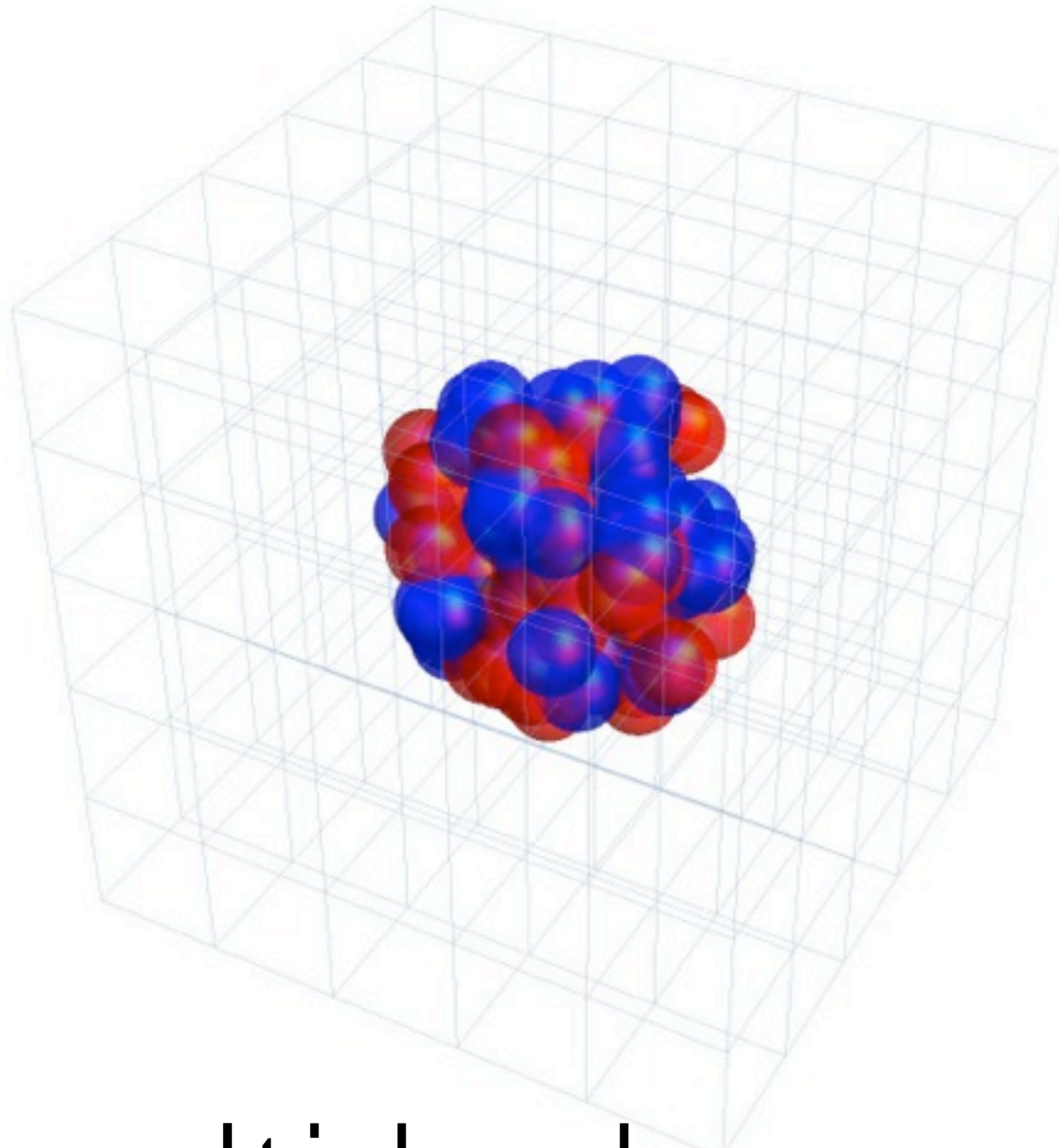


Lec 5: multi-hadron properties

William Detmold

Massachusetts Institute of Technology

Lecture content

- Multi-particle systems at finite temporal extent

- Medium effects and multi-hadron matrix elements

- Open Issues for the future

Multi-particle systems at finite temporal extent

Multi-pion correlation function

- Consider N pion correlation function

$$C_n(t) \;\propto\; \langle \left(\sum_{\mathbf{x}} \pi^-(\mathbf{x},t) \right)^n \left(\pi^+(\mathbf{0},0) \right)^n \rangle$$

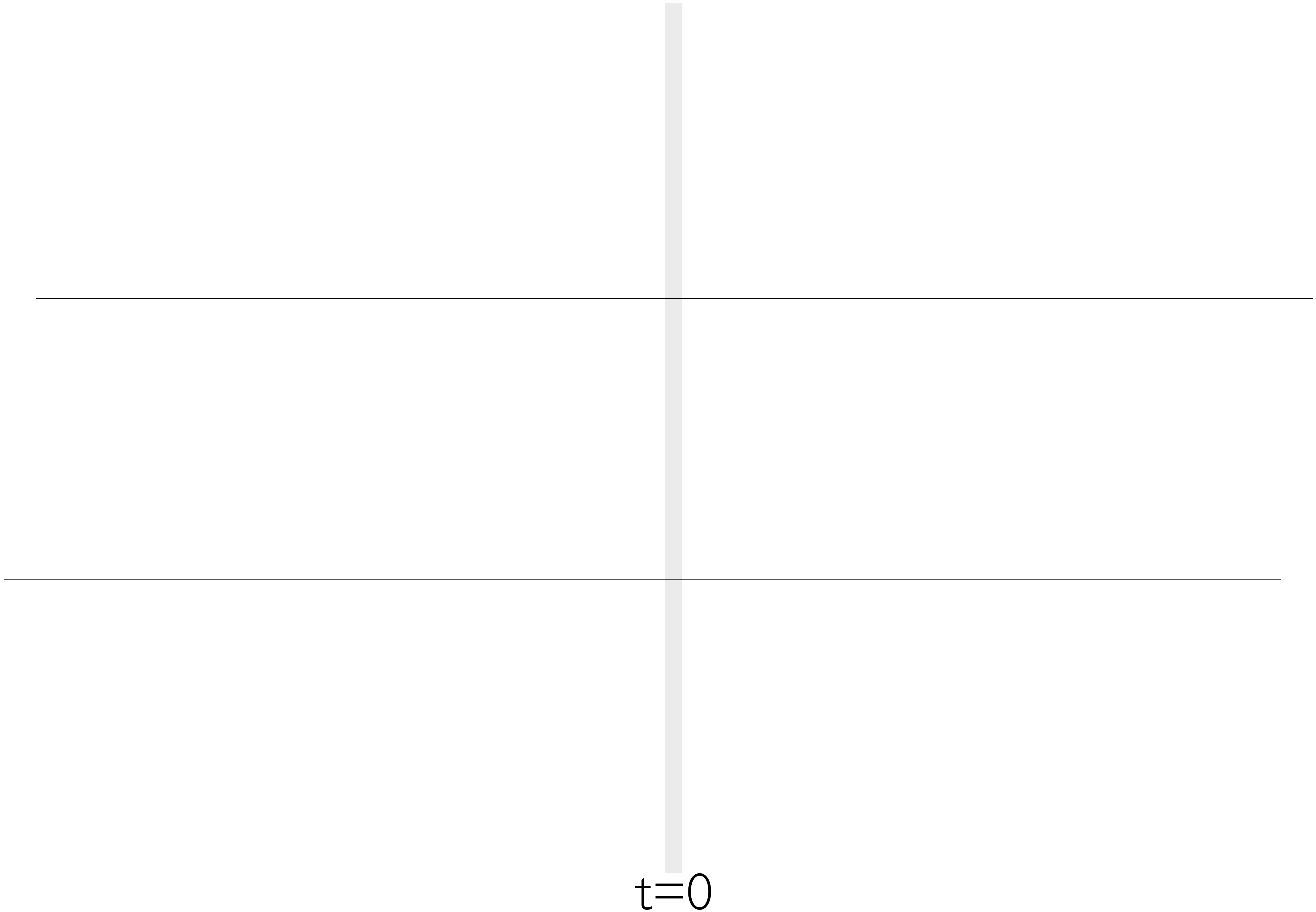
- For a lattice of temporal extent T (inverse temperature)

$$\begin{aligned}
C_n(t) &= \mathrm{Tr}\left[e^{-HT} \left(\sum_{\mathbf{x}} \pi^-(\mathbf{x},t) \right)^n \left(\pi^+(0)\right)^n \right] \\
&= \sum_m \left\langle m \left| e^{-HT} \left(\sum_{\mathbf{x}} \pi^-(\mathbf{x},t) \right)^n \left(\pi^+(0)\right)^n \right| m \right\rangle \\
&= \sum_{m,\ell} \left\langle m \left| e^{-HT} \left(\sum_{\mathbf{x}} \pi^-(\mathbf{x},t) \right)^n \right| \ell \right\rangle \langle \ell \left| \left(\pi^+(0)\right)^n \right| m \rangle \\
&= \sum_{m,\ell} \left\langle m \left| e^{-H(T-t)} \left(\sum_{\mathbf{x}} \pi^-(\mathbf{x},0) \right)^n e^{-Ht} \right| \ell \right\rangle \langle \ell \left| \left(\pi^+(0)\right)^n \right| m \rangle \\
&= \sum_{m,\ell} e^{-E_m(T-t)} e^{-E_\ell t} \mathcal{Z}_{\ell,m}
\end{aligned}$$

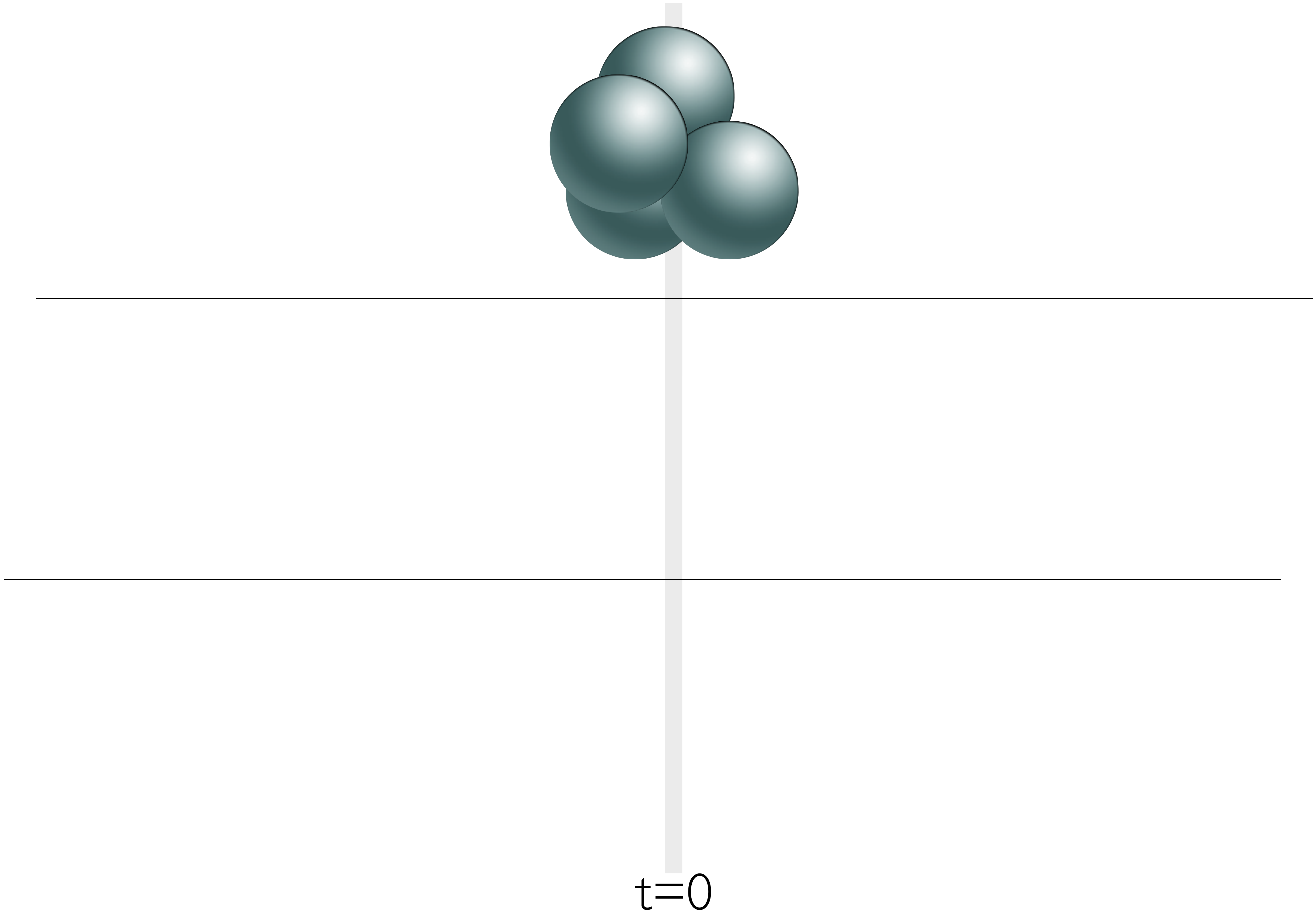
- Many states contribute (ignore excitations)

$$\{|m\rangle = |0\rangle, |\ell\rangle = |n\pi\rangle\}, \{|m\rangle = |\pi\rangle, |\ell\rangle = |(n-1)\pi\rangle\}, \ldots, \{|m\rangle = |n\pi\rangle, |\ell\rangle = |0\rangle\}$$

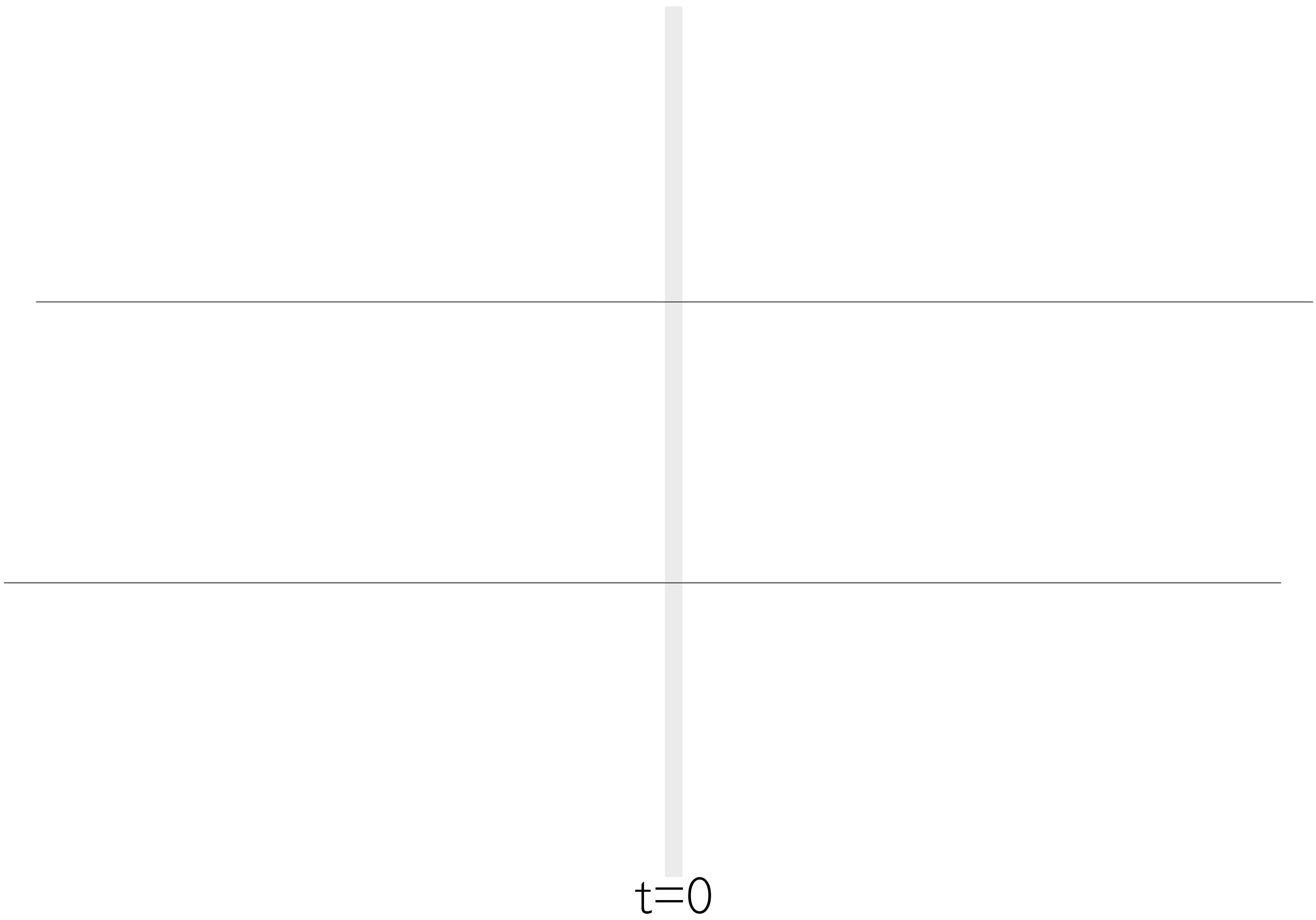
Four pion correlation



Four pion correlation



Four pion correlation



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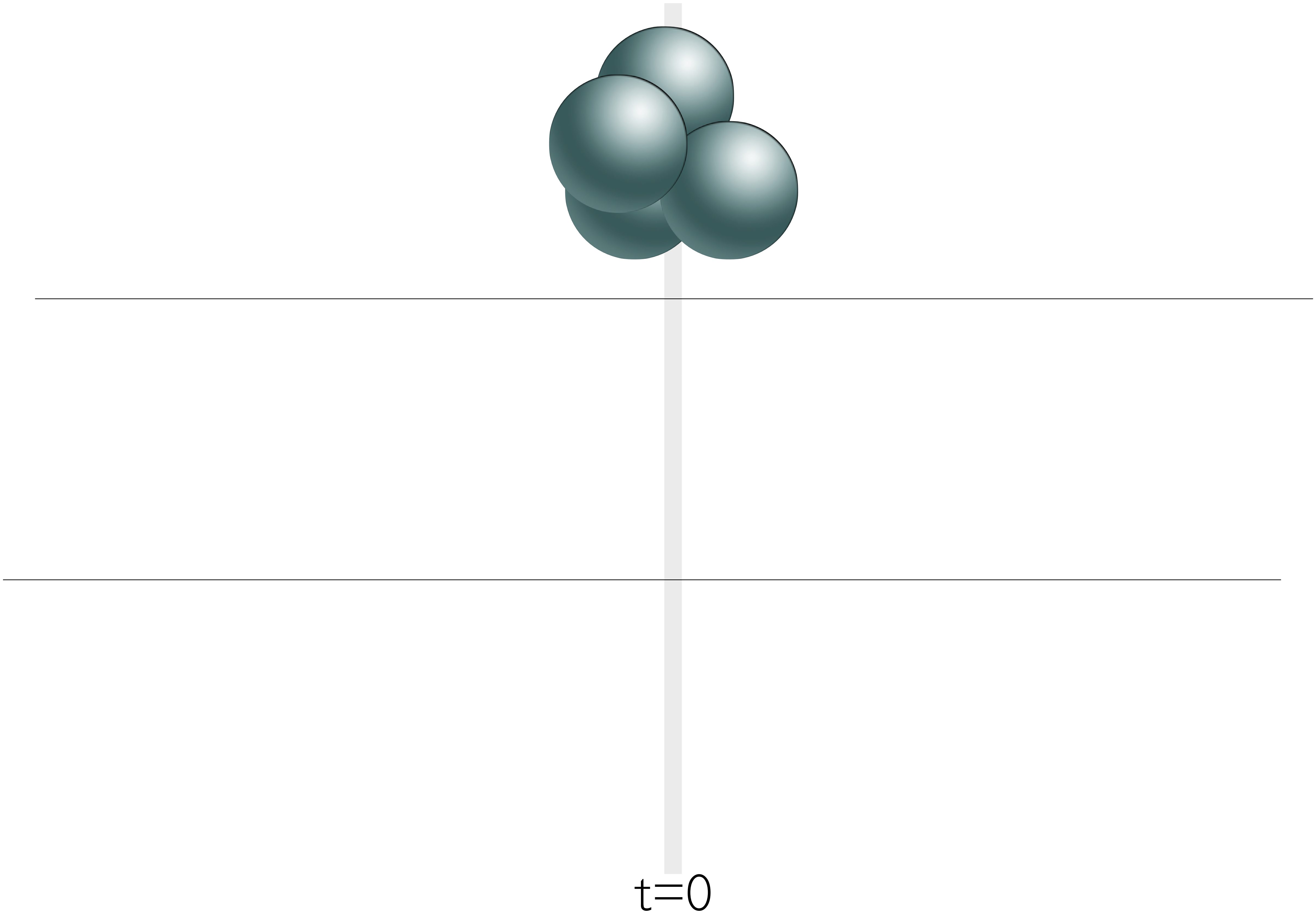

Four pion correlation

$$Z_{4\pi}\left(e^{-4E_{4\pi}t}+e^{-4E_{4\pi}(T-t)}\right)$$

t=0

Four pion correlation

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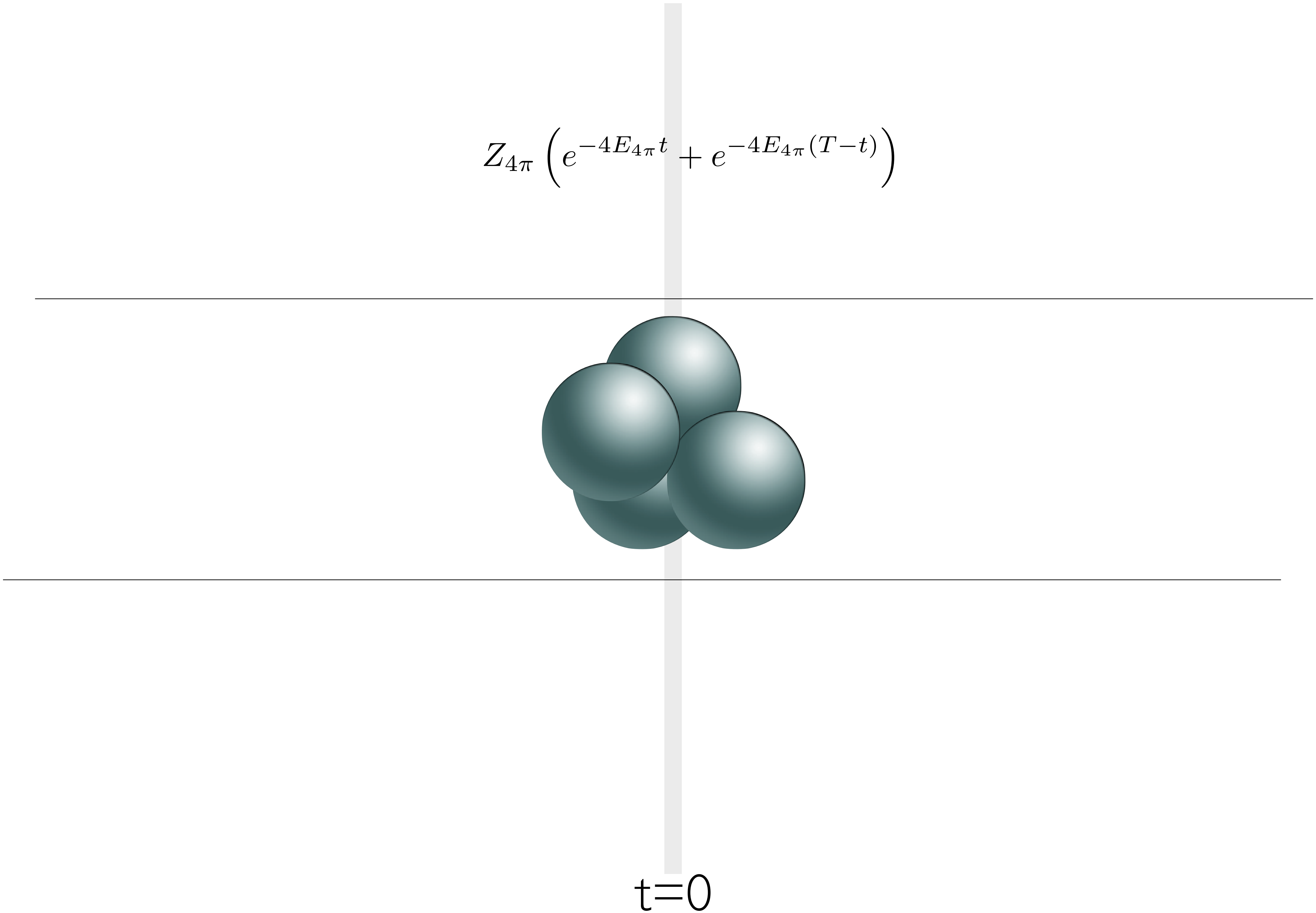

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Four pion correlation

$$Z_{4\pi}\left(e^{-4E_{4\pi}t} + e^{-4E_{4\pi}(T-t)}\right)$$

$$Z_{3/1\pi}\left(e^{-E_{3\pi}t}e^{-E_{1\pi}(T-t)} + e^{-E_{3\pi}(T-t)}e^{-E_{1\pi}t}\right)$$

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Four pion correlation

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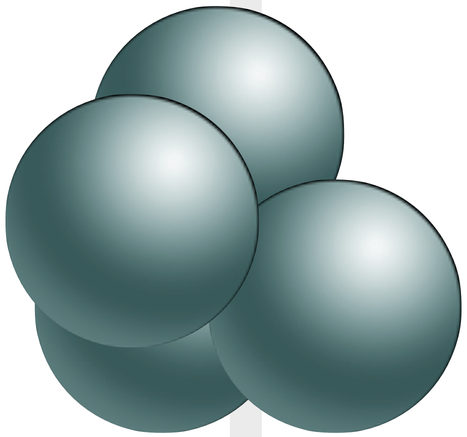

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Four pion correlation

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$$Z_{2/2\pi}e^{-E_{2\pi}t}e^{-E_{2\pi}(T-t)} = Z_{2/2\pi}e^{-E_{2\pi}T}$$

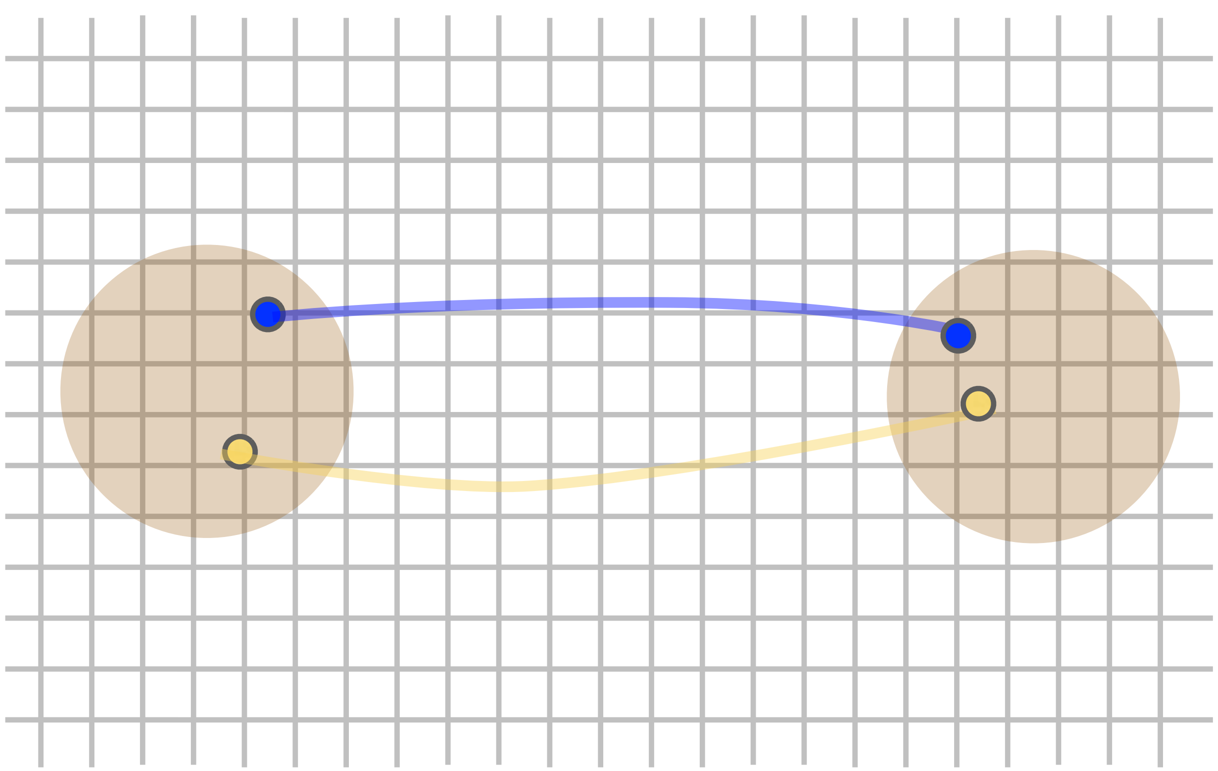
t=0

Many meson 2-point correlator

- Consider π^+ correlator (m_u=m_d)

$$C^{(1)}(t) = \left\langle 0 \left| \sum_{\mathbf{x}} \overline{d}\gamma_5 u(\mathbf{x},t)\overline{u}\gamma_5 d(\mathbf{0},0) \right| 0 \right\rangle$$

$$\xrightarrow{t\to\infty} A_1 e^{-E_1 t}$$



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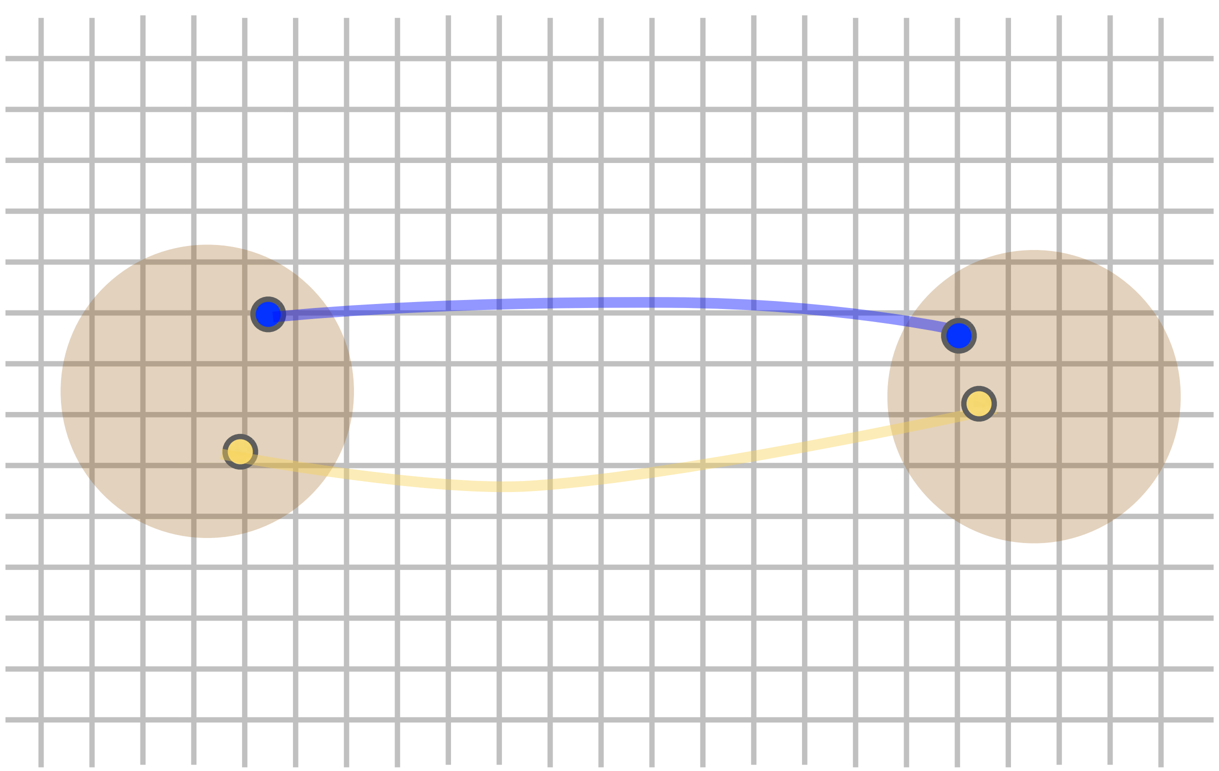

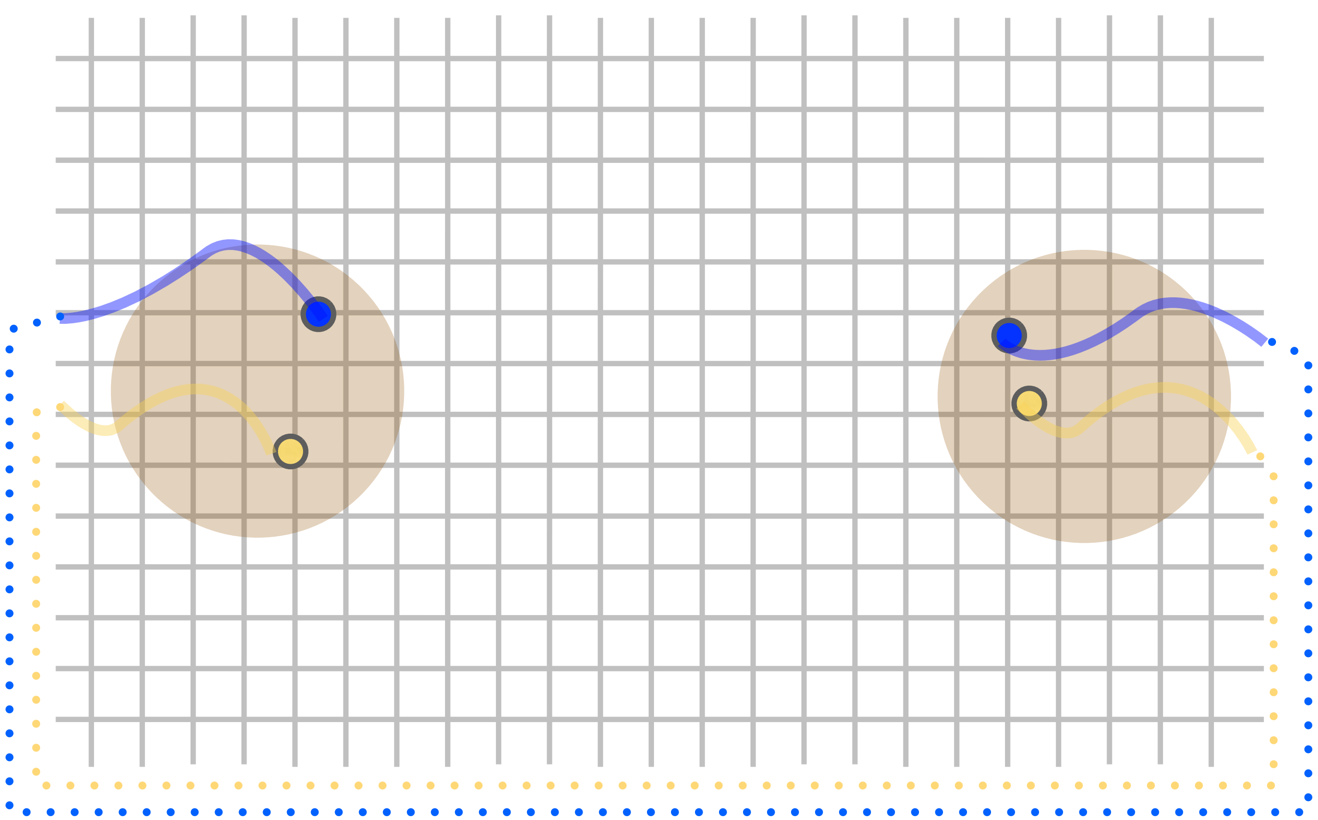

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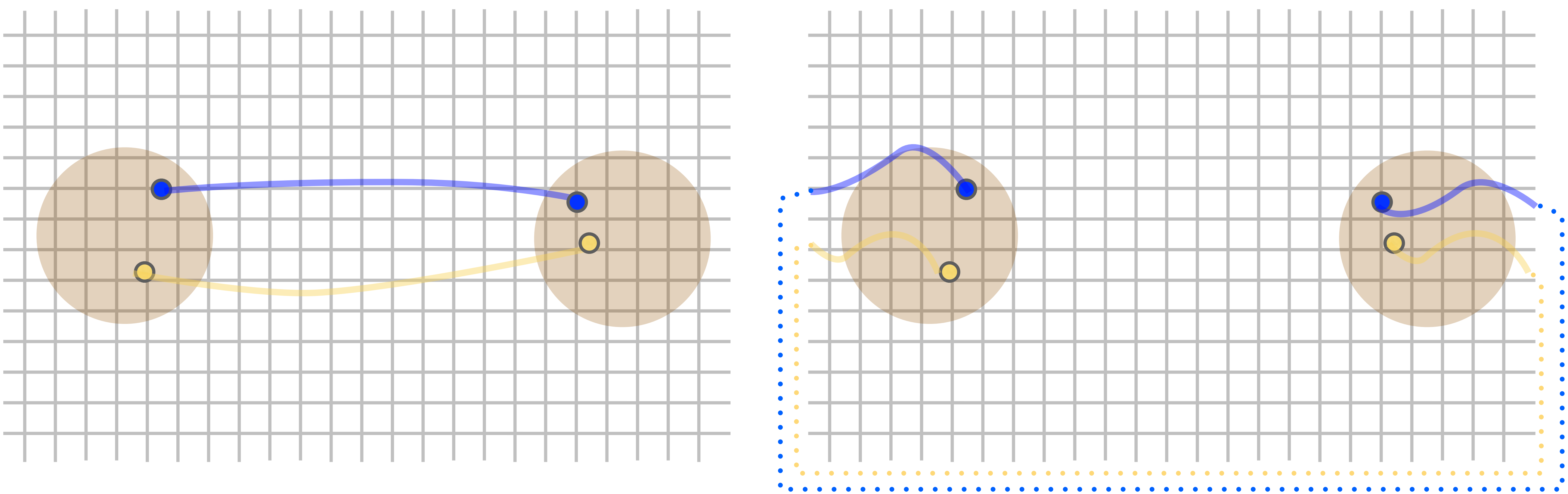
$$\xrightarrow{t \text{ large}} A_1 e^{-E_1 T} \cosh(E_1(t - T/2))$$

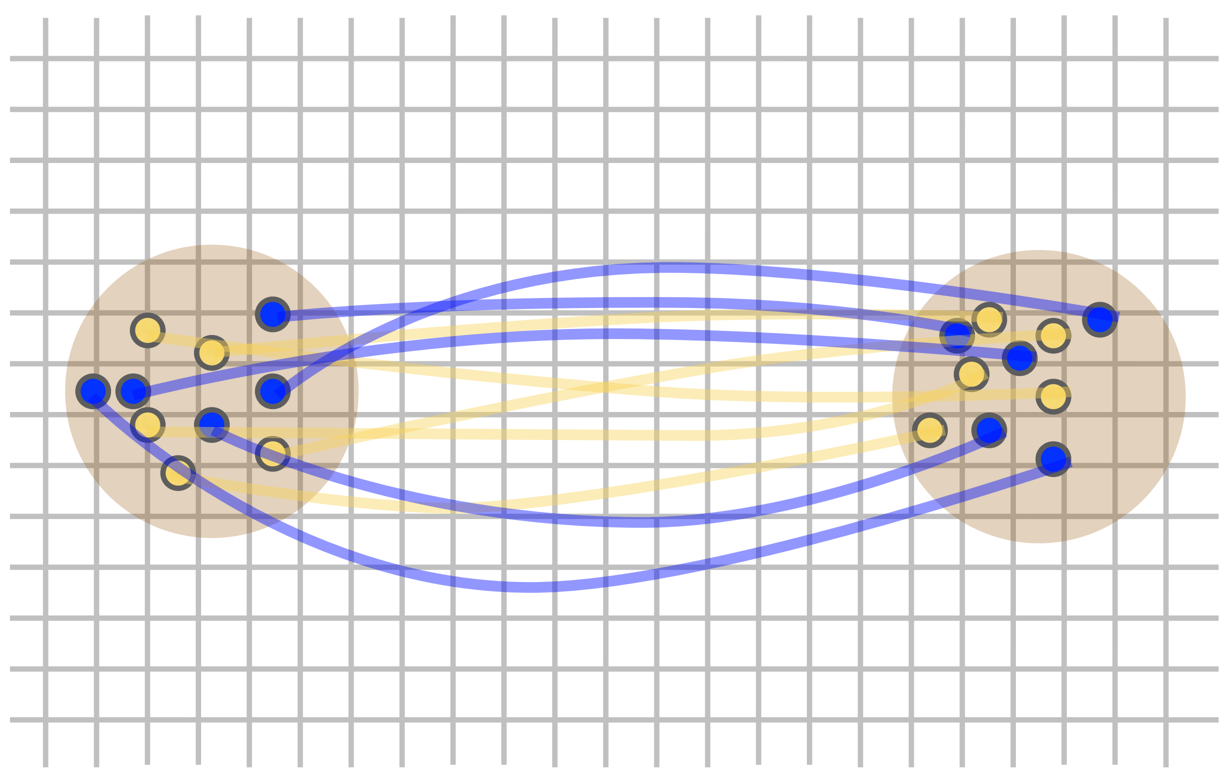


Many meson 2-point correlator

- Now an n $\boldsymbol{\pi}^+$ correlator ($m_u = m_d$)

$$C^{(n)}(t) = \left\langle 0 \left| \left[\sum_{\mathbf{x}} \bar{d}\gamma_5 u(\mathbf{x},t) \bar{u}\gamma_5 d(\mathbf{0},0) \right]^n \right| 0 \right\rangle$$

$$\xrightarrow{t\to\infty} A_n e^{-E_n t}$$



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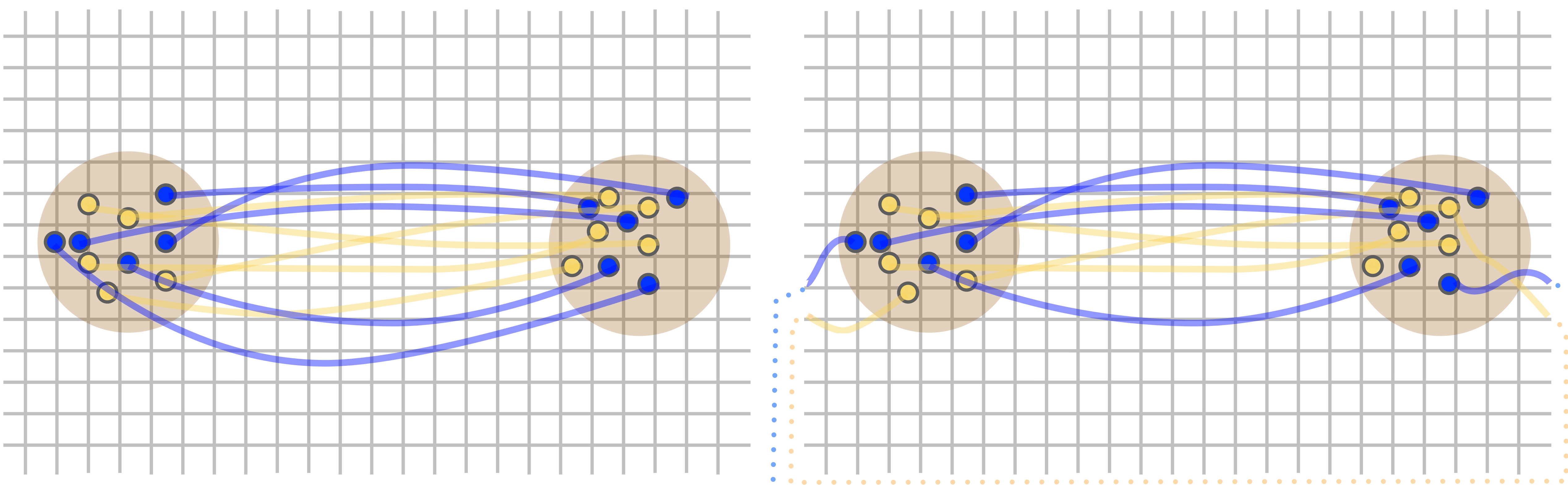

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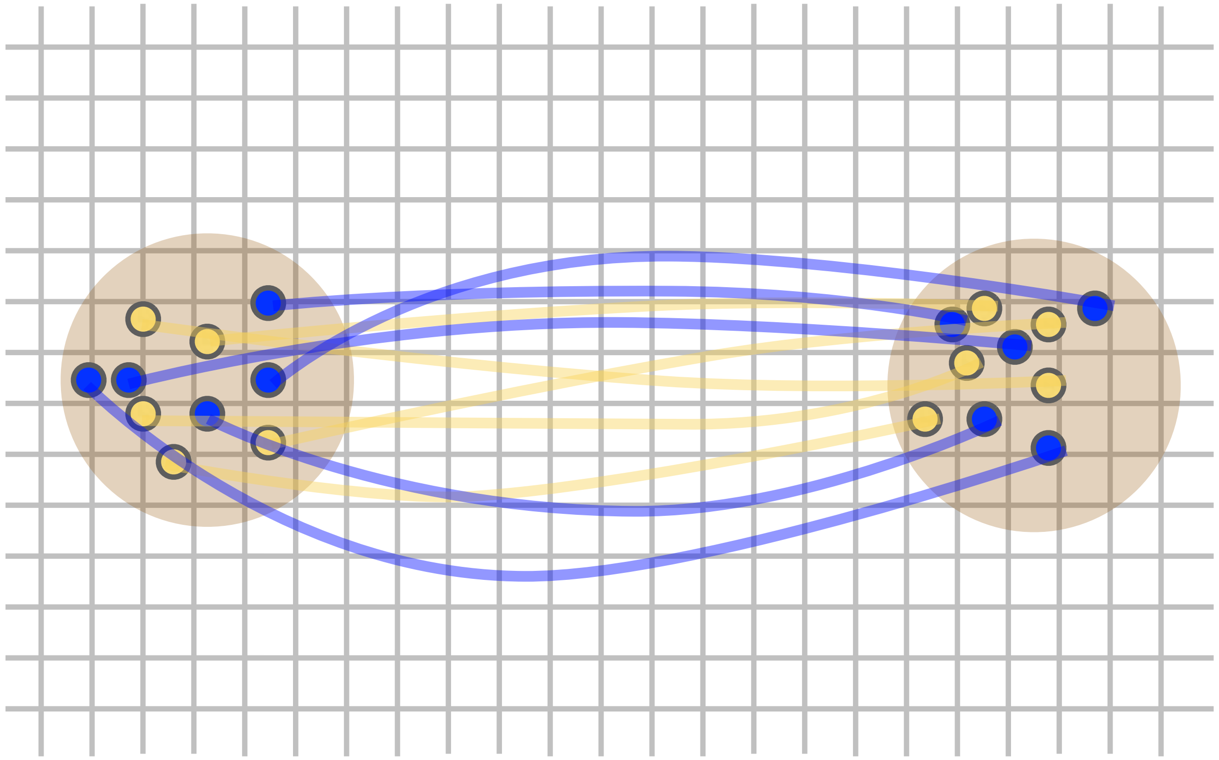
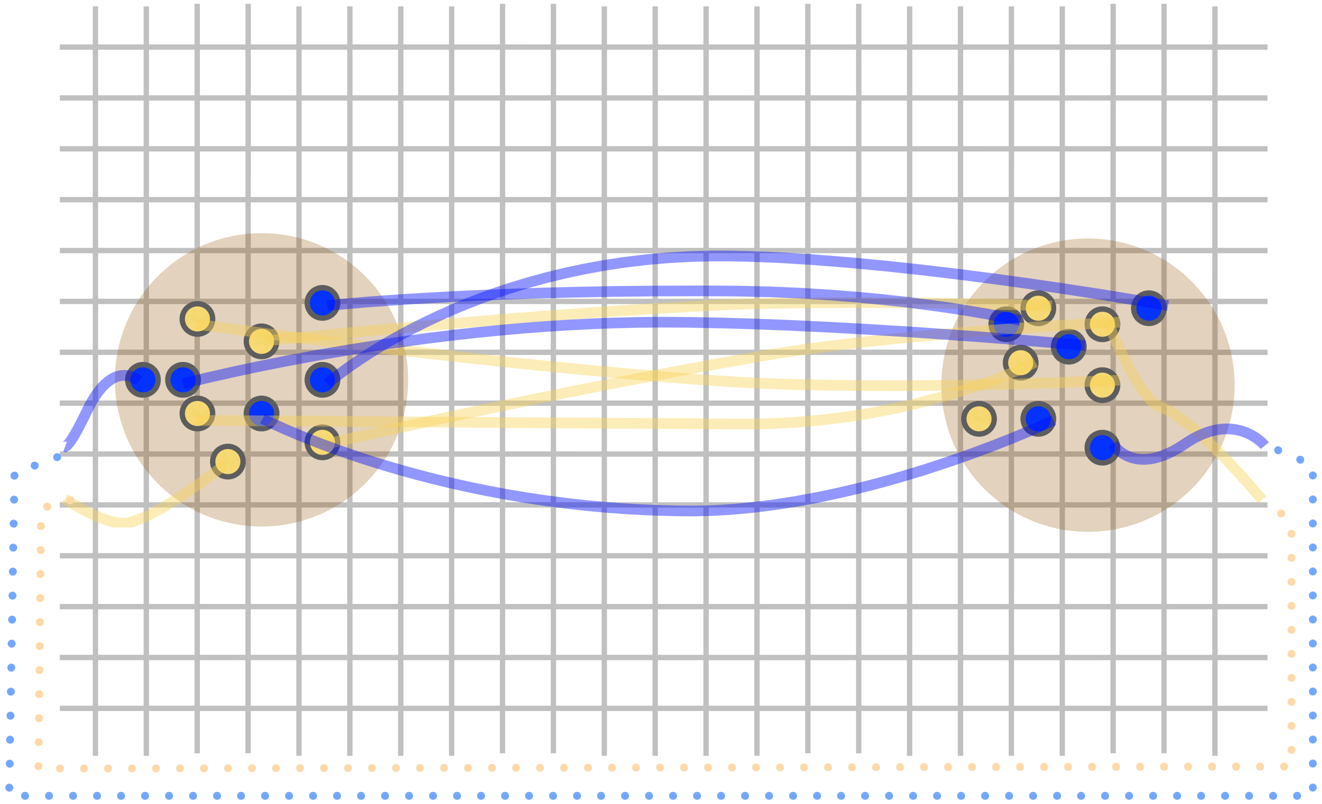
$$\xrightarrow{t \text{ large}} \sum_{m=0}^{\lfloor \frac{n}{2} \rfloor} A_{n,m} e^{-(E_m + E_{n-m})T/2} \cosh\left((E_m - E_{n-m})(t - T/2)\right)$$





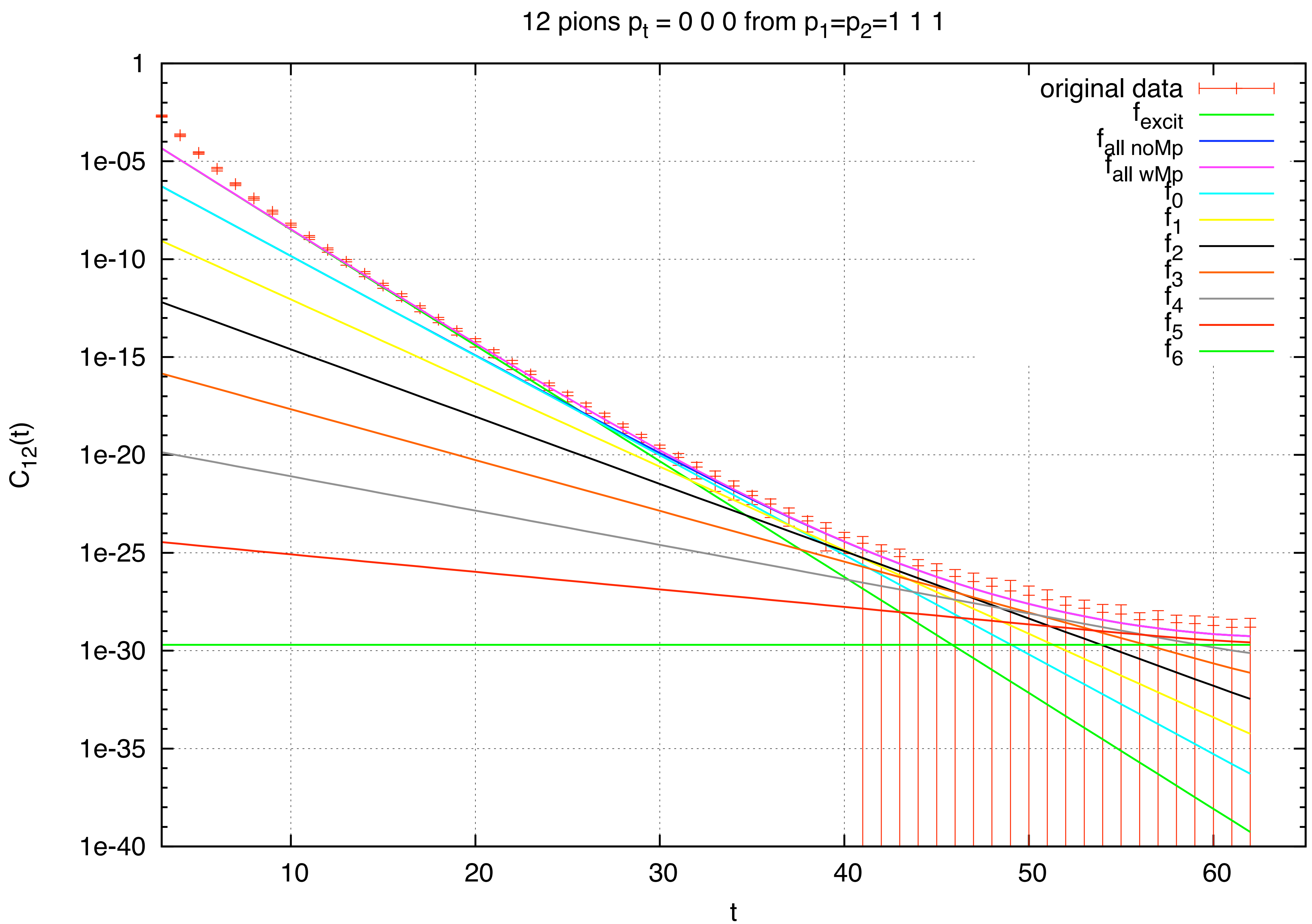
Analysis on finite T correlators

- Can rewrite the t dependence as

$$C_{n\pi}(t) = \sum_{m=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{m} A_m^n Z_m^n e^{-(E_{n-m}+E_m)T/2} \cosh((E_{n-m} - E_m)(t - T/2)) + \ldots ,$$

- Extracting the eigen-energies from these correlators is difficult
  - Many parameters appear in each correlator
  - Correlations between different C_j as the energy E_k occurs in all C_j ($j \geq k$)
  - Various ways to deal with this: eg cascading fits

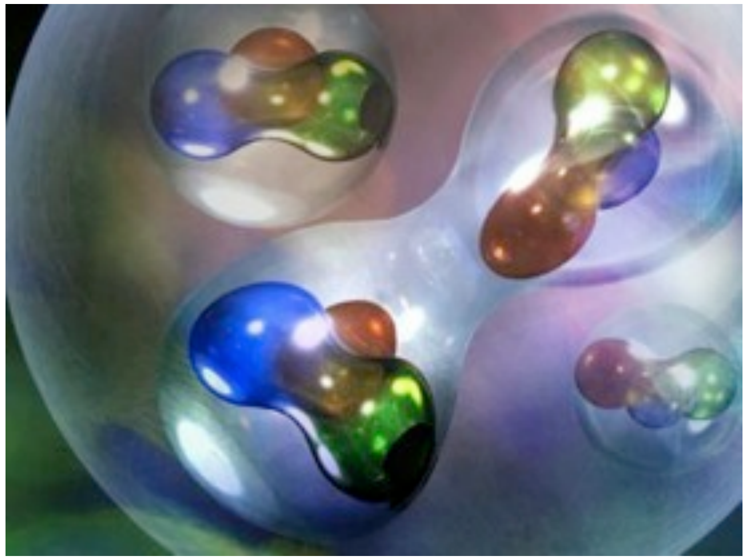
Thermal pollution



At no point does the ground state dominate the correlator!!!

Medium effects and matrix elements

- So far we have only investigated *spectroscopy* of multi-hadron systems
  - What about the *structure* and other properties of such systems?
    - Moments, form factors, polarisabilites, weak interactions....
    - Probed by matrix elements in multi-hadron eigenstates
  - What about in medium properties – how does a proton get modified in a nucleus (intrinsically not a well defined separation)?
    - Really an interpretation of the above
- Very new direction of investigation



[image from JLab]

Colour screening of static charges

- Static quark potential

[Leinweber][Bali et al.][Petreczky/Petrov]

Colour screening of static charges

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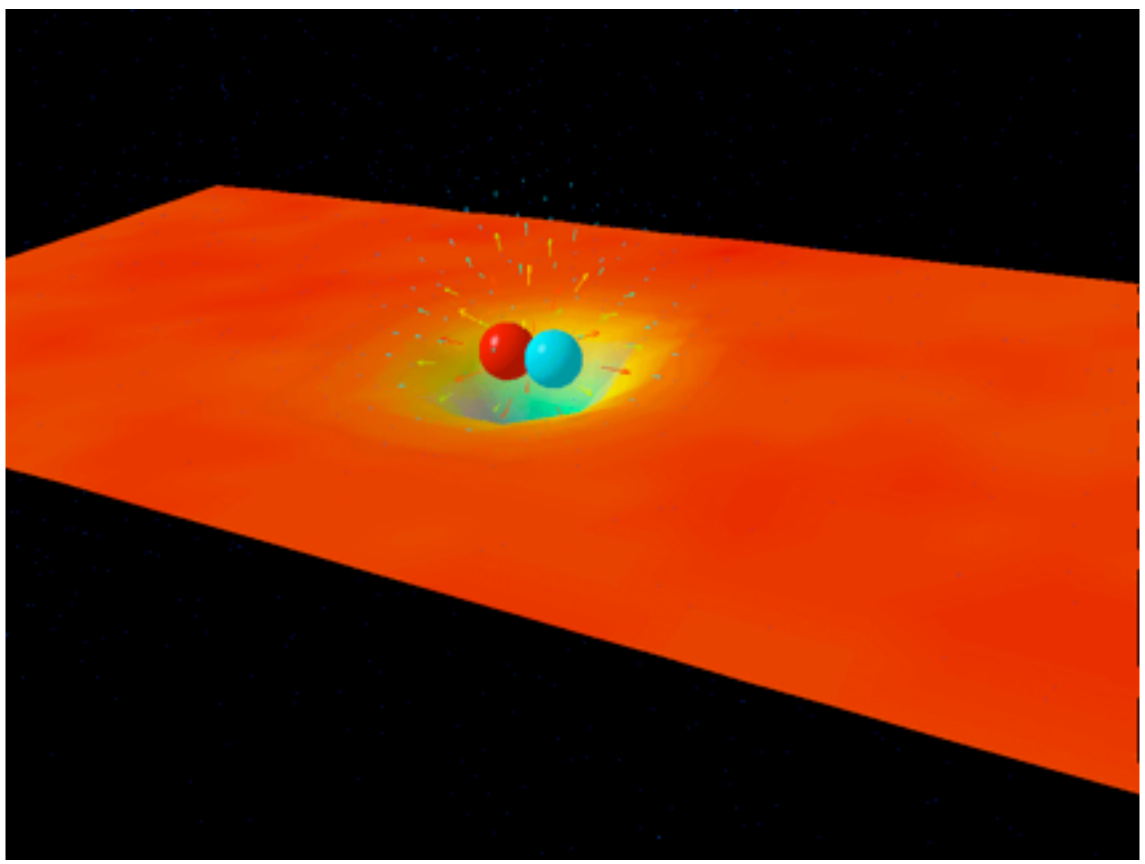

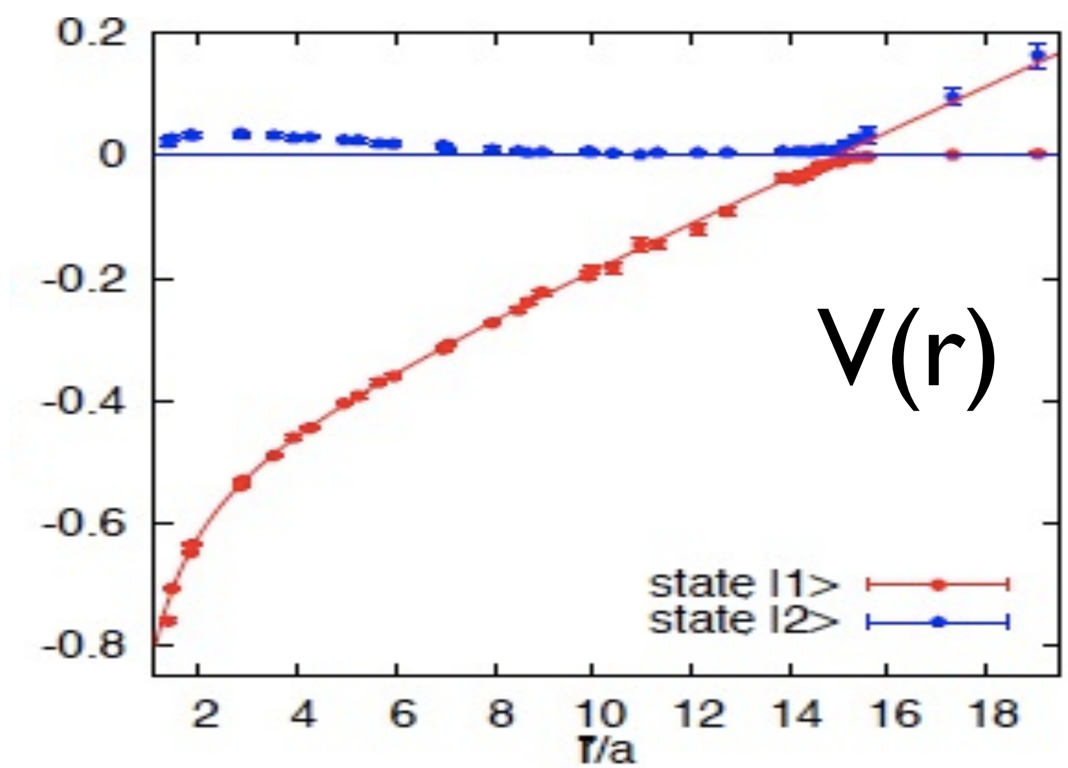

[Leinweber][Bali et al.][Petreczky/Petrov]

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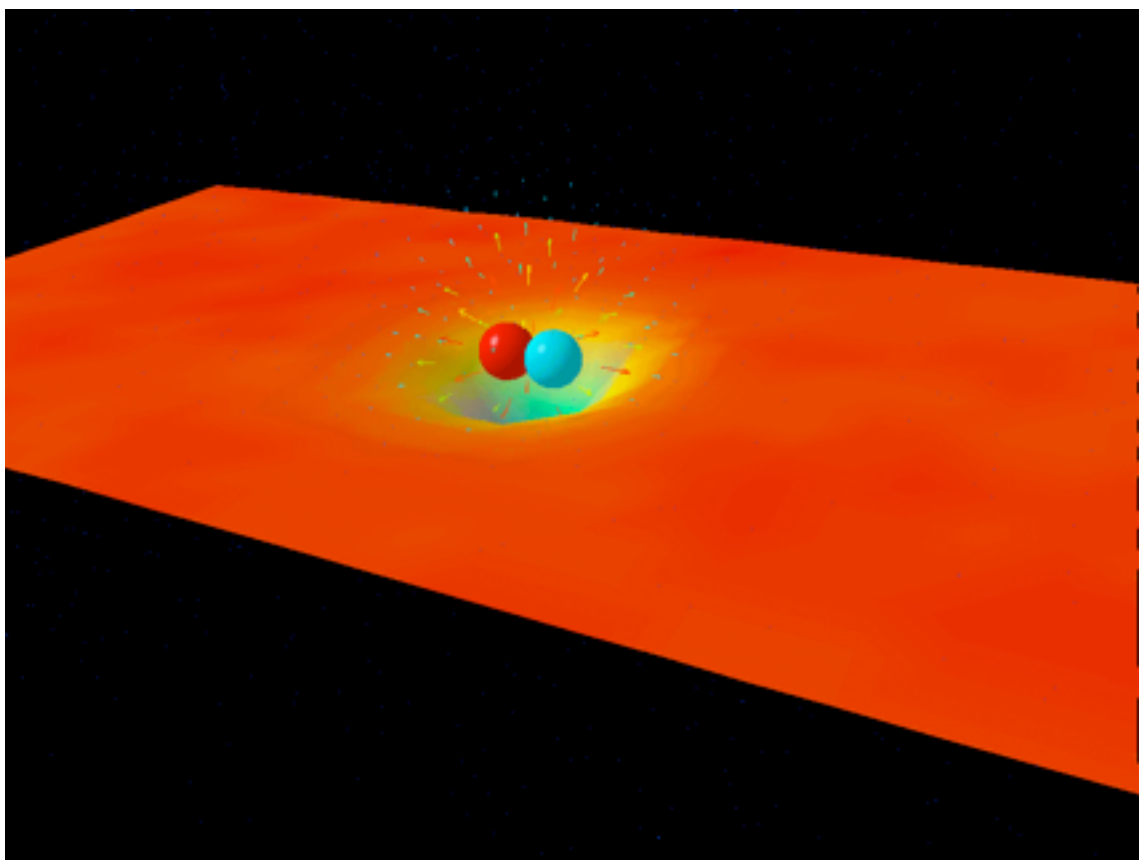
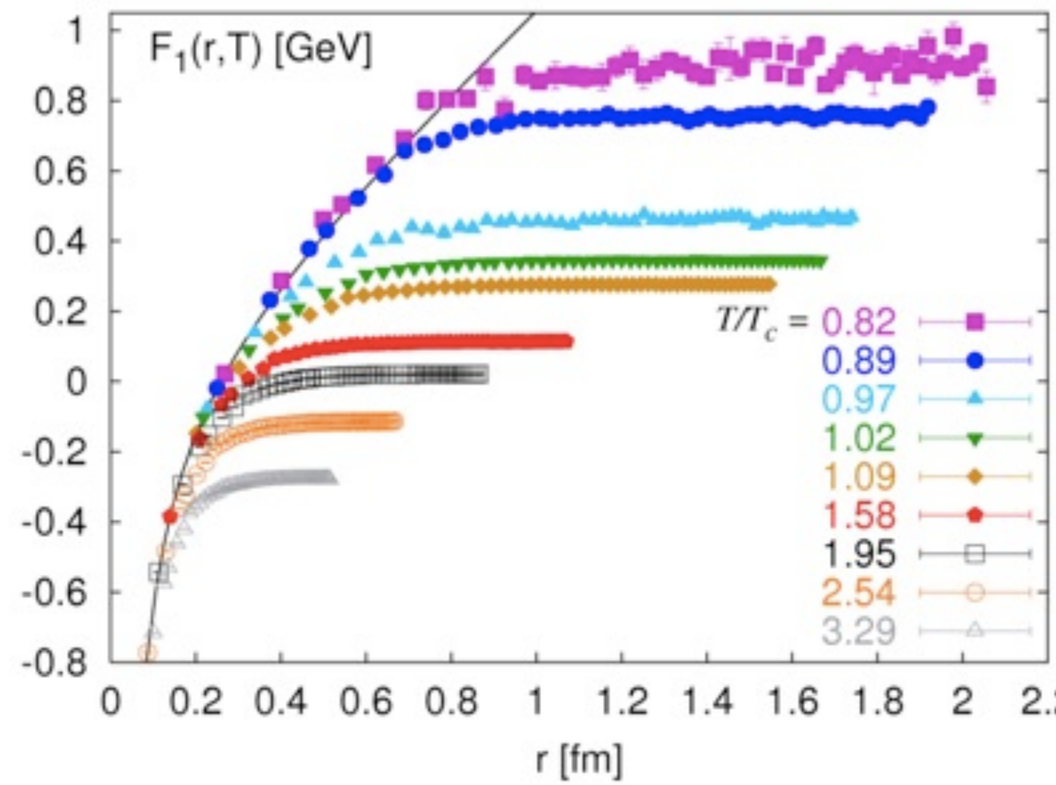




- Screening: evidence for quark-gluon plasma

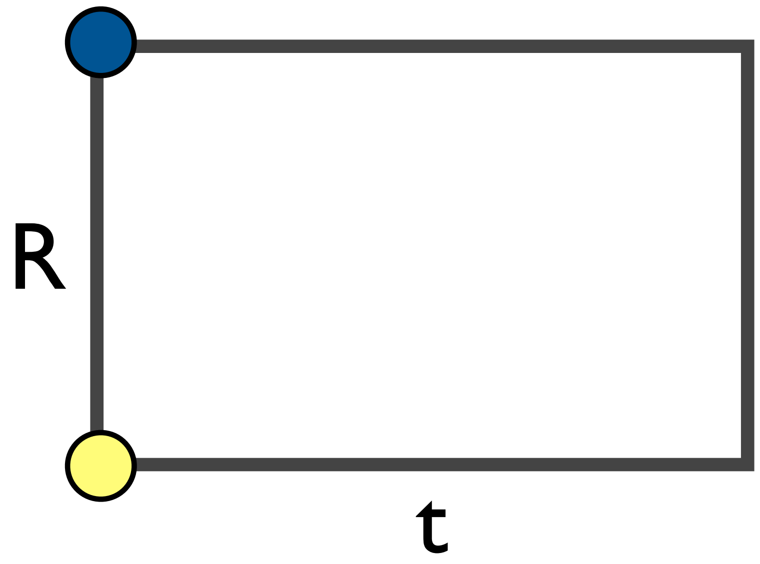
[Leinweber][Bali et al.][Petreczky/Petrov]

Color screening

- Static quark potential

$$C_W(R, t_w, t) = \left\langle 0 \left| \sum_{\mathbf{y}, |\mathbf{r}| = \mathbf{R}} \mathcal{W}(\mathbf{y} + \mathbf{r}, t; \mathbf{y}, t_w) \right| 0 \right\rangle$$

$$\longrightarrow Z \exp[-V(R)(t - t_w)]$$



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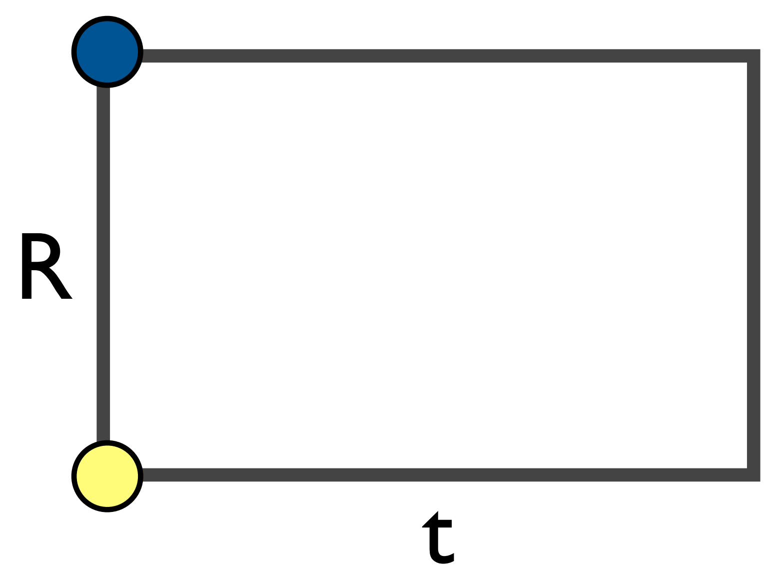

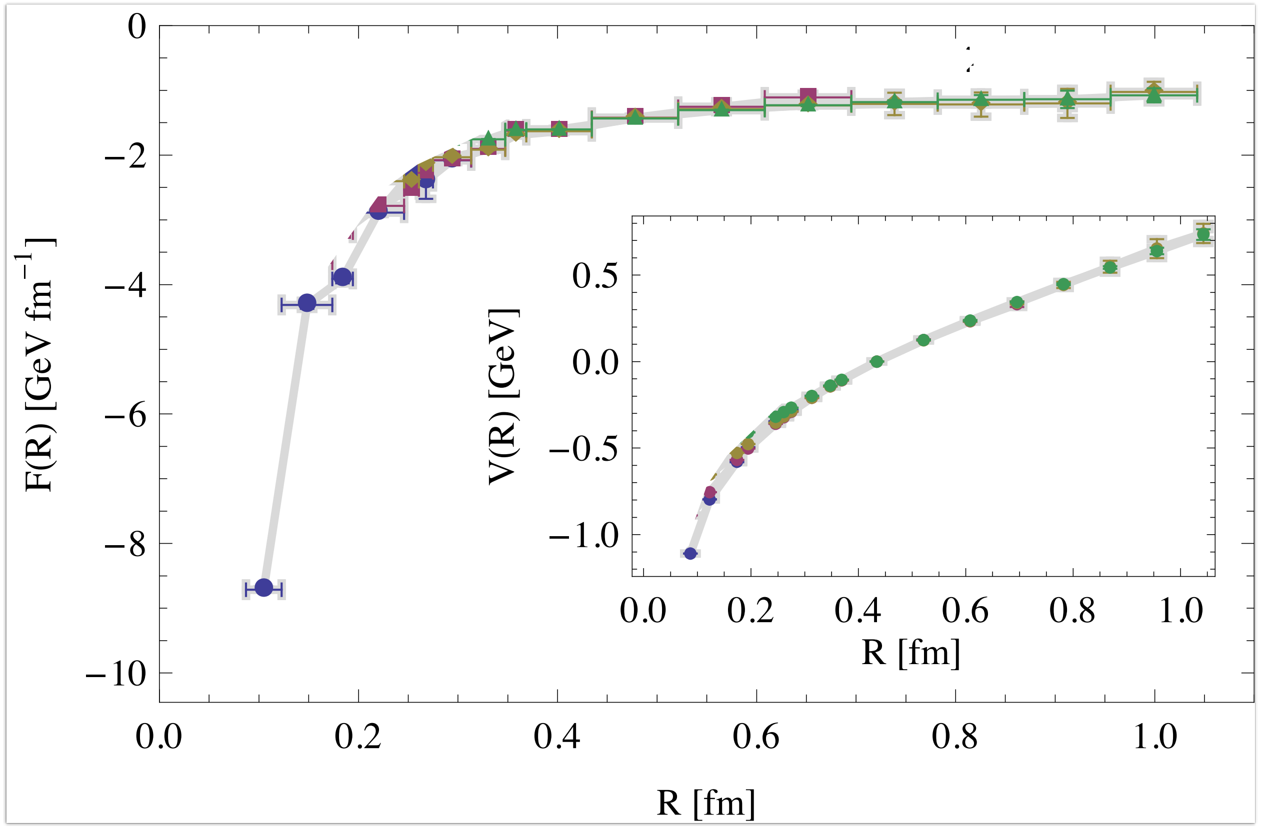

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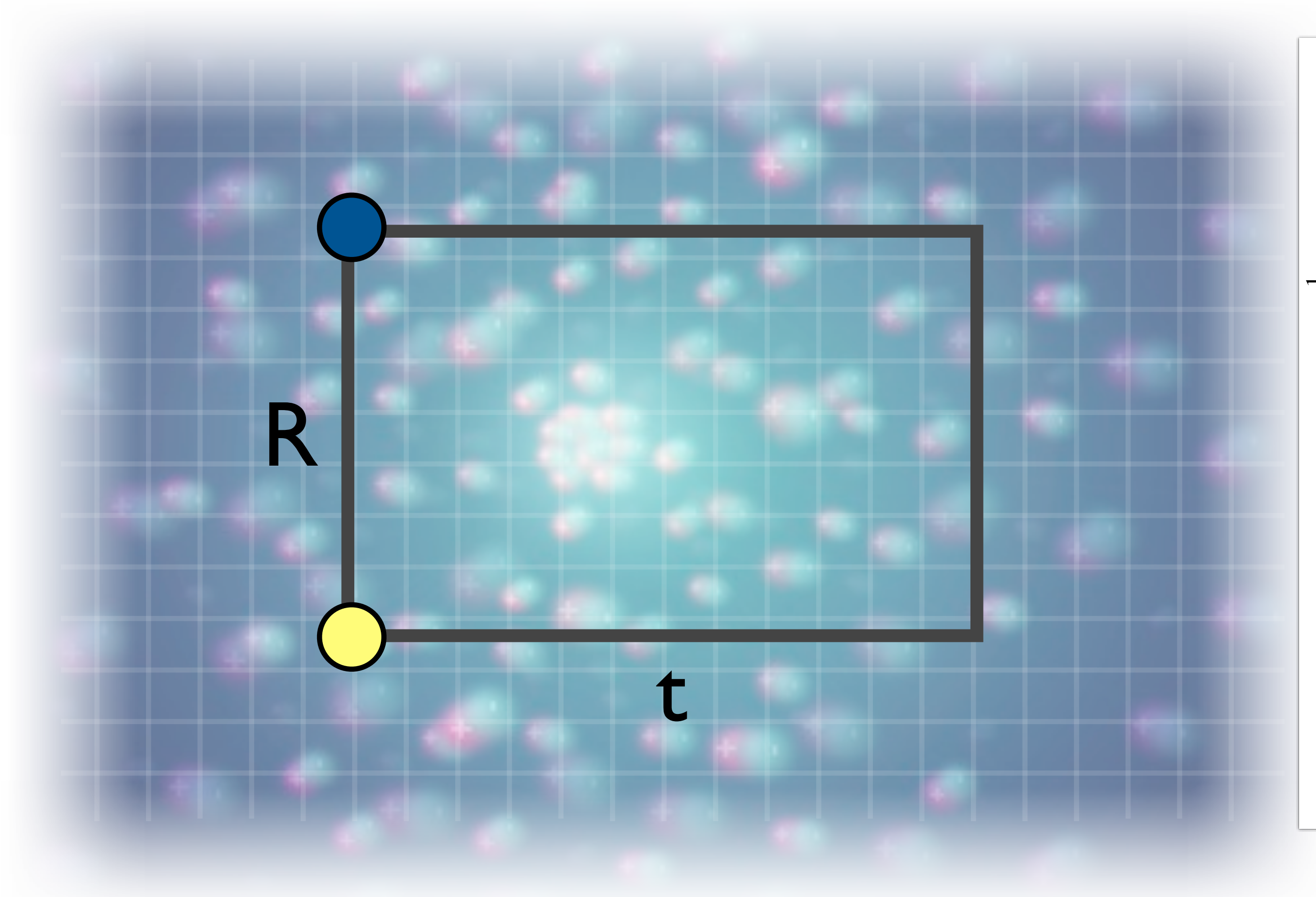

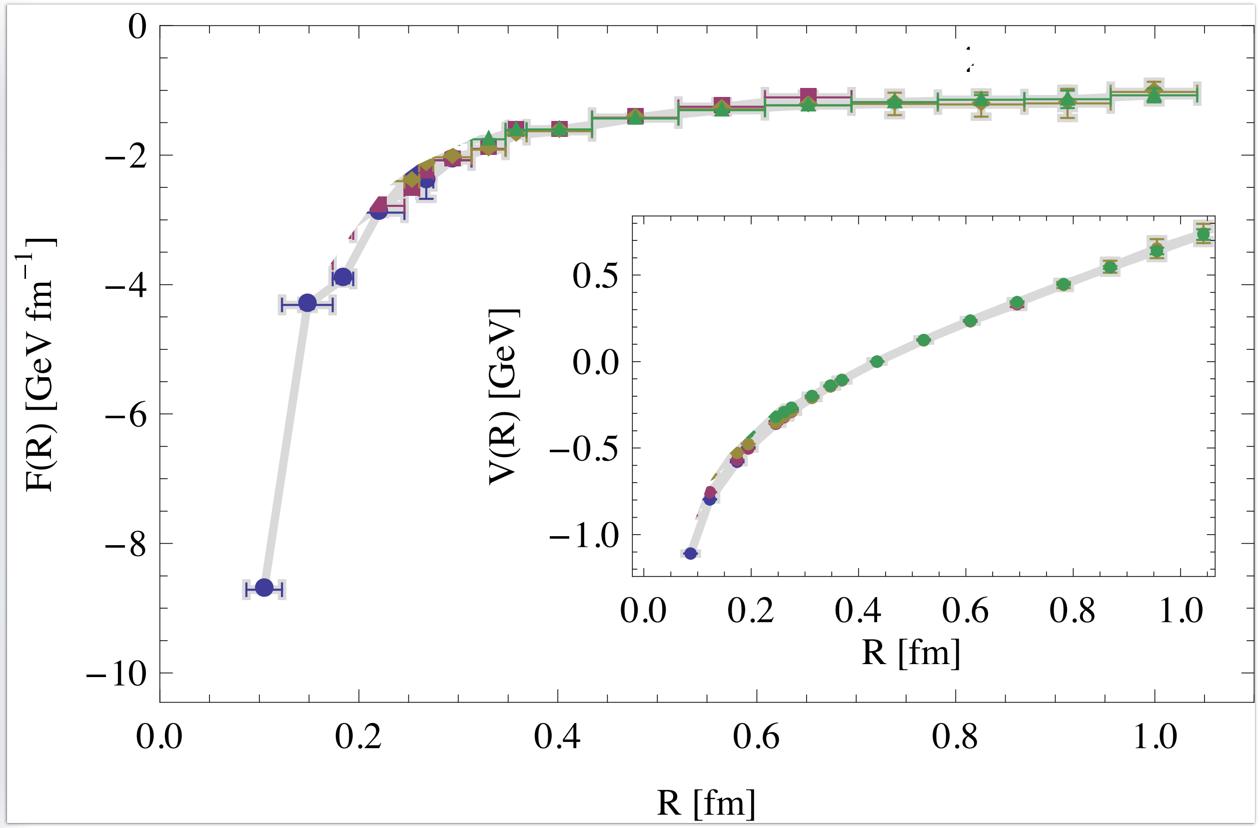

- Modified by condensate? Hadronic screening?

Color screening

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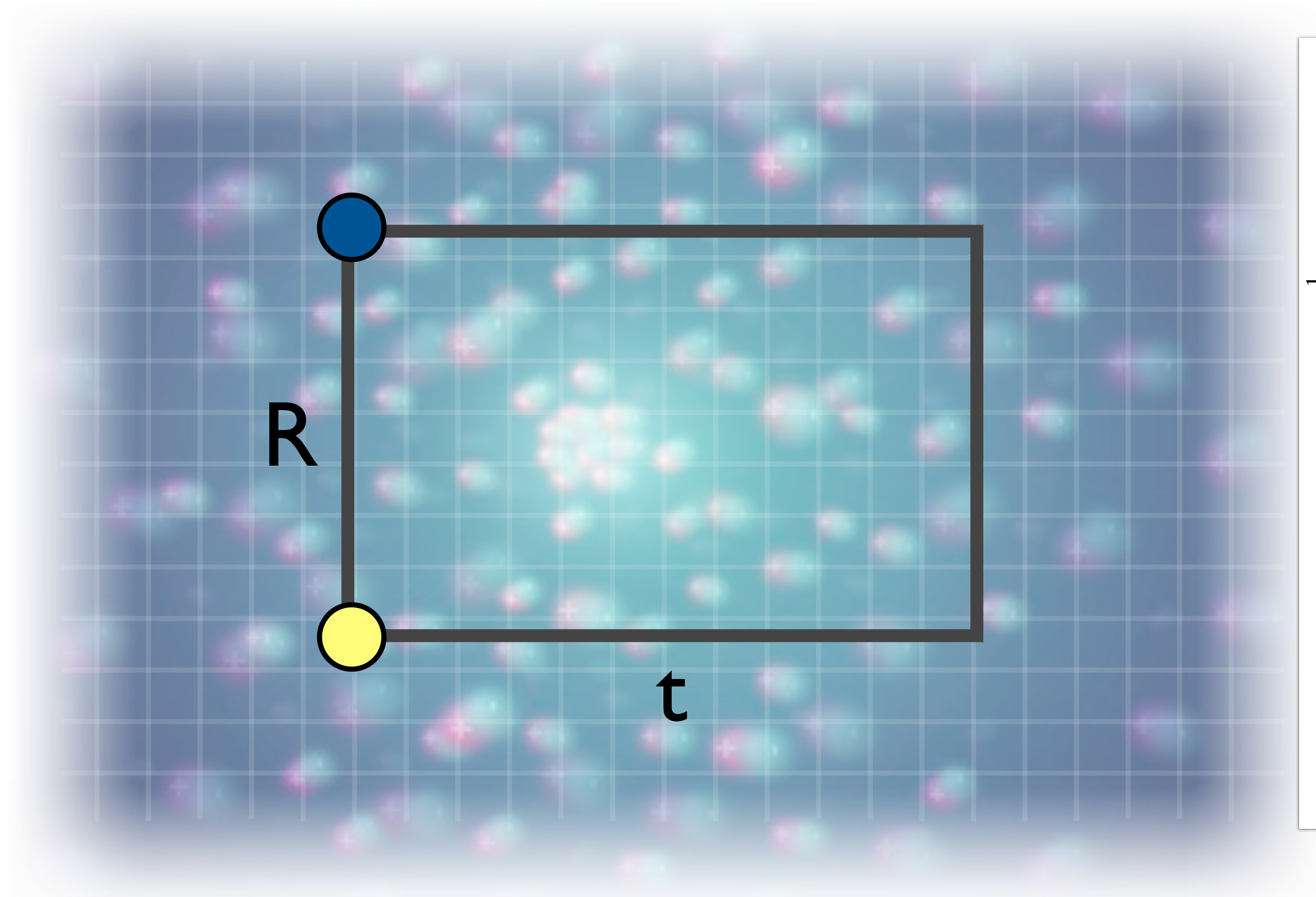
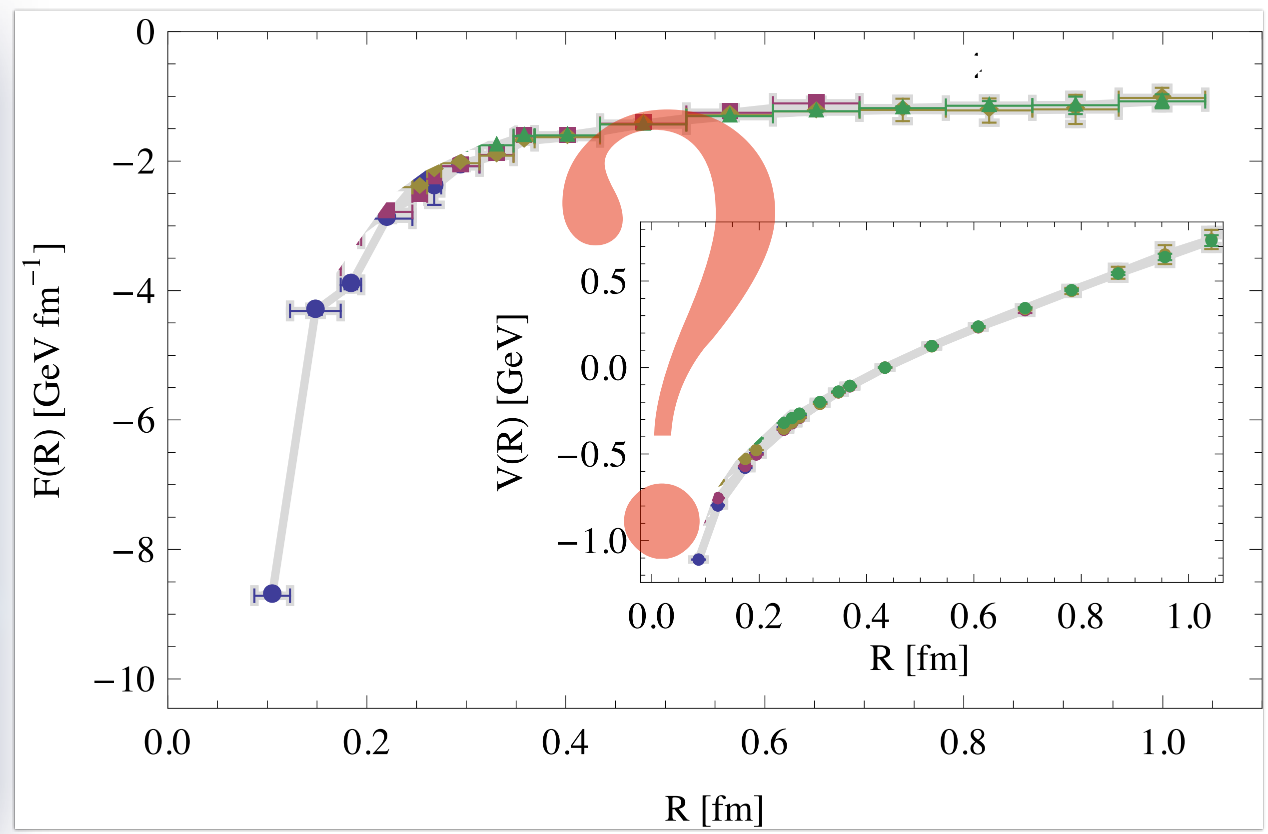




- Modified by condensate? Hadronic screening?

In medium effects

- n pion correlator

$$C_n(t_\pi, t) = \left\langle 0 \left| \left[\sum_{\mathbf{x}} \chi_{\pi^+}(\mathbf{x}, \mathbf{t}) \chi^\dagger_{\pi^+}(\mathbf{0}, \mathbf{t}_\pi) \right]^n \right| 0 \right\rangle$$

$$\longrightarrow Z' \exp[-E_{n\pi}(t - t_\pi)]$$

- Wilson loop correlator

$$C_W(R, t_w, t) = \left\langle 0 \left| \sum_{\mathbf{y}, |\mathbf{r}| = \mathbf{R}} \mathcal{W}(\mathbf{y} + \mathbf{r}, t; \mathbf{y}, t_w) \right| 0 \right\rangle$$

$$\longrightarrow Z \exp[-V(R)(t - t_w)]$$

- Pions and Wilson loop

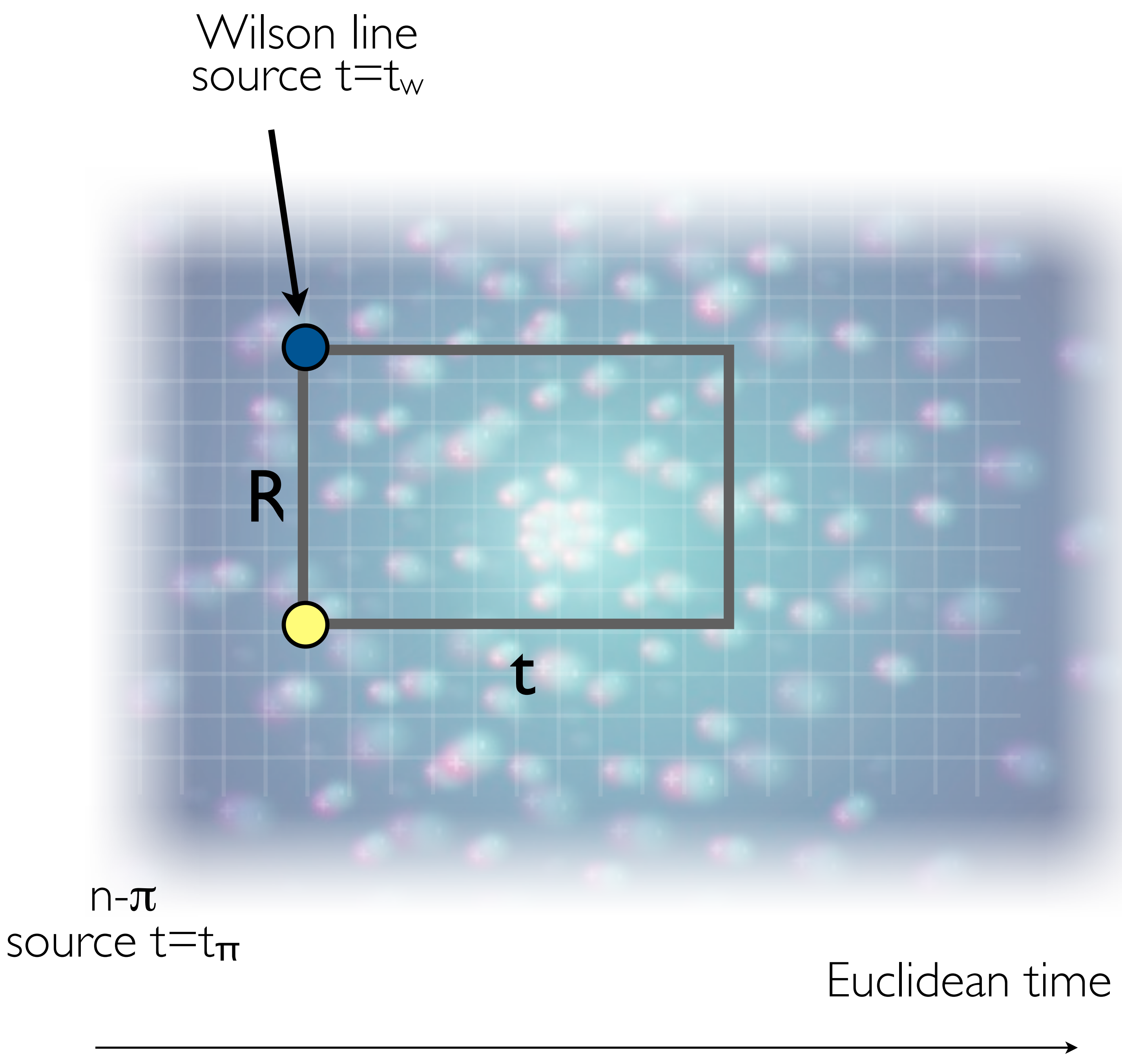
$$C_{n,W}(R, t_\pi, t_w, t) = \left\langle 0 \left| \left[\sum_{\mathbf{x}} \chi_{\pi^+}(\mathbf{x}, \mathbf{t}) \chi^\dagger_{\pi^+}(\mathbf{0}, \mathbf{t}_\pi) \right]^n \sum_{\mathbf{y}, |\mathbf{r}| = \mathbf{R}} \mathcal{W}(\mathbf{y} + \mathbf{r}, t; \mathbf{y}, t_w) \right| 0 \right\rangle$$

- Ratio gives shift in potential due to interaction of potential with pion system

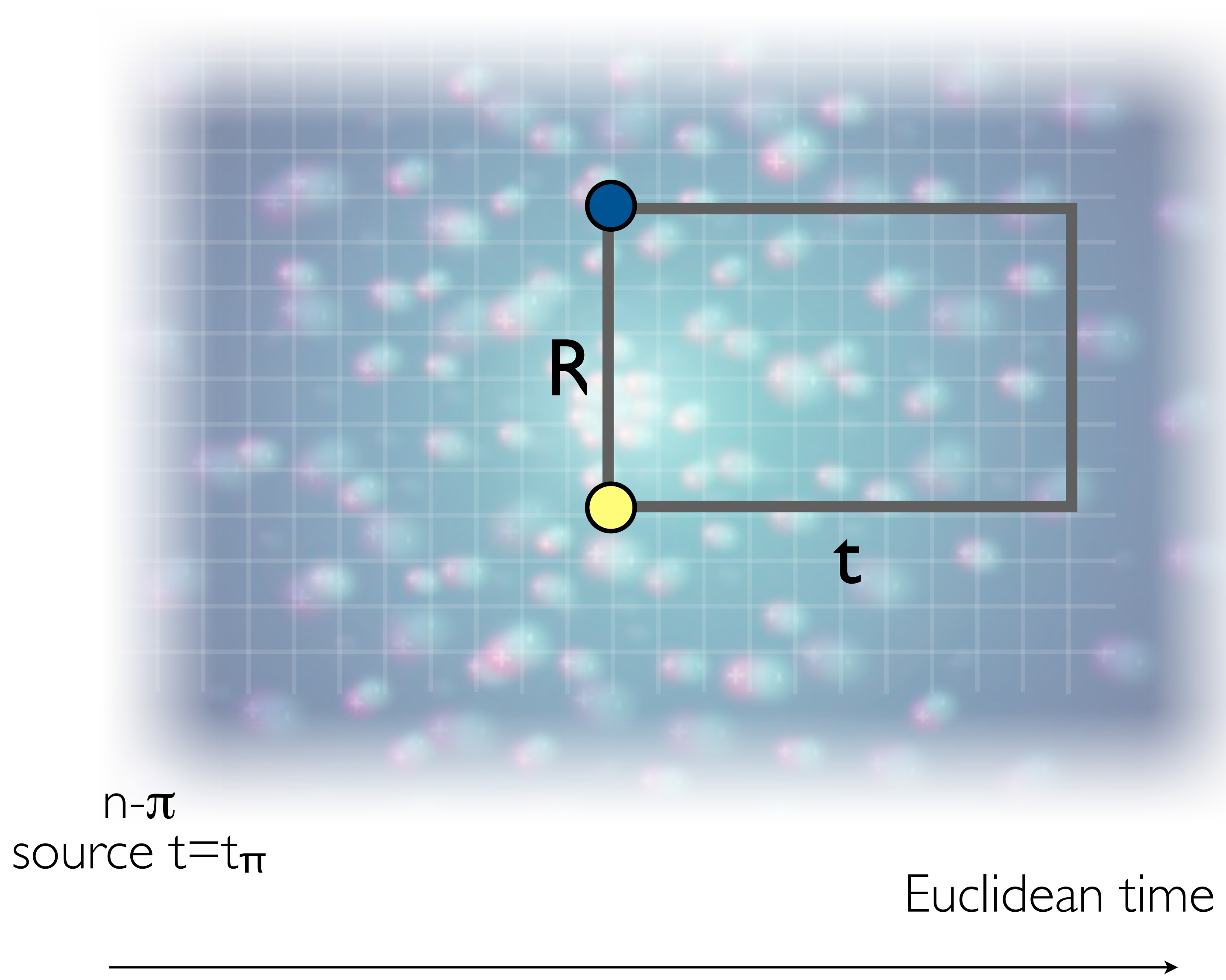
$$G_{n,W}(R, t_\pi, t_w, t) = \frac{C_{n,W}(R, t_\pi, t_w, t)}{C_n(t_\pi, t) C_W(R, t_w, t)}$$

$$\longrightarrow \# \exp\left[-\delta V(R, n)(t - t_W)\right]$$

In pictures

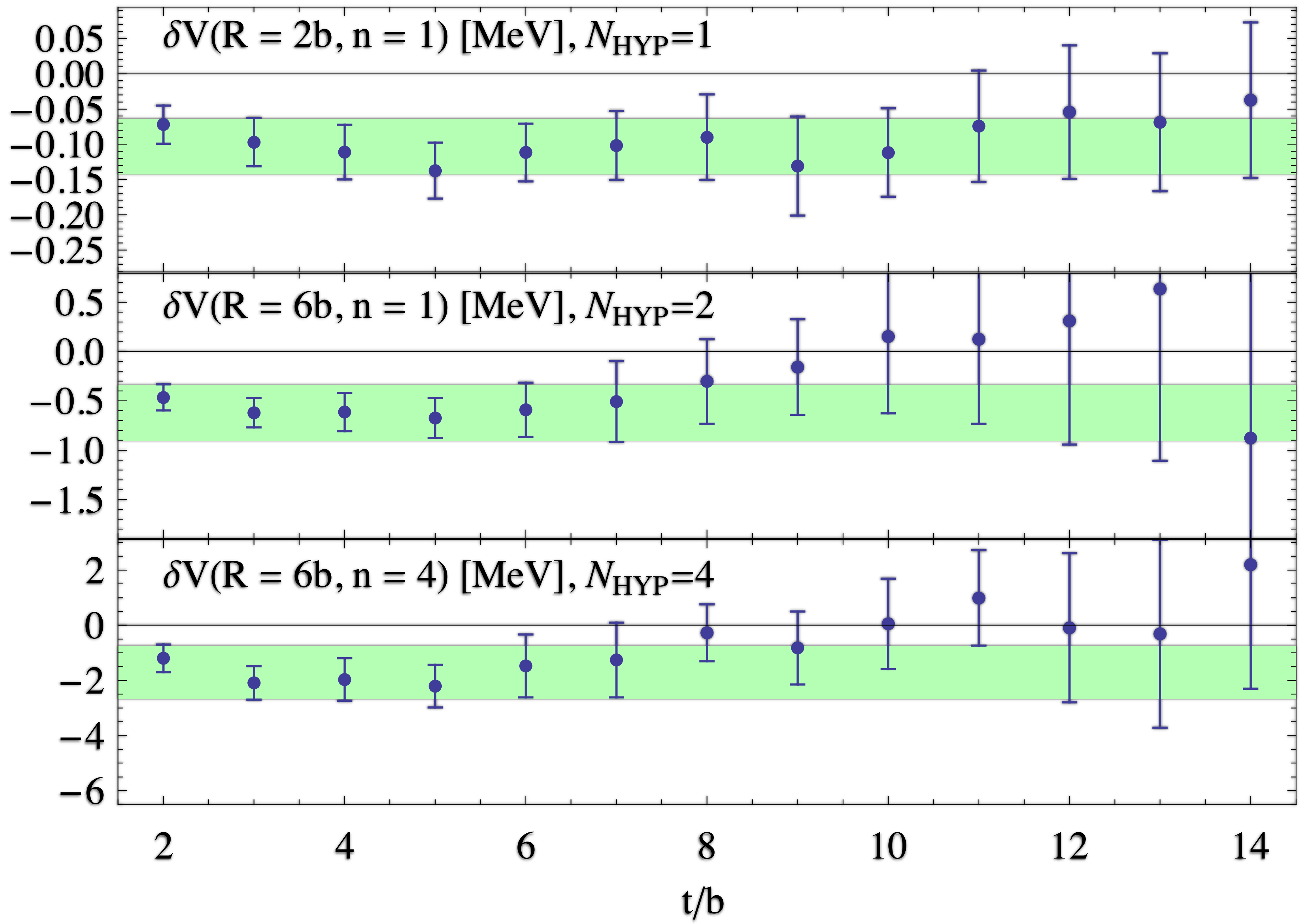


In pictures



Effective δV plots

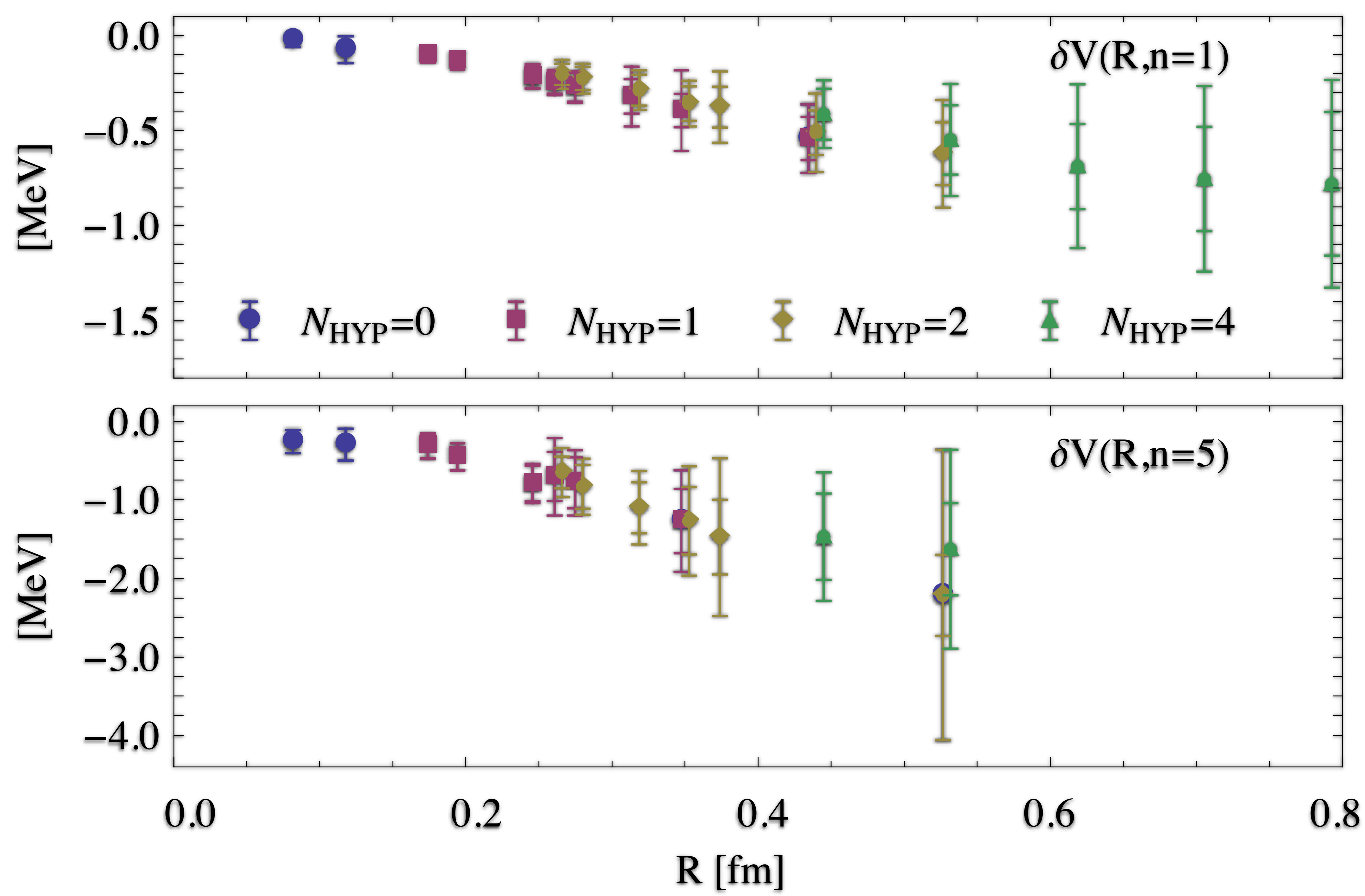
[WD & M Savage, PRL 102:032004, 2009]

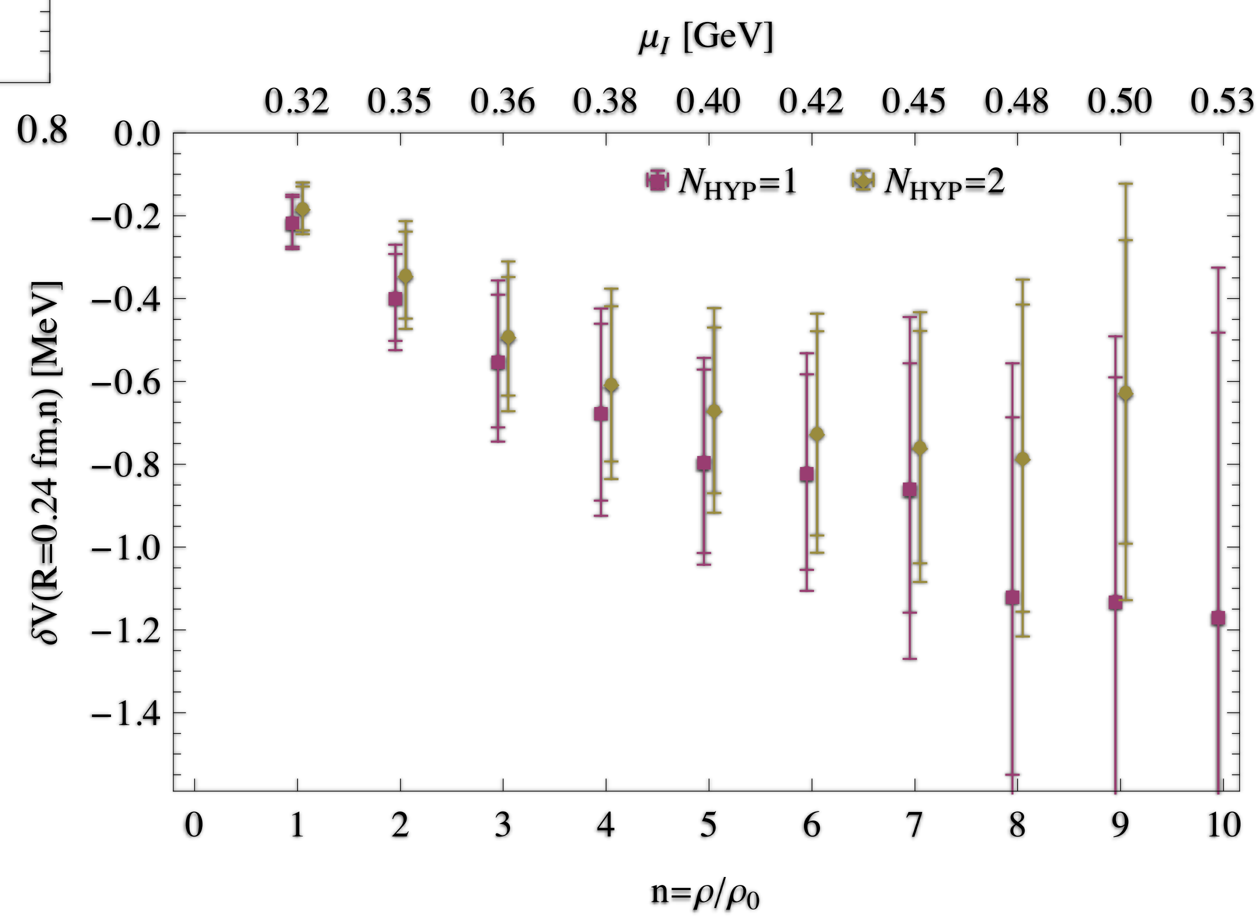


- DWF on MILC: a=0.09 fm, 28^3x96, m_π=318 MeV

δV(R, n)

[WD & M Savage, PRL 102:032004, 2009]

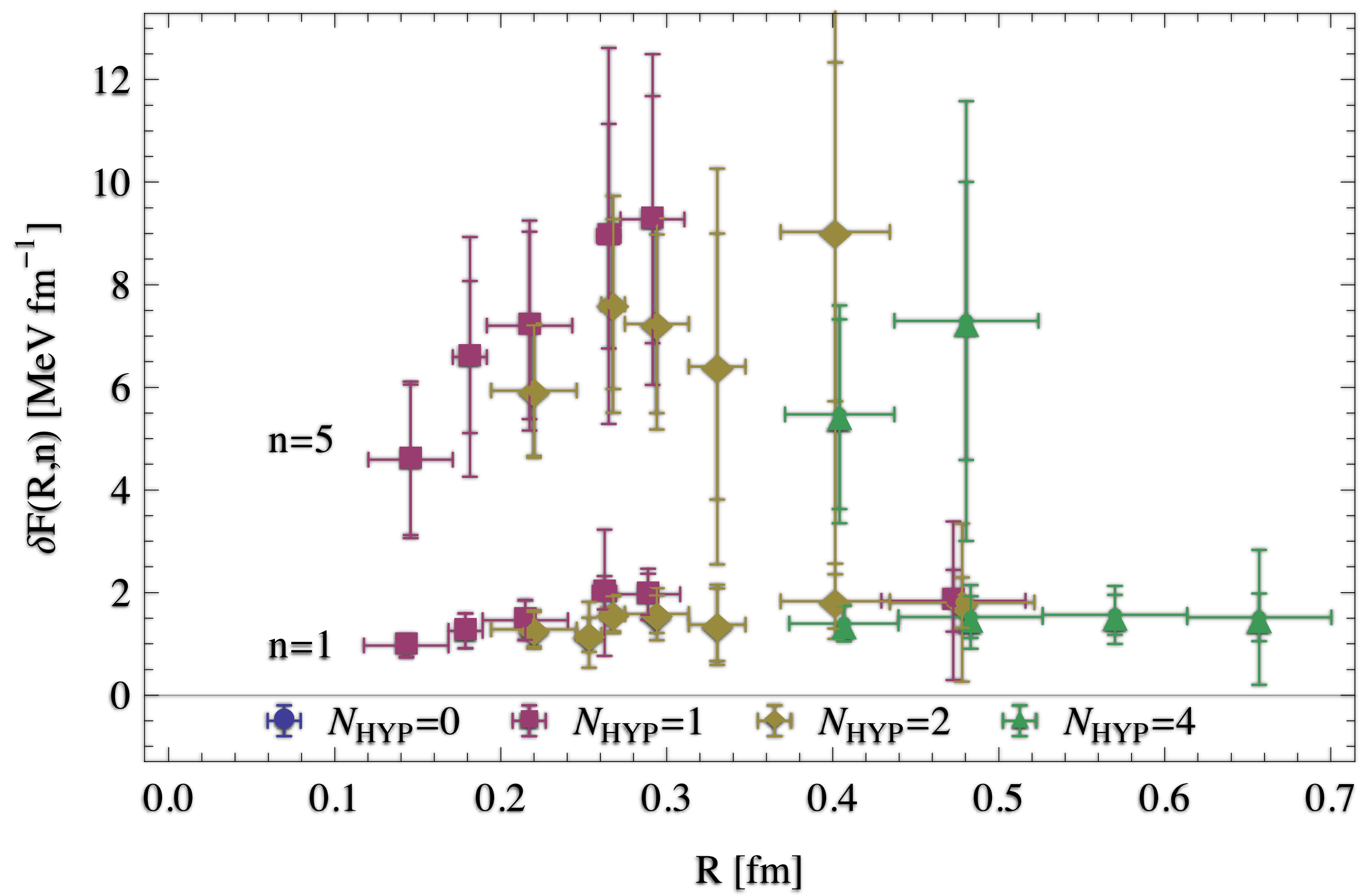




- 100 keV effect at small n, small R
- Approximately linear in isospin density

δF(R, n=1 & 5)

[WD & M Savage, PRL 102:032004, 2009]



- Small effect: δF(n=1)/F = 2/1000 at large R
- Constant at large R
  - Dielectric medium inside flux tube

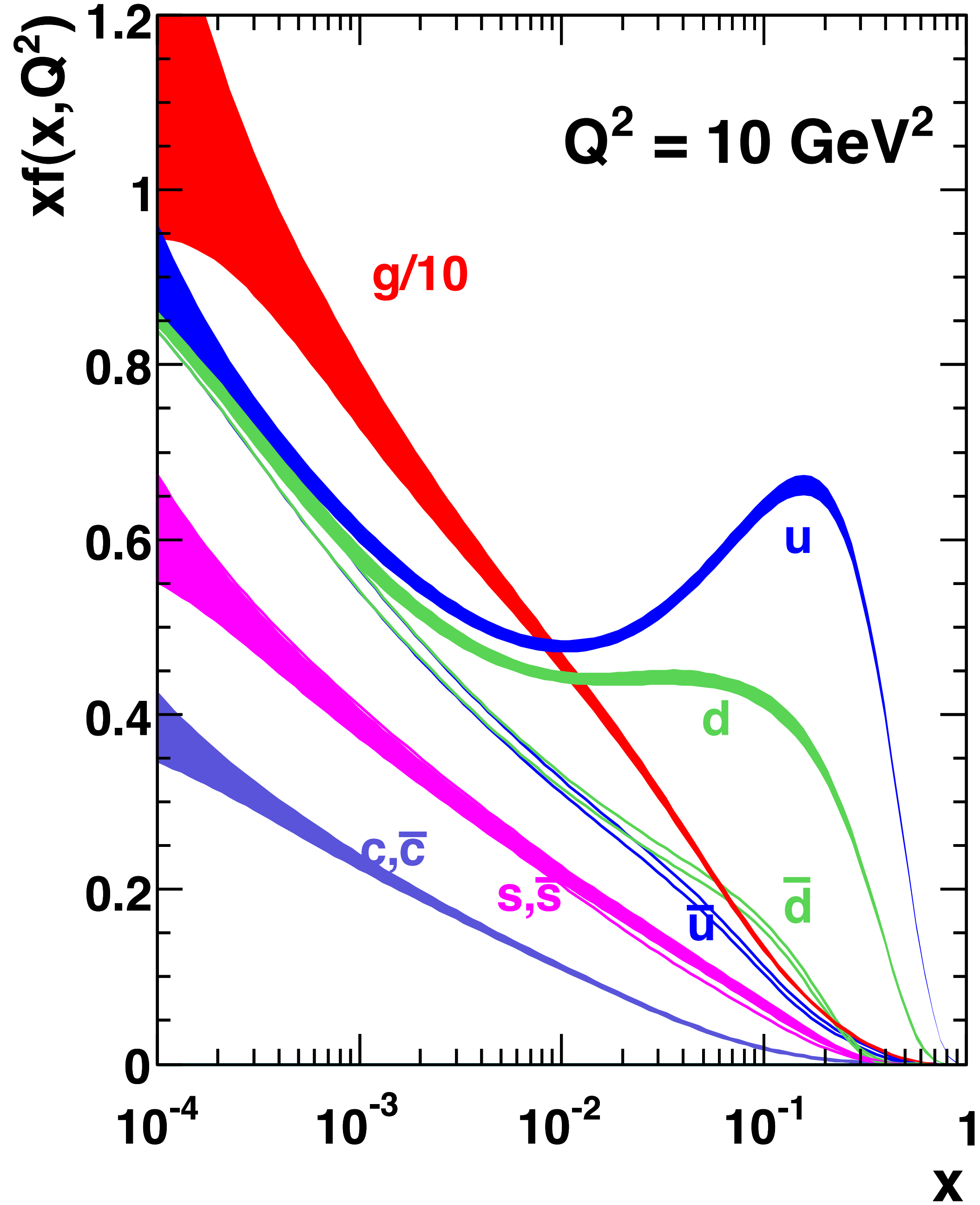
Hadron structure in QCD

- Deep inelastic scattering experiments probe parton distribution functions $q_H(x)$
  - Probability of finding a parton (q,g) in hadron h carrying longitudinal momentum fraction x
- Operator product expansion: Mellin moments of PDFs defined by forward matrix elements of local operators

$$\langle x^n \rangle_H = \int_{-1}^{1} dx\, x^n q_H(x)$$

$$\langle H | \overline{\psi} \gamma^{\{\mu_0} D^{\mu_1} \dots D^{\mu_n\}} | H \rangle = p^{\{\mu_0} \dots p^{\mu_n\}} \langle x^n \rangle_H$$

- n=1 corresponds to LC momentum fraction carried by quarks inside H
- Phenomenologically find DIS on nuclei

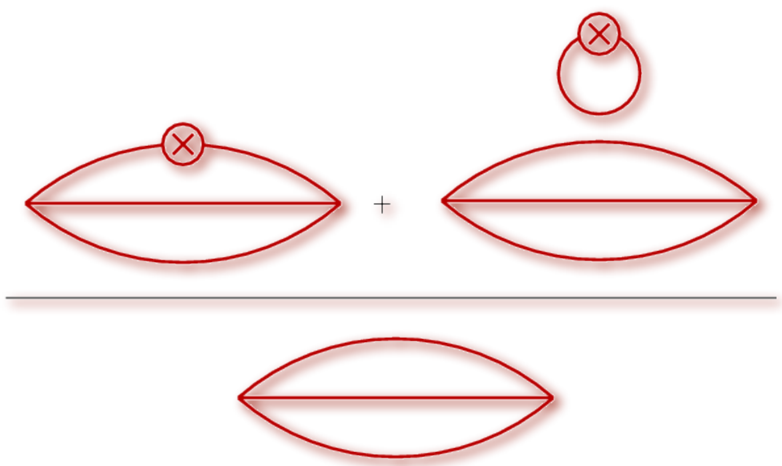


Hadron structure in QCD

- Proton structure ntensively studied in QCD using 3-pt functions (see James Zanotti's lectures next week)

$$C_2(t,\mathbf{p}) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \langle 0|\chi_H(0)\chi_H^\dagger(\mathbf{x},t)|0\rangle$$

$$C_3(t,\mathbf{p}) = \sum_{\mathbf{y},\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \langle 0|\chi_H(0)\mathcal{O}(\mathbf{y},\tau)\chi_H^\dagger(\mathbf{x},t)|0\rangle$$



$$R = \frac{C_3(t,\mathbf{p})}{C_2(t,\mathbf{p})} \xrightarrow{t\to\infty} \langle H|\mathcal{O}|H\rangle$$

- Limited to low moments by reduced lattice symmetry
- Most studies for nucleon, but also pion, rho, ...
- Disconnected term often neglected (absent for isovector quantities)
- *What about multi-baryon structure (EMC effect)?*

Many meson 3-point correlator

[WD & H-W Lin, in progress]

- Pionic analogue of EMC effect
- *n* $\boldsymbol{\pi}^+$ 3-point correlator

$$C_m^{(n)}(\tau, t, \mathbf{p}) = \left\langle 0 \right| \left[\prod_{i=1}^{m} \sum_{\mathbf{x}} e^{i\mathbf{p}_i \cdot \mathbf{x}} \chi(\mathbf{x}, t) \right] \sum_{\mathbf{y}} e^{i\mathbf{q}\cdot\mathbf{y}} \mathcal{J}(\mathbf{y}, \tau) \left[\chi^\dagger(x_0) \right]^m \left| 0 \right\rangle$$

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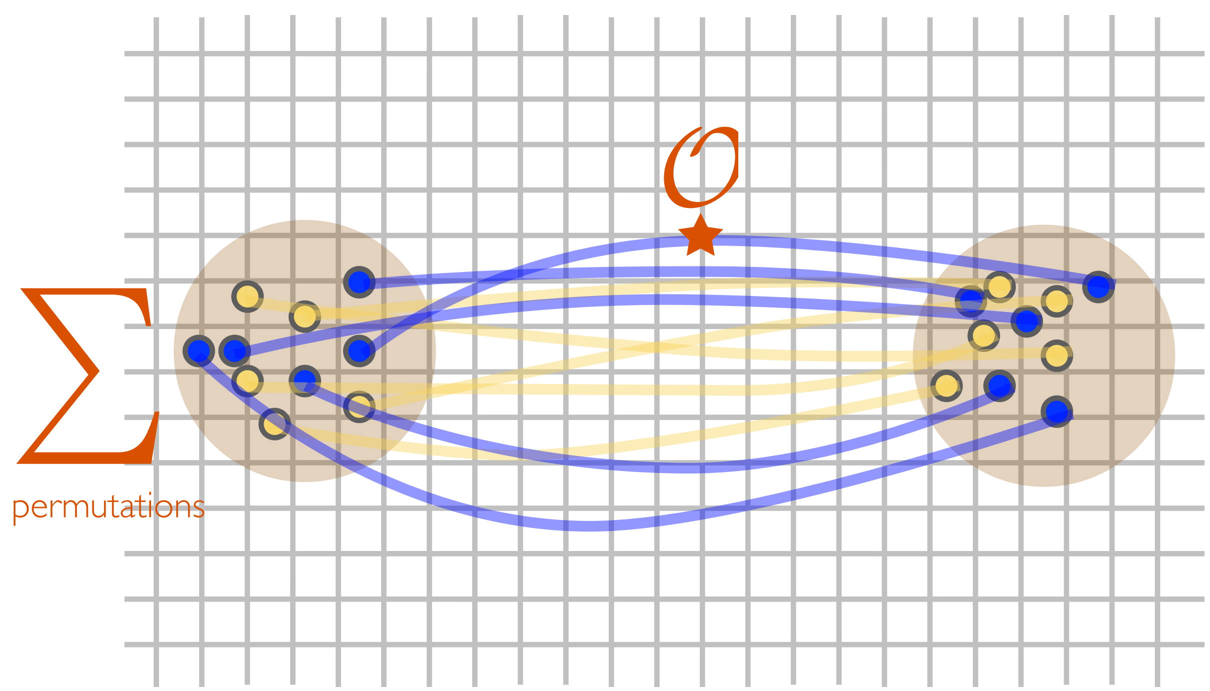

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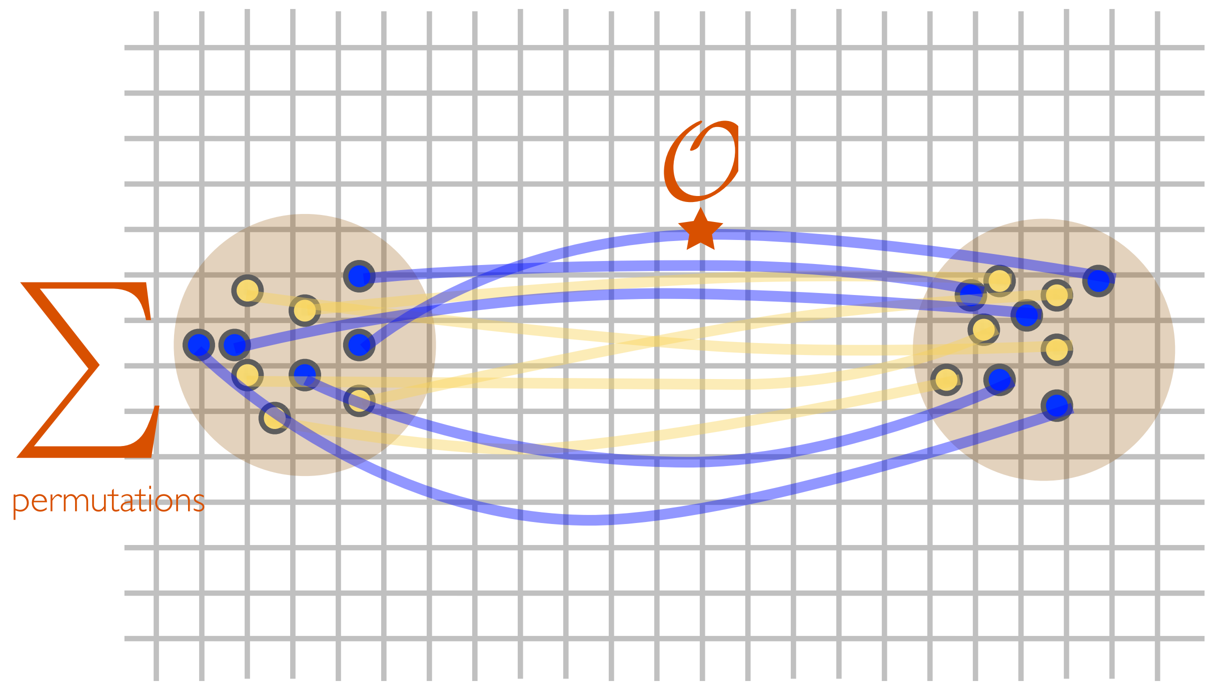
$$C_m^{(n)}(\tau, t, \mathbf{p}) = \left\langle 0 \right| \left[\prod_{i=1}^{m} \sum_{\mathbf{x}} e^{i\mathbf{p}_i \cdot \mathbf{x}} \chi(\mathbf{x}, t) \right] \sum_{\mathbf{y}} e^{i\mathbf{q}\cdot\mathbf{y}} \mathcal{J}(\mathbf{y}, \tau) \left[\chi^\dagger(x_0) \right]^m \left| 0 \right\rangle$$

$$\longrightarrow Z_m \langle \mathcal{O}_m^{(n)} \rangle e^{-E_m t}$$

where $\langle \mathcal{O}_m^{(n)} \rangle = \langle m\pi | \mathcal{J}^{(n)} | m\pi \rangle$



Many meson 3-point correlator

[WD & H-W Lin, in progress]

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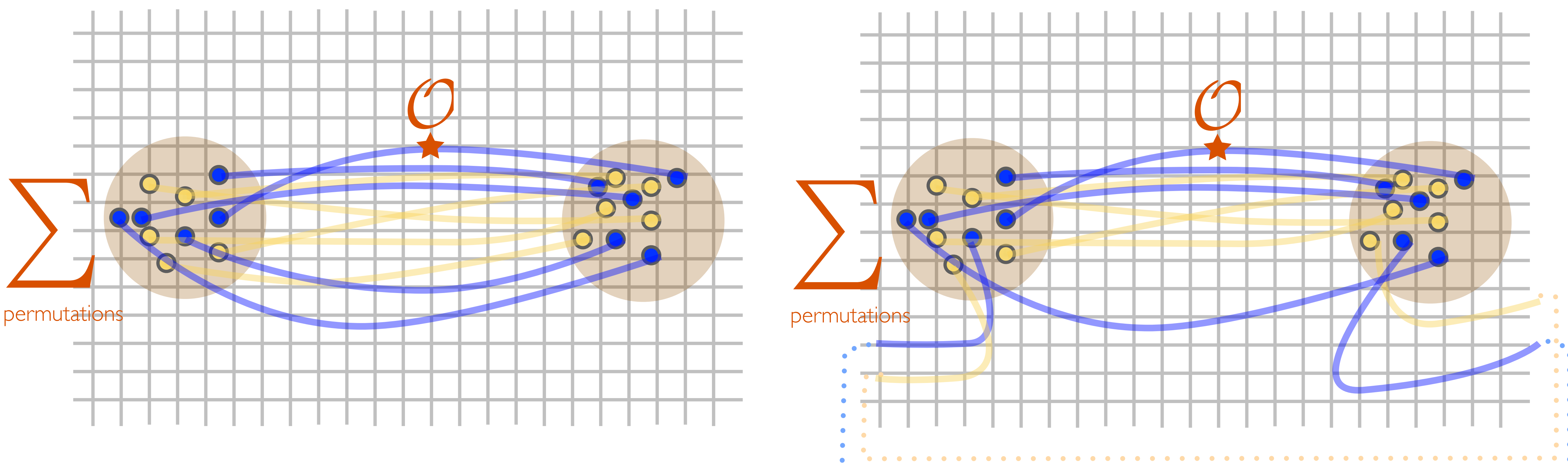

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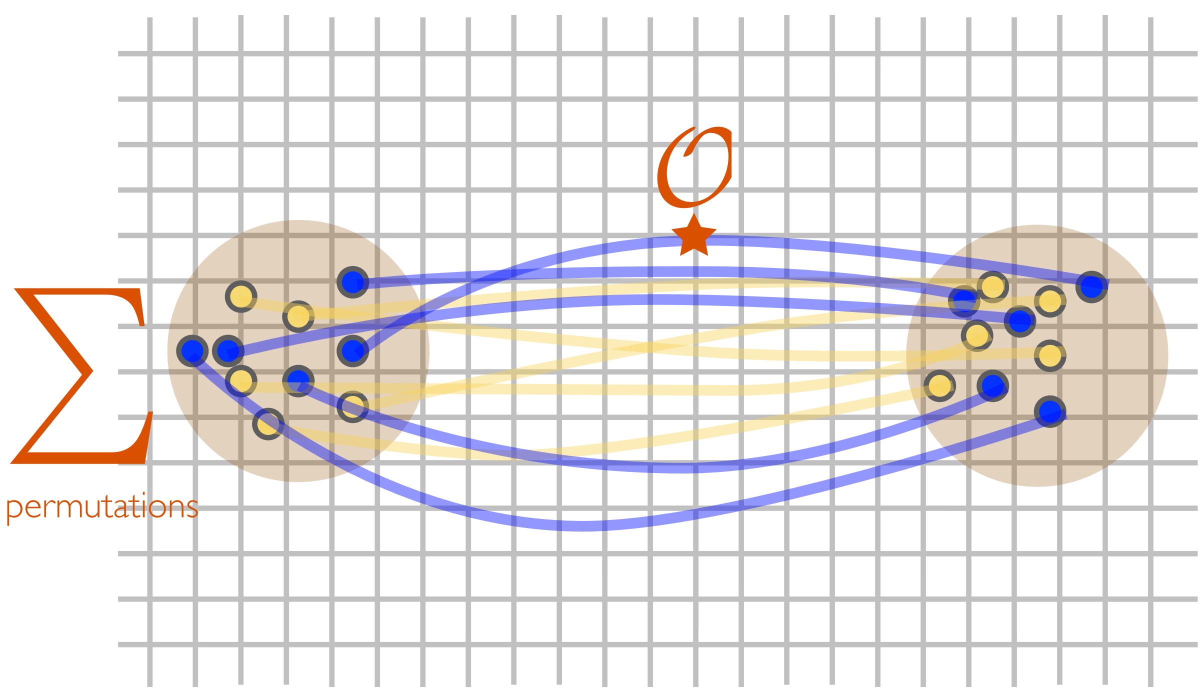
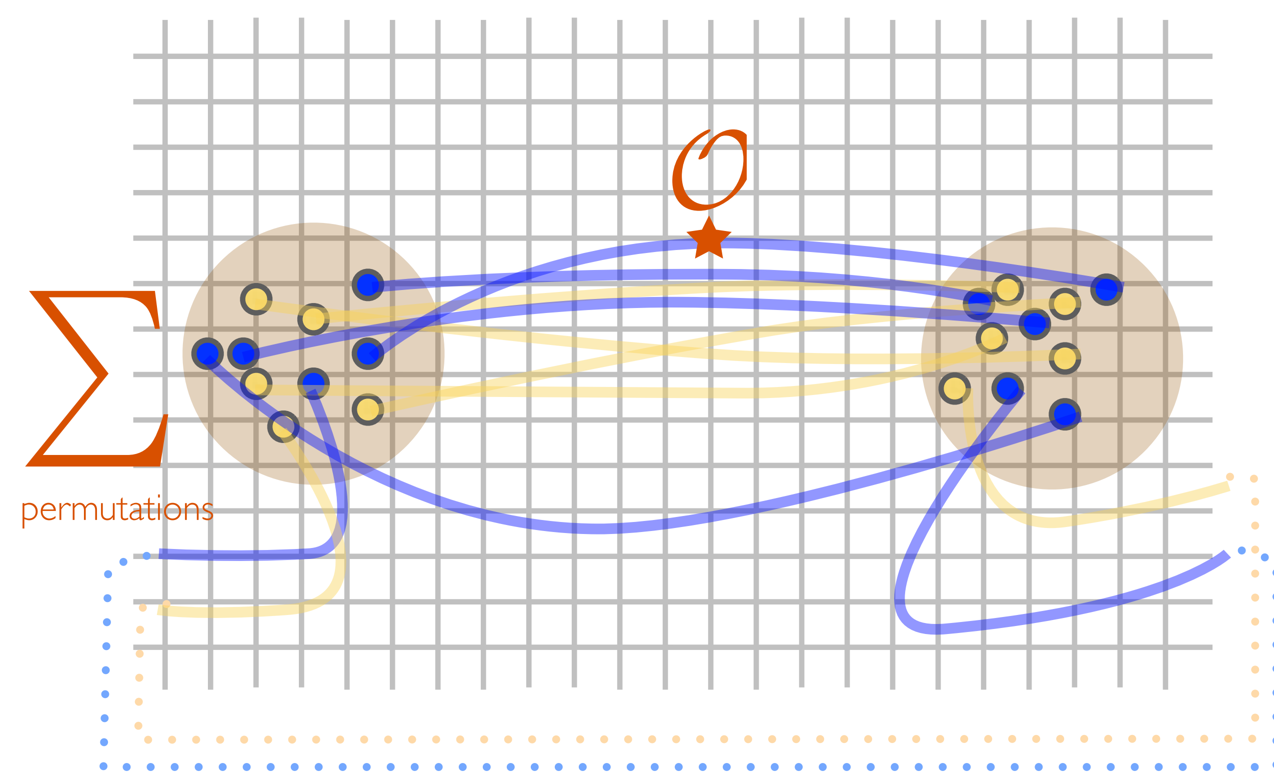
$$\longrightarrow \sum_{\ell=0}^{m} \begin{pmatrix} m \\ \ell \end{pmatrix} Z_m^{(\ell)} \langle \mathcal{O}_{m-\ell}^{(n)} \rangle e^{-E_{m-\ell} t} e^{-E_\ell (T-t)}$$

where $\langle \mathcal{O}_m^{(n)} \rangle = \langle m\pi | \mathcal{J}^{(n)} | m\pi \rangle$





Backwards propagator contamination

- Thermal contamination gets very bad near the midpoint of the temporal extent
  - Fraction of non-thermal contributions to 2pt correlator (T=64 here)

$m_\pi \mathbf{T}$=11.6
m_π=290 MeV

$m_\pi \mathbf{T}$=19.9
m_π=590 MeV

- Trying to measure three point function at t>T/4 is problematic – nothing to do with physically relevant state

Many meson 3-point correlator

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$$\longrightarrow Z_m\langle\mathcal{O}_m^{(n)}\rangle e^{-E_m t} \quad + \text{ excitations and thermal effects}$$

- Contractions performed by treating the struck meson as a separate species

Colour/Dirac structure of operator

$$\Pi = \sum_{\mathbf{x}}\gamma_5 S(\mathbf{x},t;0)\gamma_5 S^\dagger(\mathbf{x},t;0), \qquad \tilde{\Pi}_\tau =_{\mathbf{x},\mathbf{y}} \gamma_5 S(\mathbf{x},t;\mathbf{y},\tau)\Gamma_{\mathcal{O}} S(\mathbf{y},\tau;0)\gamma_5 S^\dagger(\mathbf{x},t;0)$$

- System now looks like (m-1) pions + 1 "kaon"
  - Can be written as products of traces of two matrices
    [WD & B Smigielski, arXiv:1103.4362]

Double ratio

- Define ratio to extract matrix elements (eg for momentum fraction)

$$R^{(n)}(t,\tau) = \frac{C_3^{(n)}(t;\tau)}{C_2^{(n)}(t)} \xrightarrow{t\gg\tau} \frac{1}{E_{n\pi}} \langle n\ \pi^+ | \mathcal{O}^{44} | n\ \pi^+ \rangle$$

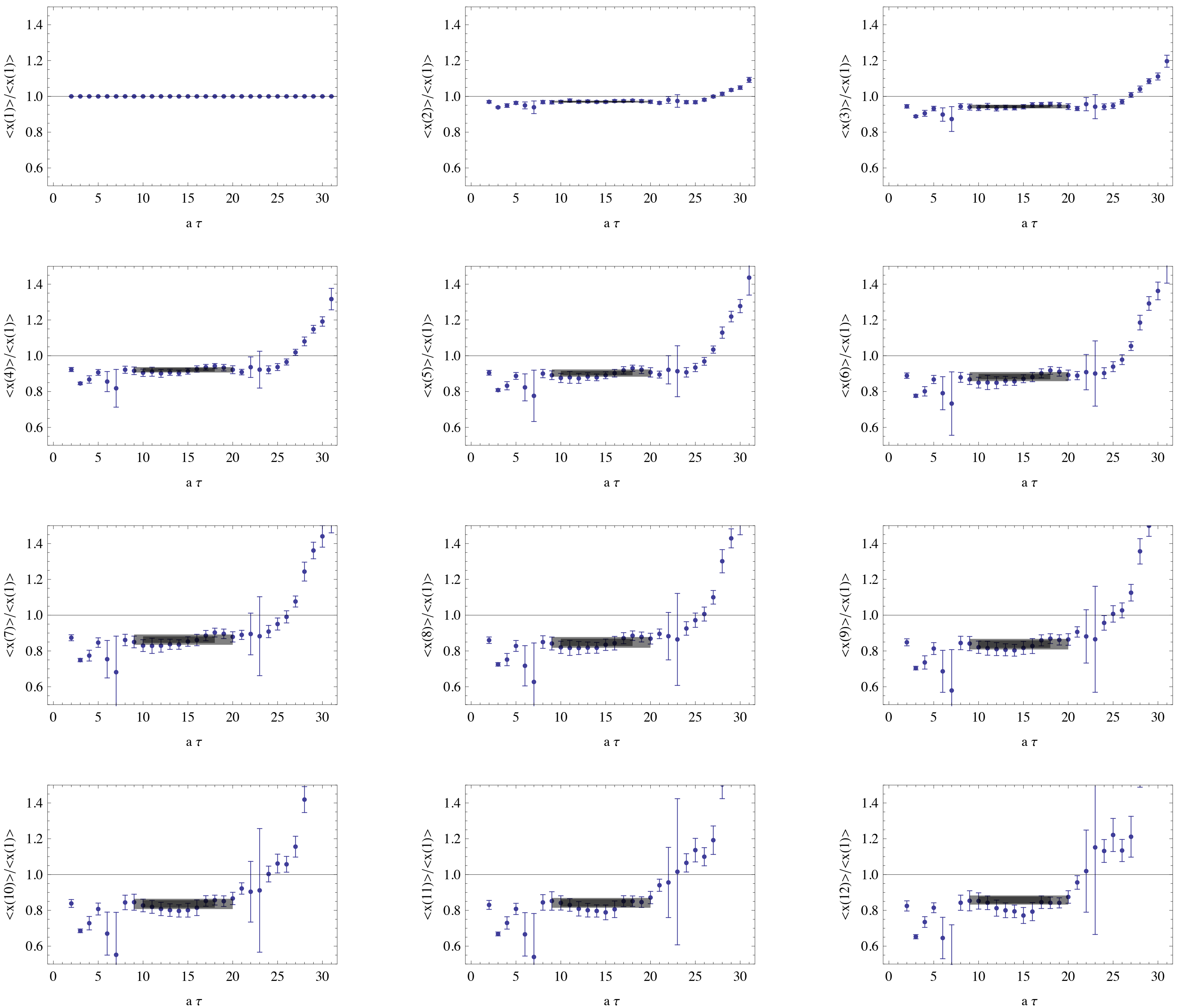
- Double ratio – allows direct investigation of ratio of moments

$$\frac{R^{(n)}(t,\tau)}{R^{(1)}(t,\tau)} \longrightarrow \frac{m_\pi\ \langle n\ \pi^+ | \mathcal{O}^{44} | n\ \pi^+ \rangle}{E_{n\pi}\ \langle \pi^+ | \mathcal{O}^{44} | \pi^+ \rangle} \longrightarrow \frac{E_{n\pi}\ \langle x \rangle_{n\pi^+}}{m_\pi \langle x \rangle_{\pi^+}}$$

  - No need to renormalise operator!

- Calculate ratios for various quark masses [DWF valence on MILC sea]

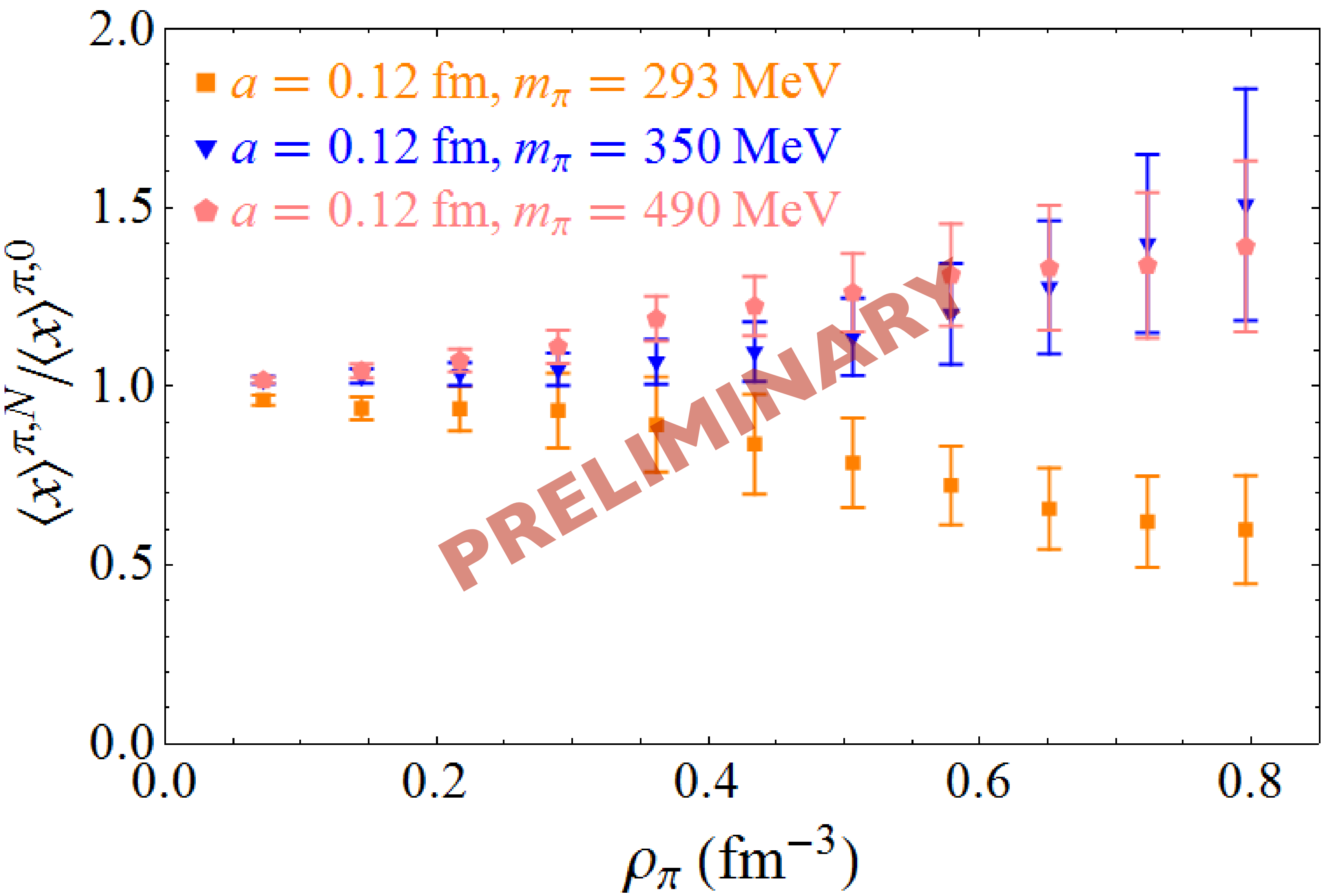
Double ratio



DWF on MILC
m_π = 350 MeV
a=0.12 fm, 20^3x64

Medium modification

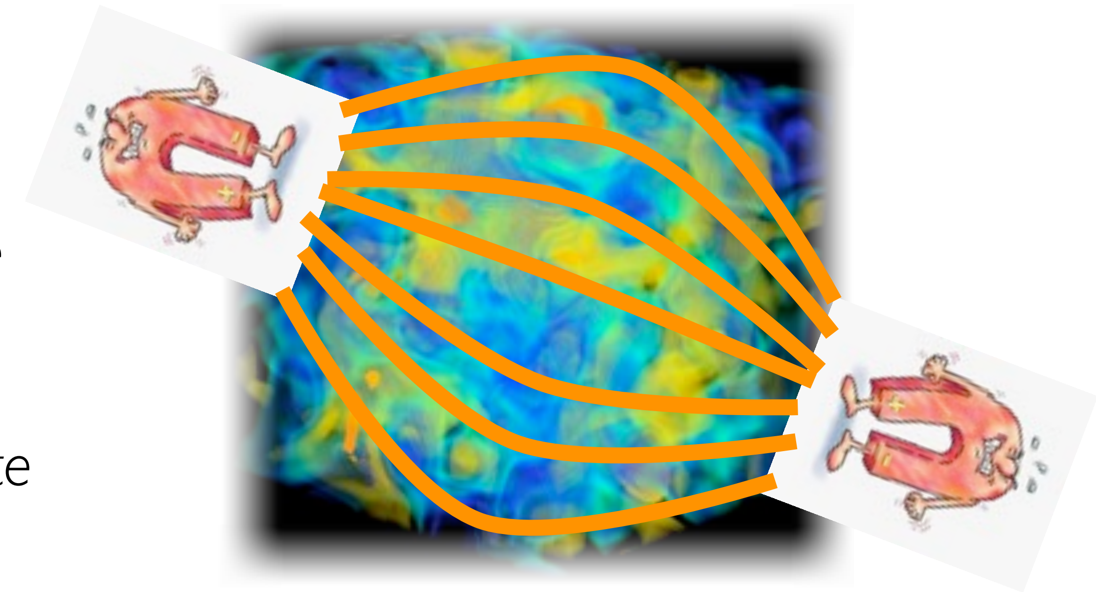
- Extracted ratio of moments is not unity – medium modification of pion stucture



- Extension to baryons certainly possible but messier as usual!

Matrix elements in multi-hadron systems

- Many pion PDF moments are one example of matrix elements of multi-hadron systems
- Other theoretical investigations
  - WD & M Savage "*Electroweak matrix elements in the two nucleon sector from lattice QCD*" hep-lat/0403005
  - H Meyer, "*Photodisintegration of a Bound State on the Torus*", 1202.6675
  - V Bernard, D Hoya, U-G Meißner & A Rusetsky, "*Matrix elements of unstable particles*" 1205.4642

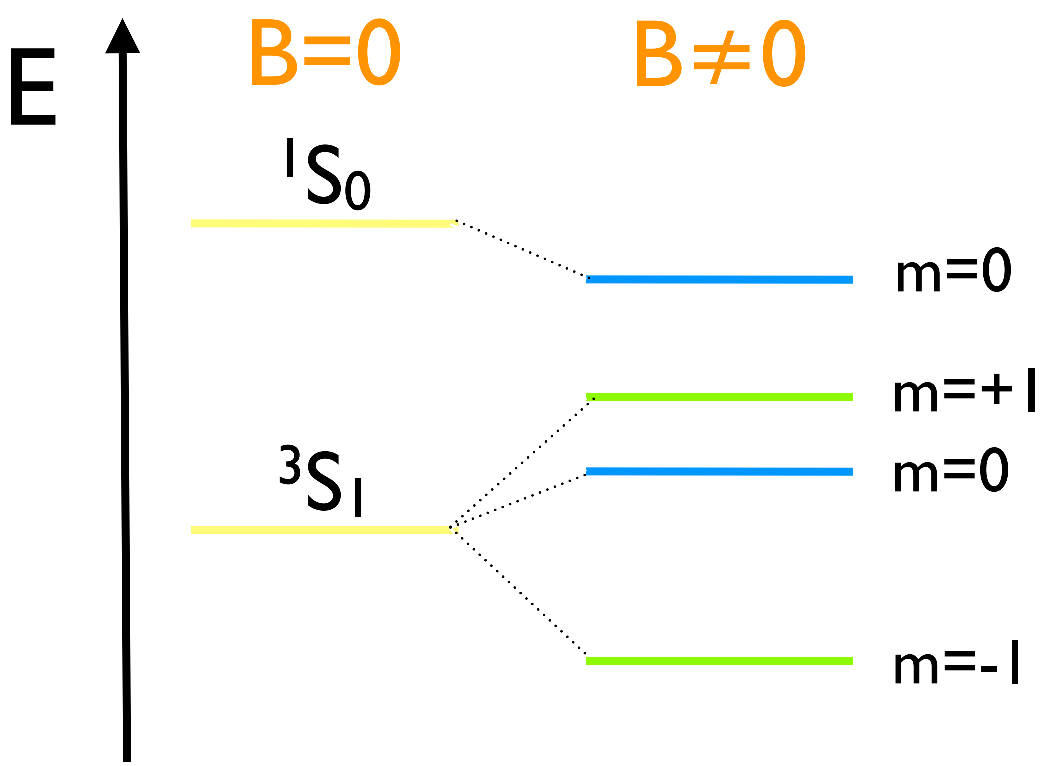


Background fields

- Consider QCD in the presence of a constant background magnetic field
  - Implement by adding term to the action (careful with boundaries)
- Shifts spin-1/2 particle masses

$$M_{\uparrow\downarrow} = M_0 \pm \mu|\mathbf{B}| + 4\pi\beta|\mathbf{B}|^2 + \ldots$$

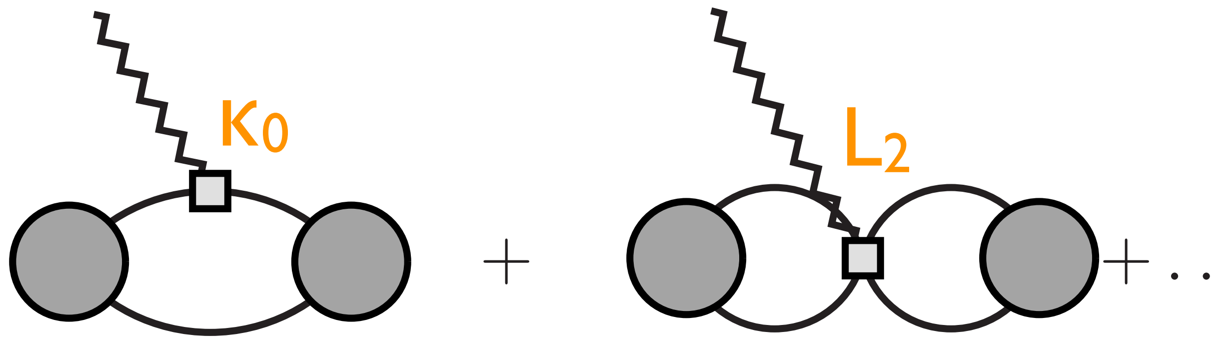
  - Changing strength of background field allows μ, β to be extracted
- Two nucleon states
  - Levels split and mix
  - Landau levels:
- Similar for electro-weak fields and twist-two fields



EFT two-body currents

- Two-body contributions

$$\langle d|\mathcal{O}|d\rangle =$$



- Magnetic moment: two body modification L2

$$\mu_d = \frac{2}{1-\gamma r_3}(\gamma L_2 + \kappa_0)$$

- Twist-two current: leading EMC effect α_n (more complicated as necessary to include pions)

$$\langle x^n\rangle_d = 2\langle x^n\rangle_N + \alpha_n\langle d|(N^\dagger N)^2|d\rangle + \ldots$$

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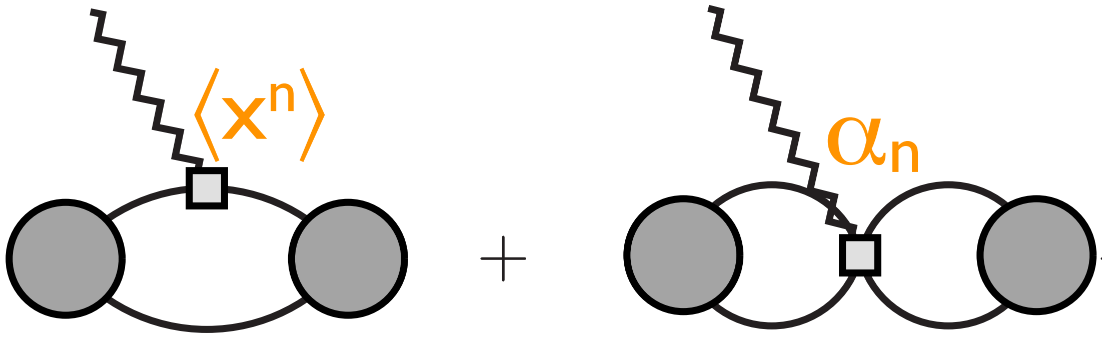
$\langle d|\mathcal{O}|d\rangle =$  $+ \ldots$

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Energy levels in BF

- Background field modifies eigenvalue equation for m=±1 states

$$p\cot\delta(p) - \frac{1}{\pi L} S\left(\frac{L^2}{4\pi^2}\left[p^2 \pm e|\mathbf{B}|\kappa_0\right]\right) \mp \frac{e|\mathbf{B}|}{2}\left(L_2 - r_3\kappa_0\right) = 0$$

- Asymptotic expansion of lowest scattering level

$$E_0^{m=\pm 1} = \mp\frac{e|\mathbf{B}|\kappa_0}{M} + \frac{4\pi A_3}{ML^3}\left[1 - c_1\frac{A_3}{L} + c_2\left(\frac{A_3}{L}\right)^2 + \ldots\right]$$

where $\frac{1}{A_3} = \frac{1}{a_3} \pm \frac{e|\mathbf{B}|L_2}{2}$

- Mixes $^1S_0$ and $^3S_1$ m=0 states (coupled channels – but perturbative)

$$\left[p\cot\delta_1(p) - \frac{S_+ + S_-}{\pi L}\right]\left[p\cot\delta_3(p) - \frac{S_+ + S_-}{\pi L}\right] = \left[\frac{e|\mathbf{B}|L_1}{2} + \frac{S_+ - S_-}{2\pi L}\right]^2$$

where

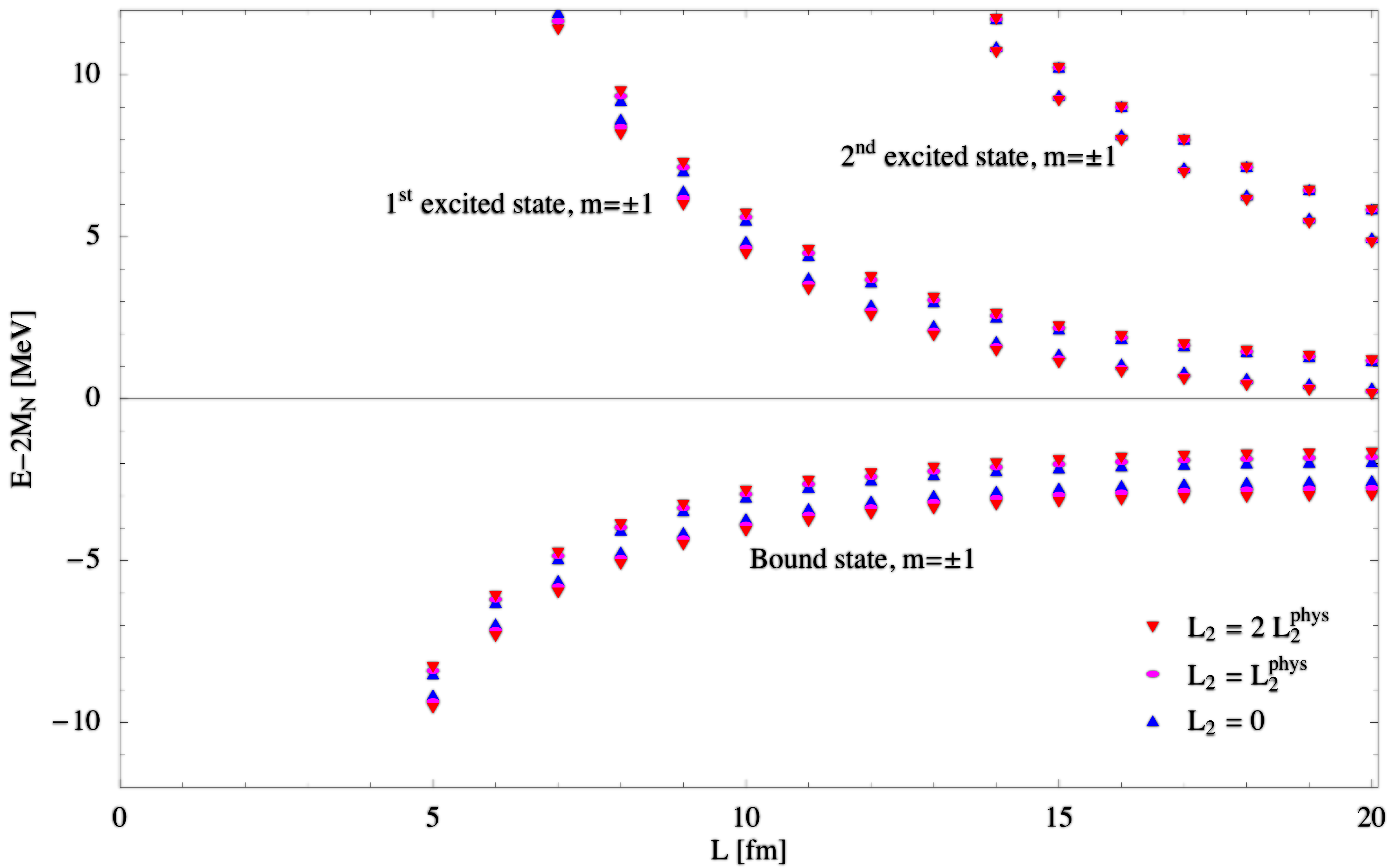
$$S_\pm = S\left(\frac{L^2}{4\pi^2}\left[p^2 \pm e|\mathbf{B}|\kappa_1\right] + \ldots\right)$$

[WD & MJ Savage Nucl Phys A 743, 170]

Energy levels in B field

EFT prediction for behaviour of m=±1 energy levels

$|e\,B| = 1000\ \mathrm{MeV}^2$



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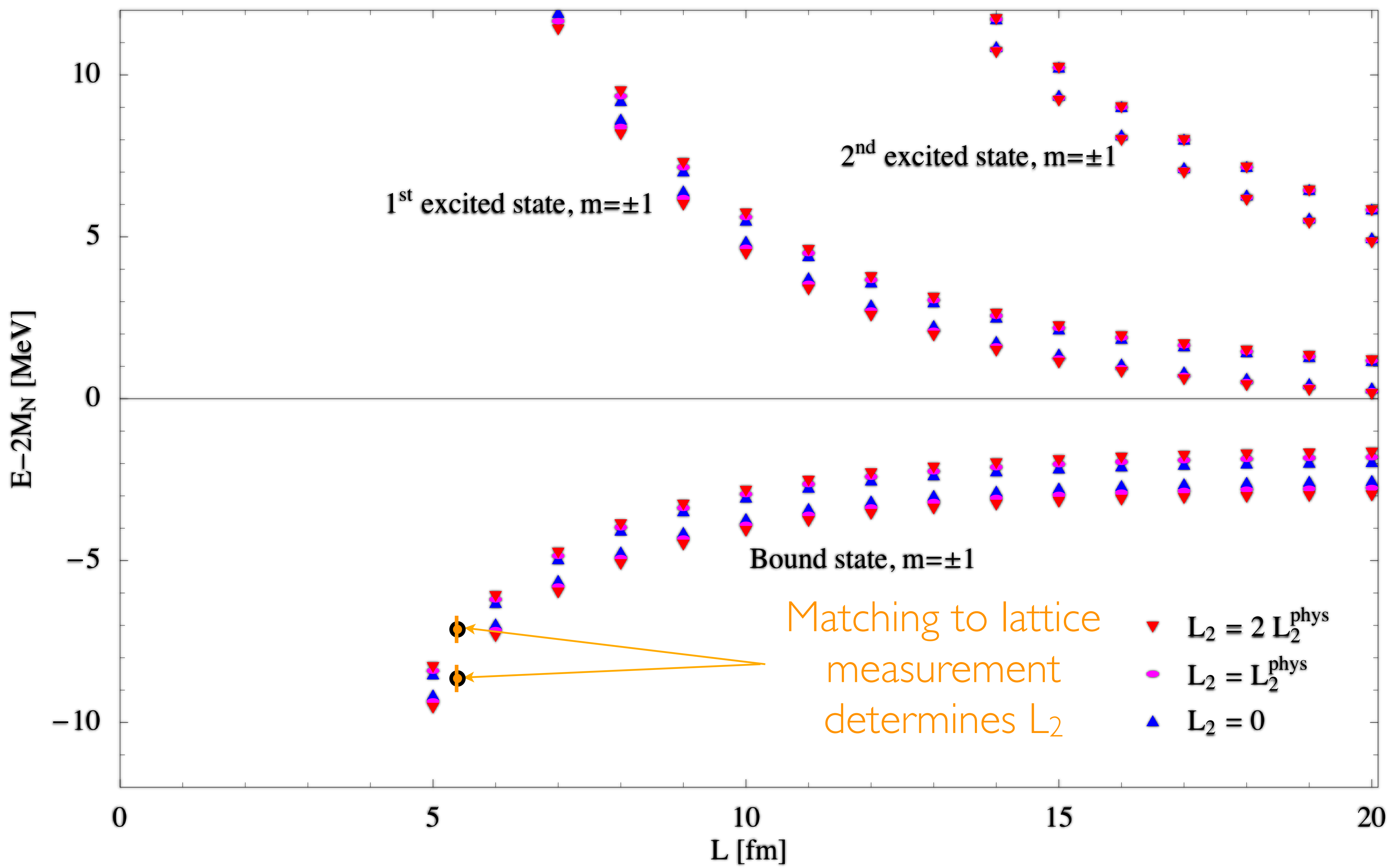
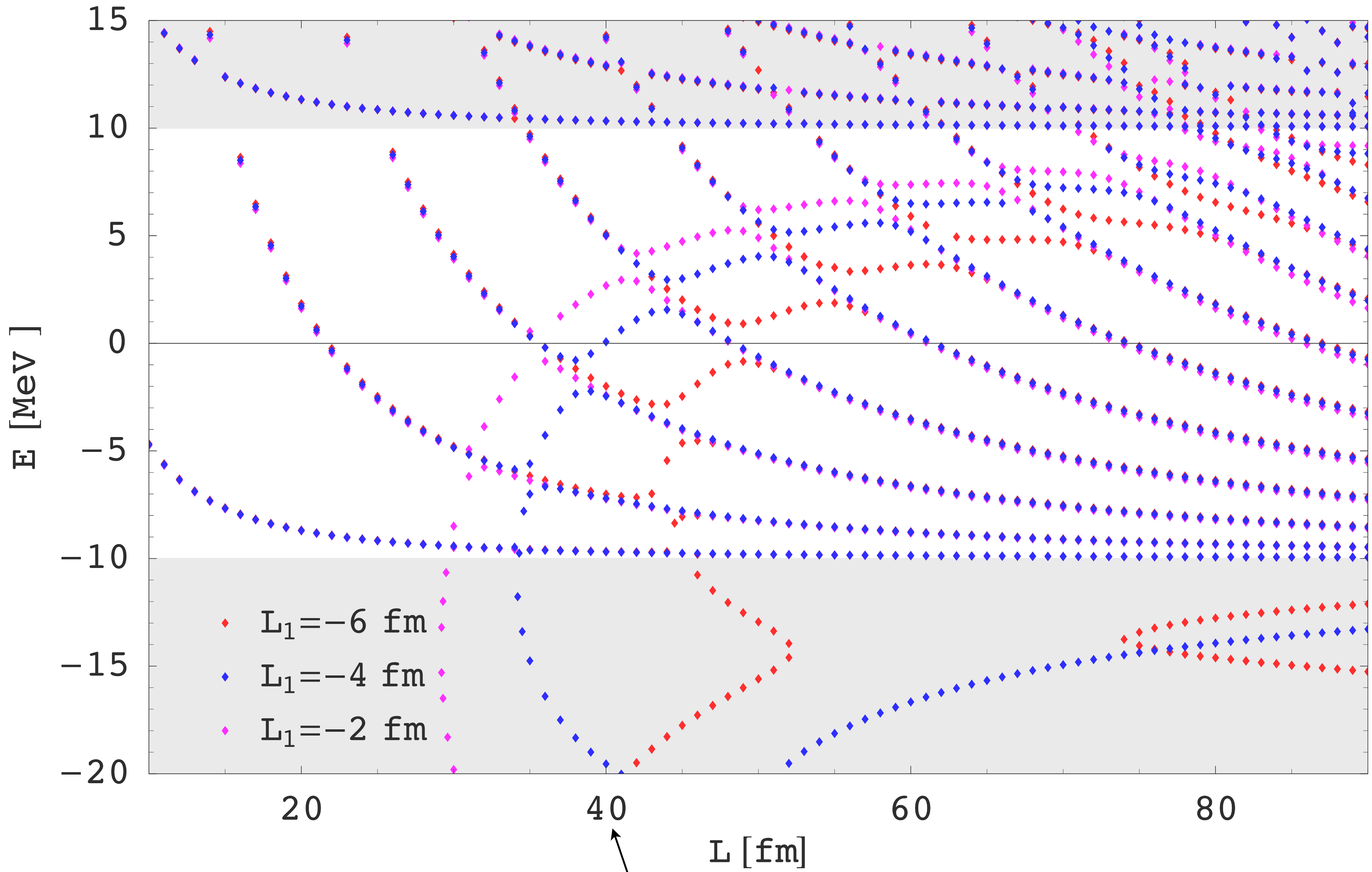


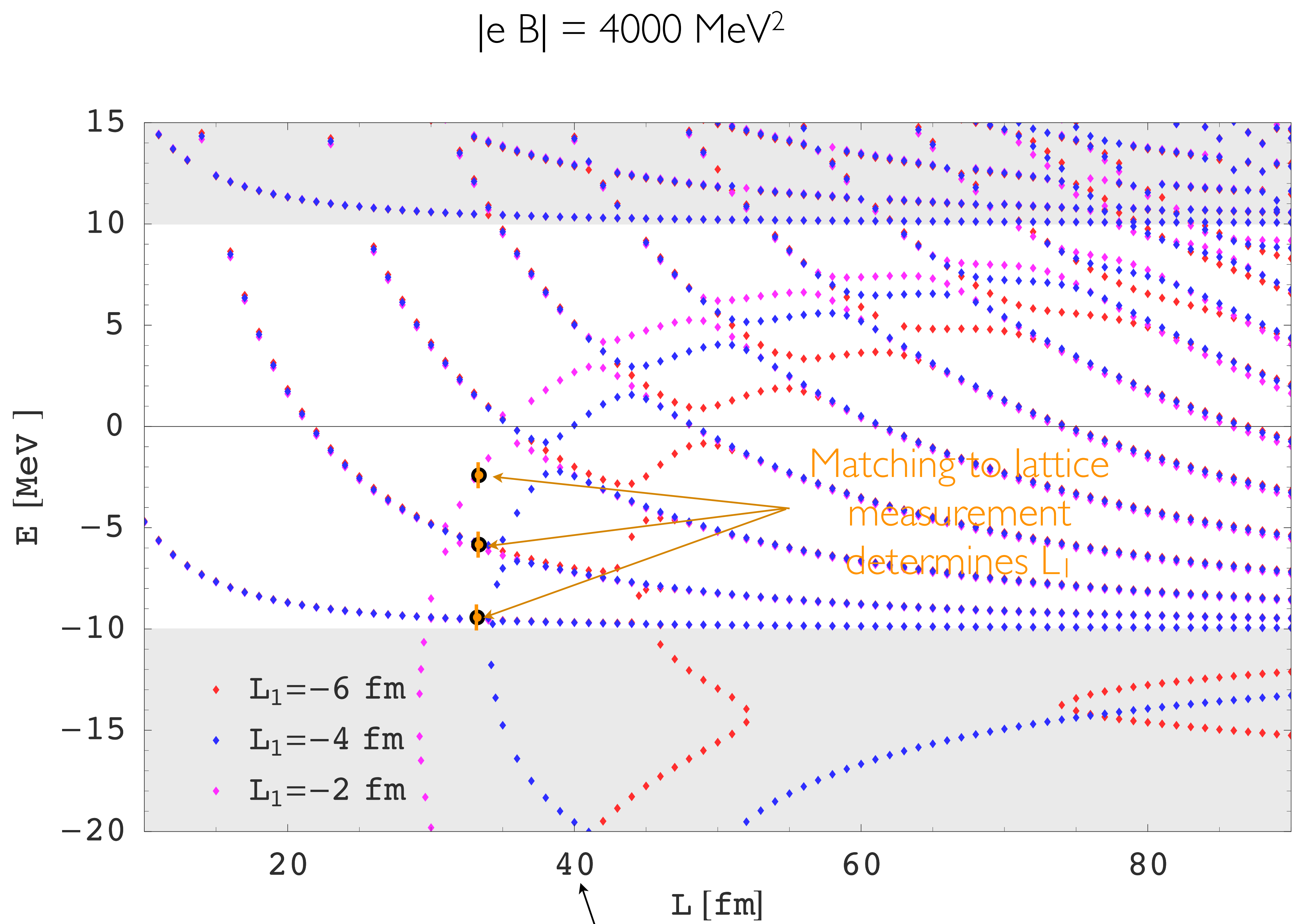
np→dγ: $^3S_1$–$^1S_0$ m=0

|e B| = 4000 MeV^2



NB: box is asymmetric: 4x4x40 fm^3

np→dγ: $^3S_1 - {}^1S_0$ m=0



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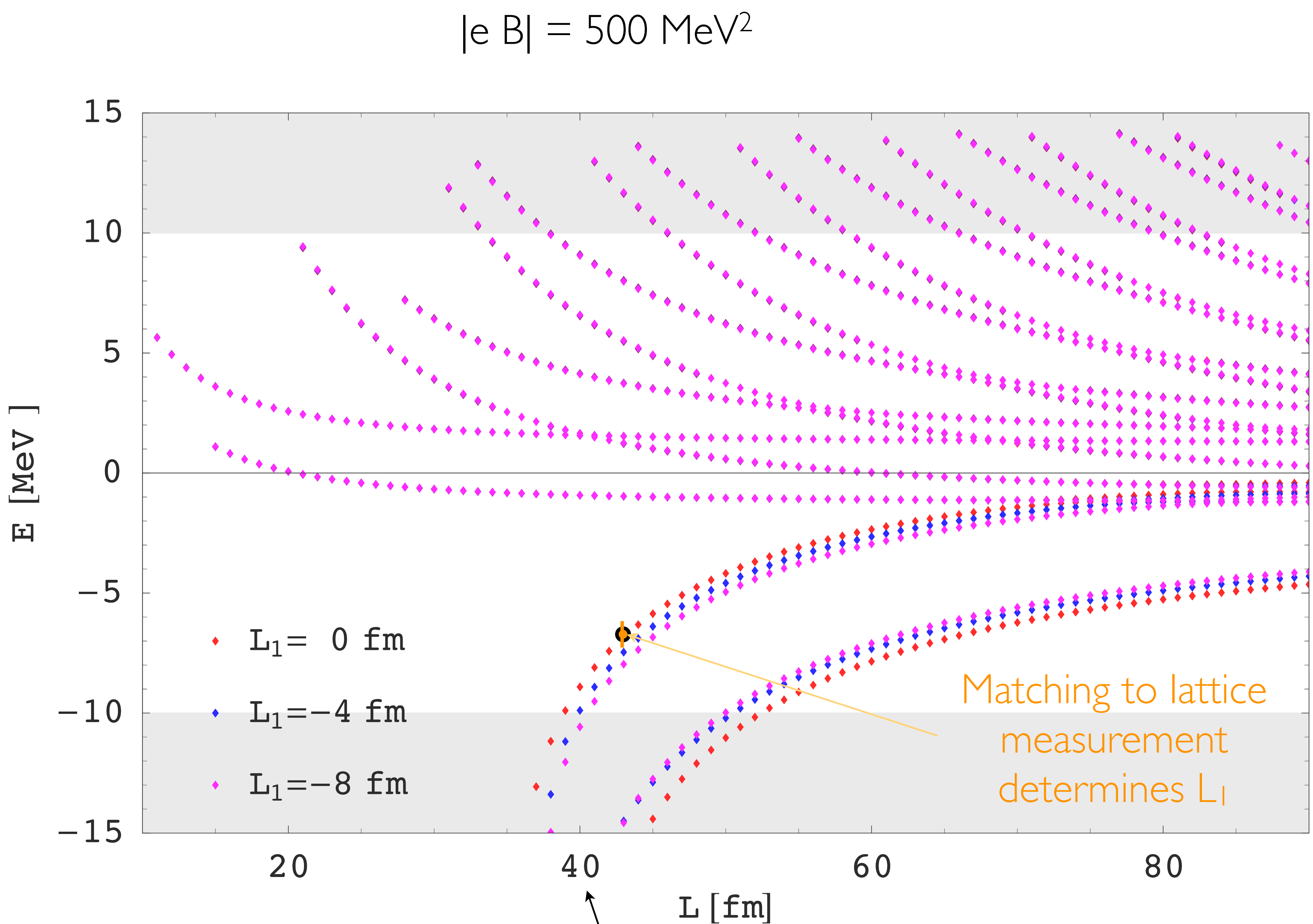

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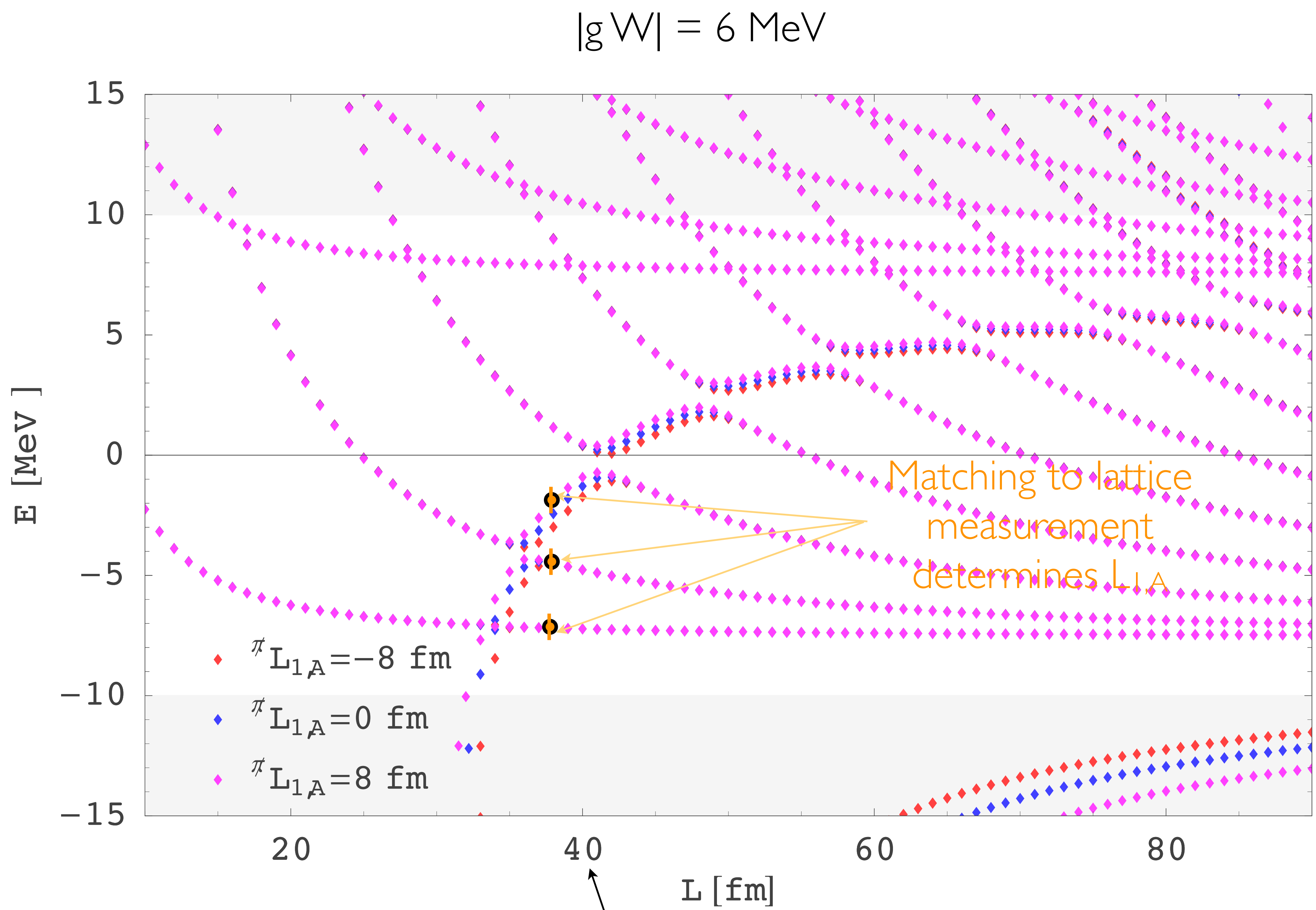
$\nu d \to np$: EW BF



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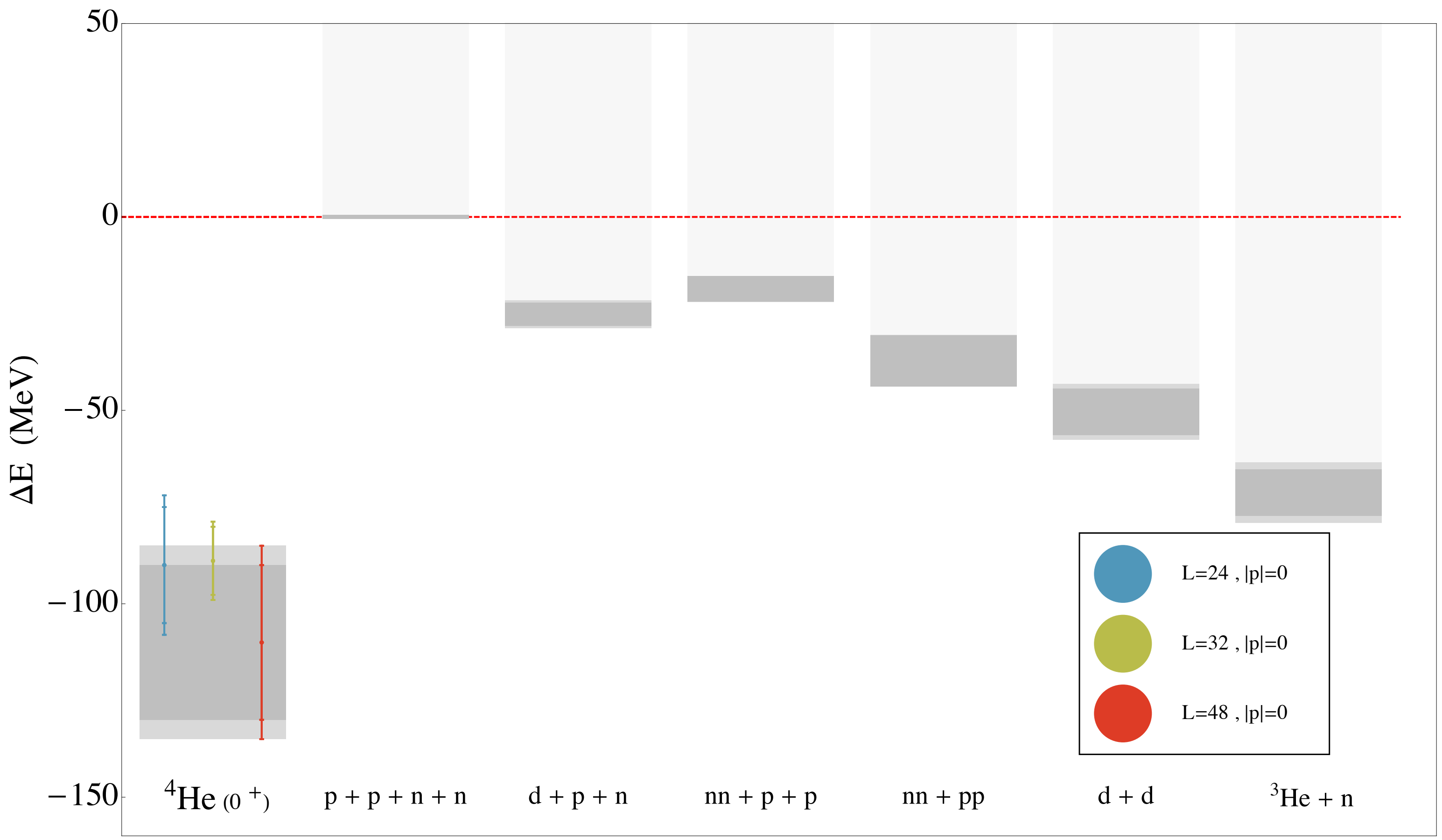
Open issues

Noise

- Noise in QCD correlators is generically a problem – somehow related to the sign problem discussed in Gert Aarts' lectures
- There are hints that we can suppress noise for certain choices of correlation functions
  - How effectively can this be systematised?
  - Can this be done for large A systems that we afford to perform contractions for?
- Are we measuring things the most sensible way?
- David K will say a lot more about noise on Friday

Density of states

- One specific issue that is a bit frightening at the moment is the density of scattering states in multi-hadron systems



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- Back to Maiani-Testa No-go Theorem

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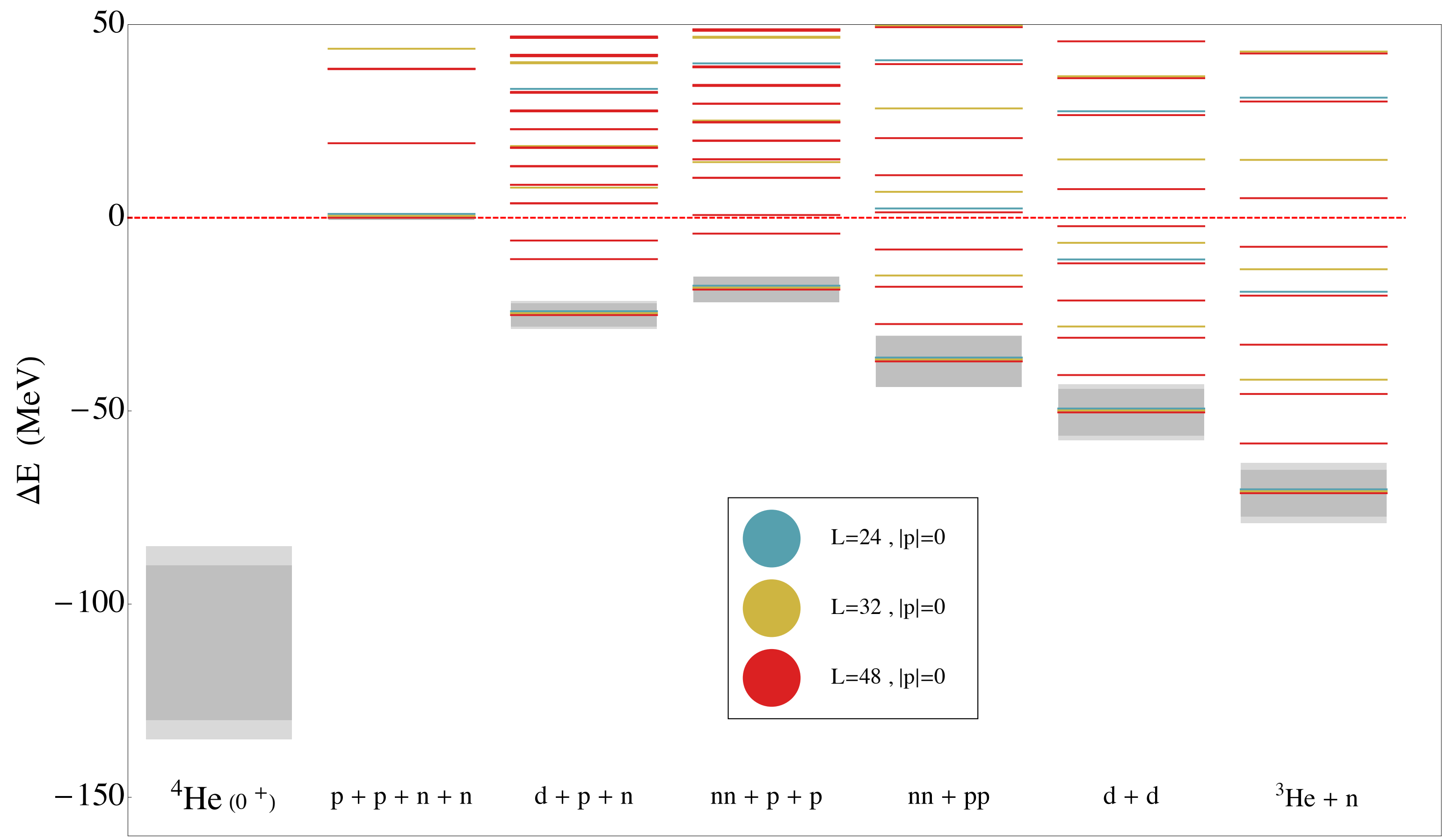

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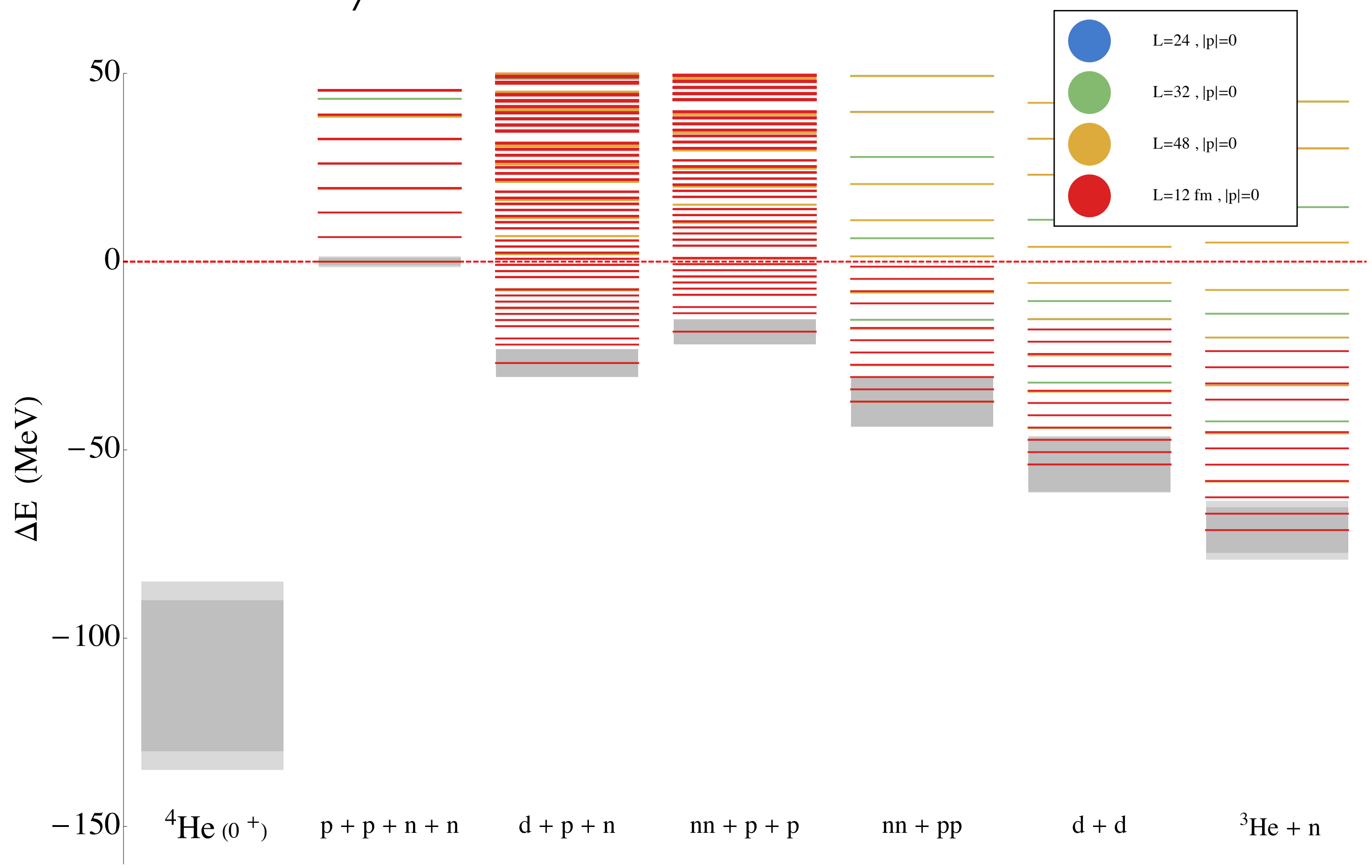

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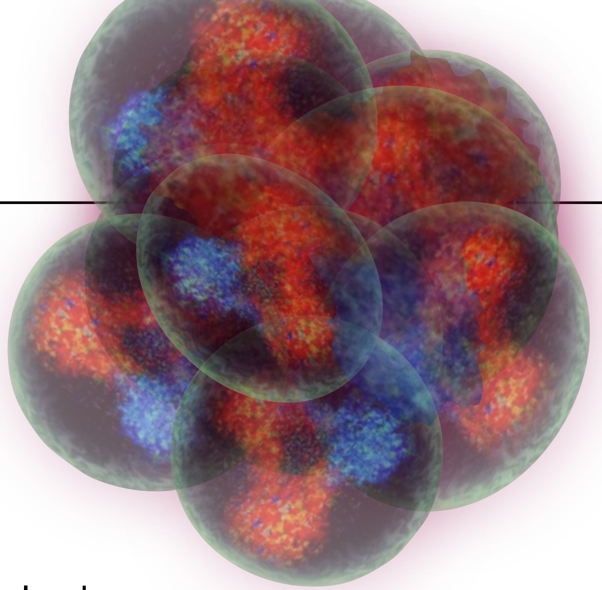
Theoretical problems

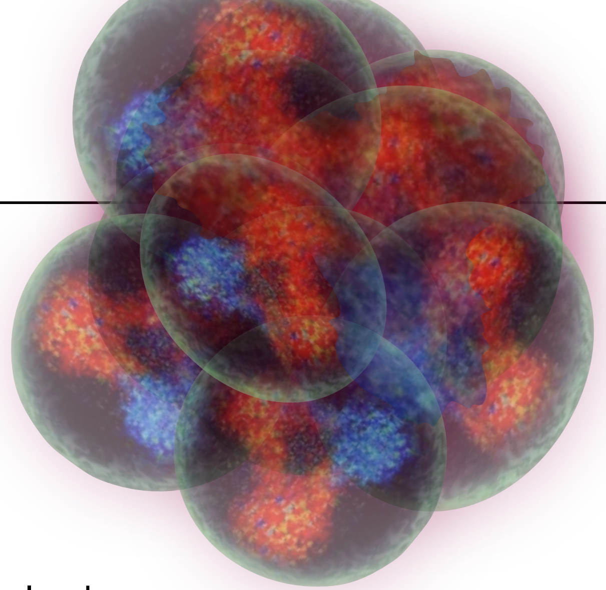
- For large A systems, how do we control the volume, lattice spacing, unphysical quark mass artefacts?
    - Maybe just empirically?
    - Can we have a better theoretical understanding?
- What other kinds of observables can we calculate?

Summary



- Nuclear physics and multi-hadron systems are a frontier for QCD calculations
  - Major advances in the last few years (A_{max}=2 → A_{max}=28)
- Definitely a difficult problem – noise, contractions, theoretical understanding,...
- Lots of possibilities for new calculations – new observables, new approaches
- Lots of room for improvements (theoretical, algorithmic and computational)
- How far can we go?

Summary



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