Chiral Perturbation Theory: Summer School Exercises

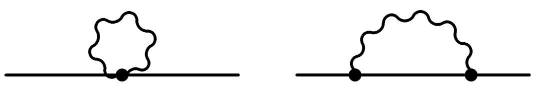
I. FOUNDATIONS

1). Consider a non-singlet axial transformation, $\psi_i \to \sum_j [e^{i\vec{\phi}\cdot\vec{\tau}\gamma_5}]_{ij}\psi_j$. Is there a corresponding symmetry group of the massless QCD action?

2). Determine the discrete symmetry properties of the pion Goldstone modes from the transformation of the coset field Σ.

3). What happens to the power-counting argument for chiral perturbation theory in $d = 2$, 6 dimensions? Do the results surprise you? What is different about $d = 3$, 5?

4). Determine the effects of strong isospin breaking, $m_u \neq m_d$, on the chiral Lagrangian. At what order in the chiral expansion does the pion isospin multiplet split?



5). The masses of hadrons are modified by electromagnetism. Construct all leading-order electromagnetic mass operators by promoting the electric charge matrix to fields transforming under the chiral group. (Notice that no photon fields will appear in the electromagnetic mass operators because there are no *external* photon lines.) Which pion masses are affected by the leading-order operators? Finally give an example of a next-to-leading order operator, or find them all.

II. LATTICE APPLICATIONS

6). Do the leading-order four-pion interactions allow mixing of zero and non-zero modes? Draw all one- and two-loop diagrams for the chiral condensate and count the powers of ε.

7). In addressing finite volume corrections, to the pion mass, for example, one typically considers lattices with finite spatial volume and infinite temporal extent. Why is this done? How are the results we derived for finite volume corrections modified? How does the pion mass scale with volume for asymptotically large volumes? Make sure to consider one-loop graphs from both $\mathcal{O}(p^2)$ vertices.

8). Let $\mathcal{M}$ transform as $\mathcal{M}_{AB} \to [\mathcal{U}\mathcal{M}\mathcal{U}^\dagger]_{AB}$ under the graded $U(M|N)$ group. Show that $\mathrm{Str}\,\mathcal{M}$ is invariant under $U(M|N)$.

9). Find the tree-level masses of all *charged* mesons in partially quenched $SU(4|2)$ chiral perturbation theory.

10). Write down all dimension-6 four-quark operators in the Symanzik action for a general mixed-action theory. Classify the operators according to symmetry. Which ones are absent in a theory describing Wilson valence quarks and overlap sea quarks?

III. BARYONS

11). Is the trace of the energy-momentum tensor the divergence of a current?

12). Integrate out the remaining heavy components of the fermion field to find the first-order correction to the static fermion Lagrangian. The result should not surprise you.

13). The non-relativistic projectors reduce the spin algebra to that of Pauli matrices. Show that the axial-vector fermion bilinear reduces to the spin-density operator up to a constant, i.e. $\overline{N}_v\gamma_\mu\gamma_5 N_v = c\overline{N}_v S_\mu N_v$. The relation $\mathcal{P}_+ N_v = N_v$ will prove useful, as will the definition of the spin vector $S_\mu = -i\epsilon_{\mu\nu\rho\sigma}\sigma_{\nu\rho}k_\sigma/4M$ which satisfies $S_\mu S_\mu = \frac{1}{2}\left(\frac{1}{2}+1\right)$. What are $v_\mu S_\mu$ and $[S_\mu, S_\nu]$?

14). Write down all strong isospin breaking mass operators for the nucleon up to second order in the quark mass. What effect does isospin breaking in the pion mass have on the nucleon mass? Deduce the behavior of the nucleon mass splitting as a function of the quark masses.

15). In the chiral limit, the isovector axial current is a conserved current. Is there a constraint on the quark isovector axial charge due to the non-renormalization of this current? What about on the nucleon axial charge?

IV. CONVERGENCE

16). In the strong isospin limit, there are two different quark masses but three meson masses of the pseudoscalar octet. Use the three-flavor chiral Lagrangian to derive the constraint $\Delta_{\mathrm{GMO}} = \frac{4}{3}m_K^2 - m_\eta^2 - \frac{1}{3}m_\pi^2 = 0$, which was originally found by Gell-Mann and Okubo. What happens away from the strong isospin limit?

17). Revisit electromagnetic mass corrections in three-flavor chiral perturbation theory. Find all leading and next-to-leading order electromagnetic mass operators. Ignoring the up and down quark masses, which octet masses are affected by leading and next-to-leading order operators?

18). Accounting for strong and electromagnetic isospin breaking to leading order, determine the mass spectrum of the meson octet, and devise a way to compute the quark mass ratios, m_u/m_d and m_d/m_s, using the experimentally measured meson masses.

19). Recall that the relation between the nucleon sigma term, given by

$$\sigma_N = \frac{m_q}{2M_N} \langle N(\vec{k}\,)|\overline{u}u + \overline{d}d|N(\vec{k}\,)\rangle,$$

and the nucleon strangeness fraction, defined by

$$y = \frac{\langle N(\vec{k}\,)|\overline{s}s|N(\vec{k}\,)\rangle}{\frac{1}{2}\langle N(\vec{k}\,)|\overline{u}u + \overline{d}d|N(\vec{k}\,)\rangle},$$

has the form

$$\left(\frac{m_s}{m_q} - 1\right)(1-y)\,\sigma_N = \frac{m_s - m_q}{2M_N} \left\langle N(\vec{k}\,)\left|\overline{u}u + \overline{d}d - 2\overline{s}s\right| N(\vec{k}\,)\right\rangle .$$

Using the $SU(3)$ baryon chiral Lagrangian at tree level, calculate the matrix element on the right-hand side and express in terms of the octet baryon masses. Finally combine the result of the previous problem with a large N_c estimate of the strangeness content to deduce a value for the sigma term.

20). Use $SU(2)$ chiral perturbation theory to construct a low-energy theory for the kaons and η.

21). Find a process involving strange baryons for which a description in terms of $SU(2)$ chiral perturbation theory certainly must fail.

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