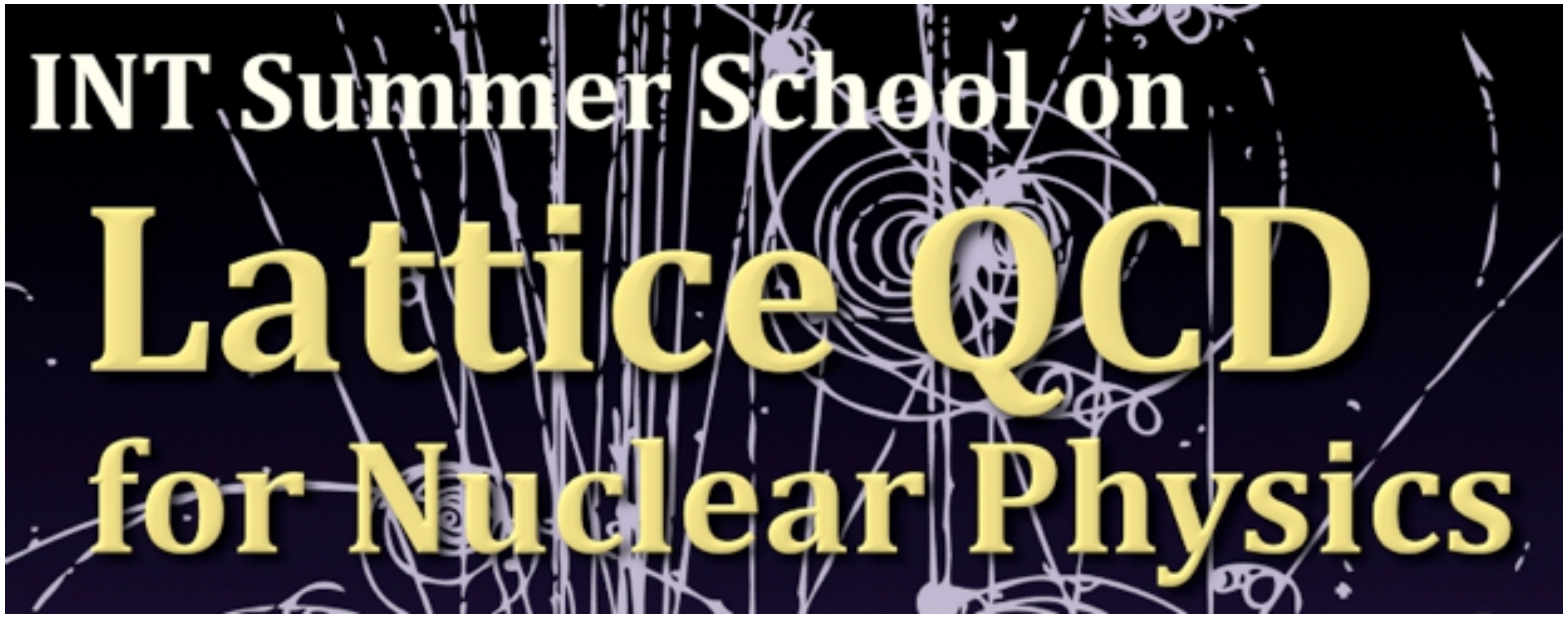

Lectures on Chiral Perturbation Theory

I. Foundations
II. Lattice Applications
III. Baryons
IV. Convergence

Brian Tiburzi

NSF
the City College of New York
CUNY
RIKEN BNL Research Center

Chiral Perturbation Theory

IV. Convergence

Reminder: Asymptotic Expansions

- Non-analytic quark mass dependence implies asymptotic expansion (but obviously so: zero radius of convergence)

Toy Model $$F(x) = \int_0^\infty ds \frac{e^{-s}}{1+sx} \qquad 0 < x \ll 1$$

No series expansion about x=0

$$F(x) = \int_0^\infty ds\, e^{-s} \left(\sum_{j=0}^{\infty} (-s\,x)^j \right) \overset{!}{=} \sum_{j=0}^{\infty} (-x)^j \left(\int_0^\infty ds\, s^j\, e^{-s} \right)$$

Suggests approximation $$F_N(x) = \sum_{j=0}^{N} (-)^j j!\, x^j$$

$$|F(x) - F_N(x)| = x^{N+1} \int \frac{s^{N+1} e^{-s}\, ds}{1+s\,x} \leq x^{N+1} (N+1)!$$

Large N $$\approx \sqrt{2\pi N} (xN)^N e^{-N} \sim \sqrt{\frac{2\pi}{x}} e^{-\frac{1}{x}}$$

Minimize

$$x \sim 1/N$$

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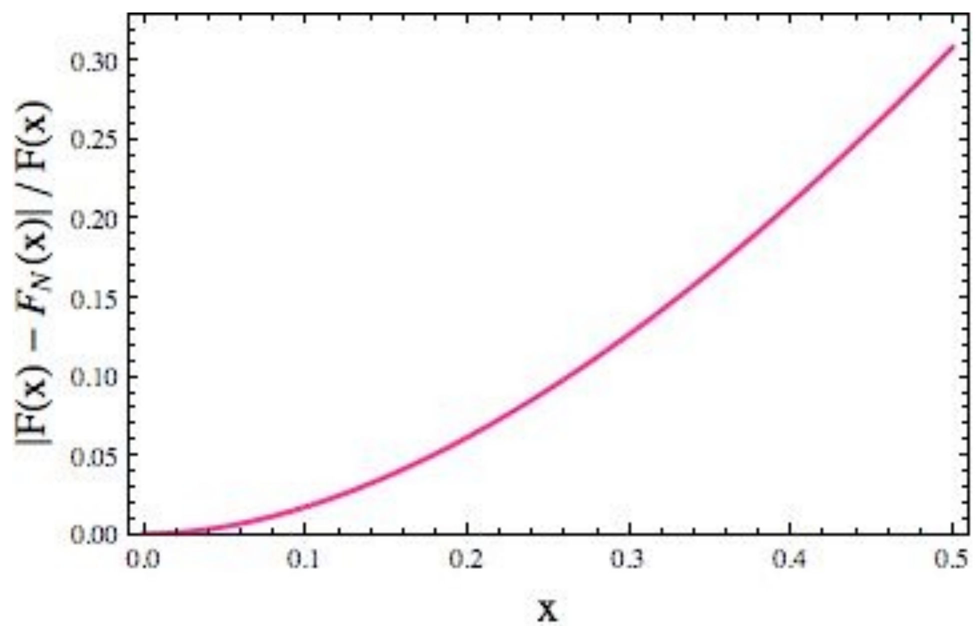

N=1

$$|F(x) - F_N(x)| \lesssim \sqrt{\frac{2\pi}{x}} e^{-\frac{1}{x}}$$

Minimize

$x \sim 1/N$

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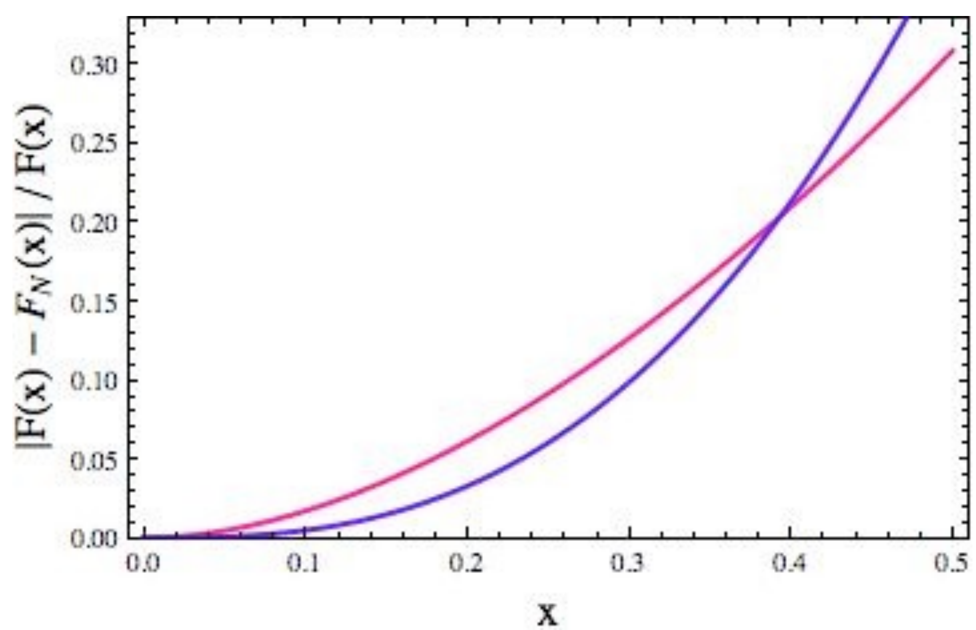

$$|F(x) - F_N(x)| \lesssim \sqrt{\frac{2\pi}{x}} e^{-\frac{1}{x}}$$

Minimize $x \sim 1/N$

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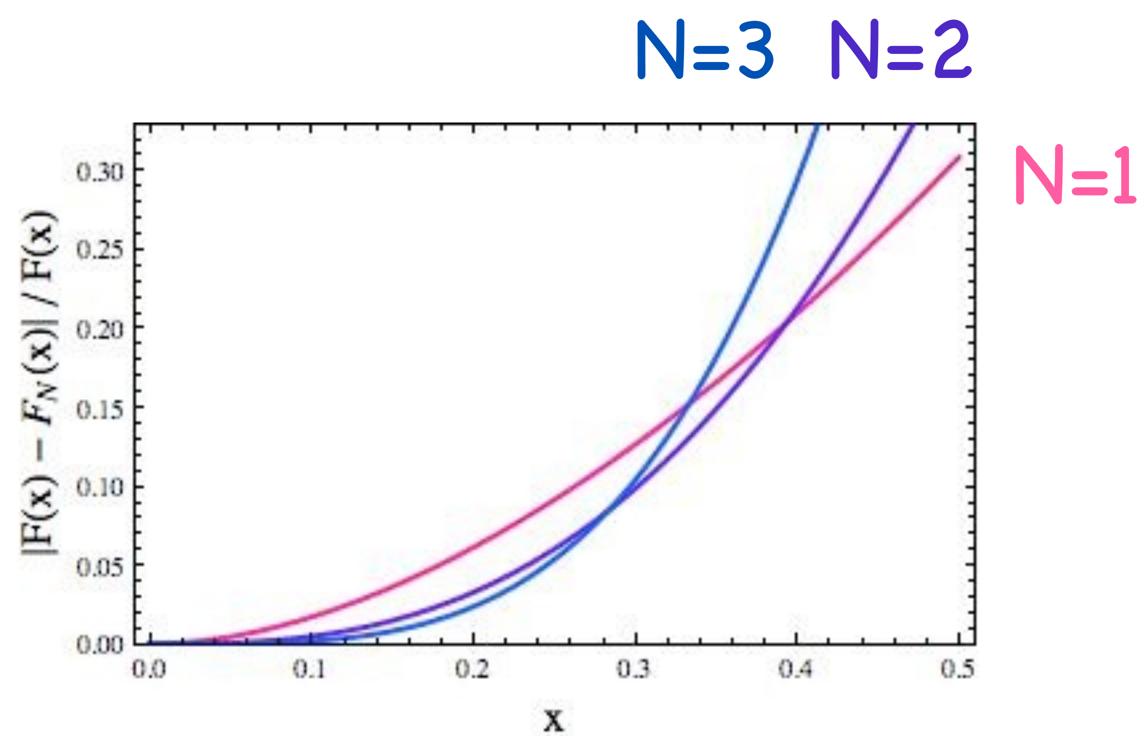

$$|F(x) - F_N(x)| \lesssim \sqrt{\frac{2\pi}{x}} e^{-\frac{1}{x}}$$

Minimize

$x \sim 1/N$

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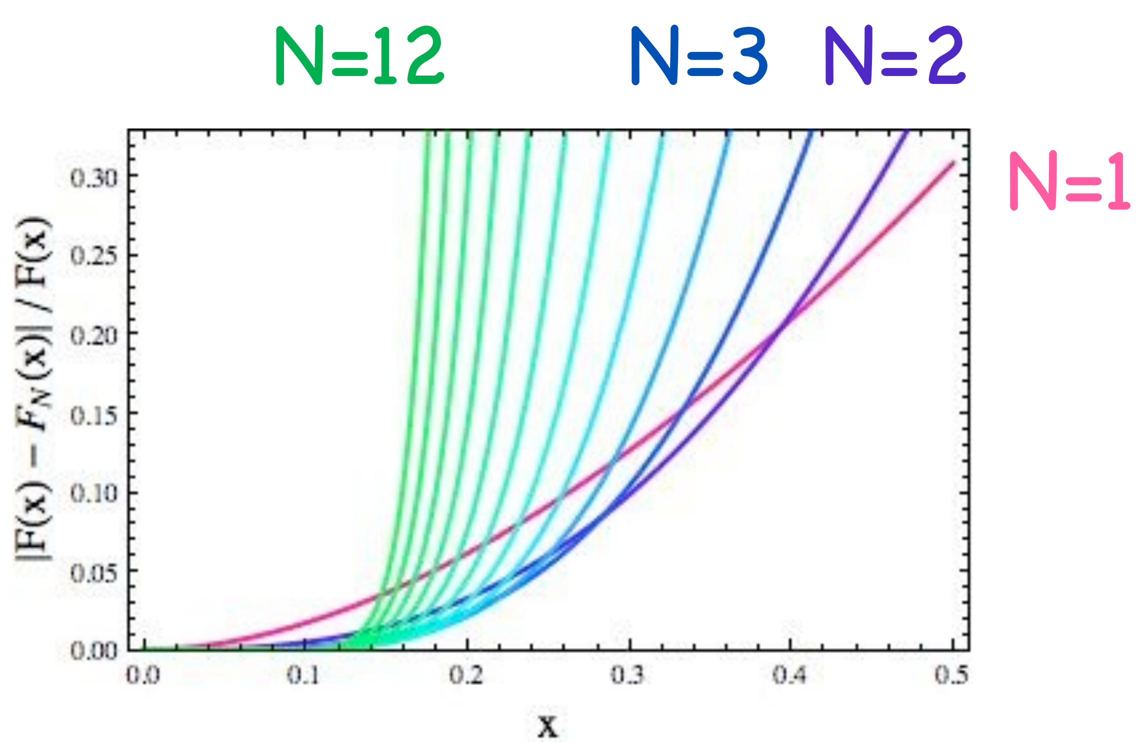
Toy Model $$F(x) = \int_0^\infty ds \frac{e^{-s}}{1+sx} \qquad 0 < x \ll 1$$

No series expansion about x=0

Include more terms: limits to smaller x



Make better at larger x by dropping terms

$$|F(x) - F_N(x)| \lesssim \sqrt{\frac{2\pi}{x}} e^{-\frac{1}{x}}$$

Minimize

$x \sim 1/N$

Reminder: Asymptotic Expansions

- Non-analytic quark mass dependence implies asymptotic expansion (but obviously so: zero radius of convergence)

Chiral expansion $m_\pi^2/\Lambda_\chi^2 \sim 0.02$ (may even be OK for larger pion masses)

$m_\pi^2/m_\rho^2 \sim 0.03$

$m_\pi^2/m_\sigma^2 \sim 0.08$

Heavy nucleon expansion $M \sim 800\,\mathtt{MeV}$ $m_\pi/M \sim 0.2$

Delta resonance contributions $m_\pi/\Delta \sim 0.5$

$\Delta/\Lambda_\chi \sim 0.3$

- Higher orders introduce more parameters (low-energy constants)

- Makes addressing convergence difficult without knowing the chiral limit values of these parameters

Three-Flavor Chiral Limit

$$\mathcal{L}_\psi = \sum_{i=1}^{3} \overline{\psi}_i \left(\not{D} + m_i \right) \psi_i$$

$$m_q/\Lambda_{\mathrm{QCD}} \sim 0.01$$

$$m_s/\Lambda_{\mathrm{QCD}} \sim 0.3$$

Ignore the warning signs

Three-Flavor Chiral Limit

Symmetries and their breaking

$$\mathcal{L}_\psi = \sum_{i=1}^{3} \overline{\psi}_i \not{D} \psi_i + \ldots \qquad \langle \overline{\psi}\psi \rangle = \langle \overline{\psi}_R \psi_L \rangle + \langle \overline{\psi}_L \psi_R \rangle \neq 0$$

$$U(1)_V \otimes SU(3)_L \otimes SU(3)_R \longrightarrow U(1)_V \otimes SU(3)_V$$

$$\Sigma_{ij} \sim \langle \overline{\psi}_{jR} \psi_{iL} \rangle \qquad \Sigma_{ij}(x) = \delta_{ij} + \ldots \qquad SU(3)_L \otimes SU(3)_R / SU(3)_V$$

Goldstone modes (embedded similarly to before)

$$\Sigma = e^{2i\phi/f} \qquad \Sigma \to L\Sigma R^\dagger \qquad \Sigma \to V\Sigma V^\dagger \qquad \phi \to V\phi V^\dagger$$

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

Three-Flavor Chiral Perturbation Theory

Explicit breaking $\overline{\psi}_L \, m \, \psi_R + \overline{\psi}_R \, m \, \psi_L$

$$m = \begin{pmatrix} m_q & & \\ & m_q & \\ & & m_s \end{pmatrix}$$

$$U(1)_V \otimes SU(3)_L \otimes SU(3)_R \longrightarrow U(1)_V \otimes SU(3)_V$$

$\mathcal{O}(p^2)$

$$\mathcal{L}_\chi = \frac{f^2}{8} \mathrm{Tr}\left(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger\right) - \lambda \, \mathrm{Tr}\left(m\Sigma + m\Sigma^\dagger\right)$$

Chiral perturbation theory (constructed similarly to before)

$$\Sigma = e^{2i\phi/f} \qquad \Sigma \to L\Sigma R^\dagger \qquad \Sigma \to V\Sigma V^\dagger \qquad \phi \to V\phi V^\dagger$$

$\mathcal{O}(p^4)$

Seven Gasser-Leutwyler coefficients,
a few more when external fields are included

Exercises

In the strong isospin limit, there are two different quark masses but three meson masses of the pseudoscalar octet. Use the three-flavor chiral Lagrangian to derive the constraint $\Delta_{\text{GMO}} = \frac{4}{3}m_K^2 - m_\eta^2 - \frac{1}{3}m_\pi^2 = 0$, which was originally found by Gell-Mann and Okubo. What happens away from the strong isospin limit?

Revisit electromagnetic mass corrections in three-flavor chiral perturbation theory. Find all leading and next-to-leading order electromagnetic mass operators. Ignoring the up and down quark masses, which octet masses are affected by leading and next-to-leading order operators?

Accounting for strong and electromagnetic isospin breaking to leading order, determine the mass spectrum of the meson octet, and devise a way to compute the quark mass ratios, $m_u/m_d \quad m_d/m_s$, using the experimentally measured masses.

Three-Flavor Chiral Perturbation Theory

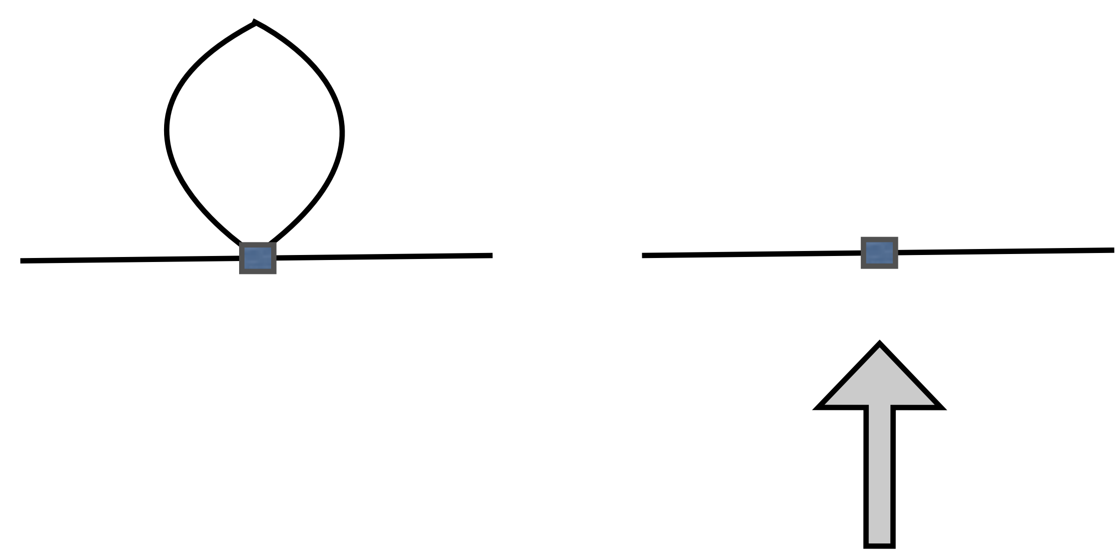
Gell-Mann Okubo mass relation $\Delta_{\text{GMO}} = \frac{4}{3}m_K^2 - m_\eta^2 - \frac{1}{3}m_\pi^2 = 0$

$$m_{\pi^0} = 135.0\,\texttt{MeV}$$
$$m_{K^0} = 497.6\,\texttt{MeV}$$
$$m_\eta = 547.9\,\texttt{MeV}$$

$$\Delta_{\text{GMO}}/\overline{m}_\phi^2 \approx 15\%$$

Next-to-leading order corrections: $\mathcal{O}(p^2)$ one-loop + local terms from $\mathcal{O}(p^4)$

$$0 = \mathcal{O}(p^4) \sim \frac{m_\phi^4}{\overline{m}_\phi^2 \Lambda_\chi^2}$$



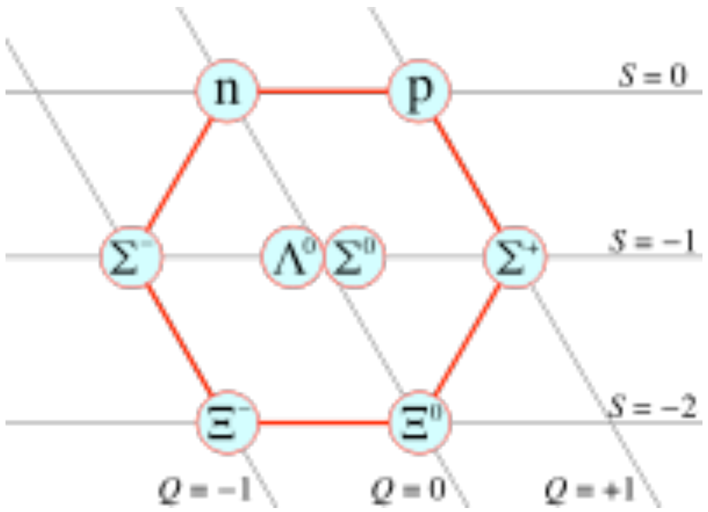
η most worrisome $\sim 35\%$

introduces free parameter

$$m_\pi^2/\Lambda_\chi^2 \sim 0.02$$
$$m_K^2/\Lambda_\chi^2 \sim 0.23$$
$$m_\eta^2/\Lambda_\chi^2 \sim 0.27$$

Pending numerical factors, $\mathcal{O}(p^6)$ contributions (which include two-loop diagrams) should be ~10%

Heavy Baryon Chiral Perturbation Theory



Lowest lying spin-half baryons form an octet of $SU(3)_V$

$$B \to V B V^\dagger$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

Couple to the Goldstone modes via $\xi = \sqrt{\Sigma} = e^{i\phi/f}$

$$\xi \to L \xi U^\dagger = U \xi R^\dagger$$

Free to choose the chiral transformation of baryon octet of the form $B \to U B U^\dagger$

$$\mathcal{A}_\mu \to U \mathcal{A}_\mu U^\dagger$$

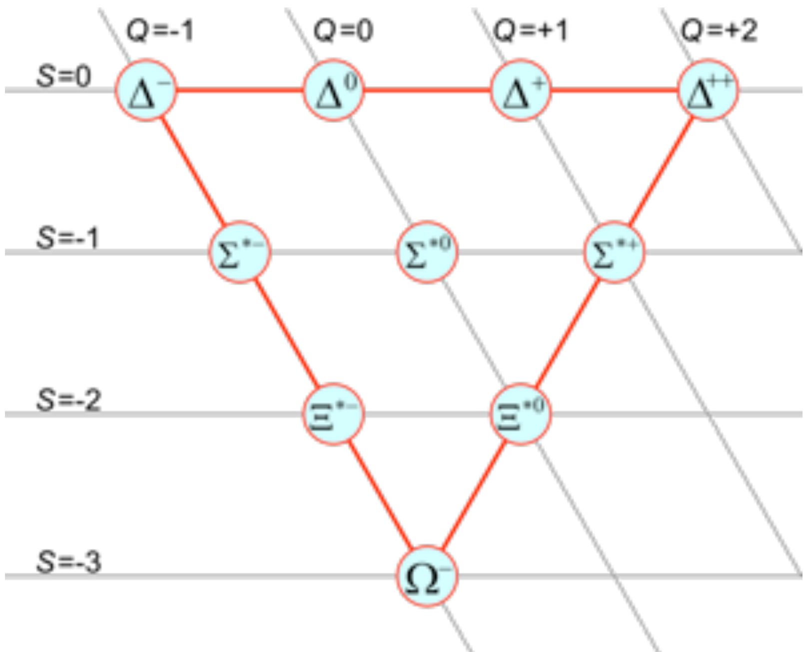
$$\mathcal{V}_\mu \to U \mathcal{V}_\mu U^\dagger + U \partial_\mu U^\dagger$$

$$D_\mu B = \partial_\mu B + [\mathcal{V}_\mu, B]$$

$\mathcal{O}(p)$ $$\mathcal{L} = \mathrm{Tr}\left(\overline{B} iv \cdot D\, B\right) + 2D\,\mathrm{Tr}\left(\overline{B}\vec{S} \cdot \{\vec{A}, B\}\right) + 2F\,\mathrm{Tr}\left(\overline{B}\vec{S} \cdot [\vec{A}, B]\right)$$

Phased away SU(3) chiral limit mass $M_B(m_q = m_s = 0)$

Heavy Baryon Chiral Perturbation Theory

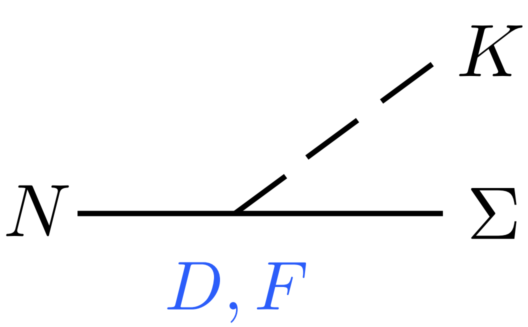


Lowest lying spin three-half baryons form a decuplet of $SU(3)_V$

$$T_{ijk} \to V_i{}^{i'} V_j{}^{j'} V_k{}^{k'} T_{i'j'k'}$$

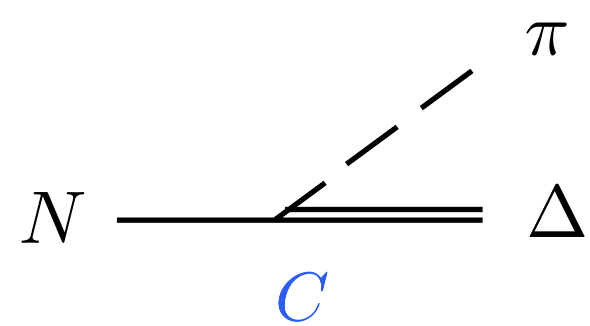
$$\Delta \equiv M_T - M_B = 270\,\mathtt{MeV}$$

No question about inclusion for three flavors



$m_K \sim 500\,\mathtt{MeV}$

$M_\Sigma \sim 1200\,\mathtt{MeV}$



$m_\pi \sim 140\,\mathtt{MeV}$

$M_\Delta \sim 1230\,\mathtt{MeV}$

$\mathcal{O}(p)$ $$\mathcal{L} = \overline{T}_\mu \left(iv \cdot D + \Delta \right) T_\mu + 2H\, \overline{T}_\mu \vec{S} \cdot \vec{A} T_\mu + 2C \left(\overline{T}_\mu A_\mu B + \overline{B} A_\mu T_\mu \right)$$

Quark Mass Dependence of the Octet Baryons

- Now turn on explicit chiral symmetry breaking due to the quark masses

$$\Delta\mathcal{L} = \overline{\psi}_L s\, \psi_R + \overline{\psi}_R s^\dagger \psi_L \qquad s \to L\, s\, R^\dagger \qquad s = m + \dots$$

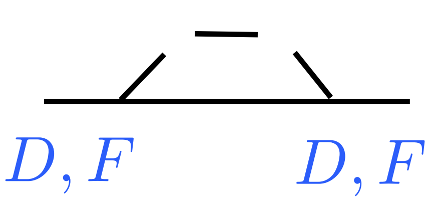
- Dress the scalar source with pions $\mathcal{M}_\pm = \frac{1}{2}\left(\xi s^\dagger \xi \pm \xi^\dagger s\, \xi^\dagger\right) \to U \mathcal{M}_\pm U^\dagger$

$$\mathcal{O}(p^2) \qquad \mathcal{L}_m = b_D \mathrm{Tr}\left(\overline{B}\{\mathcal{M}_+, B\}\right) + b_F \mathrm{Tr}\left(\overline{B}[\mathcal{M}_+, B]\right) + \sigma\, \mathrm{Tr}\left(\overline{B}B\right) \mathrm{Tr}\left(\mathcal{M}_+\right)$$

- Similar Gell-Mann Okubo constraint on octet baryon masses from tree-level

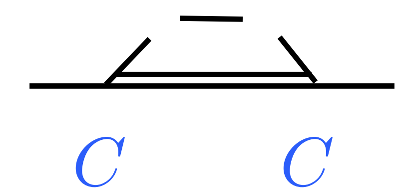
$$M_{\mathrm{GMO}} = M_\Lambda + \frac{1}{3}M_\Sigma - \frac{2}{3}M_N - \frac{2}{3}M_\Xi \stackrel{!}{=} 0 \qquad M_{\mathrm{GMO}}/\overline{M}_B \sim 1\%$$

- One-loop correction predicted in terms of axial couplings C, D, F



$$M_{\mathrm{GMO}} = \frac{4\pi}{3(4\pi f)^2}(D^2 - 3F^2)\left(\frac{4}{3}m_K^3 - m_\eta^3 - \frac{1}{3}m_\pi^3\right) + \text{Decuplet}$$

Anatomy of Decuplet Contribution



- Intermediate-state angular momentum pion p-wave $p^{2\ell}$

$$\sim \frac{C^2}{f^2} \int d^4p \, \frac{\vec{p}^{\,2}}{[ip_4 + \Delta]\,[(p_4)^2 + \vec{p}^{\,2} + m_\pi^2]} \qquad p_4 = \begin{cases} i\Delta \\ \pm iE_\pi \end{cases} \qquad E_\pi = \sqrt{\vec{p}^{\,2} + m_\pi^2}$$

poles

- Contour integration puts pion on shell

$$\sim \frac{C^2}{f^2} \int d\vec{p} \, \frac{\vec{p}^{\,2}}{E_\pi(E_\pi + \Delta)} \sim \frac{C^2}{f^2} \int_{m_\pi}^{\infty} dE_\pi \, \frac{(E_\pi^2 - m_\pi^2)^{3/2}}{E_\pi + \Delta}$$

two-body phase space near threshold $\sqrt{s - m_\pi^2}$

- Divergences

$$\int^{\Lambda} dE\, E^2 \left(1 - \frac{\Delta}{E} + \frac{\Delta^2}{E^2} - \frac{\Delta^3}{E^3} + \ldots\right)\left(1 - \frac{3}{2}\frac{m_\pi^2}{E^2} + \ldots\right)$$

$$\Lambda^3 + \Delta\Lambda^2 + \Delta^2\Lambda + \Delta^3 \log\Lambda$$
$$+ m_\pi^2\Lambda + \Delta m_\pi^2 \log\Lambda + \text{finite}$$

Renormalization condition

$$M_{N,\Sigma,\Lambda,\Xi}\Big|_{m_q=0} = M_B$$

Gell-Mann Okubo Relation to One-Loop

$$M_{\rm GMO} = \frac{4\pi}{3(4\pi f)^2}\left[\pi(D^2 - 3F^2)\Delta_{\rm GMO}(m_\phi^3) - \frac{1}{6}C^2\Delta_{\rm GMO}\big(\mathcal{F}(m_\phi, \Delta, \mu)\big)\right]$$

$$\mathcal{F}(m,\delta,\mu) = (m^2-\delta^2)\left[\sqrt{\delta^2 - m^2}\log\left(\frac{\delta - \sqrt{\delta^2 - m^2 + i\epsilon}}{\delta + \sqrt{\delta^2 - m^2 + i\epsilon}}\right) - \delta\log\frac{m^2}{\mu^2}\right] - \frac{1}{2}\delta m^2 \log\frac{m^2}{\mu^2}$$

Scale dependence?

Chiral limit?

M_{GMO}/M_B	Source	D	F	C
0.79%	ChPT	0.61	0.40	1.2
1.12%	Lattice QCD	0.72	0.45	1.6
1.29%	SU(6)	3/4	1/2	3/2

Experiment $M_{\rm GMO}/\overline{M}_B \sim 1\%$

- BUT: One-loop chiral corrections to the individual masses are LARGE

$$\delta M_N(\mu = \Lambda_\chi)/M_N = -39\%$$
$$\delta M_\Lambda(\mu = \Lambda_\chi)/M_\Lambda = -67\%$$
$$\delta M_\Sigma(\mu = \Lambda_\chi)/M_\Sigma = -89\%$$
$$\delta M_\Xi(\mu = \Lambda_\chi)/M_\Xi = -98\%$$

Expansion stranger with increasing strangeness

Heavy baryons

$$m_\pi/M_B \sim 0.1$$
$$m_K/M_B \sim 0.5$$
$$m_\eta/M_B \sim 0.5$$

Exercise:

Recall the relation between the nucleon sigma term and strangeness.

$$\left(\frac{m_s}{m_q}-1\right)(1-y)\sigma_N = \frac{m_s - m_q}{2M_N}\langle N(\vec{k})|\overline{u}u + \overline{d}d - 2\overline{s}s|N(\vec{k})\rangle$$

Using the baryon chiral Lagrangian at tree level, calculate the matrix element on the right-hand side and express in terms of the octet baryons masses. Finally estimate the size of the sigma term.

Confronting SU(3): the strange quark mass

$SU(3)_L \otimes SU(3)_R$ $\quad m_u, m_d \sim m_s \ll \Lambda_{\rm QCD}$

- Unless you're exceptionally lucky, the strange quark mass is probably too large for the success of SU(3) chiral expansion...

- One approach: integrate out the heavy strange quark mass to use an SU(2) theory

$SU(2)_L \otimes SU(2)_R$ $\quad m_u, m_d \ll m_s \sim \Lambda_{\rm QCD}$

- For the nucleon (and pion) this is just SU(2) chiral perturbation theory. **Done**

- For the nucleon, we treated it as a heavy external flavor source. Nothing stops us from having strangeness in such a source.

... SU(2) chiral perturbation theory for strange hadrons

Limited predictive power, but ideal for lattice applications

Integrating out the strange quark

- Use the kaon mass to exemplify

$$m_K^2 = \frac{4\lambda}{f^2}(m_q + m_s) + \ldots = \frac{1}{2}m_\pi^2 + M_K^2 + \ldots = M_K^2\left(1 + \frac{m_\pi^2}{2M_K^2}\right) + \ldots$$

$$M_K \equiv m_K\Big|_{m_q=0}$$

Estimate using SU(3) and pion, kaon masses

$$M_K = 0.486(5)\,\texttt{GeV} \qquad m_{K^0} = 0.497\,\texttt{GeV}$$

- Consider the SU(3) expansion of the Sigma baryon mass, schematically

$$M_\Sigma = M_B + a m_K^2 + b m_K^3 + \ldots$$

Expand out the strange quark contribution

$$M_\Sigma = M_B + a' M_K^2 + a'' m_\pi^2 + b' M_K^3 + b'' M_K m_\pi^2 + b''' \frac{1}{M_K} m_\pi^4 + \ldots$$

Reorganize into SU(2) chiral limit expansion

$$M_\Sigma = M_\Sigma^{(2)} + \sigma_{\pi\Sigma}\, m_\pi^2 + A m_\pi^3 + B m_\pi^4\left(\log m_\pi^2 + C\right) + \ldots$$

E.g. SU(2) Chiral Perturbation Theory for Hyperons

	$SU(3)$	$SU(2)_{S=0}$	$SU(2)_{S=1}$	$SU(2)_{S=2}$	$SU(2)_{S=3}$
Expansion	$p\ m_\pi\ m_K\ m_\eta\ \Delta$	$p\ m_\pi\ \Delta_{\Delta N}$	$p\ m_\pi\ \Delta_{\Sigma\Lambda}\ \Delta_{\Sigma^*\Sigma}$	$p\ m_\pi\ \Delta_{\Xi^*\Xi}$	$p\ m_\pi$
Multiplets	$\mathbf{8}B\ \mathbf{10}T$	$\mathbf{2}N\ \mathbf{4}\Delta$	$\mathbf{1}\Lambda\ \mathbf{3}\Sigma\ \mathbf{3}\Sigma^*$	$\mathbf{2}\Xi\ \mathbf{2}\Xi^*$	$\mathbf{1}\Omega$
Couplings	$D\ F\ C\ H$	$g_A\ g_{\Delta N}\ g_{\Delta\Delta}$	$g_{\Lambda\Sigma}\ g_{\Sigma\Sigma}\ g_{\Lambda\Sigma^*}\ g_{\Sigma\Sigma^*}\ g_{\Sigma^*\Sigma^*}$	$g_{\Xi\Xi}\ g_{\Xi\Xi^*}$	

$\mathcal{O}(p^3)$

SU(3) expansion at physical pion mass

$\delta M_N(\mu = \Lambda_\chi)/M_N = -39\%$

$\delta M_\Lambda(\mu = \Lambda_\chi)/M_\Lambda = -67\%$

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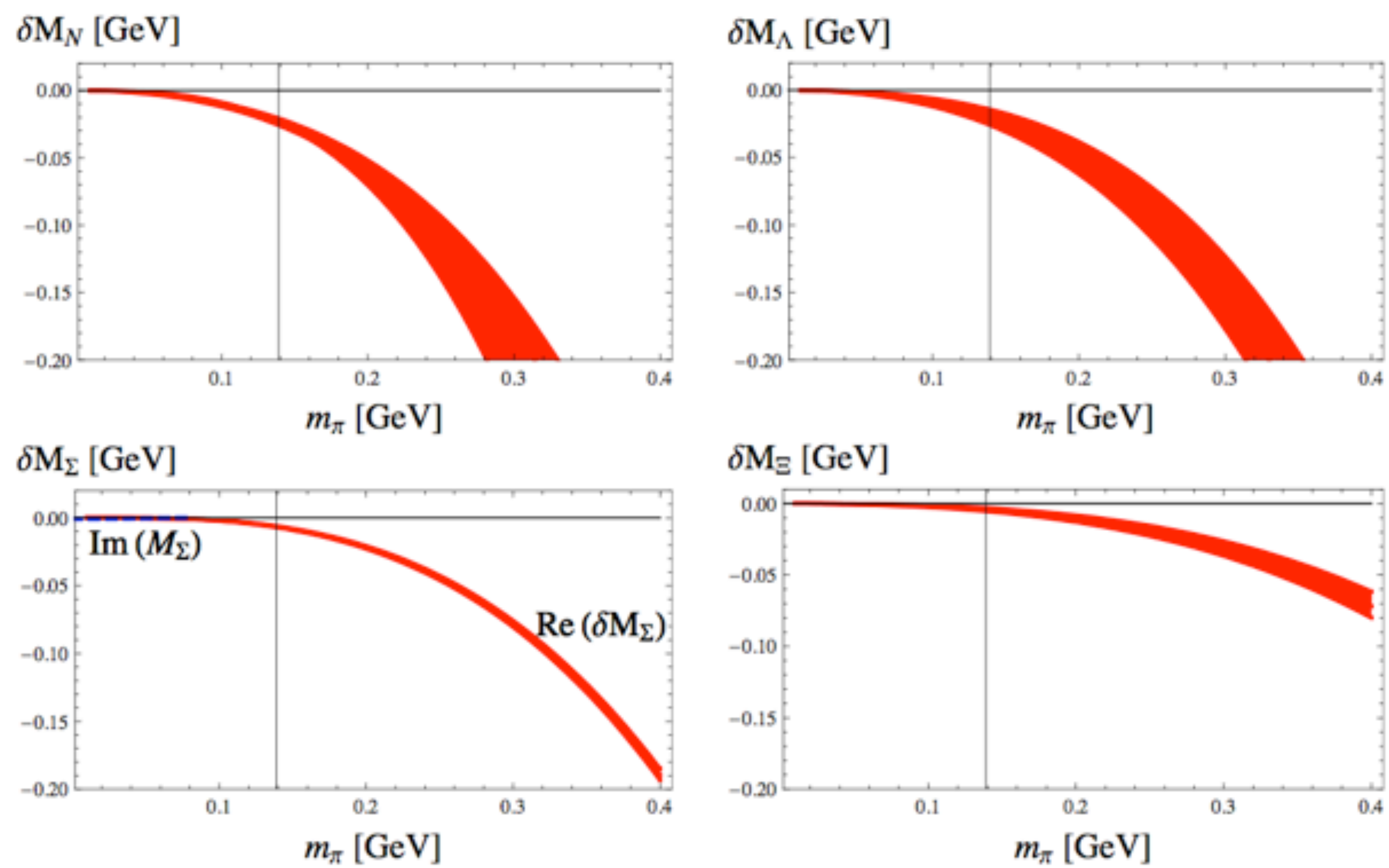


SU(2) chiral expansion $\frac{m_\pi^2}{\Lambda_\chi^2}, \quad \frac{m_\pi^2}{2M_K^2}$ Heavy baryon expansion $\frac{m_\pi}{M^{(2)}}$

Final Exercises:

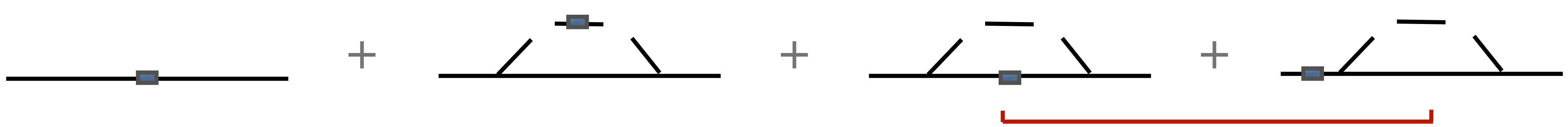
Use SU(2) chiral perturbation theory to construct a low-energy theory of kaons and the eta.

Find a process involving strange baryons for which a description in terms of SU(2) chiral perturbation theory certainly must fail.

Final Topic: Not-so-heavy Heavy Baryons

- Heavy baryons necessary for power counting, but static limit is often severe
- Can treat recoil corrections in perturbation theory, but cannot exactly capture analytic structure (poles & cuts will have approximately the correct locations)
- Heavy baryon approximation can create *unphysical* singularities

Scalar form factor of the Nucleon $\langle N(\vec{p}\,') \,|m_q(\overline{u}u + \overline{d}d)| N(\vec{p}\,) \rangle = \overline{u}(\vec{p}\,')\sigma(t)\, u(\vec{p}\,)$



$$\sigma(t = 2m_\pi^2) - \sigma(t = 0) = \frac{3\pi g_A^2 m_\pi^3}{2(4\pi f)^2} + \mathcal{O}(m_\pi^4)$$

HBChPT result at Cheng-Dashen point

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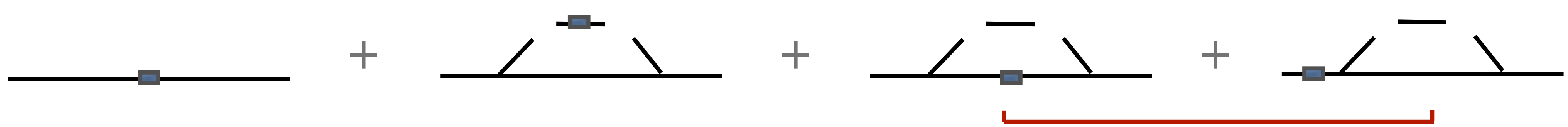

$$\sigma(t) - \sigma(0) = \frac{3\pi g_A^2 m_\pi}{4(4\pi f)^2}\left[(t-2m_\pi^2)\left[\frac{1}{2\sqrt{\tau}}\log\frac{1+\sqrt{\tau}}{1-\sqrt{\tau}} - \log\left(1+\frac{m_\pi}{2M_N\sqrt{1-\tau}}\right)\right] + 2m_\pi^2\left[1-\log\left(1+\frac{m_\pi}{2M_N}\right)\right]\right]$$

Fully relativistic calculation $\quad \tau = \dfrac{t}{4m_\pi^2} \quad$ threshold parameter

Analytic properties allow for dispersive representation

$$\sigma(t) - \sigma(0) = \frac{t}{\pi}\int_{4m_\pi^2}^{\infty} dt' \, \frac{\mathfrak{Im}\,\sigma(t')}{t'(t'-t)}$$

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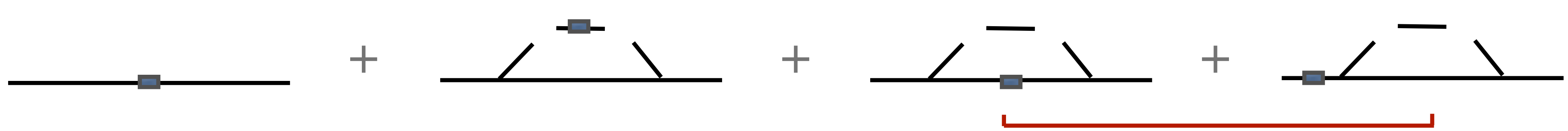

$$\sigma(t) - \sigma(0) = \frac{3\pi g_A^2 m_\pi}{4(4\pi f)^2}\left[(t - 2m_\pi^2)\left[\frac{1}{2\sqrt{\tau}}\log\frac{1+\sqrt{\tau}}{1-\sqrt{\tau}} - \qquad 0 \qquad \right] + 2m_\pi^2\,[1 - \qquad 0 \qquad]\right]$$

$$\tau = \frac{t}{4m_\pi^2}$$ threshold parameter

Expand in m_π / M_N to recover HBChPT result

Cheng-Dashen point

Final Topic: Not-so-heavy Heavy Baryons

- Heavy baryons necessary for power counting, but static limit is often severe
- Can treat recoil corrections in perturbation theory, but cannot exactly capture analytic structure (poles & cuts will have approximately the correct locations)
- Heavy baryon approximation can create *unphysical* singularities

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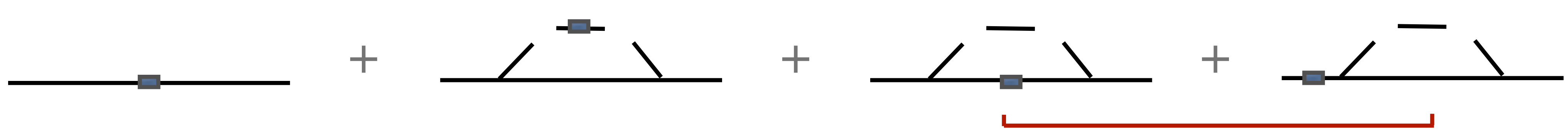

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Singular at two pion threshold $\tau = \dfrac{t}{4m_\pi^2}$

$$\frac{1}{2\sqrt{\tau}}\log\frac{1+\sqrt{\tau}}{1-\sqrt{\tau}} \to -\frac{1}{2}\log(1-\tau)$$

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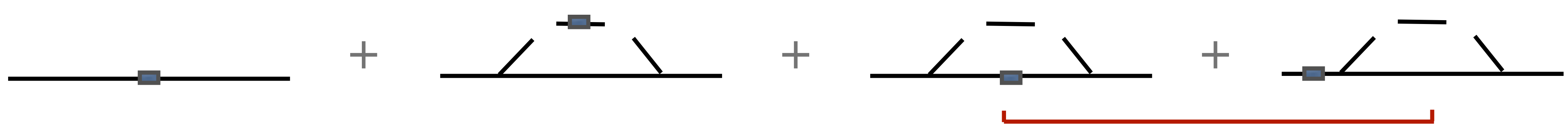

$$\sigma(t) - \sigma(0) = \frac{3\pi g_A^2 m_\pi}{4(4\pi f)^2}\left[(t - 2m_\pi^2)\left[\frac{1}{2\sqrt{\tau}}\log\frac{1+\sqrt{\tau}}{1-\sqrt{\tau}} - \log\left(1 + \frac{m_\pi}{2M_N\sqrt{1-\tau}}\right)\right] + 2m_\pi^2\left[1 - \log\left(1 + \frac{m_\pi}{2M_N}\right)\right]\right]$$

Fully relativistic calculation $\quad \tau = \dfrac{t}{4m_\pi^2}$

No unphysical singularity

$$\frac{1}{2\sqrt{\tau}}\log\frac{1+\sqrt{\tau}}{1-\sqrt{\tau}} \to -\frac{1}{2}\log(1-\tau) \qquad -\log\left(\frac{m_\pi}{2M_N\sqrt{1-\tau}}\right)$$

Final Topic: Not-so-heavy Heavy Baryons

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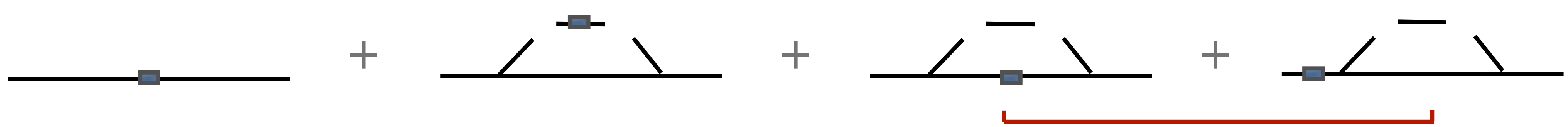

$$\sigma(t)-\sigma(0)=\frac{3\pi g_A^2 m_\pi}{4(4\pi f)^2}\left[(t-2m_\pi^2)\left[\frac{1}{2\sqrt{\tau}}\log\frac{1+\sqrt{\tau}}{1-\sqrt{\tau}}-\log\left(1+\frac{m_\pi}{2M_N\sqrt{1-\tau}}\right)\right]+2m_\pi^2\left[1-\log\left(1+\frac{m_\pi}{2M_N}\right)\right]\right]$$

Fully relativistic calculation $\tau=\dfrac{t}{4m_\pi^2}$

Two pion branch appears

New small parameter near threshold $\dfrac{M_N}{m_\pi}\sqrt{1-\tau}$ necessitates summation of $\dfrac{m_\pi}{M_N}$

Concluding Remarks

- Chiral perturbation theory provides the tool to account for light quark mass dependence of low-energy QCD observables.

- Perturbative expansion limited by size of physical quark masses: strange quark, non-relativistic baryon approximation, *etc*.

- Prior to lattice QCD, chiral perturbation theory was the only way to do precision low-energy QCD phenomenology.

- Lattice methods are testing the rigor of the chiral expansion, and currently the two in conjunction are essential.