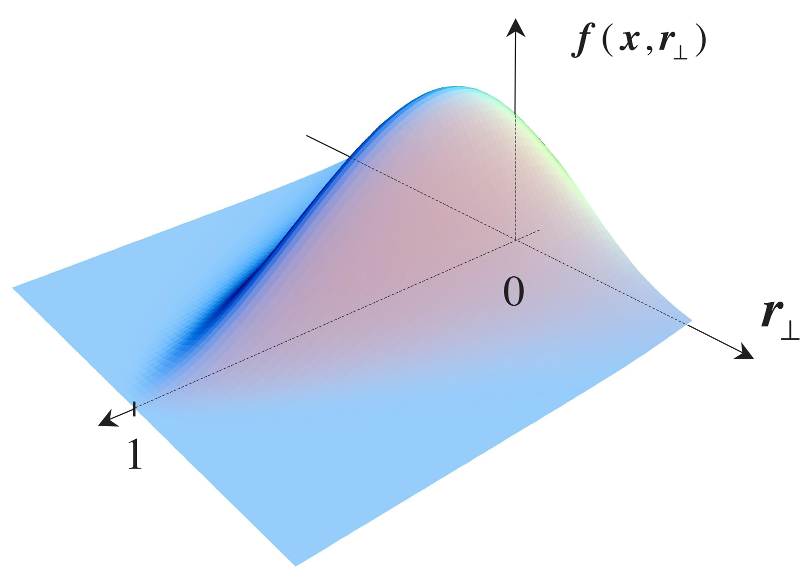




Generalised Parton Distributions

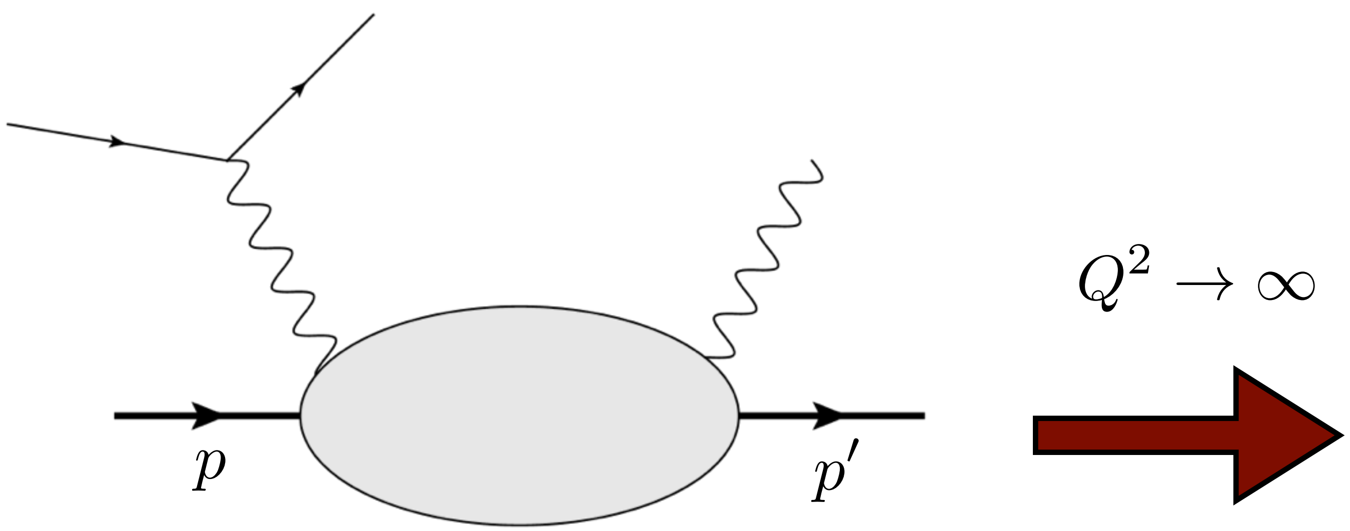
- So far we have studied
  - Form Factors
    - Information on the distribution of charge (quarks) in the transverse plane
  - (Moments) of Parton Distribution Functions
    - Distribution of momentum
- Now we want to combine these ideas into a general picture
  - Generalised Parton Distribution Functions
    - "3D" picture of the nucleon

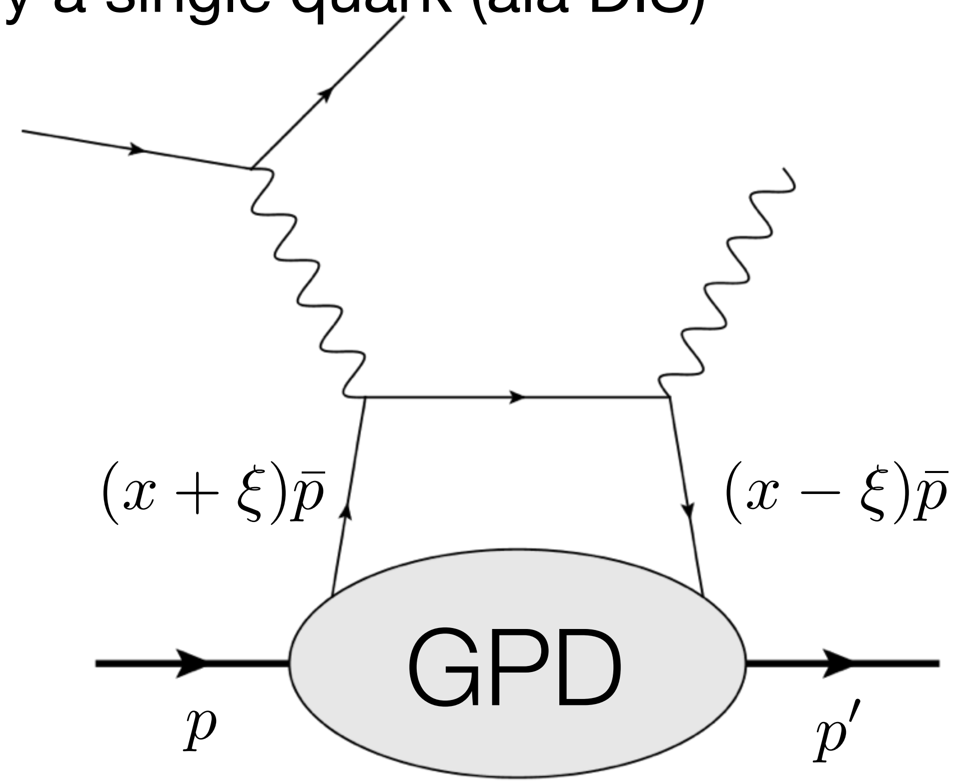




Generalised Parton Distributions

- Formal definition of GPDs:
  - Consider a process where the proton target stays intact (ala elastic)
  - But the probe has enough resolution to identify a single quark (ala DIS)
  - Known as Deeply Virtual Compton Scattering





$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p', s' | \bar{q}(-\frac{1}{2}\lambda n) \not{n} \mathcal{U} q(\frac{1}{2}\lambda n) | p, s \rangle = \bar{u}(p', s') \not{n} u(p, s) H_q(x, \xi, t) + \bar{u}(p', s') i \frac{\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} u(p, s) E_q(x, \xi, t)$$

$$\Delta (= q) = p' - p, \; t = \Delta^2, \; \bar{p} = \frac{p' + p}{2}, \; \xi = -n \cdot \Delta, \; n \cdot \bar{p} = 1$$

Generalised Parton Distributions

M. Diehl (2001): 8 real functions needed for a complete description of the nucleon quark structure at twist 2

$$H(x,\xi,t), E(x,\xi,t), \tilde{H}(x,\xi,t), \tilde{E}(x,\xi,t)$$

$$H_T(x,\xi,t), E_T(x,\xi,t), \tilde{H}_T(x,\xi,t), \tilde{E}_T(x,\xi,t)$$

- If we compare this form with the familiar matrix element of the EM current

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p', s' | \bar{q}(-\frac{1}{2}\lambda n) \not{n} \mathcal{U} q(\frac{1}{2}\lambda n) | p, s \rangle = \bar{u}(p', s') \not{n} u(p, s) H_q(x,\xi,t) + \bar{u}(p', s') i \frac{\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} u(p,s) E_q(x,\xi,t)$$

$$\langle N(p', s') | J^\mu | N(p, s) \rangle = \bar{u}(p', s') \gamma^\mu u(p, s) F_1(Q^2) + \bar{u}(p', s') i \frac{\sigma^{\mu\nu} q_\nu}{2M} u(p,s) F_2(Q^2)$$

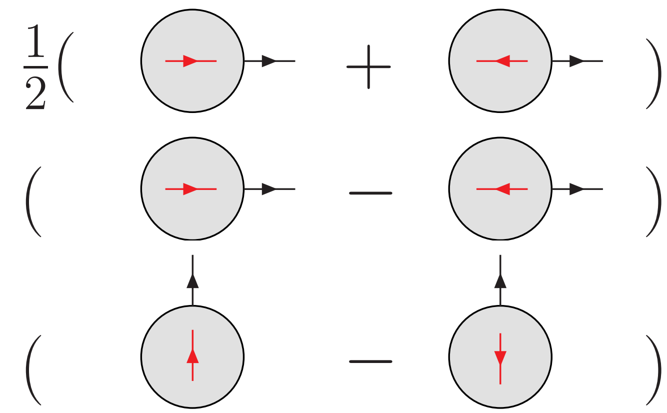
- we get the impression that the GPDs *H* and *E* are just more generalised forms of the ordinary EM form factors F_1 and F_2

Generalised Parton Distributions

Basic Properties

- Forward limit (*t=0*): reproduces the parton distributions

$$H_q(x,0,0) = q(x)$$
$$\tilde{H}_q(x,0,0) = \Delta q(x)$$
$$H_{Tq}(x,0,0) = \delta q(x)$$



- Integrating over all momentum fractions $\int dx$

→ Form Factors

Dirac	$\int dx\, H_q(x,\xi,t) = F_1(t)$
Pauli	$\int dx\, E_q(x,\xi,t) = F_2(t)$
Axial	$\int dx\, \tilde{H}_q(x,\xi,t) = g_A(t)$
Pseudoscalar	$\int dx\, \tilde{E}_q(x,\xi,t) = g_P(t)$
Tensor	$\int dx\, H_{Tq}(x,\xi,t) = g_T(t)$

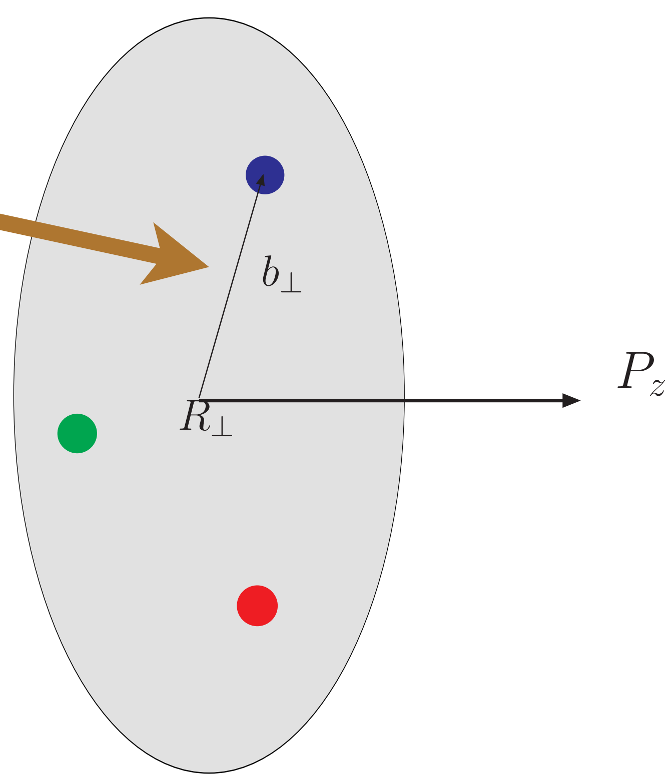
Impact Parameter GPDs (M. Burkardt, 2000)

Quark densities in the transverse plane

- Recall: Quark (charge) distribution in the transverse plane

$$q(b_\perp^2) = \int d^2\Delta_\perp \, \mathrm{e}^{-i\vec{b}_\perp \cdot \Delta_\perp} F_1(\Delta^2)$$

Distance of (active) quark to the centre of momentum in a fast moving nucleon



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No momentum transfer in longitunidal direction

- Recall: Quark (charge) distribution in the transverse plane

$$q(b_\perp^2) = \int d^2\Delta_\perp \, \mathrm{e}^{-i\vec{b}_\perp \cdot \Delta_\perp} F_1(\Delta^2)$$

- Probabilistic interpretation of $H(x,\xi,t), \tilde{H}(x,\xi,t), H_T(x,\xi,t)$ at $\xi = 0$

$$q(x, \vec{b}_\perp) = \frac{1}{(2\pi)^2} \int d^2\Delta_\perp \, \mathrm{e}^{-i\vec{b}_\perp \cdot \Delta_\perp} H(x, 0, \Delta_\perp^2)$$

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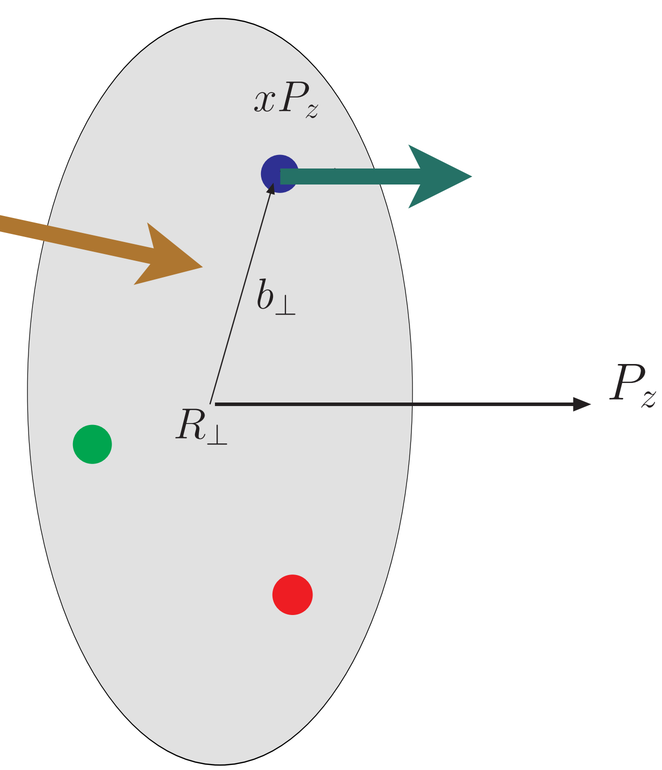

Decompose into contributions from individual quarks with momentum fraction, x

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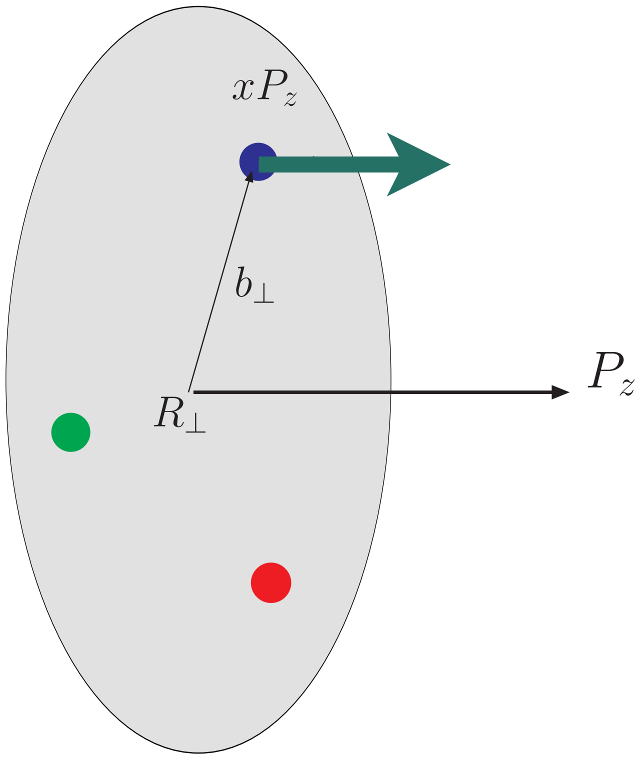
- Note that since the longitudinal momentum is fixed *(x)*
    - Longitudinal position undetermined (Heisenburg)
    - Distribution in impact parameter space meaningful



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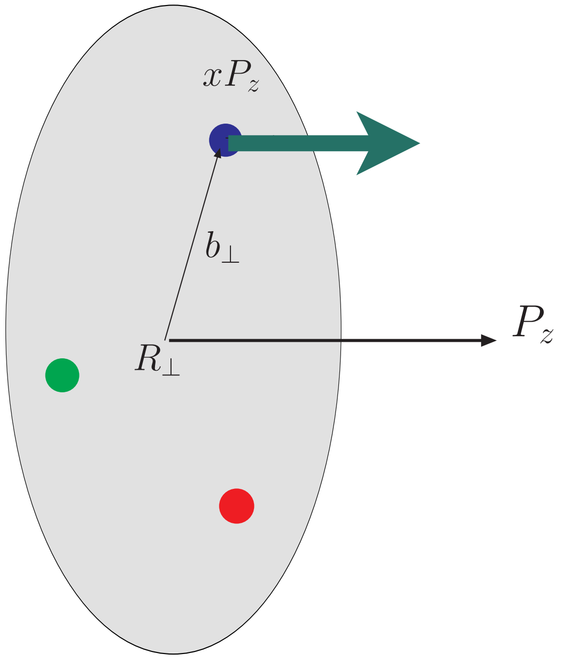
$$q(x,\vec{b}_\perp) = \frac{1}{(2\pi)^2} \int d^2\Delta_\perp \, \mathrm{e}^{-i\vec{b}_\perp \cdot \Delta_\perp} H(x,0,\Delta_\perp^2)$$

- If we consider the case where one of the quarks carries all of the nucleon's momentum



- Then the quark must be sitting at the centre-of-momentum

$$\lim_{x\to 1} q(x,\vec{b}_\perp) \propto \delta^2(\vec{b}_\perp)$$

- Hence the GPD must be independent of t

$H(x,0,\Delta_\perp^2)$: t-independent for x ⟶1

Generalised Parton Distributions

- Experimental access to GPDs is provided by
  - Deeply Virtual Compton Scattering (DVCS) $ep \to ep\gamma$
  - Meson electroproduction $ep \to ep\pi, \rho, \omega, \ldots$
- However direct (model independent) extraction from experimental data difficult (impossible?)

⇒ Additional input from, e.g. Lattice, would be extremely helpful

Moments of GPDs

- Yesterday we saw how moments of Structure Functions (or Parton Distribution Functions) can be obtained from matrix elements of local operators that could be computed on the Lattice

- Similarly, moments w.r.t *x* of GPDs are defined in terms of generalised form factors

$$\int_{-1}^{1} dx\, x^{n-1}\, H_q(x,\xi,t) = \sum_{i=0}^{\frac{n-1}{2}} A^q_{n,2i}(t)(-2\xi)^{2i} + C^q_n(t)(-2\xi)^n|_{\text{n even}}$$

$$\int_{-1}^{1} dx\, x^{n-1}\, E_q(x,\xi,t) = \sum_{i=0}^{\frac{n-1}{2}} B^q_{n,2i}(t)(-2\xi)^{2i} - C^q_n(t)(-2\xi)^n|_{\text{n even}}$$

- where the GFFs are obtained from matrix elements of local (twist-2) operators

$$\langle p', s'|\mathcal{O}^q_{\{\mu_1\cdots\mu_n\}}|p,s\rangle = \bar{u}(p',s')\gamma_{\{\mu_1} u(p,s) \sum_{i=0}^{\frac{n-1}{2}} A^q_{n,2i}(t)\Delta_{\mu_2}\ldots\Delta_{\mu_{2i+1}}\bar{p}_{\mu_{2i+2}}\ldots\bar{p}_{\mu_n\}}$$

$$- \frac{i\Delta^\nu}{2M}\bar{u}(p',s')\sigma_{\nu\{\mu_1} u(p,s) \sum_{i=0}^{\frac{n-1}{2}} B^q_{n,2i}(t)\Delta_{\mu_2}\ldots\Delta_{\mu_{2i+1}}\bar{p}_{\mu_{2i+2}}\ldots\bar{p}_{\mu_n\}}$$

$$+ \frac{1}{M}\bar{u}(p',s')u(p,s)C^q_n(t)\Delta_{\mu_1}\ldots\Delta_{\mu_n\}}|_{\text{n even}}$$

$$\mathcal{O}^q_{\mu_1\cdots\mu_n} = \bar{q}\,\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n}\,q$$

Moments of GPDs

- Similar equations exist for the polarised case for matrix elements of the operators

$$\mathcal{O}_q^{5;\mu_1\cdots\mu_n} = \overline{q}\,\gamma^{\mu_1}\gamma^5\,\overleftrightarrow{D}^{\mu_2}\ldots\overleftrightarrow{D}^{\mu_n}\,q$$

- in terms of the GFFs $\tilde{A},\ \tilde{B}$

- and also for the tensor operators $\mathcal{O}_q^{\sigma;\mu\nu\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)\overline{q}\,i\sigma_{\mu\nu}\,\overleftrightarrow{D}^{\mu_1}\ldots\overleftrightarrow{D}^{\mu_n}\,q$

- with GFFs $A_T,\ B_T,\ \tilde{A}_T,\ \tilde{B}_T$

- It is also possible to construct matrix elements for towers of gluonic operators

- Note the relation to the more familiar form factors

$$A_{10}(t) = F_1(Q^2)$$
$$B_{10}(t) = F_2(Q^2)$$
$$\tilde{A}_{10}(t) = G_A(Q^2)$$

etc...

Moments of GPDs

- Matrix elements are then extracted from the lattice three-point functions as before using ratios

$$R(t,\tau;\vec{p}\,',\vec{p};\mathcal{O},\Gamma) \;=\; \frac{G_\Gamma(t,\tau;\vec{p}\,',\vec{p},\mathcal{O})}{G_2(t,\vec{p}\,')}\left[\frac{G_2(\tau,\vec{p}\,')G_2(t,\vec{p}\,')G_2(t-\tau,\vec{p})}{G_2(\tau,\vec{p})G_2(t,\vec{p})G_2(t-\tau,\vec{p}\,')}\right]^{\frac{1}{2}}$$

- and the coefficients of the generalised form factors are computed using

$$F(\Gamma,\mathcal{J}) = \frac{1}{4}\mathrm{Tr}\left\{\Gamma\left(\gamma_4 - i\frac{\vec{p}\,'\cdot\vec{\gamma}}{E_{\vec{p}\,'}} + \frac{m}{E_{\vec{p}\,'}}\right)\mathcal{J}\left(\gamma_4 - i\frac{\vec{p}\cdot\vec{\gamma}}{E_{\vec{p}}} + \frac{m}{E_{\vec{p}}}\right)\right\}$$

Moments of GPDs

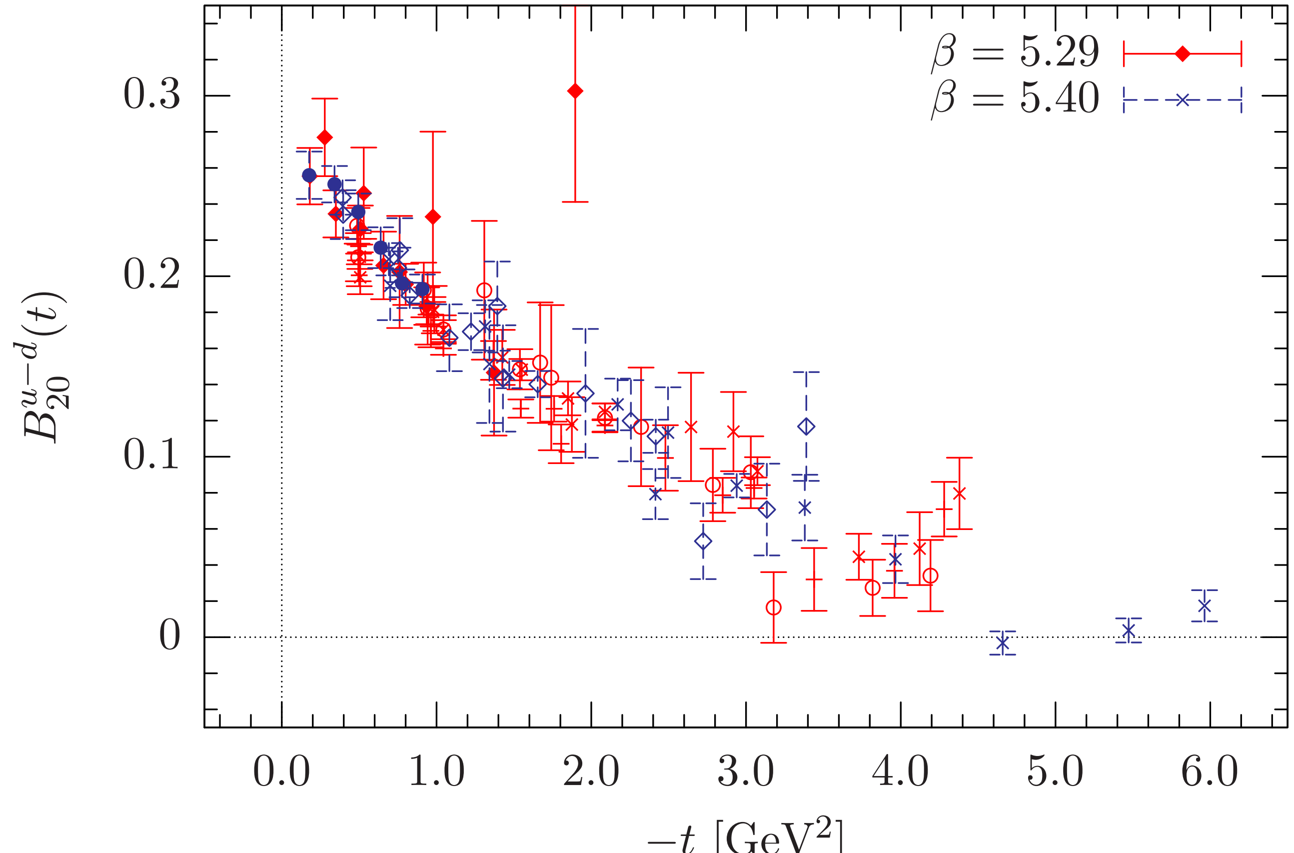
A.Sternbeck (QCDSF) Lattice 2011

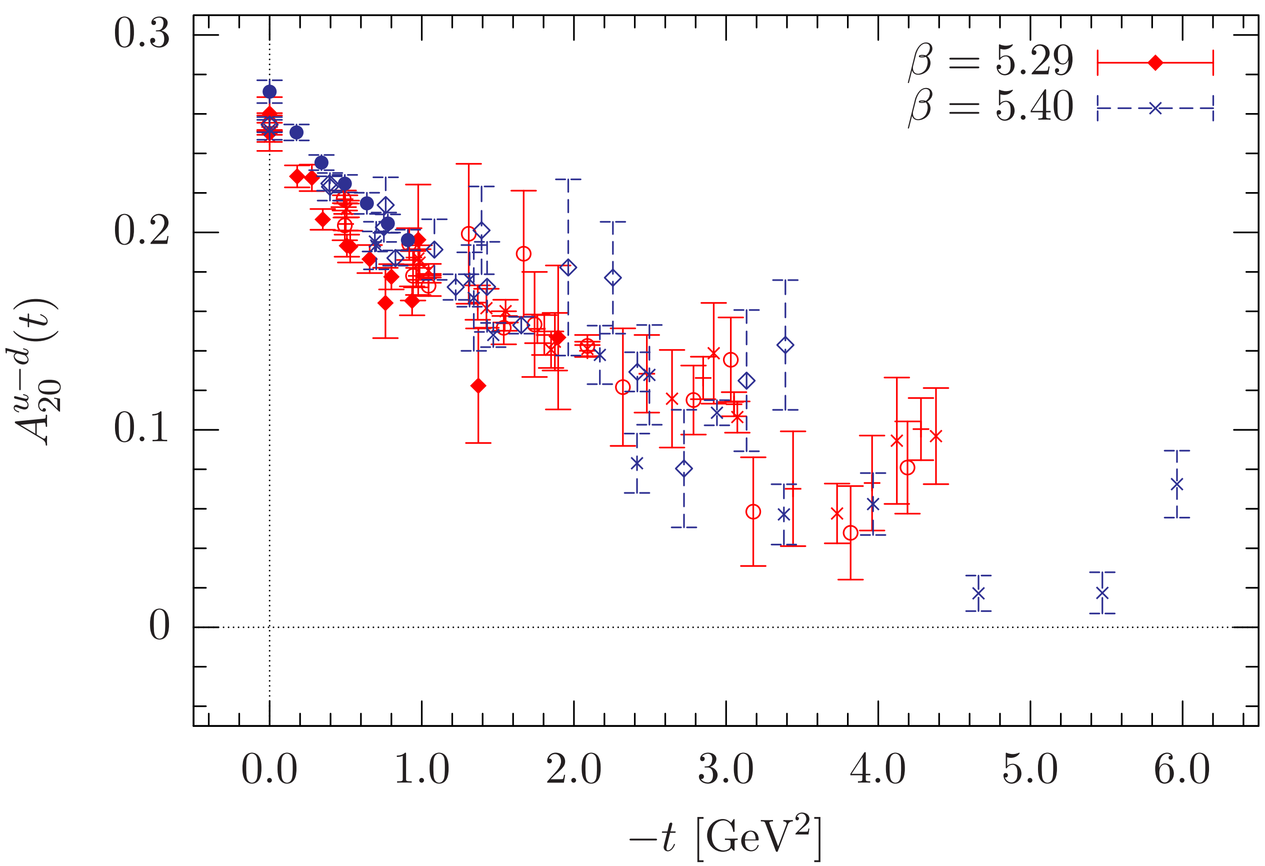
Example

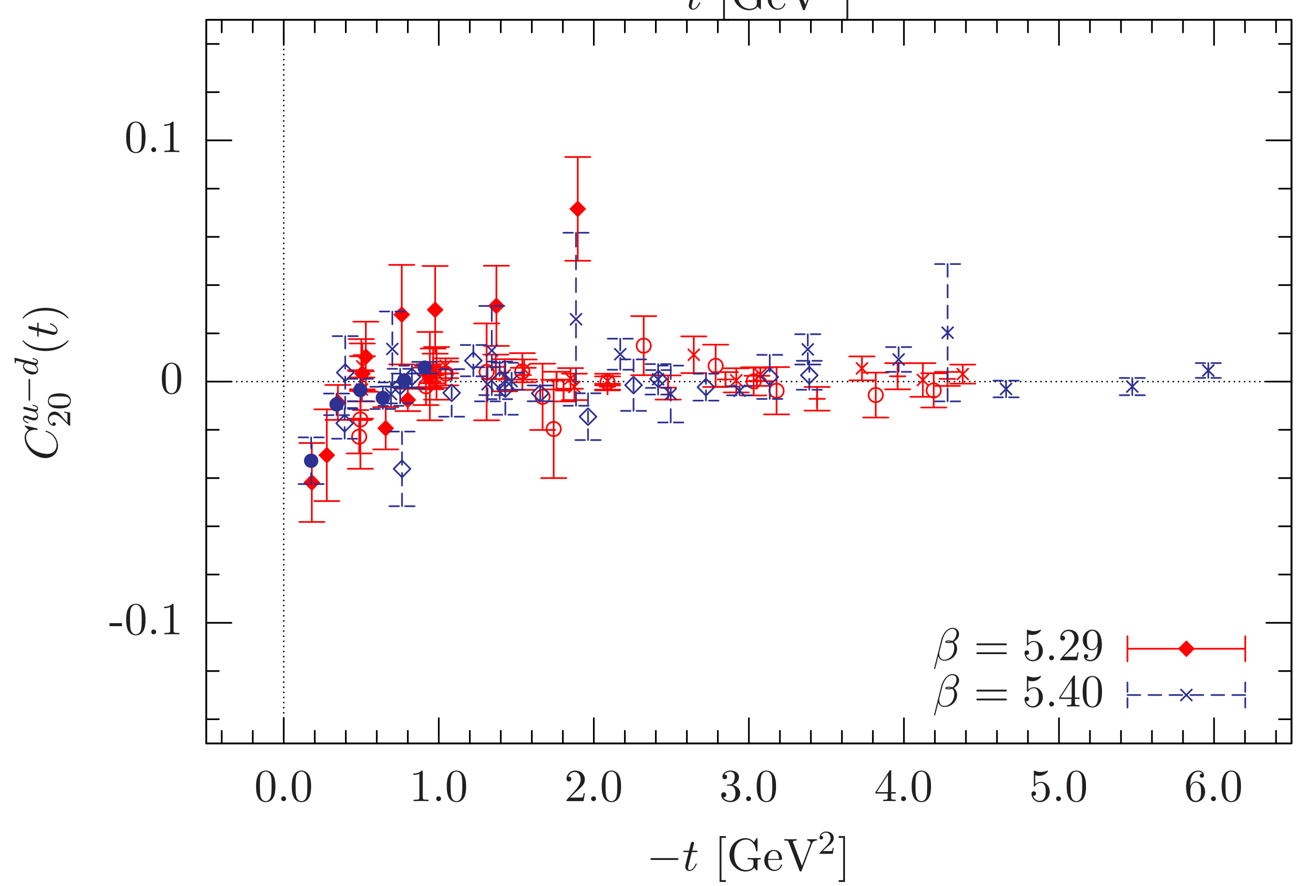
- Results for generalised form factors relevant for the second moments of H and E

$$\int_{-1}^{1} dx\, x H_q(x,\xi,t) = A_{20}^q(t) + 4\xi^2 C_2^q(t)$$

$$\int_{-1}^{1} dx\, x E_q(x,\xi,t) = B_{20}^q(t) - 4\xi^2 C_2^q(t)$$







Impact Parameter GPDs (M. Burkardt, 2000)

- Recall in our discussion about GPDs in the impact parameter plane

$$\lim_{x\to 1} q(x,\vec{b}_\perp) \propto \delta^2(\vec{b}_\perp)$$

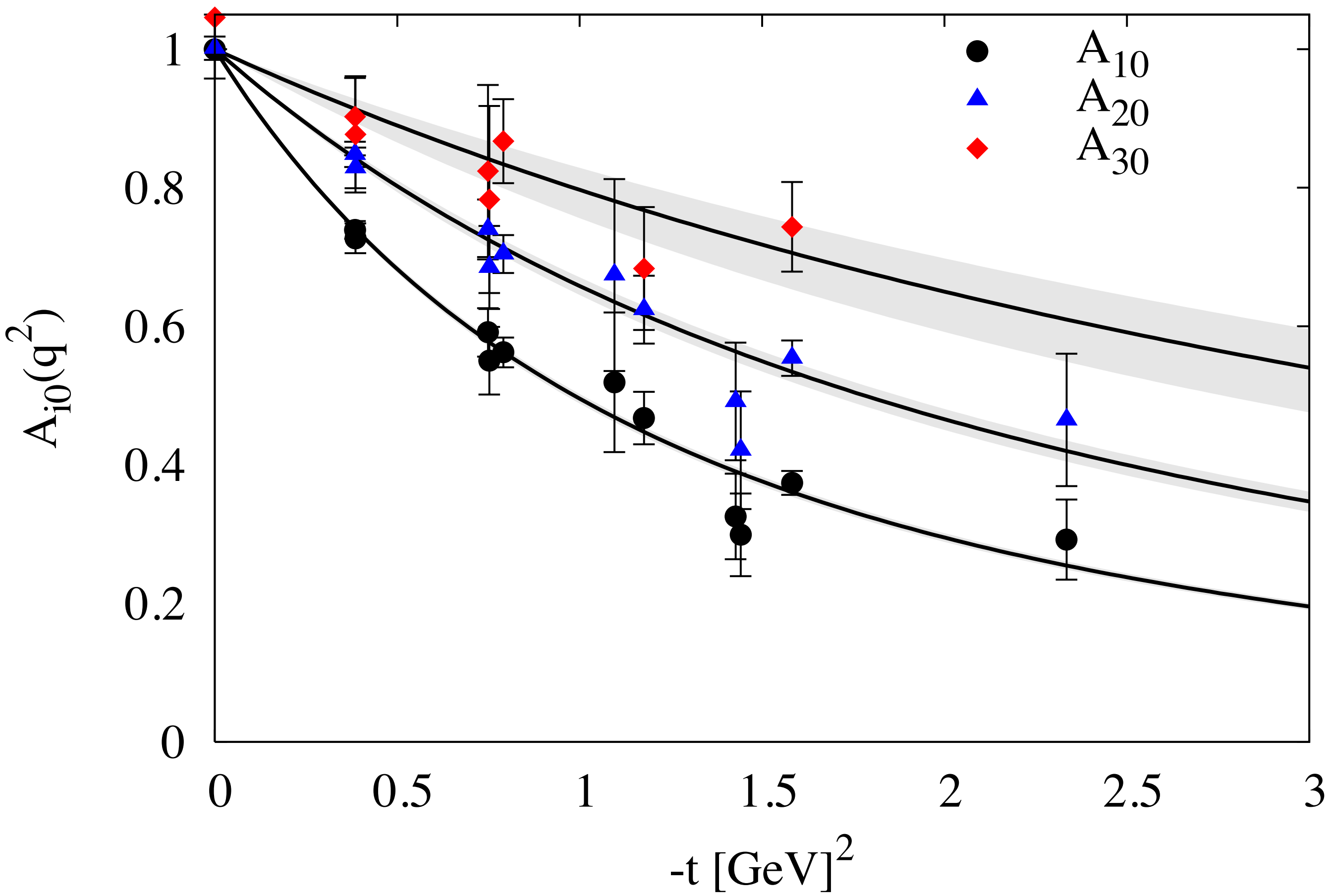
$H(x,0,\Delta_\perp^2)$: t-independent for x $\longrightarrow$ 1

- Since higher moments are weighted more by the larger-*x* range $\int_{-1}^{1} dx\, x^{n-1} H_q(x,\xi,t)$

$\Rightarrow$ Slope of GFFs should decrease as *n* increases

Impact Parameter GPDs

GFFs A_{10}, A_{20}, A_{30} normalised to unity at t=0



- GFFs are fitted with a dipole form

$$A^q_{n0}(t) = \frac{A^q_{n0}(0)}{(1 - t/M_n^2)^2} = \frac{\langle x^{n-1} \rangle}{(1 - t/M_n^2)^2}$$

- Form factors flatten as *n* grows
- Dipole mass grows with *n*

$$\int_{-1}^{1} dx\, x^{n-1}\, H_q(x, \xi, t) = A^q_{n0}(t)$$

- *H_q* (as a function of *t*) becomes wider as *x* grows

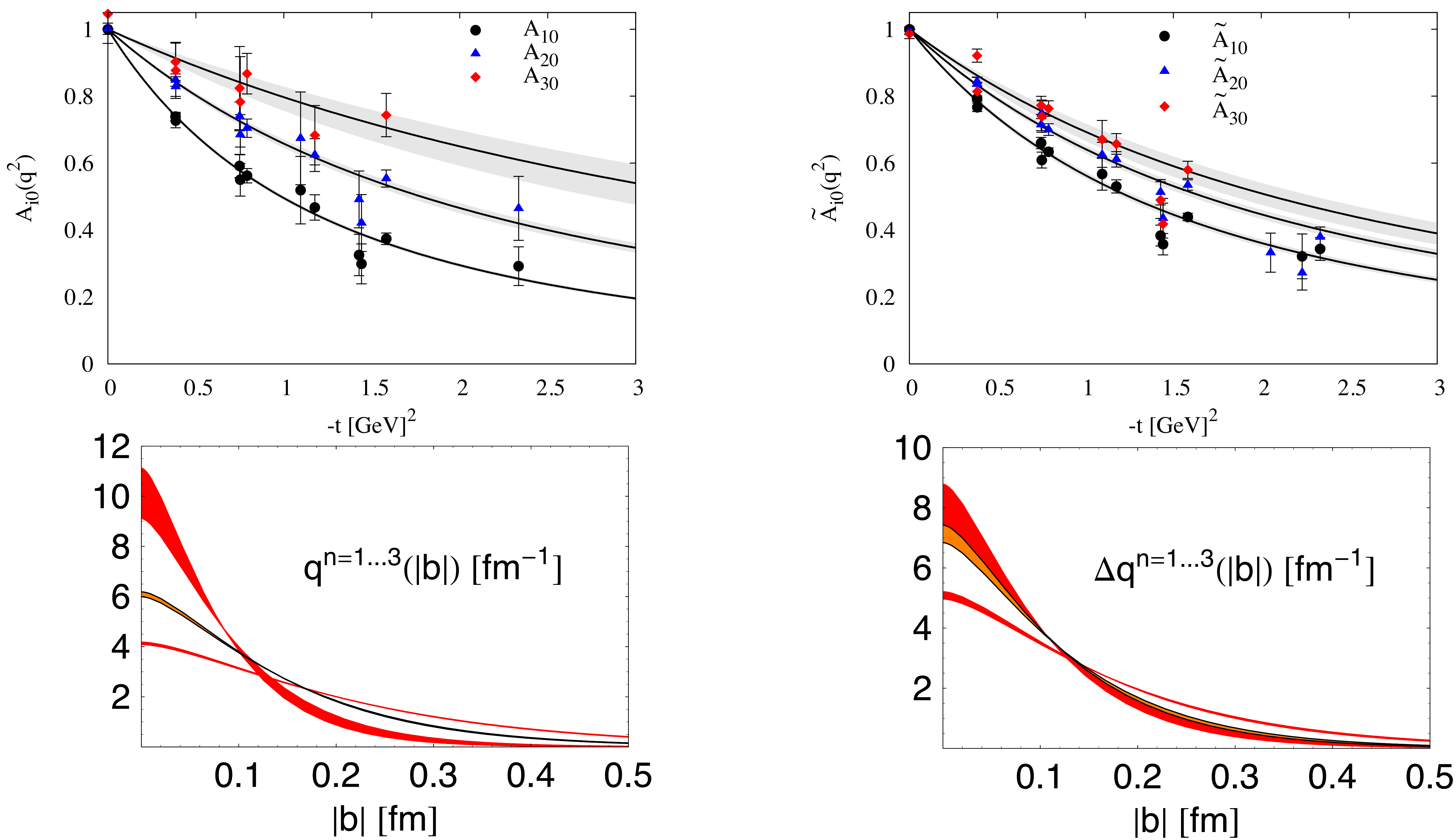
$$q(x, \vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, \mathrm{e}^{-i\vec{b}_\perp \cdot \Delta_\perp}\, H(x, 0, \Delta_\perp^2)$$

- *q* (as a function of $b_\perp$) becomes narrower as *x* grows

Impact Parameter GPDs

QCDSF: hep-lat/0609001

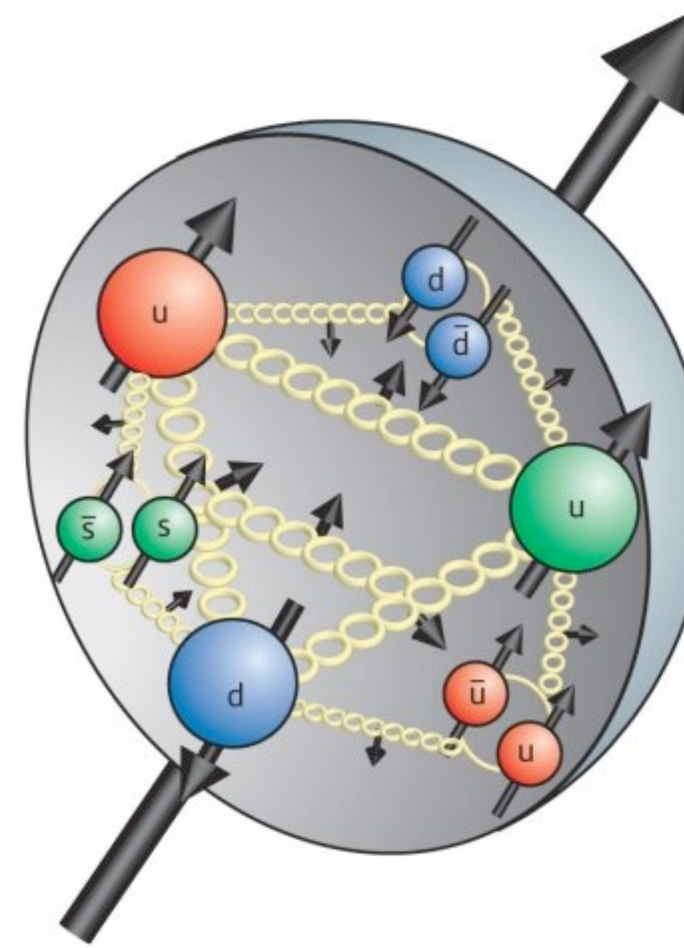
Fourier transform the fitted dipole forms to impact parameter space



Nucleon Spin

- Polarised DIS: only ~30% of the proton's spin due to quark spins

⇨ "Spin crisis"



- Only a "Spin puzzle" - remaining 70% made up of
    - quark orbital angular momentum
    - gluon spin
    - gluon orbital angular momentum
- Exact decomposition unknown
- How are they defined/measured?
- Ongoing controversy/discussion in the field regarding decomposition of nucleon spin

Nucleon Spin

- Total angular momentum well defined $J^p = \frac{1}{2}$
- Ambiguities arise when decomposing J into contributions from different constituents
- Gauge theories: changing gauge may also shift angular momentum between various degrees of freedom
  - Decomposition of angular momentum in general depends on scheme
- Need to be aware of this scheme-dependence in the physical interpretation of experimental/lattice/model results in terms of spin vs. OAM
- Two common decompositions:

$$J^z = \frac{1}{2}\Delta\Sigma + \sum_q L_q + J_g$$

Ji (1997)

$$J^z = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

Jaffe & Manohar (1990)

- $\Delta\Sigma = \Delta u + \Delta d + \Delta s$ total fraction of the helicity carried by the quarks

Nucleon Spin

$$J^z = \frac{1}{2}\Delta\Sigma + \sum_q L_q + J_g$$

Ji

$$J^z = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

Jaffe & Manohar

- Only $\Delta\Sigma$ common between the two decompositions
- In general $L_q \neq \mathcal{L}_q$ or $J_g \neq \Delta G + \mathcal{L}_g$
- ΔG measured in p-p scattering
- Controversy surrounds: Is there a gauge-invariant separation of J_g into ΔG and L_g
- Ji: Δq and J_q determined from experiment or Lattice
- Local operator exists for $L_q = q^\dagger\left(\vec{r} \times i\vec{D}\right)q$ but $L_q = J_q - \frac{1}{2}\Delta q$ easier
- J_g accessible from gluon GPDs, but $J_g = \frac{1}{2} - J_q$ easier

Ji's Spin Sum Rule

- Spin decomposed in terms of quark and gluon angular momentum

$$\frac{1}{2} = \sum_q J_q(\mu^2) + J_g(\mu^2)$$

- Further decomposition into spin and orbital angular momentum

$$J^z = \tfrac{1}{2}\Delta\Sigma + \textstyle\sum_q L_q + J_g$$

- Also expressed in terms of moments of GPDs

$$J_{q/g} = \frac{1}{2}[A_{20}^{q/g}(\Delta^2=0) + B_{20}^{q/g}(\Delta^2=0)]$$

- Matrix elements of the energy momentum tensor

$$\langle P'|T^{\mu\nu}|P\rangle = \overline{U}(P')\left\{\gamma^\mu \overline{P}^\nu A_{20}(\Delta^2) + \frac{i\sigma^{\mu\rho}\Delta_\rho \overline{P}^\nu}{2m_N} B_{20}(\Delta^2) + \frac{\Delta^\mu\Delta^\nu}{m_N} C_{20}(\Delta^2)\right\} U(P)$$

Momentum conservation:

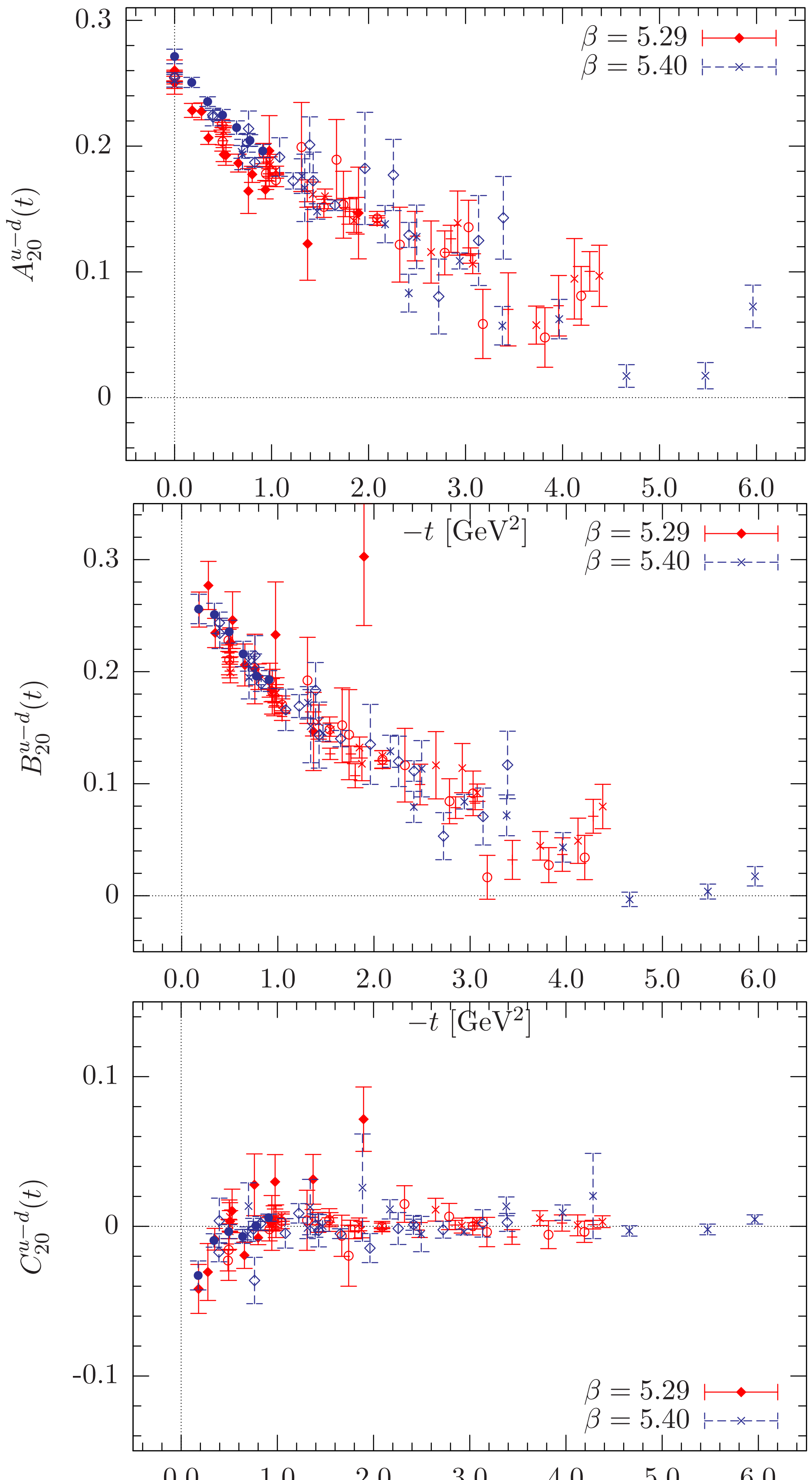
$$\begin{aligned} 1 &= \sum_q A_{20}^q(0) + A_{20}^g(0) \\ &= \sum_q \langle x\rangle_q + \langle x\rangle_g \end{aligned}$$

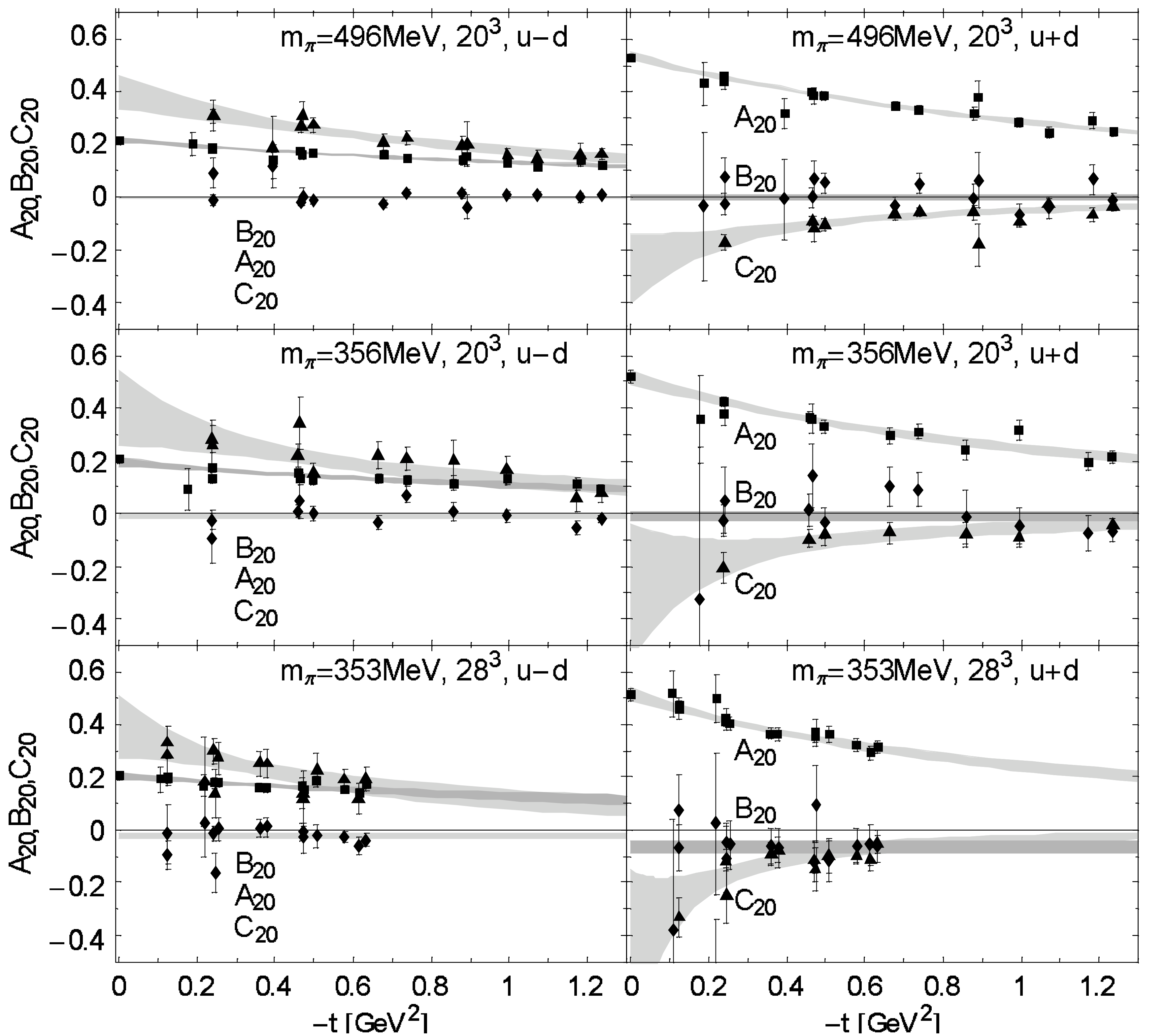
Anomalous gravitomagnetic moment

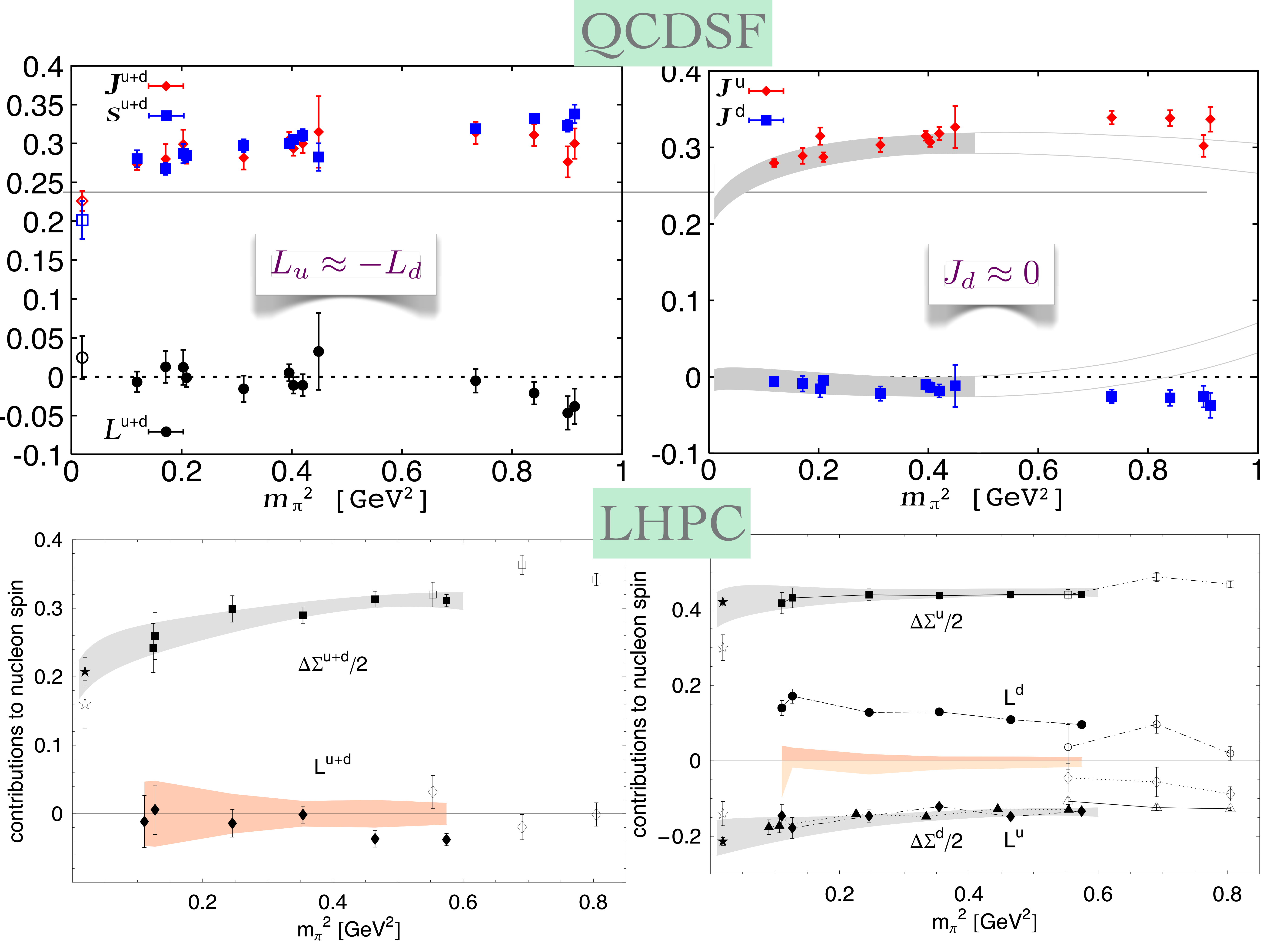
$$0 = \sum_q B_{20}^q(0) + B_{20}^g(0)$$

Generalised Form Factors A_2, B_2, C_2

A.Sternbeck (QCDSF) Lattice 2011 LHPC: PRD 77, 094502(2008), 0705.4295






QCDSF
J^{u+d}
S^{u+d}
L^{u+d}
$L_u \approx -L_d$
m_π^2 [GeV^2]
J^u
J^d
$J_d \approx 0$
m_π^2 [GeV^2]
LHPC
contributions to nucleon spin
$\Delta\Sigma^{u+d}/2$
L^{u+d}
m_π^2 [GeV^2]
contributions to nucleon spin
$\Delta\Sigma^u/2$
L^d
$\Delta\Sigma^d/2$
L^u
m_π^2 [GeV^2]

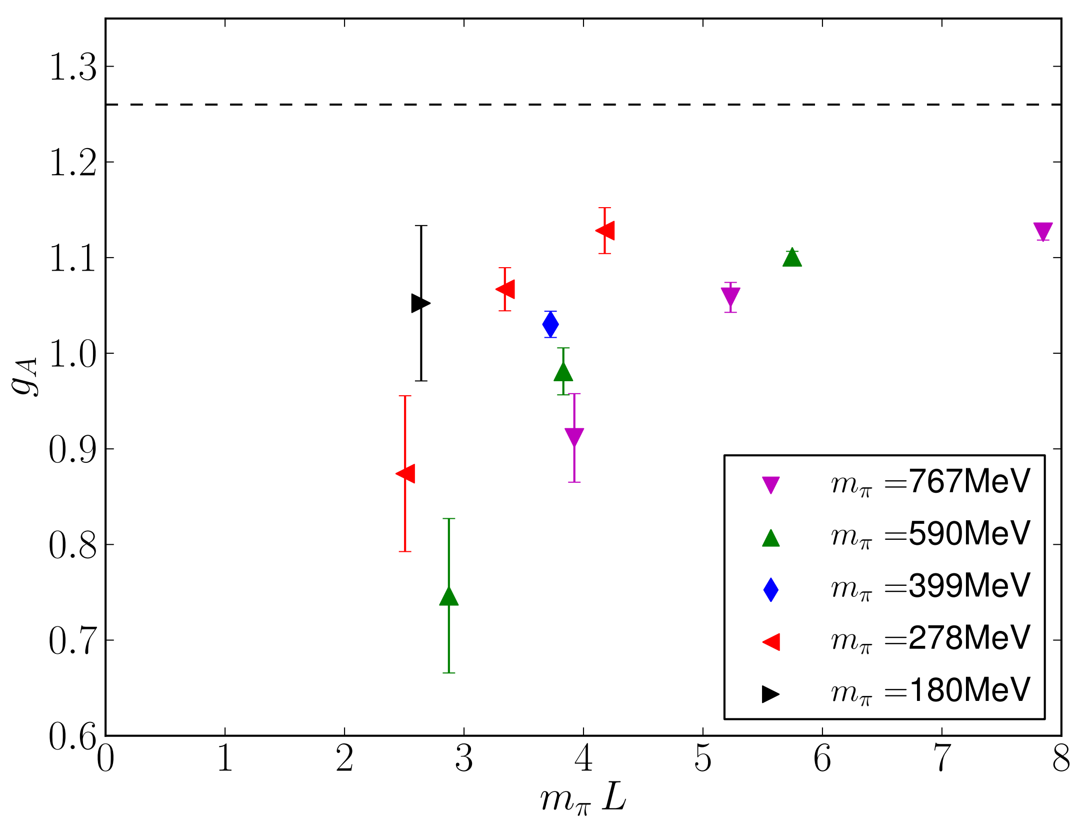
Nucleon Spin

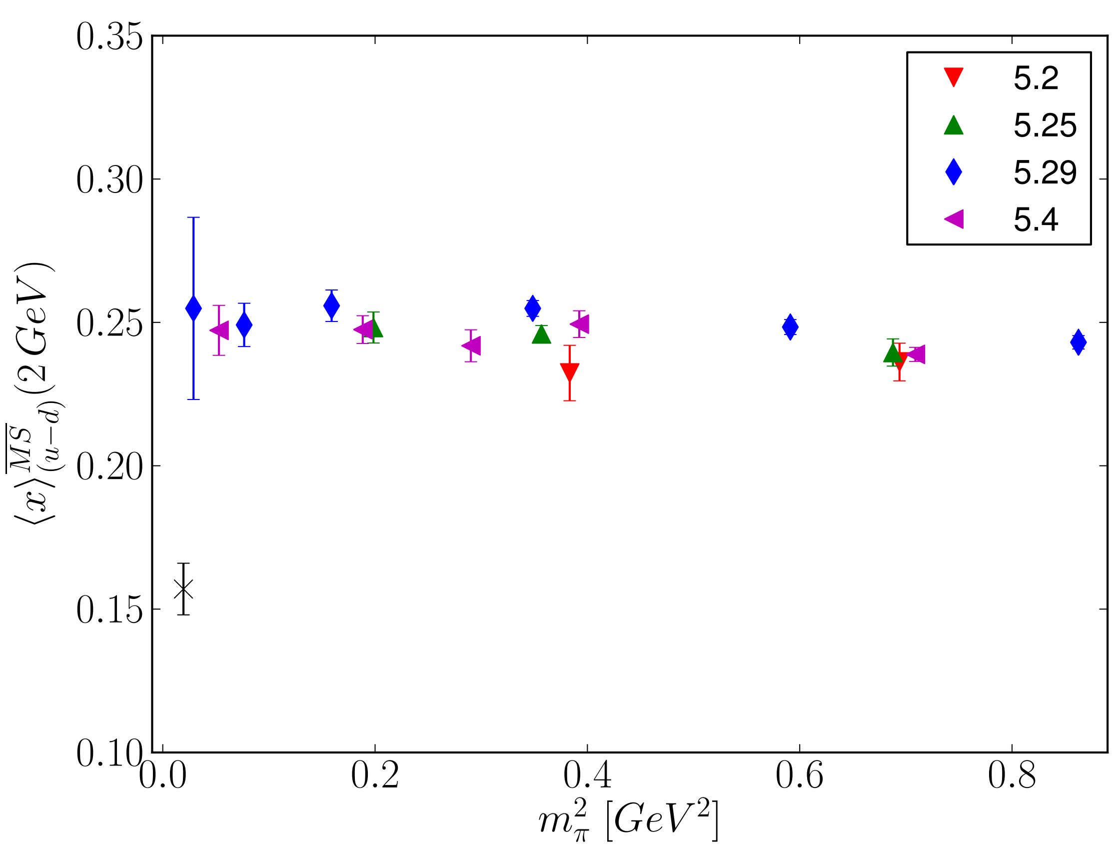
- However from our lessons yesterday regarding the systematic errors in g_A and $\langle x \rangle$ we must be careful when making precision statements from the current lattice data

  - $\Delta u, \ \Delta d$ likely to suffer from finite size effects

  - $A^q_{20}(0) = \langle x \rangle^q$ may suffer from excited state contamination

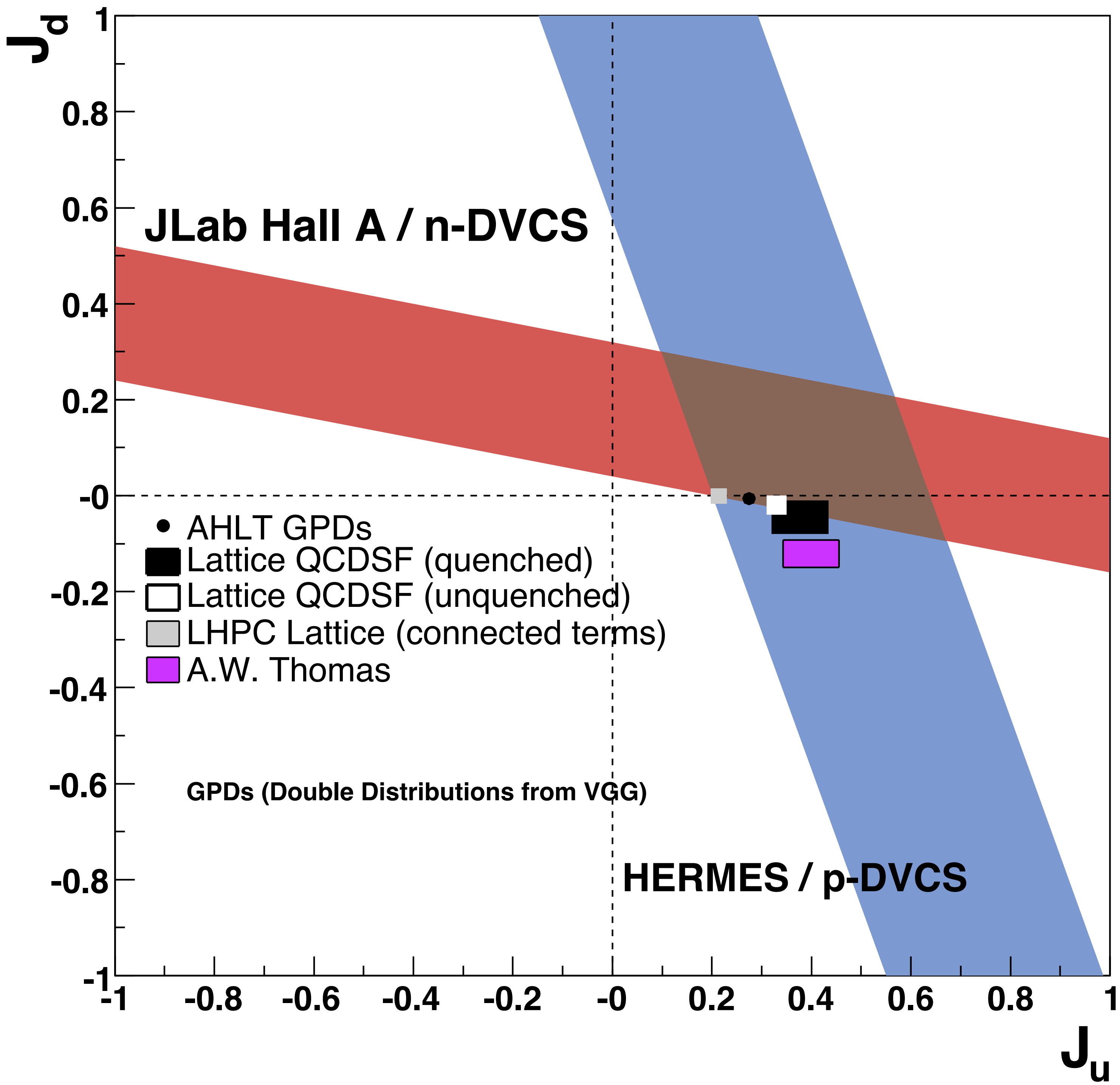
  - They both are likely to have non-trivial chiral extrapolations





Nucleon Spin

- Comparison of current lattice determinations of J_u and J_d with experimental constraints



Transverse Spin Structure of the Nucleon

- Transverse densities:

$$\rho^n(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx\, x^{n-1} \rho(x, b_\perp, s_\perp, S_\perp) = \frac{1}{2}\Big\{ A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{Tn0}(b_\perp^2) - \frac{1}{4m^2} \Delta_{b_\perp} \widetilde{A}_{Tn0}(b_\perp^2) \right)$$
$$+ \frac{b_\perp^j \epsilon^{ji}}{m} \left(S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \overline{B}'_{Tn0}(b_\perp^2) \right) + s_\perp^i (2 b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}) S_\perp^j \frac{1}{m^2} \widetilde{A}''_{Tn0}(b_\perp^2) \Big\}$$

[Diehl & Haegler, 2005] [Burkardt, 2005]

$$F(b_\perp^2) = \int d^2\Delta_\perp\, \mathrm{e}^{-i\vec{b}_\perp \cdot \Delta_\perp} F(\Delta_\perp^2) = \int d^2\Delta_\perp\, \mathrm{e}^{-i\vec{b}_\perp \cdot \Delta_\perp} \frac{F(0)}{(1 - \Delta_\perp^2/M^2)^p}$$

Transverse Spin Structure of the Nucleon

- Aim to gain insights into the spin density of quarks inside the nucleon

- Transversity $\delta q(x) = h_1(x)$: prob to find transversely polarised q with mom fraction x in a transversely polarised nucleon

- Sivers, $f_{1T}^{\perp}(x, k_{\perp}^2)$: measures correlation of intrinsic q trans. momentum and trans. nucl. spin

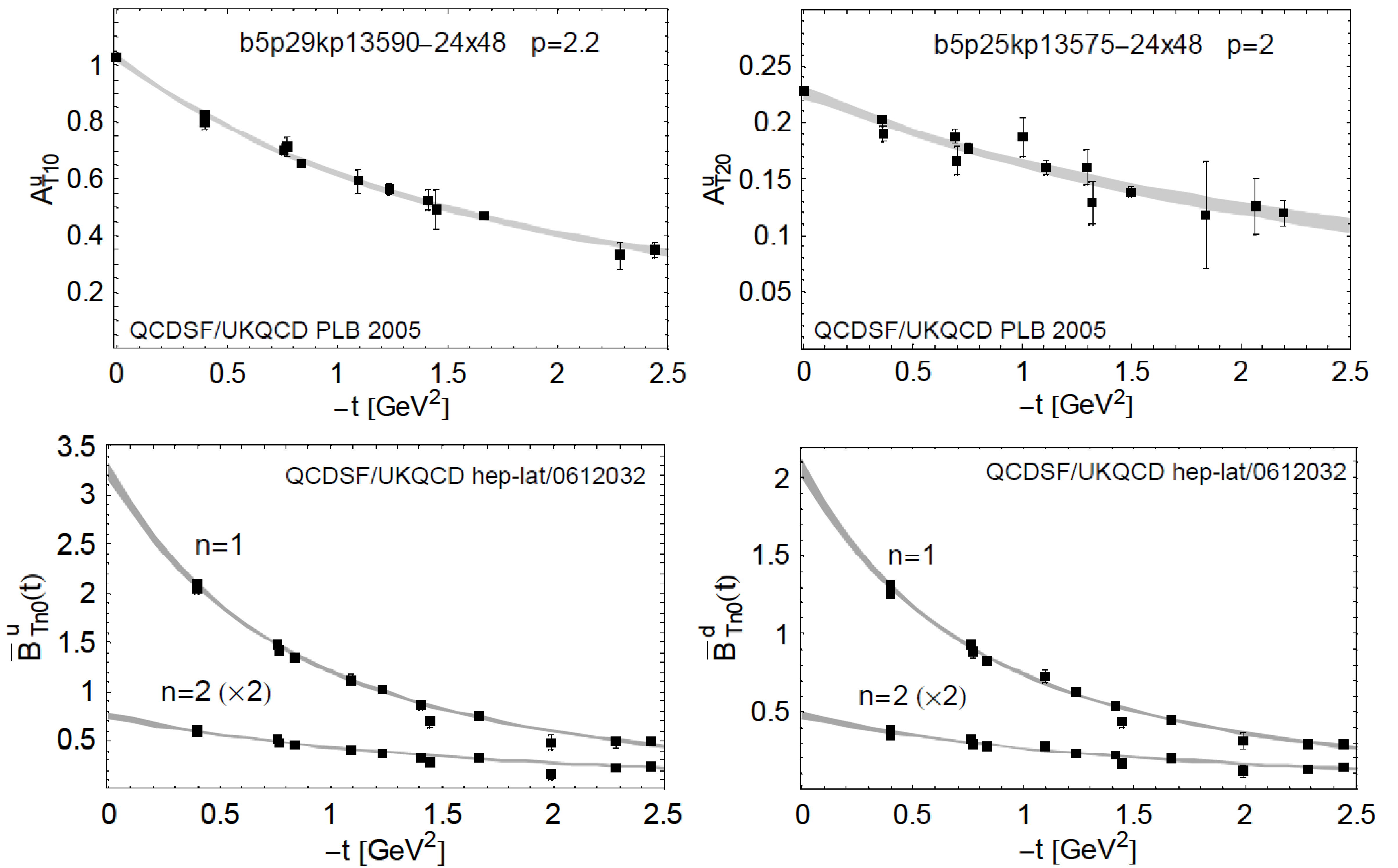
- Boer-Mulders $h_1^{\perp}(x, k_{\perp}^2)$: measures correlation of intrinsic q trans. momentum and trans q spin

Non-vanishing ⟶ interesting experimental observables

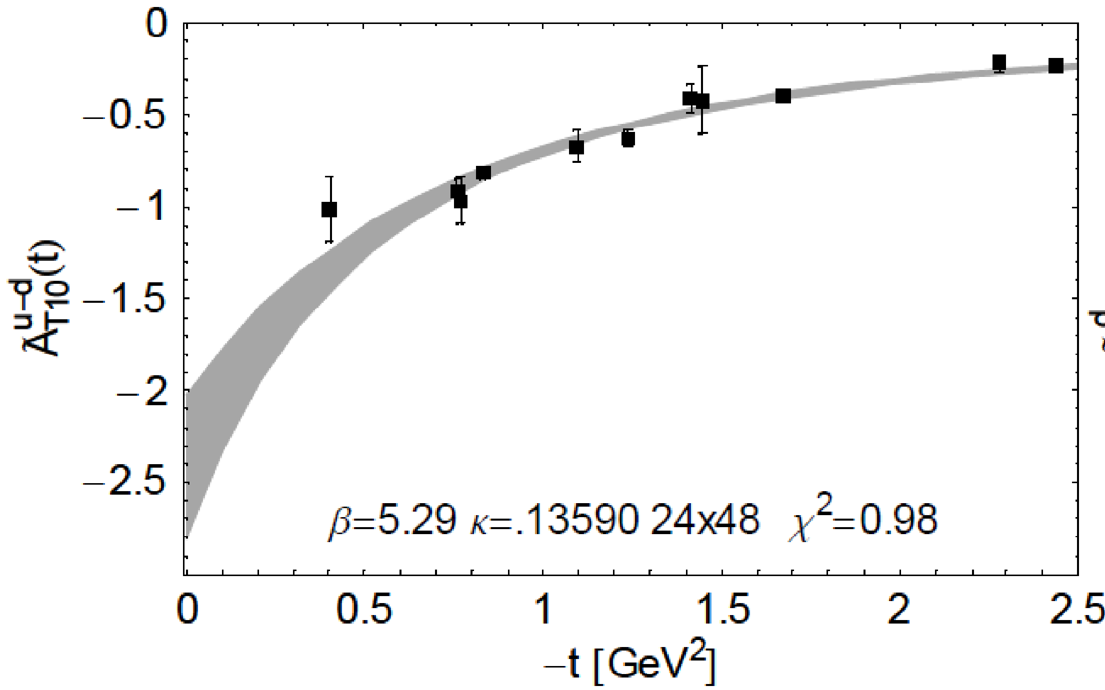
eg. single spin asymmetries [HERMES]

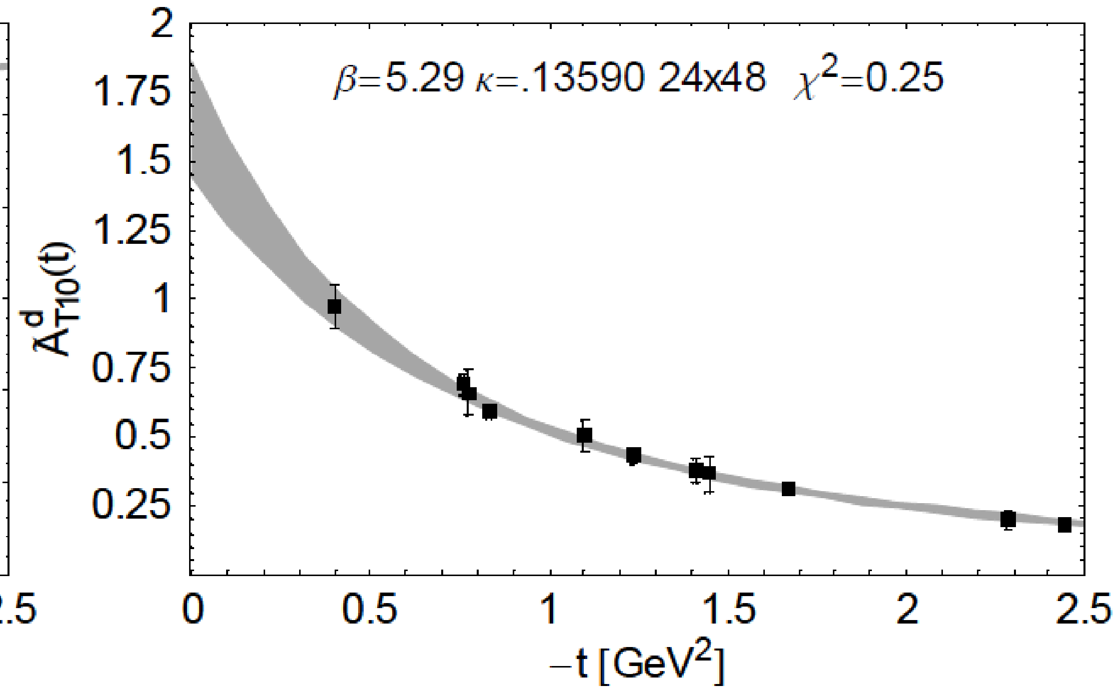
Tensor Form Factors

$\overline{B}_{Tn0}(t)$ *are remarkably large*

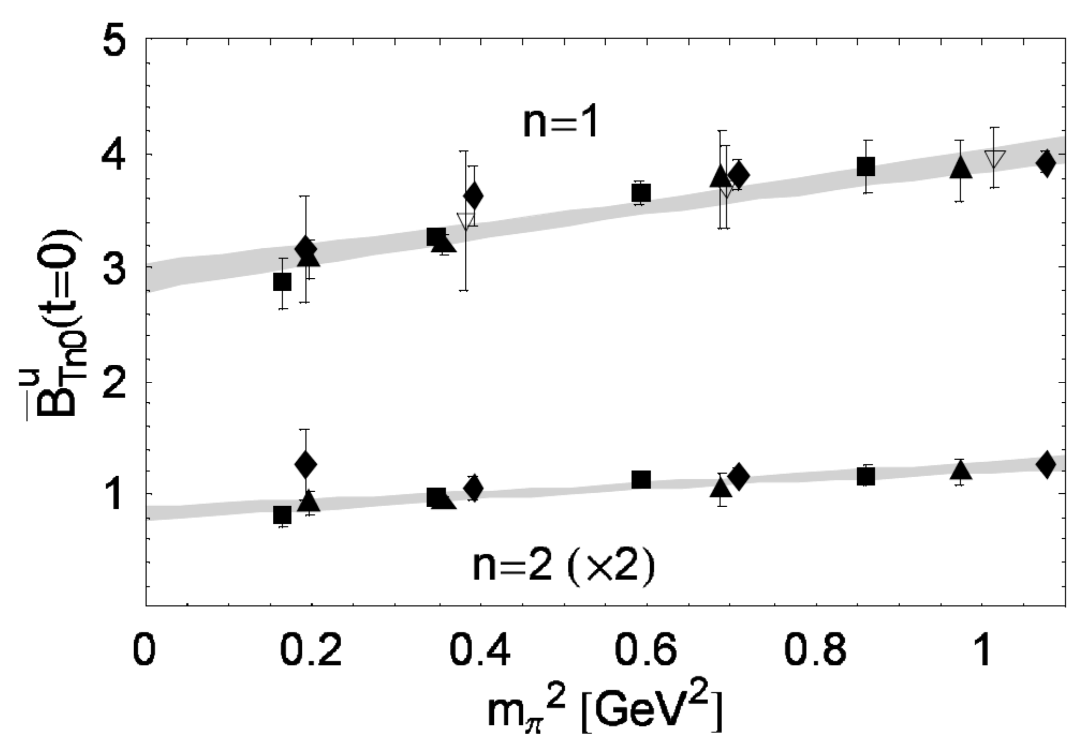


Tensor Form Factors





$\overline{A}^d_{Tn0}(t)$ is sizeable while $\overline{A}^u_{Tn0}(t) \approx 0$



Anomalous tensor magnetic moment

$$\kappa = \int dx E_T(x,\xi,0) = B_{10}(0) = F_2(0)$$

$$\begin{aligned} \kappa_u^{\text{exp}} &\approx 1.67 \\ \kappa_d^{\text{exp}} &\approx -2.03 \end{aligned}$$

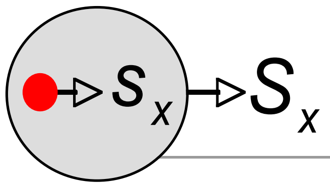
$$\kappa_T = \int dx \overline{E}_T(x,\xi,0) = \overline{B}_{T10}(0)$$

$$\left.\begin{aligned} \kappa_{Tu}^{\text{latt}} &\approx 3.13 \\ \kappa_{Td}^{\text{latt}} &\approx 1.94 \end{aligned}\right\}$$

Both positive

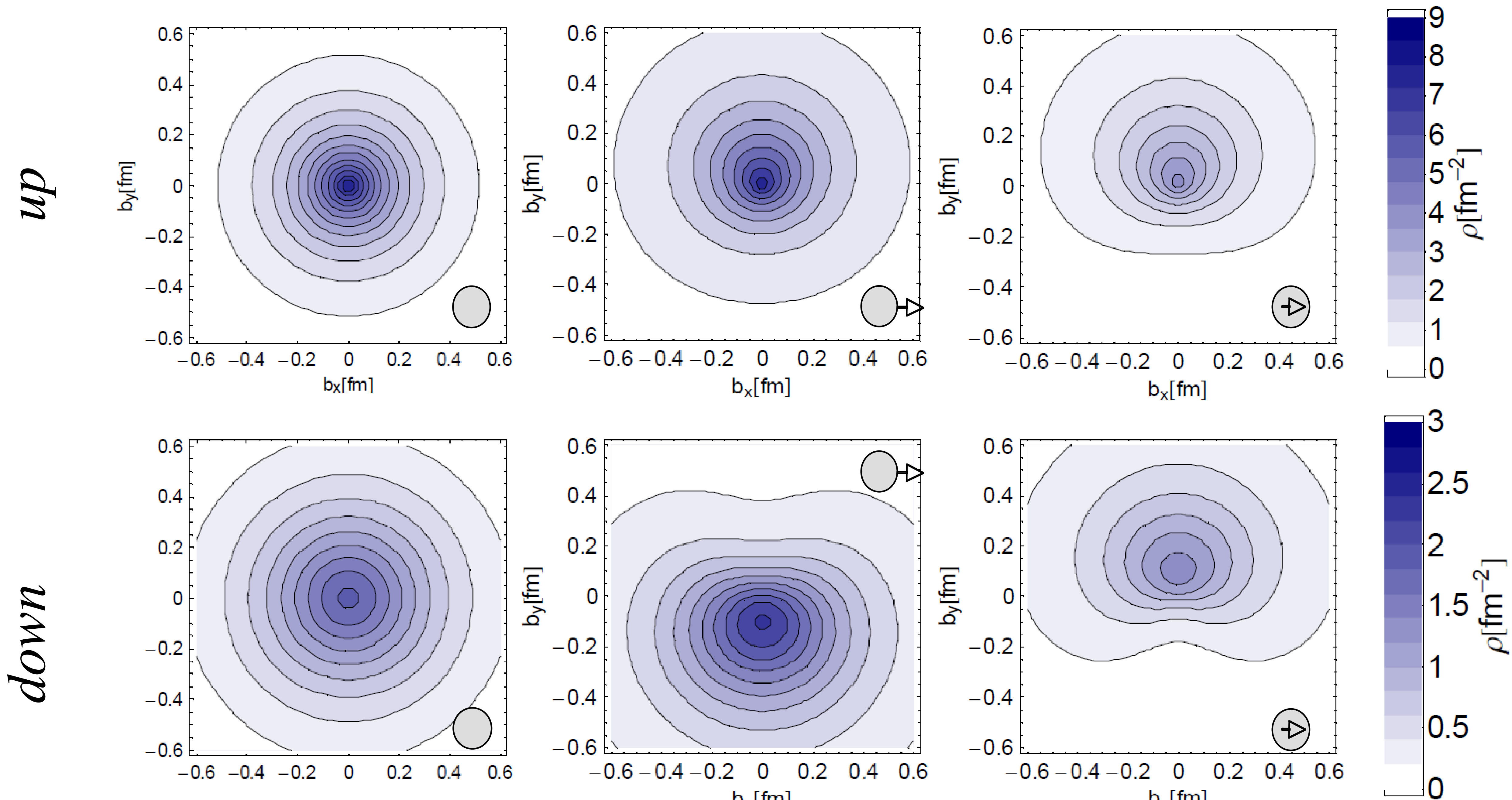
Deformed Spin Densities

Nucleon



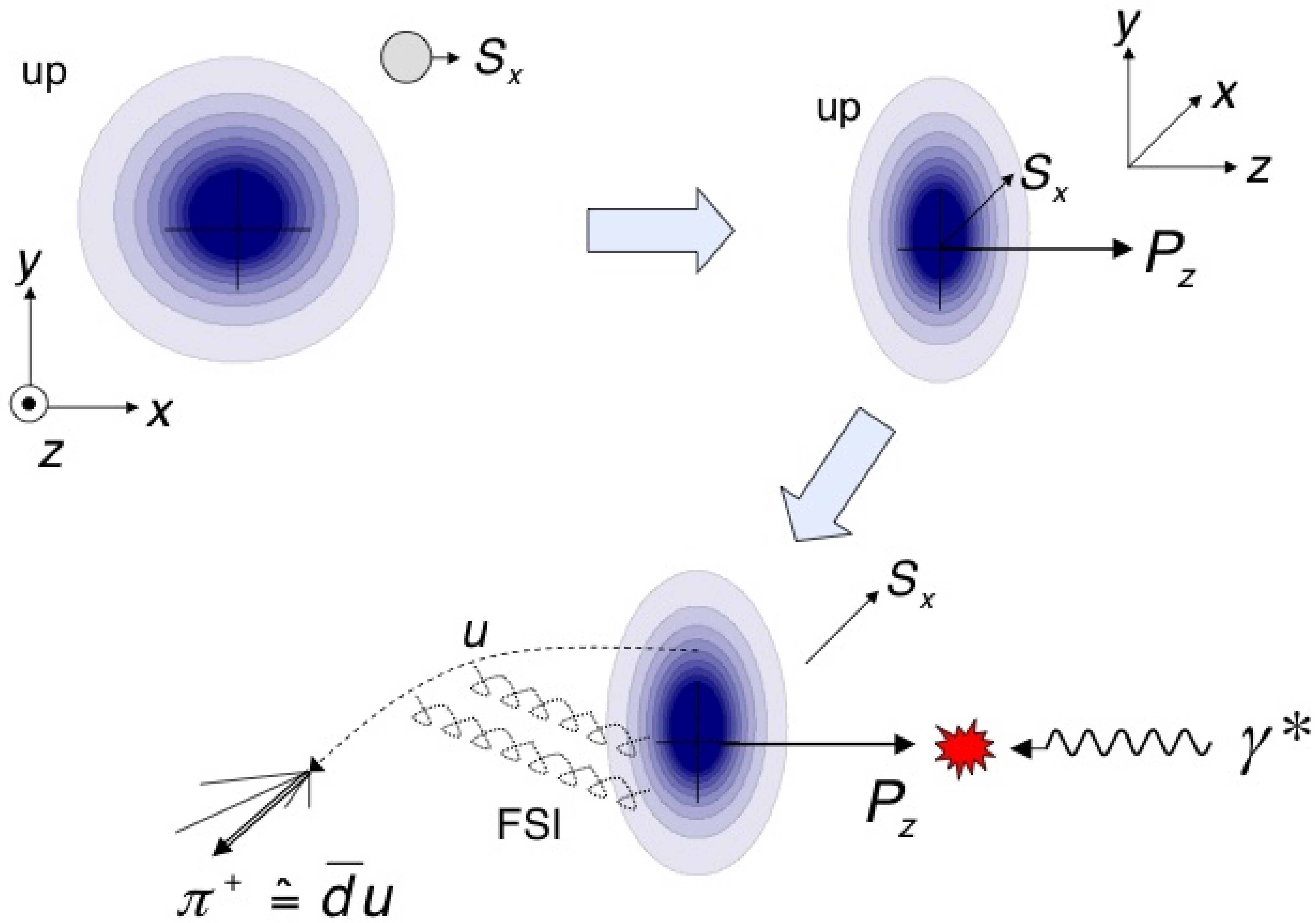
(n=1)

Ph. Hägler (QCDSF) [PRL 98, 222001 (2007)]



$r^d_{1,2} > r^u_{1,2}$ $\quad f^{\perp}_{1T}(x, k^2_{\perp})$ $\quad h^{\perp}_{1}(x, k^2_{\perp})$

Sivers Effect



Expect sizeable effect with opposite sign for up and down quarks (Sivers effect)

Transverse Spin Structure of the Nucleon

- Transverse densities:

$$\begin{aligned}\rho^n(b_\perp, s_\perp, S_\perp) &= \int_{-1}^{1} dx\, x^{n-1} \rho(x, b_\perp, s_\perp, S_\perp) = \frac{1}{2}\Bigg\{ A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{Tn0}(b_\perp^2) - \frac{1}{4m^2}\Delta_{b_\perp} \widetilde{A}_{Tn0}(b_\perp^2) \right) \\ &+ \frac{b_\perp^j \epsilon^{ji}}{m} \left(S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \overline{B}'_{Tn0}(b_\perp^2) \right) + s_\perp^i (2 b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}) S_\perp^j \frac{1}{m^2} \widetilde{A}''_{Tn0}(b_\perp^2) \Bigg\}\end{aligned}$$

[Diehl & Haegler, 2005] [Burkardt, 2005]

$$F(b_\perp^2) = \int d^2\Delta_\perp\, \mathrm{e}^{-i\vec{b}_\perp\cdot\Delta_\perp} F(\Delta_\perp^2) = \int d^2\Delta_\perp\, \mathrm{e}^{-i\vec{b}_\perp\cdot\Delta_\perp} \frac{F(0)}{(1-\Delta_\perp^2/M^2)^p}$$

Transverse Spin Structure of the ~~Nucleon~~ Pion

- Transverse densities:

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Transverse Spin Structure of the Nucleon

Pion

- Transverse densities:

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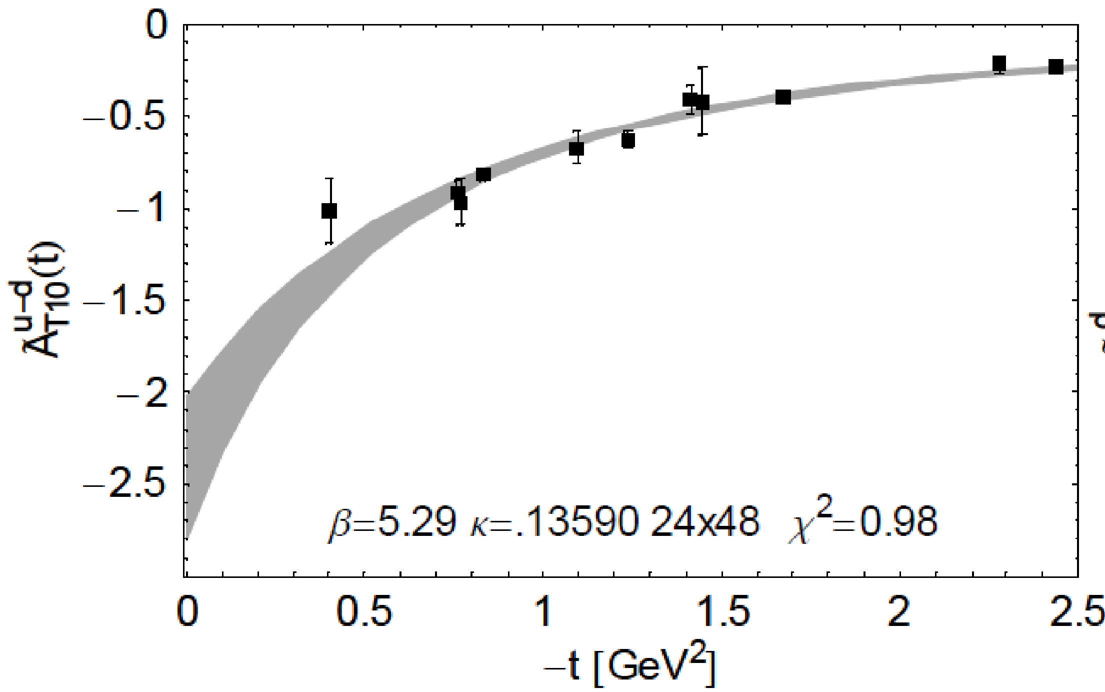
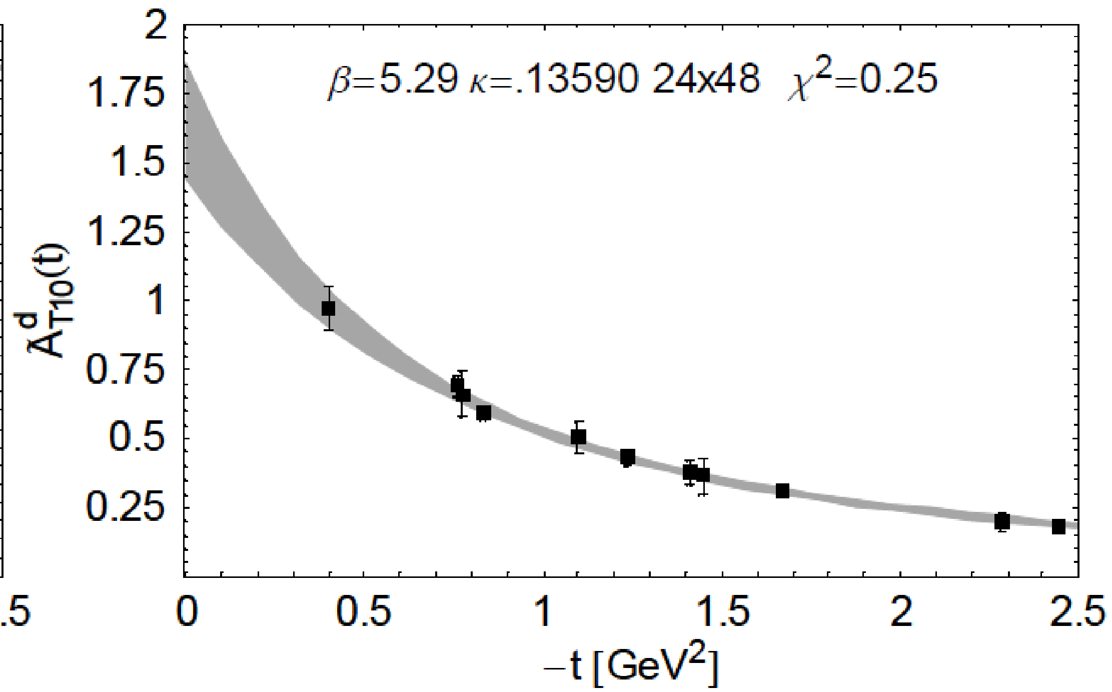
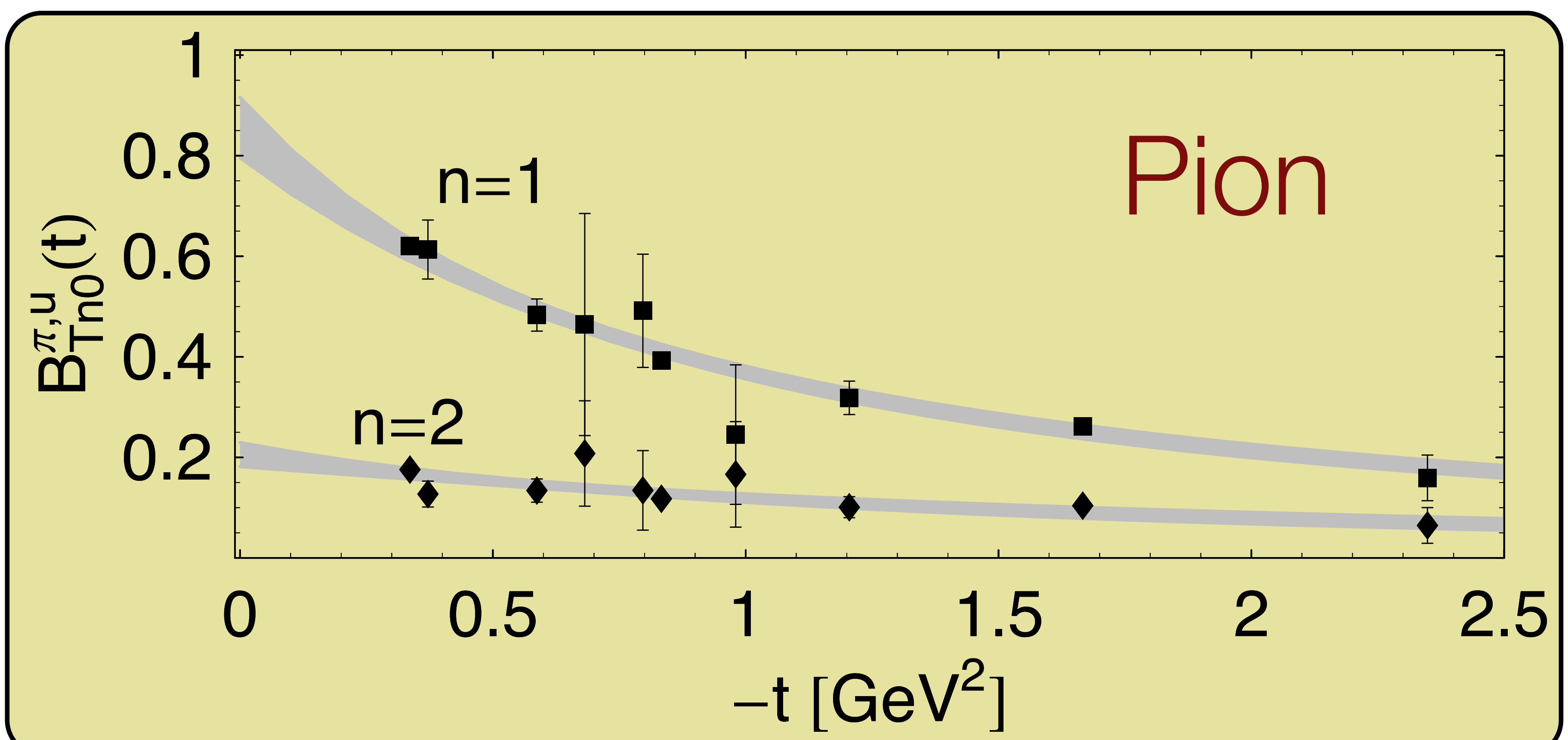
Tensor Form Factors



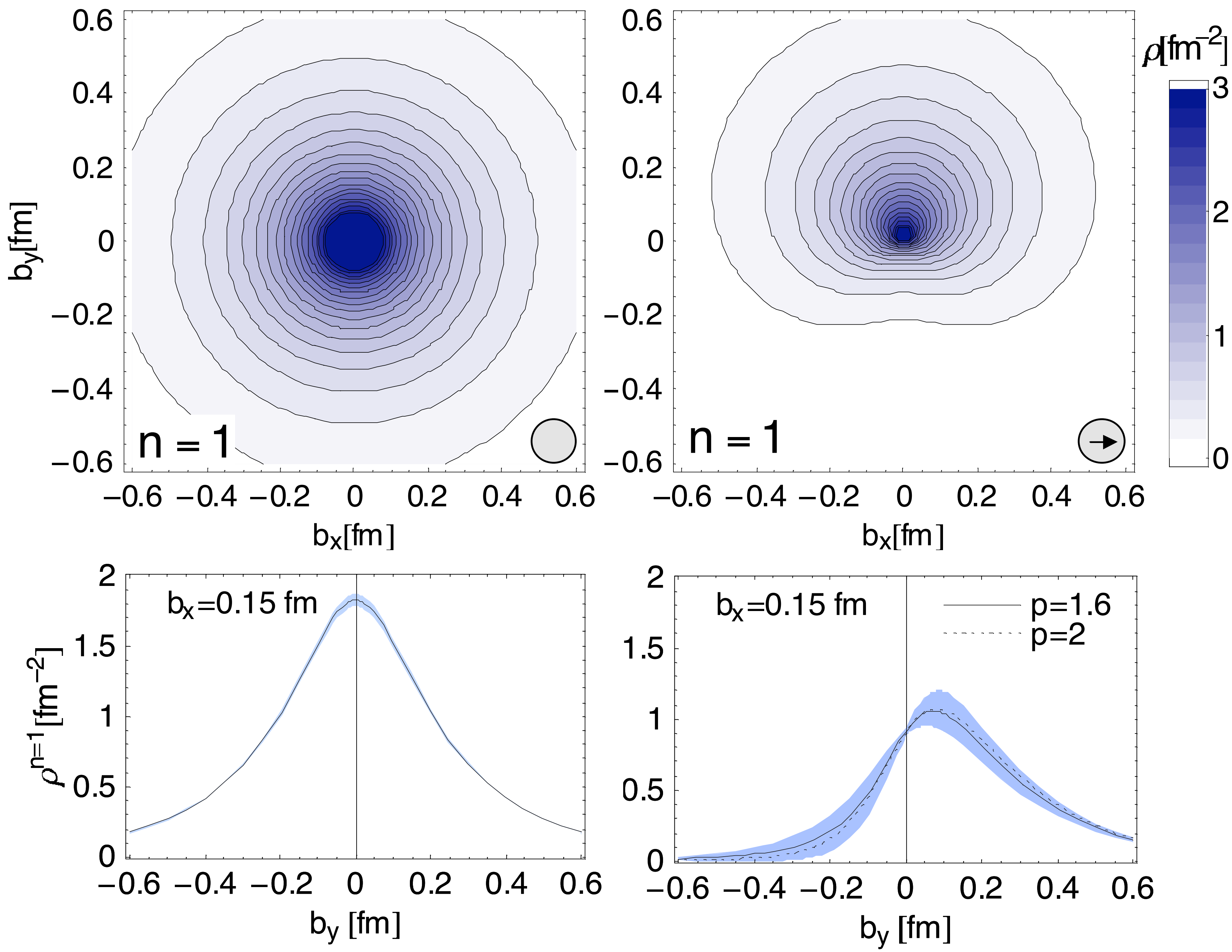


$\overline{A}^{d}_{Tn0}(t)$ ***is sizeable while*** $\overline{A}^{u}_{Tn0}(t) \approx 0$



Deformed Spin Densities
Pion

D. Brömmel (QCDSF) [arXiv:0708.2249]



Transverse Momentum Distributions

- Transverse Momentum Dependent Parton Distribution Functions (TMDs) provide a complimentary approach to studying the distribution of partons in the nucleon

    - GPDs ⇨ spatial distributions in the transverse plane

    - TMDs ⇨ intrinsic transverse motion of partons

- When combined, we can obtain a full 3-D imaging of the nucleon

- TMDs are intimately tied with the orbital motion of quarks in the nucleon

- Spin-orbit couplings lead to asymmetries in scattering experiments

Transverse Momentum Distributions

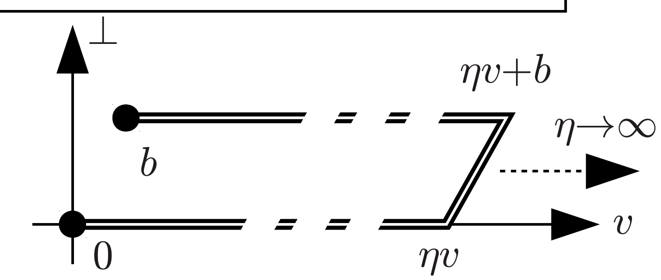
- The simplest TMD is the unpolarised function $f_1^q(x, k_\perp)$

  ⇒ the probability to find a quark carrying the longitudinal momentum fraction *x* and a transverse momentum $k_\perp = |\vec{k}_\perp|$

- The ordinary quark PDF is recovered when integrating over the transverse momentum

$$\int d^2\vec{k}_\perp f_1^q(x, k_\perp) = f_1^q(x) \;\; (= q(x))$$

gauge-link operator



- TMDs are obtained from matrix elements

$$\Phi_q^{[\Gamma]}(x, \vec{k}_\perp, \vec{S}) = \int \frac{db^- d^2 b_\perp}{(2\pi)^3} e^{ik\cdot b} \langle p, s|\bar{q}(0)\Gamma\mathcal{W}(0,b)q(b)|p, s\rangle|_{b^+=0}$$

- via (for example)

$$\Phi_q^{[\gamma^+]}(x, \vec{k}_\perp, \vec{S}) = f_1^q(x, k_\perp) - \frac{\epsilon^{jk} k_\perp^j S_T^k}{M} f_{1T}^{\perp q}(x, k_\perp)$$

Transverse Momentum Distributions

B. Musch, PhD Thesis: 0907.2381

Musch et al.: 1111.4249

- On the lattice, requires the moments of TMDs are obtained by the computation of a non-local matrix element, where the quark fields are separated by some distance b and are joined by a staple of gauge links

- Vary the distance b and length of staple η

Transverse Momentum Distributions

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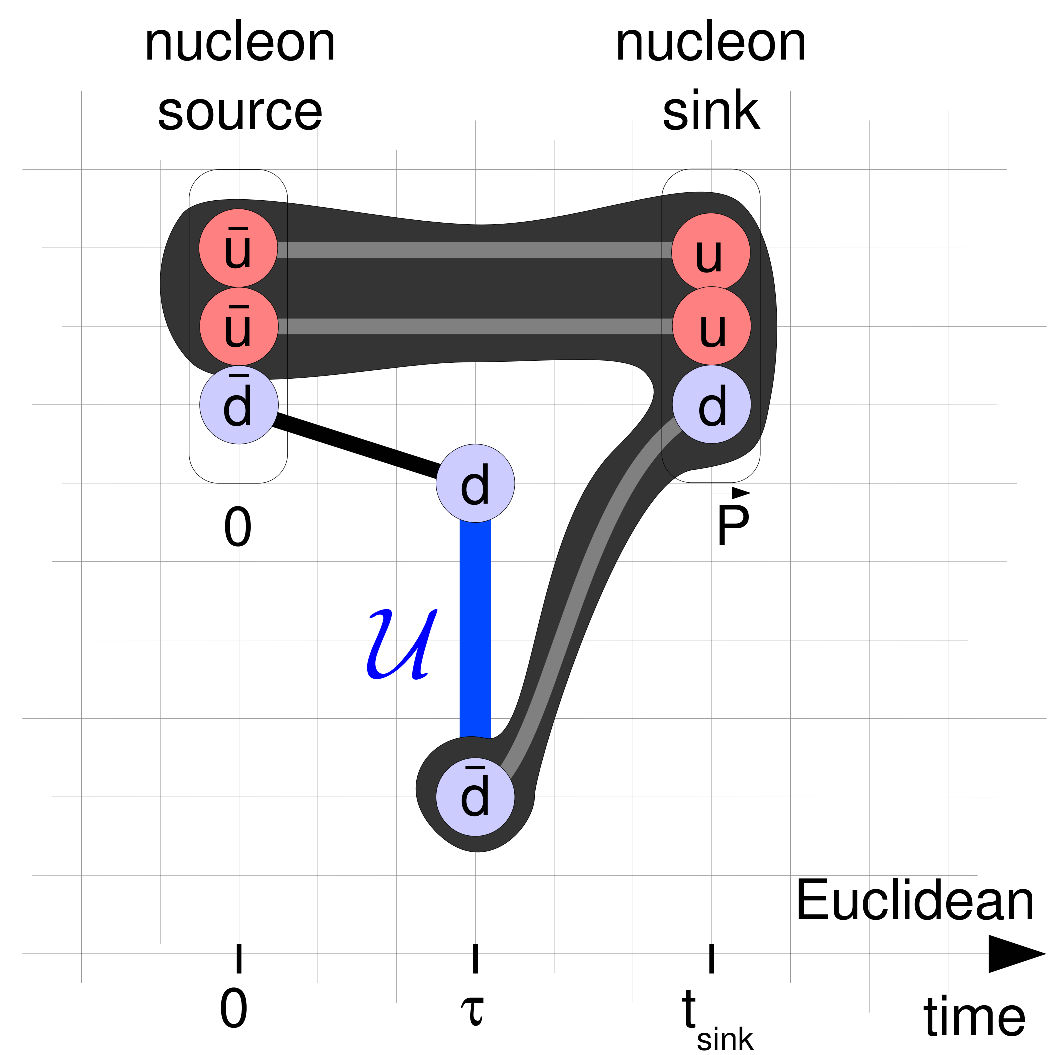
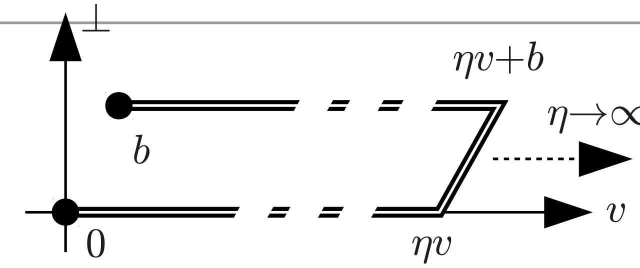
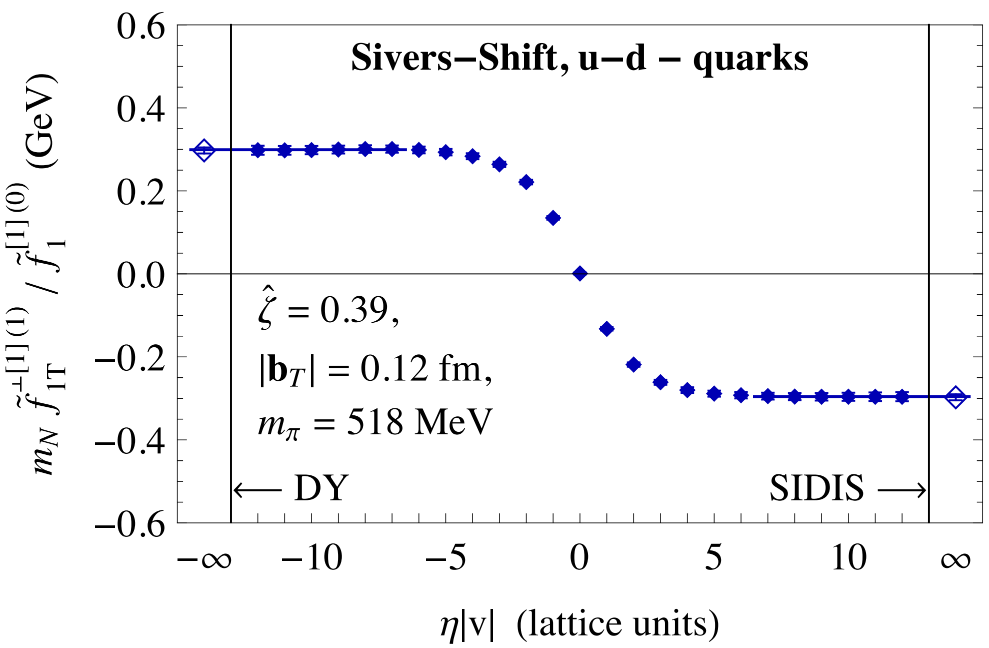
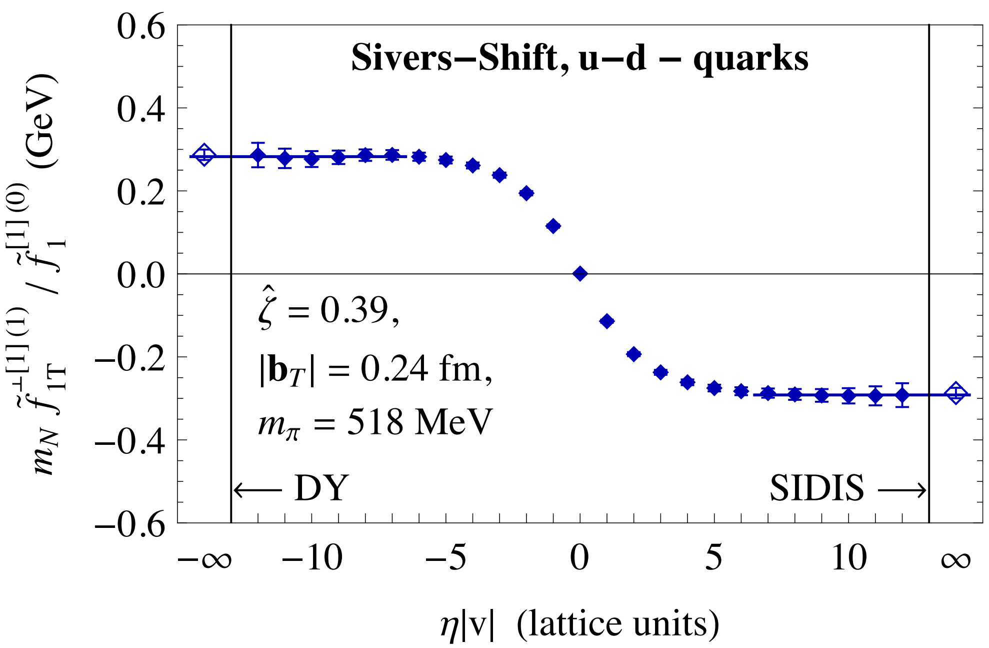


Transverse Momentum Distributions

- At fixed *b*, extrapolate to $\eta \to \infty$

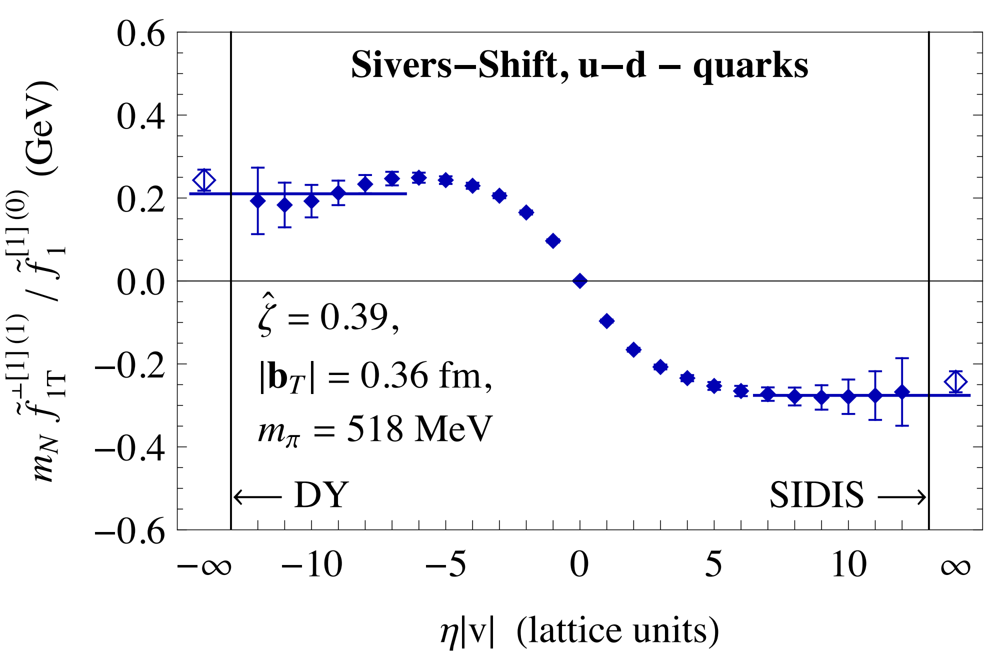


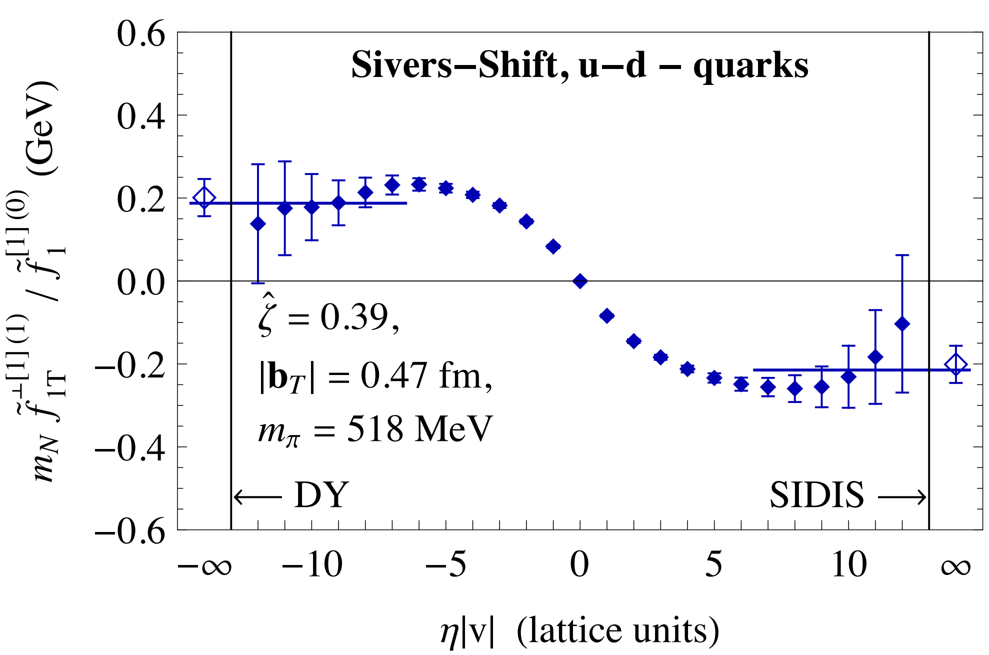




(c)

(d)





Transverse Momentum Distributions

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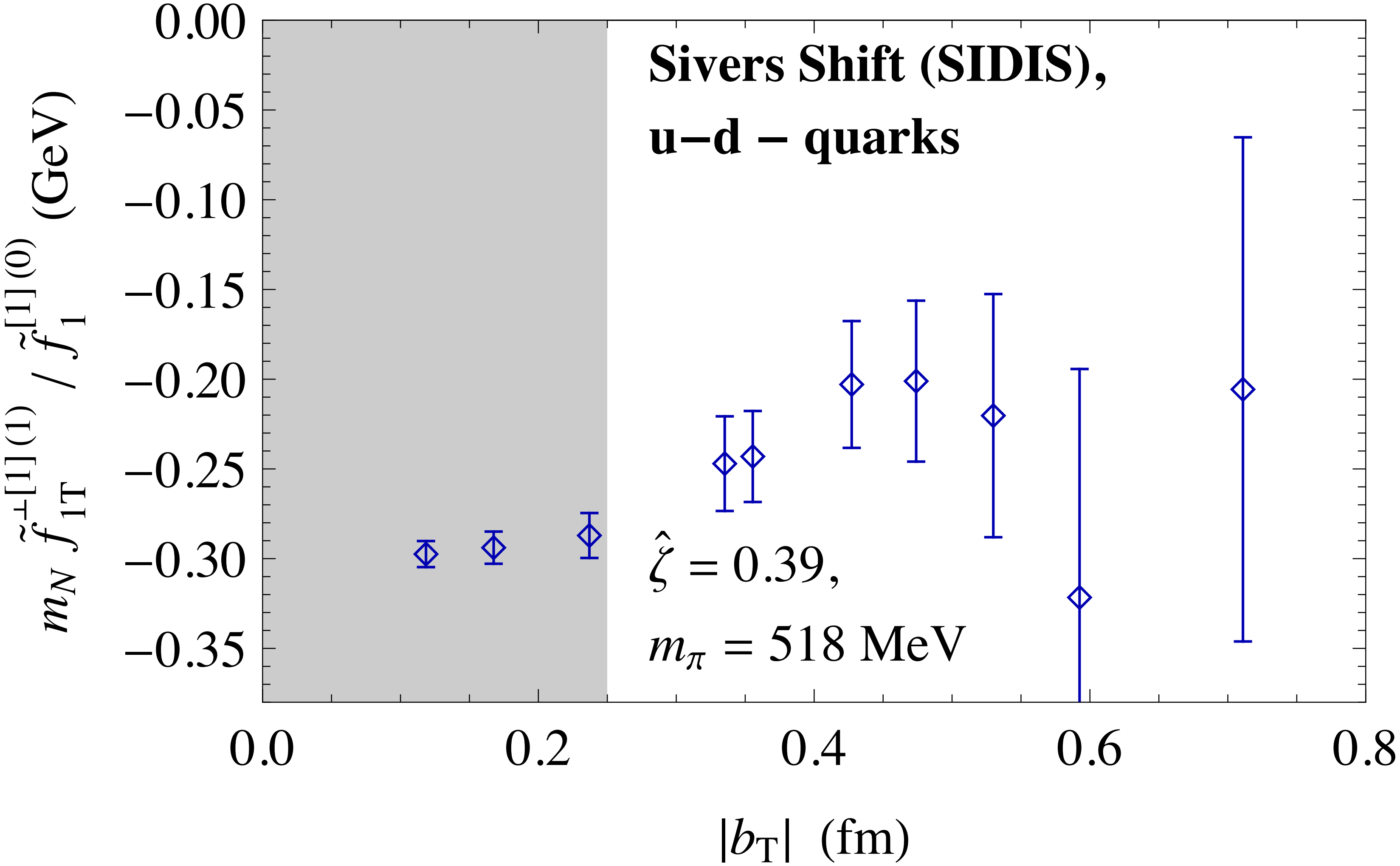
Dependence on $b_\perp$



GPDs and TMDs

- Lattice results for moments of Generalised Parton Distributions
  - Provide information on the spatial distribution of quarks in the transverse plane
  - Interesting correlations between spin and coordinate degrees of freedom
  - Via Ji's sum rule, they can provide access to total quark contribution to the nucleon's spin
    - Also a decomposition into helicity and orbital angular momentum contributions
- An exploratory study has shown that it is also possible to extract moments of Transverse Momentum Dependent Parton Distribution Functions from the lattice
  - Provides the possibility for a full "3-D" image of the nucleon