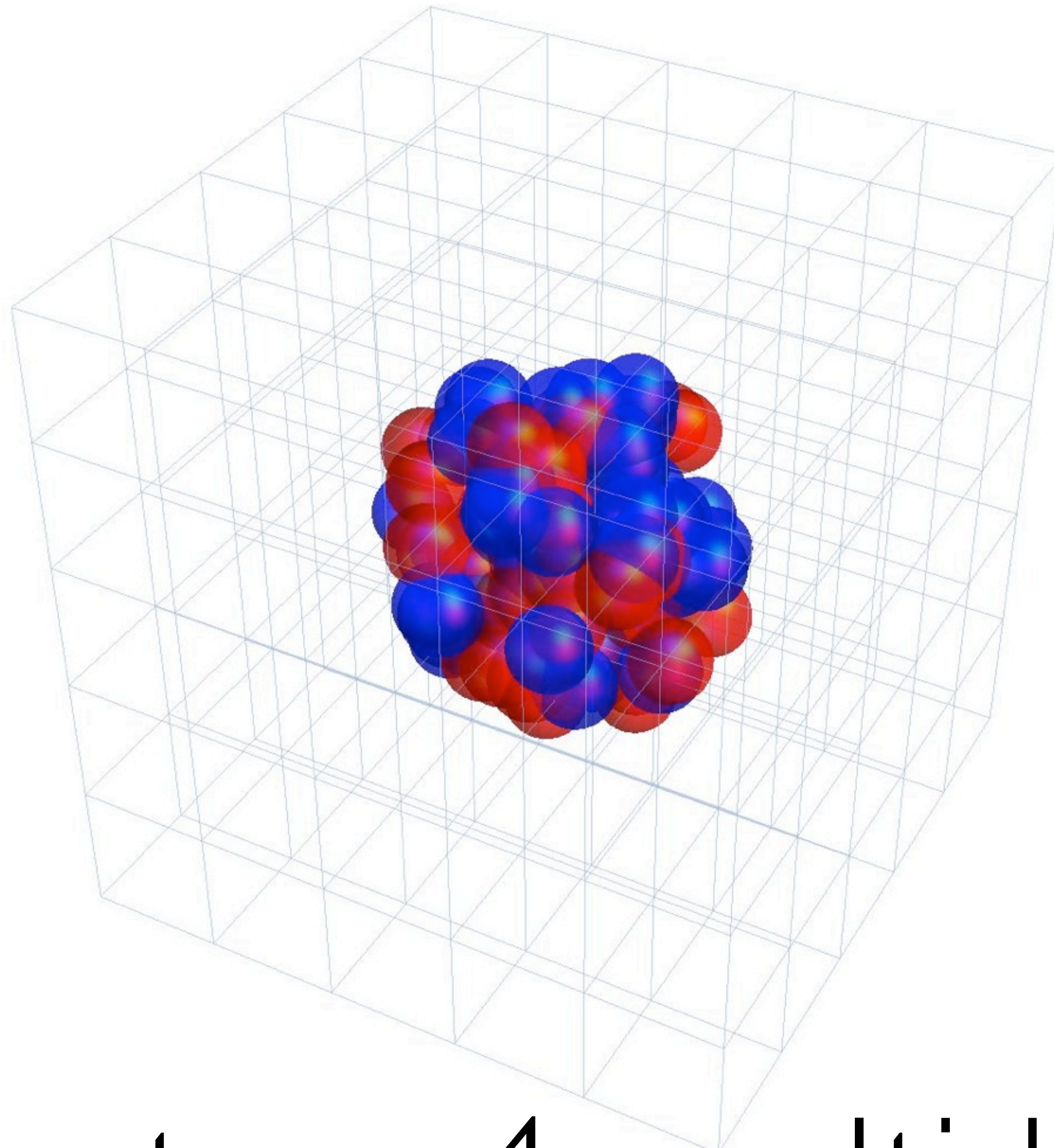

Lecture 4: multi-body

William Detmold

Massachusetts Institute of Technology

Lecture content

- Many body numerical investigations
  - Mesons
  - Few baryons
  - Many Baryons...
- Multi-particle systems at finite temperature

Multi meson systems

Multi-boson energies

- Result for shift to $1/L^7$ is

Two-body interaction

Geometric coefficients

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$$\mathcal{K} = 8.4019240$$
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$$\mathcal{T}_0 = -4116.2338$$
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$$\begin{aligned}\Delta E_0(n,L) = \frac{4\pi a}{M L^3}\binom{n}{2}\Big\{1 &- \left(\frac{a}{\pi L}\right)\mathcal{I} + \left(\frac{a}{\pi L}\right)^2\left[\mathcal{I}^2 + (2n-5)\mathcal{J}\right] \\ &- \left(\frac{a}{\pi L}\right)^3\left[\mathcal{I}^3 + (2n-7)\mathcal{I}\mathcal{J} + \left(5n^2 - 41n + 63\right)\mathcal{K}\right] \\ &+ \left(\frac{a}{\pi L}\right)^4\Big[\mathcal{I}^4 - 6\mathcal{I}^2\mathcal{J} + (4+n-n^2)\mathcal{J}^2 + 4(27-15n+n^2)\mathcal{I}\,\mathcal{K} \\ &\qquad + (14n^3 - 227n^2 + 919n - 1043)\mathcal{L}\Big]\Big\} \\ &+ \binom{n}{2}\frac{8\pi^2 a^3 r}{M L^6}\left[1 + \left(\frac{a}{\pi L}\right)3(n-3)\mathcal{I}\right] \\ &+ \binom{n}{3}\frac{1}{L^6}\left[\eta_3(\mu) + \frac{64\pi a^4}{M}\left(3\sqrt{3} - 4\pi\right)\log(\mu L) - \frac{96 a^4}{\pi^2 M}\mathcal{S}\right]\left[1 - 6\left(\frac{a}{\pi L}\right)\mathcal{I}\right] \\ &+ \binom{n}{3}\left[\frac{192\, a^5}{M\pi^3 L^7}\left(\mathcal{T}_0 + \mathcal{T}_1\, n\right) + \frac{6\pi a^3}{M^3 L^7}(n+3)\,\mathcal{I}\right] + \mathcal{O}\left(L^{-8}\right) .\end{aligned}$$

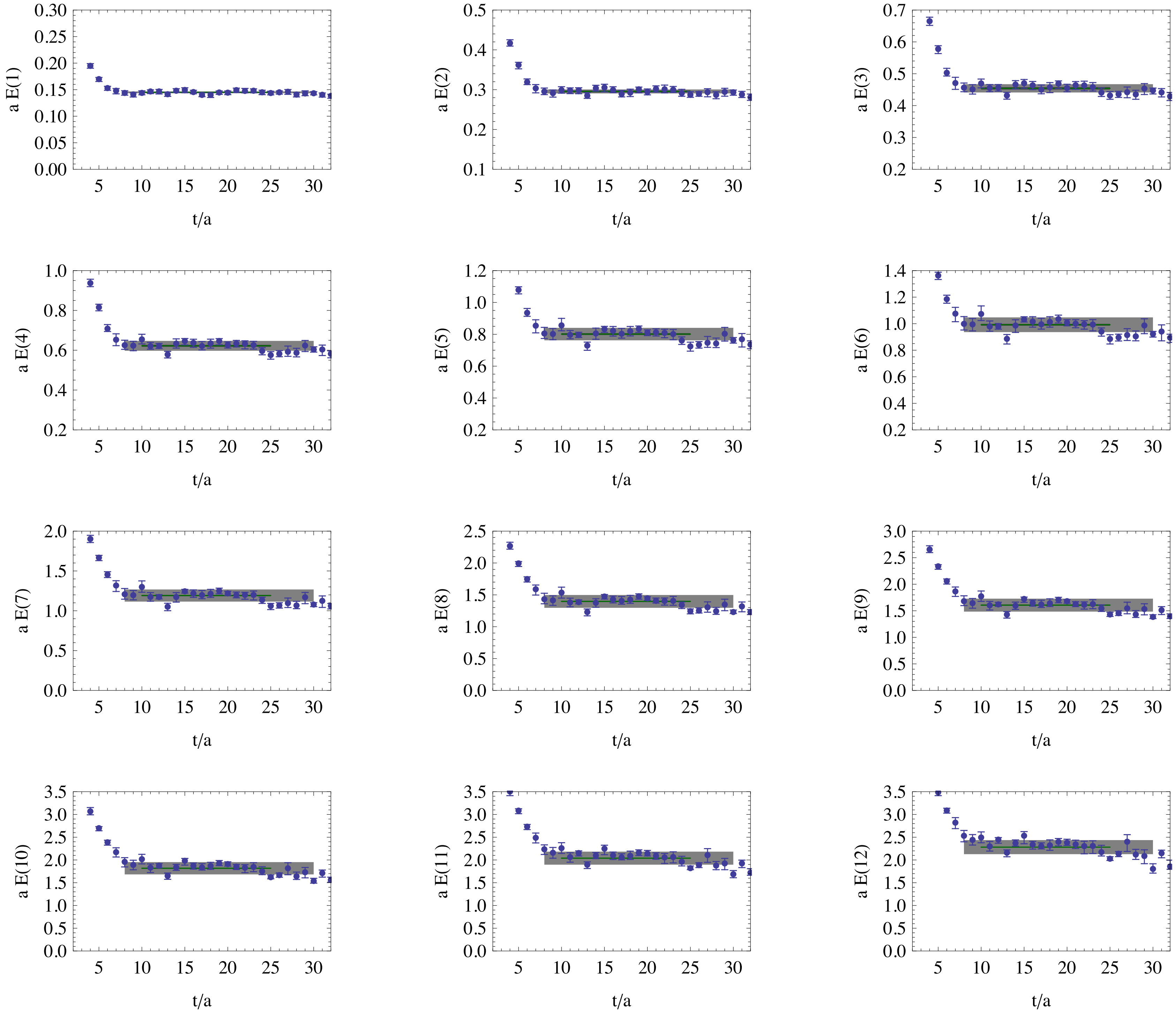
Three-body interaction

- n=2: reproduces expansion of Lüscher
- Can include higher partial waves, higher body, excited states, fermions
- Measurement of energies allows extraction of interaction parameters

[WD & M Savage, see also S Tan 07]

n=1,...,12 pion energies

- Effective energy plots: $\log[C_n(t)/C_n(t+1)]$



DWF on MILC
m_π = 319 MeV
a=0.09 fm, 28^3x96

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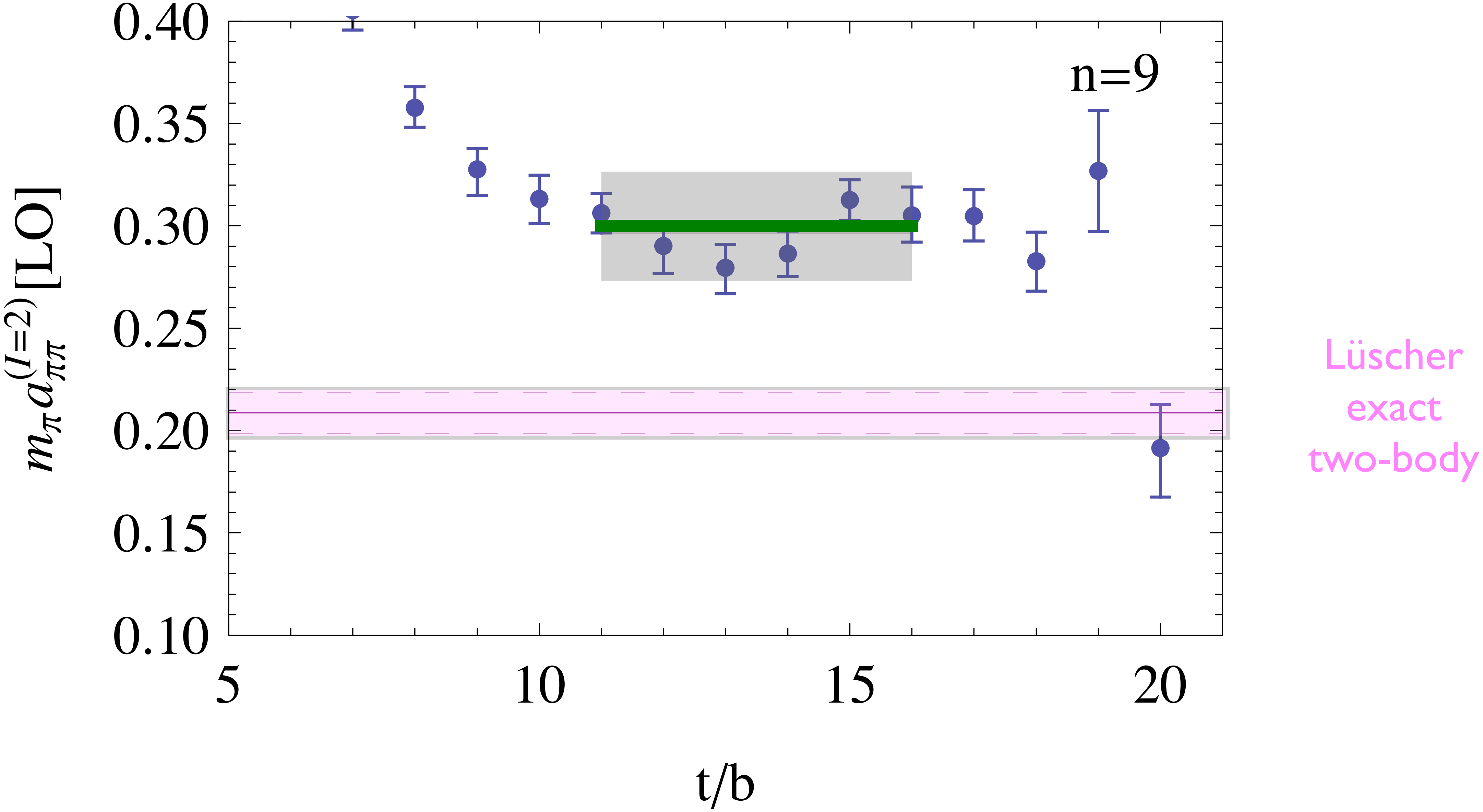
Pion scattering

- Extractions of $m_\pi a$ from four orders in L



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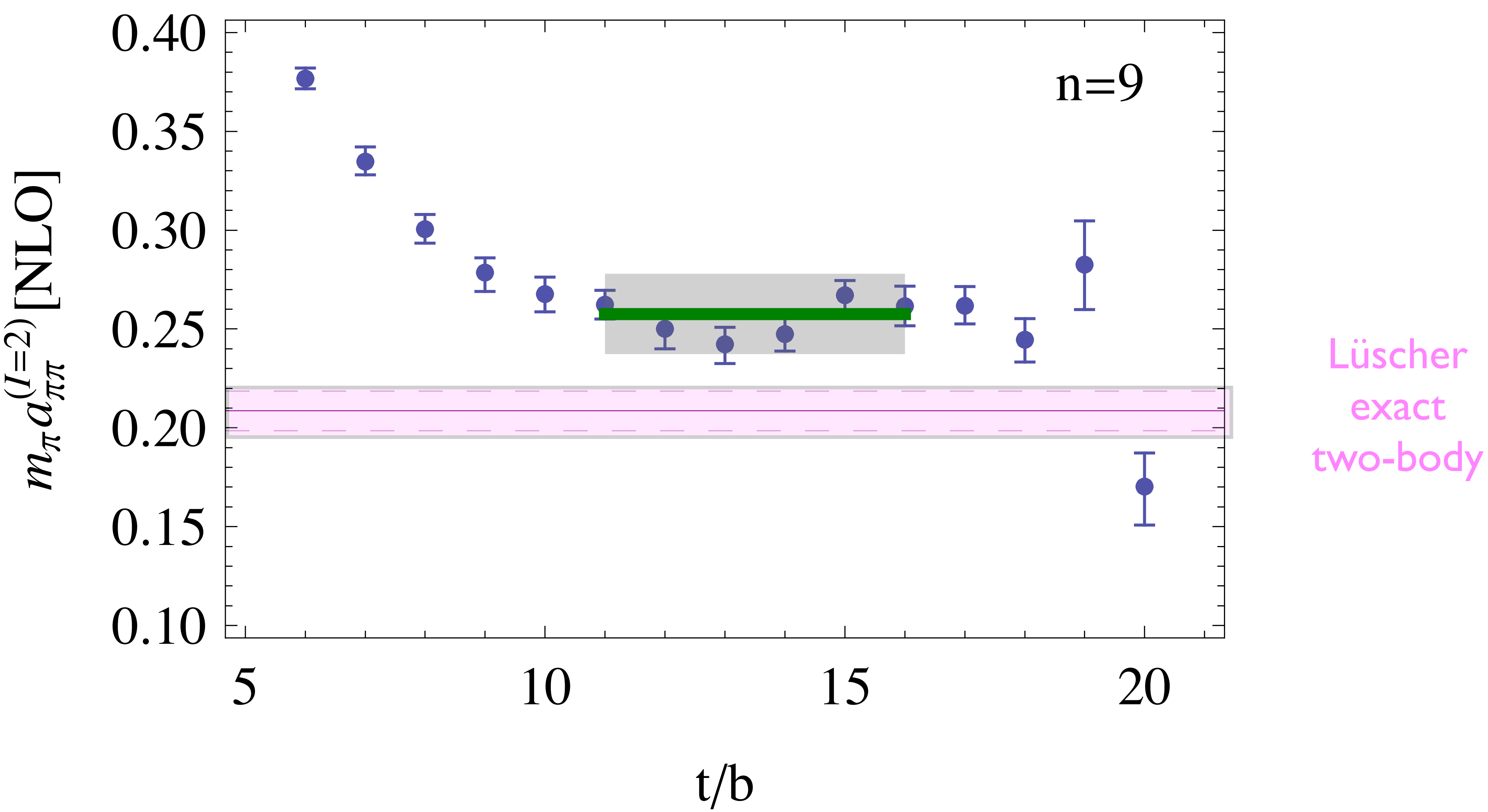
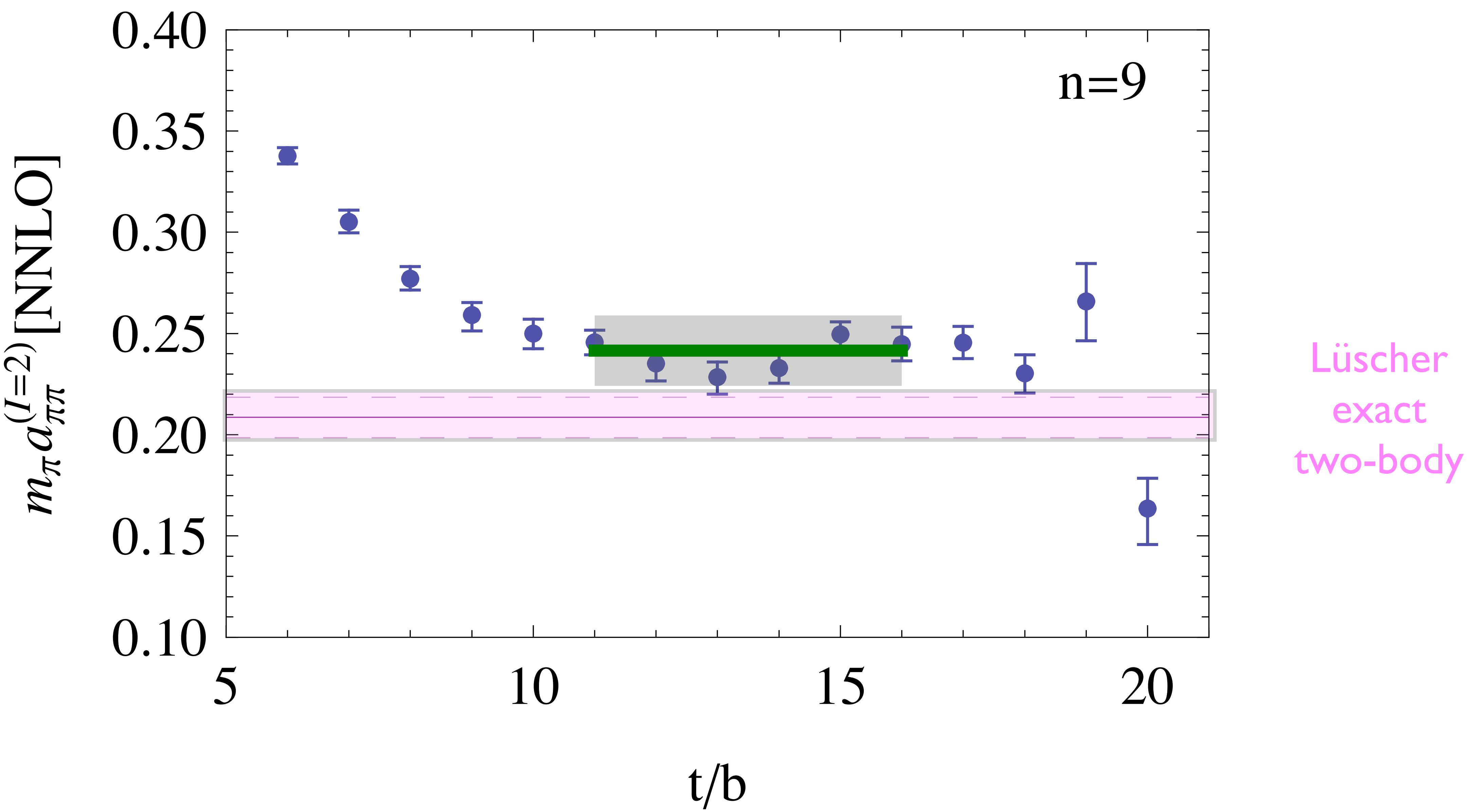


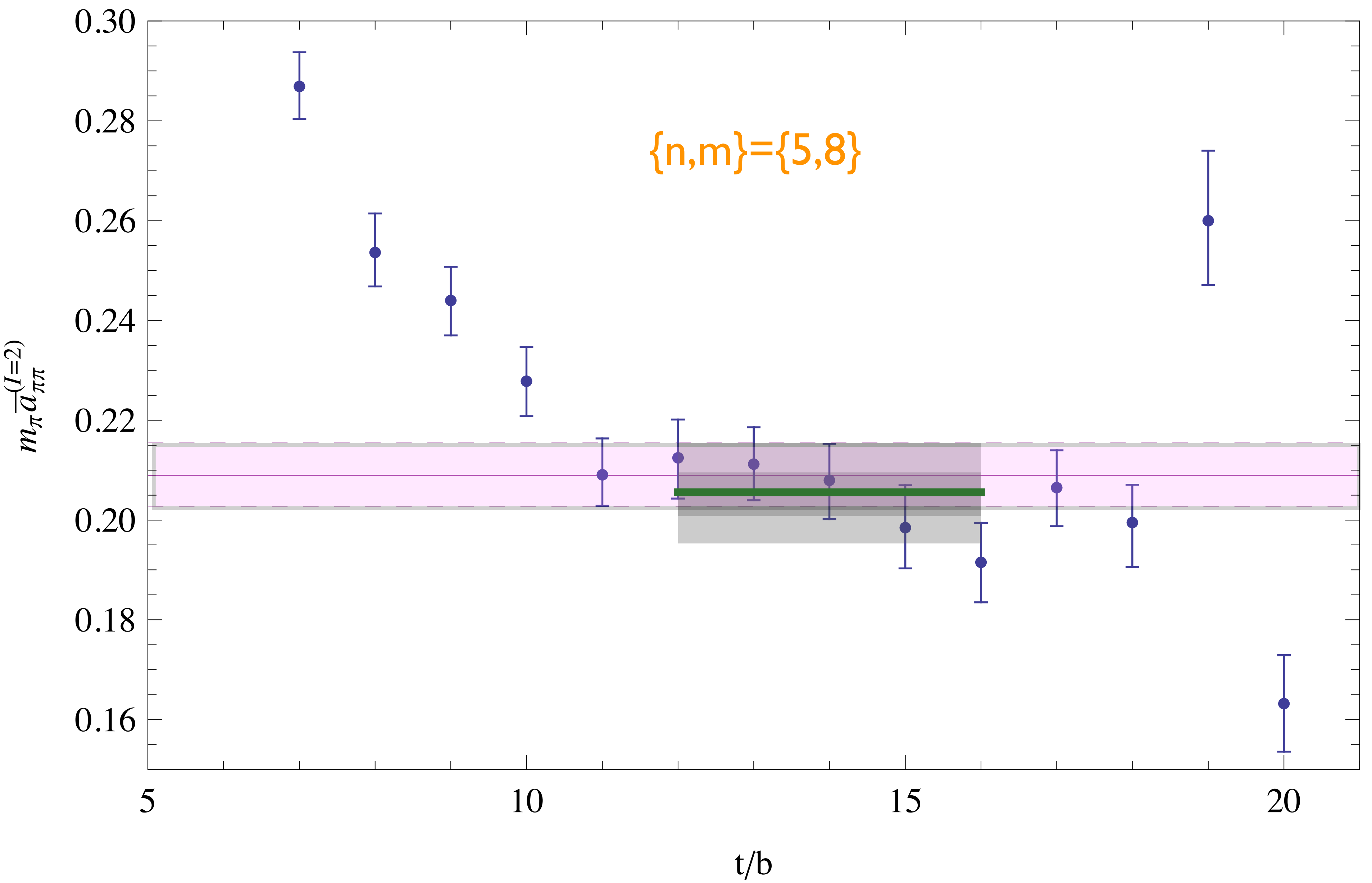
Pion scattering

- Extractions of $m_\pi a$ from four orders in L



N^{3}LO: $1/L^6$

- Two energies to cancel 3-body: 45 combinations



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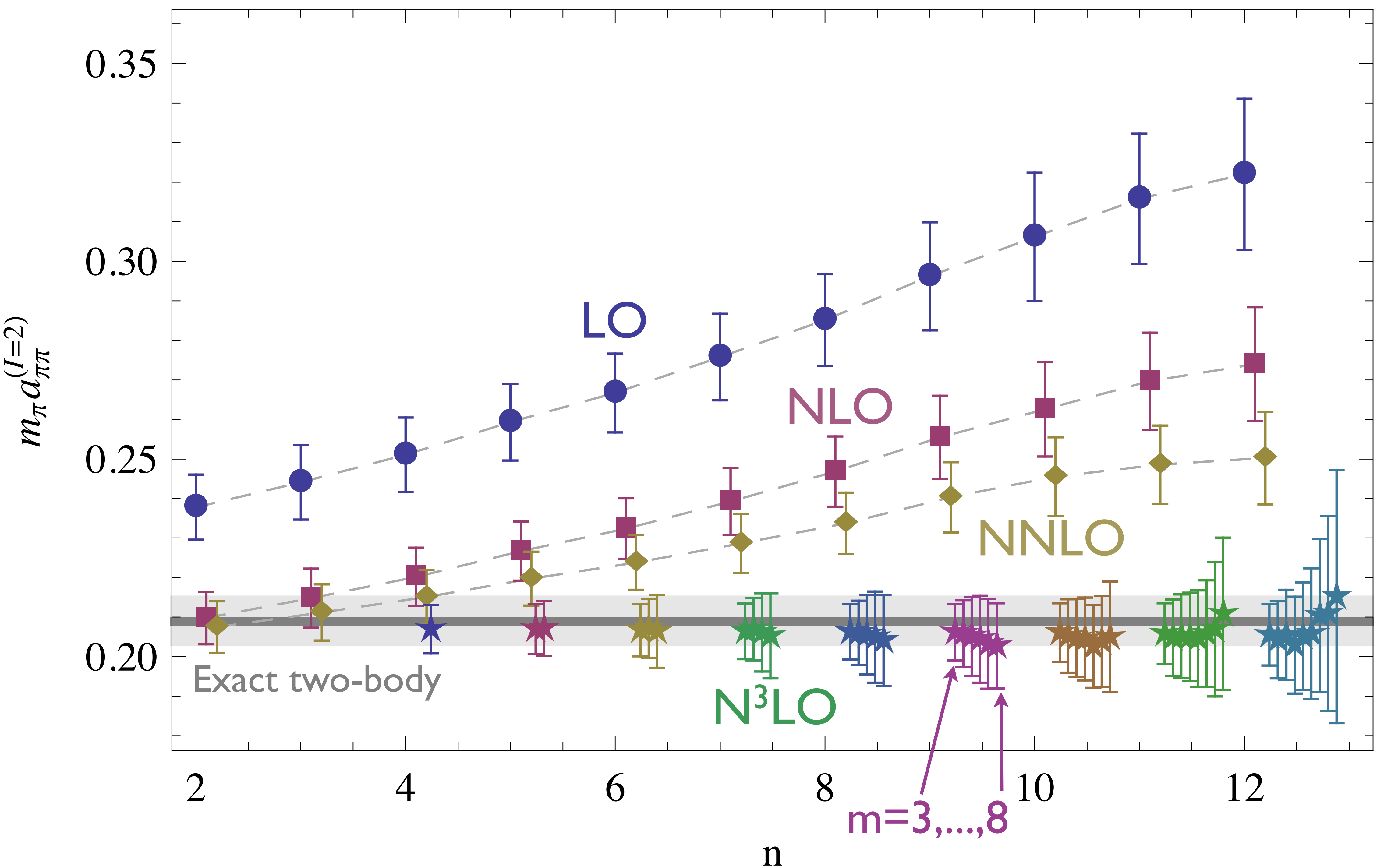
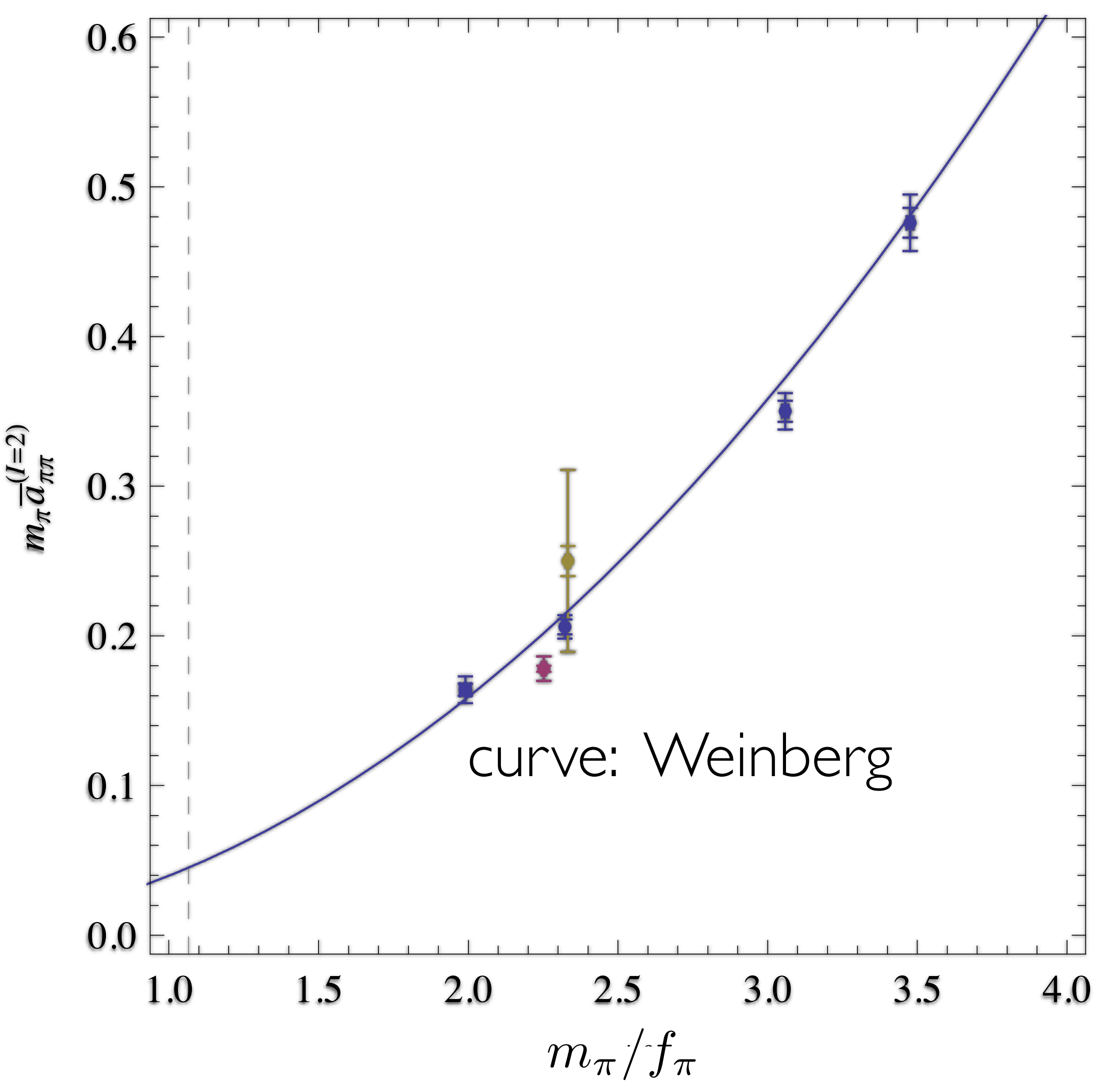
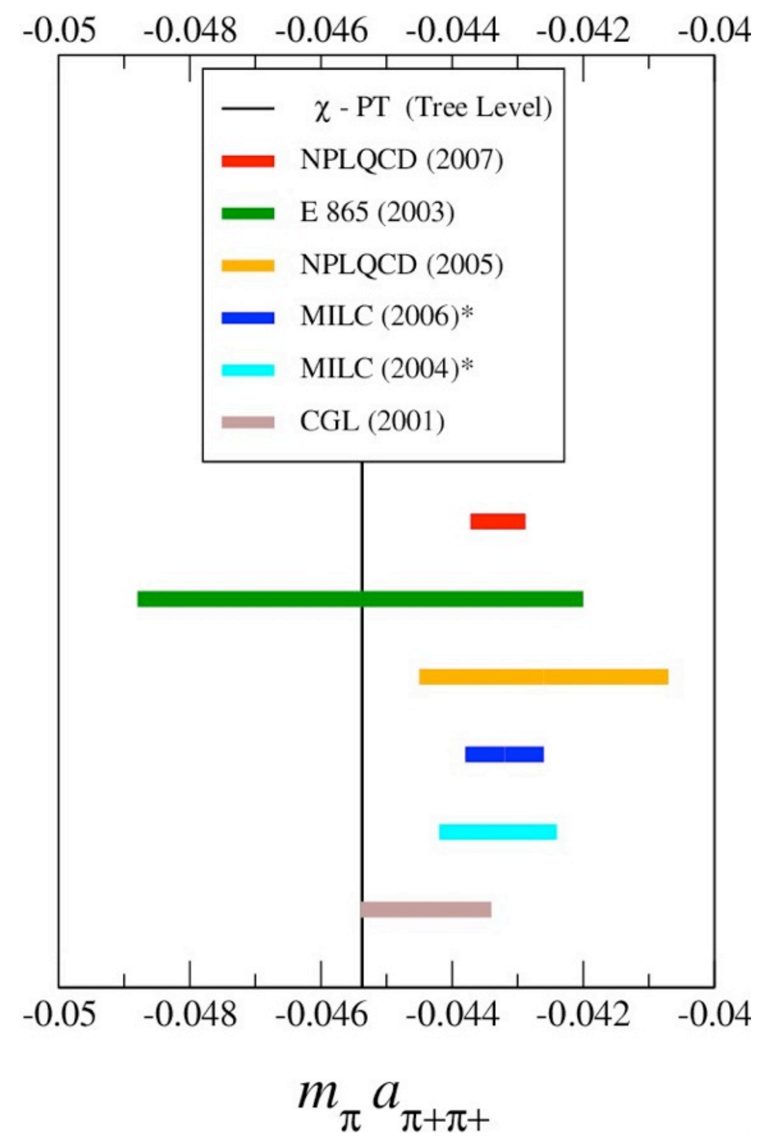


Expansion seems to be working for n<13

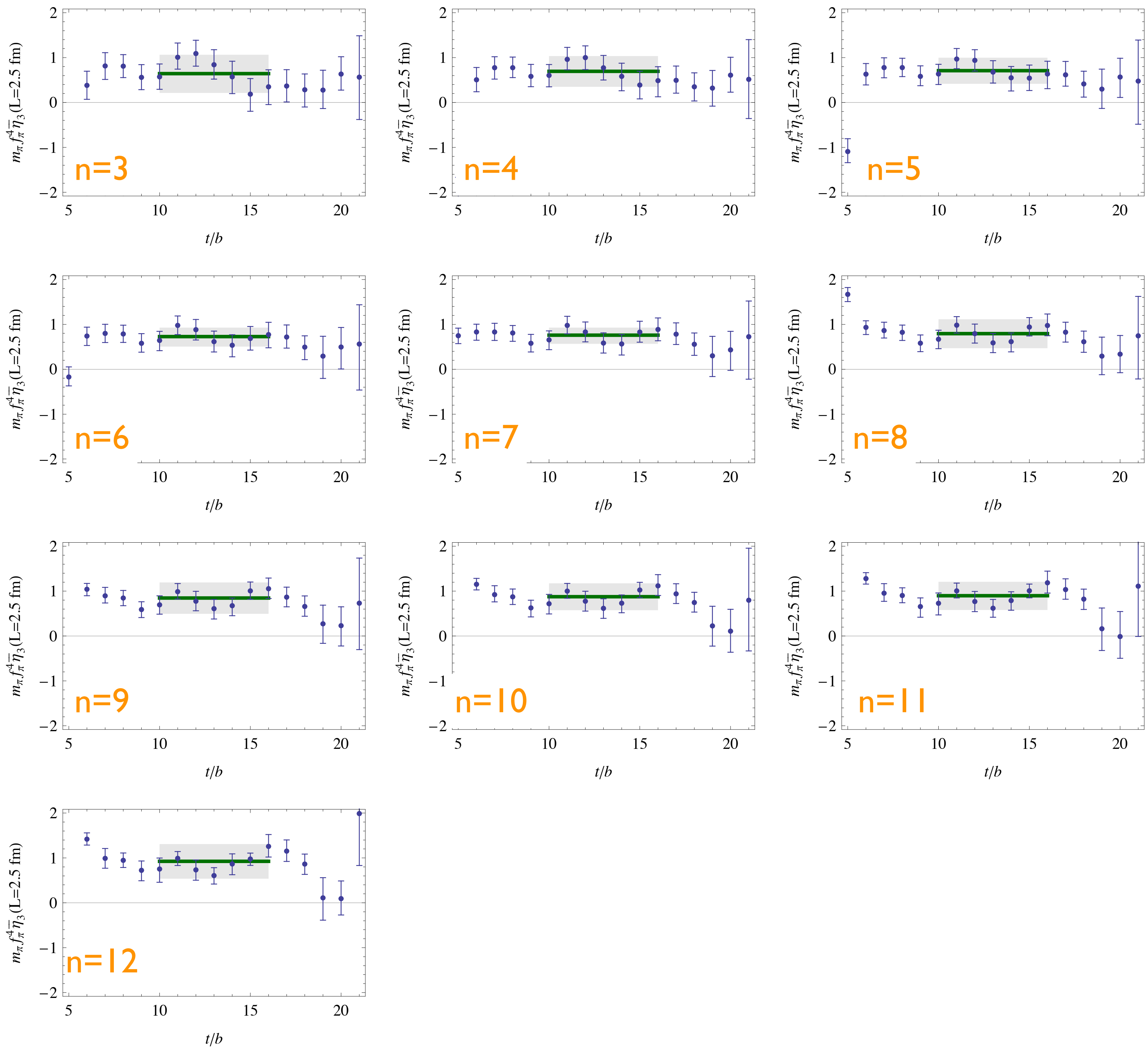
$2\pi^+$ interaction

- Scattering lengths equally well extracted for two mesons or ten mesons and compares well with experiment
- Well described by analytic prediction – shows presence of contribution that scales as $\binom{n}{3}$
  - varies by two-orders of magnitude





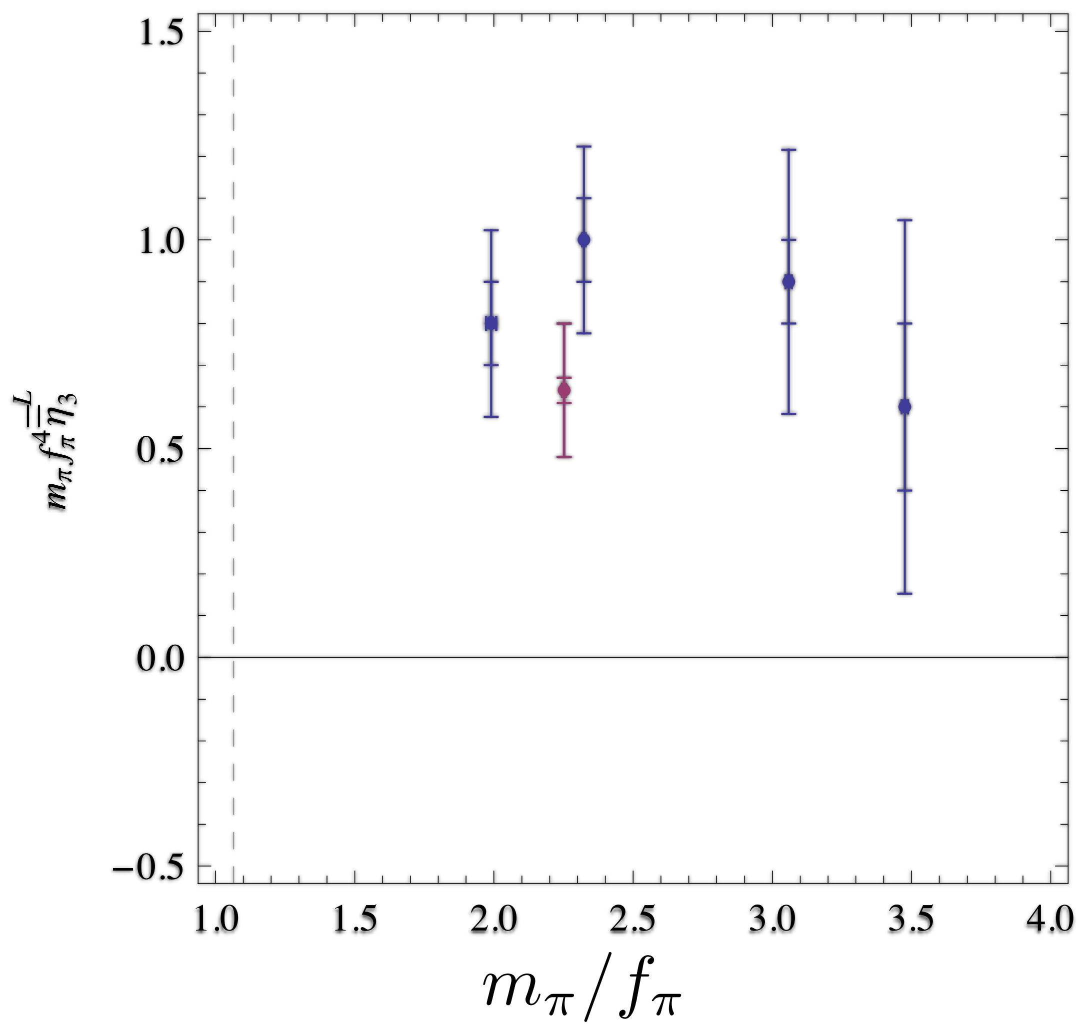
$\pi^+\pi^+\pi^+$ interaction



m_π = 352 MeV

$3\pi^+$ interaction

- First QCD determination of three body interaction
  - Final extraction from combined fit to all energies



Units such that naïve dimension analysis: 1

n pions and *m* kaons

[WD, B Smigielski 1103.4362]

- Just how complex can we go?
- Weakly interacting two species systems: pions and kaons
  - $E_{n,m}$ of *n* pions and *m* kaons depends on three 2-body and four 3-body interaction parameters
  - Perturbative form is known for weakly interacting case [Smigielski & Wasem '08]
  - Matching to lattice energies allows for extraction of interaction parameters
- Extend single species construction (project to p_{tot}=0 at sink)

$$C_{N,M}(t) = \left\langle \left(\sum_x \pi^-(x,t) \right)^N \left(\sum_x K^-(x,t) \right)^M \left(\pi^+(0,t) \right)^N \left(K^+(0,t) \right)^M \right\rangle$$

where

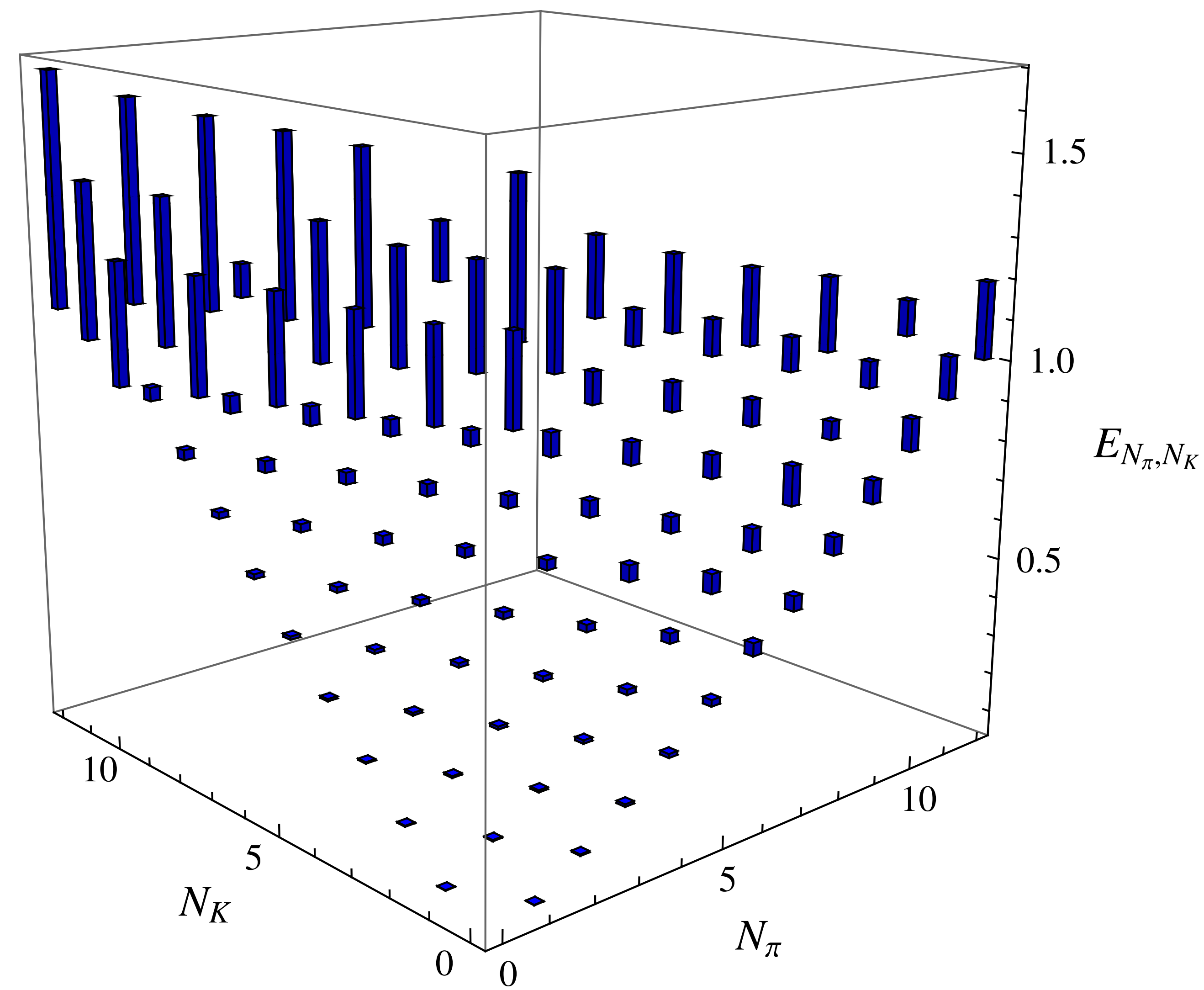
$$\pi^+ = \overline{u}\gamma_5 d, \qquad K^+ = \overline{u}\gamma_5 s$$

- Reduced symmetry: contractions significantly more complex
  Eg: n=6 pions, m=6 kaons: 1500 terms vs n=12 pions: 90 terms

n pions and m kaons

[WD, B Smigielski 1103.4362]

- 90 observables to analyse
- Boxes correspond to extracted energies and their uncertainties
- Dependence on N_π and N_K determines 2 & 3 body interactions



$$
\begin{aligned}
m_K \bar{a}_{KK} &= 0.461 \pm 0.010,\\
m_\pi \bar{a}_{\pi\pi} &= 0.271 \pm 0.021,\\
m_{\pi K} \bar{a}_{\pi K} &= 0.166 \pm 0.016,\\
m_K \bar{\eta}_{3,KKK} f_K^4 &= -0.08 \pm 0.12,\\
m_\pi \bar{\eta}_{3,\pi\pi\pi} f_\pi^4 &= 0.68 \pm 0.33,\\
\frac{m_\pi m_K}{m_\pi + 2m_K} \bar{\eta}_{3,\pi KK} f_{\pi KK}^4 &= 0.22 \pm 0.17,\\
\frac{m_\pi m_K}{2m_\pi + m_K} \bar{\eta}_{3,\pi\pi K} f_{\pi\pi K}^4 &= 0.45 \pm 0.26.
\end{aligned}
$$

Many meson systems

WD, K Orginos, Z Shi 1205.4224

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WD, K Orginos, Z Shi 1205.4224

- Calculate correlation functions for systems containing very large isospin charge I_z=72 (~numbers of mesons)
  - Improved contraction techniques and propagators from multiple source locations

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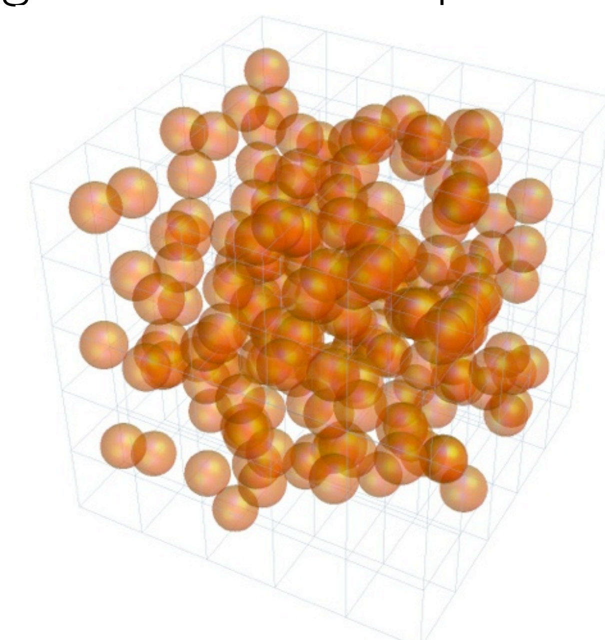

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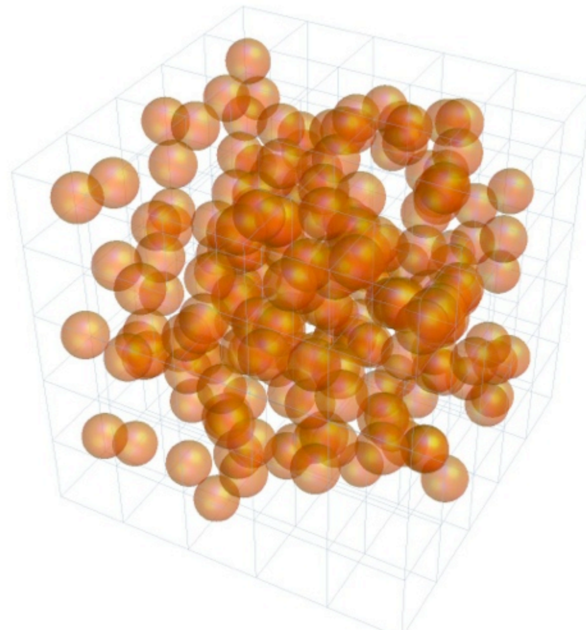

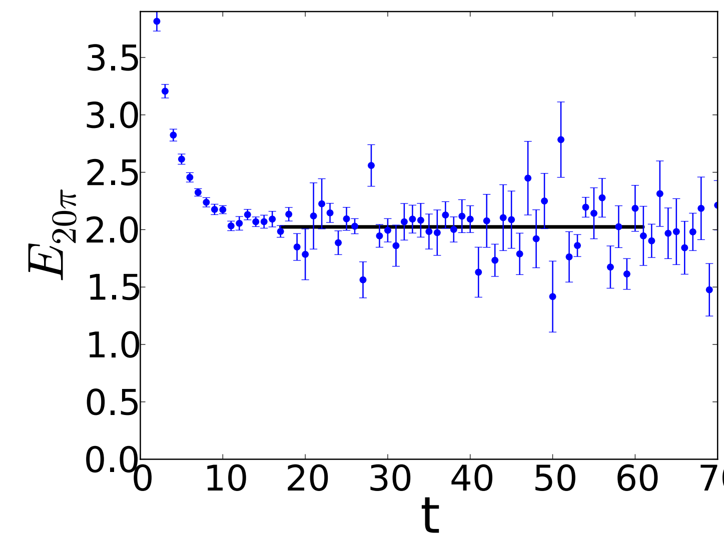

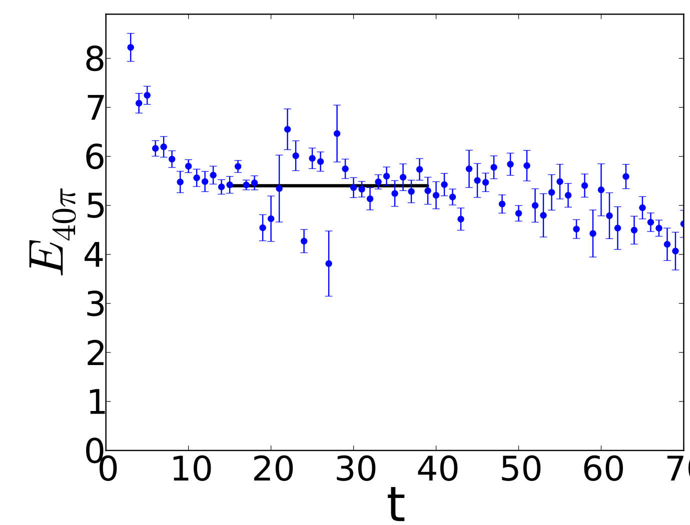

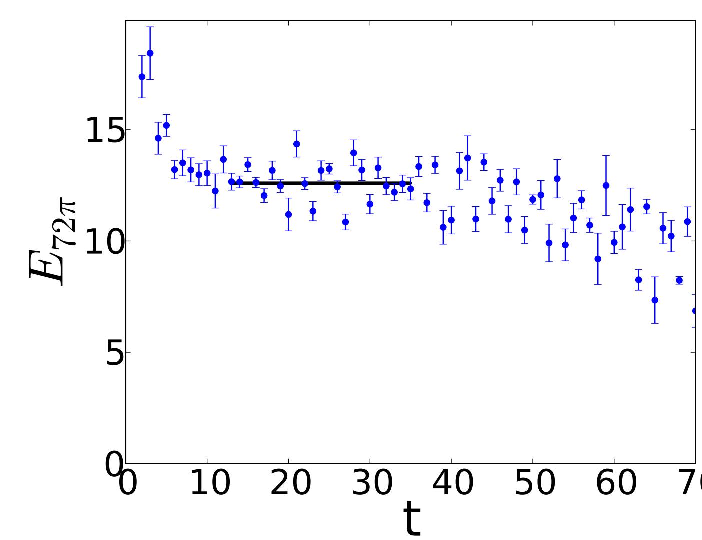

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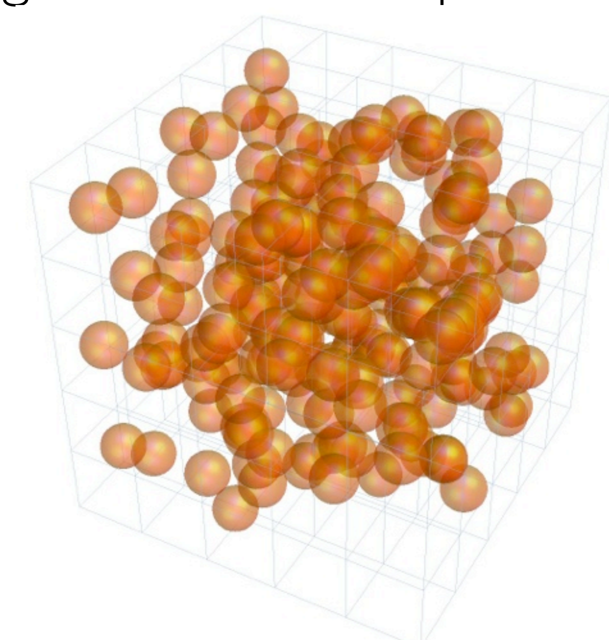

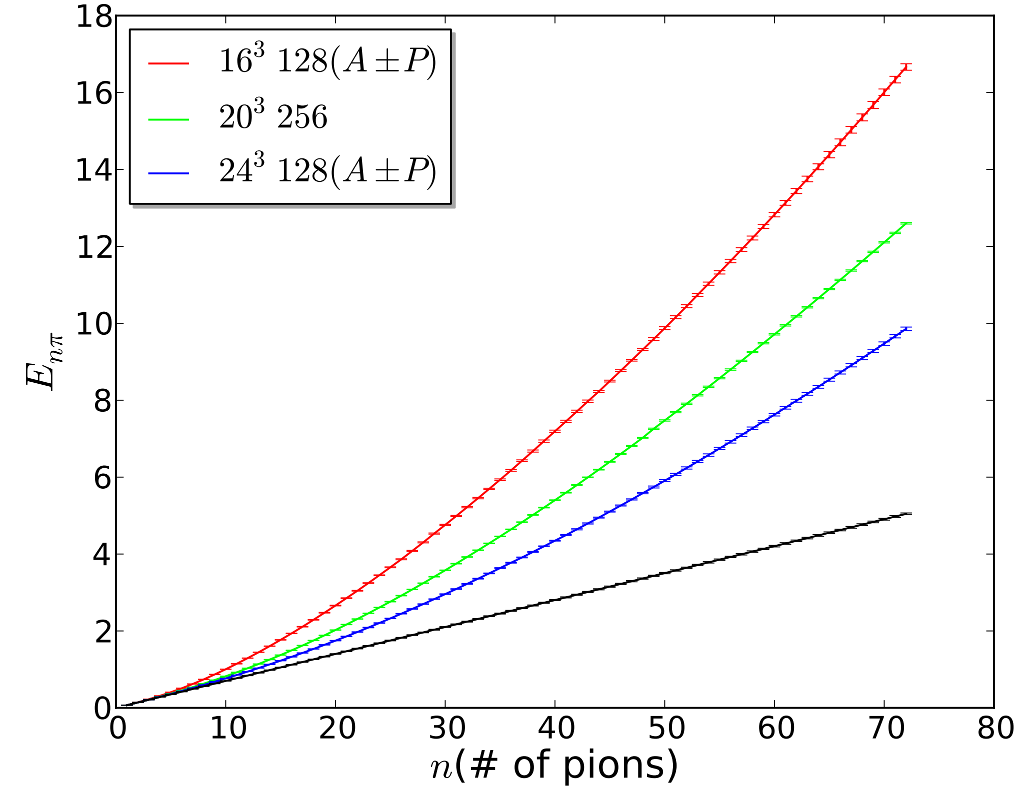

- Energy of these systems becomes enormous
  - Dominated by repulsive interactions

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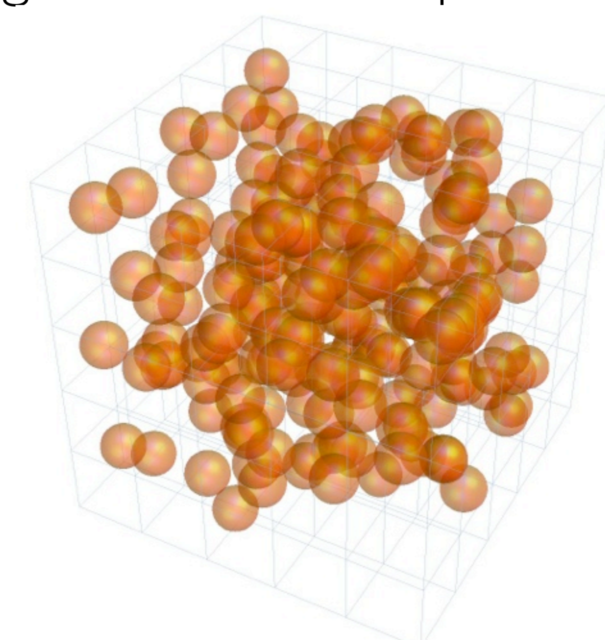
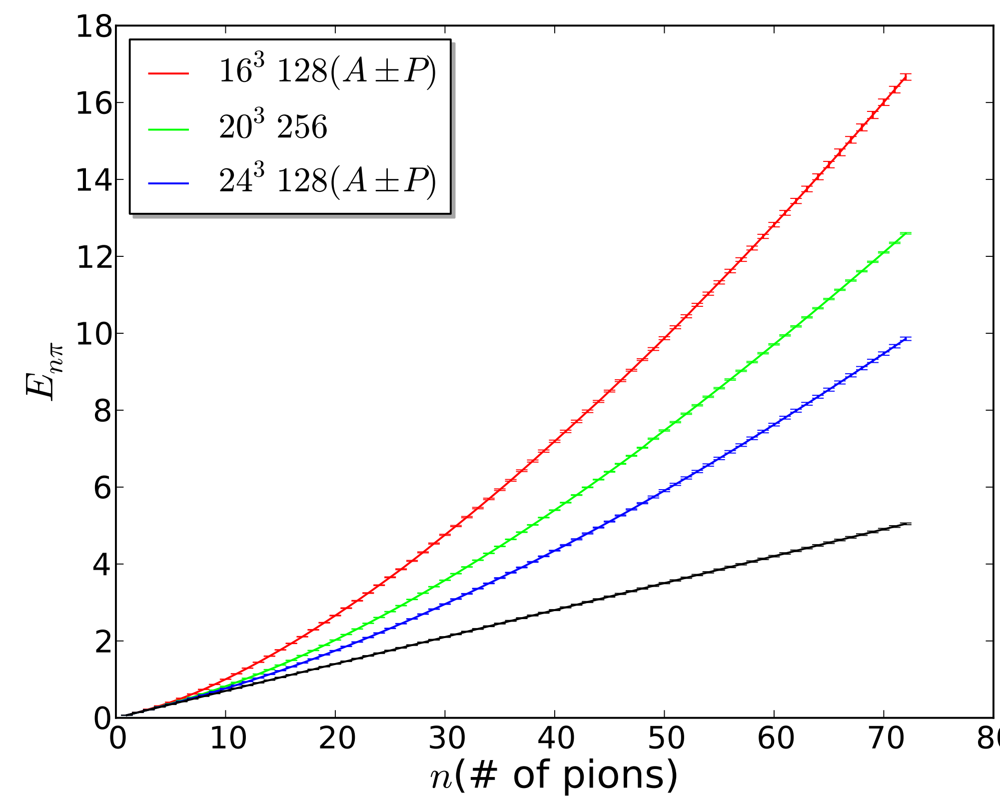
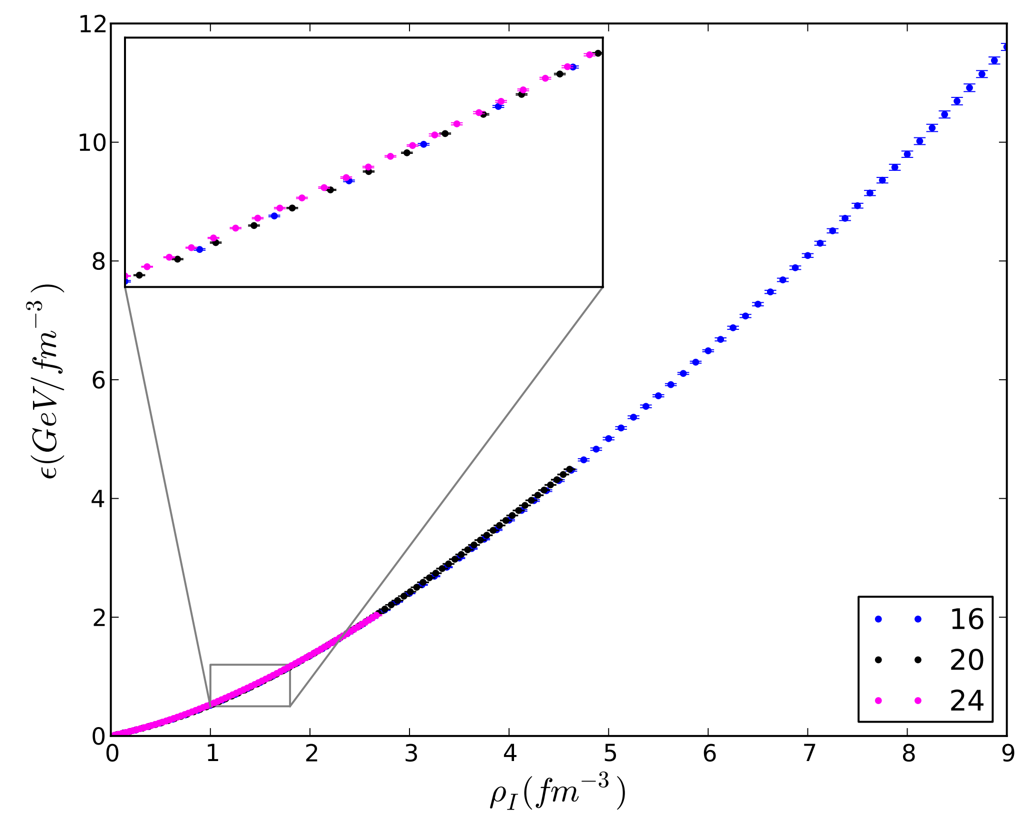


- Energy of these systems becomes enormous
    - Dominated by repulsive interactions
- More useful to think in terms of isospin density and energy density





Bose-Einstein condensation

WD, K Orginos, Z Shi 1205.4224

Bose-Einstein condensation

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- Characterise the thermodynamic properties of the system

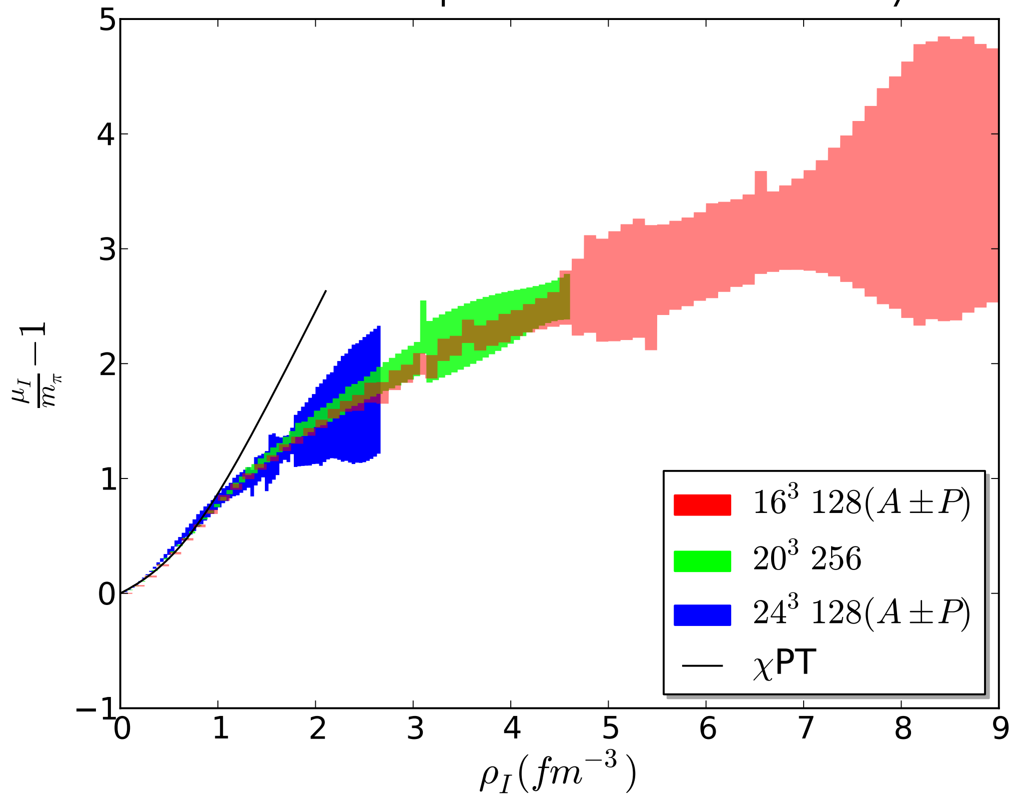
Bose-Einstein condensation

WD, K Orginos, Z Shi 1205.4224

- Characterise the thermodynamic properties of the system
- Isospin chemical potential

$$\mu_I = \left.\frac{dE}{dn}\right|_V$$

Chemical potential vs density



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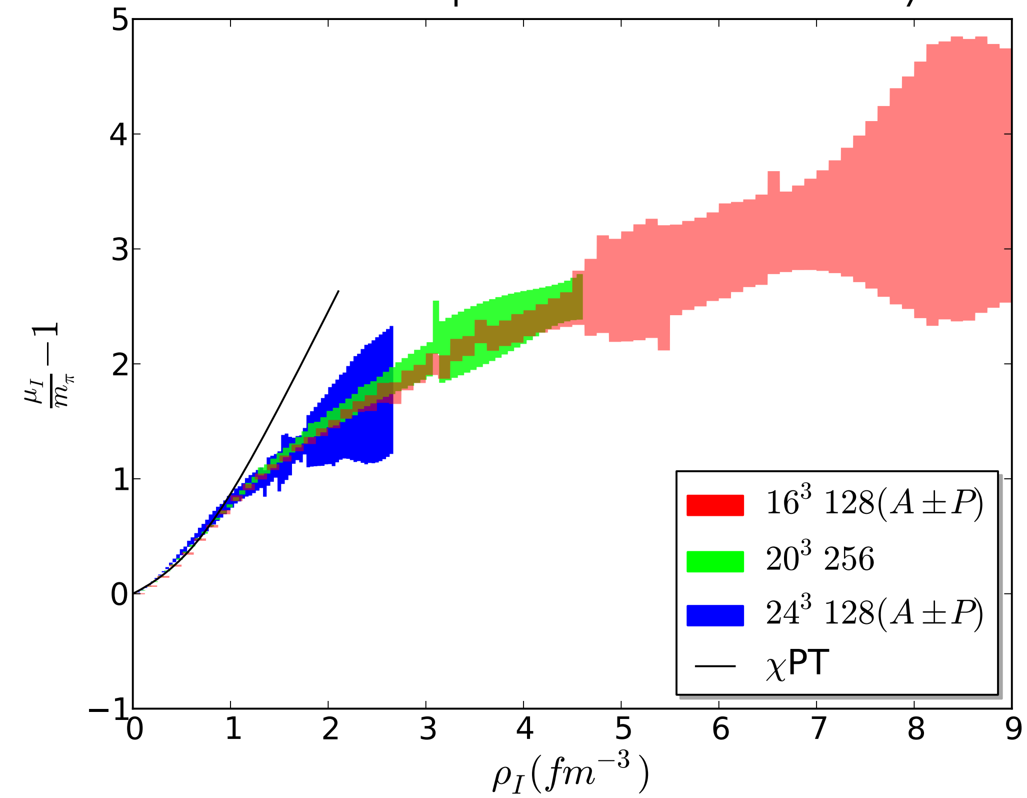
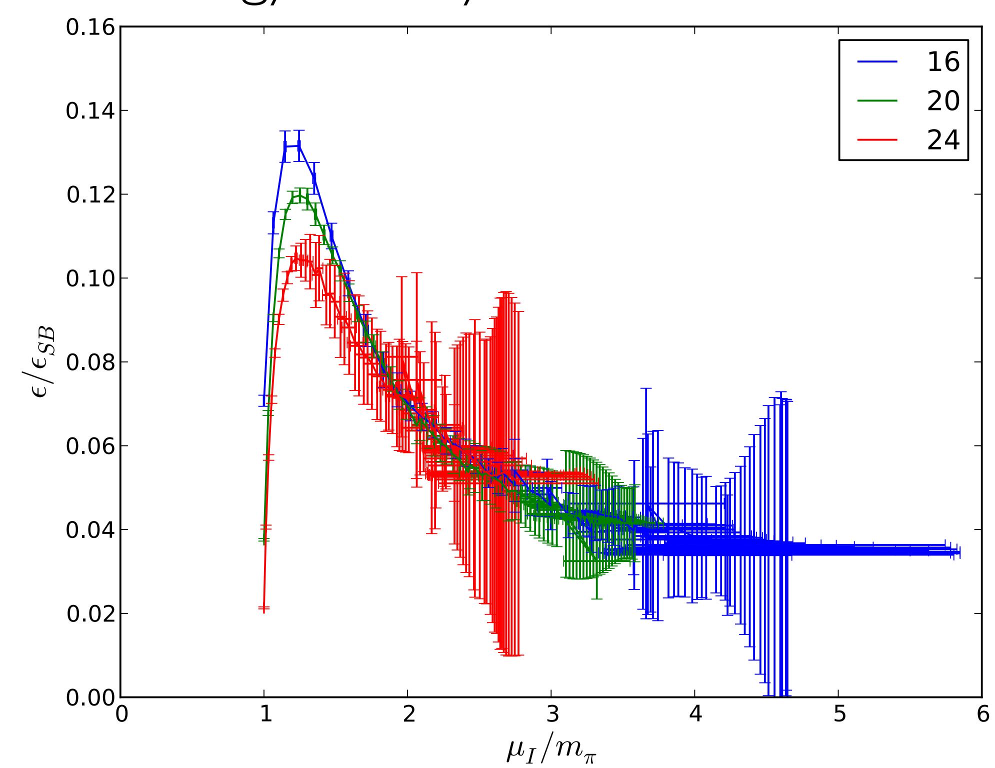


Energy density vs Stefan-Boltzmann



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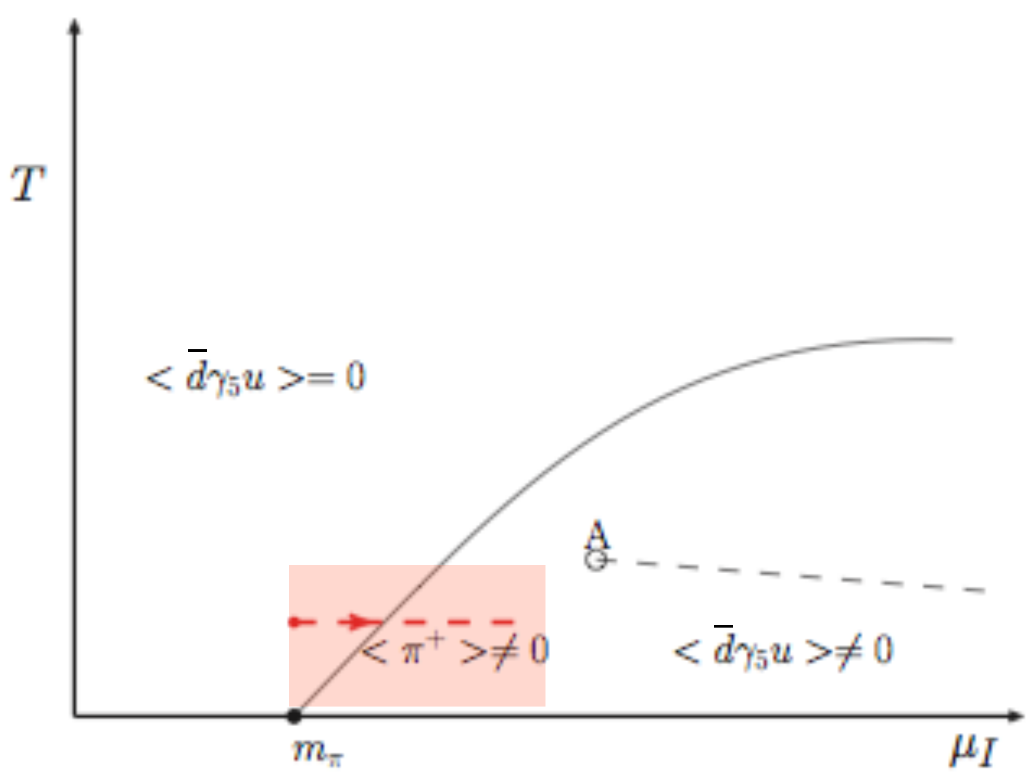

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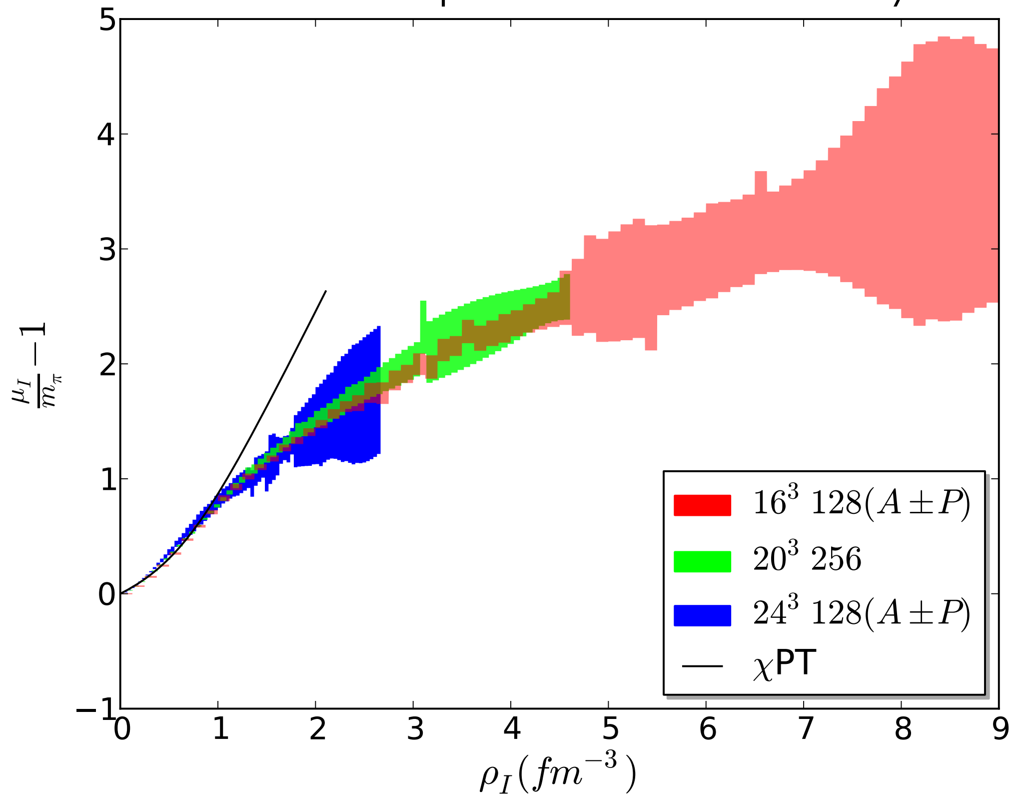

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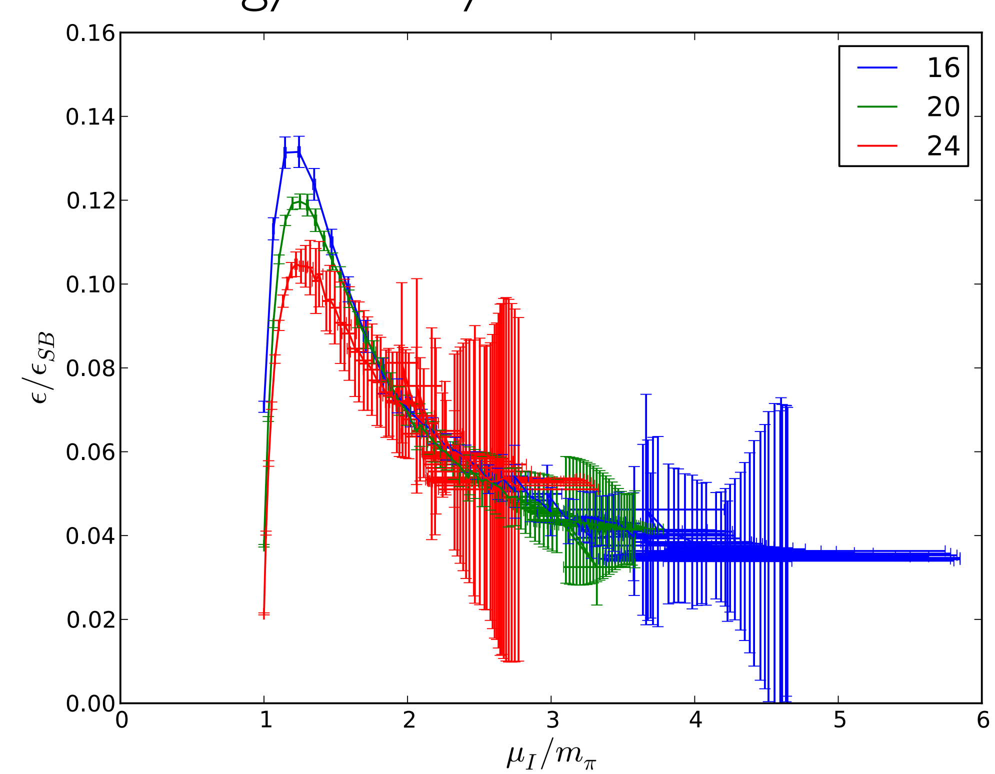

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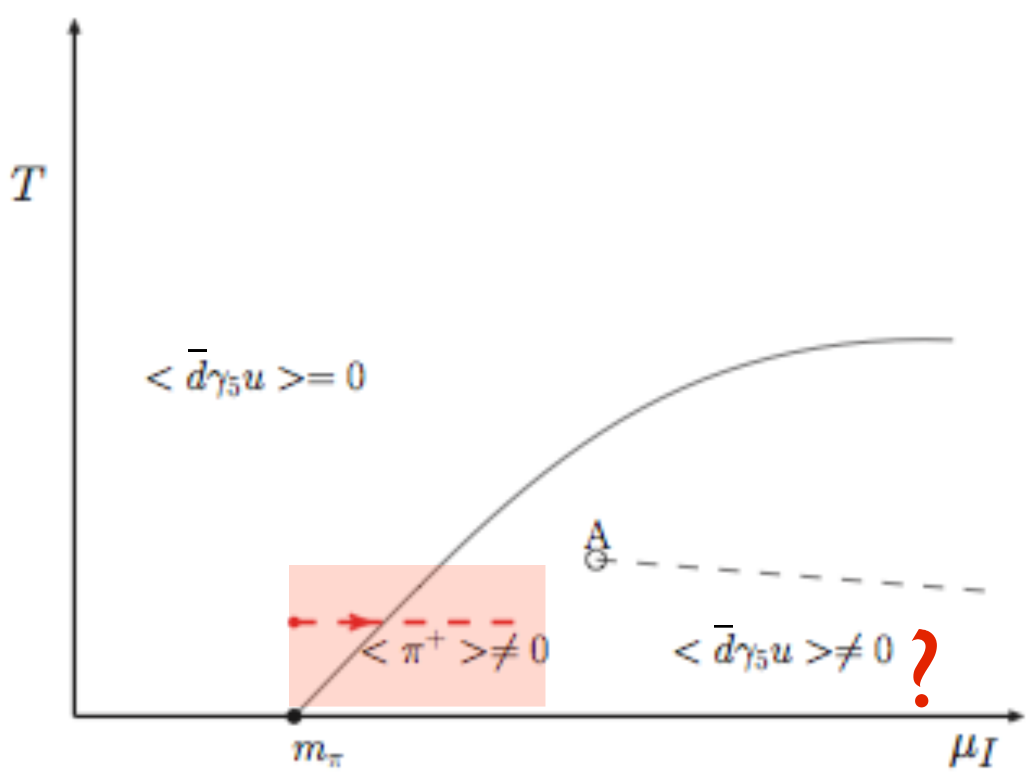

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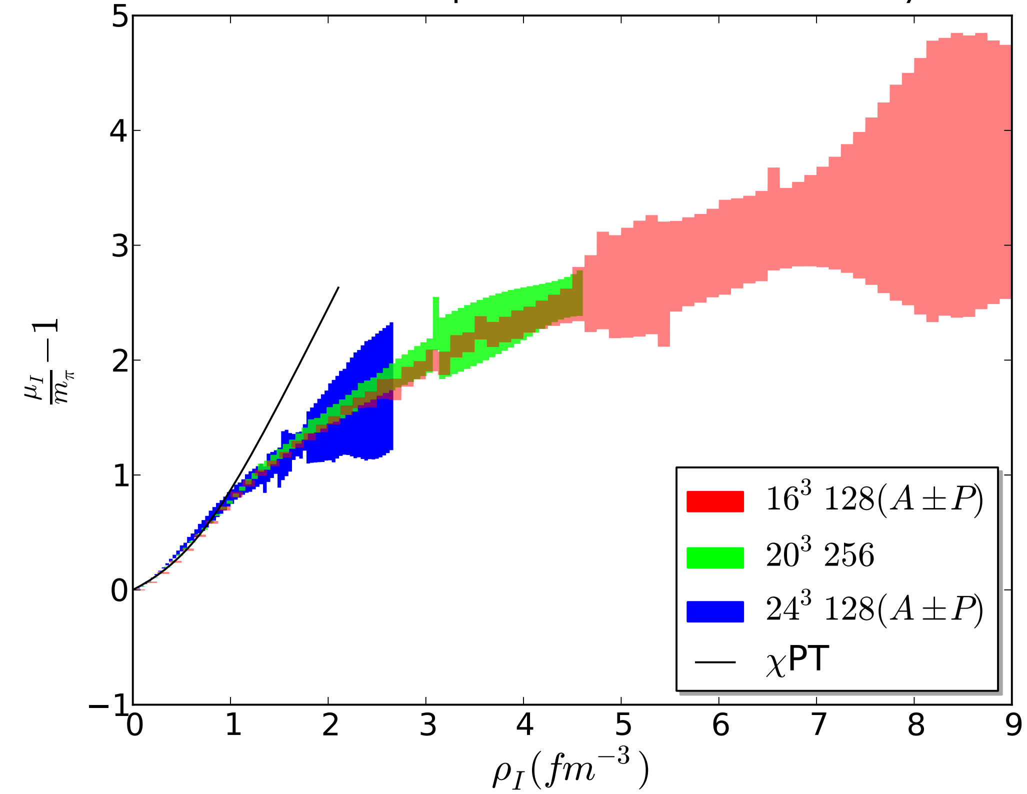

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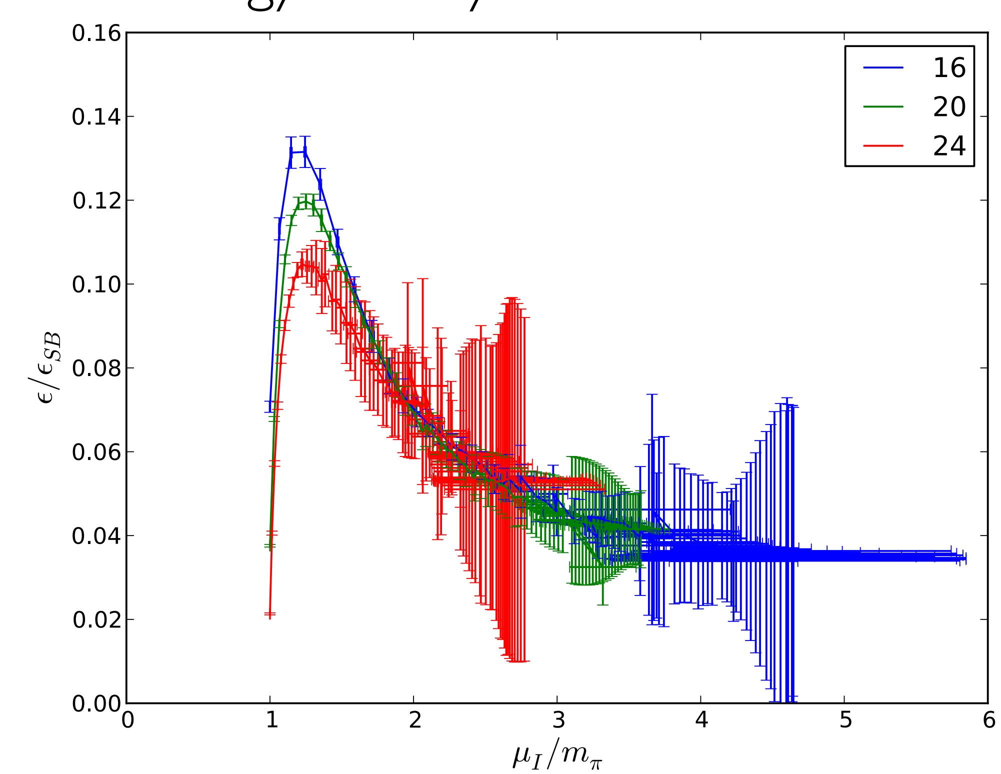
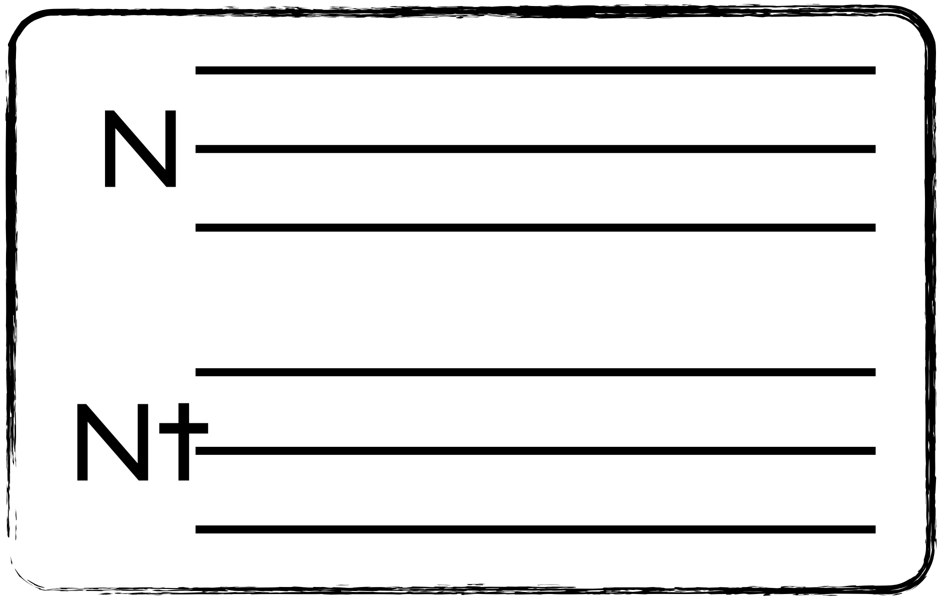


Multi baryon systems

The trouble with baryons

- Importance sampling of QCD functional integrals
  ➤ correlators determined stochastically
- Variance in single nucleon correlator (C) determined by

$$\sigma^2(C) = \langle C C^\dagger \rangle - |\langle C \rangle|^2$$



- For nucleon:

$$\frac{\text{signal}}{\text{noise}} \sim \exp\left[-(M_N - 3/2 m_\pi)t\right]$$

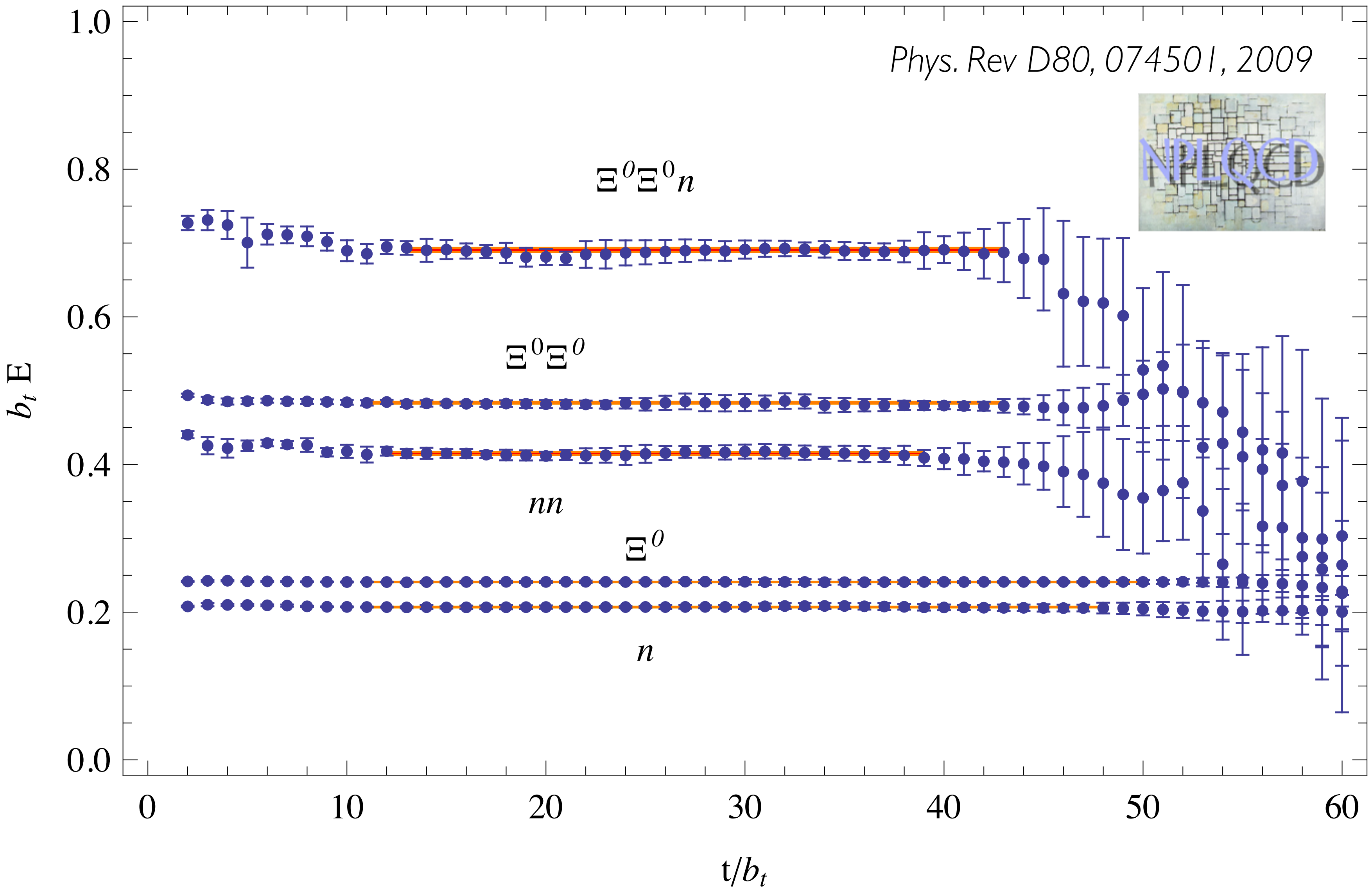
- For nucleus A:

$$\frac{\text{signal}}{\text{noise}} \sim \exp\left[-A(M_N - 3/2 m_\pi)t\right]$$

[Lepage '89]

The trouble with baryons

High statistics study using anisotropic lattices (fine temporal resolution)



@ m_π = 390 MeV

The trouble with baryons

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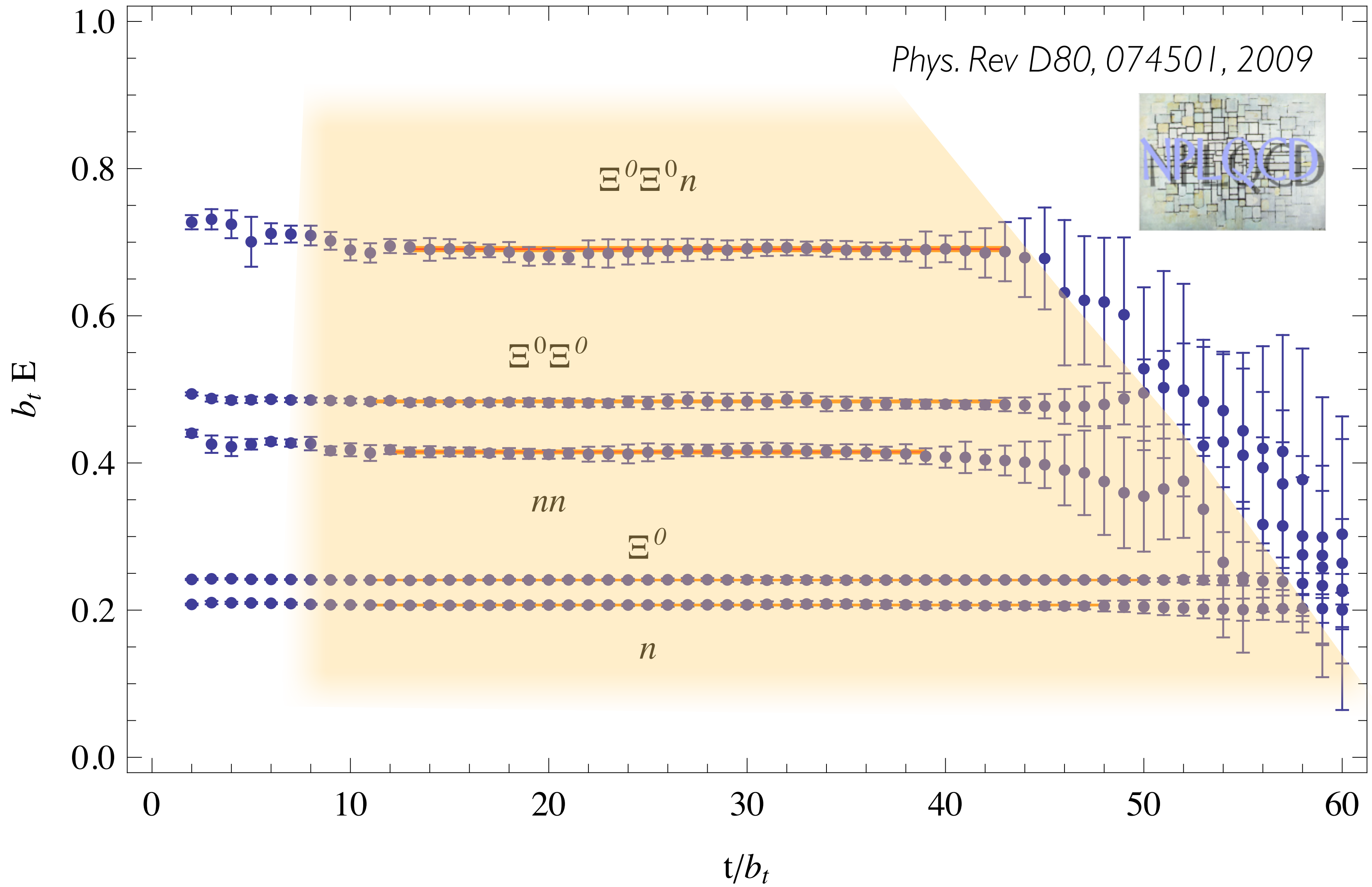

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Golden window of time-slices where signal/noise const

No? trouble with baryons

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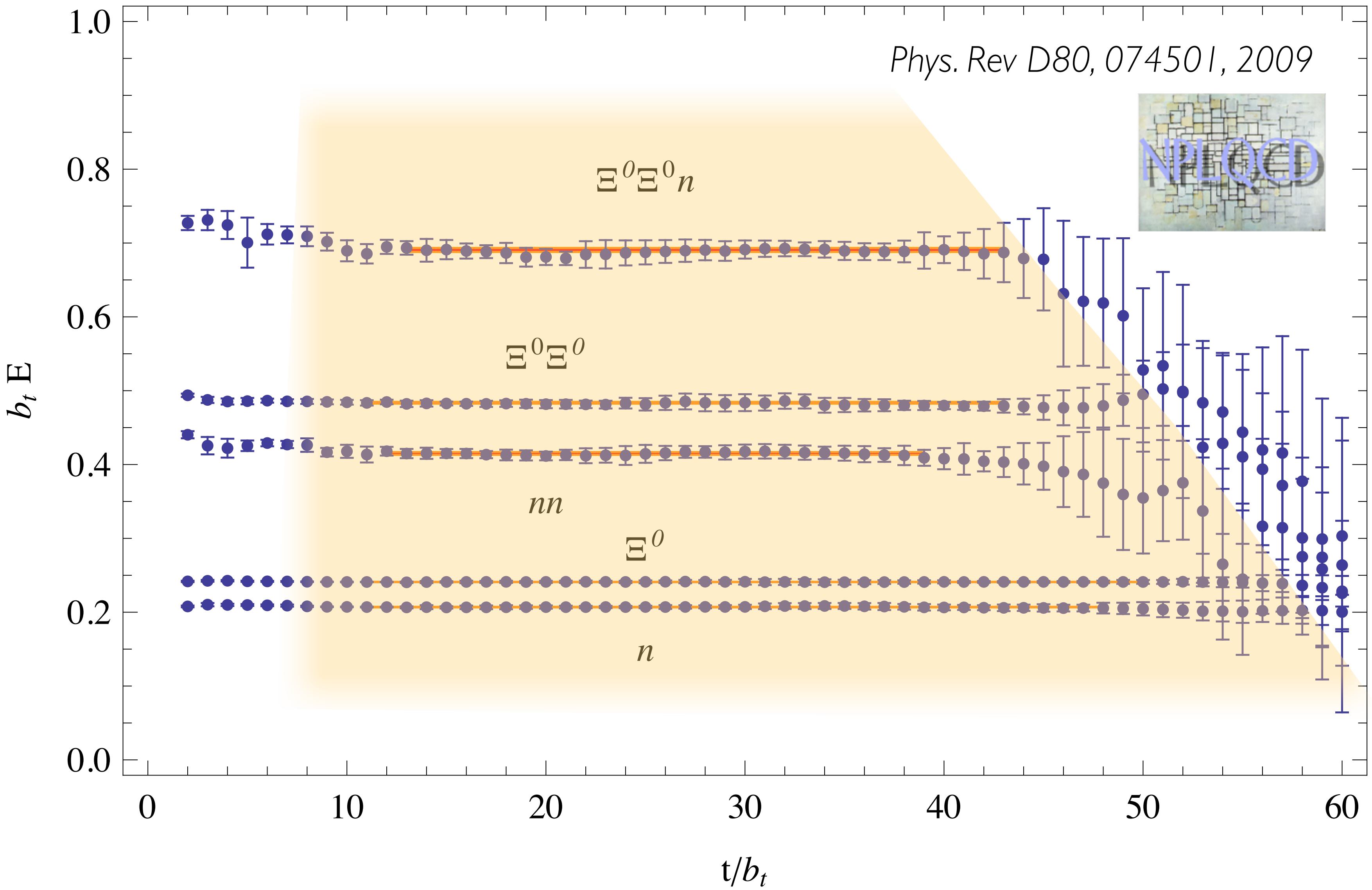

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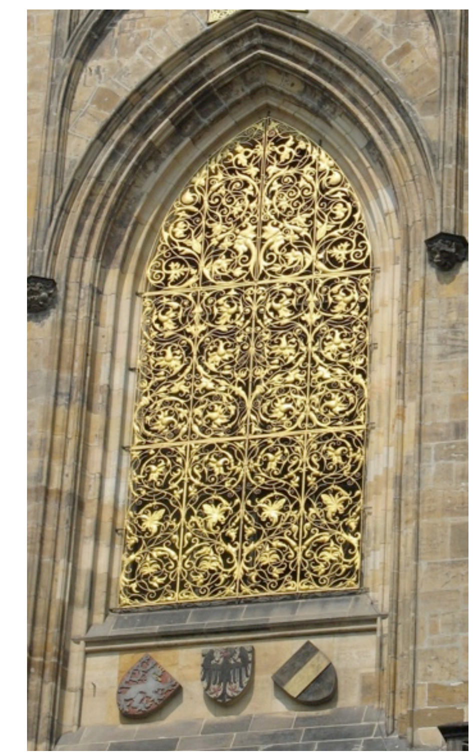

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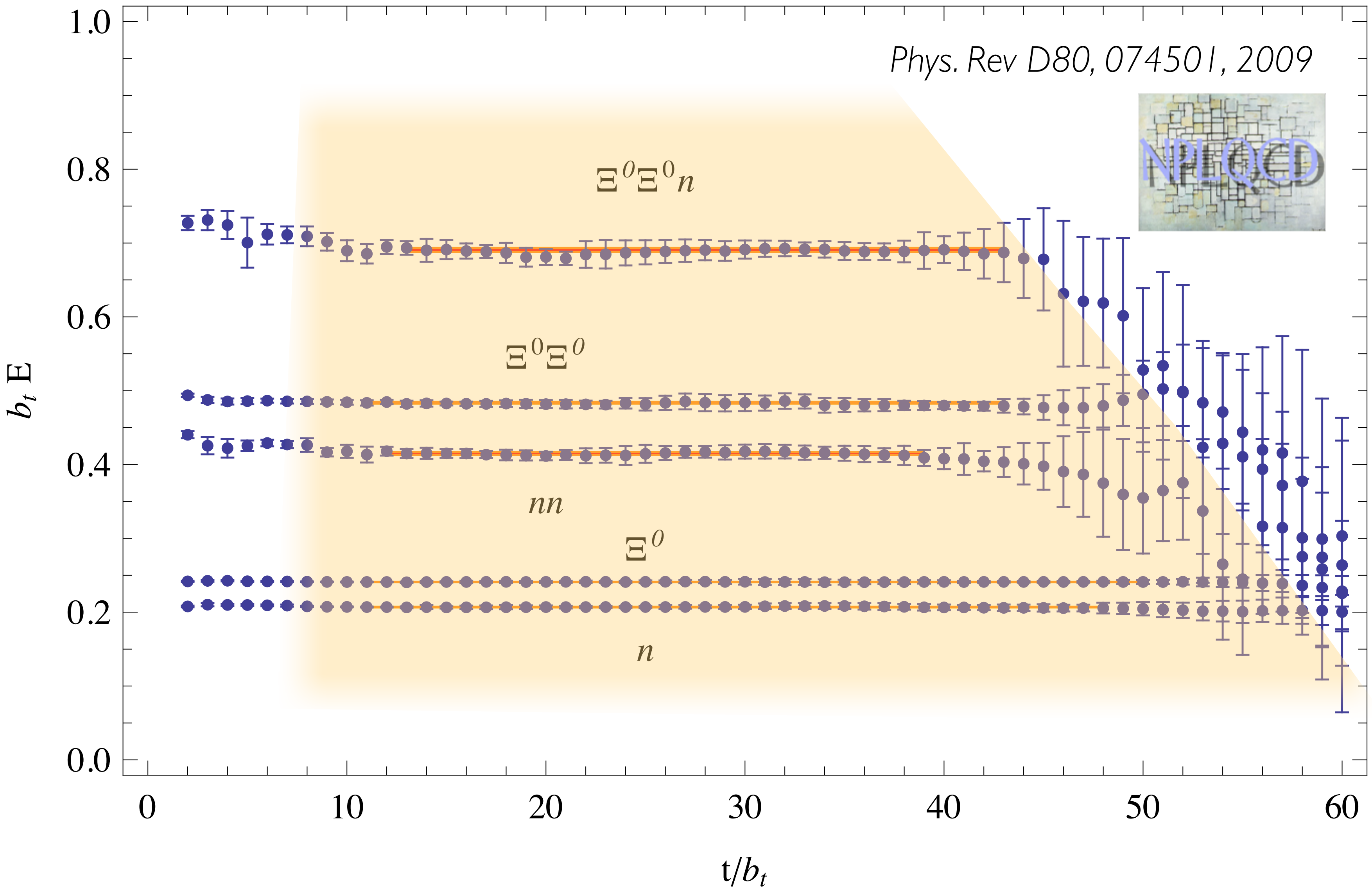

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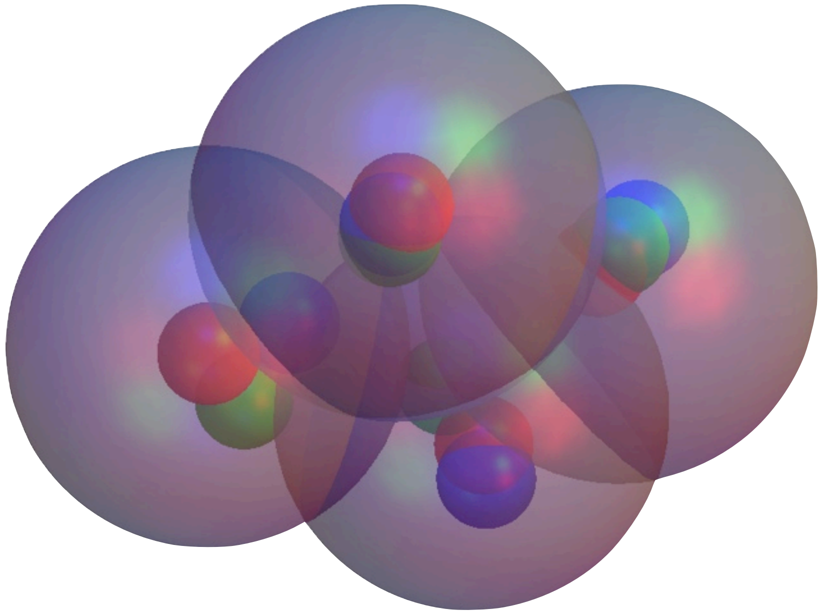
Golden window of time-slices where signal/noise const

Interpolator choice can be optimised to suppress noise

Multi-baryon systems



- ^{3}H, ^{4}He and more exotic: ^{4}He$_\Lambda$, ^{4}He$_{\Lambda\Lambda}$ (hypernuclei)
- Correlators for significantly larger A
- Caveat: at unphysical quark masses

Nuclei

- ^{3}H, ^{4}He and more exotic: $^{4}\text{He}_{\Lambda}$, $^{4}\text{He}_{\Lambda\Lambda}$ (hypernuclei)
- Correlators for significantly larger A
- Recent studies at SU(3) point (physical ms)
    - Isotropic clover lattices
    - Single lattice spacing: 0.145 fm
    - Multiple volumes: 3.4, 4.5, 6.7 fm
    - High statistics

Label	L/b	T/b	β	$b\ m_q$	b [fm]	L [fm]	T [fm]	m_π [MeV]	$m_\pi\ L$	$m_\pi\ T$	N_{cfg}	N_{src}
A	24	48	6.1	-0.2450	0.145	3.4	6.7	806.5(0.3)(0)(8.9)	14.3	28.5	3822	48
B	32	48	6.1	-0.2450	0.145	4.5	6.7	806.9(0.3)(0.5)(8.9)	19.0	28.5	3050	24
C	48	64	6.1	-0.2450	0.145	6.7	9.0	806.7(0.3)(0)(8.9)	28.5	38.0	1212	32

SU(3) symmetric world

- In flavour SU(3) symmetric case, multi-baryon states cone in multiplets

$$\mathbf{8}\otimes\mathbf{8} \;=\; \mathbf{27}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{1}$$

$$\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8} \;=\; \mathbf{64}\oplus 2\ \mathbf{35}\oplus 2\ \overline{\mathbf{35}}\oplus 6\ \mathbf{27}\oplus 4\ \mathbf{10}\oplus 4\ \overline{\mathbf{10}}\oplus 8\ \mathbf{8}\oplus 2\ \mathbf{1}$$

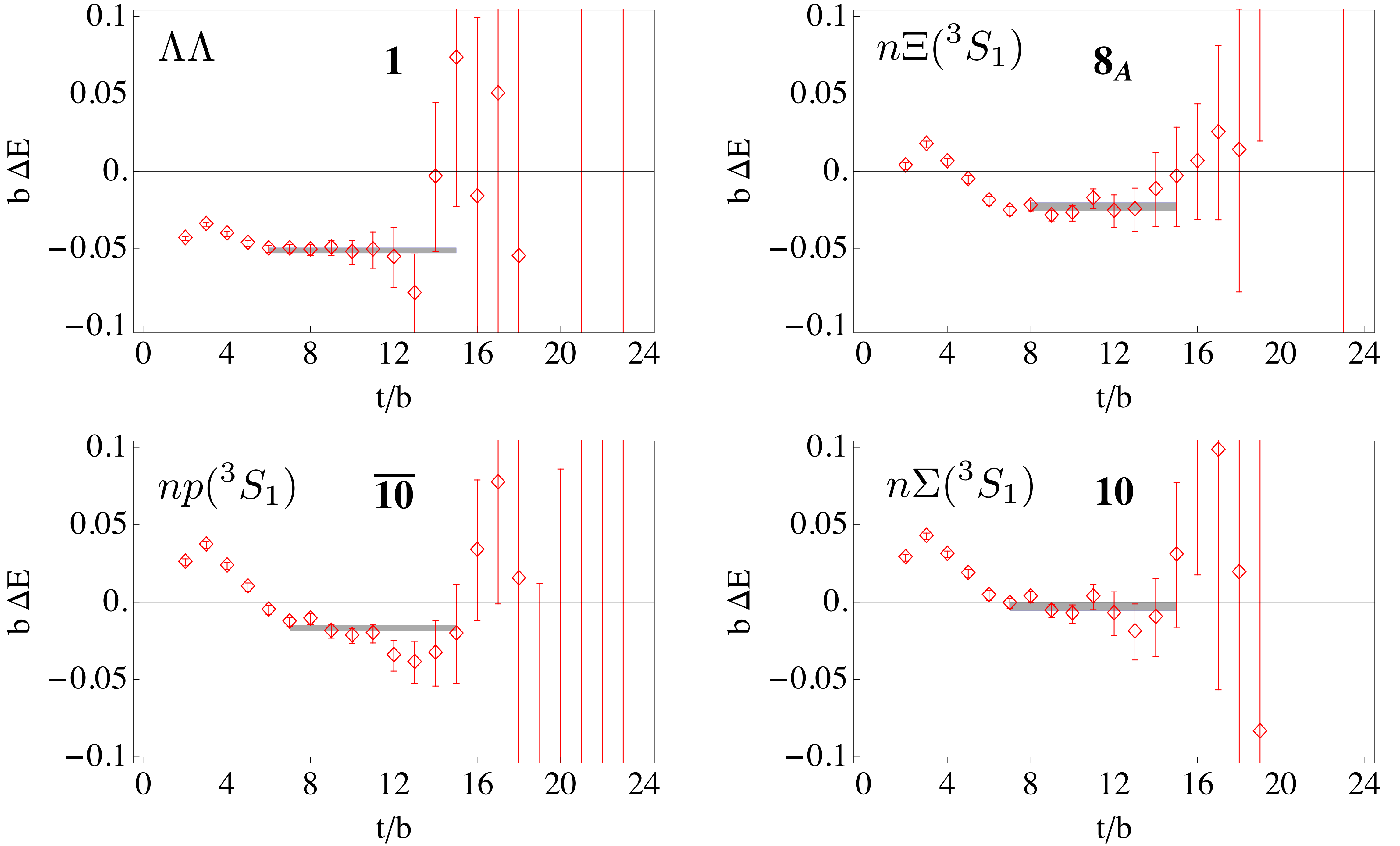
$$\begin{aligned}\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8} \;=\;& 8\ \mathbf{1}\oplus 32\ \mathbf{8}\oplus 20\ \mathbf{10}\oplus 20\ \overline{\mathbf{10}}\oplus 33\ \mathbf{27}\oplus 2\ \mathbf{28}\oplus 2\ \overline{\mathbf{28}}\oplus 15\ \mathbf{35}\oplus 15\ \overline{\mathbf{35}}\\ &\oplus 12\ \mathbf{64}\oplus 3\ \mathbf{81}\oplus 3\ \overline{\mathbf{81}}\oplus\mathbf{125}\quad ,\qquad\qquad (1\end{aligned}$$

$$\begin{aligned}\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8} \;=\;& 32\ \mathbf{1}\oplus 145\ \mathbf{8}\oplus 100\ \mathbf{10}\oplus 100\ \overline{\mathbf{10}}\oplus 180\ \mathbf{27}\oplus 20\ \mathbf{28}\oplus 20\ \overline{\mathbf{28}}\\ &\oplus 100\ \mathbf{35}\oplus 100\ \overline{\mathbf{35}}\oplus 94\ \mathbf{64}\oplus 5\ \mathbf{80}\oplus 5\ \overline{\mathbf{80}}\oplus 36\ \mathbf{81}\oplus 36\ \overline{\mathbf{81}}\\ &\oplus 20\ \mathbf{125}\oplus 4\ \mathbf{154}\oplus 4\ \overline{\mathbf{154}}\oplus\mathbf{216}\quad .\end{aligned}$$

- Unphysical symmetries manifest in spectrum

Nuclei (A=2)

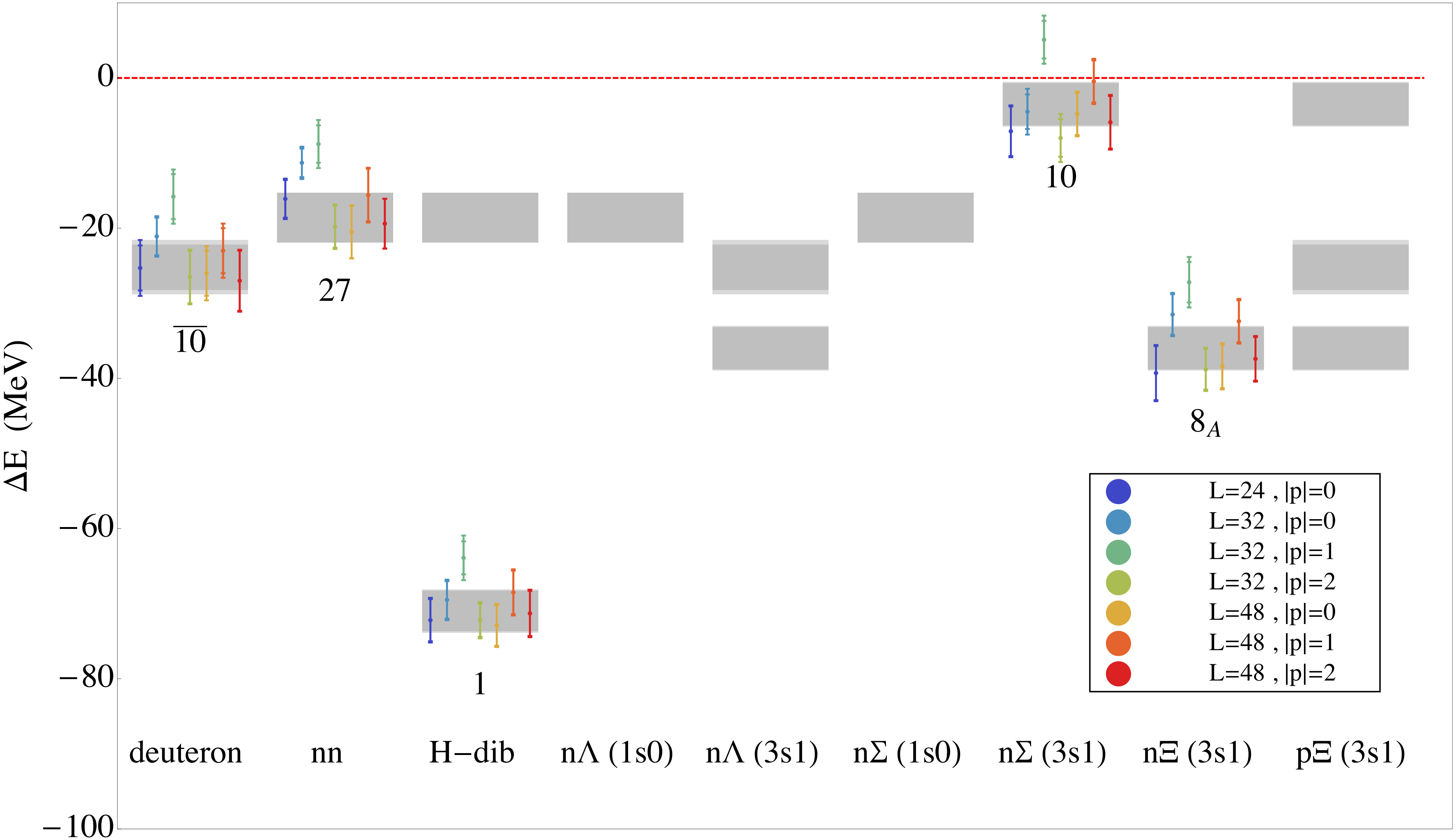
Quark-hadron contraction method



NPLQCD arXiv:1206.5219

Nuclei (A=2)

- Quark-hadron contraction method
- Multiple boost frames



NPLQCD arXiv:1206.5219

Nuclei (A=3,4)

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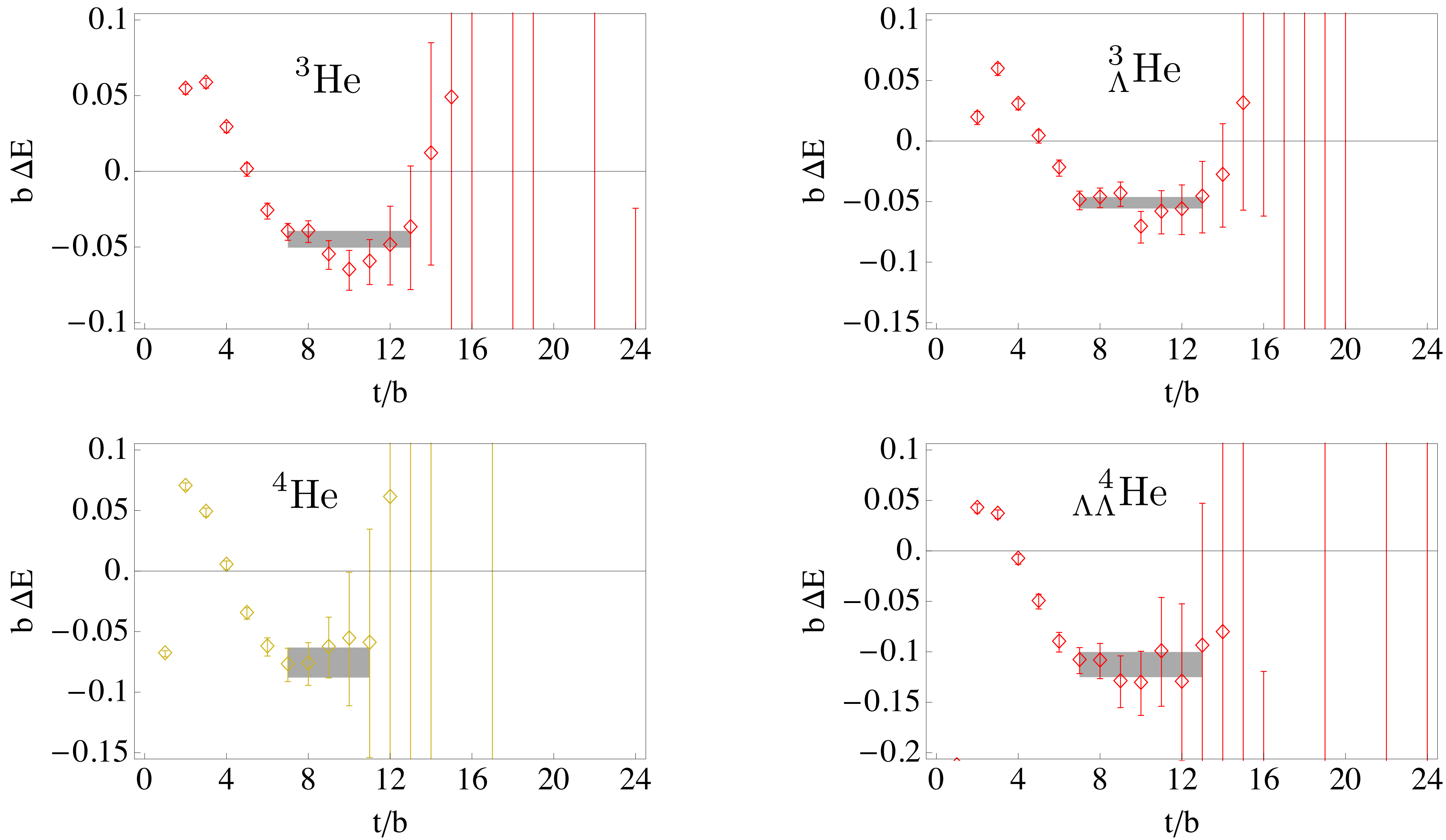

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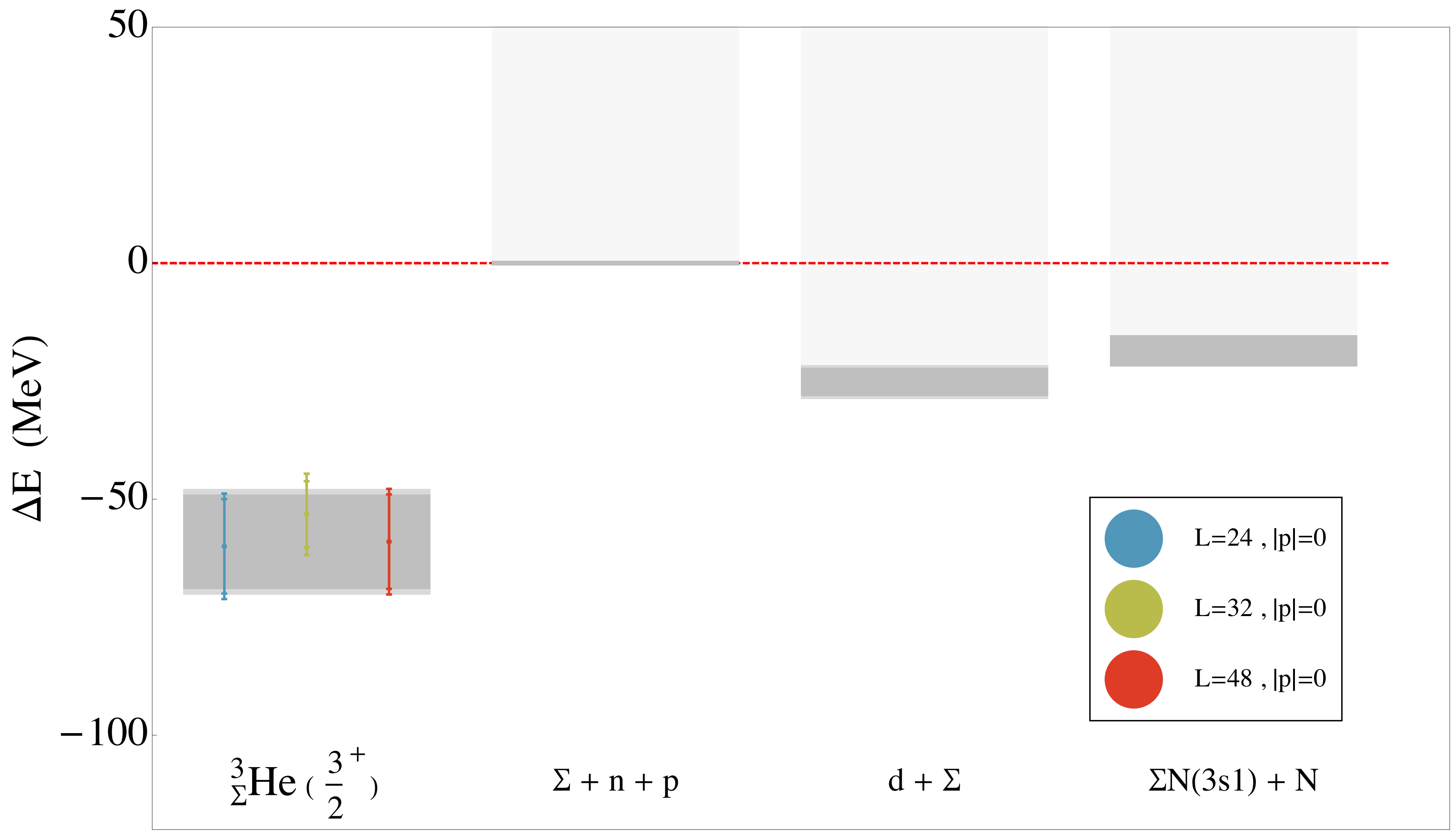
Nuclei (A=3,4)

- Need to ask if this is a 2+1 or 3+1 or 2+2 etc scattering state
- Empirically investigate volume dependence



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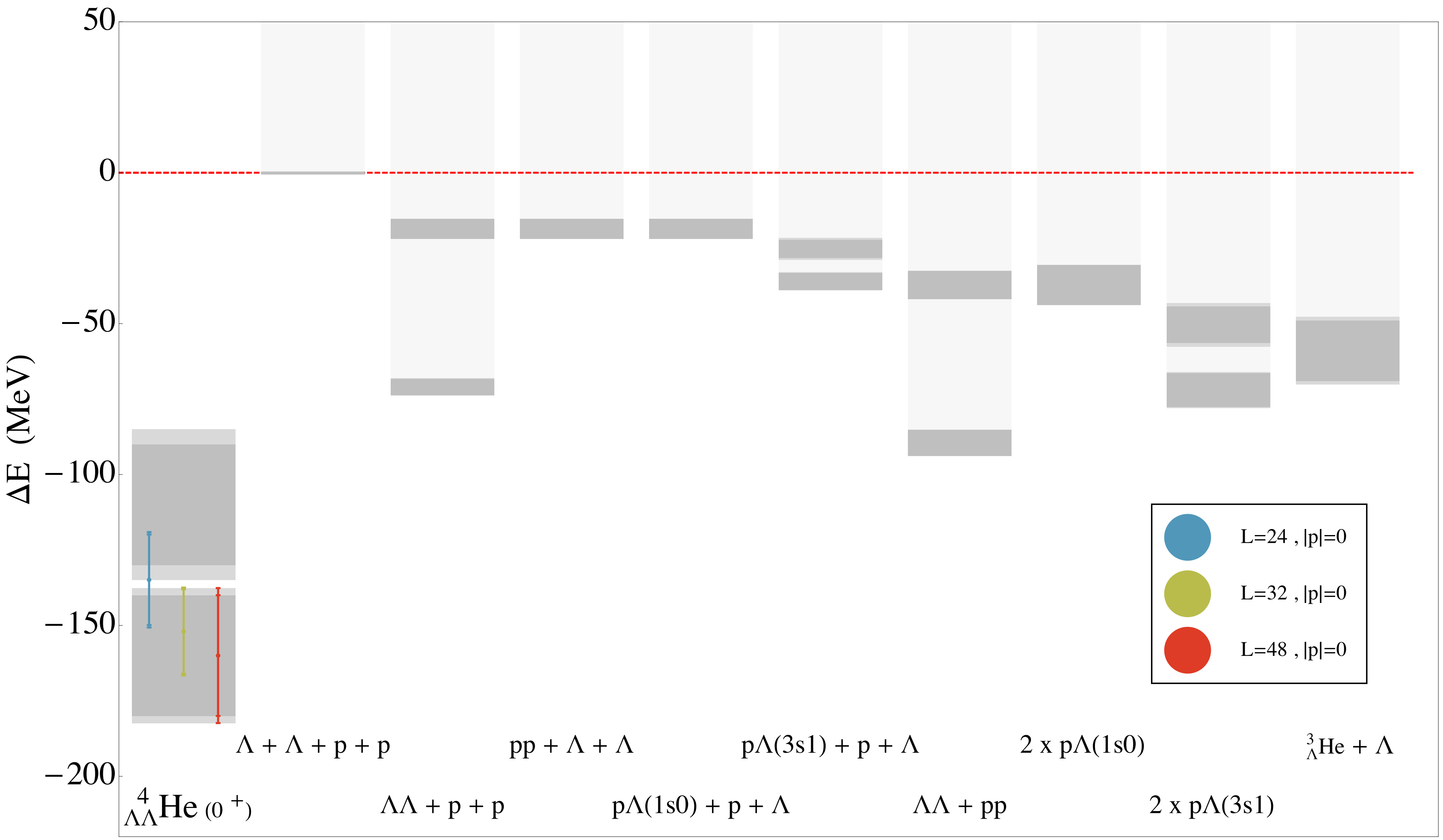

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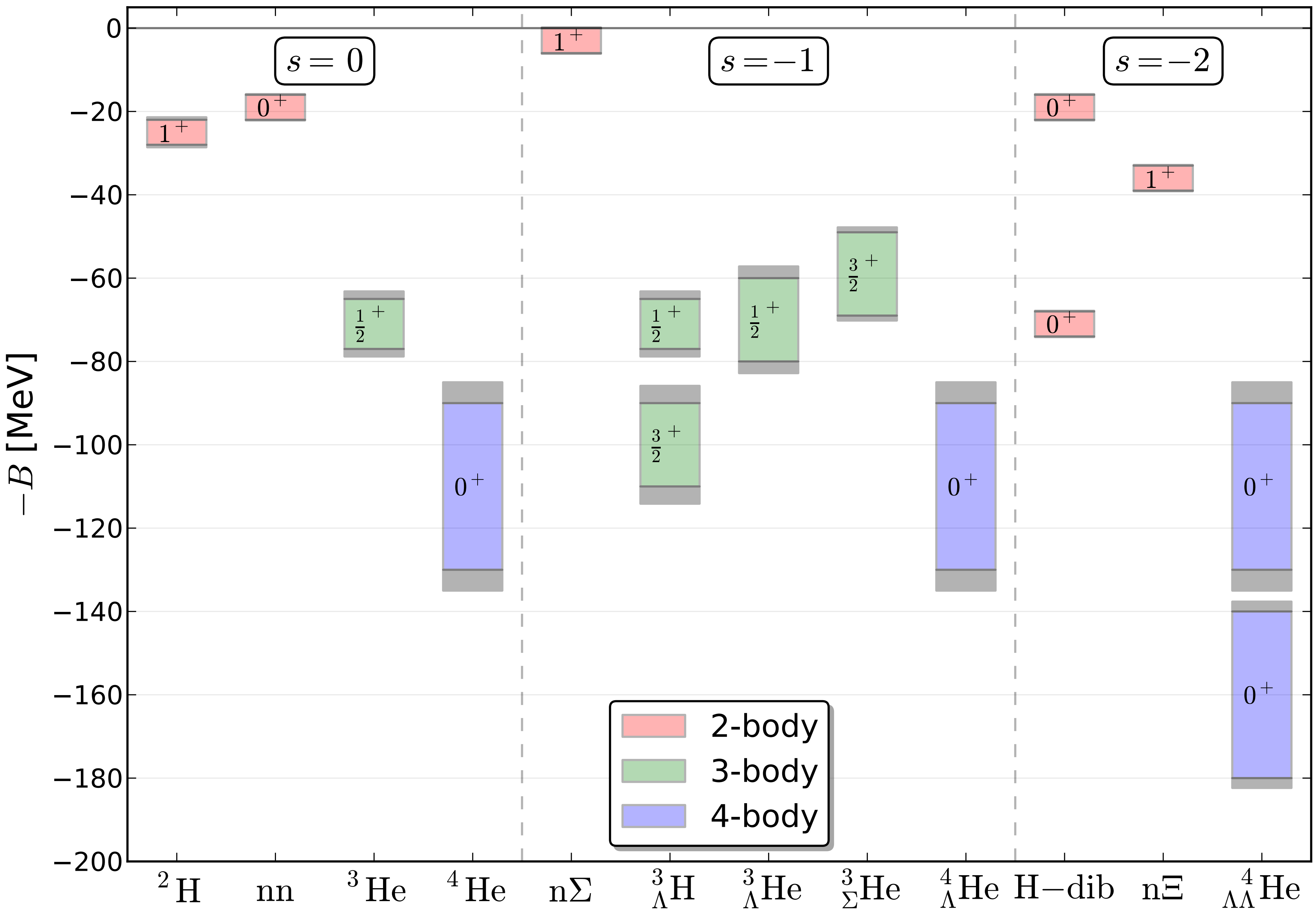
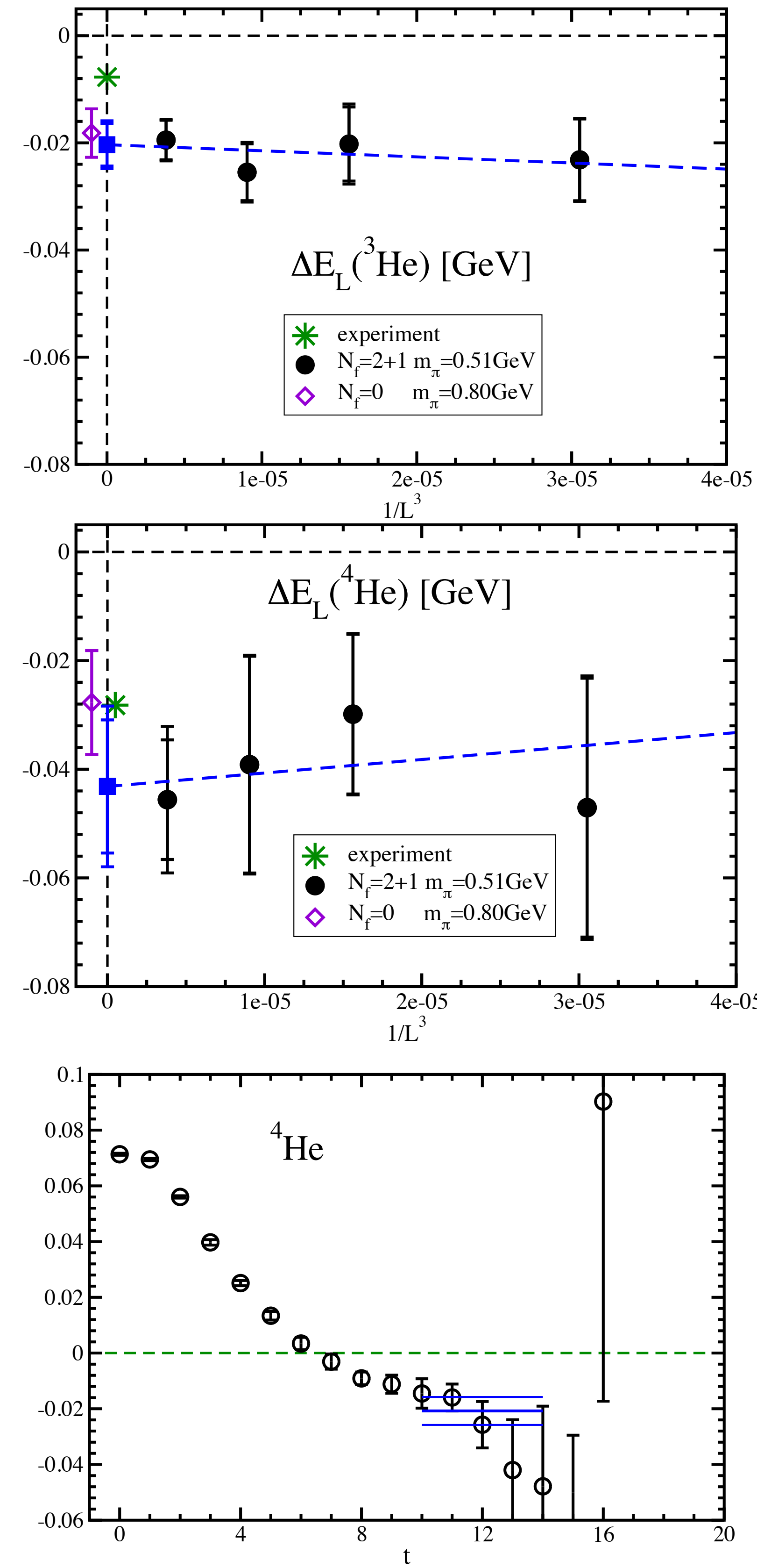


NPLQCD arXiv:1206.5219

d, nn, ^{3}He, ^{4}He

- PACS-CS: bound d,nn, ^{3}He, ^{4}He
  - Previous quenched work
  - Recent unquenched study at m_π=500 MeV
- HALQCD
  - Extract an energy-dependent NN potential
  - Strong enough to bind H, ^{4}He at m_{PS}=490 MeV SU(3) pt
  - d, nn not bound



[Yamazaki et al. 1207.4277]

Nuclei (A=4,...)

Quark-quark determinant contraction method

(low statistics, single volume)

WD, Kostas Orginos, 1207.1452

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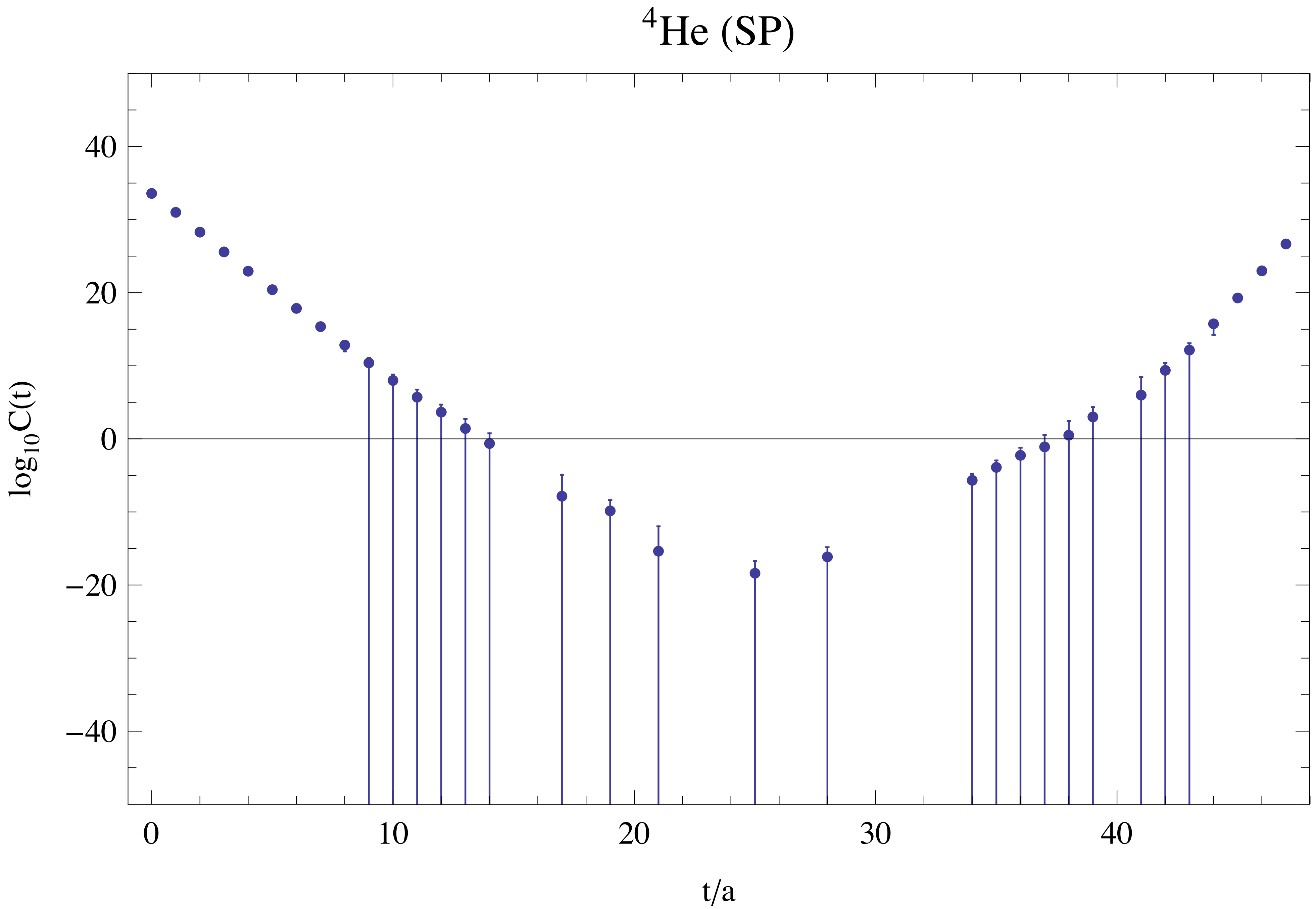
Quark-quark determinant contraction method



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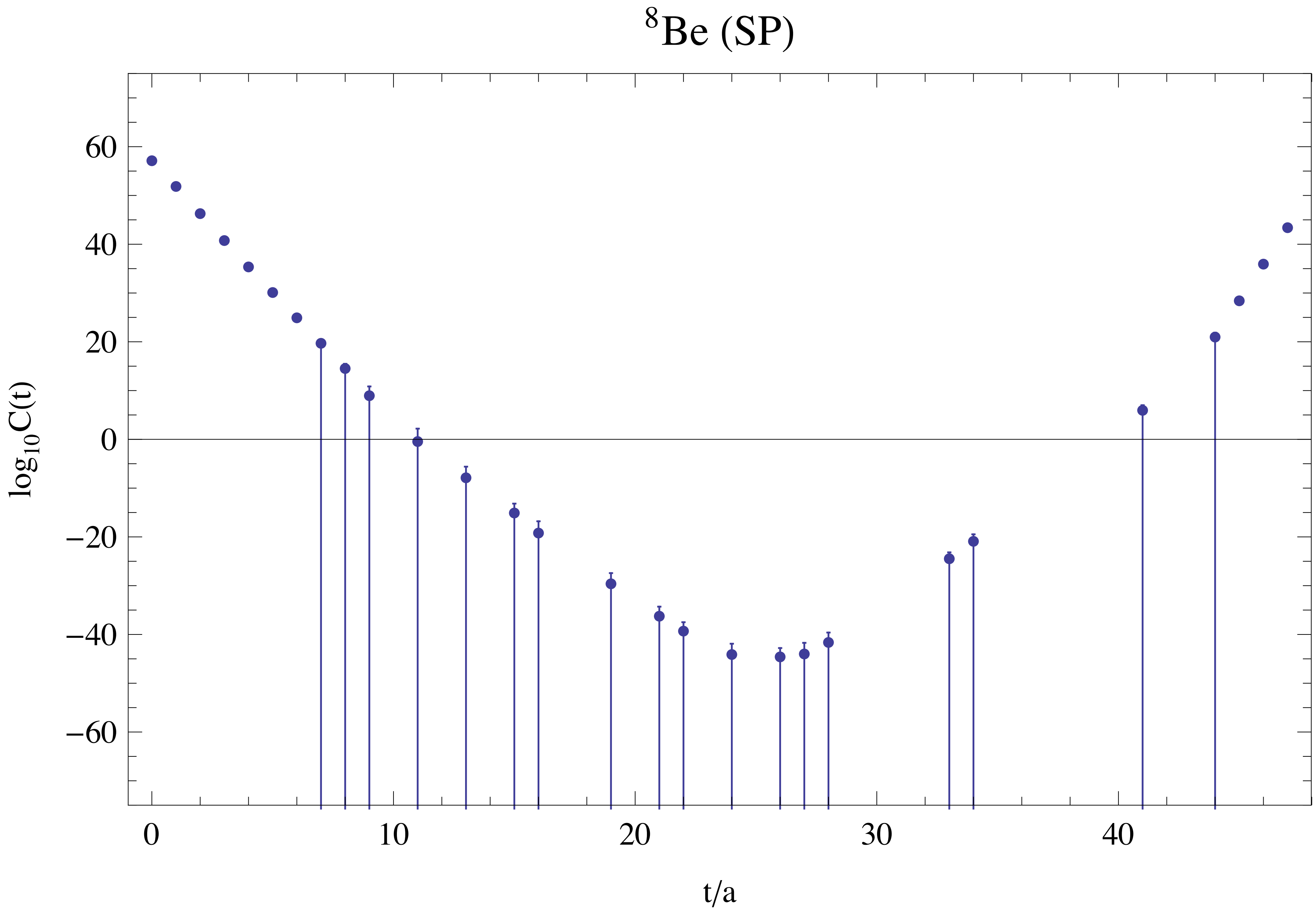

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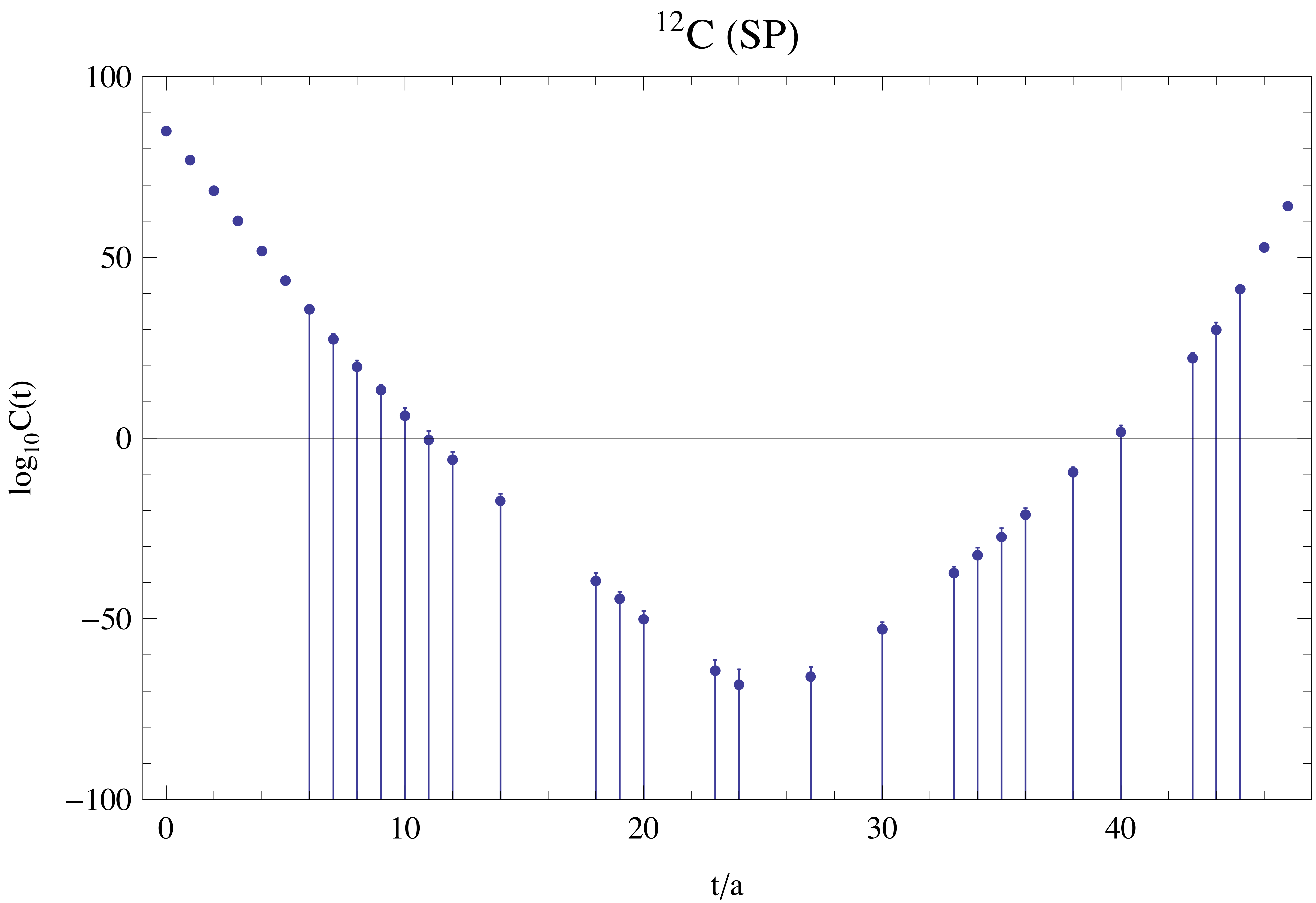

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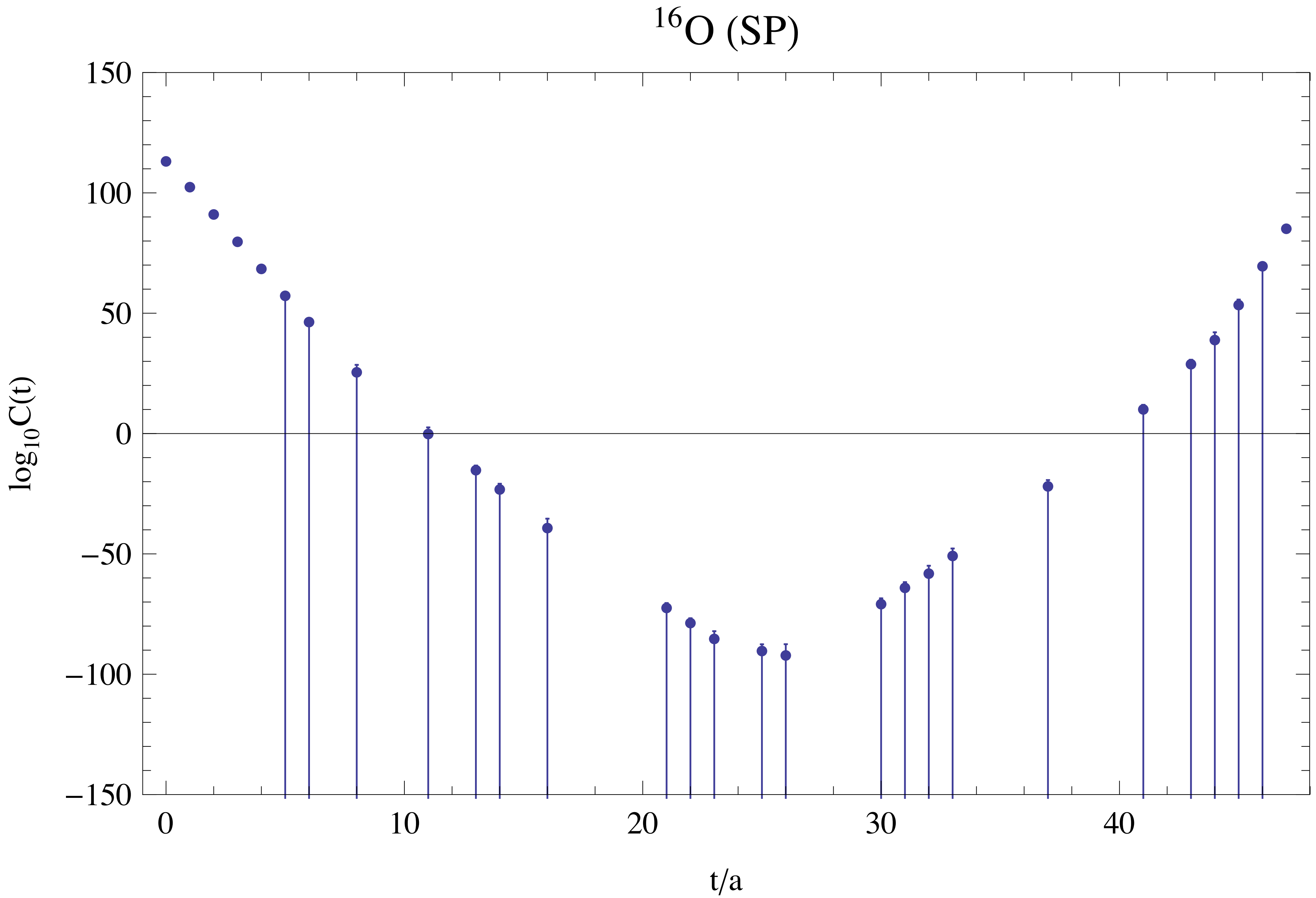

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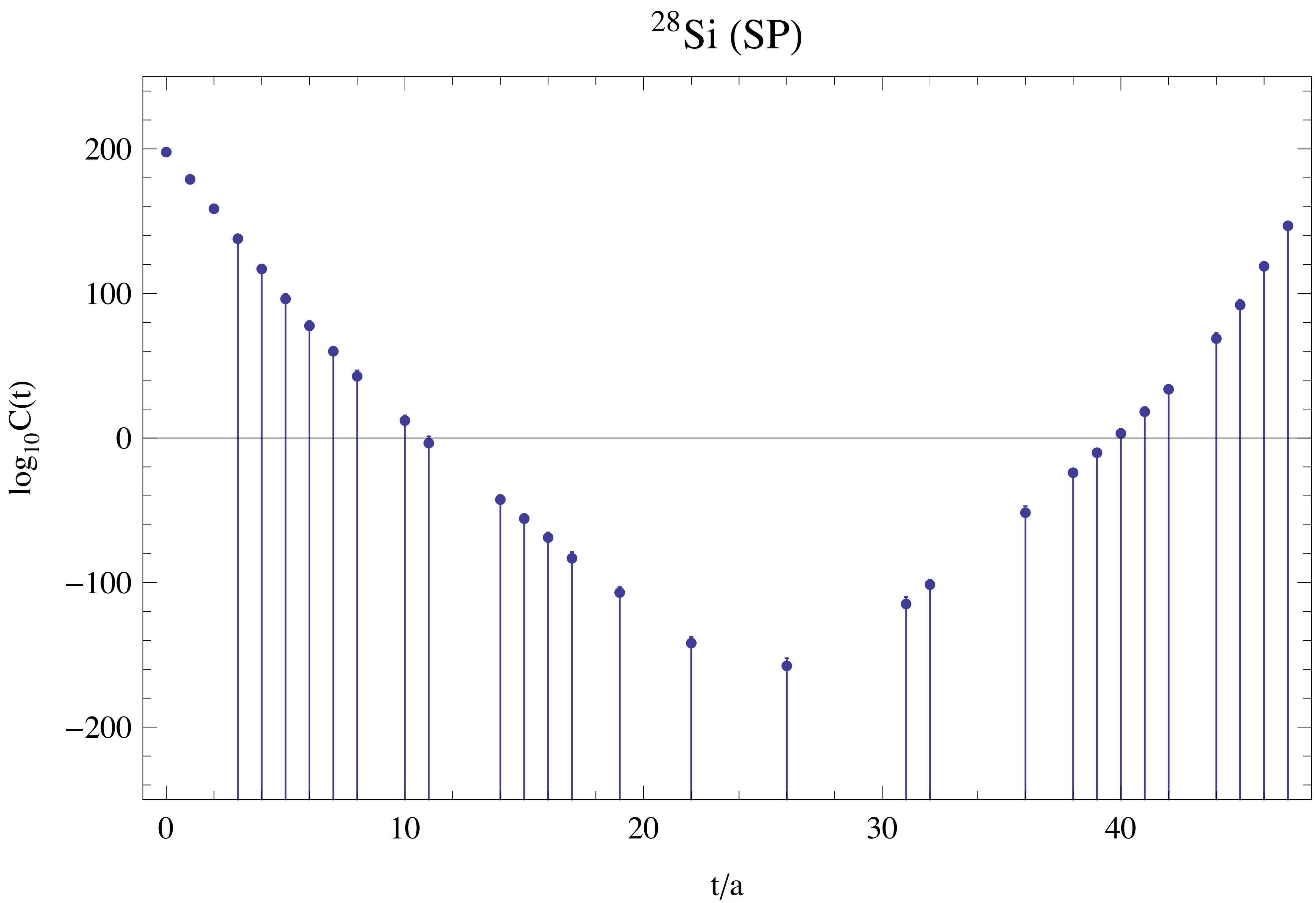


(low statistics, single volume)

WD, Kostas Orginos, 1207.1452

Multi-particle systems at finite temporal extent

Multi-pion correlation function

- Consider N pion correlation function

$$C_n(t) \;\propto\; \langle \left(\sum_{\mathbf{x}} \pi^-(\mathbf{x},t) \right)^n \left(\pi^+(\mathbf{0},0) \right)^n \rangle$$

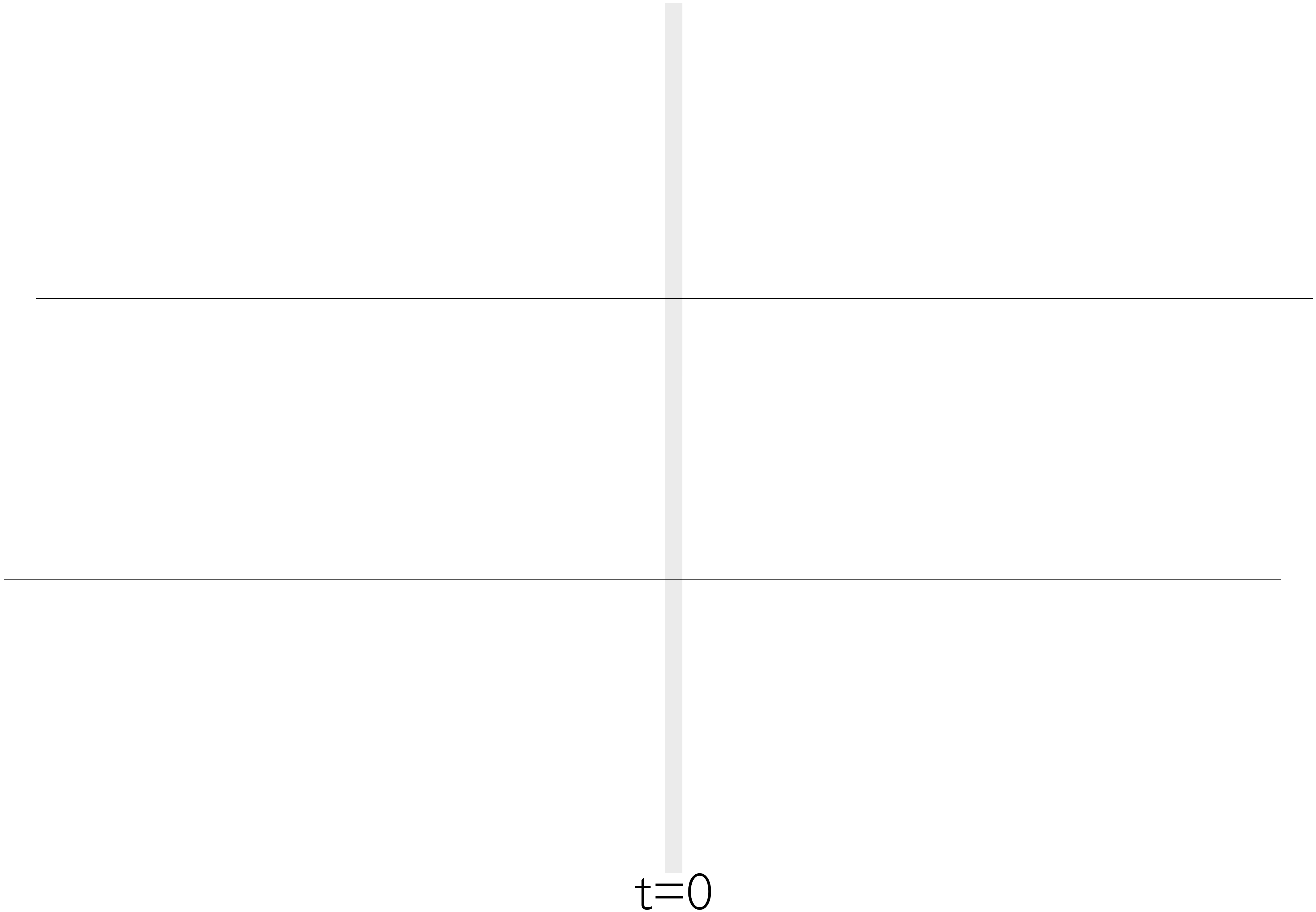
- For a lattice of temporal extent T (inverse temperature)

$$\begin{aligned}
C_n(t) &= \mathrm{Tr}\left[e^{-HT} \left(\sum_{\mathbf{x}} \pi^-(\mathbf{x},t) \right)^n \left(\pi^+(0)\right)^n \right] \\
&= \sum_m \left\langle m \left| e^{-HT} \left(\sum_{\mathbf{x}} \pi^-(\mathbf{x},t) \right)^n \left(\pi^+(0)\right)^n \right| m \right\rangle \\
&= \sum_{m,\ell} \left\langle m \left| e^{-HT} \left(\sum_{\mathbf{x}} \pi^-(\mathbf{x},t) \right)^n \right| \ell \right\rangle \langle \ell \left| \left(\pi^+(0)\right)^n \right| m \rangle \\
&= \sum_{m,\ell} \left\langle m \left| e^{-H(T-t)} \left(\sum_{\mathbf{x}} \pi^-(\mathbf{x},0) \right)^n e^{-Ht} \right| \ell \right\rangle \langle \ell \left| \left(\pi^+(0)\right)^n \right| m \rangle \\
&= \sum_{m,\ell} e^{-E_m(T-t)} e^{-E_\ell t} \mathcal{Z}_{\ell,m}
\end{aligned}$$

- Many states contribute (ignore excitations)

$$\{|m\rangle = |0\rangle, |\ell\rangle = |n\pi\rangle\}, \{|m\rangle = |\pi\rangle, |\ell\rangle = |(n-1)\pi\rangle\}, \ldots, \{|m\rangle = |n\pi\rangle, |\ell\rangle = |0\rangle\}$$

Four pion correlation



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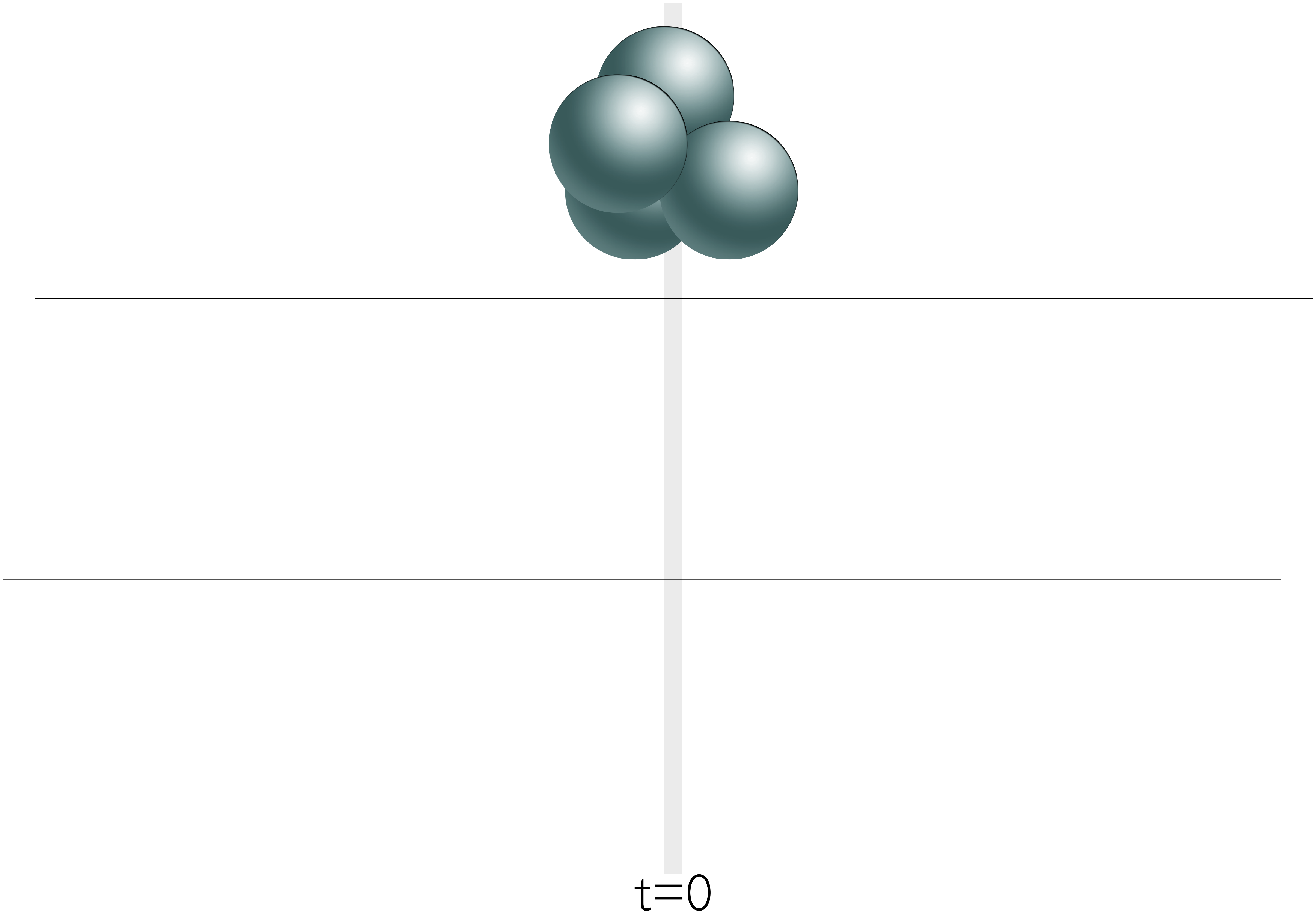

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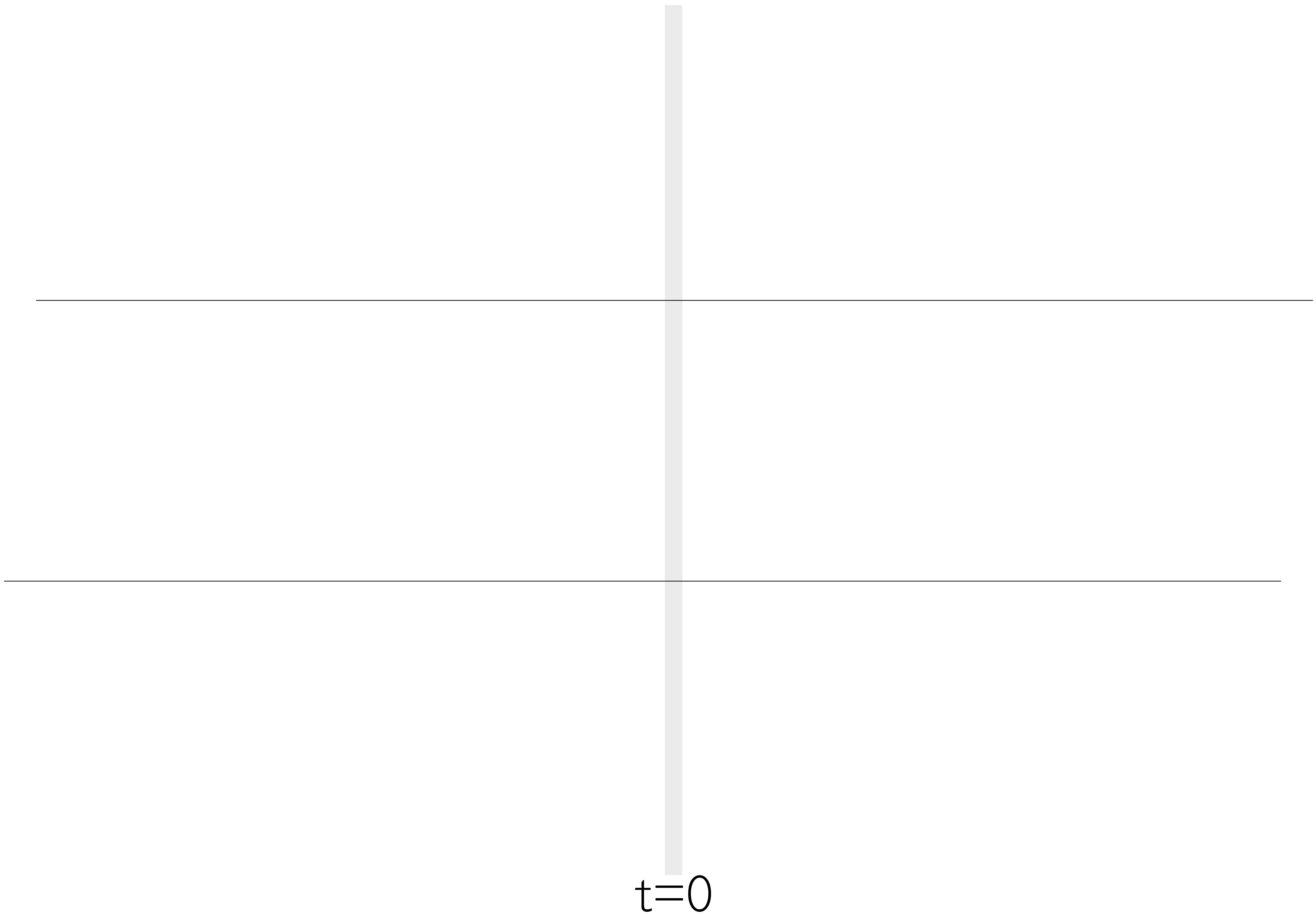

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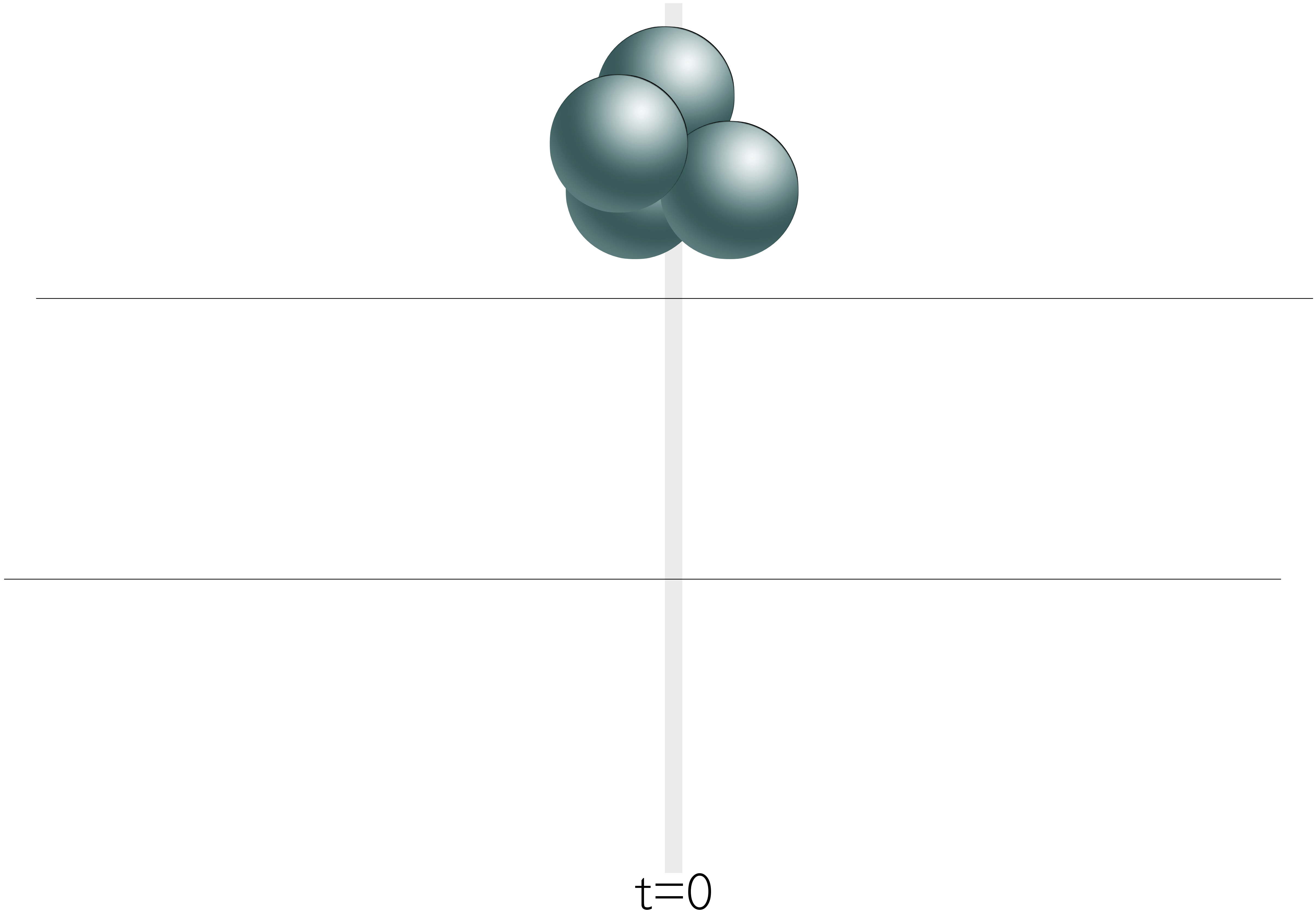
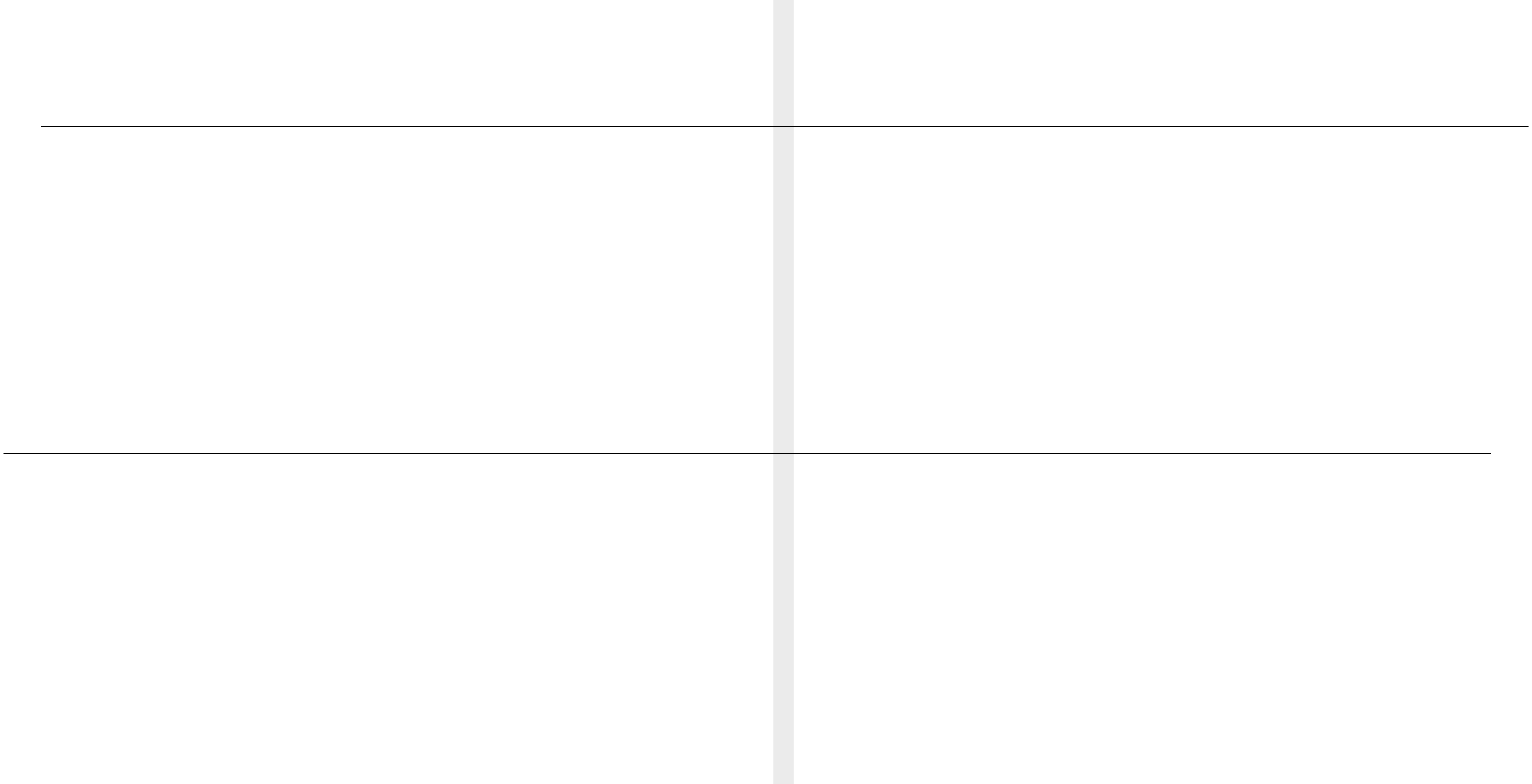


Four pion correlation

$$Z_{4\pi}\left(e^{-4E_{4\pi}t}+e^{-4E_{4\pi}(T-t)}\right)$$



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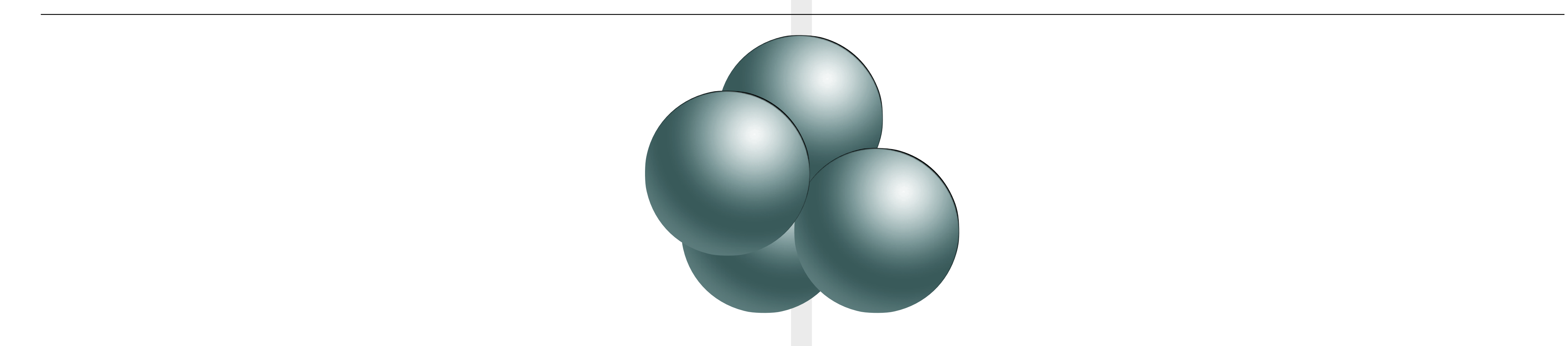

t=0

Four pion correlation

$$Z_{4\pi}\left(e^{-4E_{4\pi}t} + e^{-4E_{4\pi}(T-t)}\right)$$

t=0

Four pion correlation

$$Z_{4\pi}\left(e^{-4E_{4\pi}t}+e^{-4E_{4\pi}(T-t)}\right)$$

$$Z_{3/1\pi}\left(e^{-E_{3\pi}t}e^{-E_{1\pi}(T-t)}+e^{-E_{3\pi}(T-t)}e^{-E_{1\pi}t}\right)$$

t=0

Four pion correlation

$$Z_{4\pi}\left(e^{-4E_{4\pi}t}+e^{-4E_{4\pi}(T-t)}\right)$$

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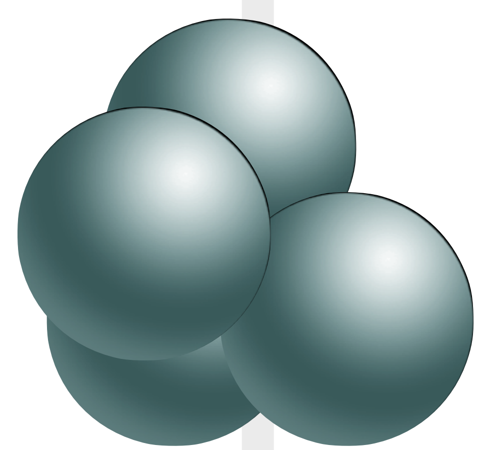

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$$Z_{2/2\pi}e^{-E_{2\pi}t}e^{-E_{2\pi}(T-t)}=Z_{2/2\pi}e^{-E_{2\pi}T}$$

t=0

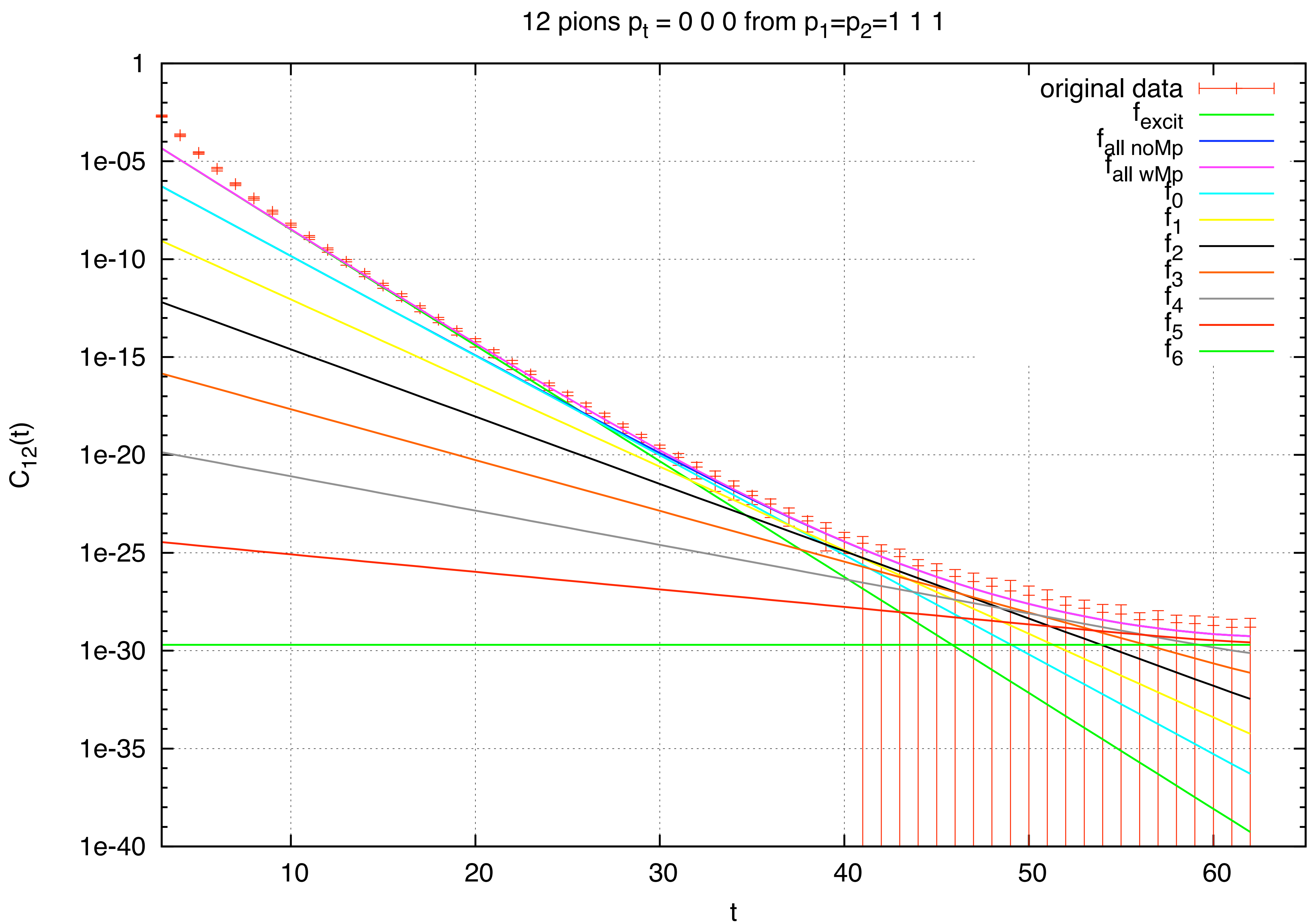
Analysis on finite T correlators

- Can rewrite the t dependence as

$$C_{n\pi}(t) = \sum_{m=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{m} A_m^n Z_m^n e^{-(E_{n-m}+E_m)T/2} \cosh((E_{n-m} - E_m)(t - T/2)) + \dots ,$$

- Extracting the eigen-energies from these correlators is difficult
  - Many parameters appear in each correlator
  - Correlations between different C_j as the energy E_k occur in all C_j ($j \geq k$) occur in multiple places
  - Various ways to deal with this: eg cascading fits

Thermal pollution



At no point does the ground state dominate the correlator!!!