Hadron Structure

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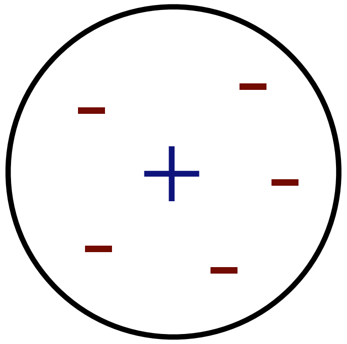
Motivation

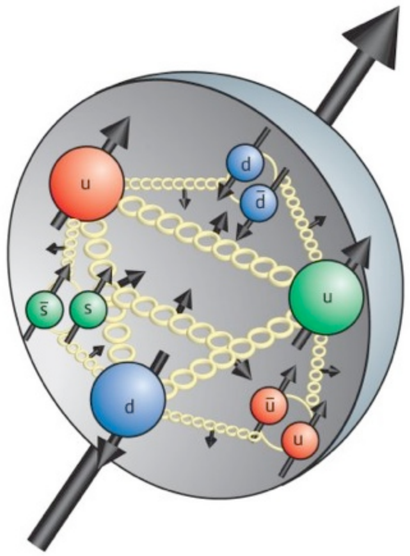
- We know the nucleon is not a point-like particle but in fact is composed of quarks and gluons

- But how are these constituents distributed inside the nucleon?

  - E.g. The neutron has zero net charge, but does it have a +/- core?



- How do they combine to produce its experimentally observed properties



- For example

  - “Spin crisis”: quarks carry on ~30% of the proton’s spin

  - gluons? orbital angular momentum?

- Understanding how the nucleon is built from its quark and gluon constituents remains one the most important and challenging questions in modern nuclear physics.

Topics to cover

- Elastic scattering ⇨ Electromagnetic form factors
- Neutron beta decay ⇨ Nucleon axial charge
- Deep Inelastic Scattering ⇨ Structure Functions and Parton Distribution Functions
- Generalised Parton Distribution Functions
- Hidden Flavour, e.g. strangeness content of the nucleon
- Focus on Nucleon and Pion

Lecture 1

- A short history
- Elastic scattering
- Form Factors
    - Density distributions in the nucleon
- Lattice methods for computing hadronic matrix elements (3pt functions)
- A taste of some lattice results

Structure of the Proton

- Until 1932, proton was considered to be an elementary particle

- In 1933, Otto Stern measured the proton's magnetic moment

$$\mu_p \approx 2.5 \quad (\text{today}: \ 2.7928456(11)) \ \frac{e}{2m_p}$$

- Deviates significantly from unity - the magnetic moment of point-like particle described by Dirac's theory of relativistic fermions

$$\mu_p = \mu_D \equiv \frac{e}{2m_p}$$

Proton is a composite particle

- The proton's constituents were later "seen" in Deep-Inelastic Scattering experiments at SLAC (1968)

Nucleon Structure

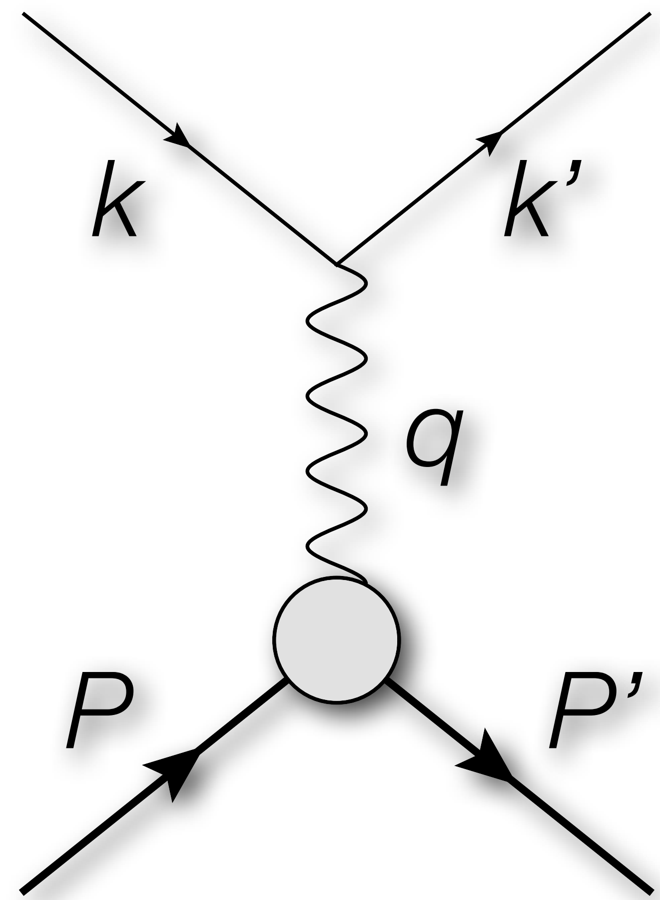
- Nucleon structure is studied experimentally by electron-proton scattering
- Electron is a good probe because:
  - QED is a "well-understood" interaction
  - $\alpha_{\rm em} = \frac{1}{137}$ ⇒ perturbation theory is valid
  - Electrons are charged and so easily accelerated
- Two types of e-p scattering:
  - Elastic scattering ⇒ Today
  - Deep-Inelastic Scattering (DIS) ⇒ Wednesday

Elastic Scattering

- Final state nucleon remains intact, but with recoil
- Map out charge and density distributions inside the nucleon
- Dominated by single-photon exchange
- 4-momentum transfer $q = k - k' = P' - P$

→ Power of the probe

- Compare cross-section with that of a point-particle



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{point}} |F(q^2)|^2$$

Form Factor

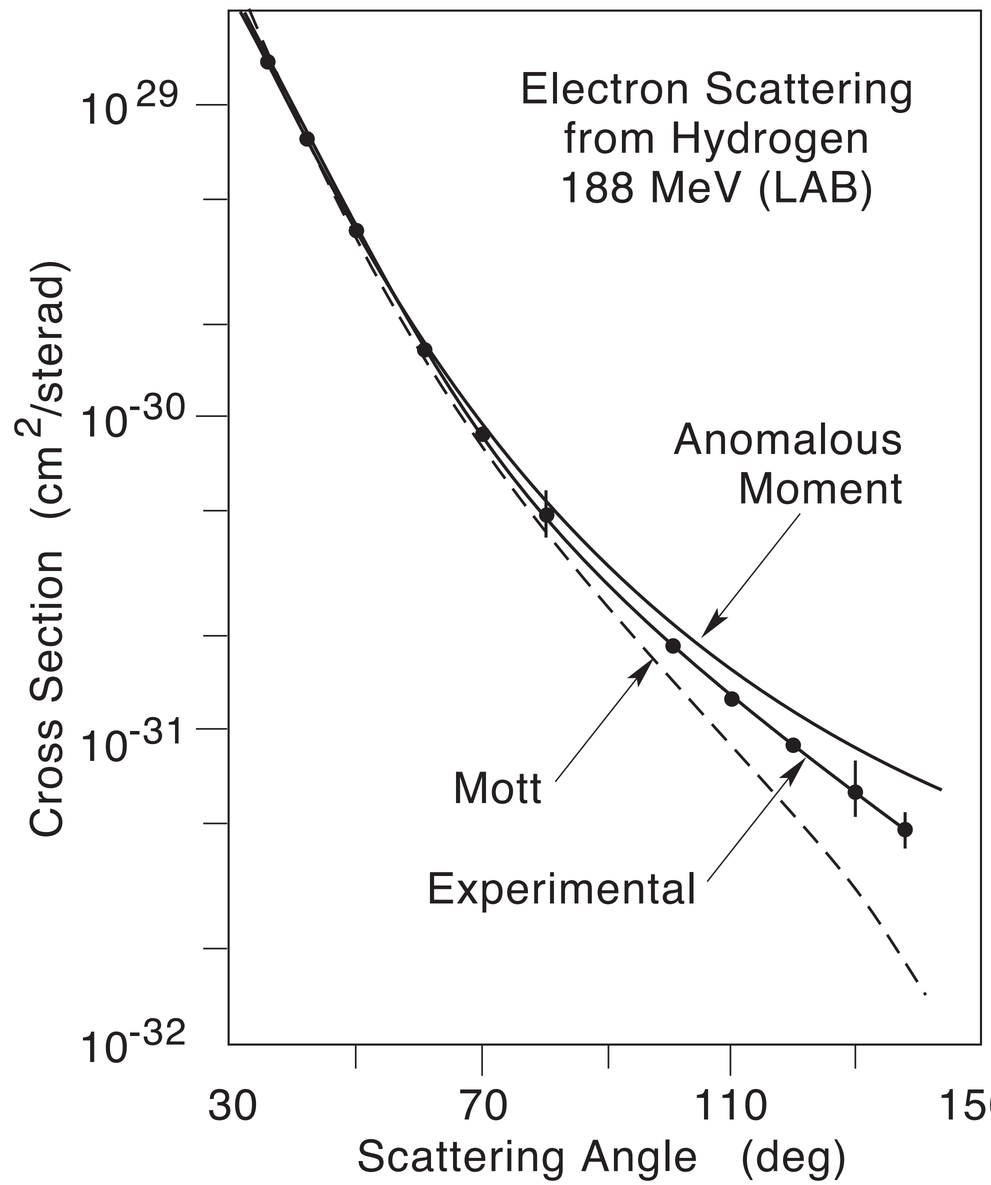
Elastic Scattering

- When using a point target, $F(q) = 1$ and

Mott cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} = \frac{(Z\alpha)^2 E^2}{4k^2 \sin^4(\theta/2)} \left(1 - \frac{k^2}{E^2}\sin^2(\theta/2)\right)$$

- E, k - energy and momentum of electron
- θ scattering angle $(q^2 = -4EE' \sin^2\theta/2)$
- But experimental cross-sections deviate from this description



Elastic Scattering

- Considering only one-photon exchange, justified because the fine structure constant is so small, the *S*-Matrix is then

$$S = (2\pi)^4\delta^4(k + P - P' - k')\overline{u}(k')(-ie\gamma^\mu)u(k)\frac{-i}{q^2}\langle P'|(ie)J^\mu|P\rangle$$
$$= -i(2\pi)^4\delta^4(k + P - P' - k')\mathcal{M}$$

Invariant amplitude

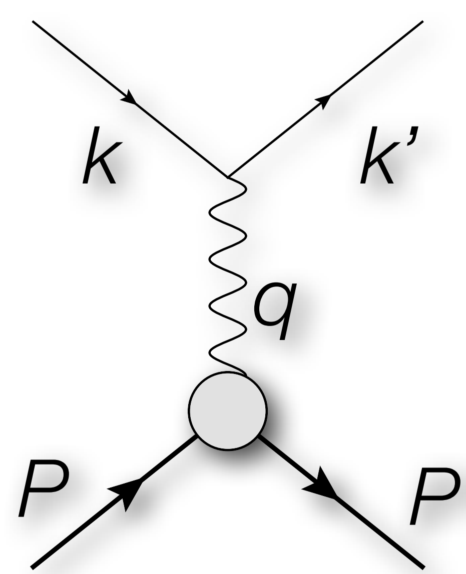
- The electromagnetic current is

$$J^\mu = \sum_i e_i\overline{\psi}_i\gamma^\mu\psi_i$$

Sum over quark flavours with $m_q \ll m_p$ (u,d,s)

- Can write cross-section in terms of invariant amplitude

$$d\sigma = \frac{E'}{2EM^2}\frac{1}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}}|\mathcal{M}|^2\frac{d\Omega}{(2\pi)^2}$$



Elastic Scattering

- The invariant amplitude squared is

$$|\mathcal{M}|^2 = \frac{e^4}{Q^4} \ell^{\mu\nu} W^{\mu\nu}$$

- Leptonic tensor $\ell^{\mu\nu} = \overline{u}(k')\gamma^\mu u(k)\overline{u}(k)\gamma^\nu u(k')$
    - Compute in QED

- Hadronic tensor $W^{\mu\nu} = \langle P|J^\nu|P'\rangle\langle P'|J^\mu|P\rangle$
    - with matrix element between nucleon states defining two Lorentz-invariant form factors

$$\langle P'|J^\mu(\vec{q})|P\rangle = \bar{u}(P')\left[\gamma^\mu F_1(q^2) + i\sigma^{\mu\nu}\frac{q_\nu}{2m}F_2(q^2)\right]u(P)$$

Dirac Pauli

Elastic Scattering

- Using the fact that both tensors are symmetric and conserved $q^\mu \ell_{\mu\nu} = q^\mu W_{\mu\nu} = 0$

- The elastic scattering cross-section in the lab frame becomes

$$\frac{d\sigma}{d\Omega} = \sigma_{\mathrm{Mott}} \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2) \tan^2 \frac{\theta}{2} \right]$$

- where

$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

$$\tau = Q^2/(4M^2)$$

- are the Sachs electric and magnetic form factors

- Rewriting in terms of the virtual photon's longitudinal polarisation

$$\epsilon^{-1} = 1 + (1+\tau) 2 \tan \theta/2$$

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{\mathrm{Mott}}}{1+\tau} \left[G_E^2(Q^2) + \frac{\tau}{\epsilon} G_M^2(Q^2) \right]$$

- Need cross sections at fixed Q^2 but different scattering angle: Rosenbluth separation

Elastic Scattering - Rosenbluth

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{\text{Mott}}}{1+\tau}\left[G_E^2(Q^2) + \frac{\tau}{\epsilon}G_M^2(Q^2)\right]$$

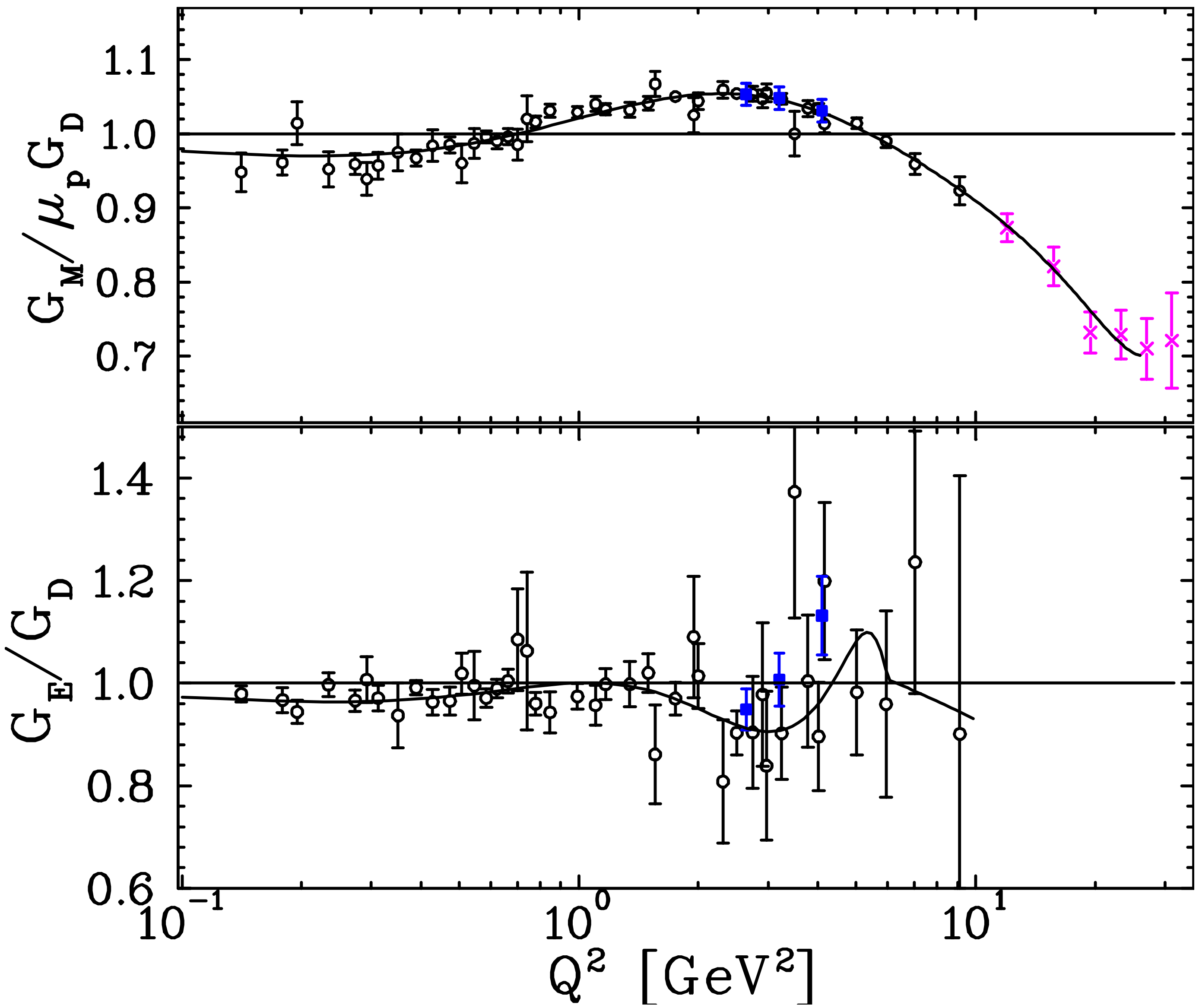
- At fixed Q^2, G_E and G_M determined from intercept and slope as a function of ϵ^{-1}

- Drawback - reduced sensitivity to G_E at large Q^2

- Both form factors reasonably well described by the dipole form

$$G_E^p(Q^2) = \frac{G_M^p(Q^2)}{\mu_p} = \frac{1}{(1+Q^2/M_D^2)^2}$$

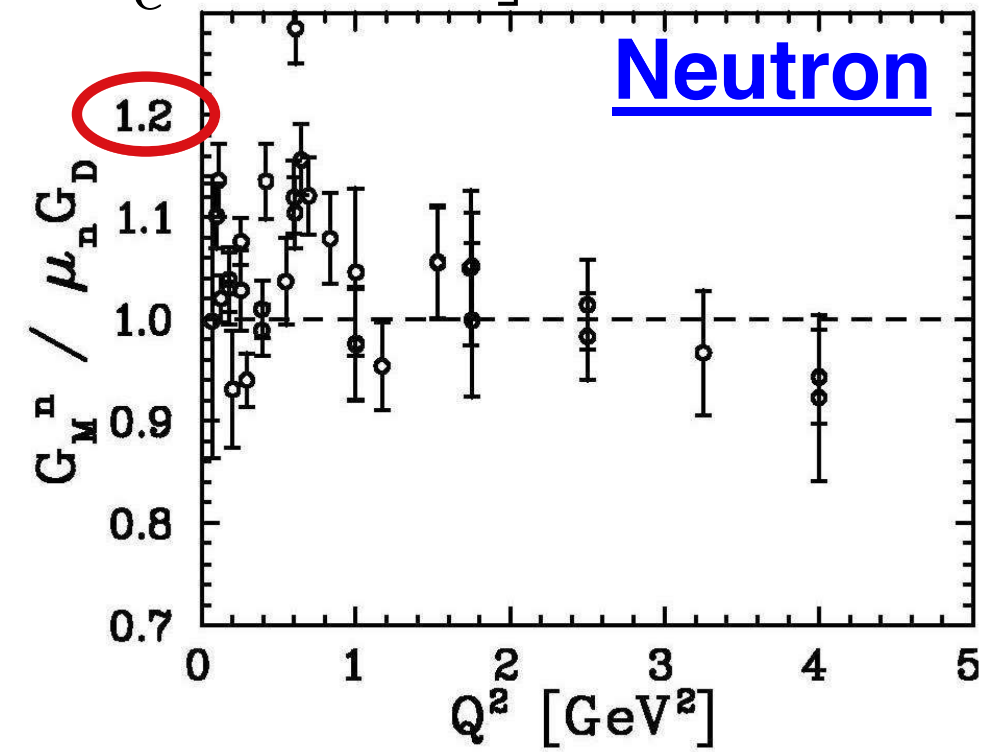
- with
$$M_D^2 \approx 0.71\,\text{GeV}^2$$

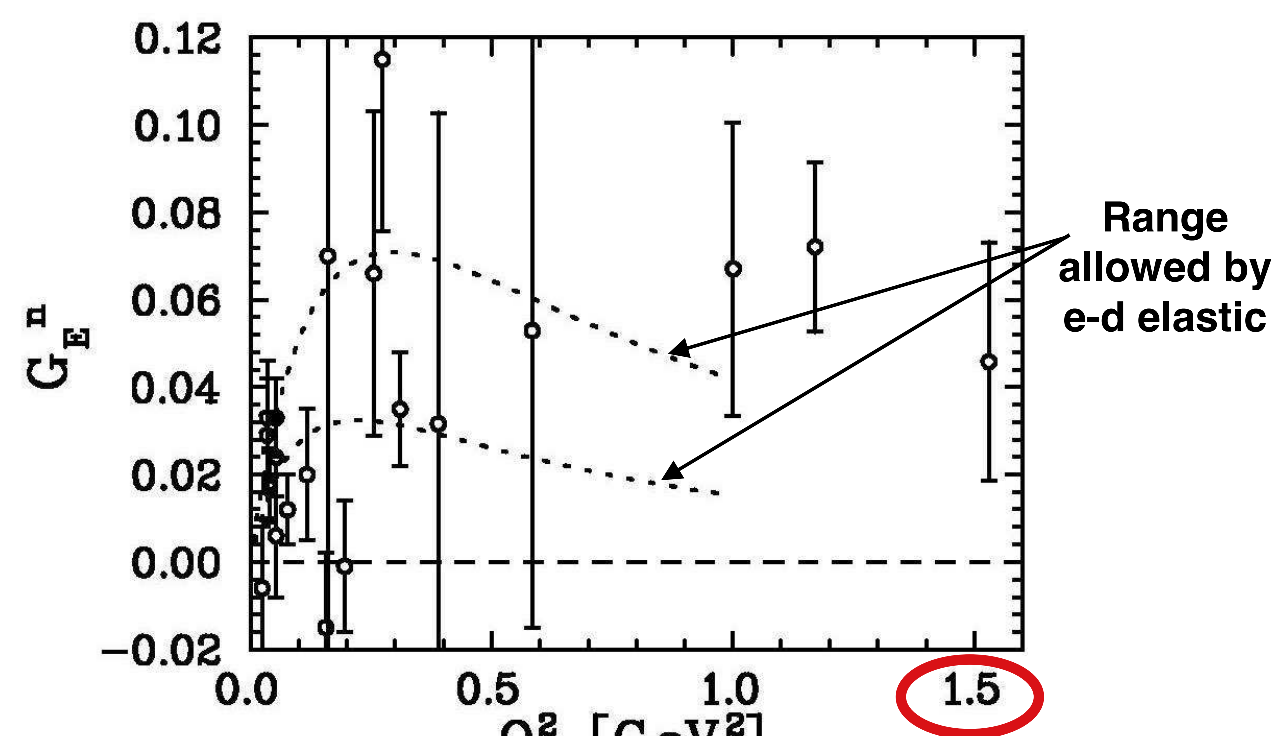


Elastic Scattering - Rosenbluth

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{\text{Mott}}}{1+\tau}\left[G_E^2(Q^2) + \frac{\tau}{\epsilon}G_M^2(Q^2)\right]$$

- For a neutron:
- G_E small, so extraction near impossible
- No neutron target so use deuterium and
    - subtract proton contribution
    - Model nuclear effects





Elastic Scattering - Polarisation Transfer

- Difficulties with unpolarised scattering ⟹ new techniques necessary
- Mid-‘90s brought
  - High luminosity, highly polarised electron beams
  - Polarised targets (^{1}H, ^{2}H, ^{3}He)
  - Large, efficient neutron detectors
- Polarisation transfer experiments provide access to the ratio G_E/G_M directly from ratio of polarisation transverse and parallel to the momentum of the nucleon

$$\frac{G_E}{G_M} = -\frac{P_t}{P_l}\frac{E+E'}{2M}\tan\frac{\theta}{2}$$

- Combine with previous accurate results for G_M to also determine G_E

Elastic Scattering - Polarisation Transfer

- Precise results now available up to 8-9 GeV^2
- Does G_E^p change sign?
- What is the origin of the linear fall-off?

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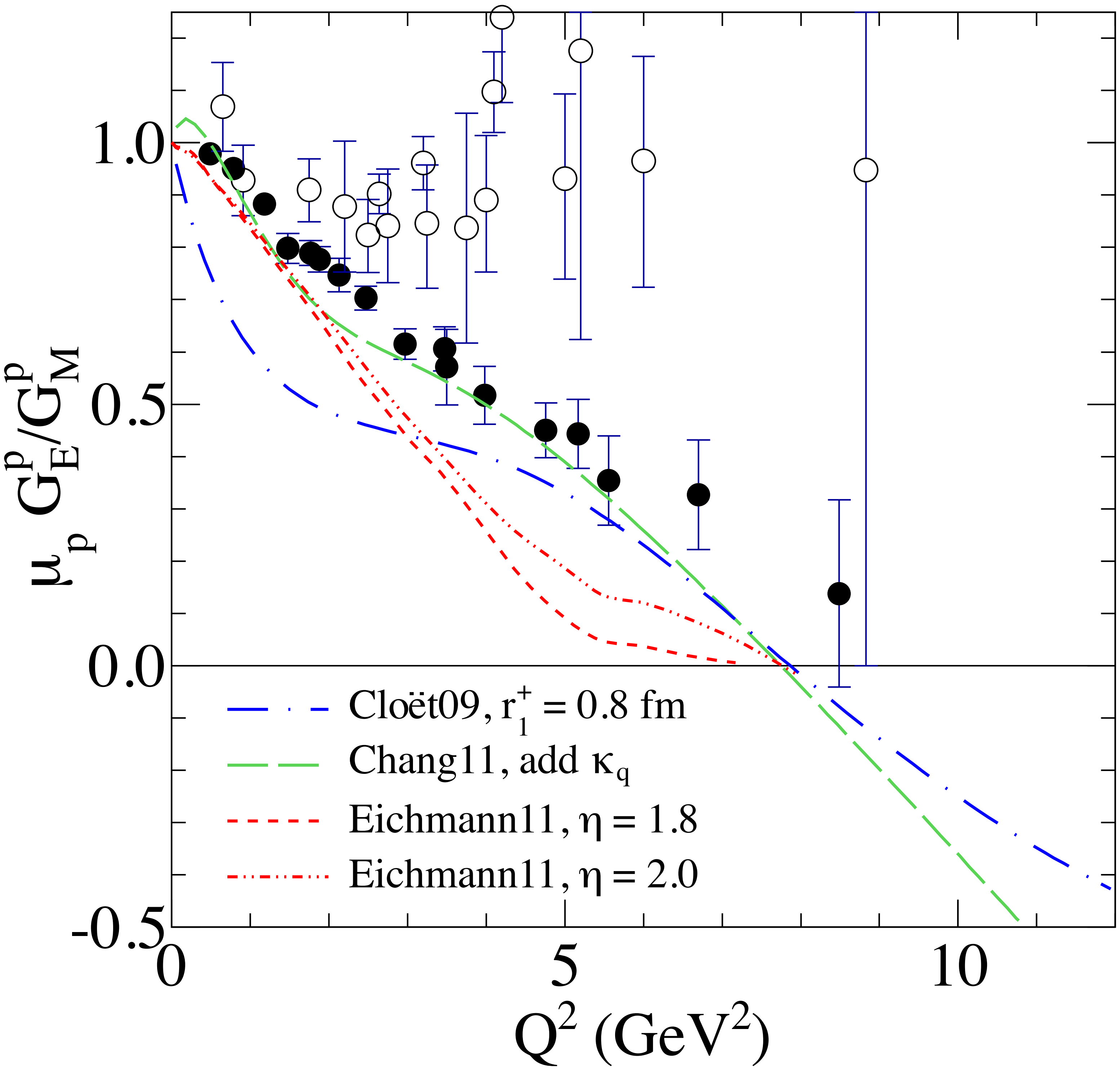

Elastic Scattering - Polarisation Transfer

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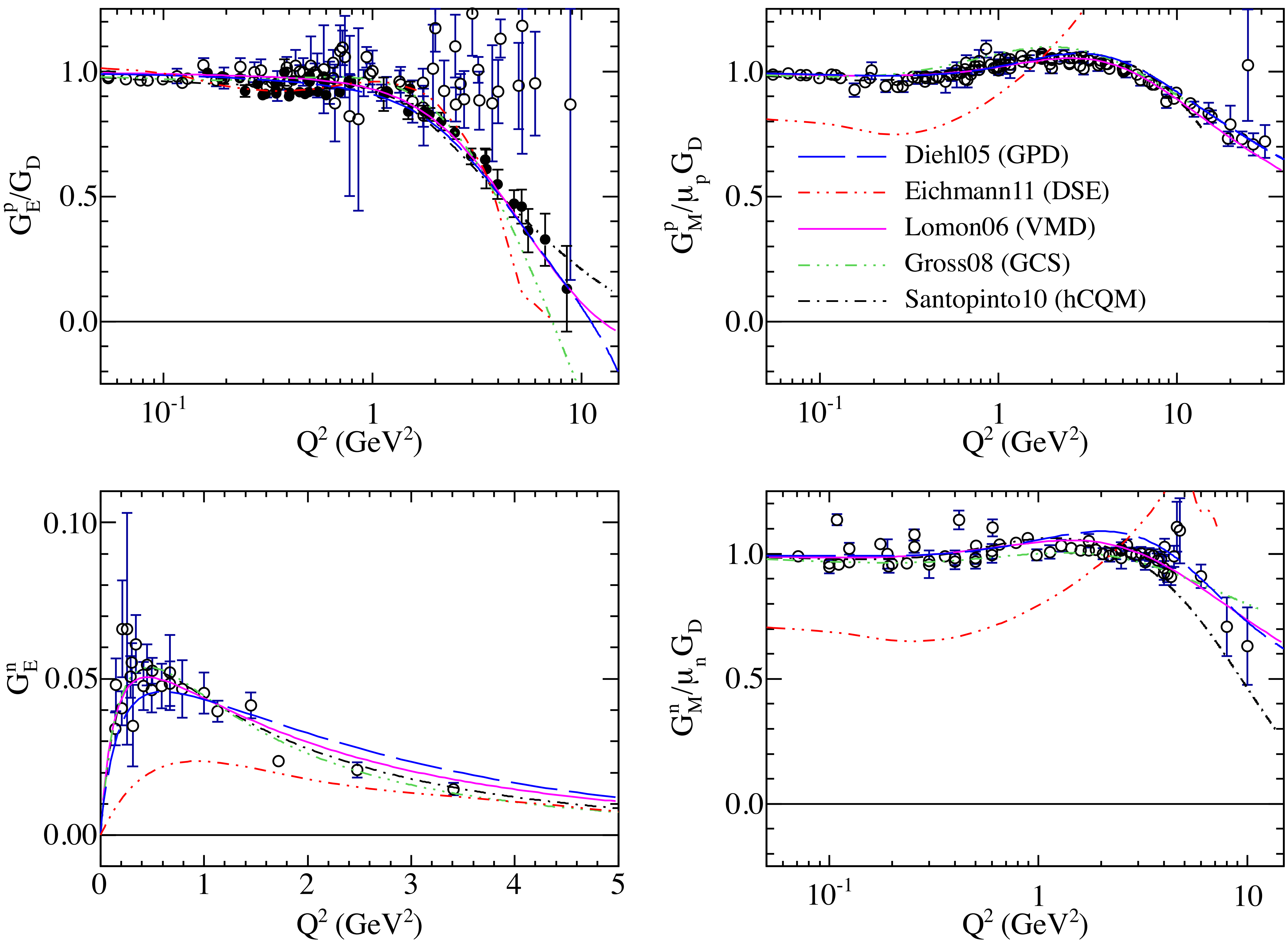


Insights into Nucleon Structure

- $G_E \neq G_M$ ⇨ different charge and magnetisation distributions
- If $M \to \infty$ initial and final nucleons are fixed at the same location $Q^2 \ll M^2$
- Initial and final states have same internal state

  ⇨ Fourier transformation of form factors are density distributions

- But *M* is finite so need to consider nucleon recoil effects
- Initial and final states now sampled in different frames ⇨ Lorentz contraction
- No model independent way to separate internal structure and recoil effects
- Work around: Breit frame or infinite momentum frame

Density Distributions

- Consider the Breit frame: $|P| = |P'|$
  - initial and final states have momenta with equal magnitude, hence similar Lorentz contraction
  - $G_E(Q^2)$ can be interpreted as the Fourier transformation of the charge distribution

$$G_E(Q^2) = \int e^{i\vec{q}\vec{x}} \rho(r) d^3 r$$

  - expanding at small Q^2

$$G_E(Q^2) = Q_e - \frac{1}{6} Q^2 \langle r^2 \rangle + \ldots$$

  - defines the charge radius of the nucleon

$$\langle r^2 \rangle = -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$

Size of the Proton

- $> 5\sigma$ discrepancy between muonic hydrogen and *e*-*p* scattering

  - r_p=0.84184(67) fm **[Nature 466, 213 (2010)]**

  - r_p=0.875(8)(6) fm **[arXiv:1102.0318]**

Transverse Spatial Distributions

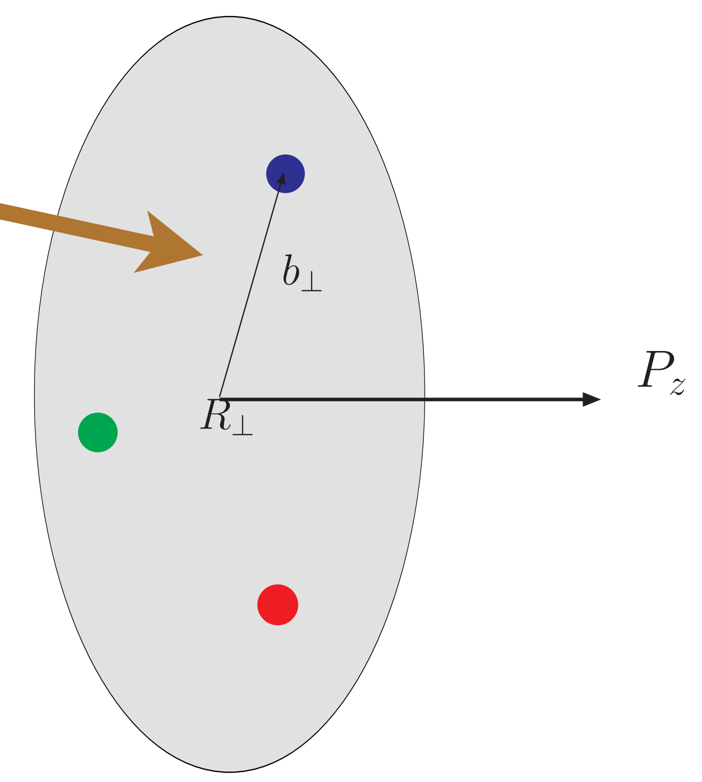
- Model independent relation between form factors and transverse spatial distributions occurs in the infinite momentum frame

- Quark (charge) distribution in the transverse plane

$$q(b_\perp^2) = \int d^2 q_\perp \, \mathrm{e}^{-i\vec{b}_\perp \cdot q_\perp} F_1(q^2)$$

Distance of (active) quark to the centre of momentum in a fast moving nucleon

Provide information on the size and internal charge densities



Electromagnetic Form Factors

- Can some of these questions be answered by a calculation from QCD?
- Form factors are nonperturbative quantities

Lattice QCD

- Need to determine

$$\langle p',\, s' | J^\mu(\vec{q}) | p,\, s\rangle = \bar{u}(p',\, s')\left[\gamma^\mu F_1(q^2) + i\sigma^{\mu\nu}\frac{q_\nu}{2m}F_2(q^2)\right]u(p,\, s)$$

Calculating Matrix Elements

$$\langle H' | \mathcal{O} | H \rangle$$

$$H, H' : \ \pi, \ K \, p, \ n, \ \ldots$$

$$\mathcal{O} : \ V_\mu, \ A_\mu, \ \ldots$$

Calculating Matrix Elements

$$q^2 = -Q^2 = (p' - p)^2$$

$$P^\mu = p'^\mu + p^\mu$$

Spin-0

$$\langle \pi(p') | J^\mu(\vec{q}) | \pi(p) \rangle = P^\mu F_\pi(q^2)$$

Spin-1/2

$$\langle N(p',\, s') | J^\mu(\vec{q}) | N(p,\, s) \rangle = \bar{u}(p',\, s') \left[\gamma^\mu F_1(q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{2m} F_2(q^2) \right] u(p,\, s)$$

Spin-1

$$\langle \rho(p', s') | J^\mu(\vec{q}) | \rho(p, s) \rangle =$$

$$-(\epsilon'^* \cdot \epsilon) P^\mu G_1(Q^2) - [(\epsilon'^* \cdot q)\epsilon^\mu - (\epsilon \cdot q)\epsilon'^{*\mu}] G_2(Q^2) + (\epsilon \cdot q)(\epsilon'^* \cdot q) \frac{P^\mu}{(2m_\rho)^2} G_3(Q^2)$$

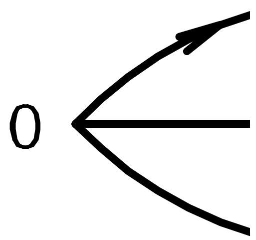
Spin-3/2

$$\langle \Delta(p', s') | J^\mu(\vec{q}) | \Delta(p, s) \rangle =$$

$$\bar{u}_\alpha(p', s') \left\{ -g^{\alpha\beta} \left[\gamma^\mu a_1(Q^2) + \frac{P^\mu}{2M_\Delta} a_2(Q^2) \right] - \frac{q^\alpha q^\beta}{(2M_\Delta)^2} \left[\gamma^\mu c_1(Q^2) + d \frac{P^\mu}{2M_\Delta} c_2(Q^2) \right] \right\} u_\beta(p, s)$$

Lattice 3pt Functions

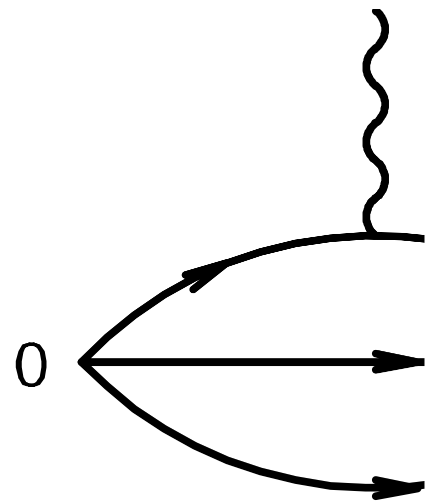
$$\langle\Omega|T\,(\qquad \overline{\chi}_\beta(0))\,|\Omega\rangle$$



- Create a state (with quantum numbers of the proton) at time *t=0*

Lattice 3pt Functions

$$\langle\Omega|T\,(\qquad\qquad \mathcal{O}(\vec{x}_1,\tau)\,\overline{\chi}_\beta(0))\,|\Omega\rangle$$

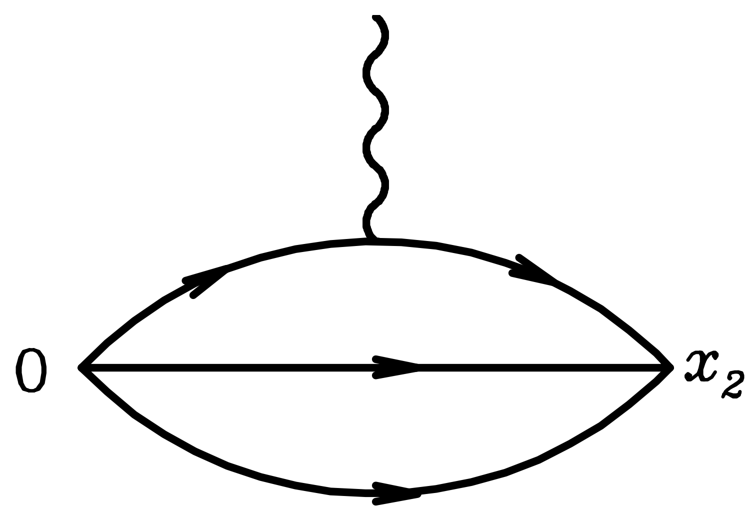


- Create a state (with quantum numbers of the proton) at time *t=0*

- Insert an operator, $\mathcal{O}$, at some time τ

Lattice 3pt Functions

$$\langle \Omega | T \left(\chi_\alpha(\vec{x}_2, t)\, \mathcal{O}(\vec{x}_1, \tau)\, \overline{\chi}_\beta(0) \right) | \Omega \rangle$$



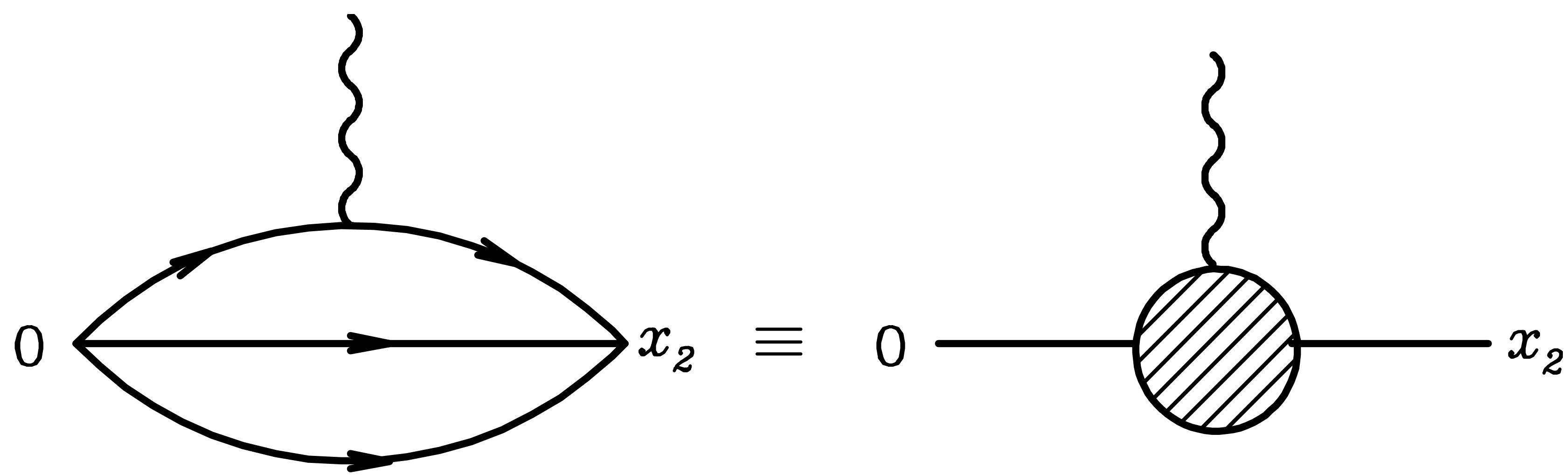
- Create a state (with quantum numbers of the proton) at time *t=0*

- Insert an operator, $\mathcal{O}$, at some time τ

- Annihilate state at final time *t*

Lattice 3pt Functions

$$G(t,\tau,p,p') = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}'\cdot(\vec{x}_2-\vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1} \Gamma_{\beta\alpha} \langle\Omega| T\left(\chi_\alpha(\vec{x}_2,t)\,\mathcal{O}(\vec{x}_1,\tau)\,\overline{\chi}_\beta(0)\right)|\Omega\rangle$$



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- Insert complete set of states

$$I = \sum_{B',p',s'} |B',p',s'\rangle\langle B',p',s'| \qquad I = \sum_{B,p,s} |B,p,s\rangle\langle B,p,s|$$

- Make use of translational invariance

$$\chi(\vec{x},t) = e^{\hat{H}t} e^{-i\hat{\vec{P}}\cdot\vec{x}} \chi(0) e^{i\hat{\vec{P}}\cdot\vec{x}} e^{-\hat{H}t}$$

$$G(t,\tau,\vec{p},\vec{p}\,') = \sum_{B,B'} \sum_{s,s'} e^{-E_{B'}(\vec{p}\,')(t-\tau)} e^{-E_B(\vec{p})\tau} \Gamma_{\beta\alpha}$$
$$\times \langle\Omega|\chi_\alpha(0)|B',p',s'\rangle \langle B',p',s'|\mathcal{O}(\vec{q})|B,p,s\rangle \langle B,p,s|\overline{\chi}_\beta(0)|\Omega\rangle$$

- Evolve to large Euclidean times to isolate ground state $0 \ll \tau \ll t$

$$G(t,\tau,\vec{p},\vec{p}\,') = \sum_{s,s'} e^{-E_{\vec{p}\,'}(t-\tau)} e^{-E_{\vec{p}}\tau} \Gamma_{\beta\alpha} \langle\Omega|\chi_\alpha(0)|N(p',s')\rangle \langle N(p',s')|\mathcal{O}(\vec{q})|N(p,s)\rangle \langle Np,s)|\overline{\chi}_\beta(0)|\Omega\rangle$$

Lattice 3pt Functions

pion

- Consider a pion 3pt function

$$G(t,\tau,p,p') = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}'\cdot(\vec{x}_2-\vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1} \langle\Omega\middle| T\left(\chi(\vec{x}_2,t)\,\mathcal{O}(\vec{x}_1,\tau)\,\chi^\dagger(0)\right)\middle|\Omega\rangle$$

- With interpolating operator $\chi(x) = \bar{d}(x)\gamma_5 u(x)$

- And insert the local operator (quark bi-linear) $\bar{q}(x)\mathcal{O}q(x)$

$\mathcal{O}$: Combination of γ matrices and derivatives

$$-\bar{d}(x_2)\gamma_5 u(x_2)\bar{u}(x_1)\mathcal{O}u(x_1)\bar{u}(0)\gamma_5 d(0)$$

u-quark

Lattice 3pt Functions

pion

- Consider a pion 3pt function

$$G(t,\tau,p,p') = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}'\cdot(\vec{x}_2-\vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1} \langle\Omega| T\left(\chi(\vec{x}_2,t)\,\mathcal{O}(\vec{x}_1,\tau)\,\chi^\dagger(0)\right)|\Omega\rangle$$

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u-quark

Lattice 3pt Functions

pion

u-quark

$$-\bar{d}^a_\beta(x_2)\gamma_{5\beta\gamma}u^a_\gamma(x_2)\bar{u}^b_\rho(x_1)\Gamma_{\rho\delta}u^b_\delta(x_1)\bar{u}^c_\xi(0)\gamma_{5\xi\alpha}d^c_\alpha(0)$$

- all possible Wick contractions

Lattice 3pt Functions

pion

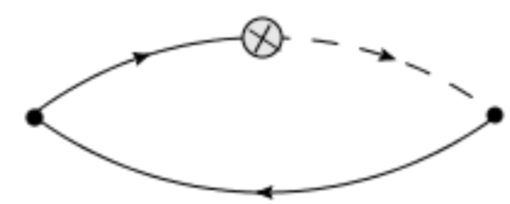
u-quark

$$-\bar{d}^a_\beta(x_2)\gamma_{5\beta\gamma}u^a_\gamma(x_2)\bar{u}^b_\rho(x_1)\Gamma_{\rho\delta}u^b_\delta(x_1)\bar{u}^c_\xi(0)\gamma_{5\xi\alpha}d^c_\alpha(0)$$

- all possible Wick contractions

- connected

$$S^{ca}_{d\alpha\beta}(0,x_2)\gamma_{5\beta\gamma}S^{ab}_{u\gamma\rho}(x_2,x_1)\Gamma_{\rho\delta}S^{bc}_{u\delta\xi}(x_1,0)\gamma_{5\xi\alpha}$$



Lattice 3pt Functions

pion

u-quark

$$-\bar{d}^a_\beta(x_2)\gamma_{5\beta\gamma}u^a_\gamma(x_2)\bar{u}^b_\rho(x_1)\Gamma_{\rho\delta}u^b_\delta(x_1)\bar{u}^c_\xi(0)\gamma_{5\xi\alpha}d^c_\alpha(0)$$

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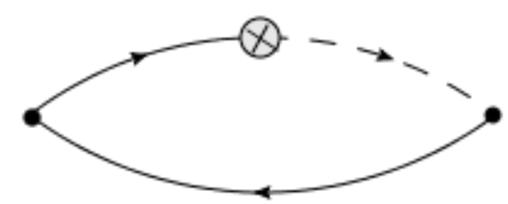

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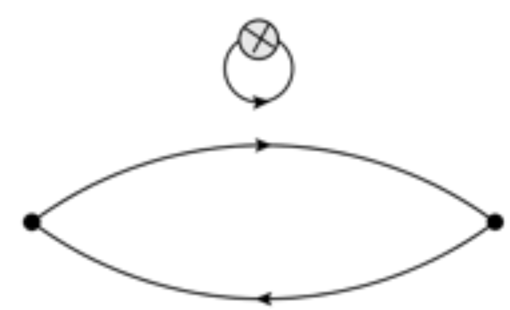
$$-S^{ca}_{d\alpha\beta}(0,x_2)\gamma_{5\beta\gamma}S^{ac}_{u\gamma\xi}(x_2,0)\gamma_{5\xi\alpha}S^{bb}_{u\delta\rho}(x_1,x_1)\Gamma_{\rho\delta}$$



Lattice 3pt Functions

pion

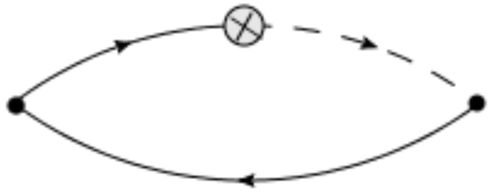
u-quark

$$-\bar{d}^a_\beta(x_2)\gamma_{5\beta\gamma}u^a_\gamma(x_2)\bar{u}^b_\rho(x_1)\Gamma_{\rho\delta}u^b_\delta(x_1)\bar{u}^c_\xi(0)\gamma_{5\xi\alpha}d^c_\alpha(0)$$

- all possible Wick contractions

- connected

$$\mathrm{Tr}\left[S_d(0,x_2)\gamma_5 S_u(x_2,x_1)\Gamma S_u(x_1,0)\gamma_5\right]$$



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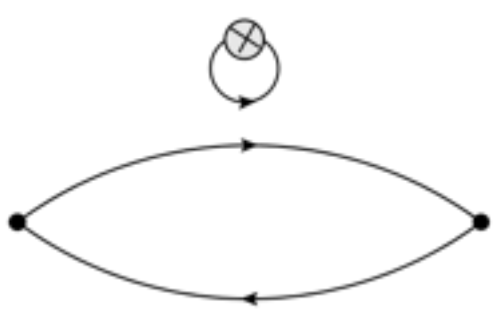


$$\mathrm{Tr}\left[-S_d(0,x_2)\gamma_5 S_u(x_2,0)\gamma_5\right]\mathrm{Tr}\left[S_u(x_1,x_1)\Gamma\right]$$

Lattice 3pt Functions

pion

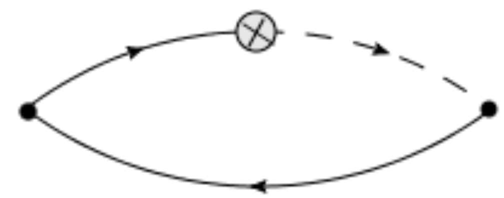
$$-\bar{d}^a_\beta(x_2)\gamma_{5\beta\gamma}u^a_\gamma(x_2)\bar{u}^b_\rho(x_1)\Gamma_{\rho\delta}u^b_\delta(x_1)\bar{u}^c_\xi(0)\gamma_{5\xi\alpha}d^c_\alpha(0)$$

γ_5-hermiticity

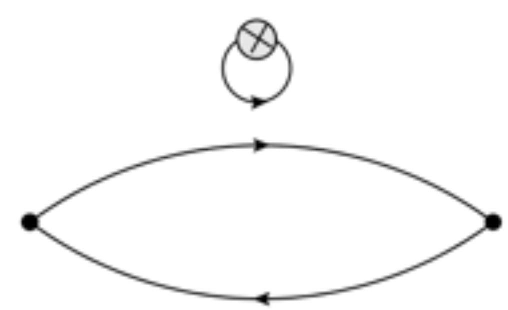
$$S^\dagger(x,0) = \gamma_5 S(0,x)\gamma_5$$

- all possible Wick contractions
- connected

$$\mathrm{Tr}\left[S^\dagger_d(x_2,0)S_u(x_2,x_1)\Gamma S_u(x_1,0)\right]$$



- disconnected



$$\mathrm{Tr}\left[-S^\dagger_d(x_2,0)S_u(x_2,0)\right]\mathrm{Tr}\left[S_u(x_1,x_1)\Gamma\right]$$

- all-to-all propagators

Lattice 3pt Functions

proton

$$G_\Gamma(t,\tau;\vec{p}',\vec{p}) = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}'\cdot(\vec{x}_2-\vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1}\, \Gamma_{\beta\alpha} \langle\Omega| T\left[\chi_\alpha(t,\vec{x}_2)\, \mathcal{O}(\tau,\vec{x}_1)\, \overline{\chi}_\beta(0)\right] |\Omega\rangle$$

- Use the following interpolating operator to create a proton

$$\chi_\alpha(x) = \epsilon^{abc}\left(u^{Ta}(x)\ C\gamma_5\ d^b(x)\right) u^c_\alpha(x)$$

- And insert the local operator (quark bi-linear) $\bar{q}(x)\mathcal{O}q(x)$

$\mathcal{O}$: Combination of γ matrices and derivatives

- Perform all possible (connected) Wick contractions

u-quark (4 terms)

$$\epsilon^{abc}\epsilon^{a'b'c'}\left(u^{Ta}(x_2)\ C\gamma_5\ d^b(x_2)\right) u^c_\alpha(x_2)\bar{u}(x_1)\mathcal{O}u(x_1)\bar{u}^{c'}(0)\left(\bar{d}^{b'}(0)C\gamma_5\bar{u}^{Ta'}(0)\right)$$

Lattice 3pt Functions

proton

$$G_\Gamma(t,\tau;\vec{p}',\vec{p}) = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}'\cdot(\vec{x}_2-\vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1}\, \Gamma_{\beta\alpha}\langle\Omega|T\left[\chi_\alpha(t,\vec{x}_2)\,\mathcal{O}(\tau,\vec{x}_1)\,\overline{\chi}_\beta(0)\right]|\Omega\rangle$$

- Use the following interpolating operator to create a proton

$$\chi_\alpha(x) = \epsilon^{abc}\left(u^{Ta}(x)\;C\gamma_5\;d^b(x)\right)u^c_\alpha(x)$$

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$$\epsilon^{abc}\epsilon^{a'b'c'}\left(u^{Ta}(x_2)\;C\gamma_5\;d^b(x_2)\right)u^c_\alpha(x_2)\bar{u}(x_1)\mathcal{O}u(x_1)\bar{u}^{c'}(0)\left(\bar{d}^{b'}(0)C\gamma_5\bar{u}^{Ta'}(0)\right)$$

Lattice 3pt Functions

proton

$$G_\Gamma(t,\tau;\vec{p}',\vec{p}) = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}'\cdot(\vec{x}_2-\vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1}\,\Gamma_{\beta\alpha}\langle\Omega|T\left[\chi_\alpha(t,\vec{x}_2)\,\mathcal{O}(\tau,\vec{x}_1)\,\overline{\chi}_\beta(0)\right]|\Omega\rangle$$

- Use the following interpolating operator to create a proton

$$\chi_\alpha(x) = \epsilon^{abc}\left(u^{Ta}(x)\;C\gamma_5\;d^b(x)\right)u^c_\alpha(x)$$

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u-quark (4 terms)

$$\epsilon^{abc}\epsilon^{a'b'c'}\left(u^{Ta}(x_2)\;C\gamma_5\;d^b(x_2)\right)u^c_\alpha(x_2)\bar{u}(x_1)\mathcal{O}u(x_1)\bar{u}^{c'}(0)\left(\bar{d}^{b'}(0)C\gamma_5\bar{u}^{Ta'}(0)\right)$$

Lattice 3pt Functions

proton

$$G_\Gamma(t,\tau;\vec{p}',\vec{p}) = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}'\cdot(\vec{x}_2-\vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1}\, \Gamma_{\beta\alpha}\langle\Omega|T\left[\chi_\alpha(t,\vec{x}_2)\,\mathcal{O}(\tau,\vec{x}_1)\,\overline{\chi}_\beta(0)\right]|\Omega\rangle$$

- Use the following interpolating operator to create a proton

$$\chi_\alpha(x) = \epsilon^{abc}\left(u^{Ta}(x)\; C\gamma_5\; d^b(x)\right) u^c_\alpha(x)$$

- And insert the local operator (quark bi-linear) $\bar{q}(x)\mathcal{O}q(x)$

$\mathcal{O}$: Combination of γ matrices and derivatives

- Perform all possible (connected) Wick contractions

u-quark (4 terms)

$$\epsilon^{abc}\epsilon^{a'b'c'}\left(u^{Ta}(x_2)\; C\gamma_5\; d^b(x_2)\right) u^c_\alpha(x_2)\bar{u}(x_1)\mathcal{O}u(x_1)\bar{u}^{c'}(0)\left(\bar{d}^{b'}(0)C\gamma_5\bar{u}^{Ta'}(0)\right)$$

Lattice 3pt Functions

proton

$$G_\Gamma(t,\tau;\vec{p}',\vec{p}) = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}'\cdot(\vec{x}_2-\vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1}\, \Gamma_{\beta\alpha}\langle\Omega|T\left[\chi_\alpha(t,\vec{x}_2)\,\mathcal{O}(\tau,\vec{x}_1)\,\overline{\chi}_\beta(0)\right]|\Omega\rangle$$

- Use the following interpolating operator to create a proton

$$\chi_\alpha(x) = \epsilon^{abc}\left(u^{Ta}(x)\; C\gamma_5\; d^b(x)\right) u^c_\alpha(x)$$

- And insert the local operator (quark bi-linear) $\bar{q}(x)\mathcal{O}q(x)$

$\mathcal{O}$: Combination of γ matrices and derivatives

- Perform all possible (connected) Wick contractions

d-quark (2 terms)

$$\epsilon^{abc}\epsilon^{a'b'c'}\left(u^{Ta}(x_2)\; C\gamma_5\; d^b(x_2)\right) u^c_\alpha(x_2)\bar{d}(x_1)\mathcal{O}d(x_1)\bar{u}^{c'}(0)\left(\bar{d}^{b'}(0)C\gamma_5\bar{u}^{Ta'}(0)\right)$$

Lattice 3pt Functions

proton

$$G_\Gamma(t,\tau;\vec{p}',\vec{p}) = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}'\cdot(\vec{x}_2-\vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1}\, \Gamma_{\beta\alpha} \langle\Omega| T\left[\chi_\alpha(t,\vec{x}_2)\,\mathcal{O}(\tau,\vec{x}_1)\,\overline{\chi}_\beta(0)\right]|\Omega\rangle$$

- Use the following interpolating operator to create a proton

$$\chi_\alpha(x) = \epsilon^{abc}\left(u^{Ta}(x)\; C\gamma_5\; d^b(x)\right) u^c_\alpha(x)$$

- And insert the local operator (quark bi-linear) $\bar{q}(x)\mathcal{O}q(x)$

$\mathcal{O}$: Combination of γ matrices and derivatives

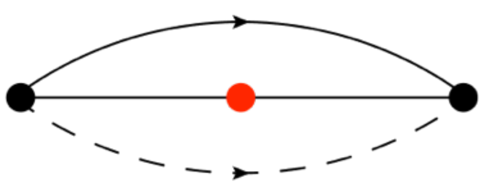
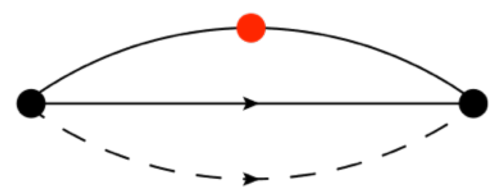
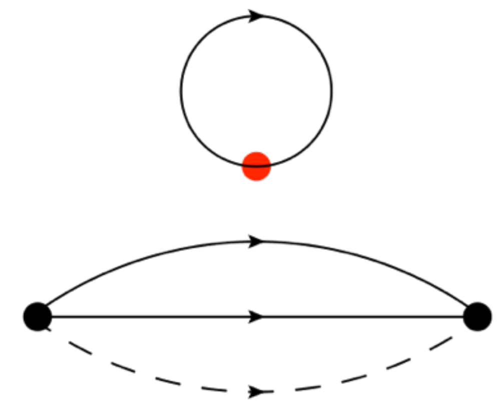
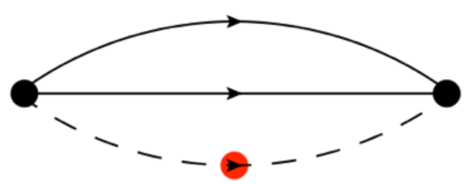
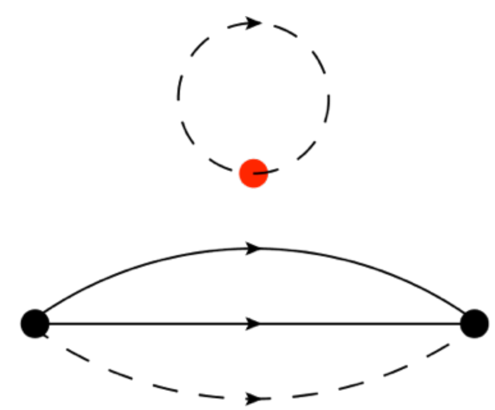
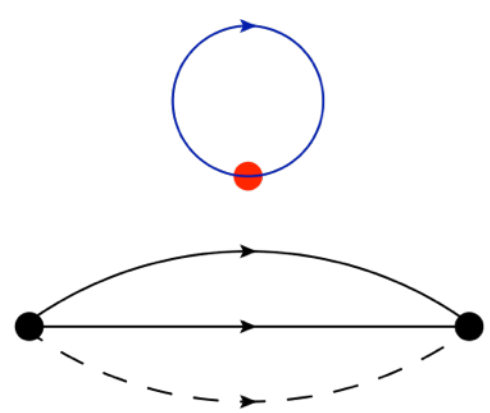
- Perform all possible (connected) Wick contractions

d-quark (2 terms)

$$\epsilon^{abc}\epsilon^{a'b'c'}\left(u^{Ta}(x_2)\; C\gamma_5\; d^b(x_2)\right) u^c_\alpha(x_2)\bar{d}(x_1)\mathcal{O}d(x_1)\bar{u}^{c'}(0)\left(\bar{d}^{b'}(0)C\gamma_5\bar{u}^{Ta'}(0)\right)$$

Lattice 3pt Functions

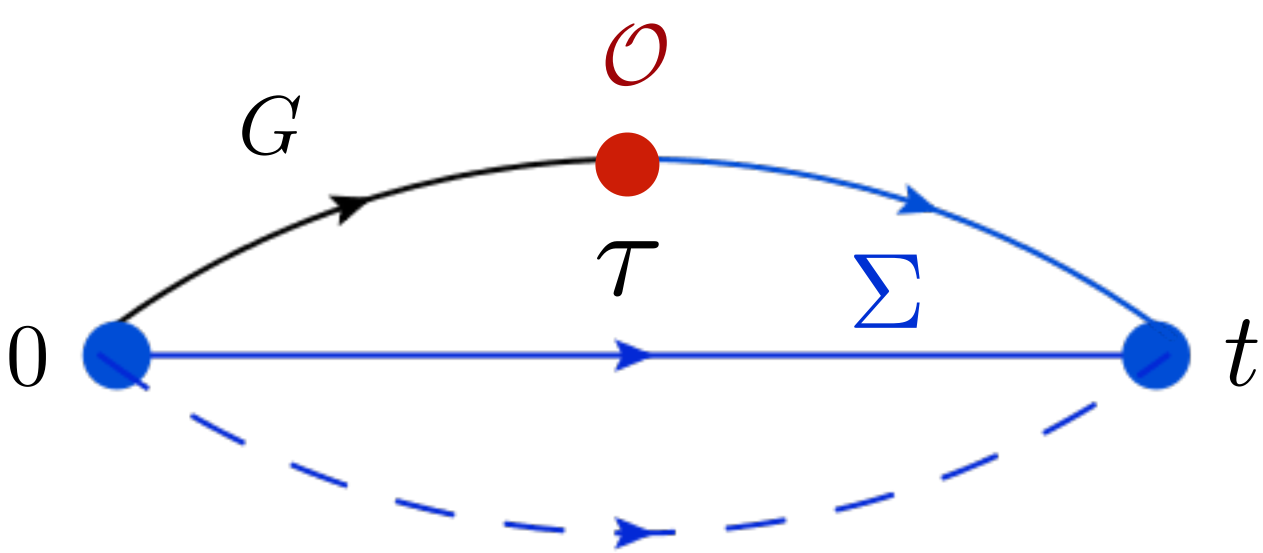
proton

- Pictorially:
  - u-quark   
  - d-quark  
  - s-quark 
- quark-line disconnected contributions drop out in isovector quantities (*u-d*) if isospin is exact ($m_u = m_d$)

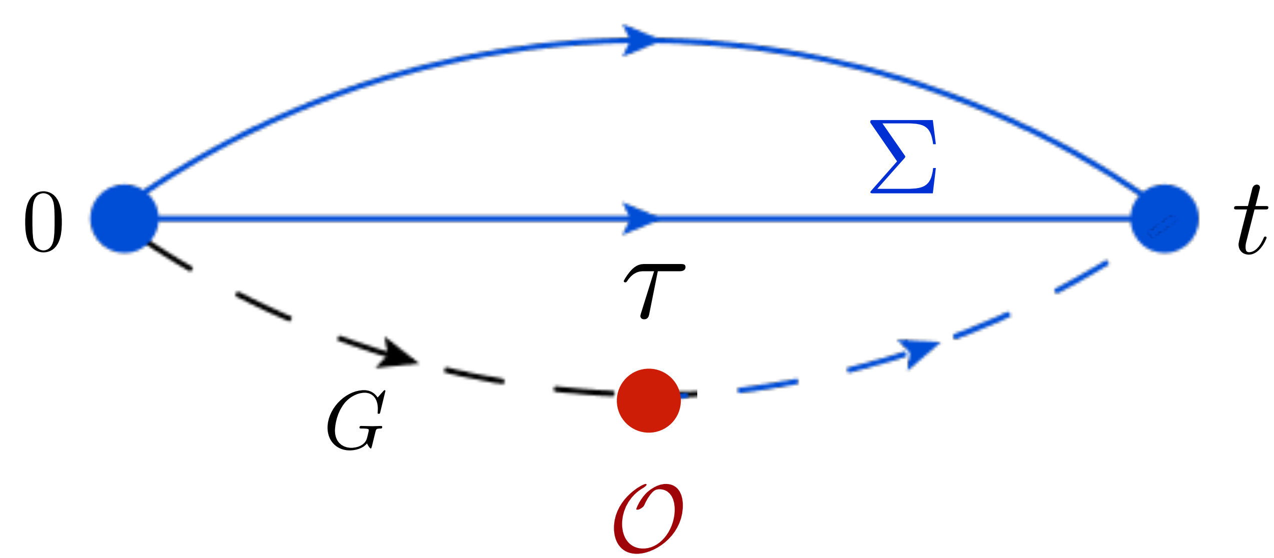
Lattice 3pt Functions at the quark level

proton

$$C_\Gamma(t,\tau;\vec{p}',\vec{p}) = \sum_{\vec{x}_1} e^{i\vec{q}\cdot\vec{x}_1} \left\langle \mathrm{Tr}\left[\Sigma_\Gamma(\vec{0},0;\vec{p}',t)\mathcal{O}(\vec{x}_1,\tau)G(\vec{x}_1,0)\right] \right\rangle_{\{U\}}$$



u-quark



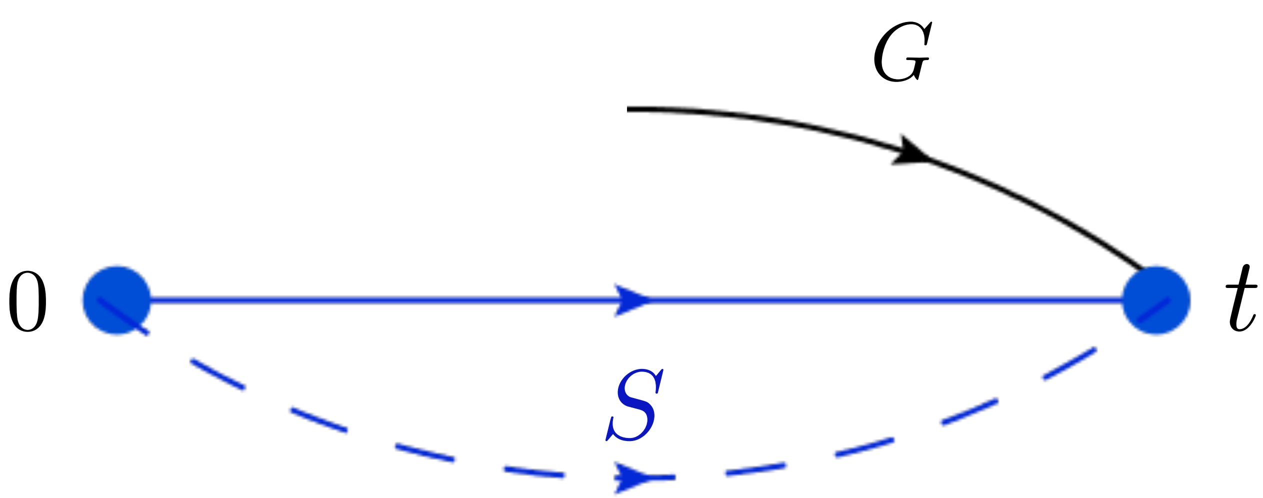
d-quark

Lattice 3pt Functions at the quark level

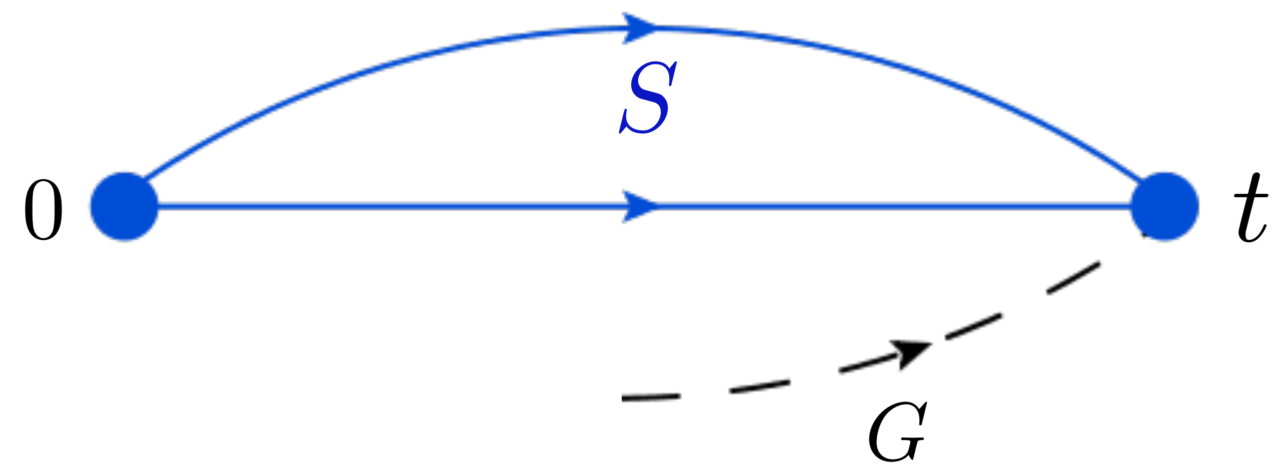
proton

$$C_\Gamma(t,\tau;\vec{p}\,',\vec{p}) = \sum_{\vec{x}_1} e^{i\vec{q}\cdot\vec{x}_1} \left\langle \mathrm{Tr}\left[\Sigma_\Gamma(\vec{0},0;\vec{p}\,',t)\mathcal{O}(\vec{x}_1,\tau)G(\vec{x}_1,0)\right] \right\rangle_{\{U\}}$$

$$\Sigma_\Gamma(\vec{0},0;\vec{x}_1;\vec{p}\,',t) = \sum_{\vec{x}_2} S_\Gamma(\vec{x}_2,t;\vec{0},0;\vec{p}\,')G(\vec{x}_2,t;\vec{x}_1)$$



u-quark



d-quark

Lattice 3pt Functions at the quark level

proton

$$C_\Gamma(t,\tau;\vec{p}',\vec{p}) = \sum_{\vec{x}_1} e^{i\vec{q}\cdot\vec{x}_1} \left\langle \mathrm{Tr}\left[\Sigma_\Gamma(\vec{0},0;\vec{p}',t)\mathcal{O}(\vec{x}_1,\tau)G(\vec{x}_1,0)\right]\right\rangle_{\{U\}}$$

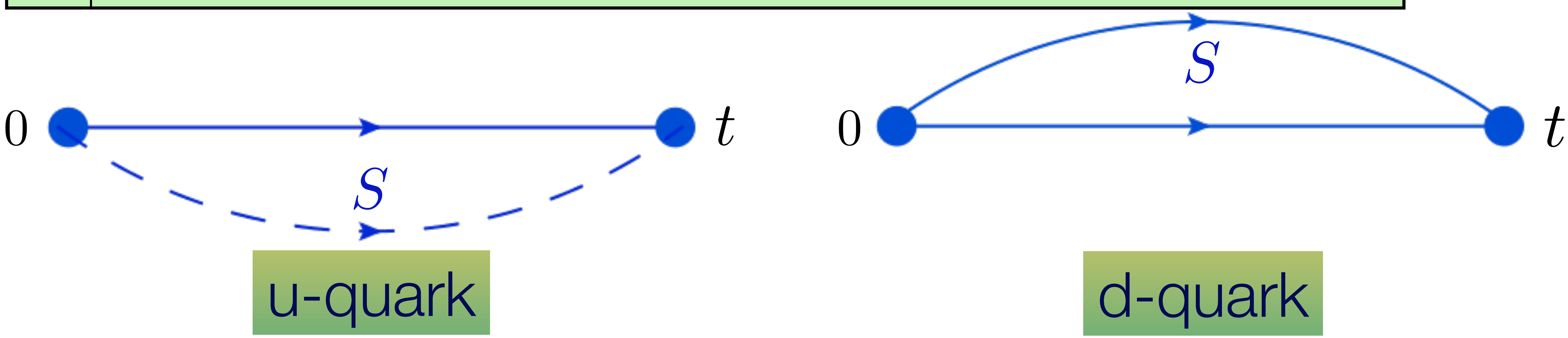
$$\Sigma_\Gamma(\vec{0},0;\vec{x}_1;\vec{p}',t) = \sum_{\vec{x}_2} S_\Gamma(\vec{x}_2,t;\vec{0},0;\vec{p}')G(\vec{x}_2,t;\vec{x}_1)$$

$$\tilde{G} = C\gamma_5 G^T \gamma_5 C$$

$$S_\Gamma^{u;a'a}(\vec{x}_2,t;\vec{0},0;\vec{p}') = e^{-i\vec{p}'\cdot\vec{x}}\epsilon^{abc}\epsilon^{a'b'c'}\times$$
$$\Big[\tilde{G}^{d;bb'}(\vec{x}_2,t;\vec{0},0)G^{u;cc'}(\vec{x}_2,t;\vec{0},0)\Gamma + \mathrm{Tr}_D[\tilde{G}^{d;bb'}(\vec{x}_2,t;\vec{0},0)G^{u;cc'}(\vec{x}_2,t;\vec{0},0)]\Gamma$$
$$+\Gamma G^{u;bb'}(\vec{x}_2,t;\vec{0},0)\tilde{G}^{d;cc'}(\vec{x},t;\vec{0},0) + \mathrm{Tr}_D[\Gamma G^{u;bb'}(\vec{x}_2,t;\vec{0},0)]\tilde{G}^{d;cc'}(\vec{x}_2,t;\vec{0},0)\Big]$$

$$S_\Gamma^{d;a'a}(\vec{x}_2,t;\vec{0},0;\vec{p}') = e^{-i\vec{p}'\cdot\vec{x}_2}\epsilon^{abc}\epsilon^{a'b'c'}\times$$
$$\Big[\tilde{G}^{u;bb'}(\vec{x}_2,t;\vec{0},0)\tilde{\Gamma}\tilde{G}^{u;cc'}(\vec{x}_2,t;\vec{0},0) + \mathrm{Tr}_D[\Gamma G^{u;bb'}(\vec{x}_2,t;\vec{0},0)\tilde{G}^{u;cc'}(\vec{x}_2,t;\vec{0},0)]\Big]$$

Exercise: Prove



Lattice 3pt Functions

- $\Sigma_\Gamma(\vec{0},0;\vec{x}_1;\vec{p}\,',t) = \sum_{\vec{x}_2} S_\Gamma(\vec{x}_2,t;\vec{0},0;\vec{p}\,')G(\vec{x}_2,t;\vec{x}_1)$ can be computed from the linear system of equations

$$\sum_v M(v',\, v)\gamma_5\Sigma^\dagger_\Gamma(\vec{0},0;v;\vec{p}\,',t) = \gamma_5 S^\dagger_\Gamma(\vec{v},t;\vec{0},0;\vec{p}\,')\delta_{v'_0,t}$$

Fermion matrix

- so $\Sigma_\Gamma(\vec{0},0;\vec{x}_1;\vec{p}\,',t)$ is a sequential propagator based on a source $S_\Gamma(\vec{x}_2,t;\vec{0},0;\vec{p}\,')$ constructed from two ordinary propagators at time ***t***

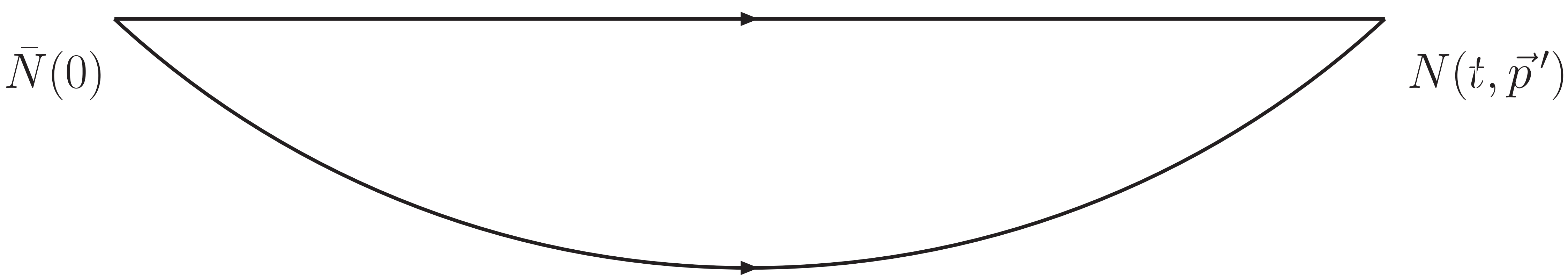
Sequential Source Technique

- First compute ordinary propagators $G(x,0)$

Sequential Source Technique

- Construct sources

$$S_{\Gamma}^{u;a'a}(\vec{x}_2, t; \vec{0}, 0; \vec{p}\,') \quad \text{or} \quad S_{\Gamma}^{d;a'a}(\vec{x}_2, t; \vec{0}, 0; \vec{p}\,')$$



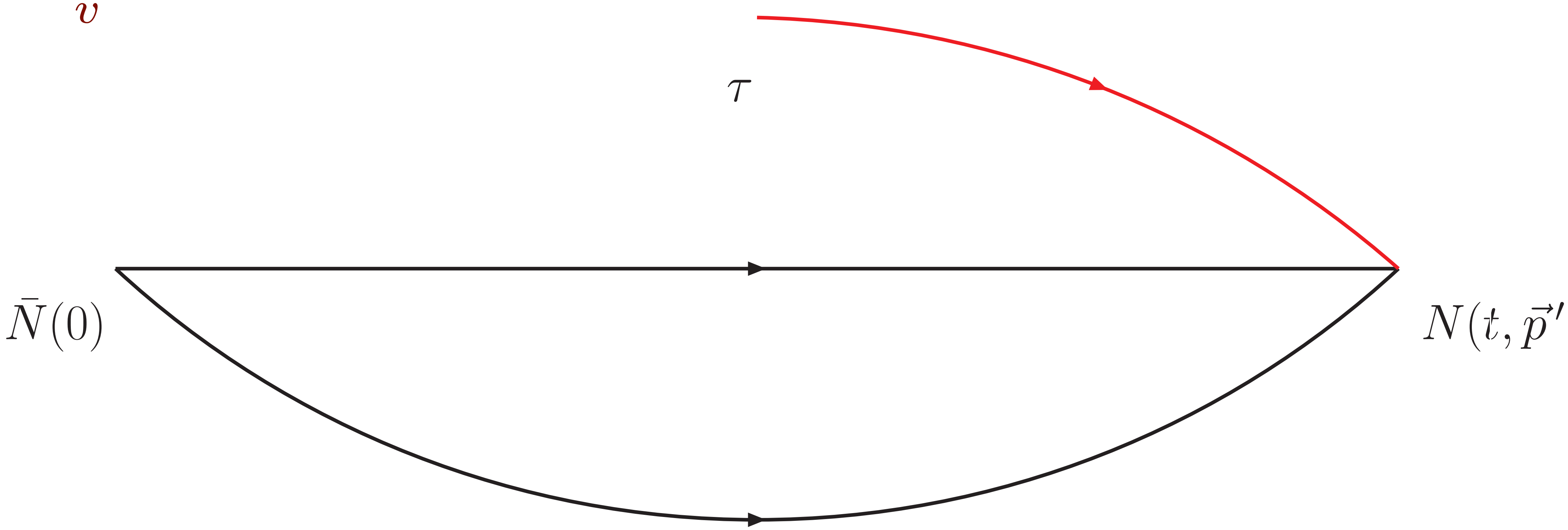
Sequential Source Technique

- Compute sequential propagators

$$\Sigma_\Gamma(\vec{0},0;\vec{x}_1;\vec{p}\,',t) = \sum_{\vec{x}_2} S_\Gamma(\vec{x}_2,t;\vec{0},0;\vec{p}\,')S(\vec{x}_2,t;\vec{x}_1)$$

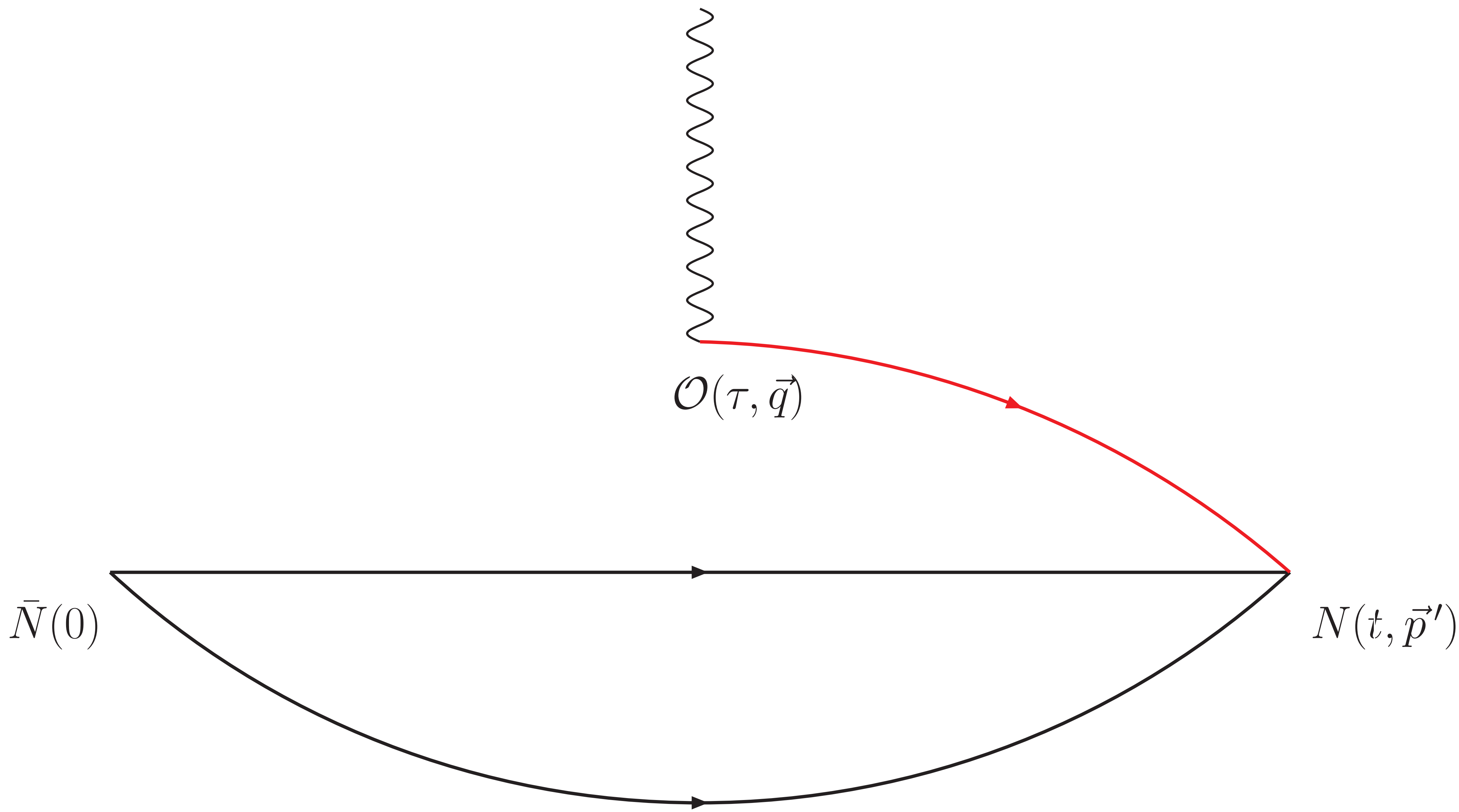
- via the second inversion

$$\sum_v M(v',\, v)\gamma_5\Sigma^\dagger_\Gamma(\vec{0},0;v;\vec{p}\,',t) = \gamma_5 S^\dagger_\Gamma(\vec{v},t;\vec{0},0;\vec{p}\,')\delta_{v'_0,t}$$



Sequential Source Technique

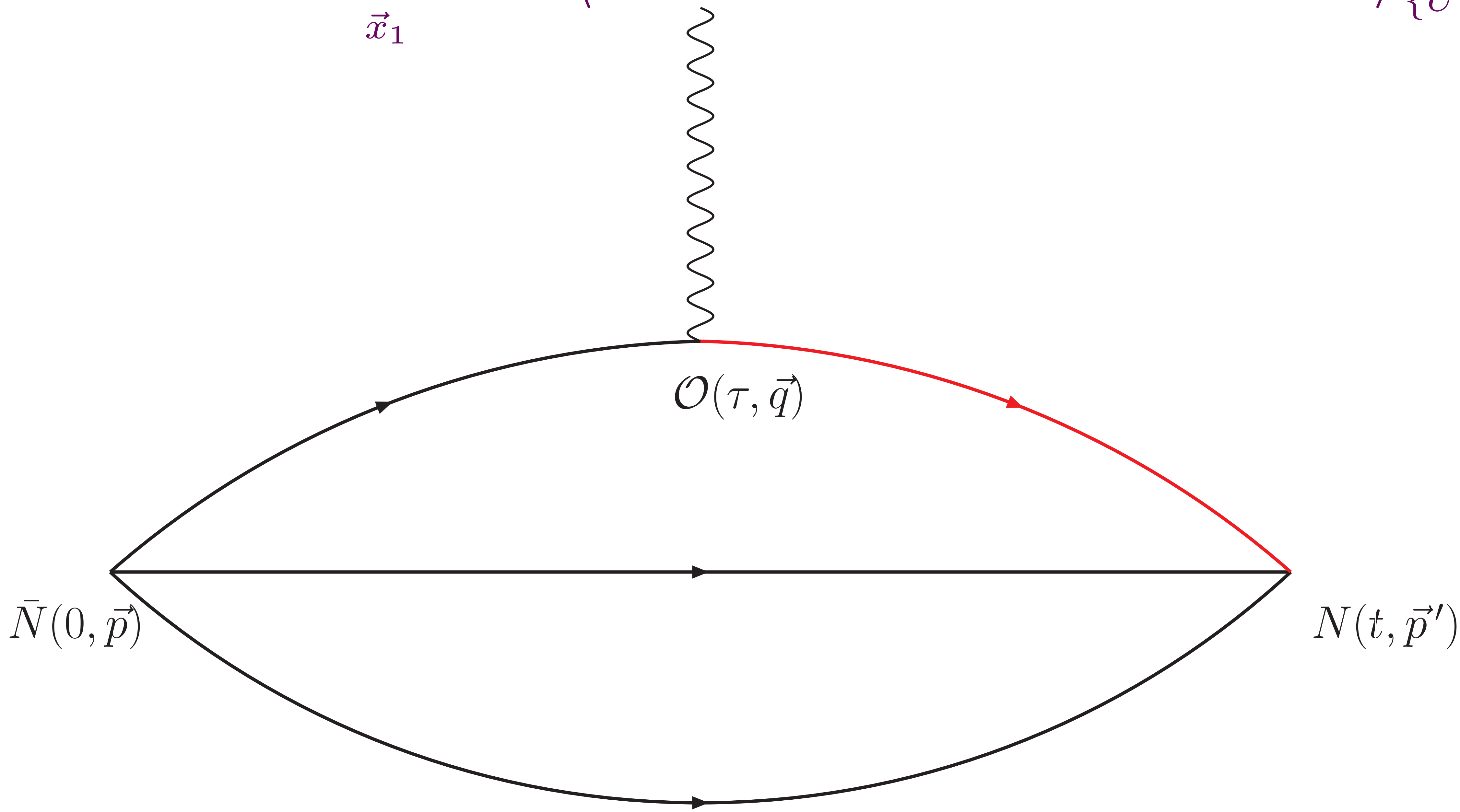
- Insert operator $\Sigma_\Gamma(\vec{0}, 0; \vec{p}\,', t)\mathcal{O}(\vec{x}_1, \tau)$



Sequential Source Technique

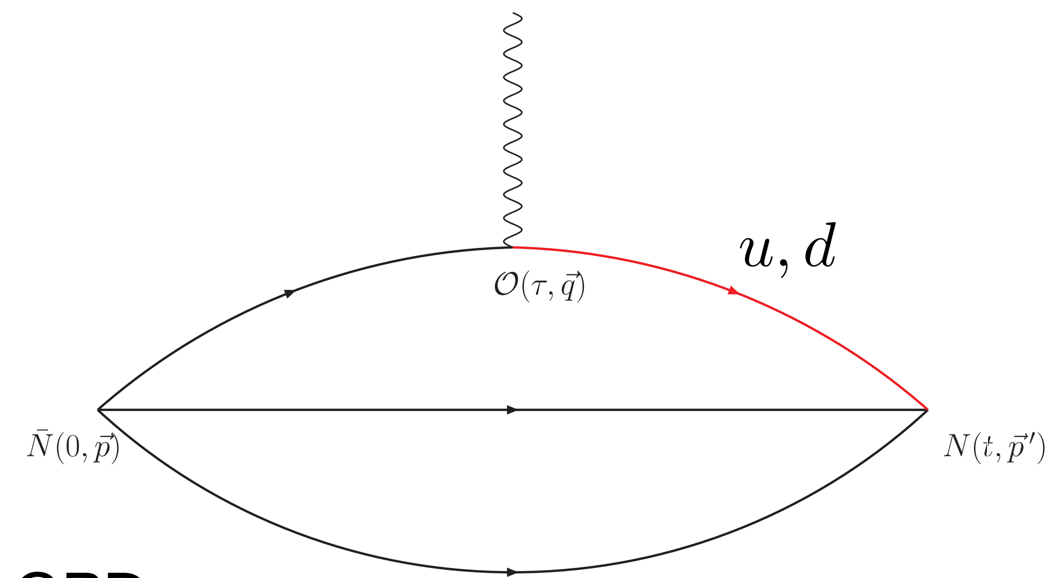
- Tie everything together with an ordinary propagator

$$C_\Gamma(t,\tau;\vec{p}\,',\vec{p}) = \sum_{\vec{x}_1} e^{i\vec{q}\cdot\vec{x}_1} \left\langle \mathrm{Tr}\left[\Sigma_\Gamma(\vec{0},0;\vec{p}\,',t)\mathcal{O}(\vec{x}_1,\tau)G(\vec{x}_1,0)\right]\right\rangle_{\{U\}}$$



Sequential Source Technique

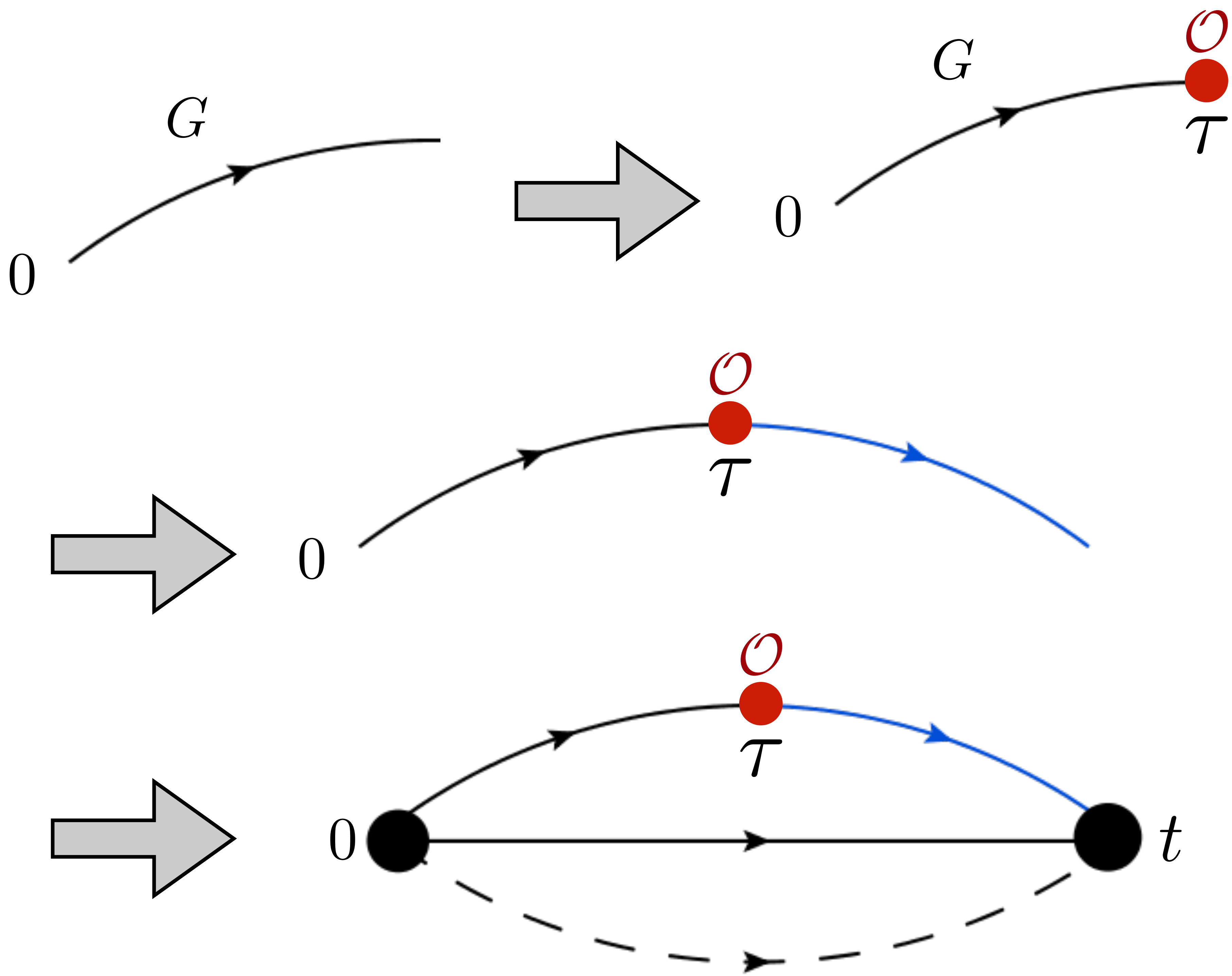
through the sink



- Advantages: Free choice of
    - Momentum transfer
    - Operator (vector/axial/tensor)
    - Ideal for Form Factors, Structure Functions, GPDs
- Disadvantages: Separate 3-pt inversion for each
    - Quark flavour
    - Hadron eg. p, Σ, Δ, π, $N \to \gamma\Delta$
    - Polarisation
    - Sink momentum

Sequential Source Technique

- Alternative method involves computing a sequential propagator "through the operator"



Sequential Source Technique

through the operator

- Advantages: Free choice of
  - Quark flavour
  - Hadron e.g. p, $\Sigma, \ \Delta, \ \pi, \ N \to \gamma\Delta$
  - Polarisation
  - Sink momentum
  - Ideal for studying flavour dependence in a hadron multiplet
- Disadvantages: Separate 3-pt inversion for each
  - Momentum transfer
  - Operator (vector/axial/tensor)

Lattice 3pt Functions in Chroma

proton

$$S_\Gamma^{u;a'a}(\vec{x}_2, t; \vec{0}, 0; \vec{p}') = e^{-i\vec{p}'\cdot\vec{x}} \epsilon^{abc}\epsilon^{a'b'c'} \times$$

$$\Big[\tilde{G}^{d;bb'}(\vec{x}_2, t; \vec{0}, 0) G^{u;cc'}(\vec{x}_2, t; \vec{0}, 0)\Gamma + \mathrm{Tr}_D[\tilde{G}^{d;bb'}(\vec{x}_2, t; \vec{0}, 0) G^{u;cc'}(\vec{x}_2, t; \vec{0}, 0)]\Gamma$$

$$+\Gamma G^{u;bb'}(\vec{x}_2, t; \vec{0}, 0)\tilde{G}^{d;cc'}(\vec{x}, t; \vec{0}, 0) + \mathrm{Tr}_D[\Gamma G^{u;bb'}(\vec{x}_2, t; \vec{0}, 0)]\tilde{G}^{d;cc'}(\vec{x}_2, t; \vec{0}, 0)\Big]$$

simple_baryon_seqsrc_w.cc

```
/* "\bar u O u" insertion in NR proton, ie.
 * "(u Cg5 d) u" */
/* Some generic T */

// Use precomputed Cg5
q1_tmp = quark_propagators[0] * Cg5;
q2_tmp = Cg5 * quark_propagators[1];
di_quark = quarkContract24(q1_tmp, q2_tmp);

// First term
src_prop_tmp = T * di_quark;

// Now the second term
src_prop_tmp += traceSpin(di_quark) * T;

// The third term...
q1_tmp = q2_tmp * Cg5;
q2_tmp = quark_propagators[0] * T;

src_prop_tmp -= quarkContract13(q1_tmp, q2_tmp) + transposeSpin(quarkContract12(q2_tmp, q1_tmp));

END_CODE();

return projectBaryon(src_prop_tmp,
                     forward_headers);
```

```
onSeqSourceFactory::Instance().registerObject(string("NUCL-NUCL_U"),
                                              barNuclNuclU);

    success &= Chroma::TheWilsonHadronSeqSourceFactory::Instance().registerObject(string("NUCL-NUCL_D"),
                                                                                  barNuclNuclD);

    success &= Chroma::TheWilsonHadronSeqSourceFactory::Instance().registerObject(string("NUCL_U_UNPOL"),
                                                                                  barNuclUUnpol);

    success &= Chroma::TheWilsonHadronSeqSourceFactory::Instance().registerObject(string("NUCL_D_UNPOL"),
                                                                                  barNuclDUnpol);

    success &= Chroma::TheWilsonHadronSeqSourceFactory::Instance().registerObject(string("NUCL_U_POL"),
                                                                                  barNuclUPol);

    success &= Chroma::TheWilsonHadronSeqSourceFactory::Instance().registerObject(string("NUCL_D_POL"),
                                                                                  barNuclDPol);

    success &= Chroma::TheWilsonHadronSeqSourceFactory::Instance().registerObject(string("NUCL_U_UNPOL_NONREL"),
                                                                                  barNuclUUnpolNR);

    success &= Chroma::TheWilsonHadronSeqSourceFactory::Instance().registerObject(string("NUCL_D_UNPOL_NONREL"),
                                                                                  barNuclDUnpolNR);

    success &= Chroma::TheWilsonHadronSeqSourceFactory::Instance().registerObject(string("NUCL_U_POL_NONREL"),
                                                                                  barNuclUPolNR);

    success &= Chroma::TheWilsonHadronSeqSourceFactory::Instance().registerObject(string("NUCL_D_POL_NONREL"),
                                                                                  barNuclDPolNR);

    success &= Chroma::TheWilsonHadronSeqSourceFactory::Instance().registerObject(string("NUCL_U_MIXED_NONREL"),
                                                                                  barNuclUMixedNR);

    success &= Chroma::TheWilsonHadronSeqSourceFactory::Instance().registerObject(string("NUCL_D_MIXED_NONREL"),
                                                                                  barNuclDMixedNR);

    success &= Chroma::TheWilsonHadronSeqSourceFactory::Instance().registerObject(string("NUCL_U_MIXED_NONREL_NEGPAR"),
                                                                                  barNuclUMixedNRnegPar);

    success &= Chroma::TheWilsonHadronSeqSourceFactory::Instance().registerObject(string("NUCL_D_MIXED_NONREL_NEGPAR"),
                                                                                  barNuclDMixedNRnegPar);

    success &= Chroma::TheWilsonHadronSeqSourceFactory::Instance().registerObject(string("XI_D_MIXED_NONREL"),
                                                                                  barXiDMixedNR);


    success &= Chroma::TheWilsonHadronSeqSourceFactory::Instance().registerObject(string("DELTA-DELTA_U"),
                                                                                  barDeltaDeltaU);

    success &= Chroma::TheWilsonHadronSeqSourceFactory::Instance().registerObject(string("DELTA-DELTA_D"),
```

Chroma xml for Sequential Source

```xml
<elem>
  <annotation>; NUCL_U_UNPOL seqsource</annotation>
  <Name>SEQSOURCE</Name>
  <Frequency>1</Frequency>
  <Param>
    <version>1</version>
    <seq_src>NUCL_U_UNPOL</seq_src>
    <t_sink>13</t_sink>
    <sink_mom>0 0 0</sink_mom>
  </Param>
  <PropSink>
    <version>5</version>
    <Sink>
      <version>2</version>
      <SinkType>SHELL_SINK</SinkType>
      <j_decay>3</j_decay>
      <SmearingParam>
        <wvf_kind>GAUGE_INV_GAUSSIAN</wvf_kind>
        <wvf_param>2.0</wvf_param>
        <wvfIntPar>5</wvfIntPar>
        <no_smear_dir>3</no_smear_dir>
      </SmearingParam>
    </Sink>
  </PropSink>
  <NamedObject>
    <gauge_id>gauge</gauge_id>
    <prop_ids>
      <elem>sh_prop_1</elem>
      <elem>sh_prop_1</elem>
    </prop_ids>
    <seqsource_id>seqsource_NUCL_U_UNPOL</seqsource_id>
  </NamedObject>
</elem>
```

u-quark in proton, unpolarised

sink timeslice

sink momentum

sink smearing

ordinary quark props required for construction of seq source (2 for u, 1 for d)

tag (to be used as source for prop calculation)

Exercise

- Using the following interpolating operator

$$\chi^{\Delta^+}(x) = \frac{1}{\sqrt{3}}\epsilon^{abc}\left[2\big(u^{T\,a}(x)C\gamma_+ d^b(x)\big)u^c(x) + \big(u^{T\,a}(x)C\gamma_+ u^b(x)\big)d^c(x)\right]$$

  perform the appropriate Wick contractions and write down the Δ^+ 3pt function and compare your result to the source implemented in Chroma

- Work out the sequential sources required for $\gamma N \to \Delta$

$$\langle\Omega|\Delta^+(x_2)j^\mu(x_1)N(0)|\Omega\rangle$$

Extracting matrix elements

- Recall hadronic form of the nucleon 3pt function

$$G(t,\tau,\vec{p},\vec{p}\,') = \sum_{s,s'} e^{-E_{\vec{p}\,'}(t-\tau)} e^{-E_{\vec{p}}\tau} \Gamma_{\beta\alpha} \langle \Omega | \chi_\alpha(0) | N(p',s') \rangle \langle N(p',s') | \mathcal{O}(\vec{q}) | N(p,s) \rangle \langle N p, s) | \overline{\chi}_\beta(0) | \Omega \rangle$$

- Need to remove time dependence and wave function amplitudes

Form a ratios with the nucleon 2pt function

$$G_2(t,\vec{p}) = \sum_s e^{-E_p t} \Gamma_{\beta\alpha} \langle \Omega | \chi_\alpha | N(p,s) \rangle \langle N(p,s) | \overline{\chi}_\beta | \Omega \rangle$$

- E.g.

$$R(t,\tau;\vec{p}\,',\vec{p};\mathcal{O}) = \frac{G_\Gamma(t,\tau;\vec{p}\,',\vec{p},\mathcal{O})}{G_2(t,\vec{p}\,')} \left[\frac{G_2(\tau,\vec{p}\,') G_2(t,\vec{p}\,') G_2(t-\tau,\vec{p})}{G_2(\tau,\vec{p}) G_2(t,\vec{p}) G_2(t-\tau,\vec{p}\,')} \right]^{\frac{1}{2}}$$

Extracting matrix elements

- Using the relation for spinors

$$\bar{u}(\vec{p},\sigma')\Gamma u(\vec{p},\sigma) = \mathrm{Tr}\Gamma(E\gamma_4 - i\vec{p}\cdot\vec{\gamma} + m)\frac{1}{2}\left(1 - \gamma_5\gamma_4\frac{\vec{p}\cdot\vec{s}}{EM} + i\gamma_5\frac{\vec{\gamma}\cdot\vec{s}}{m}\right)\delta_{\sigma\sigma'}$$

- We can write the two point function as

$$G_2(t,\vec{p}) = \sum_s \frac{\sqrt{Z^{\mathrm{snk}}(\vec{p})}\sqrt{\overline{Z}^{\mathrm{src}}(\vec{p})}}{2E_{\vec{p}}}\ \mathrm{Tr}\bar{u}(\vec{p},s)\Gamma u(\vec{p},s)[e^{-E_p t} + e^{-E'_{\vec{p}}(T-t)}]$$

+ *v*-spinor terms with opposite parity

- Use $\Gamma_4 = \frac{1}{2}(1+\gamma_4)$ to maximise overlap with positive parity forward propagating state

$$G_2(t,\vec{p}) = \sqrt{Z^{\mathrm{snk}}(\vec{p})\overline{Z}^{\mathrm{src}}(\vec{p})}\left[\left(\frac{E_{\vec{p}}+m}{E_{\vec{p}}}\right)e^{-E_p t} + \left(\frac{E'_{\vec{p}}+m'}{E'_{\vec{p}}}\right)e^{-E'_{\vec{p}}(T-t)}\right]$$

- Similarly for the three-point function

$$G_3(t,\tau;\vec{p}'\vec{p};\Gamma,\mathcal{O}) = \sqrt{Z^{\mathrm{snk}}(\vec{p}')\overline{Z}^{\mathrm{src}}(\vec{p})}F(\Gamma,\mathcal{F})e^{-E_{\vec{p}'}(t-\tau)}e^{-E_{\vec{p}}\tau}$$

- where

$$F(\Gamma,\mathcal{J}) = \frac{1}{4}\mathrm{Tr}\left(\gamma_4 - i\frac{\vec{p}'\cdot\vec{\gamma}}{E_{\vec{p}'}} + \frac{m}{E_{\vec{p}'}}\right)\mathcal{J}\left(\gamma_4 - i\frac{\vec{p}\cdot\vec{\gamma}}{E_{\vec{p}}} + \frac{m}{E_{\vec{p}}}\right)$$

- and

$$\langle N(p',\, s')|\mathcal{O}(\vec{q})|N(p,\, s)\rangle = \bar{u}(p',\, s')\mathcal{J}u(p,\, s)$$

Example

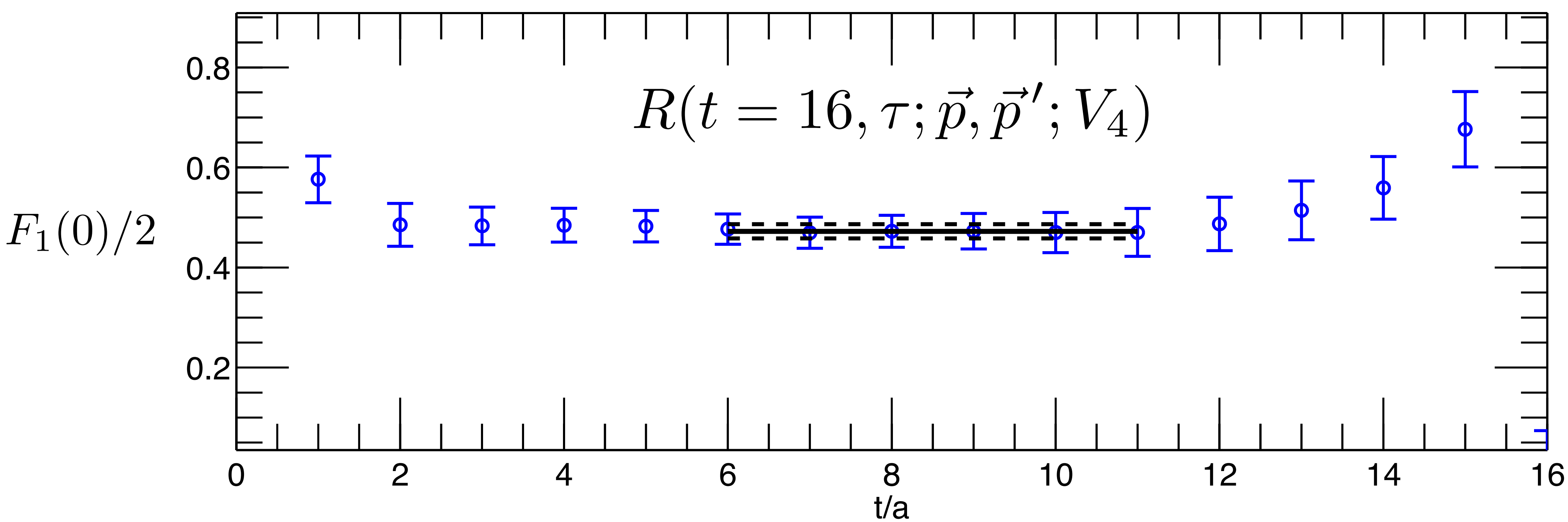
- So our ratio determines

$$R(t,\tau;\vec{p}\,',\vec{p};\mathcal{O}) = \frac{G_\Gamma(t,\tau;\vec{p}\,',\vec{p},\mathcal{O})}{G_2(t,\vec{p}\,')}\left[\frac{G_2(\tau,\vec{p}\,')G_2(t,\vec{p}\,')G_2(t-\tau,\vec{p})}{G_2(\tau,\vec{p})G_2(t,\vec{p})G_2(t-\tau,\vec{p}\,')}\right]^{\frac{1}{2}}$$

$$= \sqrt{\frac{E_{\vec{p}\,'}E_{\vec{p}}}{(E_{\vec{p}}+m)(E_{\vec{p}}+m)}}F(\Gamma,\mathcal{J}_{\mathcal{O}}(\vec{q})) \quad 0 \ll \tau \ll t \ll \frac{1}{2}T$$

$$= F_1(q^2=0)$$

$$\Gamma_{\rm unpol} = \frac{1}{2}(1+\gamma_4),\ \mathcal{O}=V_4\equiv\gamma_4,\ \vec{p}\,'=\vec{p}=0$$



Other Useful Combinations

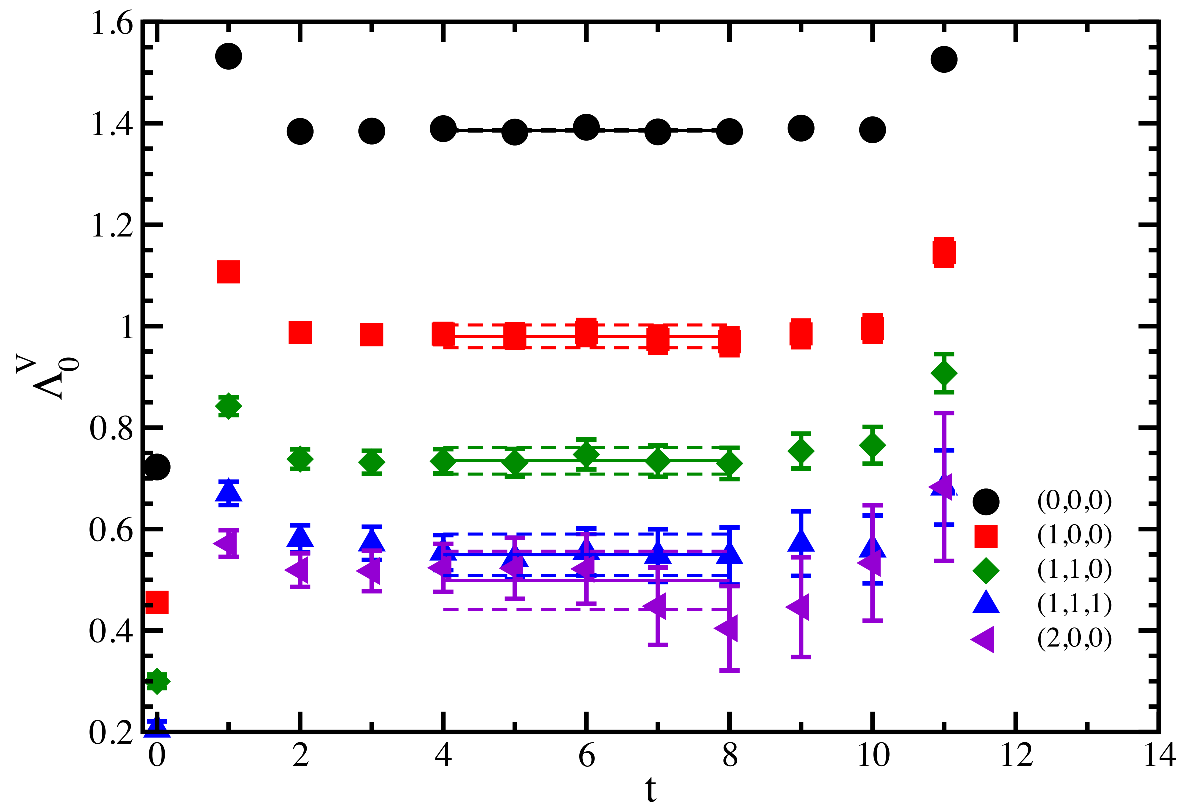
Exercise: Prove them!

$$R(t,\tau;\vec{0},\vec{p};V_4,\Gamma_4) = F_1(q^2) - \frac{E_{\vec{p}}-M}{2M}F_2(q^2) = G_E(q^2)$$

$$R(t,\tau;\vec{0},\vec{p};V_i,\Gamma_4) = -i\frac{q_i}{E+M}G_E(q^2)$$

$$R(t,\tau;\vec{0},\vec{p};V_i,\Gamma_j) = -i\epsilon_{ijk}\frac{q_k}{E+M}G_M(q^2)$$

$$\Gamma_j = \frac{1}{2}(1+\gamma_4)i\gamma_5\gamma_j$$



- Certain combinations of parameters and kinematics give access to the form factors

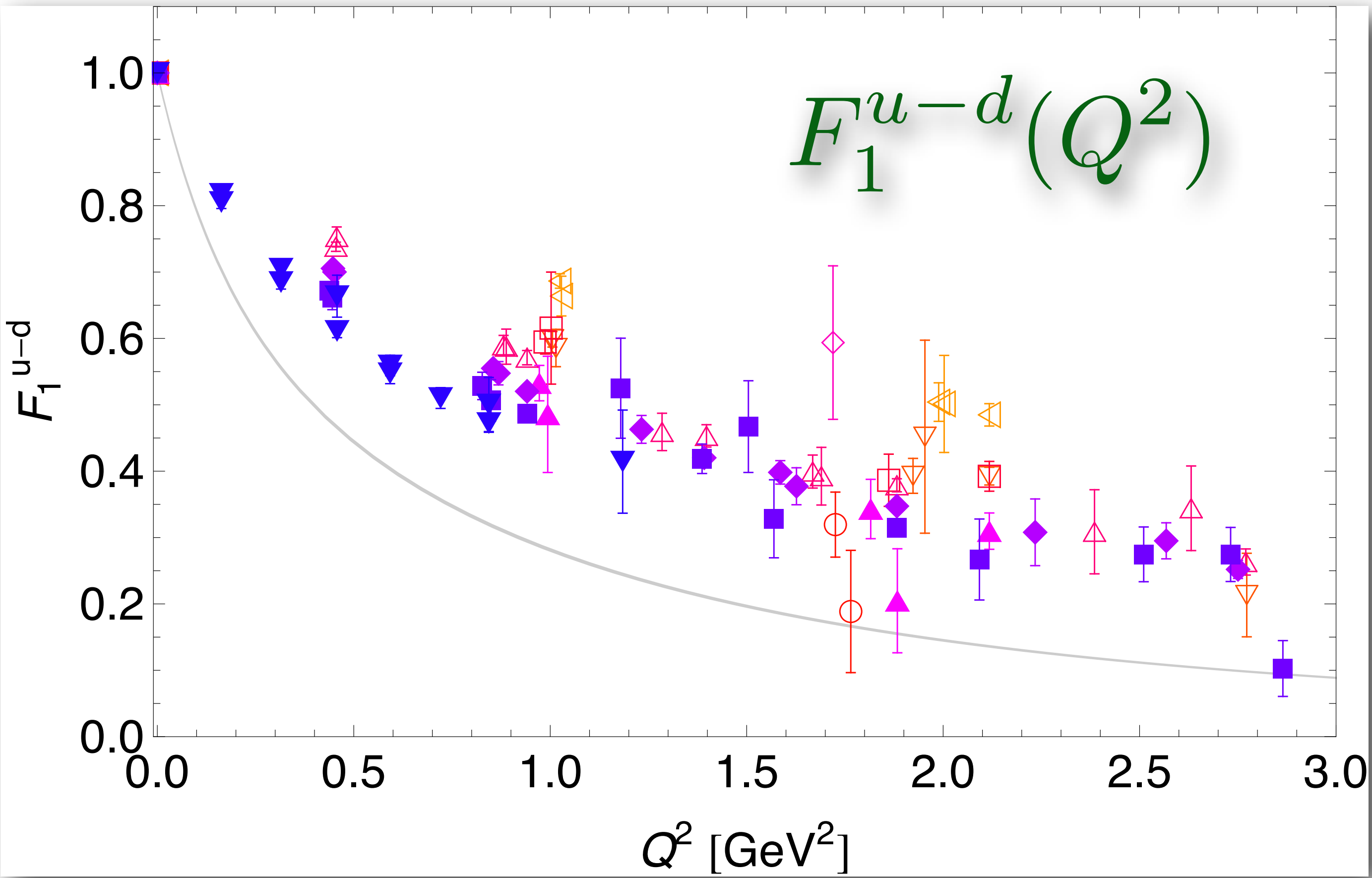
- It is possible to have several choices giving access to the form factors at a fixed Q^2

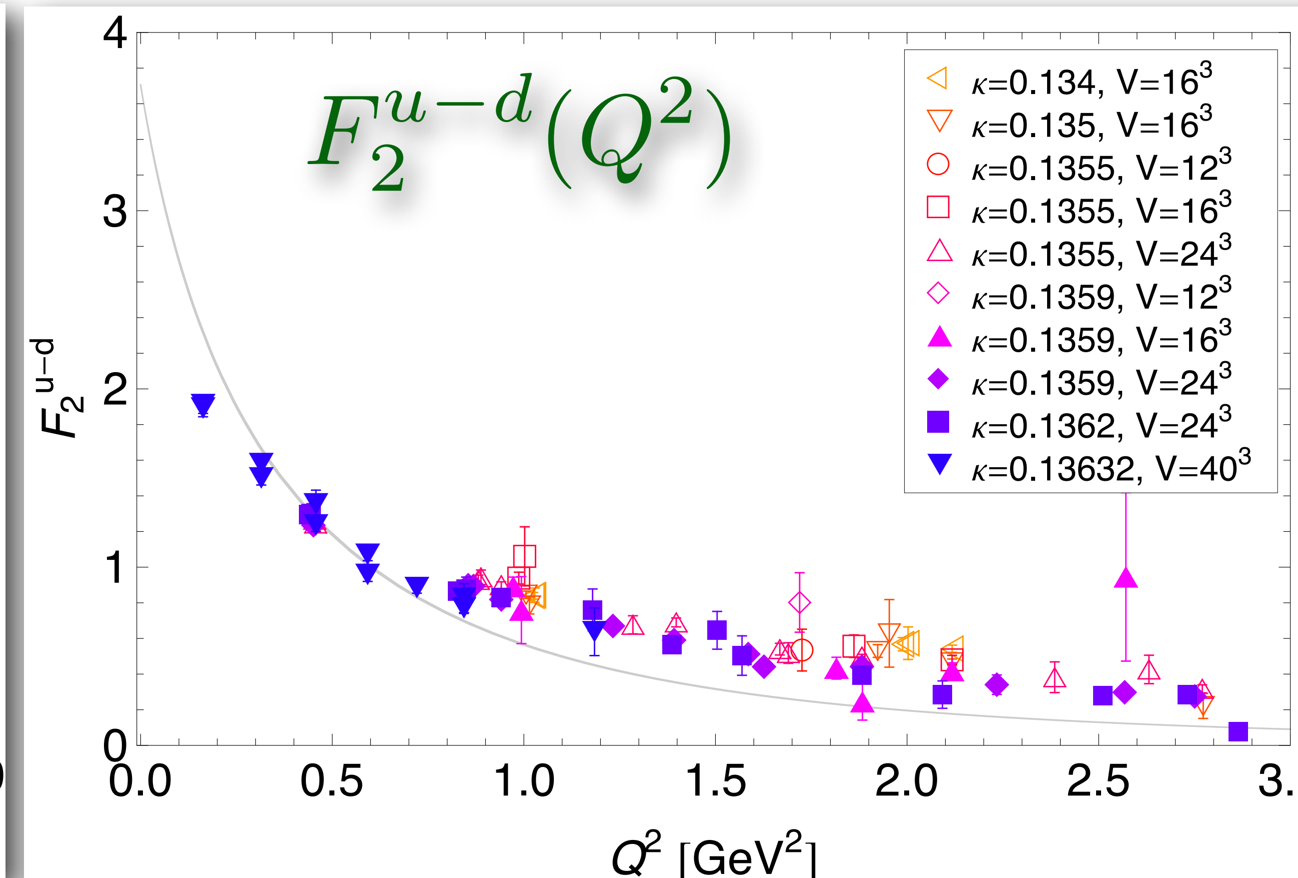
Overdetermined set of simultaneous equations that can be solved for

F_1, F_2 or G_E, G_M

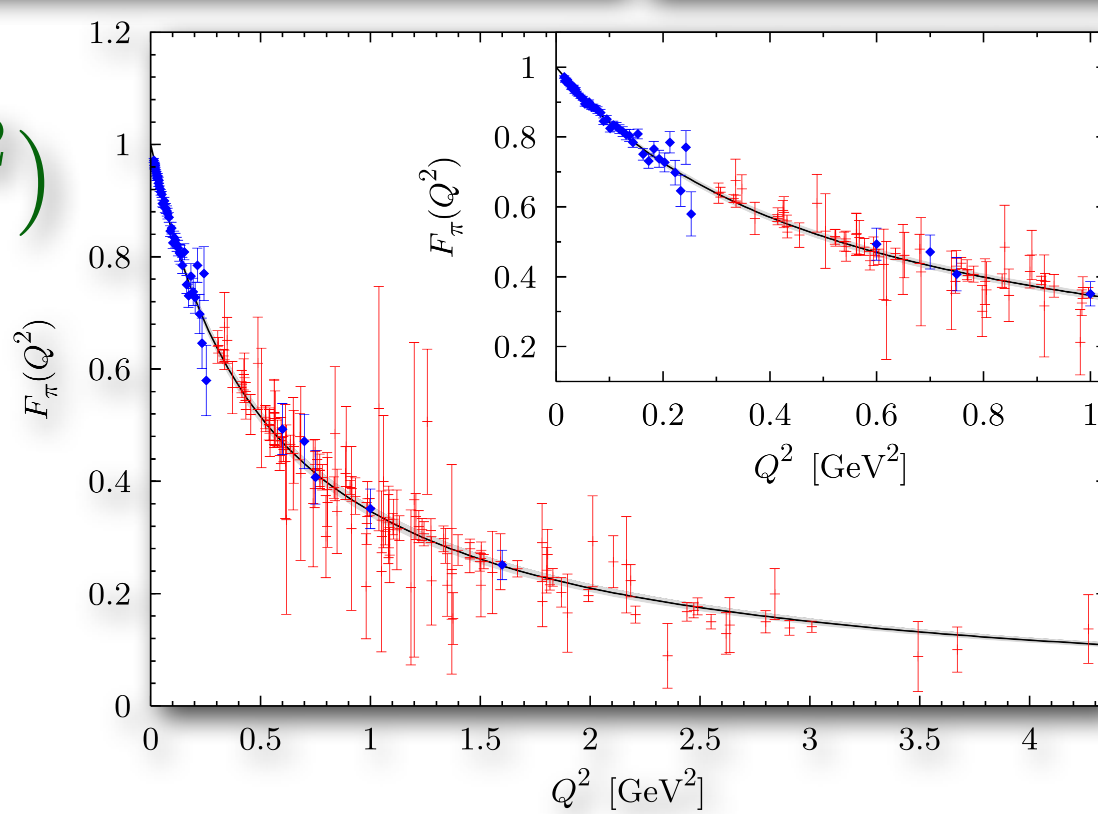
Typical Examples

More detailed look at lattice results for form factors tomorrow





$F_\pi(Q^2)$



QCDSF:
1106.3580
hep-lat/0608021

Some Recent Works

[Not an exhaustive list]

Nucleon

- Review: Ph. Hägler, 0912.5483
- QCDSF: 1106.3580
- ETMC: 1102.2208
- LHPC: 1001.3620
- RBC/UKQCD: 0904.2039
- CSSM: hep-lat/0604022

Pion

- Mainz: 1109.0196
- PACS-CS: 1102.3652
- JLQCD/TWQCD: 0905.2465
- ETMC: 0812.4042
- RBC/UKQCD: 0804.3971
- QCDSF: hep-lat/0608021