INT Summer School on

Lattice QCD for Nuclear Physics

Lectures on Chiral Perturbation Theory

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Chiral Perturbation Theory

III. Baryons

Baryons in Chiral Perturbation Theory

- This is an I***N***T summer school on lattice QCD for ***nuclear*** physics

My Definition: nuclear physics is the study of systems with baryon number > 0

E.g. the hydrogen nucleus

- ChPT is a low-energy effective theory $p^2 \ll \Lambda_\chi^2$

$$\Lambda_\chi \approx 1.1\mathtt{GeV}$$

$$M_N = 0.94\,\mathtt{GeV}$$

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- ChPT is a low-energy effective theory $p^2 \ll \Lambda_\chi^2$

$$\Lambda_\chi \approx 1.1 \texttt{GeV}$$

$$M_N = 0.94\, \texttt{GeV}$$

- Include nucleon as an external flavor source, and describe small energy fluctuations about the nucleon mass

$$p \ll M_N \sim \Lambda_\chi$$

- Account for quark mass dependence, need chiral limit nucleon mass M

Baryons in Chiral Perturbation Theory

- Digression: chiral limit mass out of nothing! $\langle N(\vec{k}\,)|T_{\mu\nu}|N(\vec{k}\,)\rangle = -\frac{k_\mu k_\nu}{M}$

  QCD energy-momentum tensor $T_{\mu\nu}$ — trace

  $$T_{\mu\mu} = m_q \overline{\psi}\psi \qquad \langle N(\vec{k}\,)|T_{\mu\mu}|N(\vec{k}\,)\rangle = M$$

  Classically $M = 0$

- Trace anomaly (QCD cannot be defined without a scale)

$$T_{\mu\mu} = \frac{\beta}{2g} G_{\mu\nu} G_{\mu\nu} + m_q \overline{\psi}\psi$$

$$M_N = \left\langle N(\vec{k}\,) \left| \frac{\beta}{2g} G_{\mu\nu} G_{\mu\nu} + m_q \overline{\psi}\psi \right| N(\vec{k}\,) \right\rangle \overset{?}{=} M + A m_q + B m_q^2 + \ldots$$

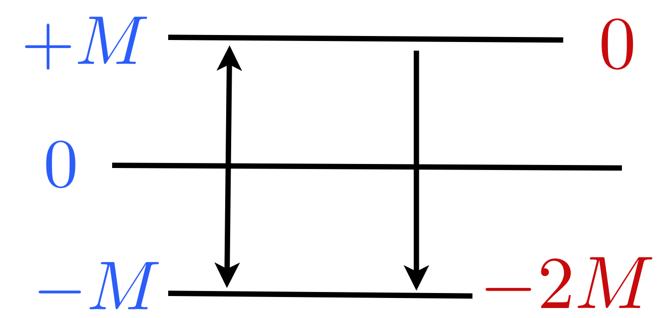
$$= M + \mathcal{A} m_\pi^2 + \mathcal{B} m_\pi^4 + \ldots$$

- Higgs doesn't have a monopoly over all masses in the universe

Exercise:

Is the trace of the energy-momentum tensor the divergence of a current?

Heavy Fermions



- Large (chiral limit) mass M: schematically $\Delta k\,\Delta x \geq \frac{1}{2}$ $\quad \Delta v\,\Delta x \geq \frac{1}{2M}$

$k_\mu = M v_\mu + p_\mu$ Separation of scales $e^{ik\cdot x} = e^{iMv\cdot x} e^{ip\cdot x}$

$$\mathcal{L} = \overline{N}\left(\not{\partial} + M\right) N$$

- Fermion propagator $\dfrac{1}{i\not{k} + M} = \dfrac{-i\not{k} + M}{k^2 + M^2} = \dfrac{-iM\not{v} - i\not{p} + M}{2Mv\cdot p + p^2} \to \mathcal{P}_+ \dfrac{1}{v\cdot p} + \mathcal{O}\left(\dfrac{p}{M}\right)$

in Feynman diagrams

Non-relativistic spin projector $\mathcal{P}_\pm = \frac{1}{2}\left(1 \mp i\not{v}\right)$

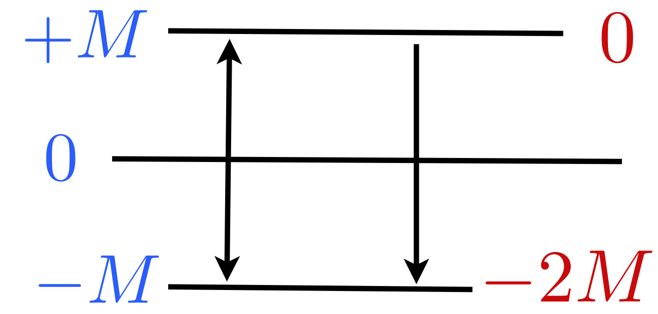
$$-\mathcal{P}_\pm \gamma_\mu \mathcal{P}_\pm = -\frac{1}{4}\left(\gamma_\mu \mp i\{\not{v}, \gamma_\mu\} - \not{v}\gamma_\mu\not{v}\right) = \pm i v_\mu \mathcal{P}_\pm$$

- Reformulate using heavy fermion fields $N(x) = e^{iMv\cdot x}\left[\mathcal{P}_+ N_v(x) + \mathcal{P}_- \mathfrak{N}_v(x)\right]$

$$\partial_\mu N(x) = iMv_\mu N(x) + e^{iMv\cdot x}\partial_\mu\left[\mathcal{P}_+ N_v(x) + \mathcal{P}_- \mathfrak{N}_v(x)\right]$$

$$\mathcal{L} = \left[\overline{N}_v \mathcal{P}_+ + \overline{\mathfrak{N}}_v \mathcal{P}_-\right] \not{\partial} \left[\mathcal{P}_+ N_v + \mathcal{P}_- \mathfrak{N}_v\right] + \overline{N}\left(-iM\not{v} + M\right) N$$

Heavy Fermions



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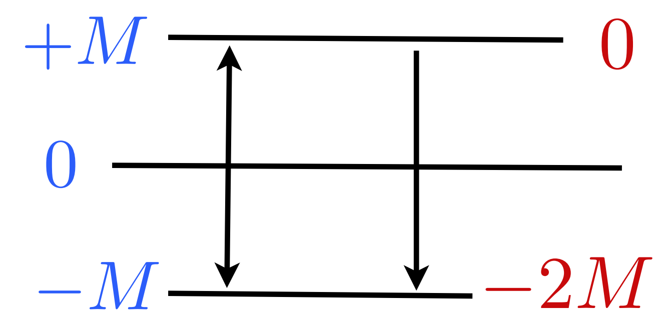
Heavy Fermions



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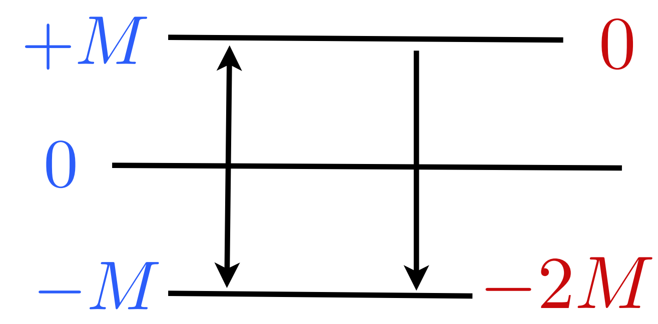
Heavy Fermions



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$$\mathcal{L} = \overline{N}_v iv\cdot\partial\,\mathcal{P}_+ N_v - \overline{\mathfrak{N}}_v\left(iv\cdot\partial - 2M\right)\mathcal{P}_- \mathfrak{N}_v + \text{mixing}$$

Exercises:

Integrate out the remaining heavy components of the fermion field to find the first-order correction to the static fermion Lagrangian. The result should not surprise you.

The non-relativistic projectors reduce the spin algebra to that of Pauli matrices. Show that the axial-vector fermion bilinear reduces to the spin density operator up to a constant, i.e. $\overline{N}_v \gamma_\mu \gamma_5 N_v = c\, \overline{N}_v S_\mu N_v$. The relation $\mathcal{P}_+ N_v = N_v$ will prove useful, as will the definition of the spin vector $S_\mu = -i\varepsilon_{\mu\nu\rho\sigma}\sigma_{\nu\rho}k_\sigma/4M$ which satisfies $S_\mu S_\mu = 1/2\,(1/2+1)$. What are $v_\mu S_\mu$ and $[S_\mu, S_\nu]$?

Heavy Nucleon Chiral Perturbation Theory

- Large chiral limit mass phased away: derivative expansion is now valid

$$\mathcal{L} = \overline{N}_v \, iv \cdot \partial \, \mathcal{P}_+ N_v \qquad\qquad \partial_\mu N_v(x) \sim p_\mu , \qquad p_\mu \ll M \sim \Lambda_\chi$$

- Combine heavy nucleon limit with chiral perturbation theory:
  quark mass dependence of nucleon properties, pion-nucleon interactions...

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \qquad N_i \xrightarrow{SU(2)_V} V_{ij} N_j$$

$$\Sigma = e^{2i\phi/f} \qquad \Sigma \xrightarrow{SU(2)_L \otimes SU(2)_R} L \Sigma R^\dagger$$

- Actually it's unknown to which chiral multiplet(s) the nucleon belongs

Assume simple $\quad N_R \to R N_R \quad N_L \to L N_L$

Dressed nucleon $\quad \tilde{N}_R = \Sigma^\dagger N_L \quad \tilde{N}_L = \Sigma N_R$

- Exploit arbitrary nature for simplicity $\quad \xi = \sqrt{\Sigma} = e^{i\phi/f}$

Heavy Nucleon Chiral Perturbation Theory

Seems complicated: $\xi \to \sqrt{L\xi^2 R^\dagger} \equiv L\xi U^\dagger$ $\quad U = U\left(L, R, \xi(x)\right)$

Vector subgroup $\quad U(L = R = V, \xi) = V \quad \xi \to V\xi V^\dagger$

$$\Sigma \to L\xi^2 R^\dagger$$
$$\xi^2 \to L\xi U^\dagger L\xi U^\dagger$$
$$\Longrightarrow \quad L\xi U^\dagger = U\xi R^\dagger$$

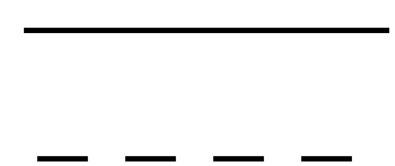
Differently dressed nucleon $\quad \breve{N}_R = \xi^\dagger N_L \quad \breve{N}_L = \xi N_R$

$$\breve{N}_R \to U\breve{N}_R \quad \breve{N}_L \to U\breve{N}_L$$

Dressed differently, chiral components transform the same way

Assume simple $\quad N_R \to RN_R \quad N_L \to LN_L$

Dressed nucleon $\quad \tilde{N}_R = \Sigma^\dagger N_L \quad \tilde{N}_L = \Sigma N_R$



N

πN

- Exploit arbitrary nature for simplicity $\quad \xi = \sqrt{\Sigma} = e^{i\phi/f}$

Heavy Nucleon Chiral Perturbation Theory

- Actually it's unknown to which chiral multiplet(s) the nucleon belongs

1). Within a given chiral multiplet, nucleon field is not unique

2). Invent nucleon field with chiral components transforming the same

$$\xi \to L\xi U^\dagger = U\xi R^\dagger$$

3). Need not know unknown $N_i \to U_{ij} N_j$ (meets known $N_i \to V_{ij} N_j$)

4). Construct heavy nucleon chiral Lagrangian based on symmetry

$$\xi^\dagger \partial_\mu \xi \to U\xi^\dagger L^\dagger \partial_\mu \left(L\xi U^\dagger \right) = U\xi^\dagger \partial_\mu \xi U^\dagger + U\partial_\mu U^\dagger$$

$$\mathcal{A}_\mu = \frac{i}{2}\left(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger\right) \quad \Longrightarrow \quad \mathcal{A}_\mu \to U\mathcal{A}_\mu U^\dagger$$

$$\mathcal{V}_\mu = \frac{1}{2}\left(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger\right) \quad \Longrightarrow \quad \mathcal{V}_\mu \to U\mathcal{V}_\mu U^\dagger + U\partial_\mu U^\dagger$$

$$D_\mu N \equiv \partial_\mu N + \mathcal{V}_\mu N \qquad D_\mu N \to U(D_\mu N)$$

Heavy Nucleon Chiral Lagrangian

$$\mathcal{L} = N^\dagger iv \cdot D\, N + 2g_A N^\dagger \vec{S} \cdot \vec{\mathcal{A}}\, N$$

$$D_\mu N = \partial_\mu N + \mathcal{V}_\mu N$$

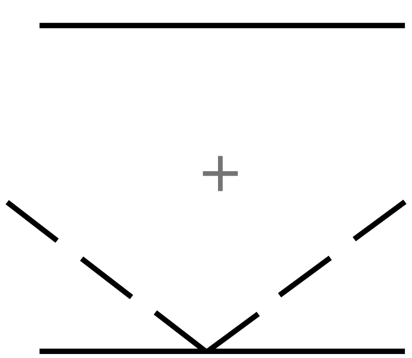
$$\mathcal{V}_\mu = \frac{1}{2}\left(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger\right)$$

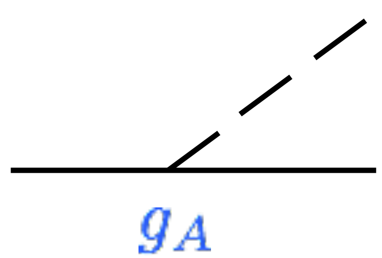
Two more invariants:

$$(N^\dagger v_\mu N)\, \mathrm{Tr}(\mathcal{V}_\mu)$$

$$(N^\dagger \vec{S} N) \cdot \mathrm{Tr}(\vec{\mathcal{A}})$$

$$\mathcal{A}_\mu = \frac{i}{2}\left(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger\right)$$





+ ...

Axial coupling free parameter in chiral limit (depends upon chiral multiplet)

Vector coupling exactly fixed by pattern of chiral symmetry breaking

Heavy Nucleon Chiral Lagrangian

$$\mathcal{L} = N^\dagger iv\cdot D\,N + 2g_A N^\dagger \vec{S}\cdot\vec{\mathcal{A}}\,N$$

$$\mathcal{V}_\mu = \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right) \qquad \mathcal{A}_\mu = \frac{i}{2}\left(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\right)$$

- Include external vector and axial-vector fields: local chiral transformation

$$\xi^\dagger\partial_\mu\xi \to U\xi^\dagger L^\dagger\partial_\mu\left(L\xi U^\dagger\right) = U\xi^\dagger(L^\dagger\partial_\mu L)\xi U^\dagger + U\xi^\dagger\partial_\mu\xi U^\dagger + U\partial_\mu U^\dagger$$

$$L_\mu \to LL_\mu L^\dagger + i(\partial_\mu L)L^\dagger \qquad \xi^\dagger D_{L\mu}\xi = \xi^\dagger(\partial_\mu + iL_\mu)\xi$$

- Vector and axial-vector pion fields become gauged

$$\mathcal{V}_\mu = \frac{1}{2}\left(\xi^\dagger D_{L\mu}\xi + \xi D_{R\mu}\xi^\dagger\right) \qquad \mathcal{A}_\mu = \frac{i}{2}\left(\xi^\dagger D_{L\mu}\xi - \xi D_{R\mu}\xi^\dagger\right)$$

- Singlet couplings can be turned on externally

$$\mathrm{Tr}\left(\mathcal{V}_\mu\right) = \frac{i}{2}\mathrm{Tr}\left(L_\mu + R_\mu\right) = i\mathrm{Tr}\left(V_\mu\right) \qquad \mathrm{Tr}\left(\mathcal{A}_\mu\right) = -\frac{1}{2}\mathrm{Tr}\left(L_\mu - R_\mu\right) = \mathrm{Tr}\left(A_\mu\right)$$

Singlet vector coupling exactly fixed by electric charge assignments

$$D_\mu N = \left[\partial_\mu + \mathcal{V}_\mu + \mathrm{Tr}(\mathcal{V}_\mu)\right]N$$

Quark Mass Dependence of the Nucleon

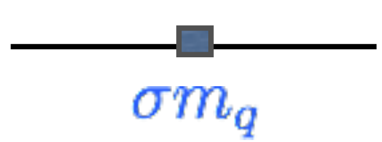
- Now turn on explicit chiral symmetry breaking due to the quark mass

$$\Delta\mathcal{L} = \overline{\psi}_L s\,\psi_R + \overline{\psi}_R s^\dagger \psi_L \qquad s \to L\,s\,R^\dagger \qquad s = m_q + \cdots$$

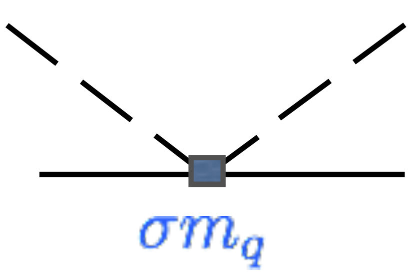
- Dress the scalar source with pions $\quad \xi \to L\xi U^\dagger = U\xi R^\dagger \qquad N \to UN$

$$\mathcal{M}_\pm = \frac{1}{2}\left(\xi s^\dagger \xi \pm \xi^\dagger s\,\xi^\dagger\right) \to U\mathcal{M}_\pm U^\dagger \qquad \mathcal{M}_\pm = m_q\left(\Sigma \pm \Sigma^\dagger\right) + \cdots$$

Leading quark mass dependence



$$\mathcal{L}_{m_q} = \sigma N^\dagger \mathcal{M}_+ N + \mathcal{O}(m_q^2)$$



$$M_N = M + \sigma m_q + \ldots$$

Recall $\quad M_N = \left\langle N(\vec{k}\,) \left| \frac{\beta}{2g} G_{\mu\nu}G_{\mu\nu} + m_q \overline{\psi}\psi \right| N(\vec{k}\,) \right\rangle$

Aside: The Pion-Nucleon Sigma Term

$$\sigma_N \equiv \frac{1}{2M_N}\langle N(\vec{k})|\, m_q\overline{\psi}\psi \,|N(\vec{k})\,\rangle$$

$$m_q = \frac{1}{2}(m_u + m_d) \quad \overline{\psi}\psi = \overline{u}u + \overline{d}d$$

- Leading-order result

$$\sigma_N = \frac{\sigma m_q}{2M_N} + \dots$$

- Sigma term relevant for: mass spectrum, strangeness content, quark mass ratios, pion-nucleon scattering, new physics searches, ...

$$\sigma_N = \frac{m_q}{2M_N}\frac{\partial M_N}{\partial m_q}$$

mass spectrum

$$y = \frac{\langle N(\vec{k})\,|\,\overline{s}s\,|\,N(\vec{k})\,\rangle}{\frac{1}{2}\langle N(\vec{k})\,\big|\,\overline{u}u + \overline{d}d\,\big|\,N(\vec{k})\,\rangle}$$

strangeness content

quark mass ratio

$$\left(\frac{m_s}{m_q} - 1\right)(1-y)\sigma_N = \frac{m_s - m_q}{2M_N}\langle N(\vec{k})|\overline{u}u + \overline{d}d - 2\overline{s}s|N(\vec{k})\rangle$$

Next lecture will be strange

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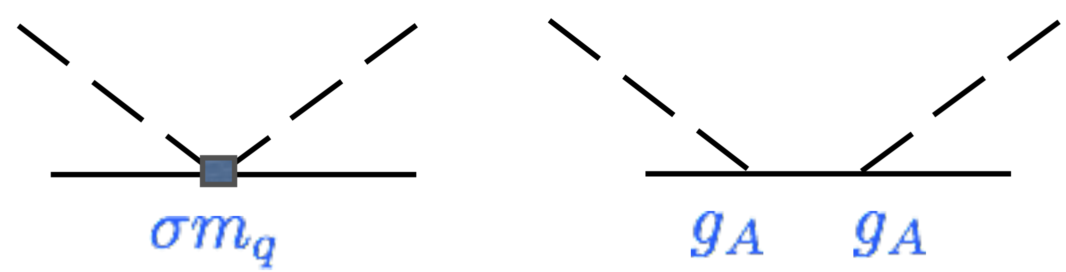
Low-Energy Theorem (Cheng-Dashen)

pion-nucleon scattering



$$t = (k' - k)^2$$

$$D^{I=0}(\nu = 0, t = 2m_\pi^2) - \mathrm{Born} = \frac{2\sigma_N}{f^2} + \ldots$$ (large corrections)

$$2M_N\sigma_N = $$

$$= \frac{2}{f^2}\left[\sigma_N(t = 2m_\pi^2) + \Delta_R\right]$$

$45(8)\,\texttt{MeV}\,[1990\text{'s}]$ $64(7)\,\texttt{MeV}\,[2000\text{'s}]$ $39(4)\,\texttt{MeV}\,[2010\text{'s}]$

(experiment coupled with ChPT analysis) (BMW nucleon specturm)

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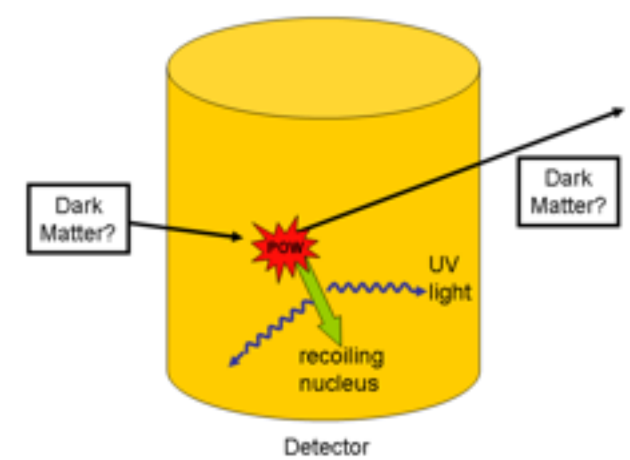
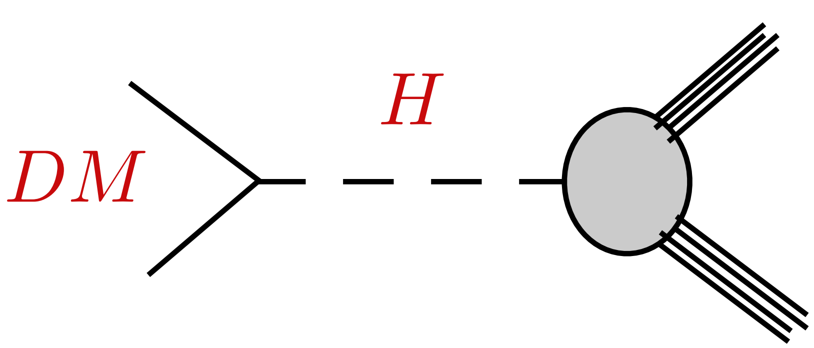
Dark matter direct detection

Dark matter-nucleus elastic scattering from, e.g., $\overline{\chi}\chi\,\overline{\psi}\psi$



Higgs mediated processes $m_Q\overline{Q}Q\,H$



$m_c\bar{c}c$

$m_b\bar{b}b$

$m_t\bar{t}t$

Higgs coupling

scalar density

pQCD

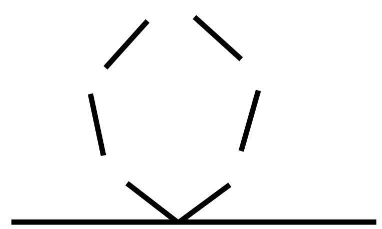
e.g. liquid Xe

$$\sim 80\,\texttt{MeV} \times \left[1 - 2\sigma_N\left(1 + \int_0^{m_s/m_q} y(x)\,dx\right)\right]$$

Anatomy of One-Loop Computations

- Chiral expansion of nucleon mass $M_N = M + \sigma m_q + \ldots$ $\mathcal{O}(p^2)$
- Power counting one-loop diagrams

$\mathcal{O}(p)$ $\mathcal{L} = N^\dagger iv \cdot D\, N + 2g_A N^\dagger \vec{S} \cdot \vec{A}\, N$

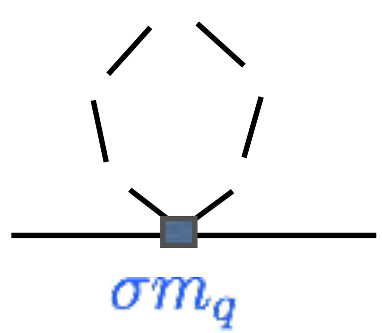


$$\sim \int d^4p\, \frac{1}{p^2}\, p = p^3$$

Odd powers!

$\mathcal{O}(p^2)$ $\mathcal{L}_{m_q} = \sigma N^\dagger \mathcal{M}_+ N$

Form all one-loop diagrams from leading-order vertices



$$\sim p^2 \int d^4p\, \frac{1}{p^2} = p^4$$

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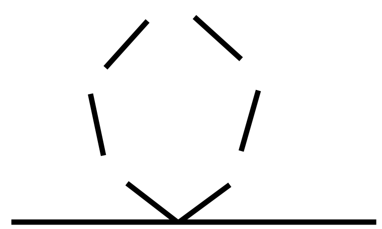
- Power counting one-loop diagrams

$\mathcal{O}(p)$ $\mathcal{L} = N^\dagger iv \cdot D\, N + 2g_A N^\dagger \vec{S} \cdot \vec{\mathcal{A}}\, N$

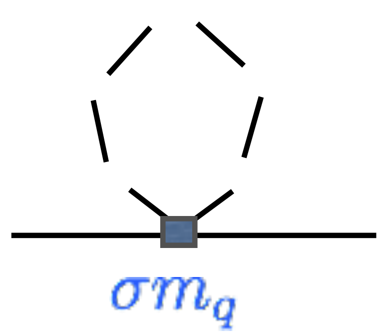


$$\sim \int d^4p\, \frac{v \cdot p}{p^2 + m_\pi^2} = 0$$

Odd powers!

$\mathcal{O}(p^2)$ $\mathcal{L}_{m_q} = \sigma N^\dagger \mathcal{M}_+ N$

Form all one-loop diagrams from leading-order vertices



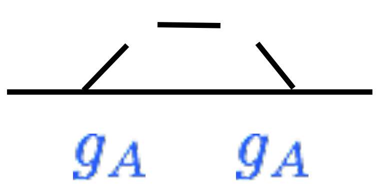
$$\sim p^2 \int d^4p\, \frac{1}{p^2} = p^4$$

Anatomy of One-Loop Computations

- Chiral expansion of nucleon mass $M_N = M + \sigma m_q + \ldots$ $\mathcal{O}(p^2)$

- Power counting one-loop diagrams

$\mathcal{O}(p)$ $\mathcal{L} = N^\dagger iv \cdot D\, N + 2g_A N^\dagger \vec{S} \cdot \vec{A}\, N$



$$\sim \int d^4p\; p\, \frac{1}{p^2}\, \frac{1}{p}\, p = p^3$$

Odd powers!

$$\sim \frac{g_A^2}{f^2} \int d^4p\, \frac{p \cdot S\;\; p \cdot S}{p \cdot v\, (p^2 + m_\pi^2)}$$

Form all one-loop diagrams from leading-order vertices

$$= 0$$

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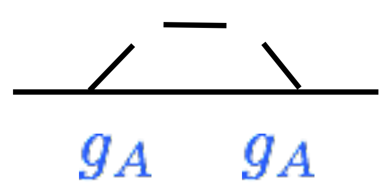

$$\sim \int d^4p\; p\,\frac{1}{p^2}\,\frac{1}{p}\,p = p^3$$

Odd powers!

$$\sim \frac{g_A^2}{f^2}\int d^4p\,\frac{p\cdot S\;\;p\cdot S}{p\cdot v\,(p^2+m_\pi^2)}$$

Form all one-loop diagrams from leading-order vertices

$$D_N(x,0) = \mathcal{P}_+ e^{-M\tau}\delta(\vec{x}\,)\theta(\tau)$$

forward propagating heavy nucleon

Rest frame $v_\mu = (0,0,0,i)$

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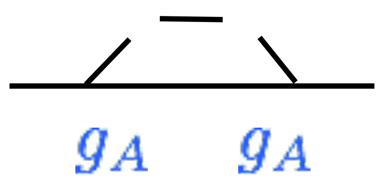

$$\sim \int d^4p\, p\, \frac{1}{p^2}\, \frac{1}{p}\, p = p^3$$

Odd powers!

$$\sim \frac{g_A^2}{f^2} \int d^4p\, \frac{p \cdot S\;\; p \cdot S}{p \cdot v\, (p^2 + m_\pi^2)}$$

Form all one-loop diagrams from leading-order vertices

$$\theta(\tau) = \int \frac{dp_4}{2\pi i} \frac{e^{ip_4\tau}}{p_4 - i\epsilon}$$

forward propagating heavy nucleon

Rest frame $v_\mu = (0, 0, 0, i)$

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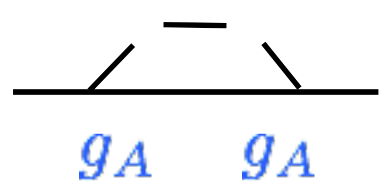

$$\sim \int d^4p\; p\,\frac{1}{p^2}\,\frac{1}{p}\,p = p^3$$

Odd powers!

$$\sim \frac{g_A^2}{f^2}\int d^4p\,\frac{p\cdot S\;\, p\cdot S}{p\cdot v\,(p^2+m_\pi^2)}$$

Form all one-loop diagrams from leading-order vertices

$$\frac{1}{p\cdot v} = \frac{-i}{p_4 - i\epsilon} = -i\,\mathrm{PV} + \pi\delta(p_4)$$

Rest frame $v_\mu = (0,0,0,i)$

Anatomy of One-Loop Computations

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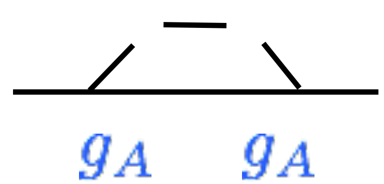

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Form all one-loop diagrams from leading-order vertices

$$\sim \int d\vec{p}\, \frac{\vec{p} \cdot \vec{\sigma}\;\; \vec{p} \cdot \vec{\sigma}}{\vec{p}^{\,2} + m_\pi^2} = \int d\vec{p}\, \frac{\vec{p}^{\,2}}{\vec{p}^{\,2} + m_\pi^2} = \int d\vec{p} \;-\; m_\pi^2 \int d\vec{p}\, \frac{1}{\vec{p}^{\,2} + m_\pi^2}$$

Λ^3 $\qquad m_\pi^2(\Lambda + \text{finite})$

Rest frame $v_\mu = (0, 0, 0, i)$

Anatomy of One-Loop Computations

- Chiral expansion of nucleon mass $M_N = M + \sigma m_q + \ldots$ $\mathcal{O}(p^2)$
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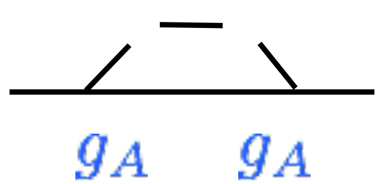


$$\sim \int d^4p\; p\, \frac{1}{p^2}\, \frac{1}{p}\, p = p^3$$

Odd powers!

$$\sim \frac{g_A^2}{f^2} \int d^4p\, \frac{p \cdot S \;\; p \cdot S}{p \cdot v\, (p^2 + m_\pi^2)}$$

Form all one-loop diagrams from leading-order vertices

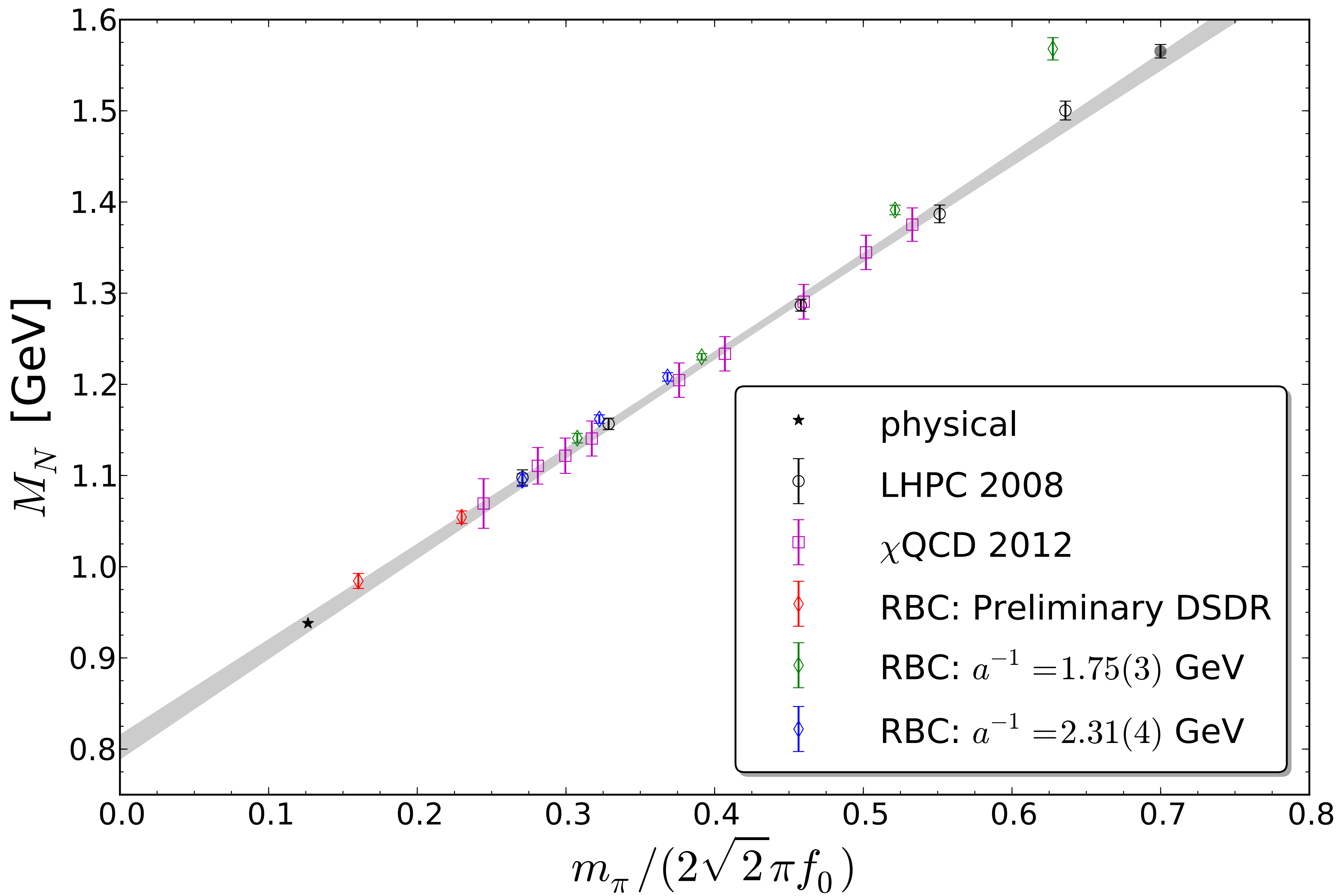
$$\sim \int d\vec{p}\, \frac{\vec{p}\cdot\vec{\sigma} \;\; \vec{p}\cdot\vec{\sigma}}{\vec{p}^{\,2} + m_\pi^2} = \int d\vec{p}\, \frac{\vec{p}^{\,2}}{\vec{p}^{\,2} + m_\pi^2} = \int d\vec{p} \; - m_\pi^2 \int d\vec{p}\, \frac{1}{\vec{p}^{\,2} + m_\pi^2}$$

Λ^3 $\quad m_\pi^2(\Lambda + \text{finite})$

Or just use dimensional regularization $= m_\pi^3$

Chiral Expansion of Nucleon Mass

$$M_N = M + \mathcal{A}\, m_\pi^2 - \frac{3\pi g_A^2}{(4\pi f)^2} m_\pi^3 + \mathcal{B}\, m_\pi^4 \left(\log \frac{\mu^2}{m_\pi^2} + C \right) + \ldots$$



Compilation courtesy of A. Walker-Loud (Chiral Dynamics 2012)

Chiral Expansion of Nucleon Properties

- Can compute chiral corrections to matrix elements of various currents. For example: quark bilinears $\overline{\psi}\,\Gamma\,\psi$, four quark operators $\left(\overline{\psi}\,\Gamma_1\,\psi\right)\left(\overline{\psi}\,\Gamma_2\,\psi\right)$.

Isovector axial current $\langle N(\vec{p}\,')|J^{+}_{5\mu}|N(\vec{p})\rangle = u'^{\dagger}\left[2S_\mu G_A(q^2) + q_\mu S\cdot q\, G_P(q^2)\right] u$

$$G_A = g_A + A m_\pi^2 \left(\log m_\pi^2 + B\right) + \dots$$

$$< r_A^2 >= r^2 + A\, m_\pi^2 \left(\log m_\pi^2 + B\right) + \dots$$

$$G_P(q^2) = \frac{g_A}{q^2 + m_\pi^2} - \frac{< r_A^2 >}{3} + \mathcal{O}(m_\pi^2)$$

Form factors and radii

$$\mathcal{G}(q^2) = \mathcal{G}(0) + \frac{1}{6} q^2 < r^2 > + \dots$$

Isovector EM current $\langle N(\vec{p}\,')|J^{+}_{\mu}|N(\vec{p})\rangle = u'^{\dagger}\left[v_\mu G_E(q^2) + \frac{\varepsilon_{ijk} q_j \sigma_k}{2M_N} G_M(q^2)\right] u$

$$< r_E^2 >= A\left(\log m_\pi^2 + B\right) + \mathcal{O}(m_\pi)$$

$$\mu = \mu_0 + A m_\pi + B\, m_\pi^2 (\log m_\pi^2 + C)$$

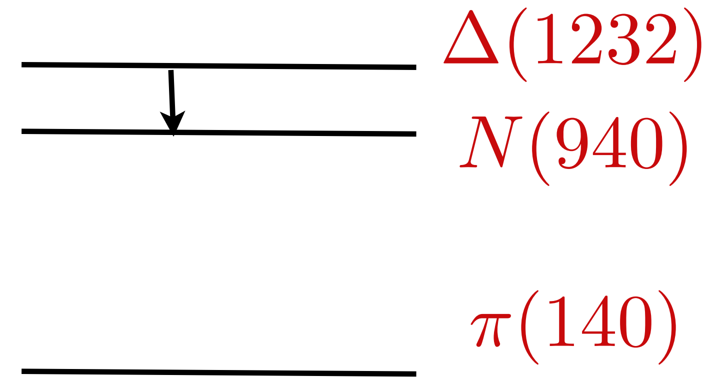
$$< r_M^2 >= A\frac{1}{m_\pi} + B\left(\log m_\pi + C\right)$$

Experimental numbers

$$\sqrt{< r_A^2 >} = 0.65\ \mathtt{fm}$$

$$\sqrt{< r_E^2 >^{p-n}} = 0.94\ \mathtt{fm}$$

Delta Resonances $I = \frac{3}{2} \quad J^P = \frac{3}{2}^+$

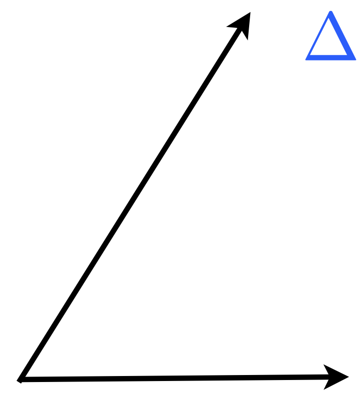


- Low-energy expansion limited by nearest-lying excluded states

$\Delta \equiv M_\Delta - M_N = 290\,\texttt{MeV}$ strong decays $\Delta \to N\pi$

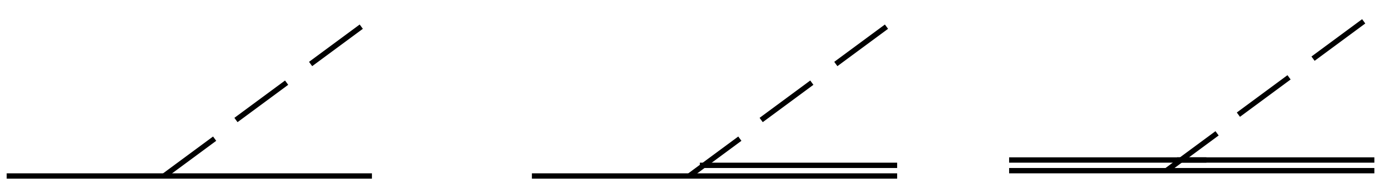
Higher resonances:
Too much momentum available for decays.

- New dimensionless parameter $\frac{m_\pi}{\Delta}$



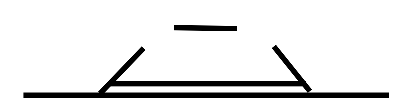
$\frac{m_\pi}{\Delta} \ll 1$ chiral limit, nucleon only

$\frac{m_\pi}{\Delta} \sim 1$ nucleon with explicit deltas



g_A	$g_{\Delta N}$	$g_{\Delta\Delta}$
1.27	~ 1.5	~ 2.2

δM_N



$\mathcal{O}(p^3)$

$$\frac{g_{\Delta N}^2}{(4\pi f)^2}\mathcal{F}\left(\frac{m_\pi}{\Delta}\right)$$

Exercises:

Write down all strong isospin breaking mass operators up to second order in the quark mass. What effect does isospin breaking in the pion mass have on the nucleon mass? Deduce the behavior of the nucleon mass splitting as a function of the quark masses.

In the chiral limit, the isovector axial current is a conserved current. Is there a constraint on the quark isovector axial charge due to the non-renormalization of this current? What about on the nucleon axial charge?