Lattice QCD+QED

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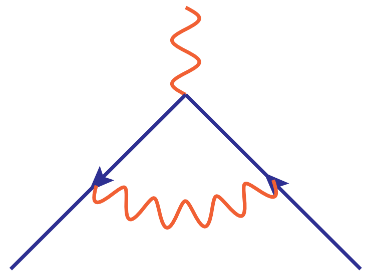
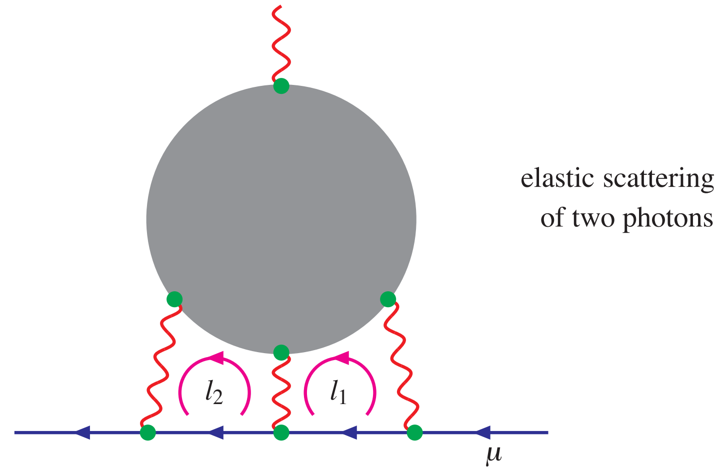
- Introduction
- lattice QED+QCD and ChPT
- up, down and strange quark masses
- Isospin breaking in PS decay constants
- Isospin breaking in baryon masses
- QED reweighting
- Conclusion

QCD+QED

- A the precision of lattice QCD improves, QED effects would be non-negligible.
- QED was the first Quantum Field Theory





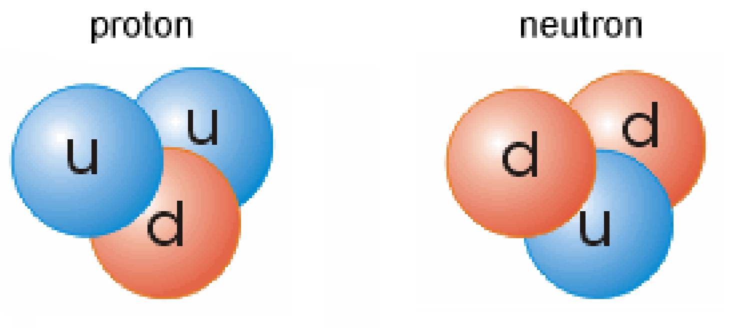
- Lattice QCD results are becomming very precise,
  $e.g.$ err(f_π), err$(f_K) \sim 1\%$, err$(f_\pi/f_K) \sim 0.5\%$. QED effects may not be negligible.
- Although QED part could be treated perturbatively ($e.g.$ hadronic vacuum poralization in $(g-2)_\mu$), not all of problems in QCD+QED system are convinently solved by non-perturbative + perturbative treatments.
- A ground work towards $(g-2)_\mu$ hadronic light-by-ligh diagram

Isospin symmetry

- In 1932, Werner Heisenberg introduced Isospin to explain the newly discovered particle, Neutron.
- Neutron's mass is nearly degenerated to Proton.
- Strong interactions of Neutron are almost equal to those of Proton.

- In the contemporary understanding, isospin symmetry is the $SU(2)_V \times SU(2)_A$ flavor symmetry between up and down quarks.

$$\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow \exp\{i(\theta_V^a + i\theta_A^a \gamma_5)\tau^a\} \begin{pmatrix} u \\ d \end{pmatrix}$$



Isospin Breakings

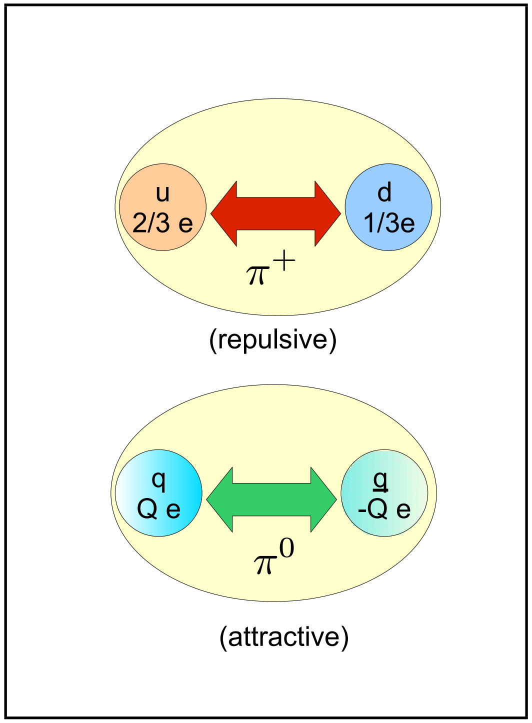
- The effect of isospin breaking due to electromagnetic (EM) and the up, down quark mass difference has phenomenological impacts for accurate hadron spectrum, quark mass determination.

- Isospin breaking's are measured very accurately :

$$m_N - m_P = 1.2933321(4)\text{MeV}$$

$$m_{\pi^\pm} - m_{\pi^0} = 4.5936(5)\text{MeV},$$

$$m_{K^\pm} - m_{K^0} = -3.937(28)\text{MeV},$$



- The positive mass difference between Neutron (udd) and Proton (uud) stabilizes proton thus make our world as it is.

- One of the limiting factors for the precise understanding of nature from the current lattice QCD, especially so for u,d quark masses. [MILC 2004]

- $m_u = 0$ is considered to be a possible solution for Strong CP problem (but also see [M. Creutz] 's arguments).

QCD+QED lattice simulation

- In 1996, Duncan, Eichten, Thacker carried out SU(3)×U(1) simulation to do the EM splittings for the hadron spectroscopy using quenched Wilson fermion on $a^{-1} \sim 1.15$ GeV, $12^3 \times 24$ lattice. [Duncan, Eichten, Thacker PRL76(96) 3894, PLB409(97) 387]

- Using $N_F = 2+1$ Dynamical DWF ensemble (RBC/UKQCD) would have benefits of chiral symmetry, such as better scaling and smaller quenching errors.

- Especially smaller systematic errors due to the the quark massless limits, $m_f \to -m_{res}(Q_i)$, has smaller Q_i dependence than that of Wilson fermions, $\kappa \to \kappa_c(Q_i)$.

- Generate Feynman gauge fixed, quenched non-compact U(1) gauge action with $\beta_{QED} = 1$. $U_\mu^{EM} = \exp[-iA_{\text{em}\,\mu}(x)]$.

- Quark propagator, $S_{q_i}(x)$ with EM charge $Q_i = q_i e$ with Coulomb gauge fixed wall source

$$D\left[(U_\mu^{EM})^{Q_i} \times U_\mu^{SU(3)}\right] S_{q_i}(x) = b_{src}, \quad (i = \text{up,down})$$

$$q_{\text{up}} = 2/3, \quad q_{\text{down}} = -1/3$$

photon field on lattice

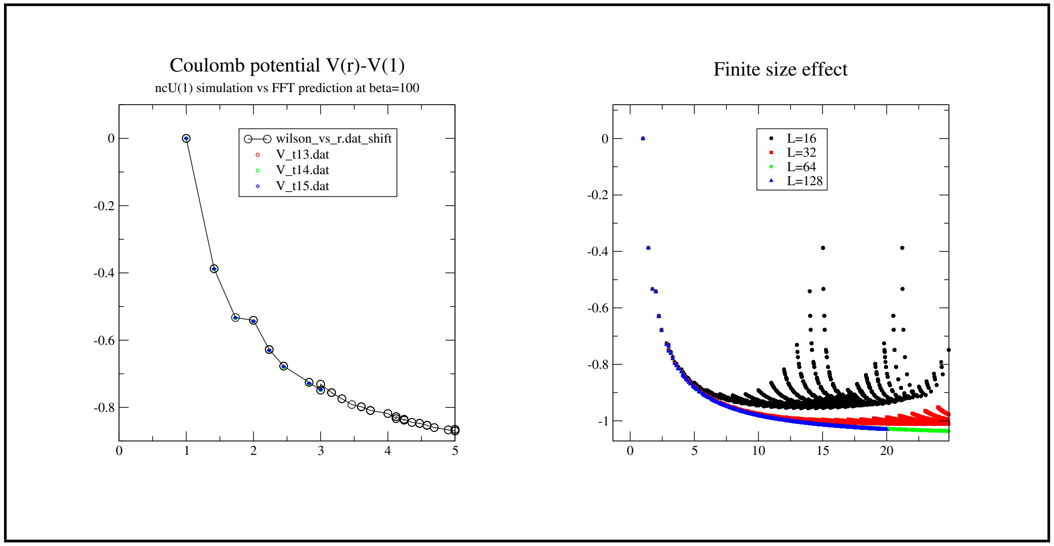
- non-compact $U(1)$ gauge is generated by using Fast Fourier Transformation (FFT). Feynman gauge with eliminating zero modes.

$$S_{\text{EM}} = \frac{1}{4e^2} \sum_{\mu\nu} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2$$

- In our quenched QED simulation, QED coupling e is set by the static Coulomb potential in infinite volume limit to be,

$$V(r) = \frac{e^2}{4\pi}\frac{1}{r} = 1/137, \quad e = 0.30286$$

- Finite volume effects is checked by two volumes.



Measurements

lat	m_{sea}	m_{val}	Trajectories	Δ	N_{meas}	t_{src}
16^3	0.01	0.01, 0.02, 0.03	500-4000	20	352	4,20
16^3	0.02	0.01, 0.02, 0.03	500-4000	20	352	4,20
16^3	0.02	0.01, 0.02, 0.03	500-4000	20	352	4,20
24^3	0.005	0.00{1,5}, 0.0{1,2,3}	900-8660	40	195	0
24^3	0.01	0.001, 0.0{1,2,3}	1460-5040	20	180	0
24^3	0.02	0.02	1800-3580	20	360	0,16,32,48
24^3	0.03	0.03	1260-3040	20	360	0,16,32,48

- $N_F = 2 + 1$ DWF QCD ensemble generated by [RBC/UKQCD, PRD78:114509(08), in prep.]
- a^{-1} =1.784 (44) GeV, $V = (16a = 1.76 \text{ fm})^3$ and $(24a = 2.65 \text{ fm})^3$
- $m_v = 0.0001$ ($\sim$ 9 MeV), 0.005 ($\sim$ 22 MeV) , 0.01 ($\sim$ 40 MeV), 0.02 ($\sim$ 70 MeV), , 0.03 ($\sim$ 100 MeV)
- $m_{res} = 0.003148(46)$ ($\sim$ 8.9 MeV)
- In total, $\sim$ 200 charge/mass combinations are measured.

$\mathcal{O}(e)$ error reduction

- On the infinitely large statistical ensemble, term proportional to odd powers of e vanishes. But for finite statistics,

$$\langle O \rangle_e = \langle C_0 \rangle + \langle C_1 \rangle\, e + \langle C_2 \rangle\, e^2 + \cdots$$

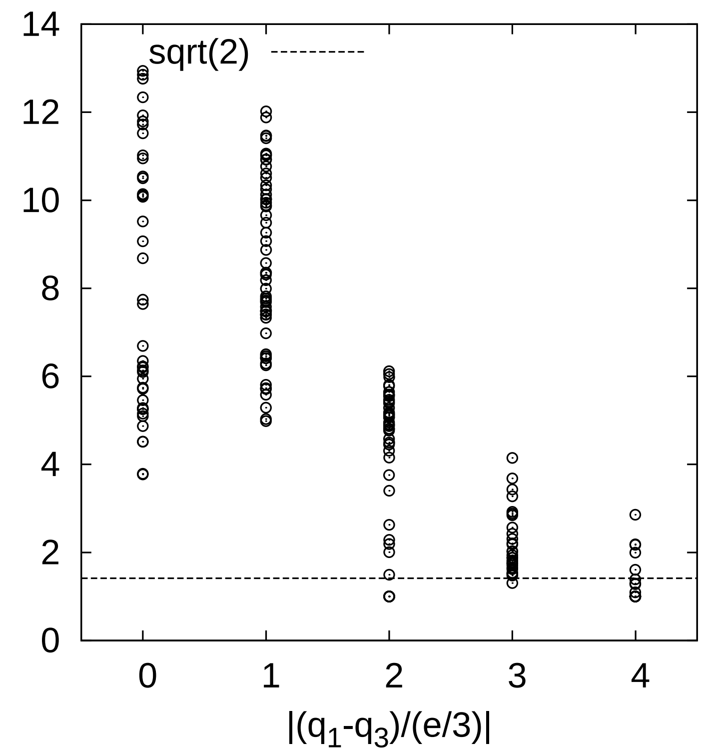
  $\langle C_{2n-1} \rangle$ could be finite and source of large statistical error as e^{2n-1} vs e^{2n}.

- By averaging $+e$ and $-e$ measurements on the same set of QCD+QED configuration,

$$\frac{1}{2}[\langle O \rangle_e + \langle O \rangle_{-e}] = \langle C_0 \rangle + \langle C_2 \rangle\, e^2 + \cdots$$

  $\mathcal{O}(e)$ is exactly canceled.

- More than a factor of 10 error reduction, corresponding to $\times 100$ measurements by only twice computational cost (vs naive reduction factor $\sqrt{2}$).

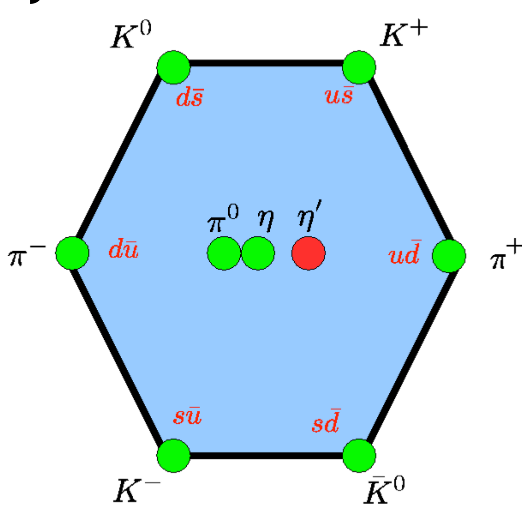


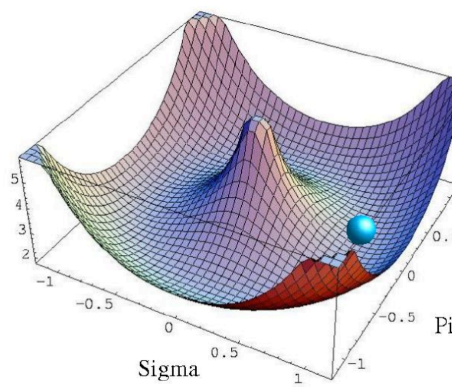
flavor-chiral symmetry in QCD

- massless 3 flavor QCD : $\mathcal{L}_{\rm QCD} = -1/4 G_a^{\mu\nu} G^a_{\mu\nu} + i\bar{q}\, \not{D} q, \quad q = (u,d,s)^T$
- $SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A$ flavor-chiral symmetry

$$q \to \exp\left(i\theta_V^a T^a + i\theta_A^a T^a \gamma_5\right) q$$

- $U(1)_A$ is broken by chiral anomaly
- spontaneous chiral symmetry breaking $\langle \bar{q} q \rangle \approx \Lambda_{\rm QCD}$
- 8 Nambu-Goldstone PS bosons
  + 1 heavy PS meson



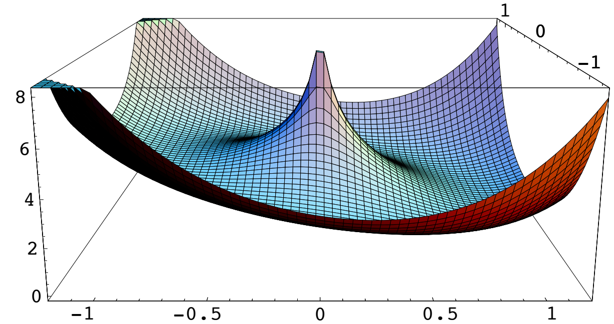


Nother current & PCAC

- Axial and Vector Ward Takahashi identity

$$\langle \mathcal{O} \delta S \rangle + \langle \delta \mathcal{O} \rangle = 0$$

$$A^a_\mu(x) = \bar{q} \gamma_\mu \gamma_5 q(x)$$



- Partially Conserved currents $\mathcal{L}_m = m \bar{q} q$

$$\partial_\mu \langle A^a_\mu(x) \mathcal{O} \rangle = 2m J^a_5(x) + \delta^a \mathcal{O}$$

$$J^a_5(x) = \bar{q}(x) T^a \gamma_5 q(x)$$

$$\Longrightarrow m^2_\pi = 2m B_0 + \cdots$$

EM splittings [B .Tiburzi]

- Axial WT identity with EM for massless quarks $(N_F = 3)$,

$$\mathcal{L}_{\text{em}} = e A_{\text{em}\,\mu}(x) \bar{q} Q_{\text{em}} \gamma_\mu q(x),\ Q_{\text{em}} = \text{diag}(2/3, -1/3, -1/3)$$

$$\partial^\mu \mathcal{A}^a_\mu = ie A_{\text{em}\,\mu}\, \overline{q}\, [T^a,\ Q_{\text{em}}]\, \gamma^\mu \gamma_5 q - \frac{\alpha}{2\pi} tr \left(Q^2_{\text{em}} T^a \right) F^{\mu\nu}_{\text{em}} \widetilde{F}_{\text{em}\,\mu\nu}\,,$$

neutral currents, four $\mathcal{A}^a_\mu(x)$, are conserved (ignoring $\mathcal{O}(\alpha^2)$ effects): $\pi^0, K^0, \overline{K^0}, \eta_8$ are still a NG bosons.

- ChPT with EM at $\mathcal{O}(p^4, p^2 e^2)$:

$$M^2_{\pi\pm} = 2mB_0 + 2e^2 \frac{C}{f_0^2} \qquad + \mathcal{O}(m^2 \log m, m^2) + I_0 e^2 m \log m + K_0 e^2 m$$

$$M^2_{\pi 0} = 2mB_0 \qquad + \mathcal{O}(m^2 \log m, m^2) + I_\pm e^2 m \log m + K_\pm e^2 m$$

Dashen's theorem :
The difference of squared pion mass is independent of quark mass up to $\mathcal{O}(e^2 m)$,

$$\Delta M^2_\pi \equiv M^2_{\pi\pm} - M^2_{\pi 0} = 2e^2 \frac{C}{f_0^2} + (I_\pm - I_0) e^2 m \log m + (K_\pm - K_0) e^2 m$$

$C, K_\pm, K_0$ is a new low energy constant. $I_\pm, I_0$ is known in terms of them.

ChPT+EM at NLO

- Double expansion of $M_{\text{PS}}^2(m_1, q_1; m_3, q_3)$ in $\mathcal{O}(\alpha), \mathcal{O}(m_q)$.
  QCD LO:
  $$M_{\text{PS}}^2 = \chi_{13} = B_0(m_1 + m_3)$$
  QCD NLO: $(1/F_0^2 \times)$
  $$(2L_6 - L_4)\chi_{13}^2 + (2L_5 - L_8)\chi_{13}\bar{\chi}_1 + \chi_{13} \sum_{I=1,3,\pi,\eta} R_I \chi_I \log(\chi_I/\Lambda_\chi^2),$$
  QED LO: (Dashen's term)
  $$\frac{2C}{F_0^2}(q_1 - q_3)^2$$
  QED NLO: ($\bar{Q}_2 = \sum q_{\text{sea}-i}^2$, no $\bar{Q}_1$ in SU(3)$_{N_F}$)
  $$-Y_1\bar{Q}_2\chi_{13} + Y_2(q_1^2\chi_1 + q_3^2\chi_3) + Y_3 q_{13}^2\chi_{13} - Y_4 q_1 q_3 \chi_{13} + Y_5 q_{13}^2\bar{\chi}_1$$
  $$+\chi_{13}\log(\chi_{13}/\Lambda_\chi^2)q_{13}^2 + \bar{B}(\chi_\gamma, \chi_{13}, \chi_{13})q_{13}^2\chi_{13} - \bar{B}_1(\chi_\gamma, \chi_{13}, \chi_{13})q_{13}^2\chi_{13} + \cdots$$

- QED LO adds mass to $\pi^\pm$ at $m_q = 0$, QED NLO changes slope, B_0, in m_q.

- Partially quenched formula ($m_{\text{sea}} \neq m_{\text{val}}$) SU(3)$_{N_F}$ [Bijnens Danielsson, PRD75 (07)] SU(2)$_{N_F}$+Kaon+FiniteV [Hayakawa Uno, PTP 120(08) 413] [RBC/UKQCD] (also [C. Haefeli, M. A. Ivanov and M. Schmid, EPJ C53(08)549])

Domain Wall fermion action [D .Kaplan's lecture]

- $S_{\mathrm{DWF}} = \psi(x,s) D_5(x,s;y,t)\psi(y,t)$

$$D_5(s,t) = \begin{cases} m_f \delta_{L_s-1,t} P_L + \delta_{s,t}(D_W+1) - \delta_{s+1,t} P_R & s=0 \\ -\delta_{s-1,t} P_L + \delta_{s,t}(D_W+1) - \delta_{s+1,t} P_R & 1 \le s \le L_s-2 \\ -\delta_{s-1,t} P_L + \delta_{s,t}(D_W+1) + m_f \delta_{0,t} P_R & s = L_s-1 \end{cases} \quad (1)$$

- Wilson-Dirac matrix, $D_W = D_W(-M_5)$ with the domain wall height, M_5,

$$D_W(x,y) = (4-M_5) - \frac{1}{2}\sum_\mu \left[(1+\gamma_\mu)U_\mu^\dagger(x-\hat\mu)\delta(x-\hat\mu,y)\right. \quad (2)$$

$$\left. +(1-\gamma_\mu)U_\mu(x)\delta(x+\hat\mu,y)\right] \quad (3)$$

- physical 4D field

$$q = P_L\psi(0) + P_R\psi(L_s-1) \quad , \quad (4)$$

$$\overline{q} = \overline{\psi}(0)P_R + \overline{\psi}(L_s-1)P_L \quad , \quad (5)$$

- Axial rotation

$$\delta^a\psi(x,s) = T^a \mathrm{sgn}(s - L_s/2) e^{i\theta}\psi(x,s) \quad (6)$$

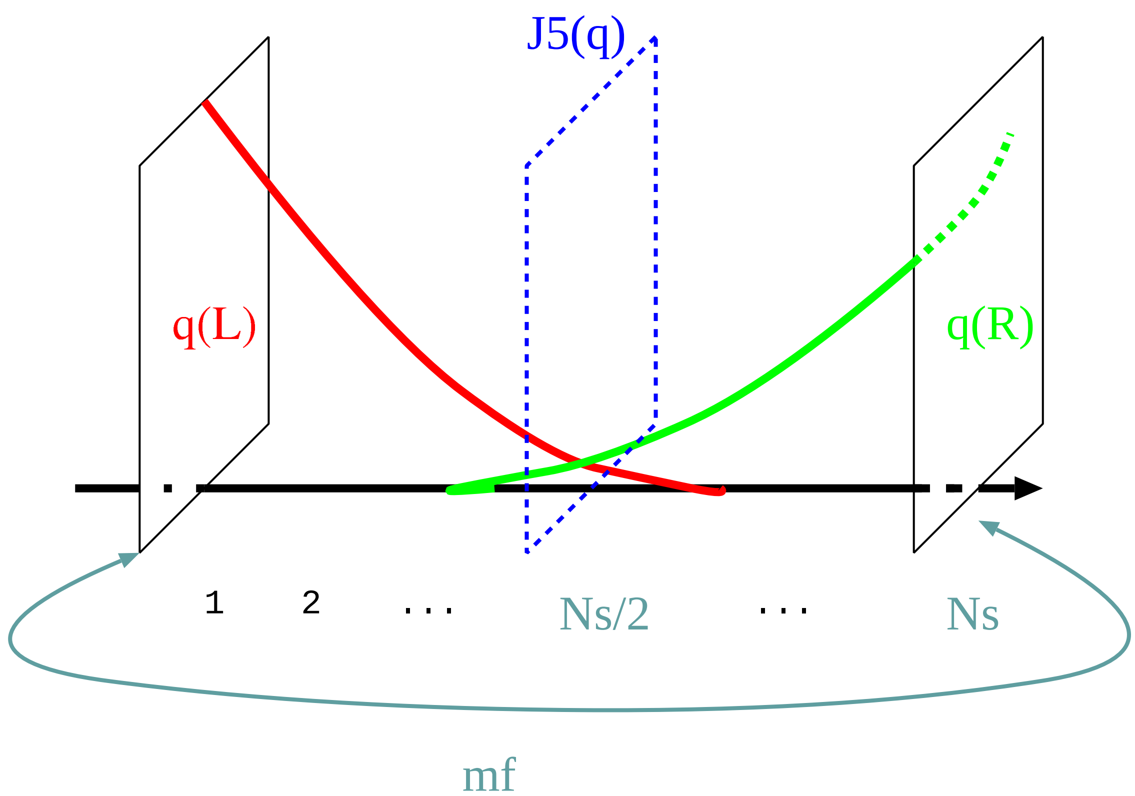
Home Work

- Derive the axial Ward-Takahashi identity for DWF

$$\partial \mathcal{A}^a_\mu(x) = 2m J^a_5(x) + J^a_{5q}(x)$$

with the conserved axial current, $\mathcal{A}^a_\mu(x)$ and the explicit breaking operator, $J_{5q}(x)$.

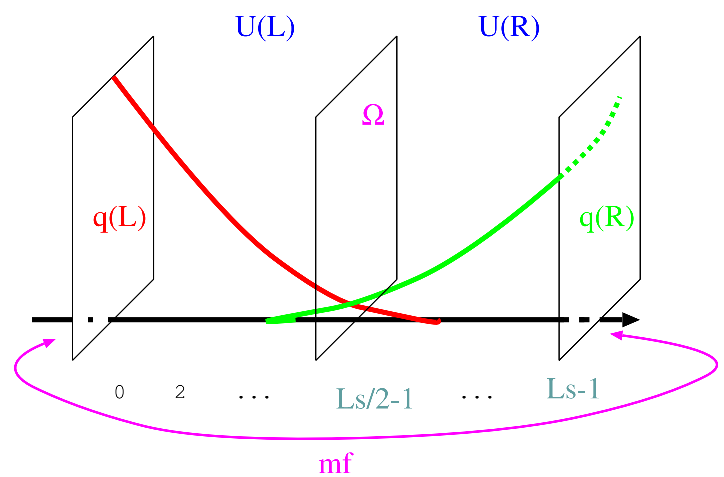
- Do the same thing for vector Ward-Takahashi identity



The residual chiral symmetry breaking in QCD+QED

- Using DWF's PCAC relation, in terms of the mid-point correlator $J_{5q}(L_s/2)$, for the flavor off-diagonal current with same EM charge quarks, q_i. Parametrize the EM charge dependence in terms of C_2:

$$m_{\text{res}}(q_i, q_i) = \frac{\left\langle \sum_x J^a_{5q}(\vec{x}, t)\pi^a(0) \right\rangle}{\left\langle \sum_x J^a_5(\vec{x}, t)\pi^a(0) \right\rangle},$$



$$m_{\text{res},i}(q_i, q_i) - m_{\text{res}}(0, 0) = e^2 C_2\, q_i^2,$$

	16^3	24^3
m_{sea}	m_{res}	m_{res}
chiral limit	0.003148(46)	0.003203(15)
0.005	N/A	0.003222(16)
0.01	0.003177(31)	0.003230(15)
0.02	0.003262(29)	0.003261(16)
0.03	0.003267(28)	0.003297(15)

L_s	$C_2 u\bar{u}$	$C_2 d\bar{d}$
	16^3 lattice size	
16	2.597(23)	2.532(22)
32	0.309(16)	0.301(16)
	24^3 lattice size	
16	2.585(7)	2.519(7)

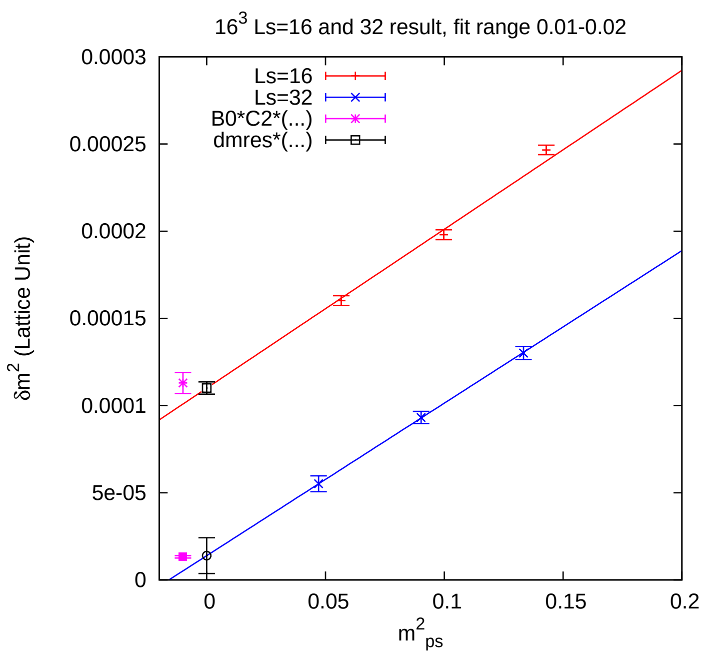
- In the massless quark limit of QCD, $m_f = -m_{res}(0,0)$, *Neutral* PS meson (should still be a NG boson upto α^2), has additive mass shift due to the additional chiral symmetry breaking from photon field, $m_{\text{res},i}(q_i, q_i) - m_{\text{res}}(0,0)$.

- This effect is expressed in the DWF-ChPT as

$$\Delta m^2 = M^2_{\text{PS}^0}(e \neq 0) - M^2_{\text{PS}^0}(e = 0) = BC_2 e^2 (q_1^2 + q_3^2),$$

where $\chi = 2Bm_q$ is the LO PS mass squared.

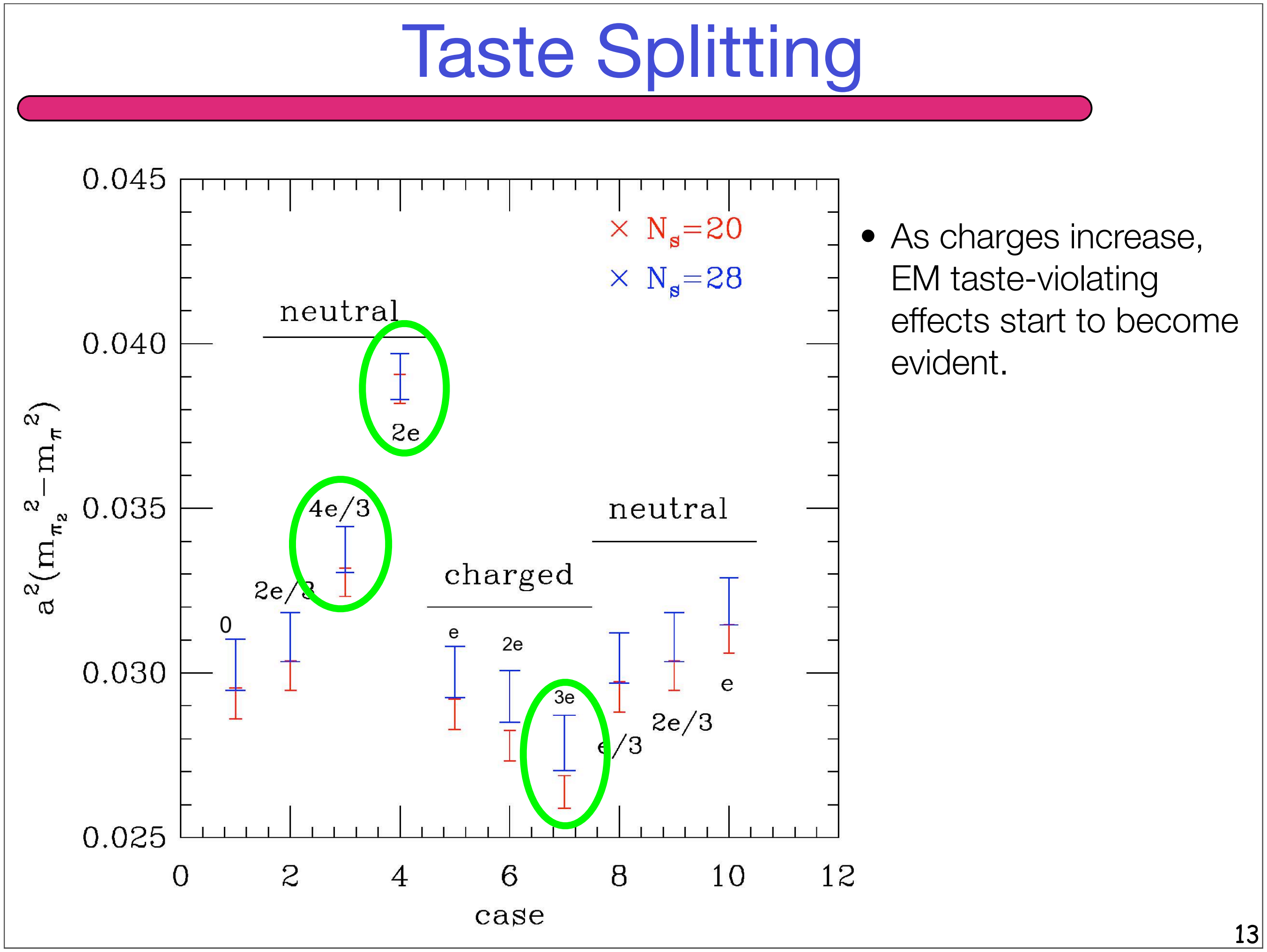
- $L_s = 16$ and 32 (partially quenched) consistent with DWF-PCAC.



Staggered case

Q^2 scaling

Similar counter part in Wilson's case $\propto Q^2/a$.



SU(3)+EM ChPT LEC

[Bijnens Danielsson, PRD75 (07)]

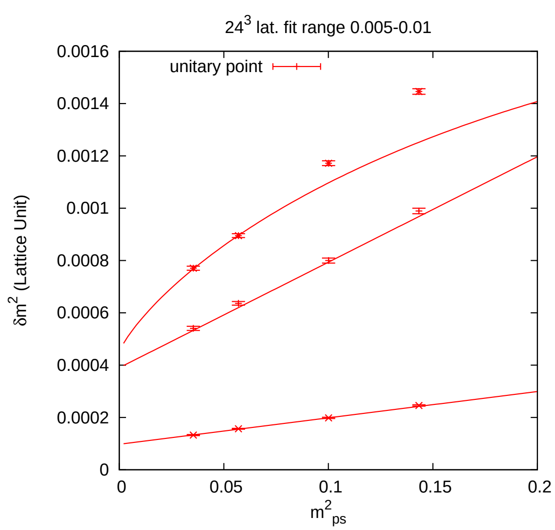
- By fitting charge splitting

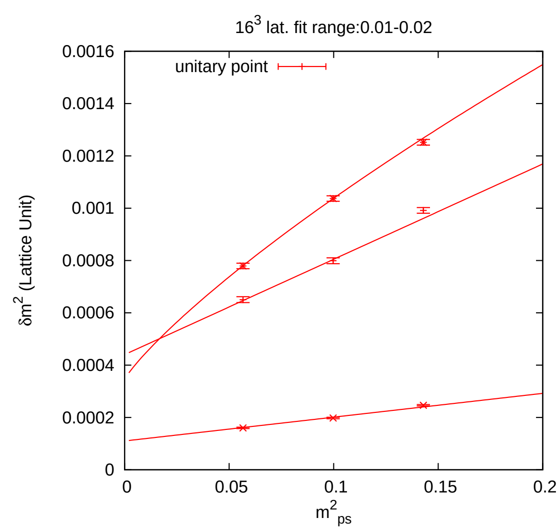
$$\delta M^2 = M^2_{\text{PS}}(m_1, q_1; m_2, q_2; m_l) - M^2_{\text{PS}}(m_1, 0; m_2, 0; m_l)$$

  by SU(3) ChPT+EM formula at NLO, 3 QCD LECs (1 LO + 2 NLO), 5 QED LECs (1 LO + 4 NLO) are determined.

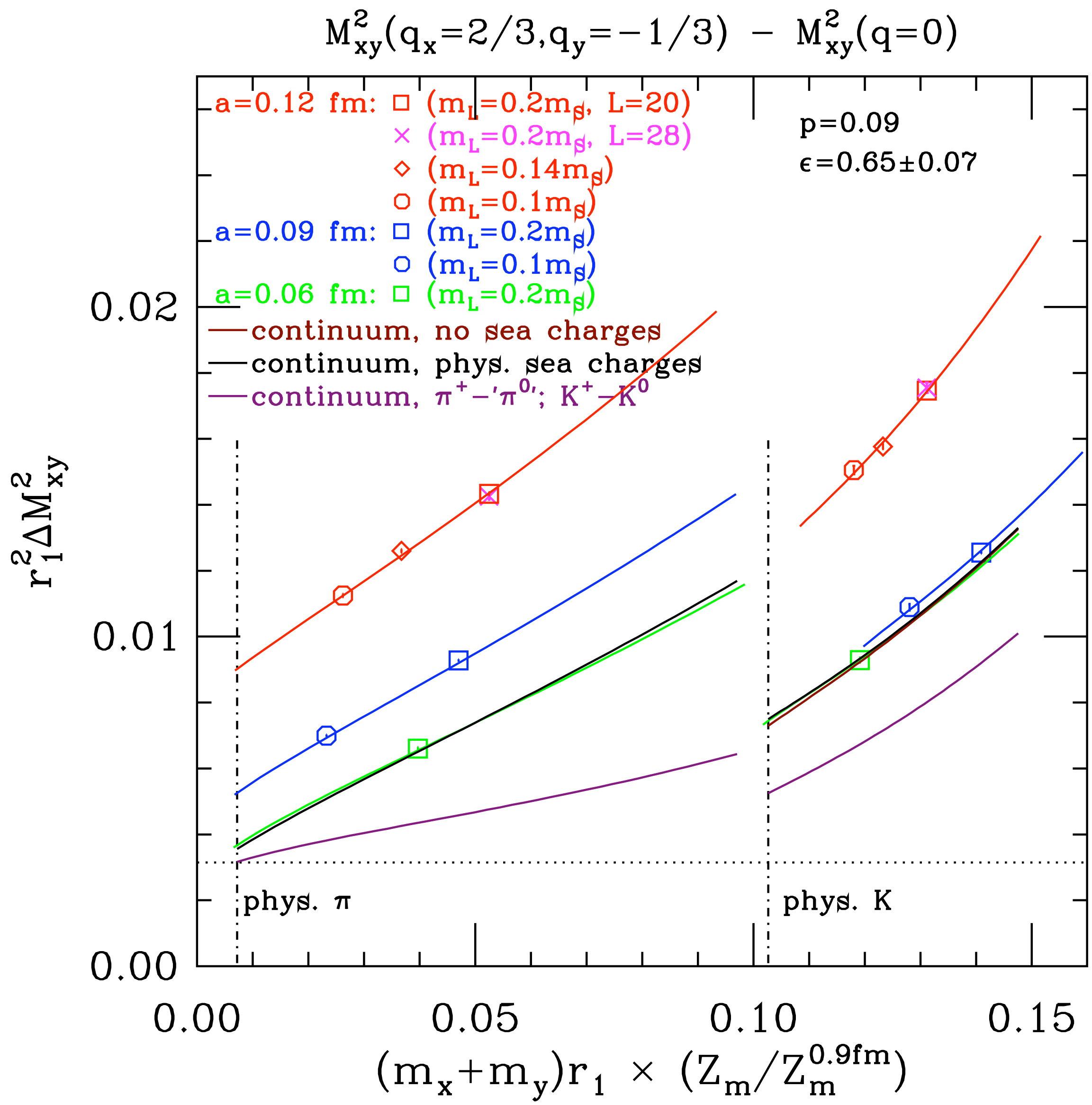
- Requiring $m_1, m_3, m_l \leq$ 0.01 (0.02), 58 (124) partially quenched data for $M_{\text{PS}}(m_1, q_1; m_2, q_2; m_l)$ are used in the fit (to see NNLO effects).

- Finite volume effects are observed by repeating the fit on (1.8 fm)^3 and and (2.7 fm)^3.





MILC-EM-ChPT fit



- NLO correction to the Dashens's theorem :

$$\Delta_{EM} = (M^2_{K^\pm} - M^2_{K^0})/(M^2_{\pi^\pm} - M^2_{\pi^0})$$

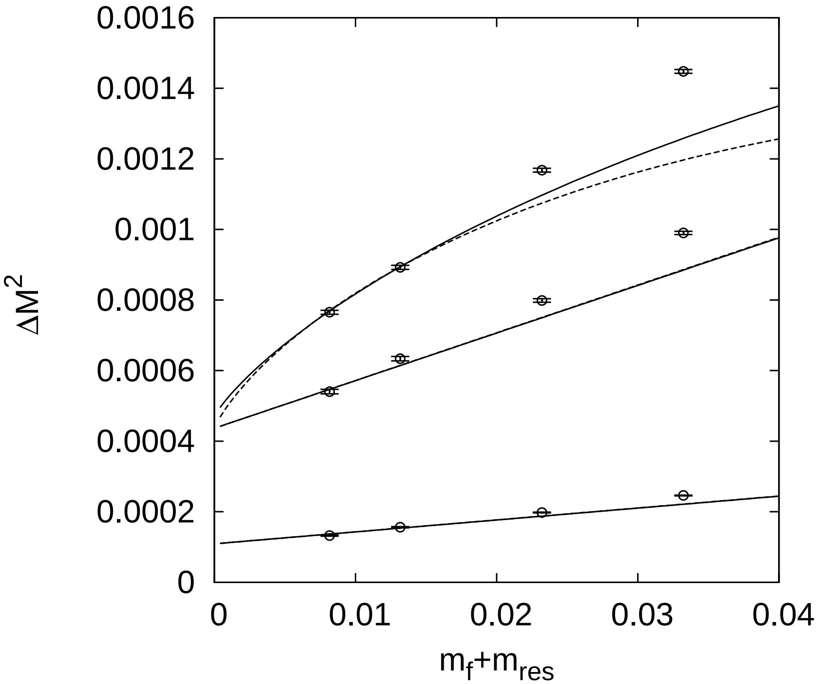
- $\Delta_{EM} = 0.65(7)(14)(\ldots)$ (MILC 2012) partial sys. error
- Blum 10 (stat error only),: $\Delta_{EM} = 0.75(5)$ for SU(3), $\Delta_{EM} = 0.63(5)$ for SU(2)
- BMW (12) partial sys. error $\Delta_{EM} = 0.70(4)(8)(\ldots)$
- Difficulty in Covariant fit

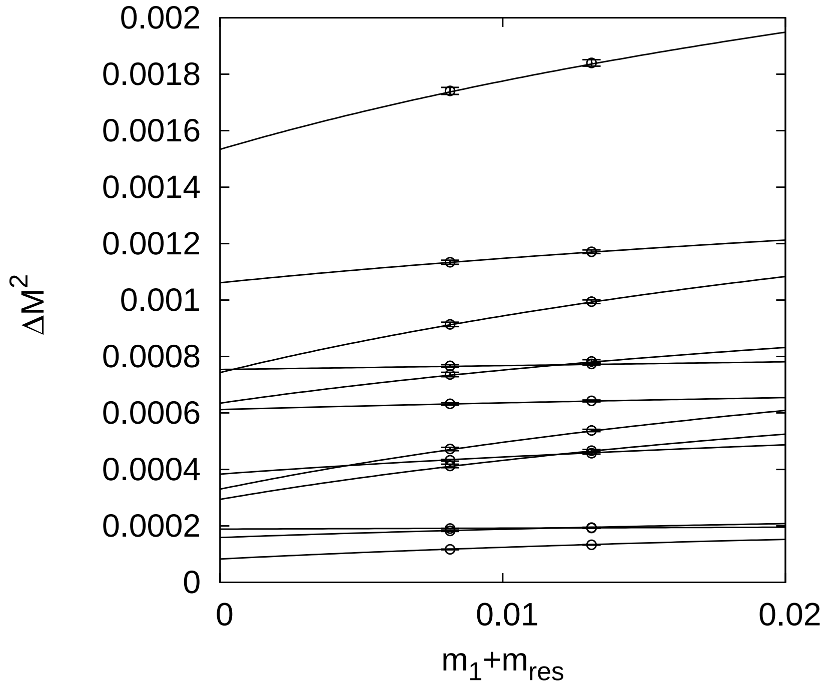
SU(2)+ Kaon+EM ChPT Fit

- $$
\begin{aligned}
M_K^2 &= M^2 - 4B(A_3 m_1 + A_4(m_4+m_5)) \\
&+e^2\left(2\left(A_5^{(1,1)} + A_5^{(2,1)}\right) q_1^2 + A_5^{(s,1,1)} q_3^2 + 2A_5^{(s,2)} q_1 q_3\right) \\
&-\frac{e^2}{(4\pi)^2F^2}\left((A_5^{(1,1)} + 3A_5^{(2,1)})q_1^2 + A_5^{(s,2)} q_1 q_3\right)\sum_{s=4,5}\chi_{1s}\log\frac{\chi_{1s}}{\mu^2} \\
&+e^2 m_1\left(x_3^{(K)}(q_1+q_3)^2 + x_4^{(K)}(q_1-q_3)^2 + x_5^{(K)}(q_1^2-q_3^2)\right) \\
&+e^2\frac{m_4+m_5}{2}\left(x_6^{(K)}(q_1+q_3)^2 + x_7^{(K)}(q_1-q_3)^2 + x_8^{(K)}(q_1^2-q_3^2)\right) \\
&+e^2\delta_{mres}(q_1^2+q_3^2),
\end{aligned}
$$

- EM splitting NLO/LO is still large ($\sim$ 50% at $m_q = 40$ MeV) for Pion but small ($\sim$ 10% at $m_q = 70$ MeV) for Kaon. But quark mass determination is stable under NLO correction.

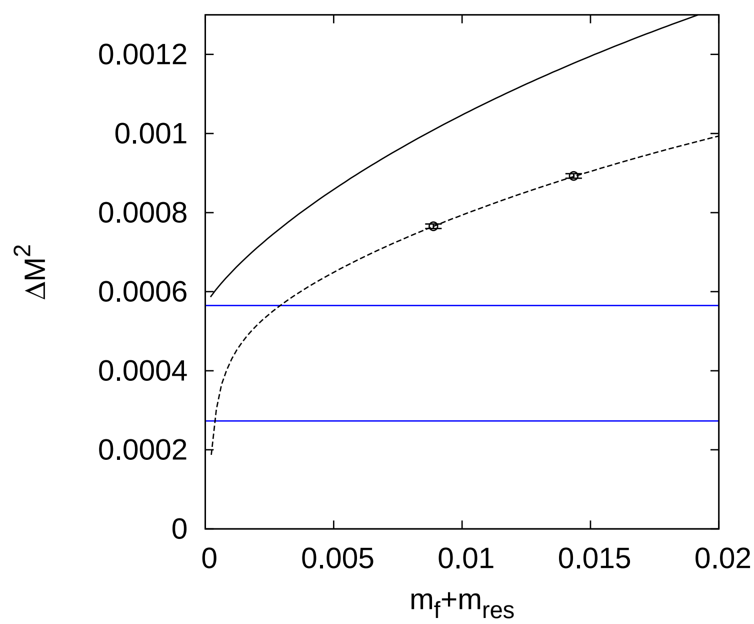
- An accidental flat direction of χ^2 function in our data set (degenerate light quark) : increase light mass range ($ml \leq 0.02$) or fix QED NLO LEC to zero to see the effects on quark mass (included in systematic error).

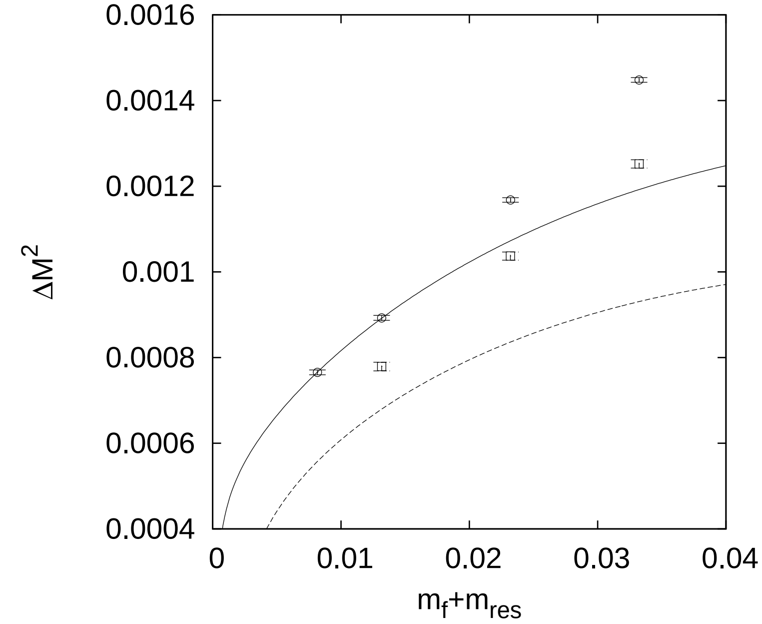




- Left: Pion fit, $\bar{u}d$, $\bar{u}u$, $\bar{d}d$ from top. SU(2) fit is in solid curve and dashed curve is SU(3) fit.
- Right: Kaon fit for various charge combinations.
- Infinite volume fit formula are shown.

Finite Volume effect on ChPT fits





- We use finite volume (FV) ChPT formula to fit data.
- Left: Pion unitary points. lower line: δm_{res}, upper line: LO (Dashen's) term
- NLO contributions at simulation points are 50-100% $\times$ LO. But only +2% contribution to $m_d - m_u$ from NLO.
- Left: Using FV fit on (2.7 fm)^3, dotted curve are predicted for (1.8 fm)^3, which overshoots the data by a factor of 2.

Quark mass determinations

- Using the LECs, B_0, F_0, L_i, C_0, Y_i, from the fit, we could determine the quark masses $m_{\text{up}}, m_{\text{dwn}}, m_{\text{str}}$ by the solving equations [PDG] :

$$M_{\pi^\pm} = M_{\text{PS}}(m_{\text{up}}, 2/3, m_{\text{dwn}}, -1/3) = 139.57018(35)\text{MeV}$$
$$M_{K^\pm} = M_{\text{PS}}(m_{\text{up}}, 2/3, m_{\text{str}}, -1/3) = 493.673(14)\text{MeV}$$
$$M_{K^0} = M_{\text{PS}}(m_{\text{dwn}}, -1/3, m_{\text{str}}, -1/3) = 497.614(24)\text{MeV}$$

- $(m_{\text{up}} - m_{\text{dwn}})$ is mainly determined by Kaon charge splittings,

$$M^2_{K^\pm} - M^2_{K^0} = B_0(m_{\text{up}} - m_{\text{dwn}}) + \frac{2C}{F_0^2}(q_1 - q_3)^2 + \text{NLO}$$

- π^0 mass is not used for now (disconnected quark loops).

- The term proportional to sea quark charge, $-Y_1\bar{Q}_2\chi_{13}$, is omitted. We will estimate the systematics by varying Y_1.

Quark mass results

- $\overline{MS}$ at 2 GeV, using NPR, RI-SMOM$_{\gamma\mu}$ scheme2 [C.Sturm et.al PRD (09) 014501, Y.Aoki, PoS LAT2009 012, L. Almeida C.Sturm arXiv:1004.4613, P.Boyle et. al. arXiv:1006.0422, RBC/UKQCD in prep.] as a intermediate scheme. (10% $\rightarrow$ 5% $\rightarrow$ 2,3% error)
- $m_1, m_3 \leq 0.01(\sim 40\text{MeV})$, $M_{ps} \leq$ 250 MeV
- SU(3)$_{N_F}$/SU(2)$_{N_F}$ in infinite/finite volume.
- Uncertainties in QED LEC have small effect to quark mass.

	SU(3)		SU(2)	
	inf.v	f.v	inf.v.	f.v.
m_u [MeV]	2.606(89)	2.318(91)	2.54(10)	2.37(10)
m_d [MeV]	4.50(16)	4.60(16)	4.53(15)	4.52(15)
m_s [MeV]	89.1(3.6)	89.1(3.6)	97.7(2.9)	97.7(2.9)
$m_d - m_u$ [MeV]	1.900(99)	2.28(11)	1.993(67)	2.155(63)
m_{ud} [MeV]	3.55(12)	3.46(12)	3.54(12)	3.44(12)
m_u/m_d	0.578(11)	0.503(12)	0.5608(87)	0.5238(93)
m_s/m_{ud}	25.07(36)	25.73(36)	27.58(27)	28.34(29)

- Only statistical error shown above.

Quark mass from QCD+QED simulation

[PRD82 (2010) 094508 [47pages]]

$$
\begin{aligned}
m_u &= 2.24 \pm 0.10 \pm 0.34 \ \text{MeV} \\
m_d &= 4.65 \pm 0.15 \pm 0.32 \ \text{MeV} \\
m_s &= 97.6 \pm 2.9 \pm 5.5 \ \text{MeV} \\
m_d - m_u &= 2.411 \pm 0.065 \pm 0.476 \ \text{MeV} \\
m_{ud} &= 3.44 \pm 0.12 \pm 0.22 \ \text{MeV} \\
m_u/m_d &= 0.4818 \pm 0.0096 \pm 0.0860 \\
m_s/m_{ud} &= 28.31 \pm 0.29 \pm 1.77,
\end{aligned}
$$

- $\overline{\text{MS}}$ at 2 GeV using NPR/SMOM scheme.
- Particular to QCD+QED, finite volume error is large: 14% and 2% for m_u and m_d.
- This would be due to photon's non-confining feature (vs gluon).
- Volume, a^2, chiral extrapolation errors are being removed.
- Applications for Hadronic contribution to $(g-2)_\mu$ in progress.

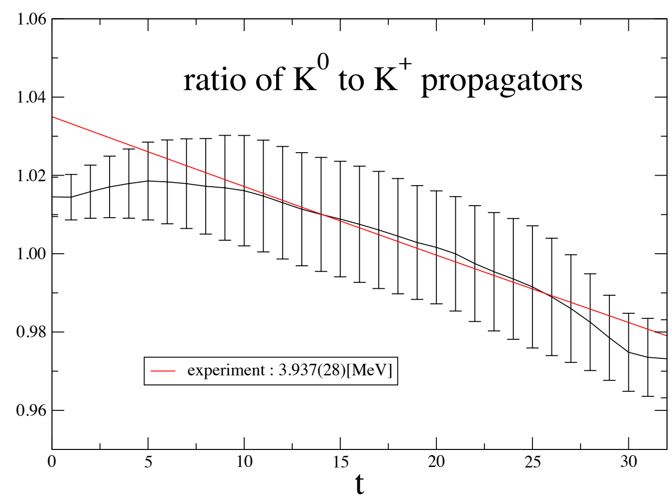
PACS- CS [arXiv:1205.2961[hep-lat]] (N. Ukita)

Table summarizes our results for quark masses renormalized at μ=2GeV.
We neglect the QED corrections to the renormalization factor.

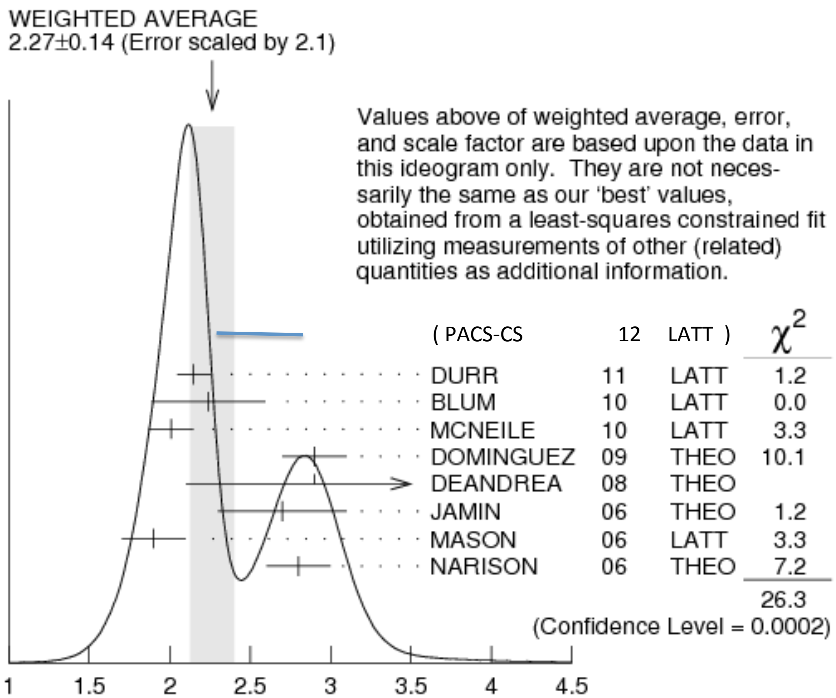
Figure shows a ratio of K^0 to K^+ propagators clarifying
K^0-K^+ mass difference, which is consistent with the experimental value.

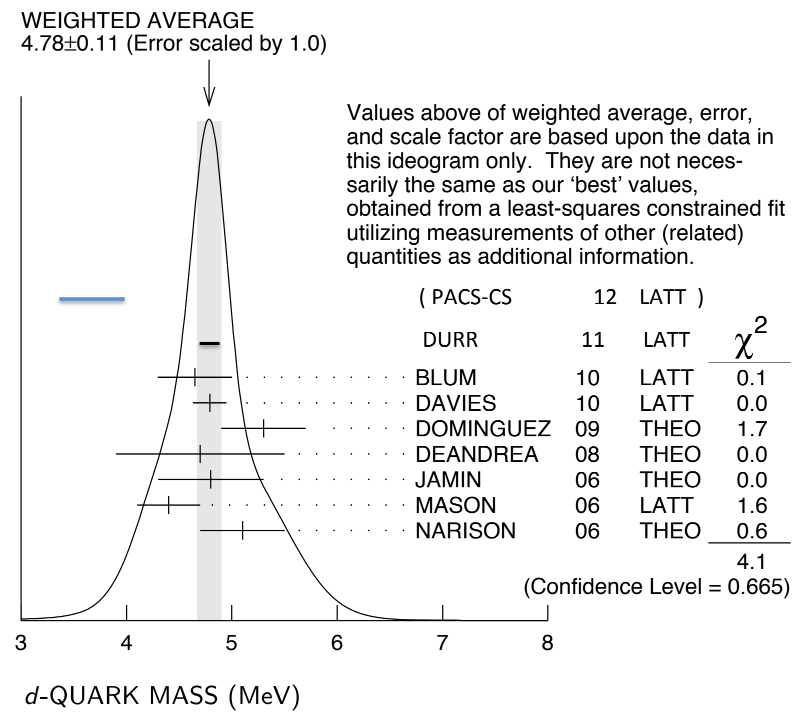
$$
\begin{aligned}
m_{\pi^+} &= 137.7(8.0)\ [\mathrm{MeV}] \\
m_{K^+} &= 492.3(4.7)\ [\mathrm{MeV}] \\
m_{K^0} &= 497.4(3.7)\ [\mathrm{MeV}] \\
m_{K^0} - m_{K^+} &= 4.54(1.09)\ [\mathrm{MeV}]
\end{aligned}
$$

$$
\begin{aligned}
m_{\mathrm{u}}^{\overline{\mathrm{MS}}} &= 2.57(26)(07)\ [\mathrm{MeV}] \\
m_{\mathrm{d}}^{\overline{\mathrm{MS}}} &= 3.68(29)(10)\ [\mathrm{MeV}] \\
m_{\mathrm{s}}^{\overline{\mathrm{MS}}} &= 83.60(58)(2.23)\ [\mathrm{MeV}] \\
m_{\mathrm{ud}}^{\overline{\mathrm{MS}}} &= 3.12(24)(08)\ [\mathrm{MeV}] \\
m_{\mathrm{u}}/m_{\mathrm{d}} &= 0.698(51) \\
m_{\mathrm{s}}/m_{\mathrm{ud}} &= 26.8(2.0)
\end{aligned}
$$



PDG2012





- New results from [BMW] , smeared-Wilson clover.
- New results from [PACS-CS] . On physics point, quenched QED + QED reweighting, as well as $m_u \neq m_d$ effects, $N_F =$ 1+1+1 colover-Wilson simulation.

Error budget

- Statistic error is small, especially for ratios.
- Chiral fit error: $m_q \leq$ 40 or 70 MeV ($M_{ps} \leq$ 250 or 420 MeV).
- Finite Volume Effect by comparing (1.9 fm)^3 and (2.7 fm)^3.

$$\frac{\Delta^{\text{EM}} M^2_{PS}(\infty)\Big|_{V.S.M}}{\Delta^{\text{EM}} M^2_{PS}(L \approx 1.9\ fm)\big|_{V.S.M}} = 1.10\,.$$

FV ChPT overestimate the FV effect. Generally quark masses are stable against $\Delta M_{PS} \sim$ O(10) %. ($M_{\pi^\pm}$, $M_{K^\pm}$, M_{K^0} inputs)

	stat. err (%)	fit(%)	fv(%)	$\mathcal{O}(a^2)$ (%)	QED qnch(%)	renorm(%)
m_u	4.5	+4.0	+14	4	2	2.8
m_d	3.3	+3.6	-2.5	4	2	2.8
m_s	3.0	+0.2	+0.1	4	2	2.8
$m_d - m_u$	2.7	+7.8	-17	4	2	2.8
m_{ud}	3.5	+2.8	+2.7	4	2	2.8
m_u/m_d	2.0	+5.5	+16	4	2	-
m_s/m_{ud}	1.0	+3.0	-2.6	4	2	-

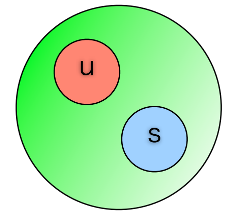
- QED Z_m $\mathcal{O}(\alpha) \sim$ 1%. Error of $m_s^{\text{sea}} \sim$ 2 %.

Origins of Isospin breaking in Kaon

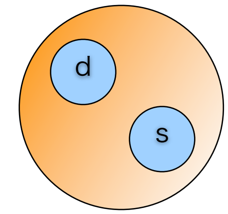
- Reason why the iso doublet, (K^+, K^0), has the mass splitting

$$M_{K^\pm} - M_{K^0} = -3.937(29) \quad \text{MeV}, \quad \text{[PDG]}$$

$(m_{\text{dwn}} - m_{\text{up}})$: makes $M_{K^+} - M_{K^0}$ negative.
$(q_u - q_d)$: makes $M_{K^+} - M_{K^0}$ positive.



Charged Kaon
(repulsive EM)



Neutral Kaon
(attractive EM)

- Using the determined quark masses and SU(3) LEC, we could isolate (to $\mathcal{O}((m_{\text{up}} - m_{\text{dwn}})\alpha)$) each of contributions,

- $$M^2_{\text{PS}}(m_{\text{up}}, 2/3, m_{\text{str}}, -1/3) - M^2_{\text{PS}}(m_{\text{dwn}}, -1/3, m_{\text{str}}, -1/3)$$
$$\simeq M^2_{\text{PS}}(m_{\text{up}}, 0, m_{\text{str}}, 0) - M^2_{\text{PS}}(m_{\text{dwn}}, 0, m_{\text{str}}, 0) \qquad [\Delta M(m_{\text{up}} - m_{\text{dwn}})]$$
$$+ M^2_{\text{PS}}(\bar{m}_{ud}, 2/3, \bar{m}_{ud}, -1/3) - M^2_{\text{PS}}(\bar{m}_{ud}, -1/3, m_{\text{str}}, -1/3) \qquad [\Delta M(q_u - q_d)]$$

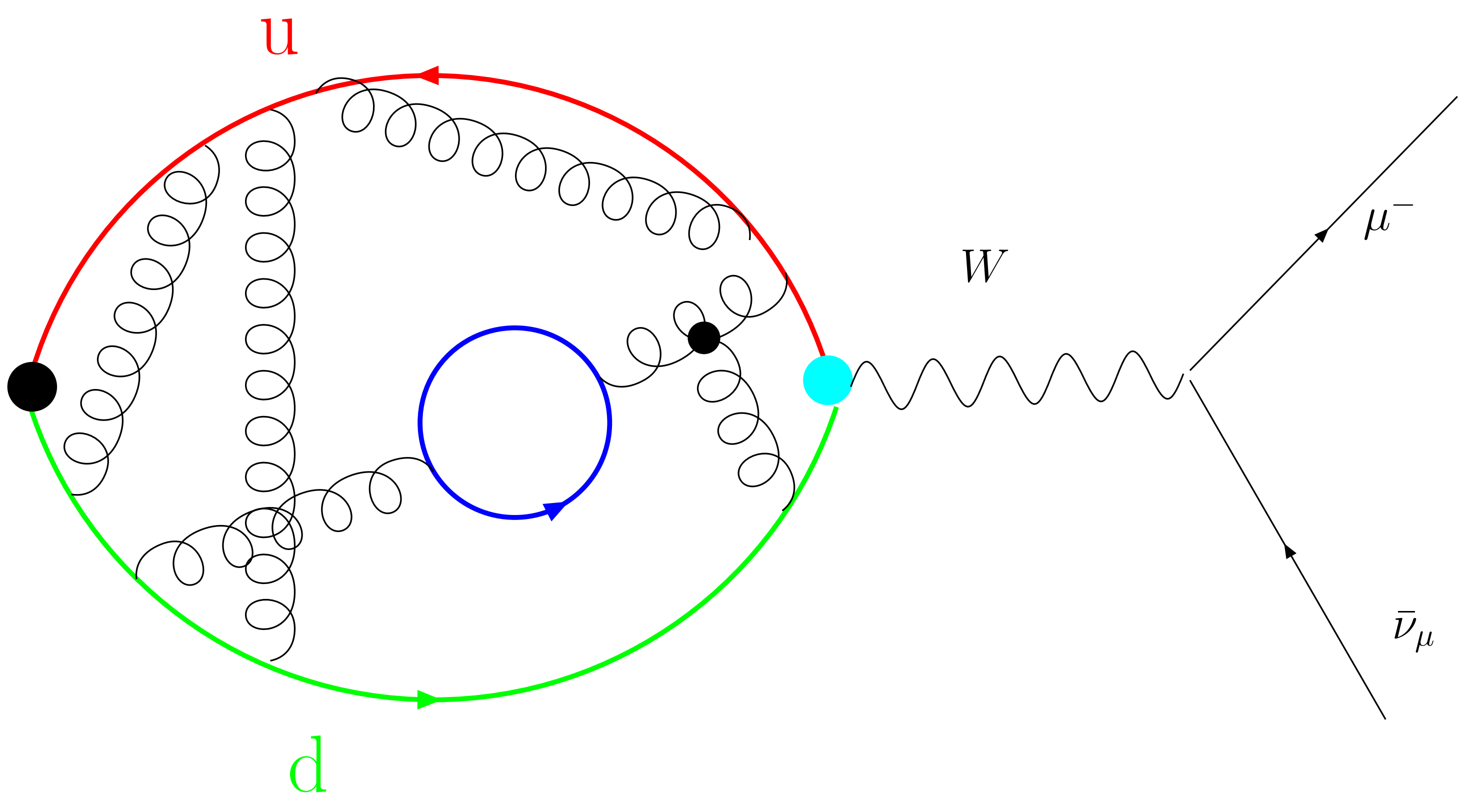
- *$\Delta M(m_{\text{up}} - m_{\text{dwn}})$ = -5.23 (14) MeV [133(4)% in $\Delta M^2(m_{\text{up}} - m_{\text{dwn}})$]*
$\Delta M(q_u - q_d)$ = 1.327(37) MeV [-34(1)% in $\Delta M^2(q_u - q_d)$]

Also SU(3) ChPT, $\Delta M(m_{\text{up}} - m_{\text{dwn}})$=-5.7(1) MeV and $\Delta M(q_u - q_d)$=1.8(1) MeV.

- Similar analysis for π is possible, but facing a difficulty of isolating sea strange quark terms. $m_{\pi^\pm} - m_{“\pi^0”} =$ 4.50(23) MeV (experiment: 4.5936(5) MeV)

Meson leptonic Decay constants, f_π, f_K

- Meson's wave function at origine



$$\langle 0|\bar{d}\gamma_5 u(0)|\pi\rangle \frac{e^{ipx}}{\sqrt{2E}} \langle \pi|\bar{u}\gamma_m u\gamma_5 d|0\rangle \times G_F V_{ud} m_\mu \bar{\nu}(1-\gamma_5)\mu$$

Isospin violation in PS leptonic decays

[discussion with A.Juttner, C.Sachrajda, G. Colangelo, L. Lellouch @LGT10, CERN]

- f_K/f_π is getting very precise:

  $f_K/f_\pi = 1.193(6)$ [0.5%] [WA by FlaviaNet Kaon WG 2010]

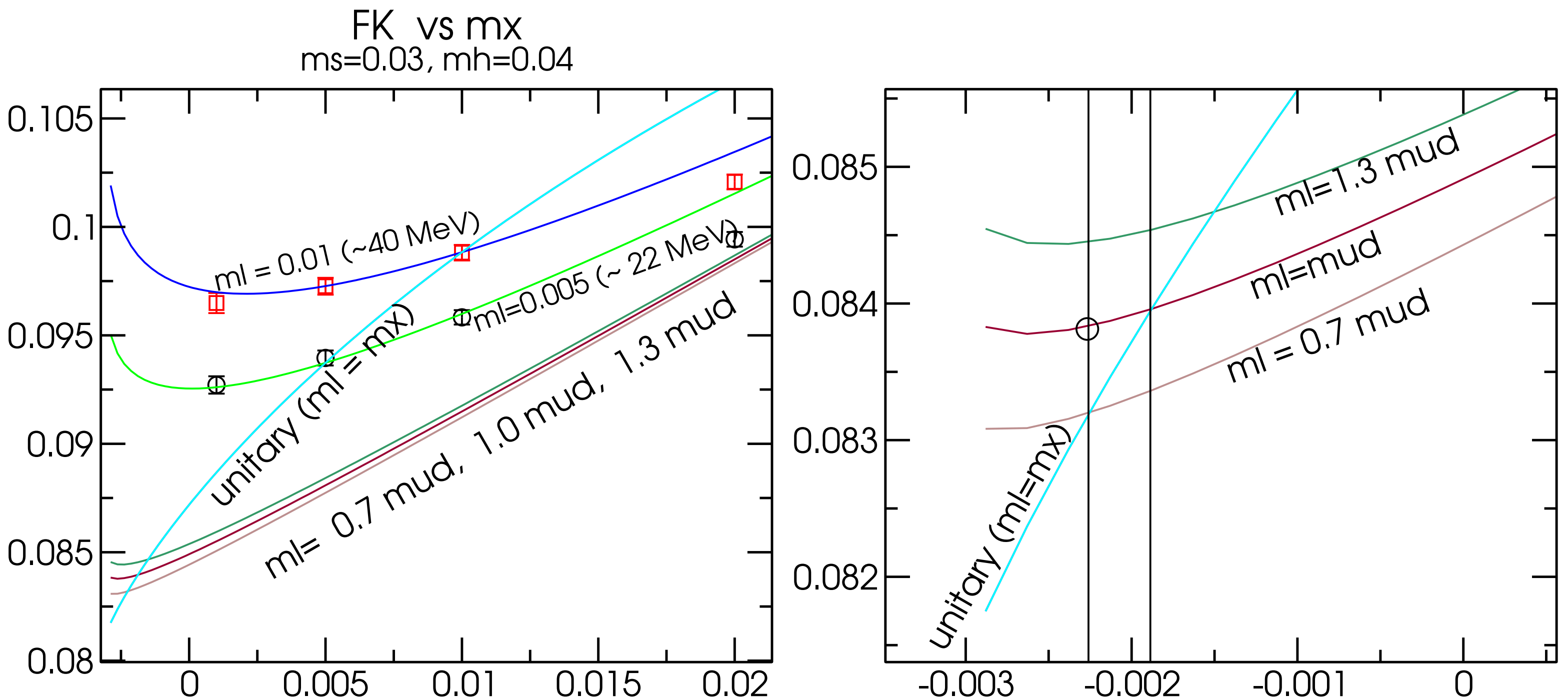
- CKM matrix elements ratio from charged π and K leptonic decay widths:

$$\frac{\Gamma(K^+ \to l^+\nu(\gamma))}{\Gamma(\pi^+ \to l^+\nu(\gamma))} = \frac{|V_{us}|^2}{|V_{ud}|^2} \times \frac{f_K^2}{f_\pi^2} \times \frac{m_K(1-m_l^2/m_K^2)^2}{m_\pi(1-m_l^2/m_\pi^2)^2} \times (1+\delta_{\mathrm{SU(2)}} + \delta_{\mathrm{EM}})$$

- At which quark masses, f_π and f_K should be computed ?

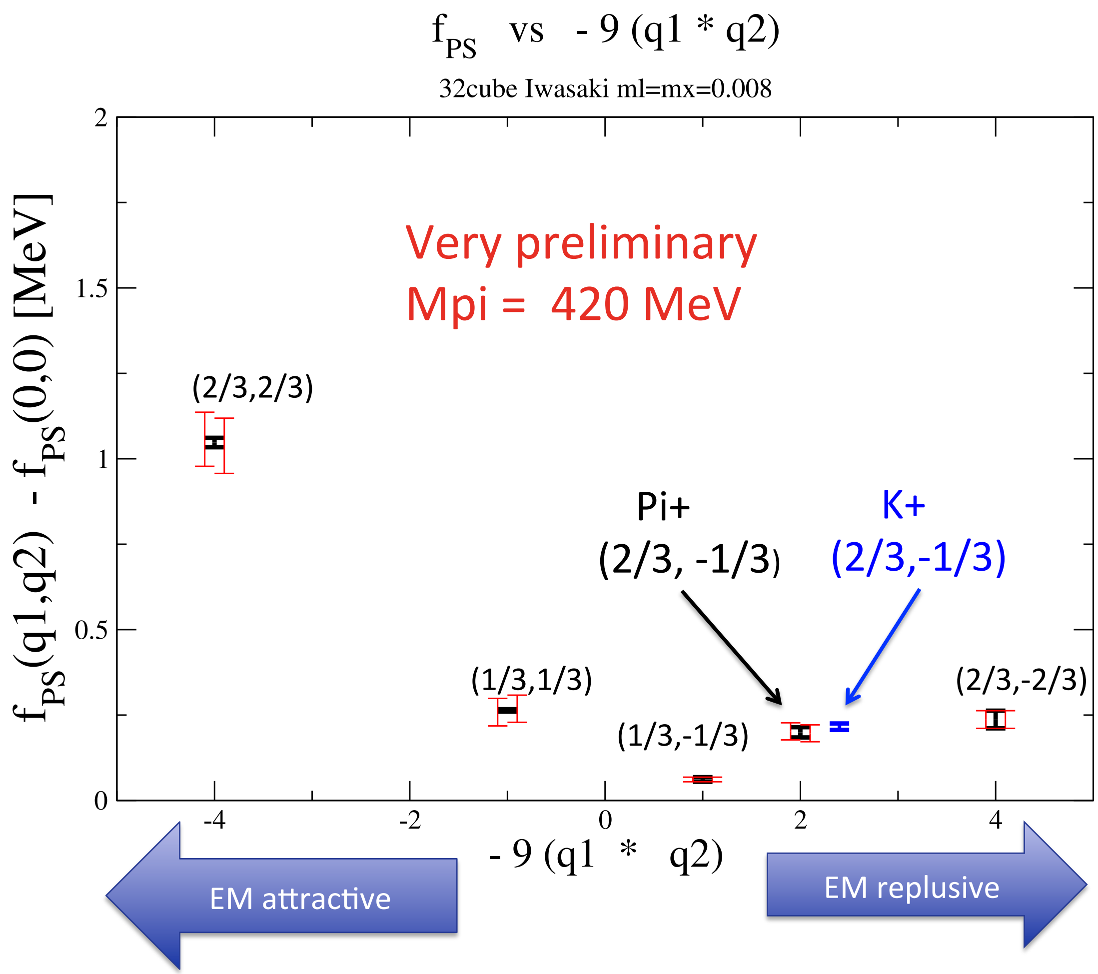
  - f_K: Should light quark mass be m_u or $m_{ud} = (m_u + m_d)/2$?
    $m_u/m_{ud} \sim 0.6 - 0.8$
  - f_π: Is the π mass shift from EM effect totally removed by δ_{EM} ?
    $m_\pi^0 = 135$ MeV vs $m_\pi^\pm = 139$ MeV ?

- Which is the best way to correct isospin breakings in the $|V_{us}/V_{ud}|$ extraction ?



- $K^+ = \bar{s}u$ (light sea quark mass: m_l, light valence quark mass : m_x)
- f_K @ $m_l = m_x = m_{ud}$: 149.6(7) MeV
- f_K @ $m_x = 0.7m_{ud}, \;\; m_l = m_{ud}$: $\delta_{SU(2)}/2 \approx$ -0.15% vs the WA error, 0.5%
- f_K @ $m_l = m_x = 0.7m_{ud}$: [-0.904%]
- ChPT analysis [Cirigliano, Neufeld 2011] says F_K/F_π would shift -0.22(6) % from $(m_u - m_d)$, while it was found to be - 0.39(4) % in Lattice study [RM123, 2012] .

EM effects on PS decay (preliminary)



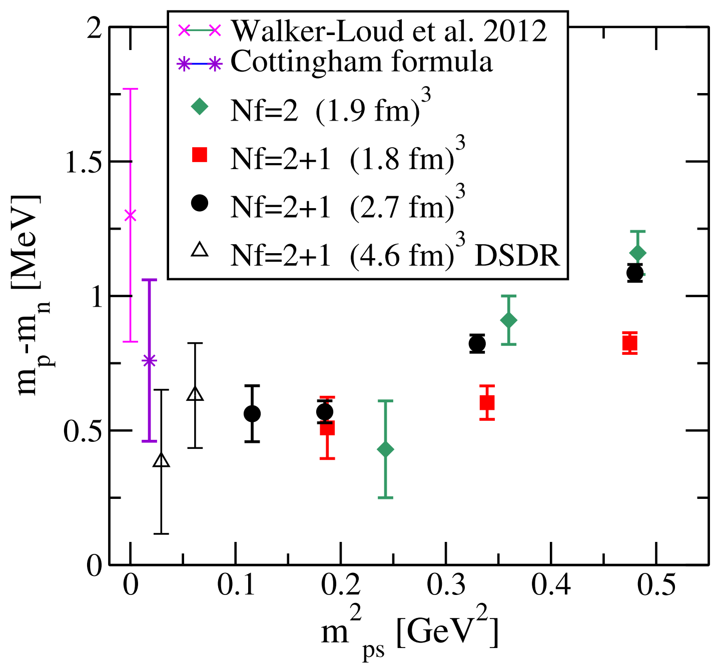
- Statistically well resolved (101 measurements) by the $+e/-e$ averaging.
- *c.f.* [Bijnens Danielsson 2006]
  $F_{\pi+,\text{NLO}}/F_0 = 0.0039$
  $F_{K+,\text{NLO}}/F_0 = 0.0056$
- our preliminary results are smaller. Note heavy M_π

- Decay constants with EM turned on, but $m_u = m_d$
- Wall-point 2pt $\langle A_4(t)P(0)\rangle$ and $\langle P(t)P(0)\rangle$
- See also B. Glaeble and G. Bali `arXivX:1111.3958`

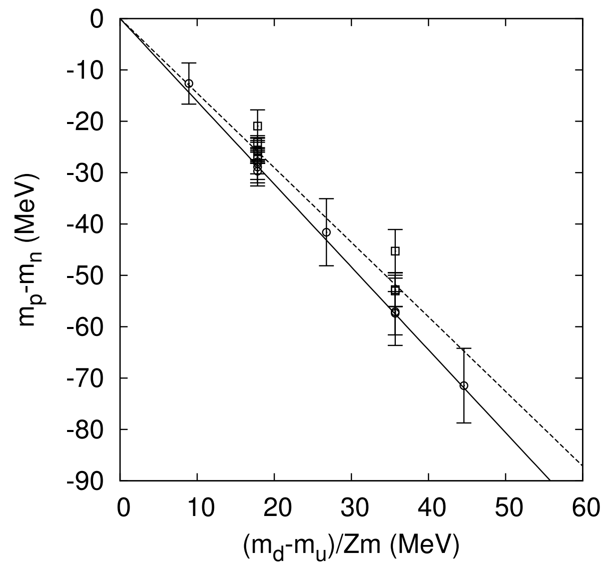
Baryon mass splitting in $N_F = 2, 2+1$

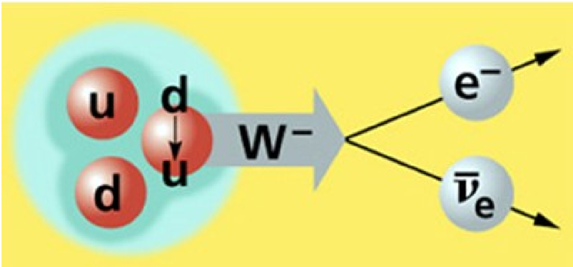
- [A. Walker-Loud *et. al*] : new estimation for QED effects
- [R. Horsley *et. al* (QCDSF-UKQCD)] , octet baryon splittings due to $(m_u - m_d)$
- preliminary N-P splitting with Iwasaki-DSDR lattice $N_F = 2+1$ DWF (4.6 fm)^3

$(q_u - q_d)$ effect



$(m_{\mathrm{up}} - m_{\mathrm{dwn}})$ effect





	$m_u - m_d$	EM
NPLQCD	2.26(72)	
BLUM	2.51(71)	0.54(24)
RM123	2.80(70)	
QCDSF-UKQCD	3.13(77)	
	2.68(35)	0.54(24)

$\Longrightarrow M_N - M_p| = 2.14(42)$ MeV
(experiment: 1.2933321(4) MeV)

- Also EM correction to Ω^- meson is found to be $1.26(6)$ MeV (statistical error only) (preliminary)
- BMWc's presenation could be found on the web.

QED reweighting

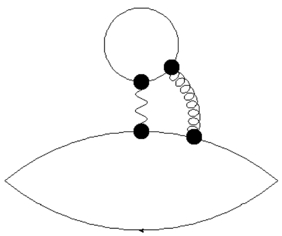
[T. Ishikawa et al. arXiv:1202.6018]

- Full QED (+QCD) from quenched QED (+QCD)
  [Duncan et. al. PRD72 094509(2005)]

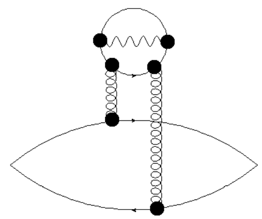
by computing the reweighting factor:

$$w[U_{\rm QCD}, A] = \frac{\det D[U_{\rm QCD} \times e^{iqeA}]}{\det D[U_{\rm QCD}]}$$

on the dynamical QCD configuration



$O(e^2)$



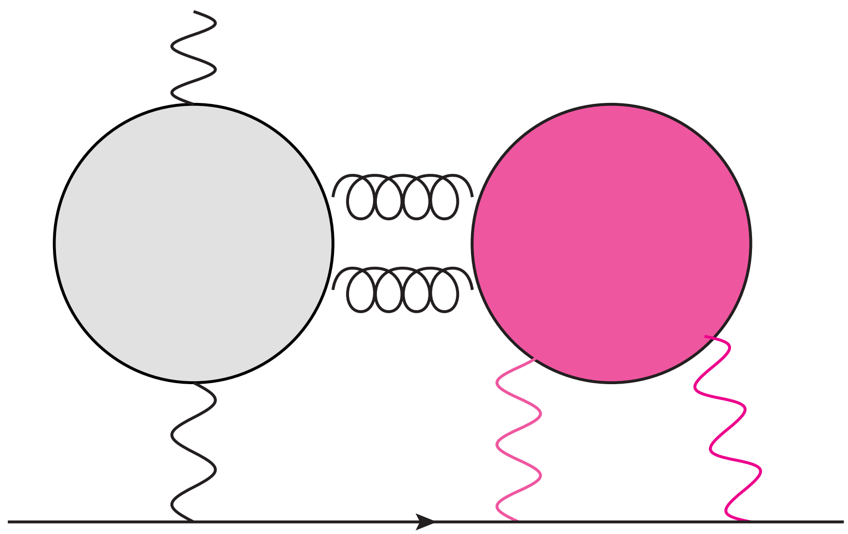
[A. Hasenfratz et.al. PRD78 (08) 014515,
M.Luscher F.Palombi PoS(LATTICE 2008)049,
PACS-CS PRD81(10) 074503]

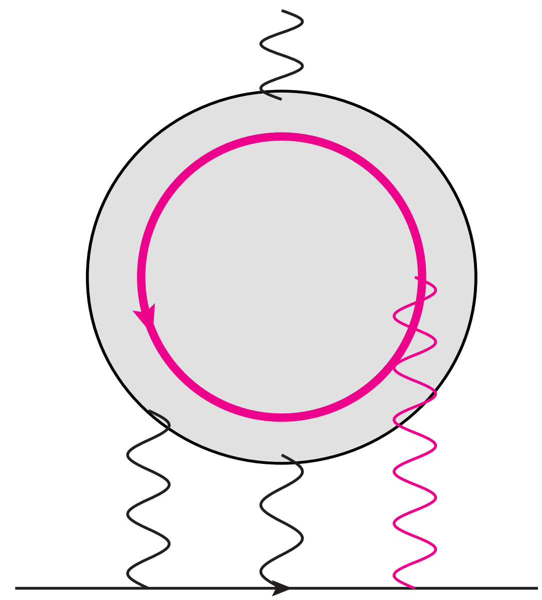
- Stochastic eval. via Root trick [T.Ishikawa et. al. 2007]

$$\det \Omega = (\det \Omega^{1/n})^n = \prod_{i=1}^{n} \langle e^{-\xi_i^\dagger (\Omega^{-1/n} - 1)\xi_i} \rangle_{\xi_i}$$

Disconnected diagrams in HLbL

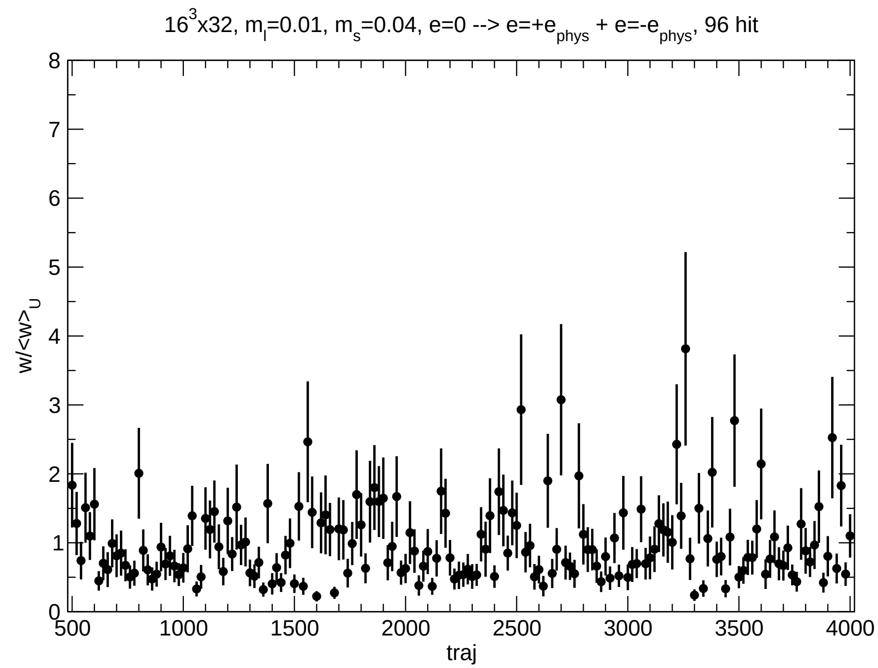
- Missing disconnected diagrams

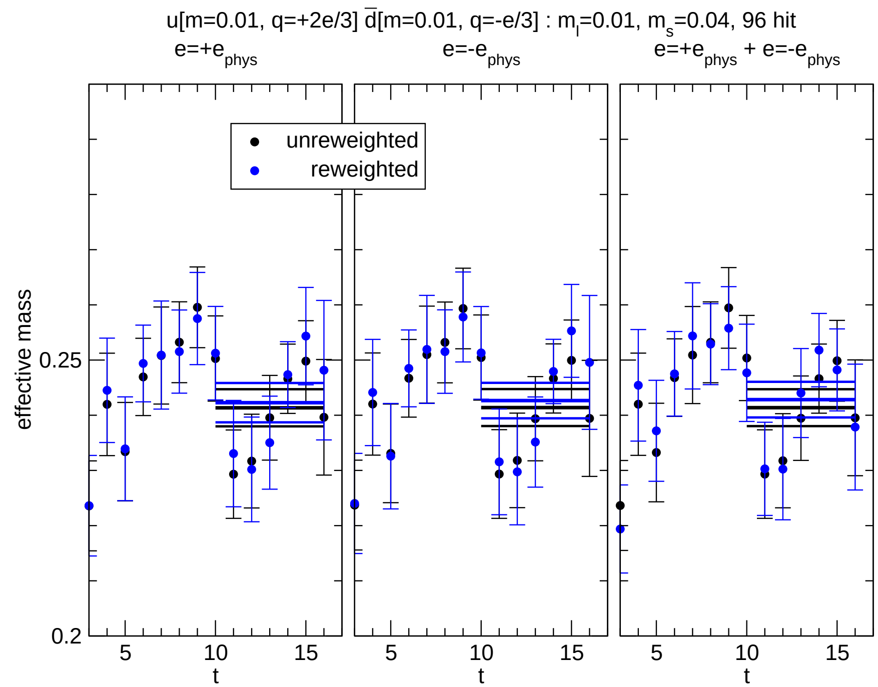


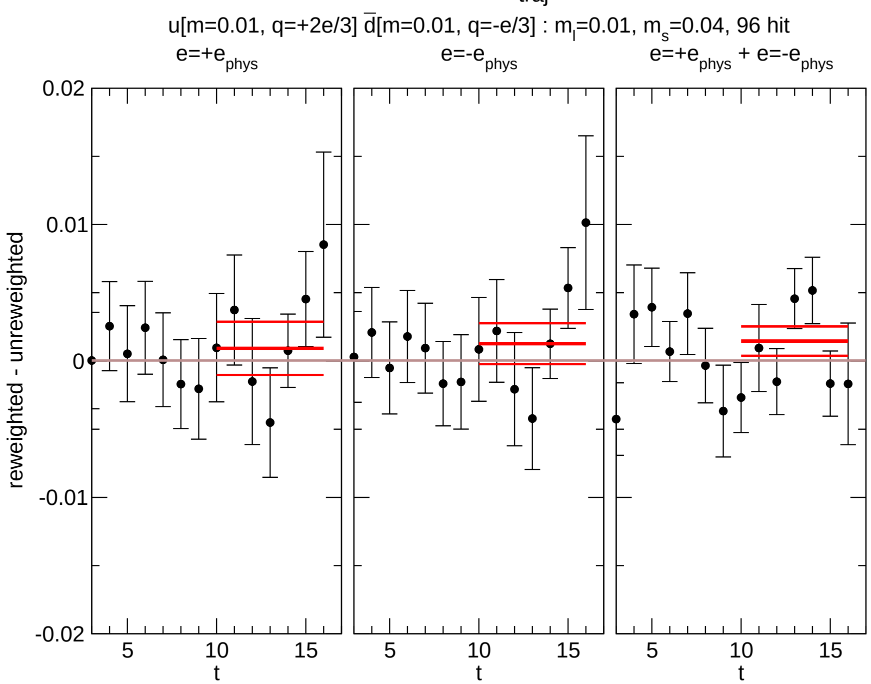


- The second quark loop could be automatically evaluated as sea quark effect, if the sea quark electric charge effect is taken into account

  $\rightarrow$ QED reweighting (or dynamics QCD+QED)



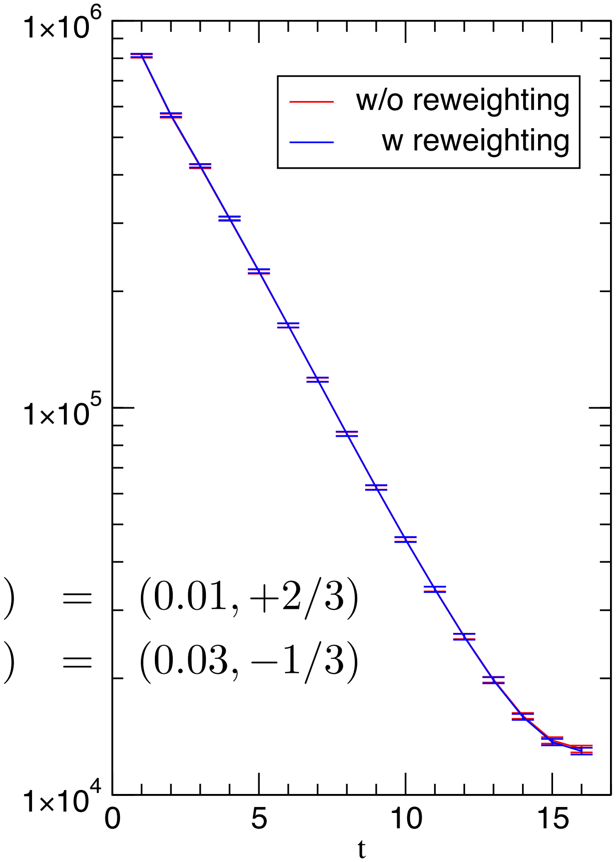


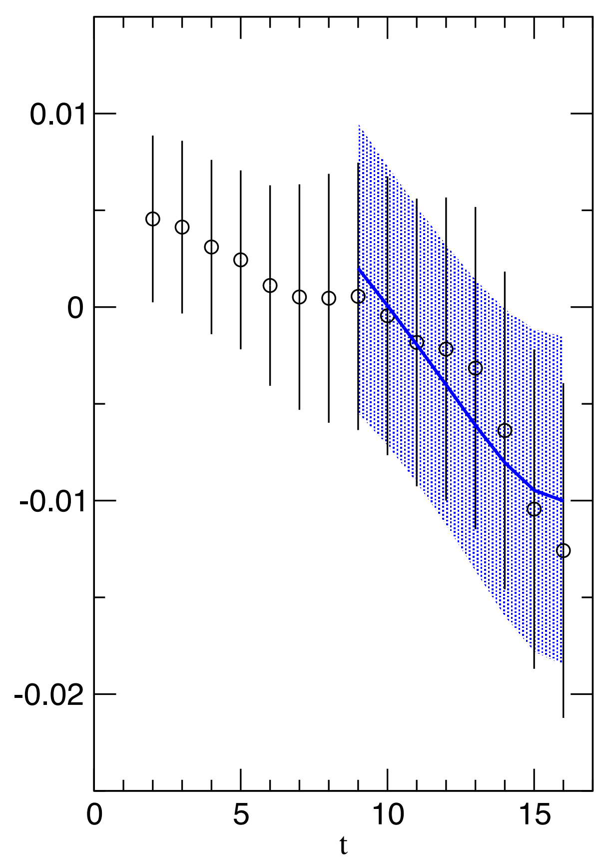


- 24-th root$\times$ 4 hits
- sea charges $q_u = 2/3, q_d = q_s = -1/3$ for $m_u = m_d$
- Size of the sea charge LEC, Y_1, is roughly a ball park of other LEC, consistent with systematic error estimate.

Full QED effect on PS meson correlator

$$C(t) = \langle P(t) P(0) \rangle \qquad \frac{C(t)[e_S = e_{phys}] - C(t)[e_S = 0]}{C(t)[e_S = 0]}$$





$$(m_{v_1}, q_{v_1}) = (0.01, +2/3)$$
$$(m_{v_2}, q_{v_2}) = (0.03, -1/3)$$

Separating the terms

- A set of transformations

$$\begin{aligned}\mathcal{T}_1 : (m_1, q_1; m_3, q_3) &\longrightarrow (m_3, q_3; m_1, q_1),\\ \mathcal{T}_2 : (m_1, q_1; m_3, q_3) &\longrightarrow (m_1, -q_1; m_3, -q_3),\\ \mathcal{T}_3 : (m_1, q_1; m_3, q_3) &\longrightarrow (m_3, -q_1; m_1, -q_3).\end{aligned}$$

e.g. SU(2) formula

$\mathcal{T}_2$ −even

$$\Delta(M_\pi^{SU(2)})^2 = -4e_{\rm s}^2 \left\{ Y_1 {\rm tr} Q_{\rm s(2)}^2 + Y_1' ({\rm tr} Q_{\rm s(2)})^2 + Y_1'' q_6 {\rm tr} Q_{\rm s(2)} \right\} \chi_{13}$$

$\mathcal{T}_2$ −odd & $\mathcal{T}_3$ −even

$$+e_{\rm s} e_{\rm v} \left\{ \frac{C}{F_0^4} \frac{1}{8\pi^2} \sum_{i=4,5} \left(\chi_{1i} \ln \frac{\chi_{1i}}{\mu^2} - \chi_{3i} \ln \frac{\chi_{3i}}{\mu^2} \right) q_i + 4(\chi_1 - \chi_3) \left(J {\rm tr} Q_{\rm s(2)} + J' q_6 \right) \right\} (q_1 - q_3)$$

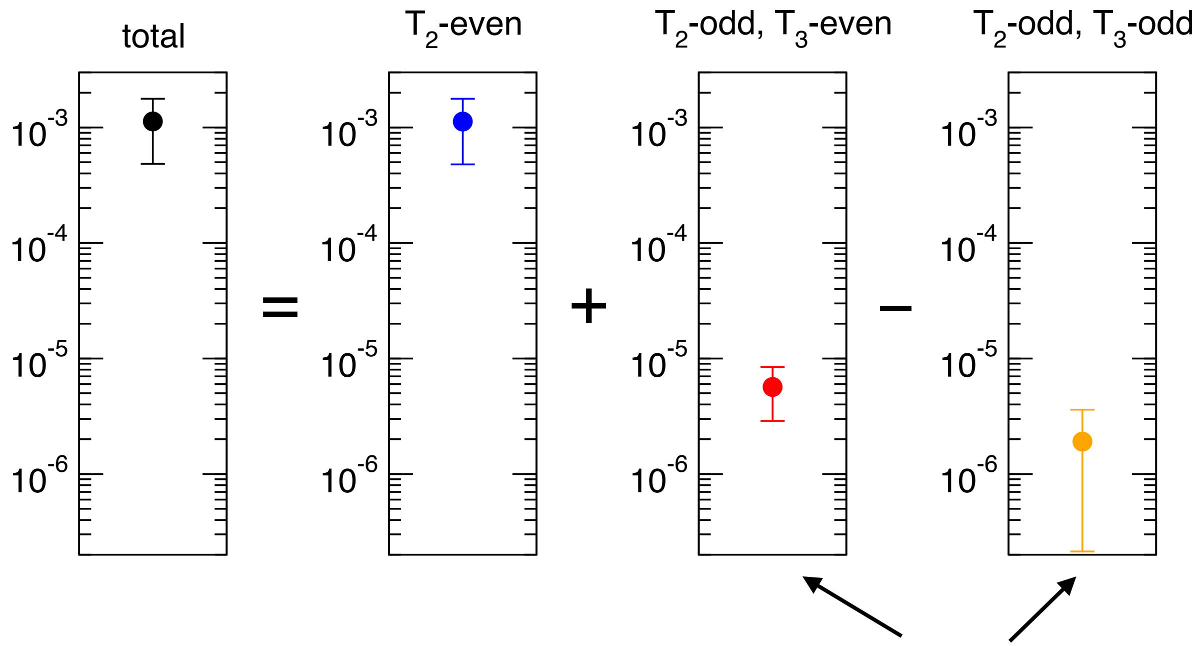
$\mathcal{T}_2$ −odd & $\mathcal{T}_3$ −odd

$$+4 e_{\rm s} e_{\rm v} \left(K {\rm tr} Q_{\rm s(2)} + K' q_6 \right) (q_1 + q_3) \chi_{13},$$

Separating the terms

- The hierarchy problem is resolved and the difficulty of multi parameter fit is reduced using even/oddness of the transformations.

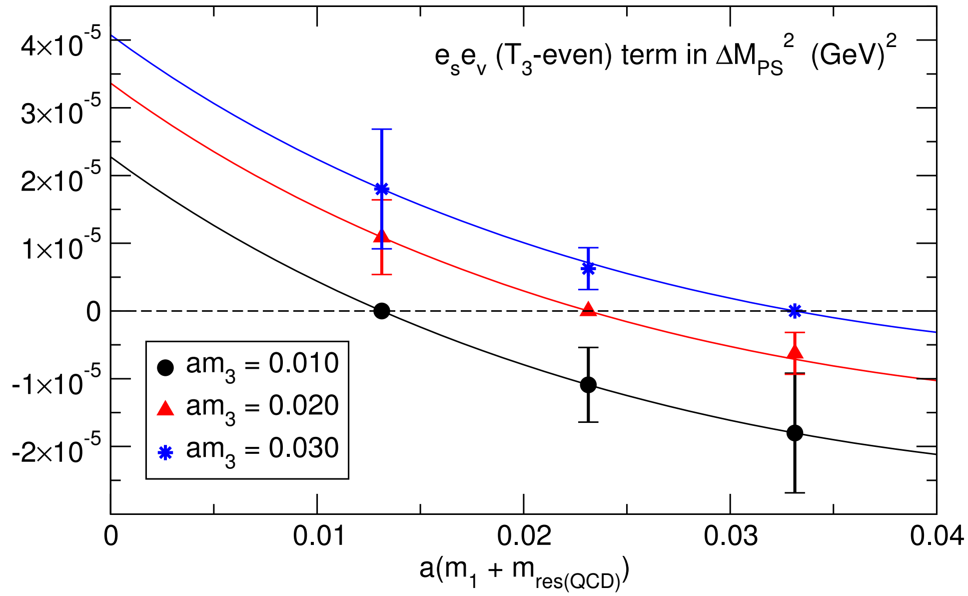
actual data: m_π^2 $(m_1, m_3) = (0.01, 0.03)$



more than $10^2\times$ suppressed
(as expected)

ChPT fit

e.g. SU(2) ChPT fit to $e_S e_V$ ($\mathcal{T}_3$−even) data



$$(q_1, q_3) = (+2/3, -1/3)$$

- Infinite volume formulae are used, because quark mass parameter in this study is not so small that finite volume effects are significant.
- Only minimal set of data with smaller valence quark masses is used in the each fit.

QED LEC's

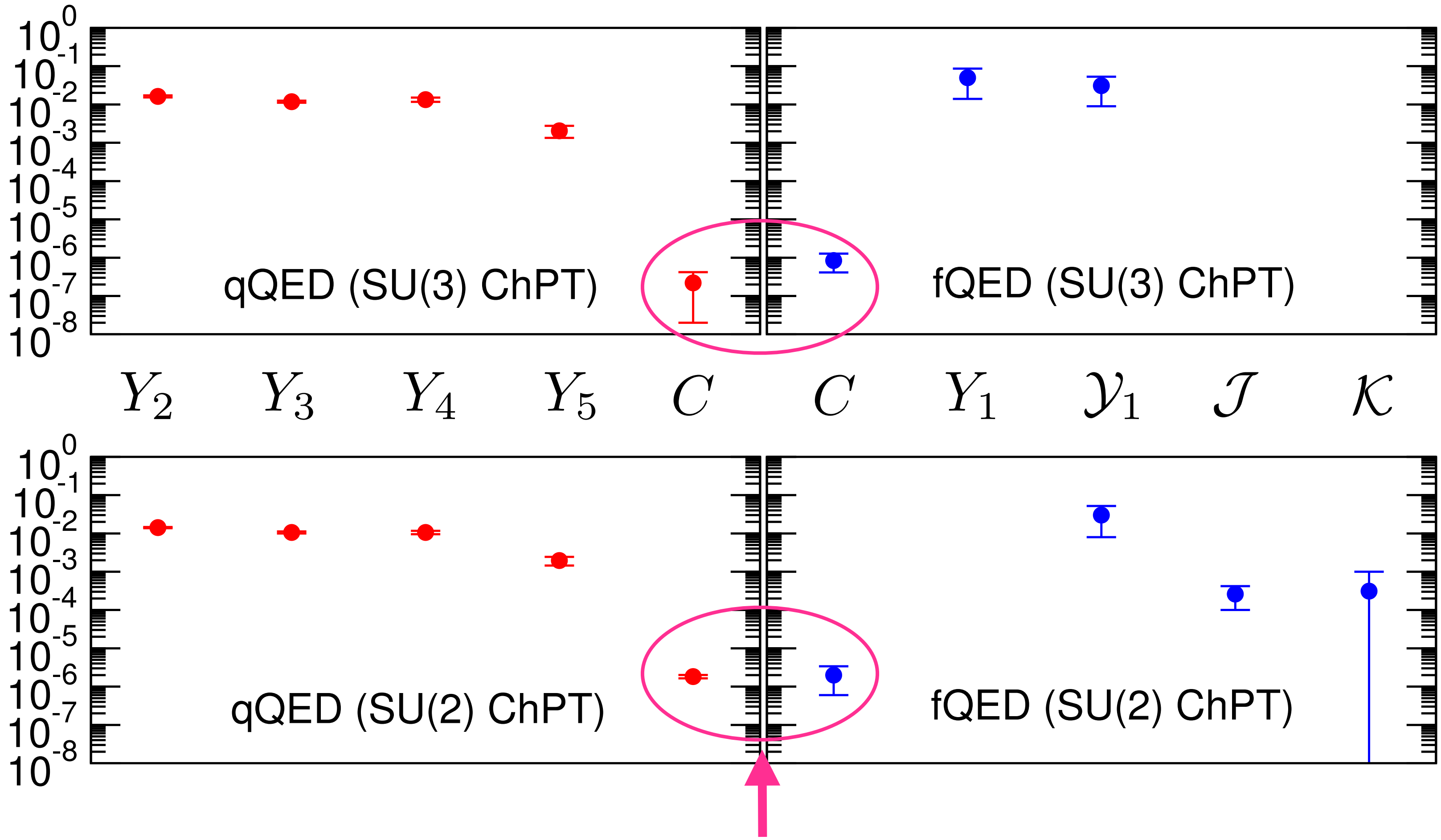
	$SU(3)$ ChPT		$SU(2)$ ChPT	
	uncorr	**corr**	**uncorr**	**corr**
$10^7 C$ (qQED)	2.2(2.0)	–	18.3(1.8)	–
$10^7 C$	8.4(4.3)	8.3(4.7)	20(14)	15(21)
$10^2 Y_1$	-5.0(3.6)	-0.4(5.6)	–	–
$10^2 \mathcal{Y}_1$	-3.1(2.2)	-0.2(3.4)	-3.0(2.2)	-0.2(3.4)
$10^4 \mathcal{J}$	–	–	-2.6(1.6)	-3.3(2.8)
$10^4 \mathcal{K}$	–	–	-3.1(6.9)	-3.7(7.8)

SU(3) $\quad \mathcal{Y}_1 = Y_1 \mathrm{tr} Q^2_{\mathrm{s}(3)}$

SU(2)
$$\begin{cases} \mathcal{Y}_1 = Y_1 \mathrm{tr} Q^2_{\mathrm{s}(2)} + Y_1' (\mathrm{tr} Q_{\mathrm{s}(2)})^2 + Y_1'' q_6 \mathrm{tr} Q_{\mathrm{s}(2)} \\ \mathcal{J} = J \mathrm{tr} Q_{\mathrm{s}(2)} + J' q_6 \\ \mathcal{K} = J \mathrm{tr} Q_{\mathrm{s}(2)} + K' q_6 \end{cases}$$

To fully obtain LEC's in SU(2) ChPT, at least 3 independent combinations of sea quark EM charges are required.

QED LEC's



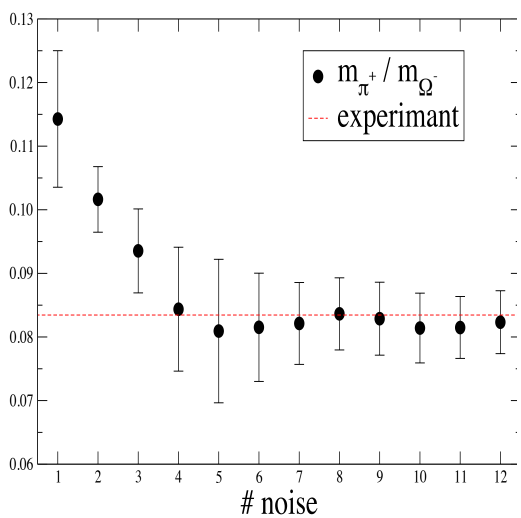
consistent : quenched QED ⟷ full QED (reweighed)

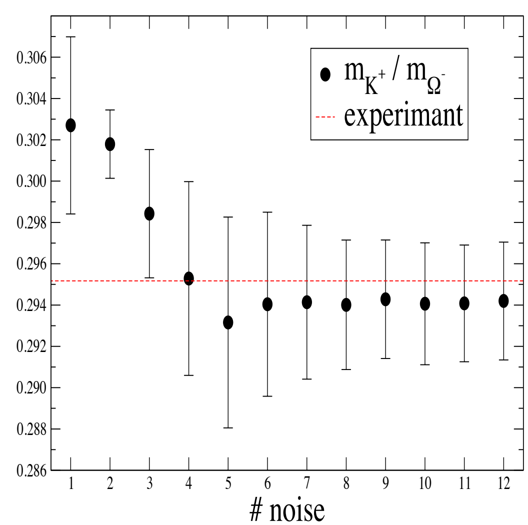
PACS-CS (N. Ukita) QED reweighting [arXiv:1205.2961[hep-lat]]

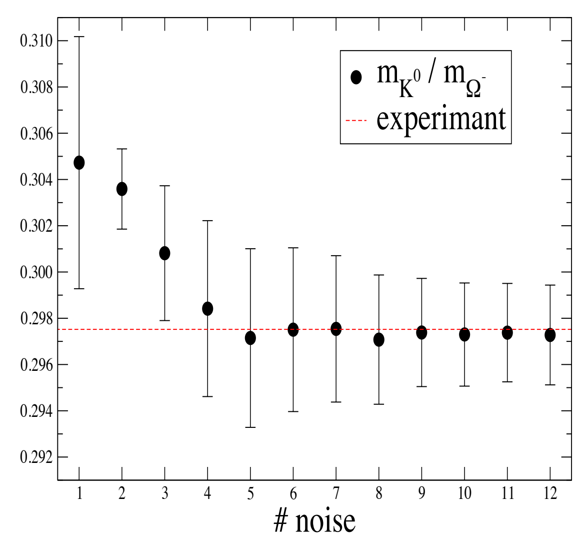
U(1) gauge confs are generated on a 64^3x128 lattice and are averaged inside the 2^4 cell to reduce local fluctuations.

- Reweighting factor : square root trick $|D'/D| = (|D'/D|^2)^{1/2}$,
  426(=400+26) determinant breakup [Hasenfratz et. al, 2008],
  12 noises for each breakup,
  block solver → factor of 3~4 speedup,
  1) 400 breakup for U(1) charges + quark masses near the physical point,
  2) 26 breakup for final tuning of hopping parameters to the phys. point.

- Hadron measurement : 16 source points for each conf.







Conclusions

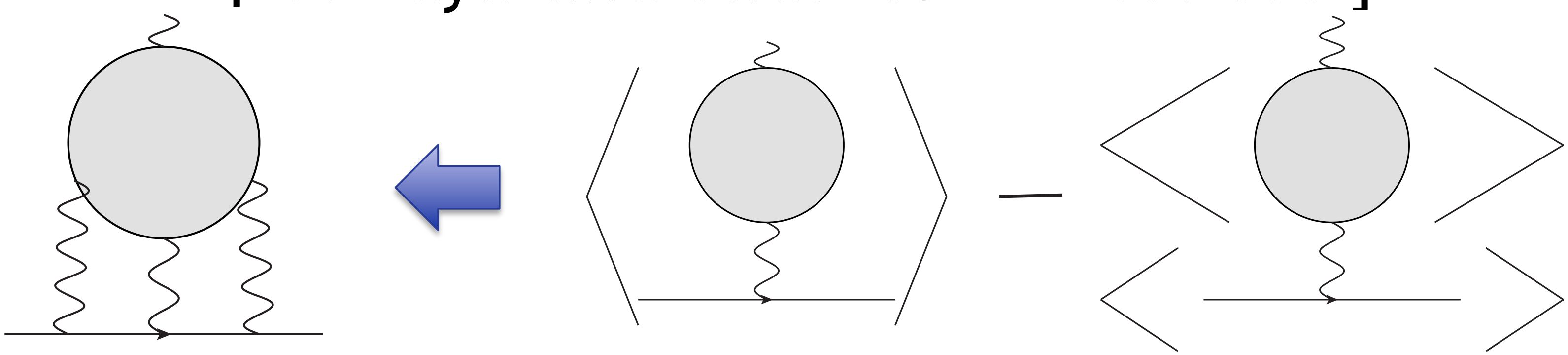
- Isospin breaking studies are interesting and inevitable as precision of lattice QCD is improved.
- quark masses
- Neutron-Pron splittings
- Other interesting quantities ?

  D,B meson mass
  $\pi^0 - \eta - \eta'$ *and* $\rho - \omega$ *mixings*
  $K_l 3$
  $\pi^0 \to \gamma\gamma$
  $K \to \pi\pi$ *and* $\Delta I = 1/2$ *rule*

- Lattice QED +QED is also a ground work for $(g-2)_\mu$ Hadronic light-by-light
- Statistical error reduction techniques are important for Lattice QED+QCD simulations: All Mode Averaging (AMA)

g-2 light-by-light

- Light-by-Light only needs the part of $O(\alpha^3)$
- Currently $O(\alpha)$, $O(\alpha^2)$, and unwanted $O(\alpha^3)$ are subtracted (T. Blum's talk)
  [M. Hayakawa et.al PoS LAT2005 353]



- QED perturbative expansion works

→ Order by Order Feynman diagram calculation on lattice : Aslash SeqSrc method

[T.Blum Lattice2012]

a_μ(HLbL) in 2+1 flavor lattice QCD+QED

- Try larger lattice 24^3 ($(2.7\ \text{fm})^3$)
- Pion mass is smaller too, $m_\pi = 329$ MeV
- Same muon mass
- two lowest values of Q^2 (0.11 and 0.18 GeV^2)
- Use All Mode Averaging (AMA) (Izubuchi, Shintani)
  - 6^3 point sources/configuration (216)
  - AMA approximation: "sloppy CG", $r_{\text{stop}} = 10^{-4}$

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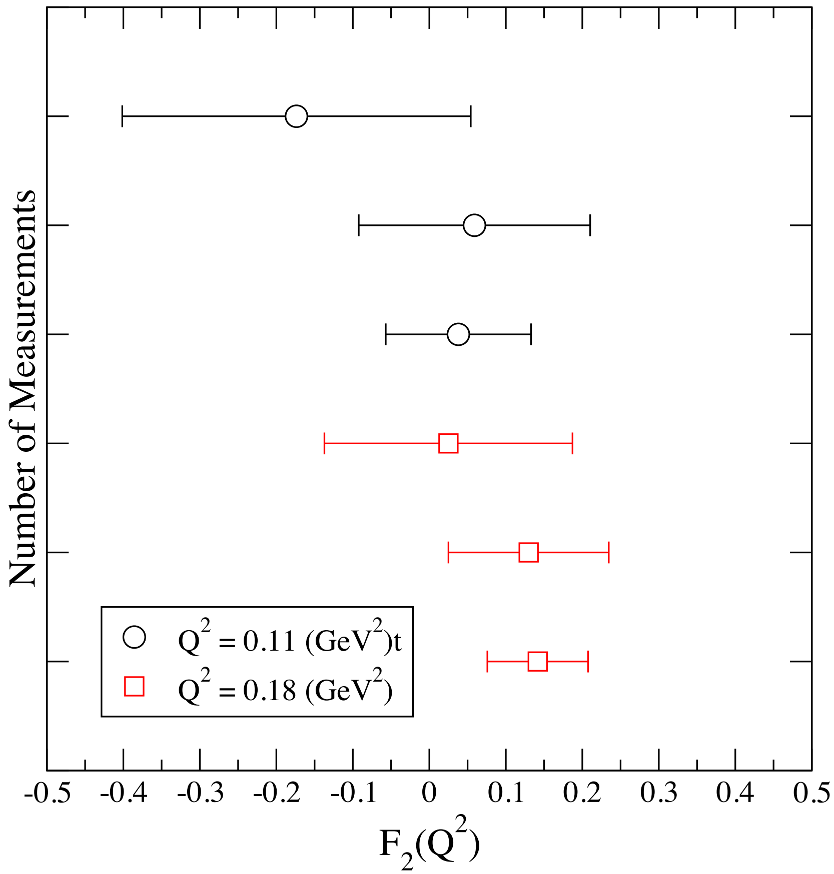
a_μ(HLbL) in 2+1f lattice QCD+QED (PRELIMINARY)

$F_2(Q^2)$ stable with additional measurements ($20 \to 40 \to 80$ configs)



24^3 lattice size

$Q^2 = 0.11$ and 0.18 GeV^2

$m_\pi \approx 329$ MeV

$m_\mu \approx 190$ MeV

Related works

- Related recent works

  [BMWc] , light hadron masses on physics point, also baryon spectrum
  [PACS-CS] , (N. Ukita et al) PS mass including QED reweighting on physics point
  [UQCDSF-UKQCD] , Octet baryon mass from $(m_u - m_d)$
  [Rome123] , Hadron mass from $(m_u - m_d)$ using perturbation
  [B. Glaeble and G. Bali] decay constants
  [A. Walker-Loud] , Isospin violation
  MILC [C.Bernard's talk] EM spectrum / ChPT fit
  [JLQCD] $\pi^0 \to \gamma\gamma$
  [A.Nyffeler] g-2 light-by-light

[T.Blum, T.Doi, M.Hayakawa, T.Izubuchi, S.Uno N.Yamada, and R.Zhou] , *"Electromagnetic mass splittings of the low lying hadrons and quark masses from 2+1 flavor lattice QCD+QED"*, Phys. Rev.D82 (2010) 094508 arXiv:1006.1311[hep-lat] (95 pages).

[T.Izubuchi] , *"Studies of the QCD and QED effects on Isospin breaking"*, PoS(KAON09) 034.

[R.Zhou, T.Blum, T.Doi, M.Hayakawa, T.Izubuchi, and N.Yamada] , *"Isospin symmetry breaking effects in the pion and nucleon masses"* PoS(LATTICE 2008) 131.

[T. Blum, T. Doi, M. Hayakawa, TI, N. Yamada] ,

"Determination of light quark masses from the electromagnetic splitting of psedoscalar meson masses computed with two flavors of domain wall fermions"

Phys. Rev.D76 (2007) 114508 (38 pages)

"The isospin breaking effect on baryons with Nf=2 domain wall fermions"

PoS(LAT2006) 174 (7 pages)

"Electromagnetic properties of hadrons with two flavors of dynamical domain wall fermions"

PoS(LAT2005) 092 (6 pages)

"Hadronic light-by light scattering contribution to the muon g-2 from lattice QCD: Methodology"

PoS(LAT2005) 353(6 pages)

Why lattice QED ?

- Since QED is weakly coupled, $\alpha = 1/137$, the perturbation theory works well. One could extract the necessary quntities as QCD's matrix elements

$$\langle \pi(x)\pi(y)\rangle_{\text{QCD+QED}} = \langle \pi(x)\pi(y)\rangle_{\text{QCD}} + \alpha \int d^4q \langle \pi(x) V_\mu(q) V_\nu(q) \pi(y)\rangle_{\text{QCD}} G^{\text{photon}}_{\mu\nu}(q) + \cdots$$

from which the QCD+QED physical observables would be obtained.

- Rather, we computed for full non-perturbative lattice QCD+QED system

$$\langle \pi(x)\pi(y)\rangle_{\text{QCD+QED}}$$

because of computational costs and higher order $\mathcal{O}(e^4)$ (see later $\not{A}$-Seq. method), its own interesting features, and as an exercises for $(g-2)_\mu$ light-by-light calculation.

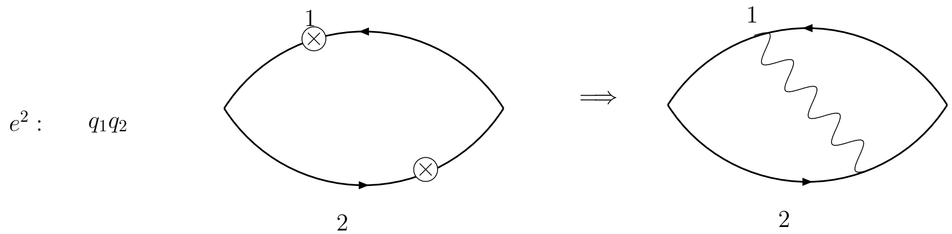
- Lattice QED has problems
  - Finite volume effects from photon
  - Landau ghost (but $\alpha(0) = 1/137$ vs $\alpha(m_Z) \sim 1/128$)

which will not be cured by switching the method to the QCD matrix element calculation.

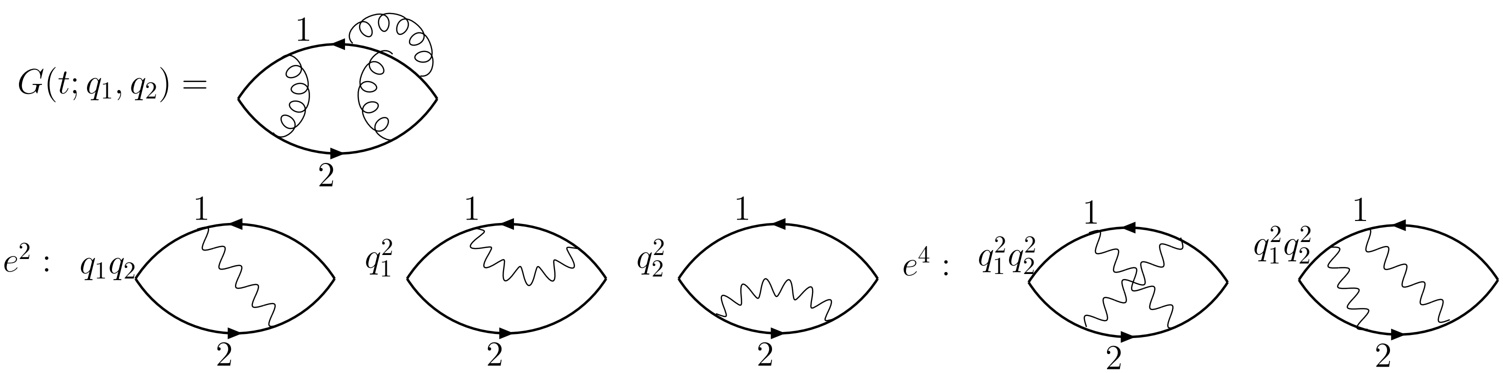
Other considerations and quantities

- $\not{A}$-Sequential source method. Compute each term of propagator in the e expansion.

$$S(e) = S(0) + ieS(0)\not{A}S(0) - e^2 S(0)\not{A}S(0)\not{A}S(0) - e^2 S(0)(\not{A})^2 S(0)\cdots$$



make the contraction to desired orders of wanted diagrams piece by piece.



** No $\mathcal{O}(e^{2n+1})$ noise to disturb $\mathcal{O}(e^{2n})$, can skip diagrams of lower orders than the target.*

** Value of q and e could be determined off-line.*

** # of solves are equal or less up to $\mathcal{O}(e^2)$, compared to the original methods, needs five solves ($q = 0, \pm 2e/3, \mp e/3$).*

** Could use the $e = 0$ Eigen values/vectors.*

- Various checks to make sure we understand systematics in light-by-light.
- The computation of quark propagators with EM will be shared among various quantities.
- $\mathcal{O}(\alpha, \alpha^2)$: Vacuum polarizations $\Pi_{\mu\nu} = \langle V_\mu V_\nu \rangle$ include the disconnected quark loops, which include.
- Quark condensate magnetic susceptibility $\langle \bar{q}\sigma_{\mu\nu} q \rangle_F = e\chi \langle \bar{q}q \rangle_0 F_{\mu\nu}$ to constraint the short distance of $\pi - \gamma - \gamma$ coupling