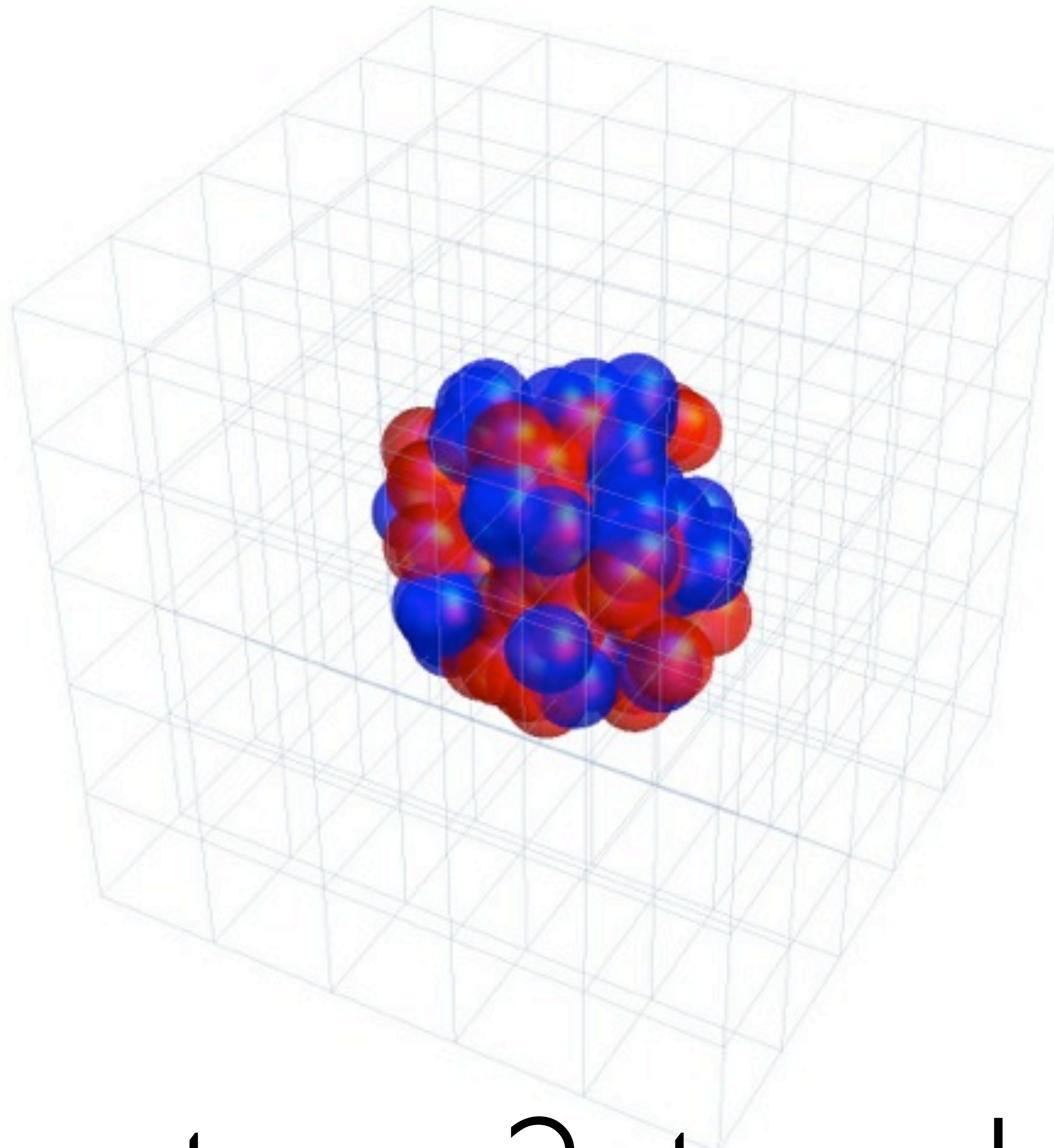


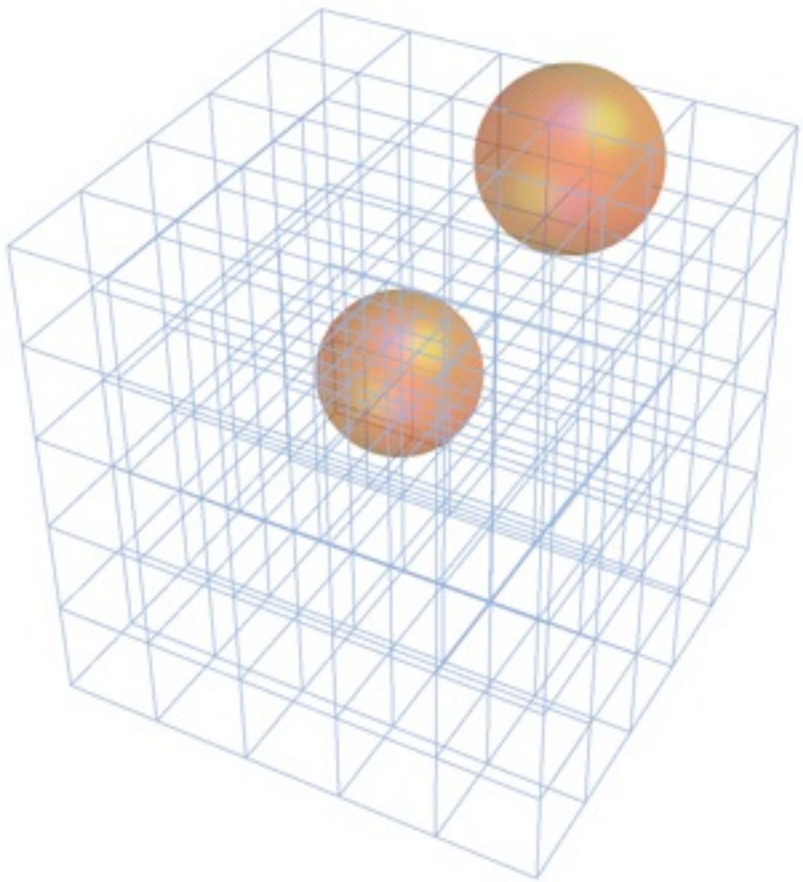
Lecture 2: two-body

William Detmold

Massachusetts Institute of Technology

Lecture content

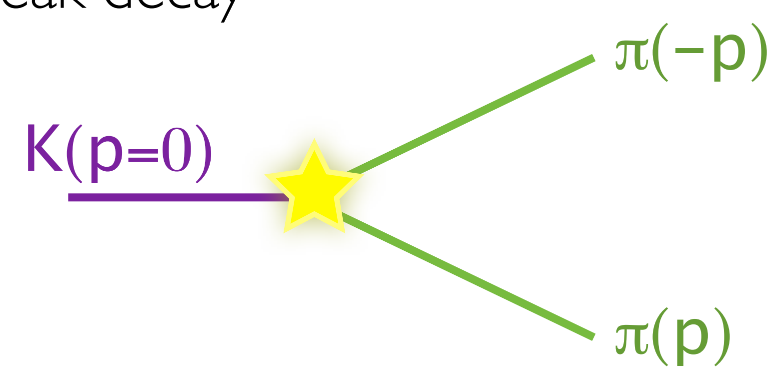
- Two particles in finite volume: Lüscher method
  - Scattering
  - Resonances
  - Two-particle bound states



Theory

Maiani-Testa no-go theorem

- Scattering and decays are real-time processes
- How can Euclidean space (imaginary time) calculations address generic Minkowski space correlations?
  - Maiani & Testa [91]: Euclidean correlators with initial/final states at kinematic thresholds allow access to physical information (matrix elements, weak decays)
  - In infinite volume away from kinematic thresholds, scattering continuum masks the physically interesting information
- Example: K → π π weak decay



  - Consider

$$C(t_1, t_2) = \langle \mathcal{O}_K(t_1) \mathcal{O}_{\text{weak}}(0) \mathcal{O}_{\pi\pi}(t_2) \rangle$$

Take large $|t_{1,2}|$ to get single state

$$\xrightarrow{?} \langle K | \mathcal{O}_{\text{weak}} | \pi(\hat{\mathbf{p}}) \pi(-\hat{\mathbf{p}}) \rangle + \ldots$$

[see also Michael 89]

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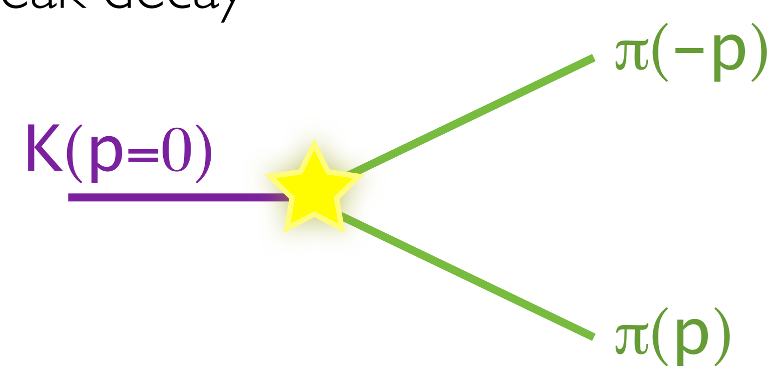

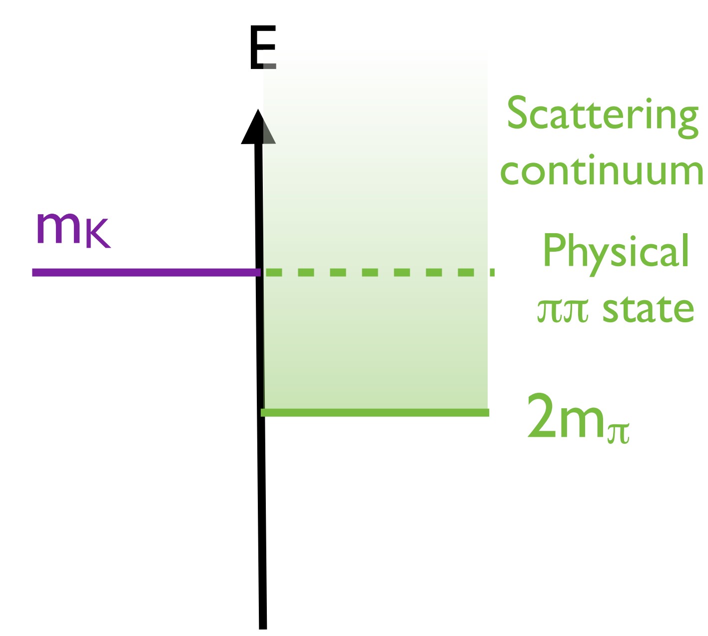

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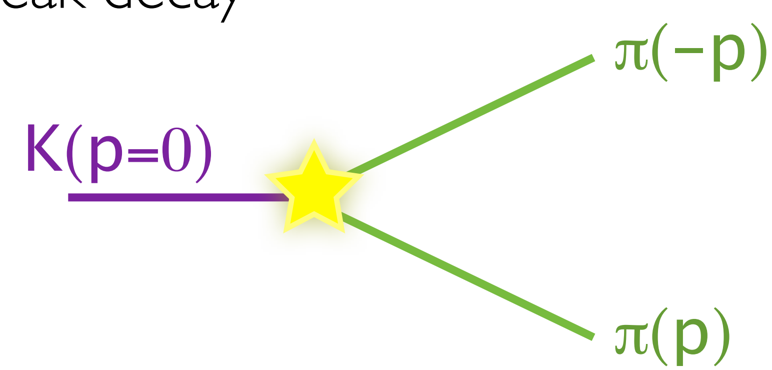

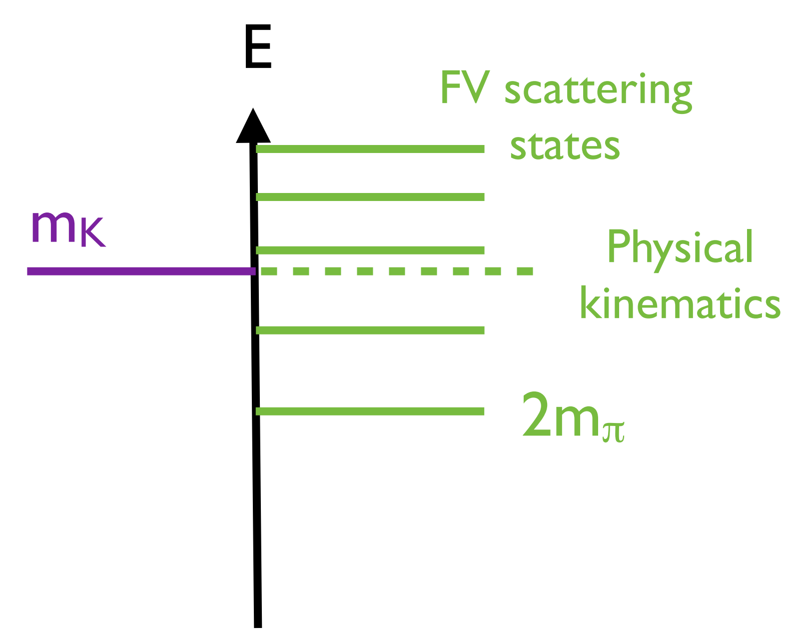

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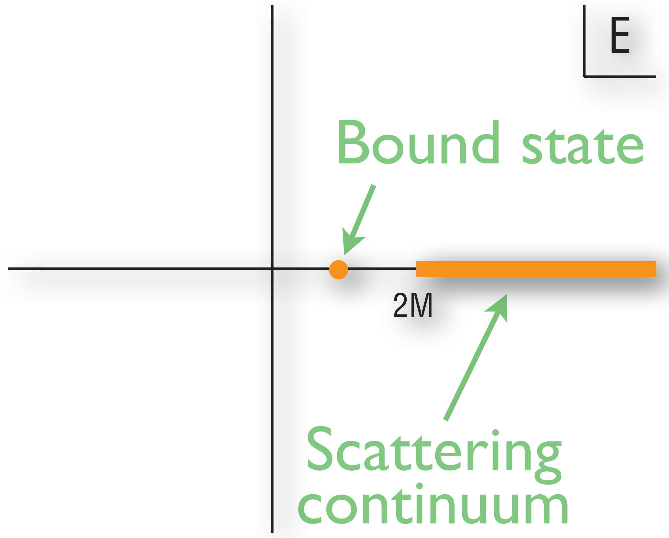
Two particles in a box

- Long realised that forcing particles to be in a finite volume shifts their energy in a way that depends on their interactions
  - Uhlenbeck 1930's; Bogoliubov 1940's; Lee, Huang, Yang 1950's, ...
  - Lüscher (1986,1991) demonstrated that this is also true in QFT up to inelastic thresholds (see also Hamber, Marinari, Parisi & Rebbi)
- Energy eigenvalues of discrete scattering states well defined – no issue in Minkowski-Euclidean connection
  - Bypasses Maiani-Testa NGT



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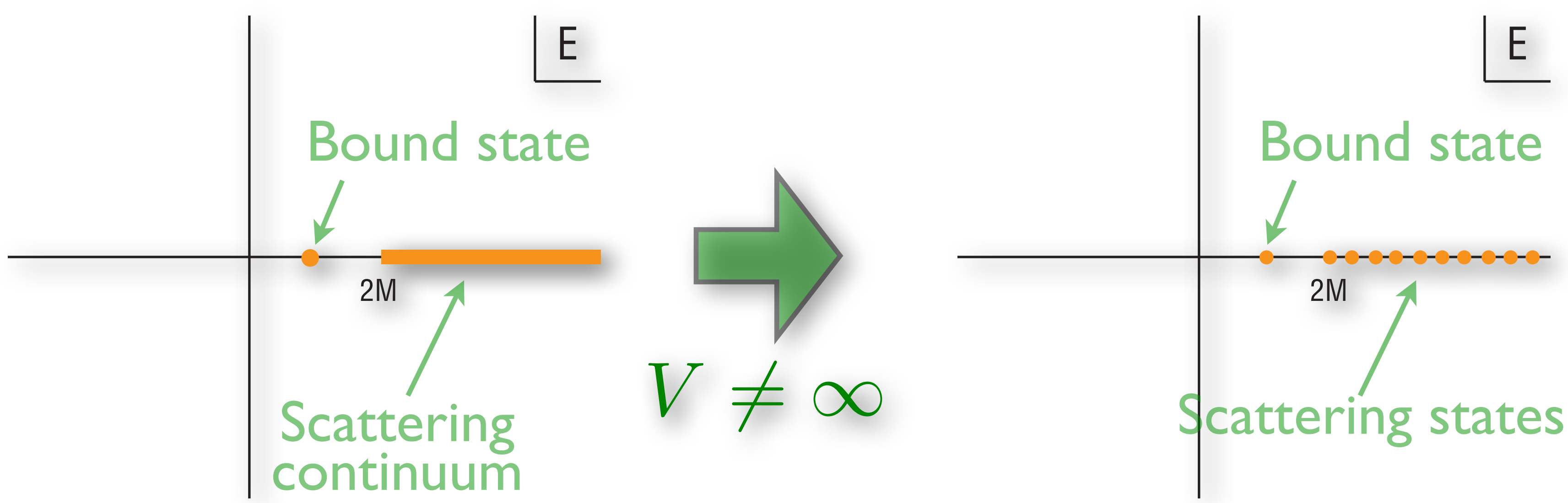
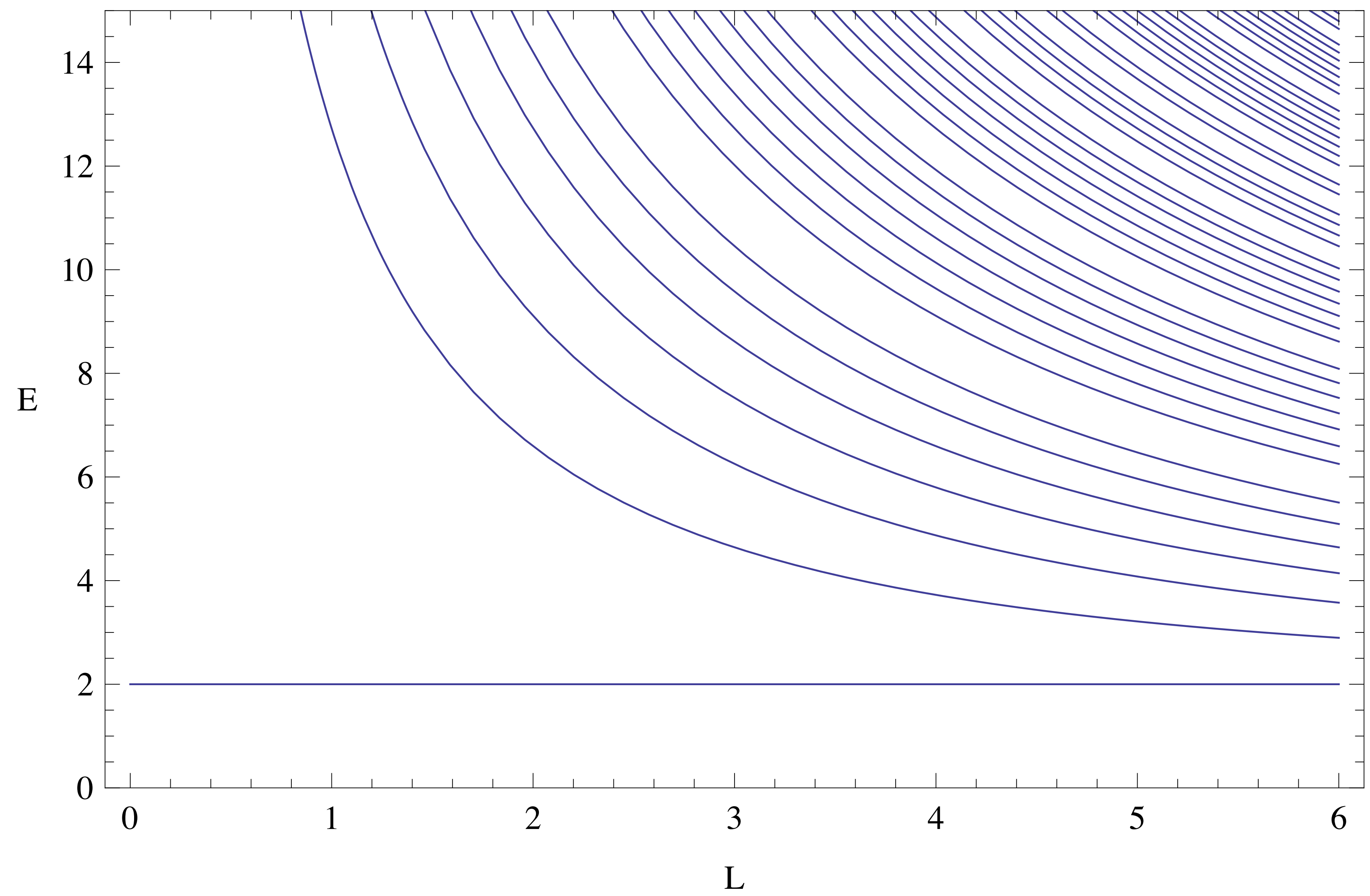


Two particles in a box

$$\mathbf{p} = \frac{2\pi}{L}\mathbf{n}, \quad n_i \in \mathbb{Z}$$

$$E_{\mathbf{n}}(L) = 2\sqrt{m^2 + \frac{4\pi^2}{L^2}|\mathbf{n}|^2}$$



Two particles in a box

- Consider first the non-interacting two particle system with particles of mass m in a box of dimensions L^3 with zero CoM momentum
- Particles constrained to have momenta

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hence the spectrum is given by

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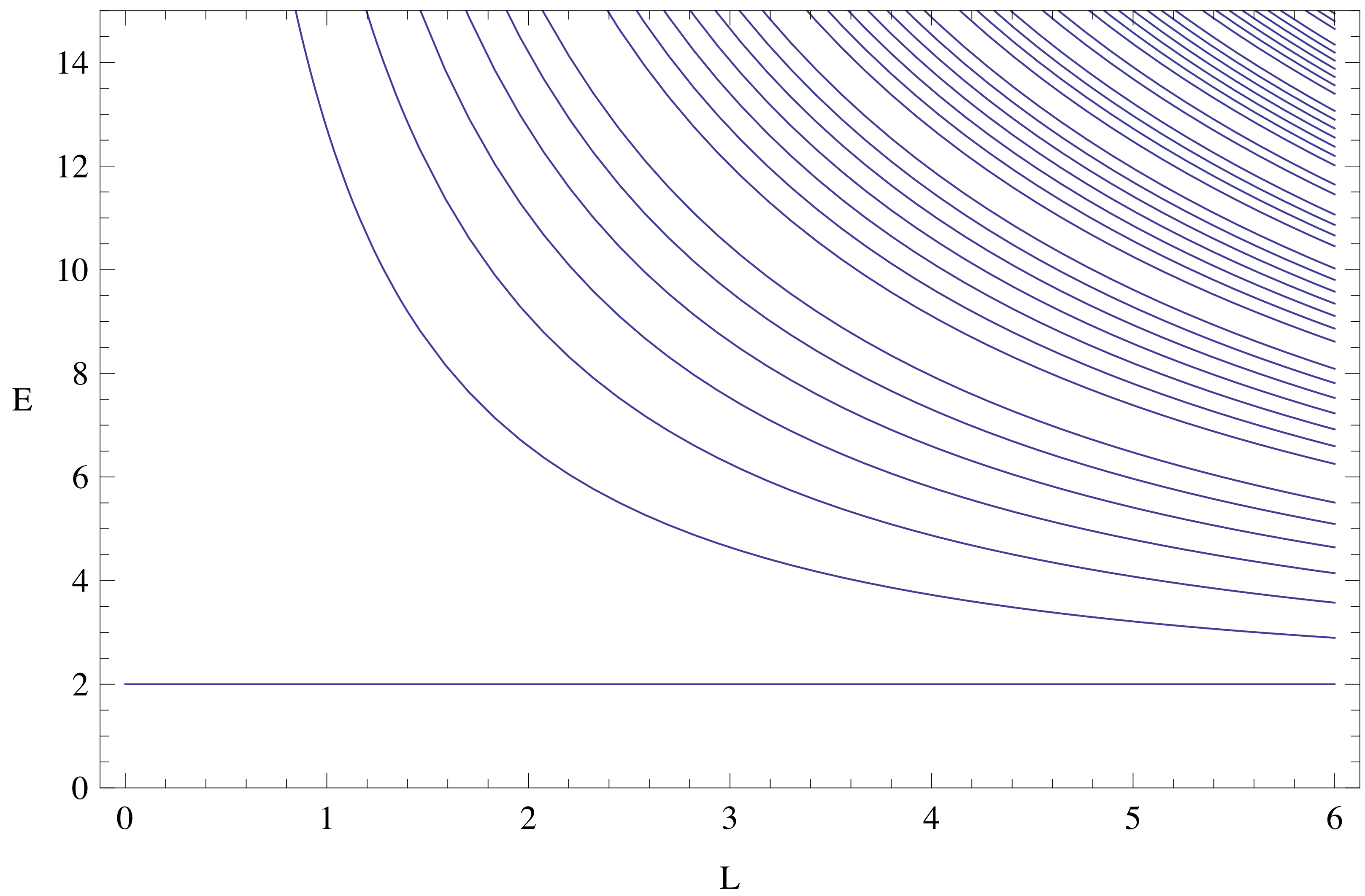

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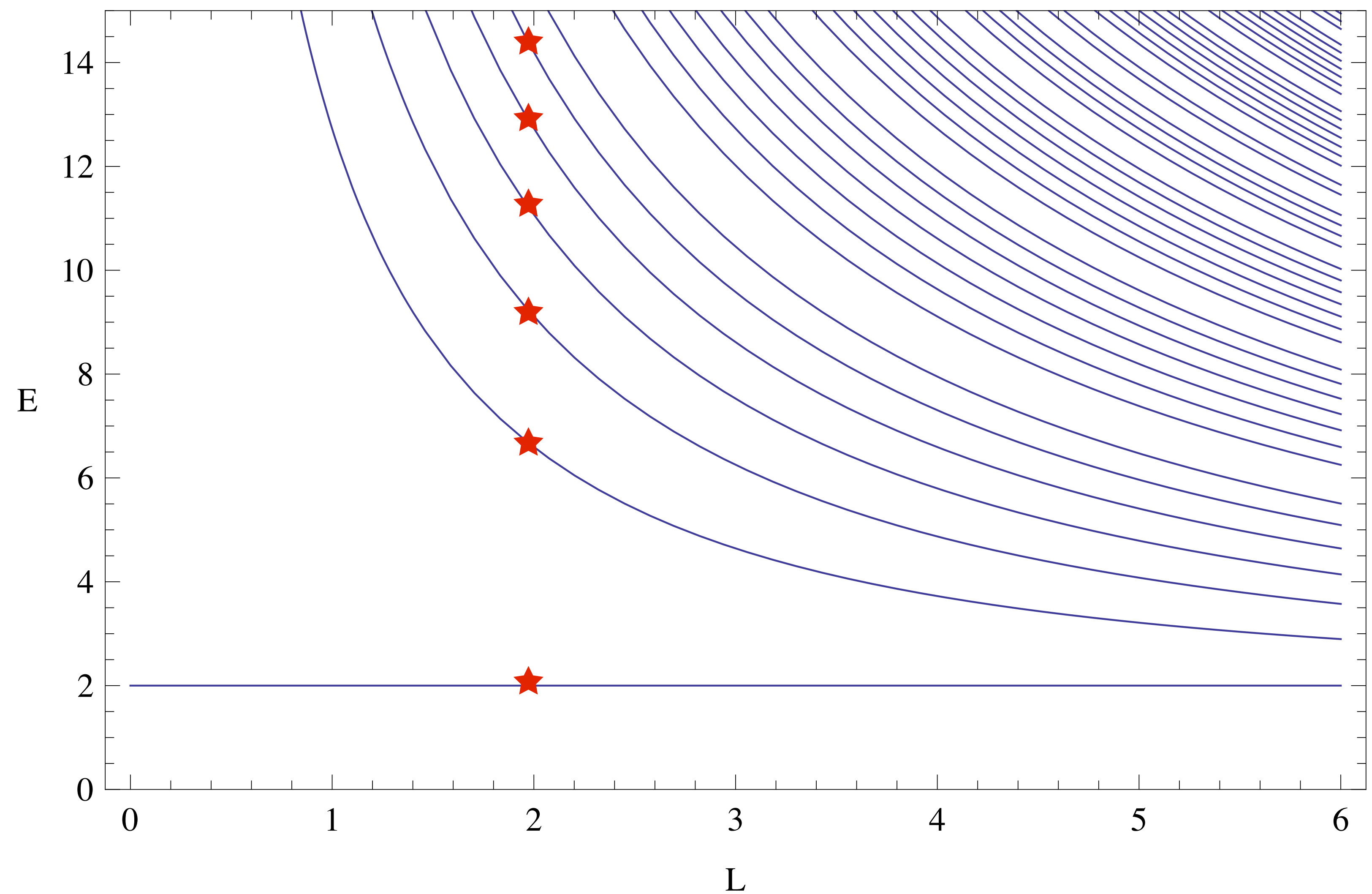

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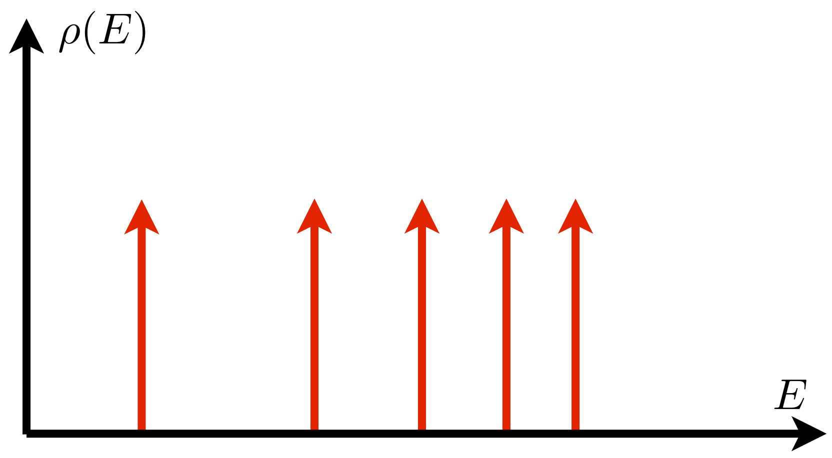

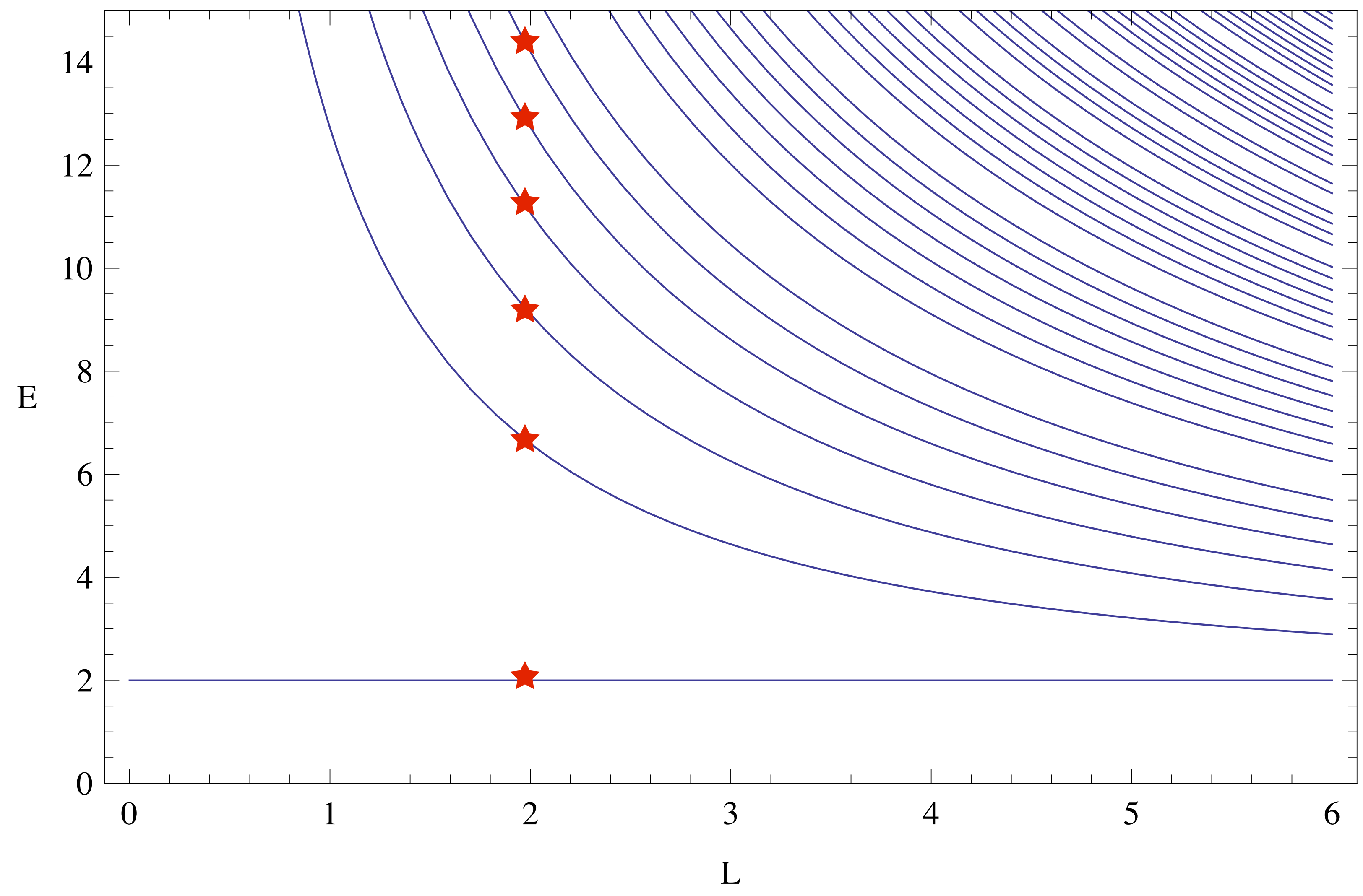

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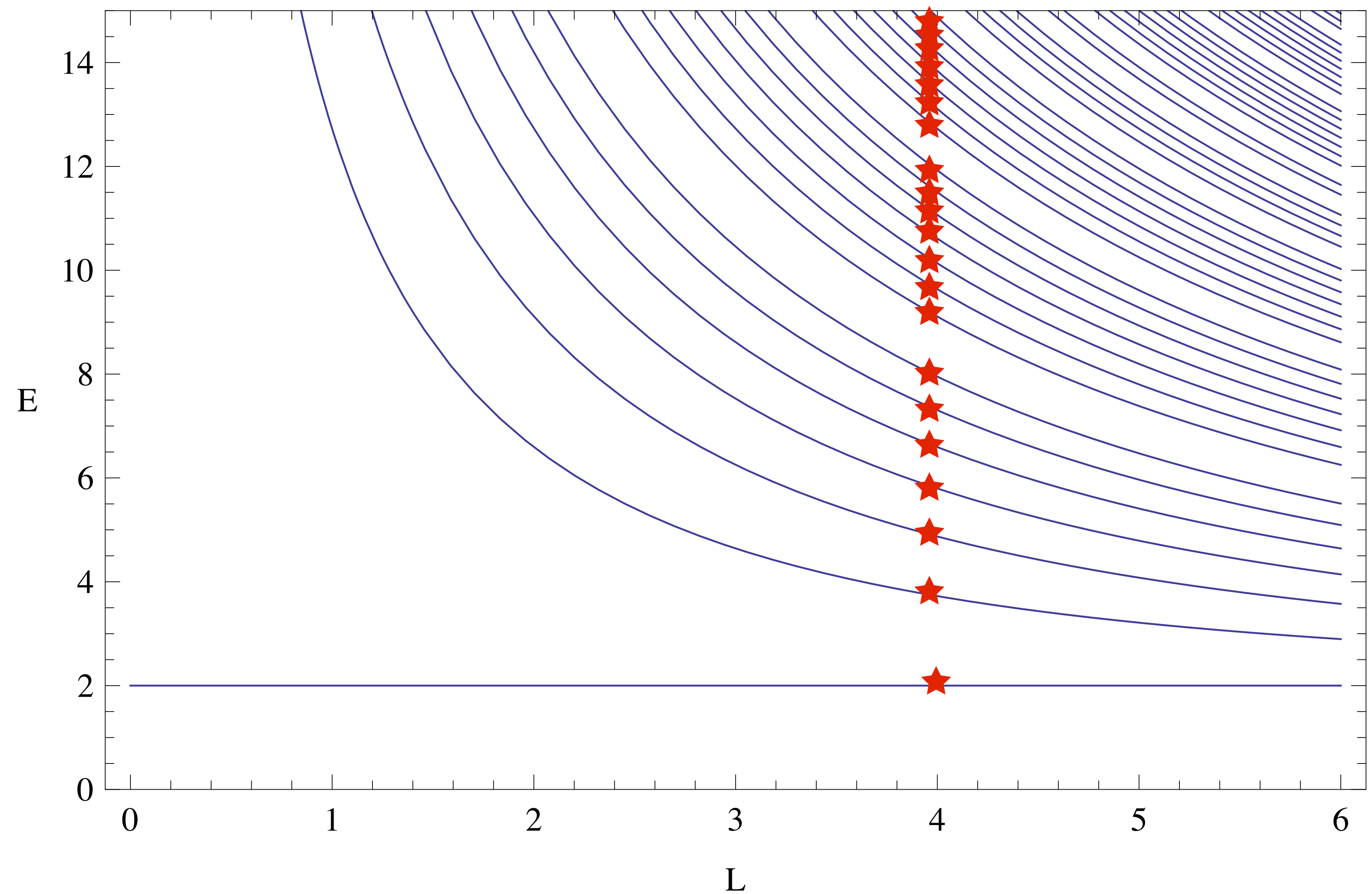

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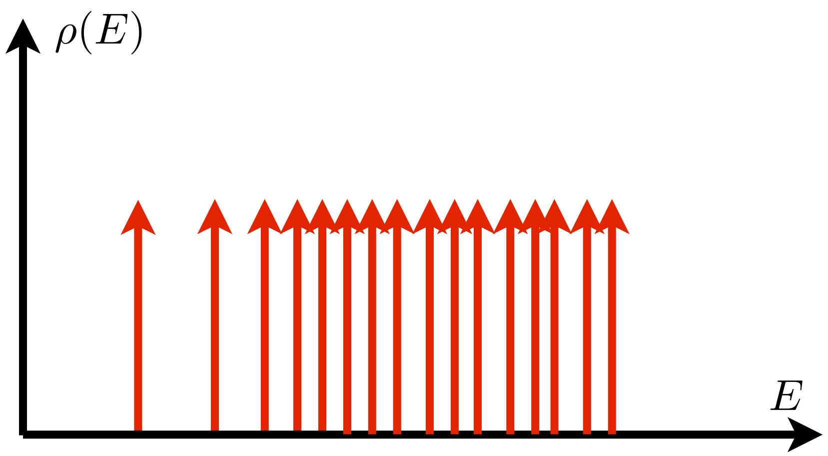

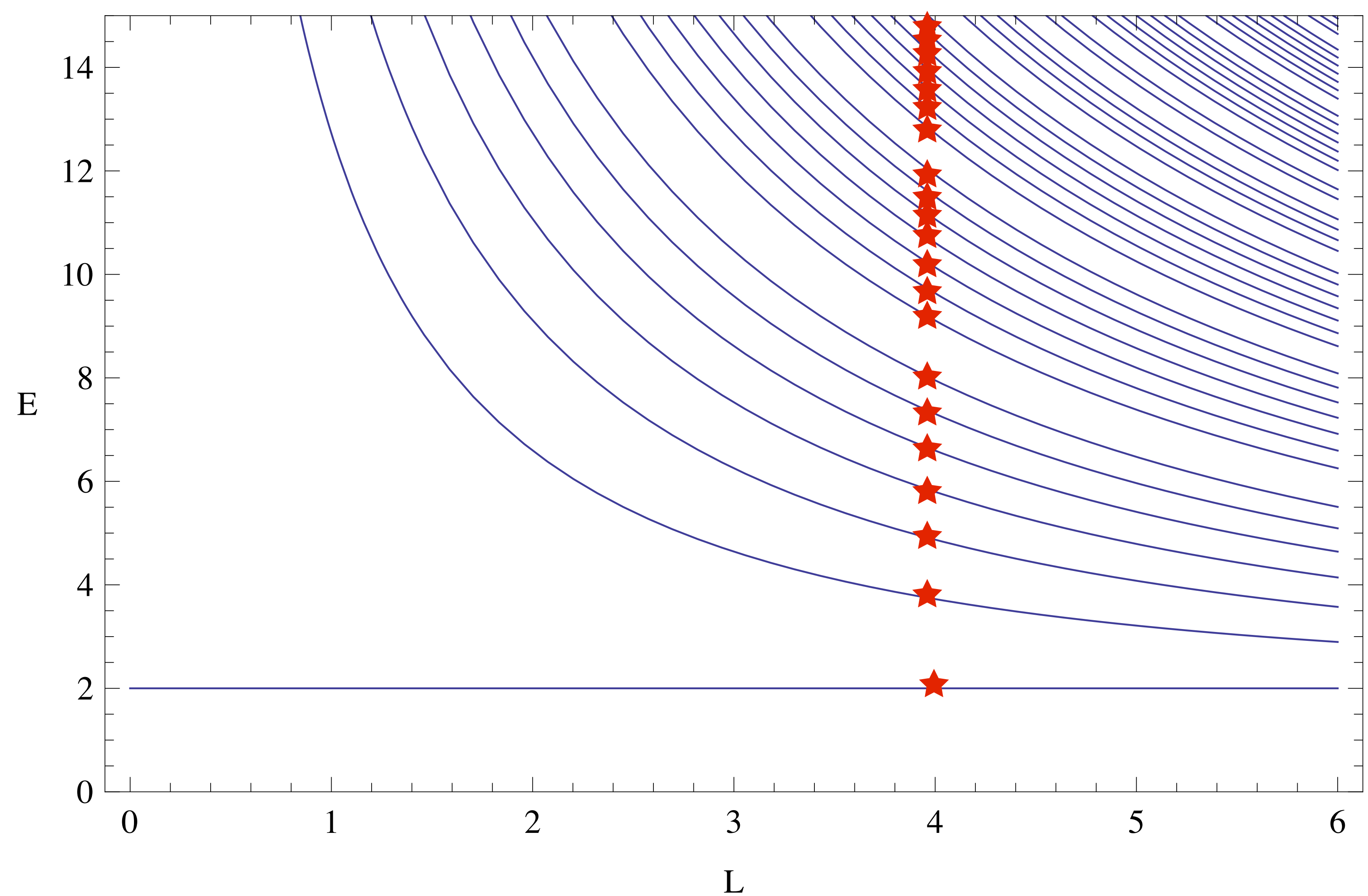

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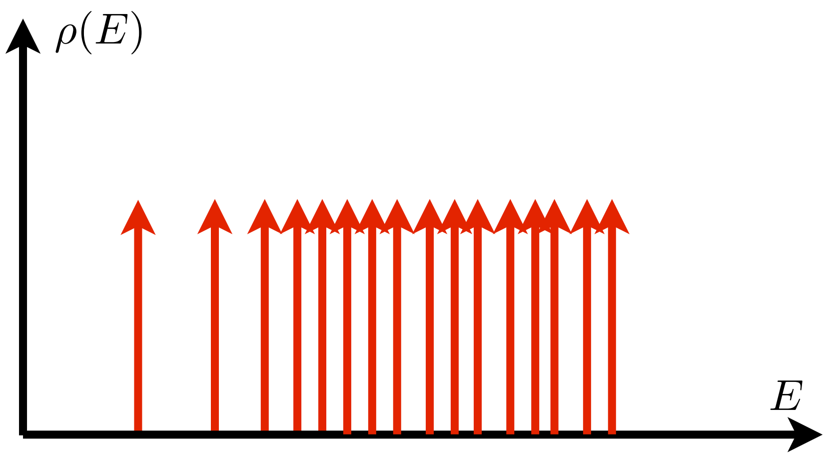
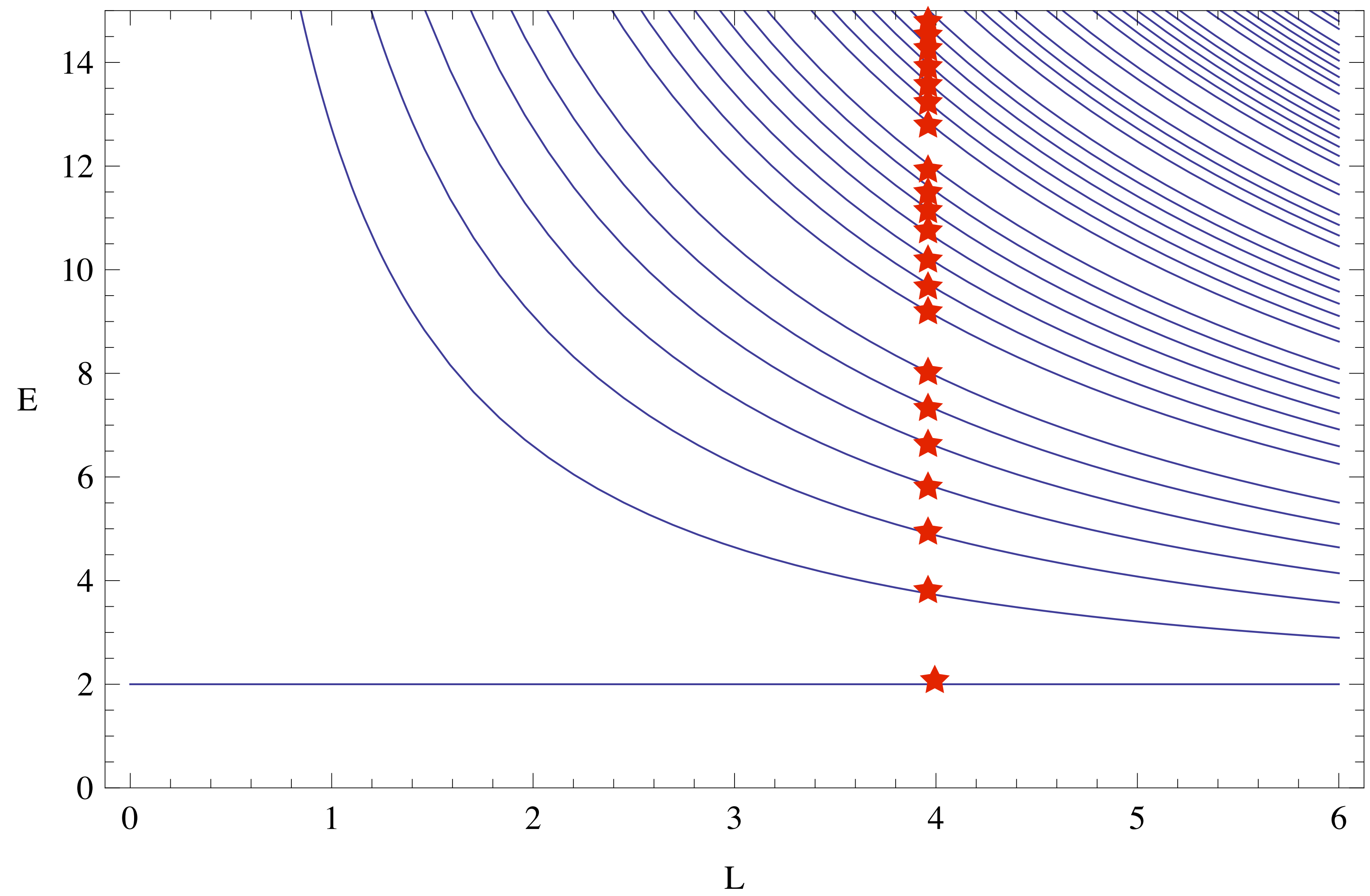




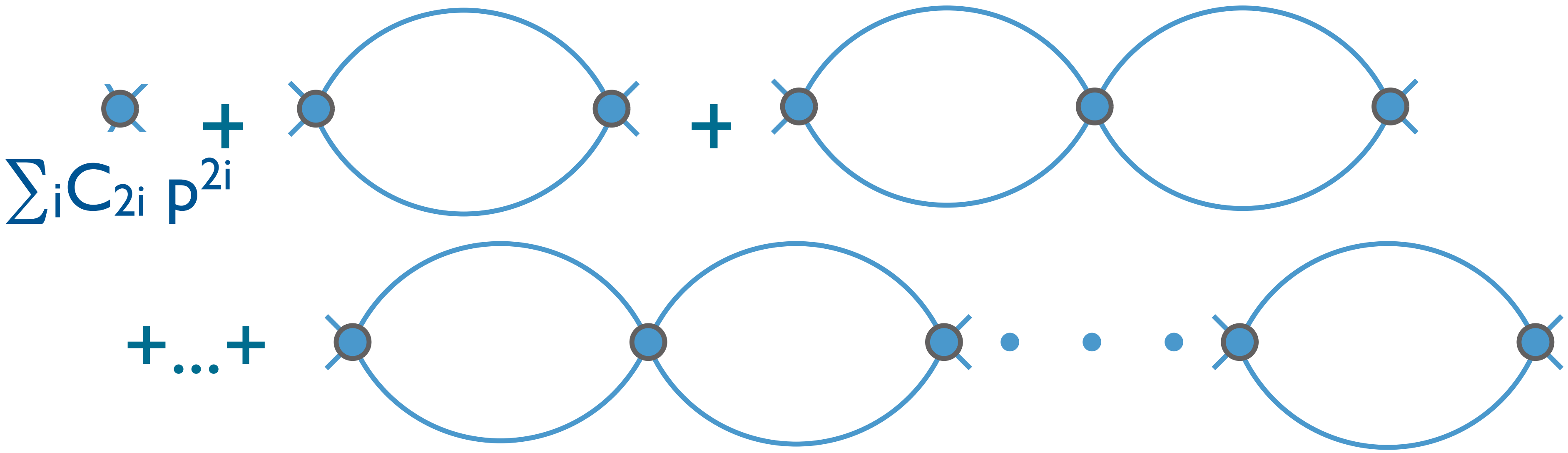
- Modified by interactions in a calculable way

Scattering at finite volume

- Consider simple effective field theory of a scalar particle interacting via contact interactions

$$\mathcal{L} = \partial\phi\partial\phi + M\phi^2 + C_0\phi^4 + C_2\phi D^2\phi\phi^2 + \ldots$$

- Scattering amplitude given by bubble sum



- In infinite volume

$$\mathcal{A} \equiv \frac{4\pi/M}{p\cot\delta(p) - i\ p} = \frac{\sum_n C_{2n}p^{2n}}{1 - I_0(p)\sum_n C_{2n}p^{2n}}$$

- Making non-relativistic approx (doing relativistically is not much harder) and using power divergent subtraction regularisation scheme [Kaplan, Savage, Wise]

$$I_0^{PDS}(p) = \mu^{4-d}\int \frac{d^{d-1}\mathbf{k}}{E - |\mathbf{k}|^2/M + i\ \epsilon} = -\frac{M}{4\pi}(\mu + i\ p)$$

Finite volume energies

- In finite volume, integral restricts to allowed mode sum

$$\mathcal{A}(L) = \frac{\sum_n C_{2n} p^{2n}}{1 - I_0^L(p) \sum_n C_{2n} p^{2n}} \qquad p \cot\delta(p) = \frac{4\pi}{M} \frac{1}{\sum_n C_{2n} p^{2n}} + \mu$$

$$0 = \mathcal{A}^{-1}(L) = p\cot\delta(p) - \frac{M\mu}{4\pi} - I_0^L(p)$$

- Define PDS regulated sum as

$$I_0^L(p) \equiv \frac{1}{L^3} \sum_{\mathbf{k}}^{PDS} \frac{1}{E - |\mathbf{k}|^2/M}$$

$$= \frac{1}{L^3} \sum_{\mathbf{k}}^{|\mathbf{k}|<\Lambda} \frac{1}{E - |\mathbf{k}|^2/M} + \int^{\Lambda} \frac{d^3\mathbf{k}/(2\pi)^3}{|\mathbf{k}|^2/M} - \int^{PDS} \frac{d^3\mathbf{k}/(2\pi)^3}{|\mathbf{k}|^2/M}$$

$$= \frac{M}{4\pi} \left[-\frac{1}{\pi L} \sum_{\mathbf{n}}^{\Lambda} \frac{1}{|\mathbf{n}|^2 - \frac{L^2 E M}{4\pi^2}} - \frac{4\Lambda}{L} - \mu \right]$$

Finite volume energies

- Energies satisfy eigenvalue equation (Lüscher's method)

$$p \cot \delta(p) - \frac{1}{\pi \, L} S\left(\frac{L^2 p^2}{4\pi^2}\right) = 0$$

where $$S(x) = \sum_{\mathbf{n}}^{\Lambda} \frac{1}{|\mathbf{n}|^2 - x} - 4\pi\Lambda$$ [3D zeta function]

- Result valid for momenta up to inelastic threshold

- Valid up to exponentially small corrections

- Eg: lowest energy level (zero rel. mom.)

known Lüscher coefficients

$$\Delta E_0 = \frac{4\pi a}{M L^3}\left[1 + c_1 \frac{a}{L} + c_2 \left(\frac{a}{L}\right)^2 + \ldots\right]$$

- Calculation of energy levels on the lattice determines scattering parameters

HW: 1. derive the energy shift ΔE_0 from the Lüscher formula above assuming small a/L.
2. Calculate the coefficient c_1.

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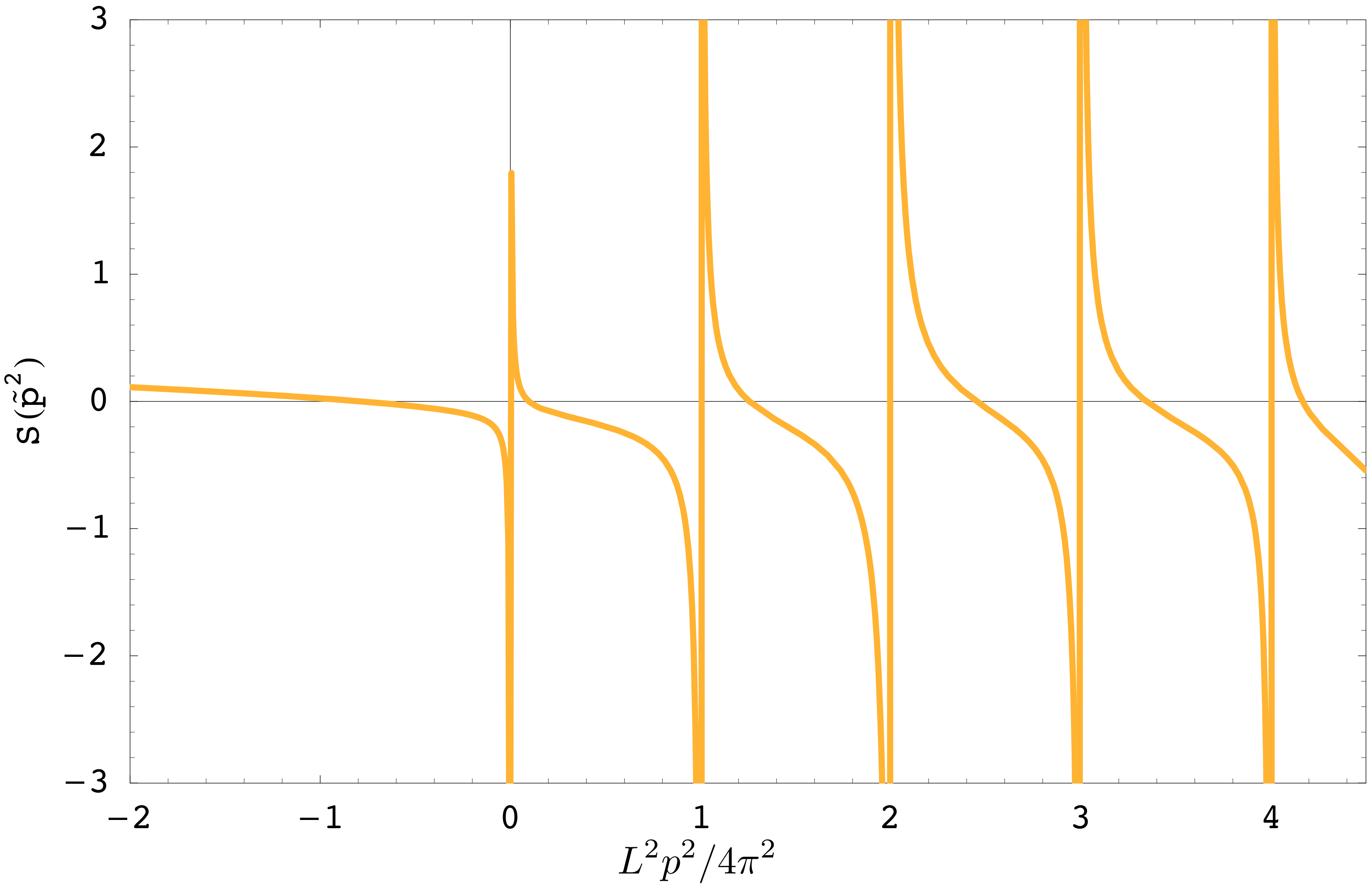
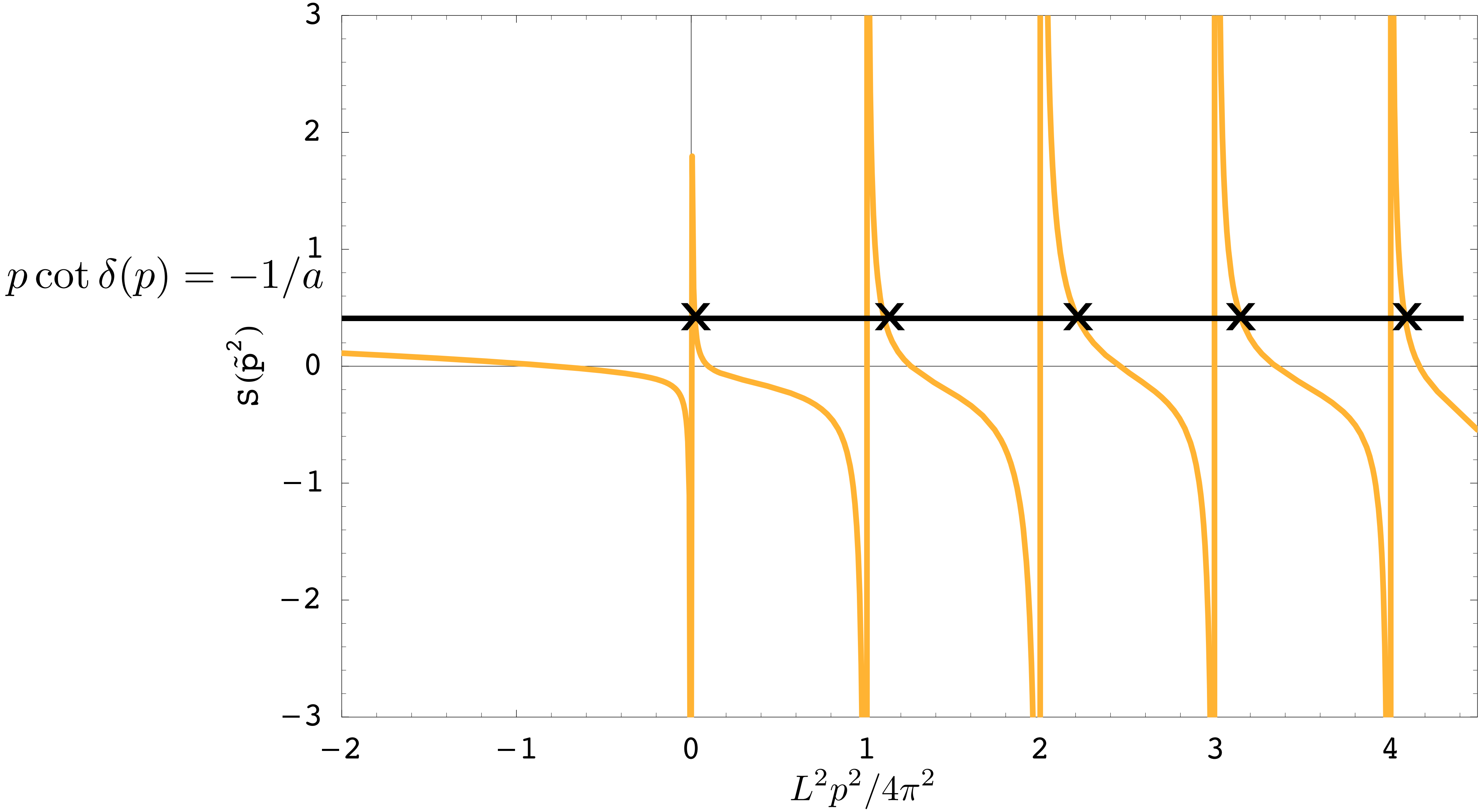
Two particle energies



Intersections correspond to eigen-energies of states in lattice volume

Two particle energies

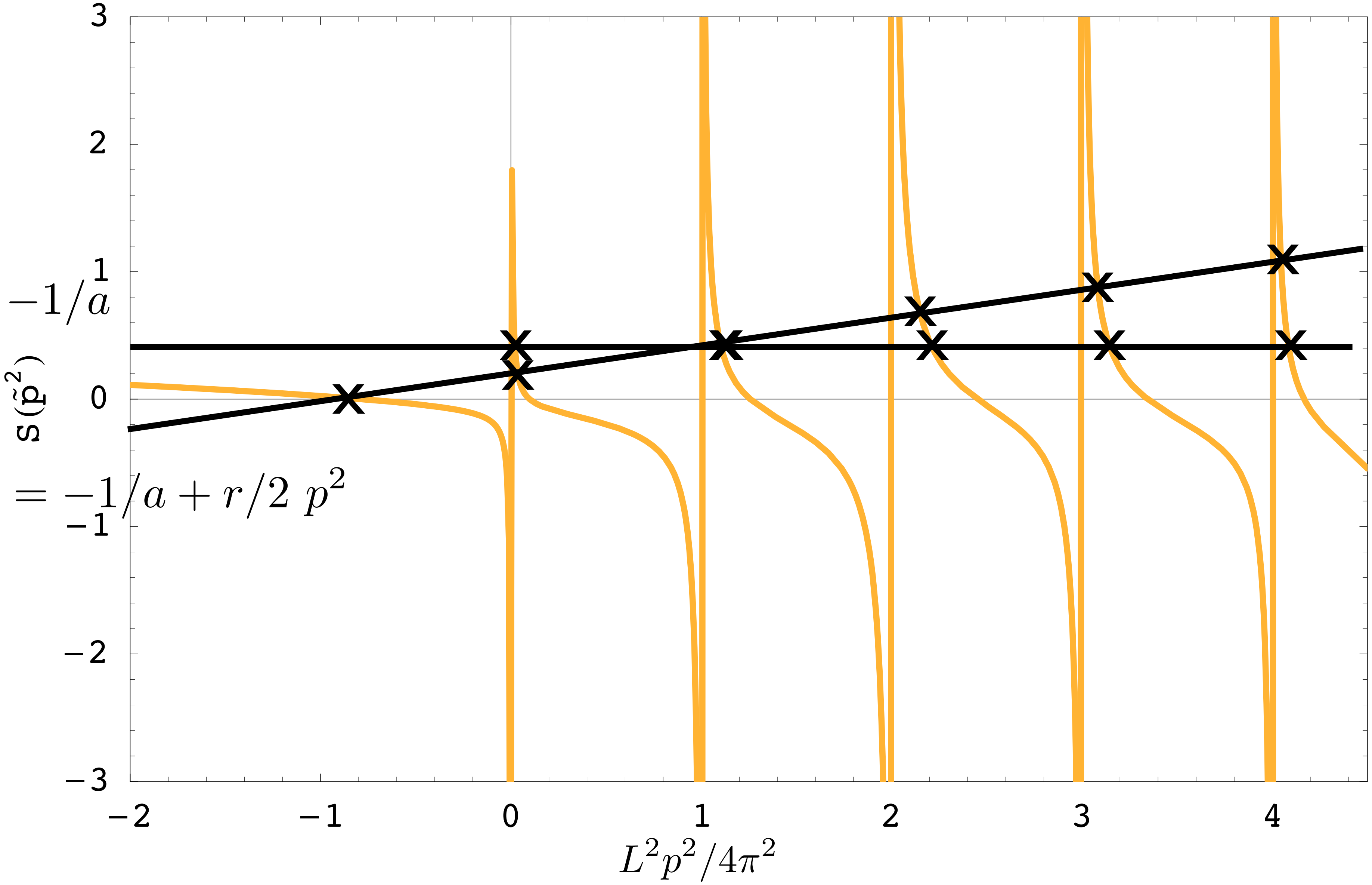


Intersections correspond to eigen-energies of states in lattice volume

Two particle energies

$$p \cot \delta(p) = -1/a$$

$$p \cot \delta(p) = -1/a + r/2\ p^2$$



Intersections correspond to eigen-energies of states in lattice volume

Asymptotic expansions

- Ground state energy shift

$$\Delta E_0 = \frac{4\pi a}{ML^3}\left[1 + c_1 \frac{a}{L} + c_2 \left(\frac{a}{L}\right)^2 + \ldots\right]$$

- First excited state energy shift

$$\Delta E_1 = \frac{4\pi}{ML^2} - \frac{12\tan\delta_0}{ML^2}\left[1 + c_1' \tan\delta_0 + c_2' \tan^2\delta_0\right] + \ldots \quad \text{where} \quad \delta_0 = \delta(p_{E_1})$$

- Each new level extracted or new volume used adds information on the phase shift at a different energy

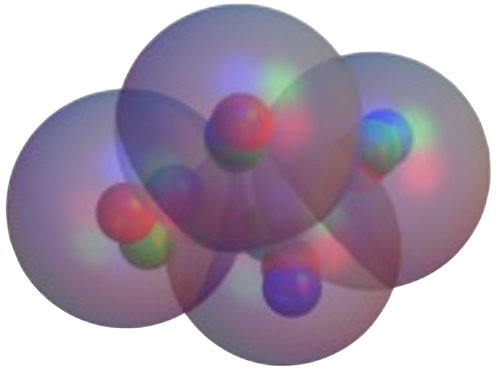
- Bound states can also be described

$$\Delta E_{-1} = -\frac{\gamma^2}{M}\left[1 + \frac{12}{\gamma_0 L(1-\gamma r_3)} e^{-\gamma L}\right] + \ldots$$

- Expansions also for L/a << 1 [Beane et al.]

Bound states at finite volume

- Lüscher eigenvalue equation also includes solutions with $p^2<0$ (bound state?)
- Two particle scattering amplitude in infinite volume



$$\mathcal{A} \equiv \frac{4\pi/M}{p\cot\delta(p) - i\ p}$$

bound state at $p^2 = -\gamma^2$ when $\cot\delta(i\gamma) = i$

  - Binding energy E_B related to binding momentum as $\gamma = \sqrt{2ME_B}$

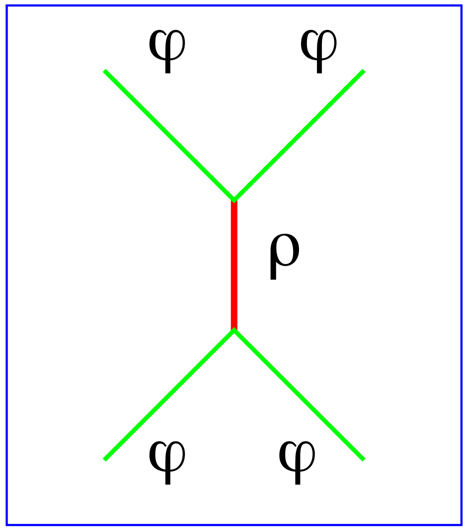
- Scattering amplitude in finite volume (another way of expanding Lüscher eqn)

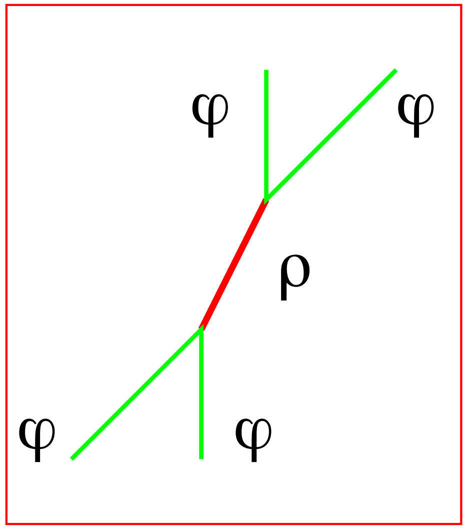
$$\cot\delta(i\kappa) = i - i\sum_{\vec{m}\neq 0}\frac{e^{-|\vec{m}|\kappa L}}{|\vec{m}|\kappa L} \qquad \kappa \xrightarrow{L\to\infty} \gamma$$

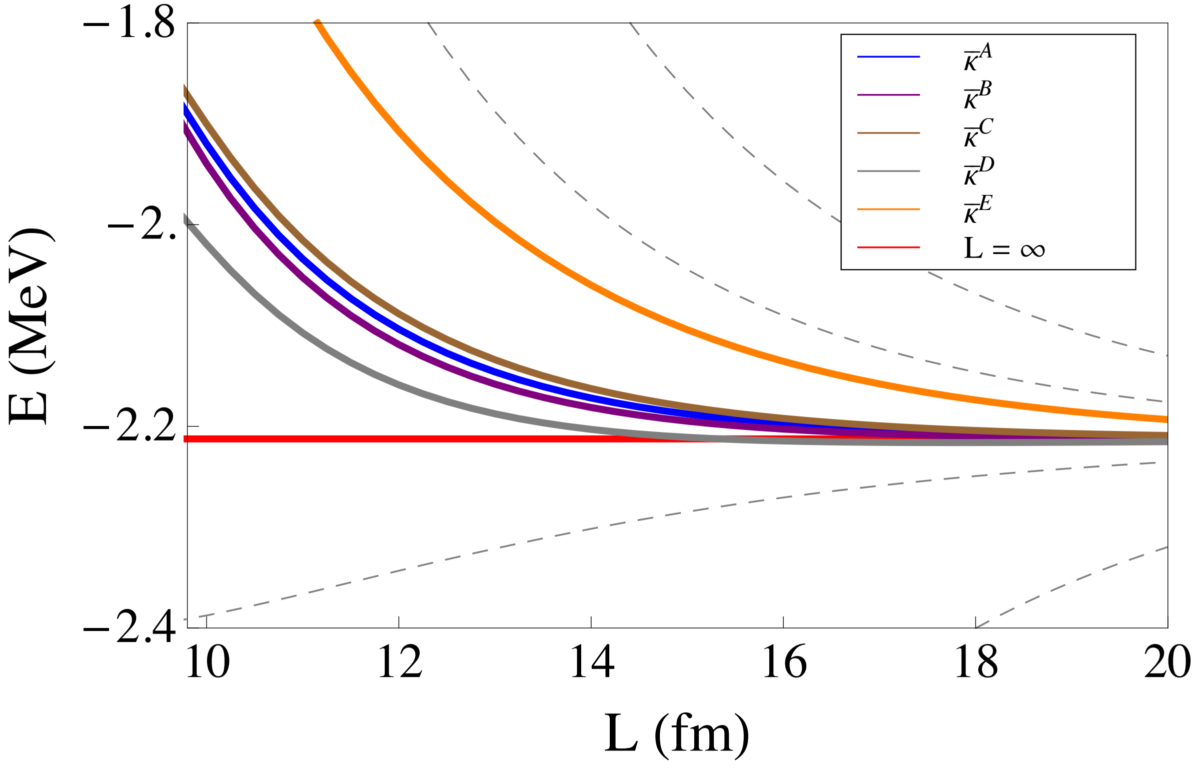
  - Multiple volumes required to show a negatively shifted state is bound

Boosted systems

- Boost of the two body system CoM relative to boundary conditions (lab) changes the effective shape of the box as seen by the interacting system
  - First studied by Rummukainen & Gottlieb [95]; Further study by Kim, Sachrajda & Sharpe [05]; Kim Christ & Sachrajda [05];
  - Generalised to other frames [Feng, Renner & Jansen]
  - *Allows access to phase shift at different momenta*
- Effects on bound states investigated [Bour et al; Davoudi & Savage]
  - Can be used to cancel leading exponential FV corrections to binding energies

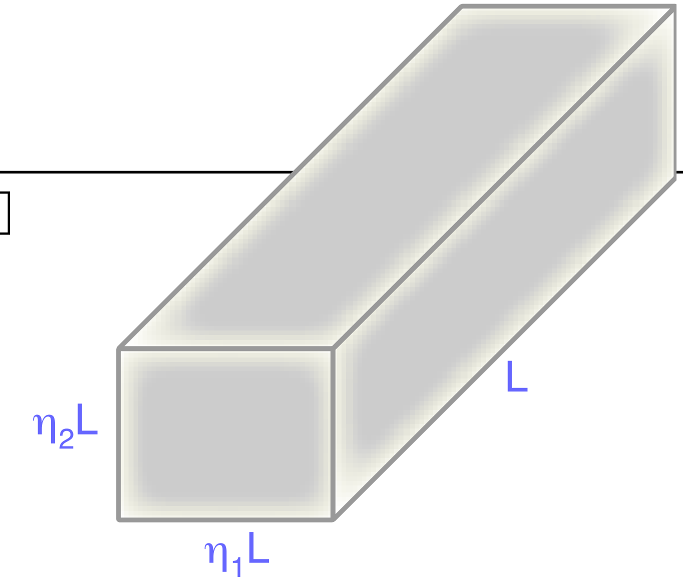






Asymmetric boxes

[Li & Liu hep-lat/0311035; WD & Savage hep-lat/0403005]



- Asymmetry box of geometry $\eta_1 L \times \eta_2 L \times L$
- Eigenvalue equation modified

$$S(\tilde{p}^2) \longrightarrow S(\tilde{p}^2, \eta_1, \eta_2) = \frac{1}{\eta_1 \eta_2} \sum_{\tilde{\mathbf{n}}}^{\Lambda_n} \frac{1}{|\tilde{\mathbf{n}}|^2 - \tilde{p}^2 - 4\pi\Lambda_n} \quad \text{where} \quad \tilde{\mathbf{n}} = \left(\frac{n_1}{\eta_1}, \frac{n_2}{\eta_2}, n_3\right)$$

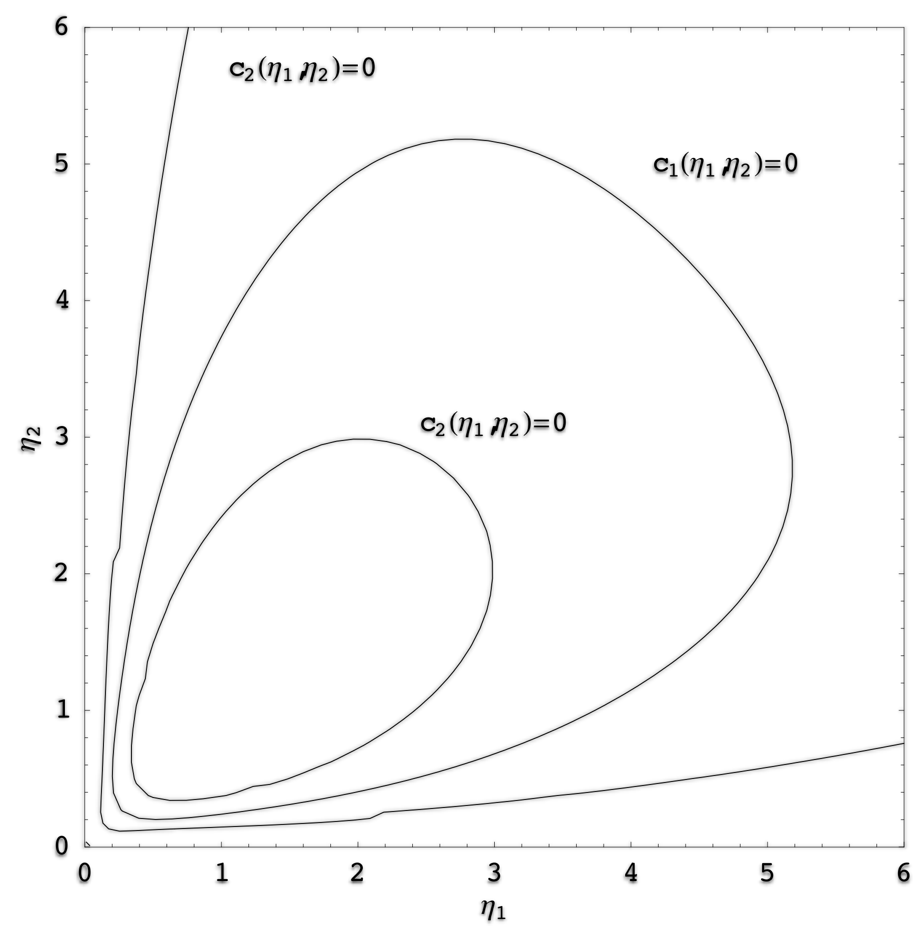
- Asymptotic expansion

$$\Delta E_0 = \frac{4\pi a}{\eta_1 \eta_2 M L^3} \left[1 + c_1(\eta_{1,2}) \frac{a}{L} + c_2(\eta_{1,2}) \left(\frac{a}{L}\right)^2 + \dots\right]$$

- Geometric coefficients

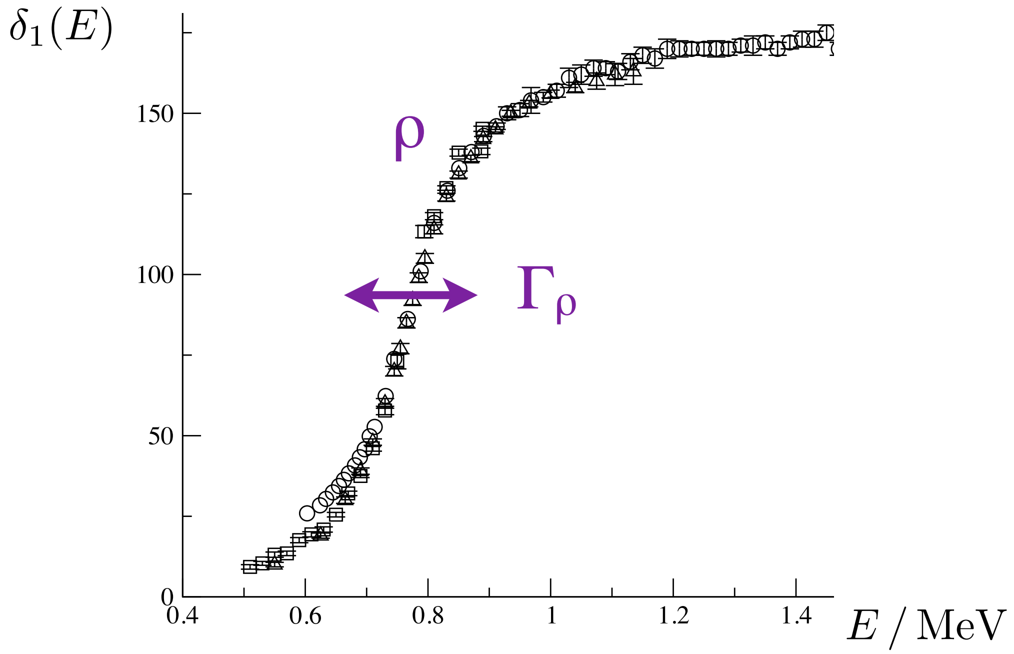
$$c_1(\eta_{1,2}) = \frac{1}{\pi} \left(\frac{1}{\eta_1 \eta_2} \sum_{\tilde{\mathbf{n}} \neq 0}^{\Lambda_n} \frac{1}{|\tilde{\mathbf{n}}|^2} - 4\pi\Lambda_n \right)$$

  - Asymmetries exist where sub-leading FV effects are suppressed: $c_i(\eta_1,\eta_2)=0$



Resonances at finite volume

- Pion is light so very few stable hadrons in the real world – often the lightest particle of a given set of conserved quantum numbers and not much else
  - Ex: ρ(770) decays to ππ and to ππππ; Δ(1232) decays to Nπ
- Ex: I=1 π π phase shift



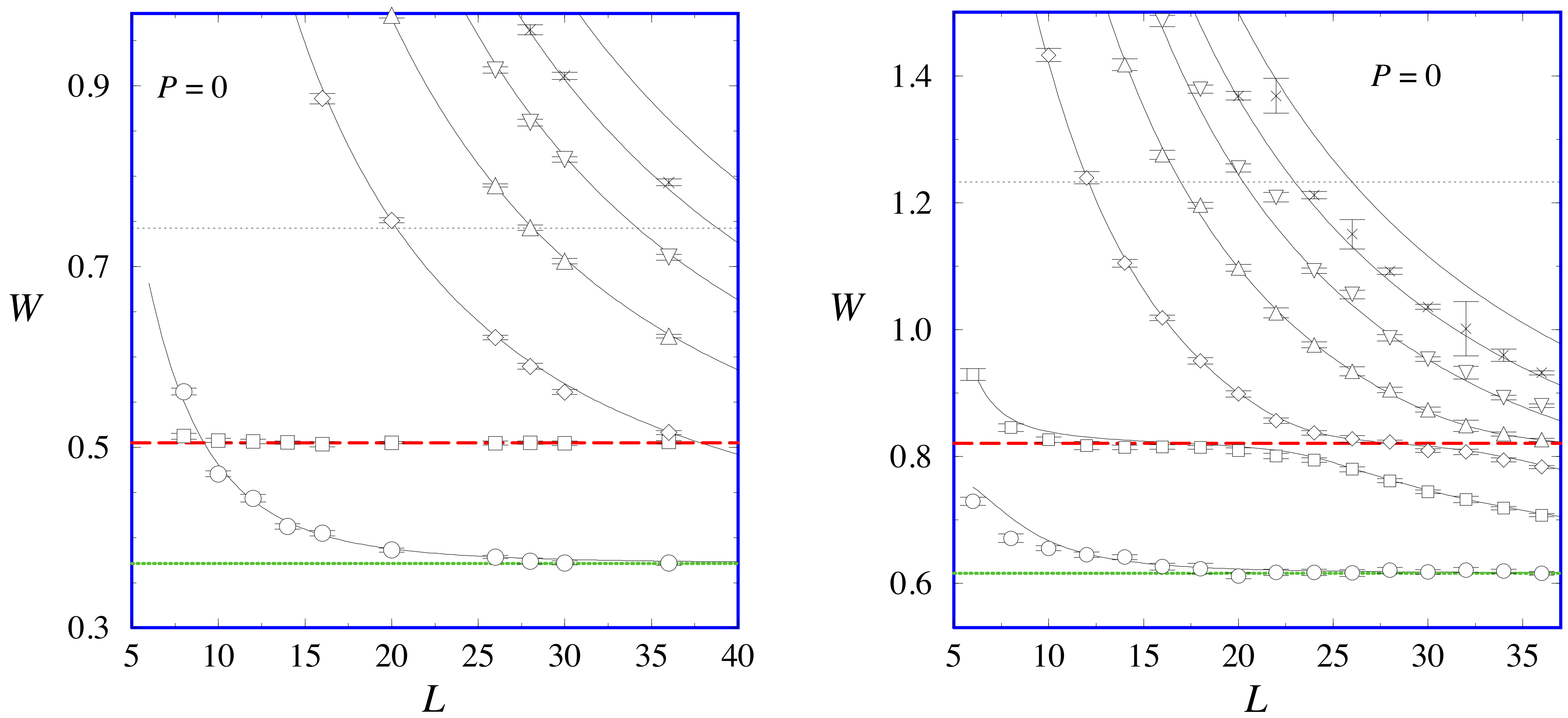
  - Extensive experimental efforts to understand excited spectrum of hadrons
- At finite volume, spectrum is discrete: how do resonances manifest?
  - Spectrum gets large modifications as a function of volume near resonance energy – embodies in solutions of Lüscher eigenvalue equation
- See excellent lectures of Jo Dudek at HUGS 2012 for details [http://www.jlab.org/hugs/program.html]

Resonances at finite volume

- Consider simple spin model in 3+1 D [Rummukainen & Gottlieb hep-lat/9509088]

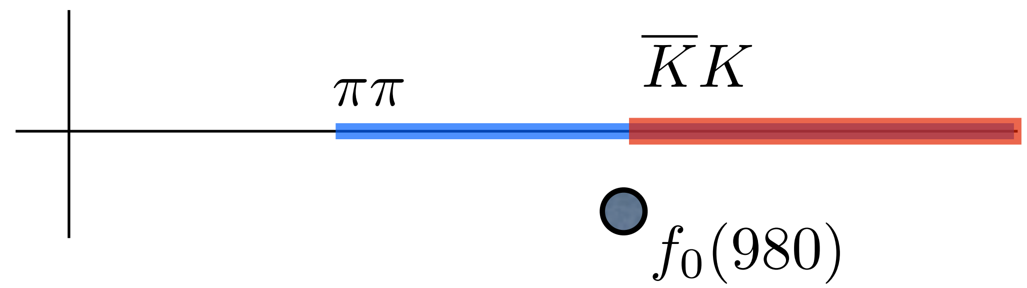
$$S = -\kappa_\phi \sum_{x;\,\hat{\mu}} \phi_x \phi_{x+\hat{\mu}} - \kappa_\rho \sum_{x;\,\hat{\mu}} \rho_x \rho_{x+\hat{\mu}} + g \sum_{x;\,\hat{\mu}} \rho_x \phi_x \phi_{x+\hat{\mu}}$$

- Left hand case g=0 (no resonance); right hand case g=0.021 (ρ appears as resonance in $\phi\phi$ channel)



Inelastic scattering

- In most physical scattering processes, inelastic contributions are important
  Ex: I=0 $\pi\pi$

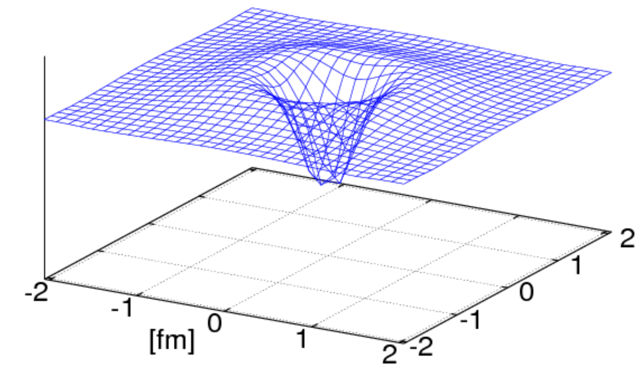


- Derivation of Lüscher method breaks down at inelastic thresholds
- Various attempts to get around this
    - Treat the system purely quantum mechanically [He, Liu et al.]
    - Effective field theory at finite volume [Bernard et al.; Döring et al.; Briceno&Davoudi; Hansen&Sharpe]
    - Introduces some level of systematic uncertainty (high order effects, ...)
- Active area of research

Bethe-Salpeter wave functions

- An alternate way of learning about scattering is based on determination of (Nambu-)Bethe-Salpeter wavefunction [Lüscher; Lin et al.; Aoki et al.; HALQCD]

$$\psi_{\mathbf{k}}(x-y) = \#\langle 0|T[\phi(x)\phi(y)]|\phi(\mathbf{k})\phi(-\mathbf{k})\rangle_{\rm in}$$



chosen interpolating operator probes content of state

- Satisfies Schrödinger equation for non-local BS kernel U(x,y)

$$\frac{1}{2\mu}\left[\nabla^2 + |\mathbf{k}|^2\right]\psi_{\mathbf{k}}(\mathbf{x}) = \int d^3\mathbf{y}U(\mathbf{x},\mathbf{y})\psi_{\mathbf{k}}(\mathbf{y}) \qquad (*)$$

- Provided U(x,y)=0 for large |x-y| asymptotic behaviour of partial waves given by

$$\psi^{\ell}_{\mathbf{k}}(\mathbf{x}) \to A_\ell \frac{\sin(|\mathbf{k}|\mathbf{x}| - \ell\pi/2 + \delta_\ell(k))}{|\mathbf{k}|\mathbf{x}|}$$

can be used to determine phase shift

- HALQCD method: invert (*) to determine a potential by approximating

$$U(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\nabla)\delta^3(\mathbf{x}-\mathbf{y}) = \delta^3(\mathbf{x}-\mathbf{y})\left[V(\mathbf{x}) + \mathcal{O}(\nabla^2)\right]$$

giving

$$V(\mathbf{x}) = \frac{1}{2\mu}\frac{\left[\nabla^2 + |\mathbf{k}|^2\right]\psi_{\mathbf{k}}(\mathbf{x})}{\psi_{\mathbf{k}}(|bfx)}$$

Dropping these terms renders V(x) energy dependent [perhaps weakly]

Bethe-Salpeter wave functions

- BS wavefunctions can be determined from LQCD correlation functions

$$
\begin{aligned}
C(\mathbf{r},t) &= \langle 0|T[\phi(\mathbf{x}+\mathbf{r},t)\phi(\mathbf{x},t)]\mathcal{J}^\dagger(0)|0\rangle \\
&= \sum_n \langle 0|T[\phi(\mathbf{x}+\mathbf{r},t)\phi(\mathbf{x},t)]|\phi\phi^{(n)}\rangle\langle\phi\phi^{(n)}|\mathcal{J}^\dagger(0)|0\rangle \\
&= \sum_n Z_n e^{-E_n t}\psi_{\mathbf{k}_n}(\mathbf{r}) \\
&\xrightarrow{t\to\infty} Z_0 e^{-E_0 t}\psi_{\mathbf{k}_0}(\mathbf{r})
\end{aligned}
$$

Eigen-energies contain scattering information

- Various extensions considered – see recent review [Aoki et al. 1206.5088]
- Potentials obtained from this method are:
  - Energy dependent (weakly in some cases – see [Murano et al, 1103.0619])
    - Only guaranteed to reproduce phase shift at E_0
  - Sink dependent – not an issue as observable is phase shift at measured energy
  - Extraction of phase shift from lattice potential introduces model dependence as a functional form must be fit to finite lattice data – probably a mild problem

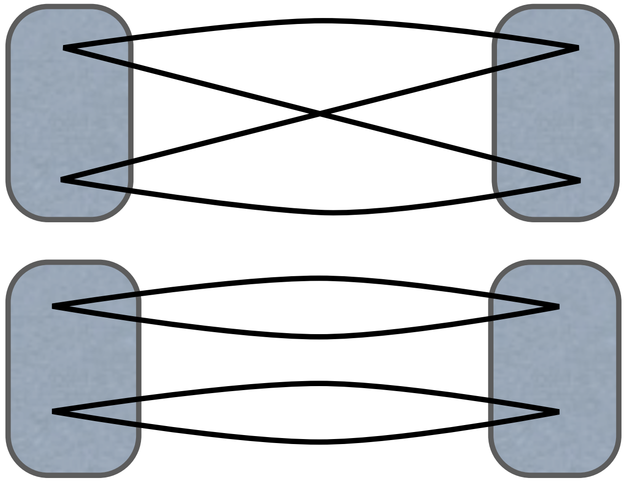
Numerical Investigations

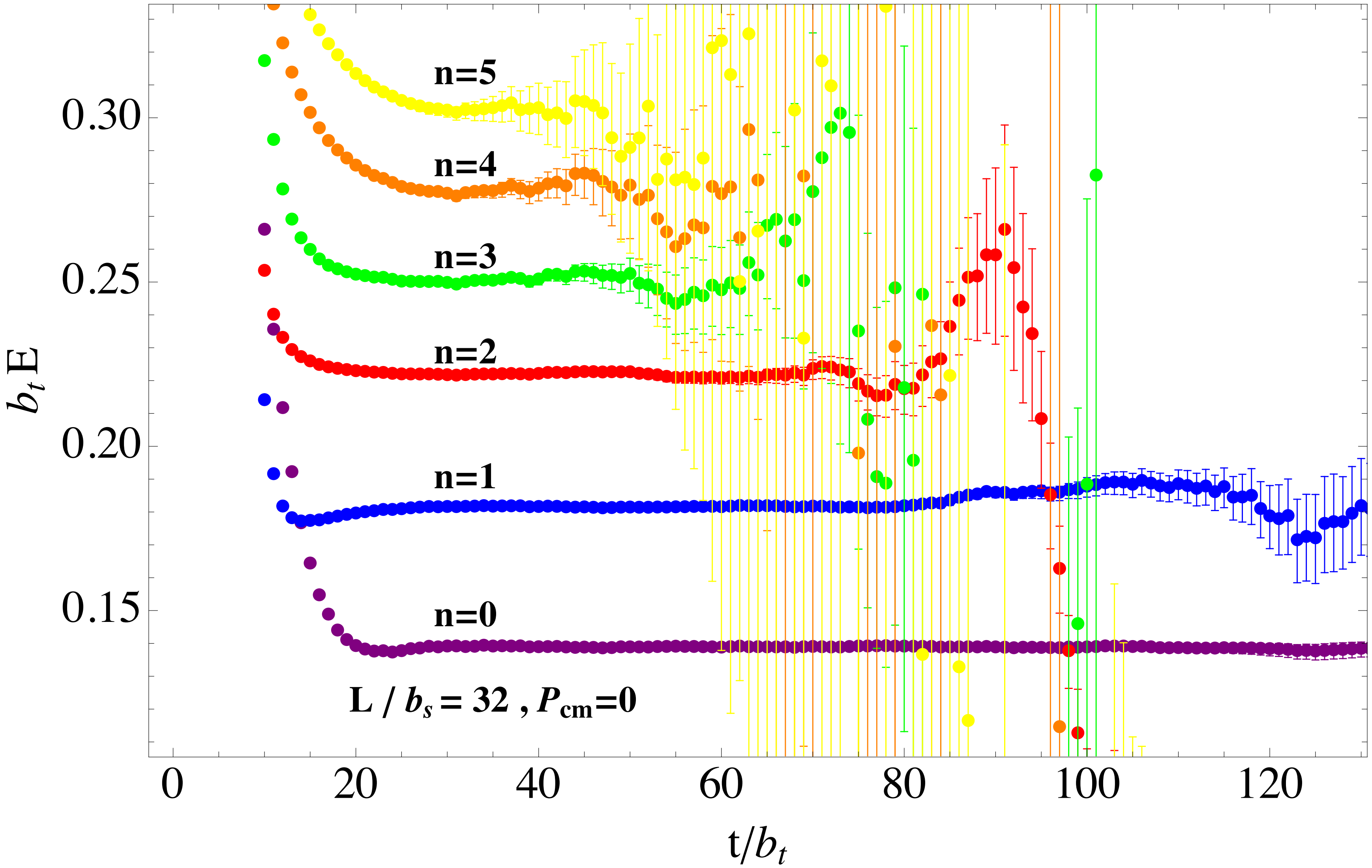
Two body scattering studies

- Meson-meson
  - π–π (I=2,1,0): [CP-PACS; NPLQCD; Feng et al; HadSpec; Fu; .... many others]
  - π–K (I=1/2, 3/2): [NPLQCD; Z Fu; Nagata et al; PACS-CS; Lang et al.]
  - K–K (I=1): [NPLQCD; Z Fu]
- Meson-baryon
  - Five simple octet baryon – octet meson channels studied [NPLQCD]
  - J/ψ–nucleon [Liu et al; Kawanai & Sasaki]
- Baryon-baryon
  - Various octet baryon – octet baryon scattering [HALQCD, NPLQCD]
  - Omega (sss)–Omega scattering *[Buchoff, Luu & Wasem]*
- A rapidly growing field

Example: I=2 ππ

- I=2 π–π "easy" as no disconnected contractions
- Measure multiple energy levels of two pions in a box for multiple volumes and with multiple P_{CM}





@ m_π = 390 MeV

1107.5023 [prd]

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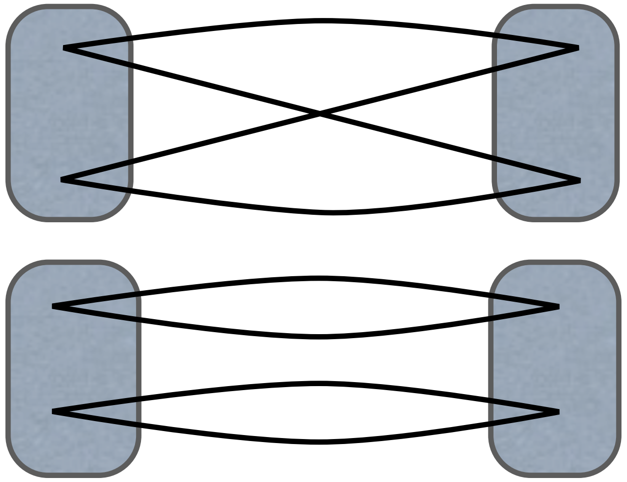
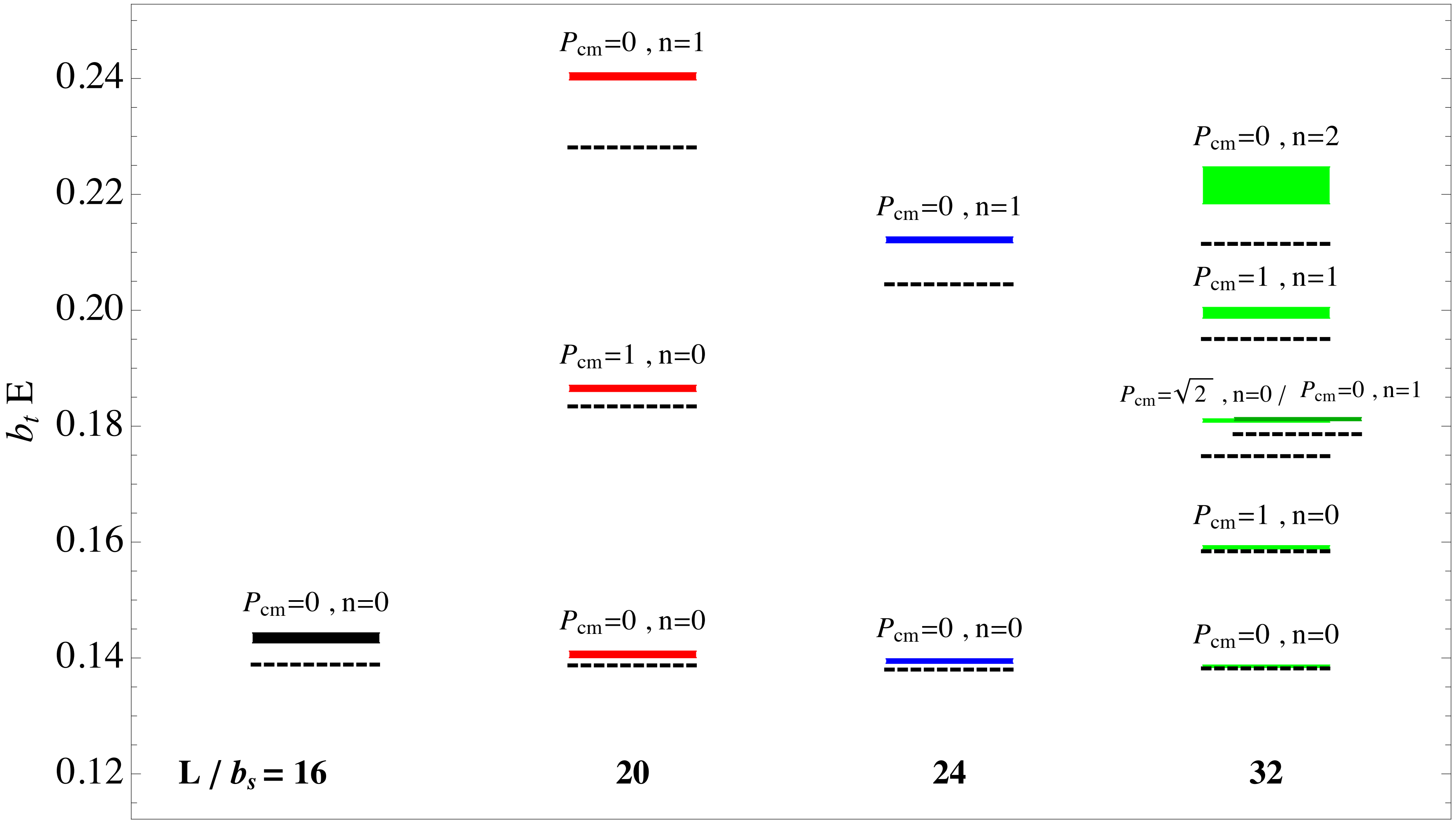




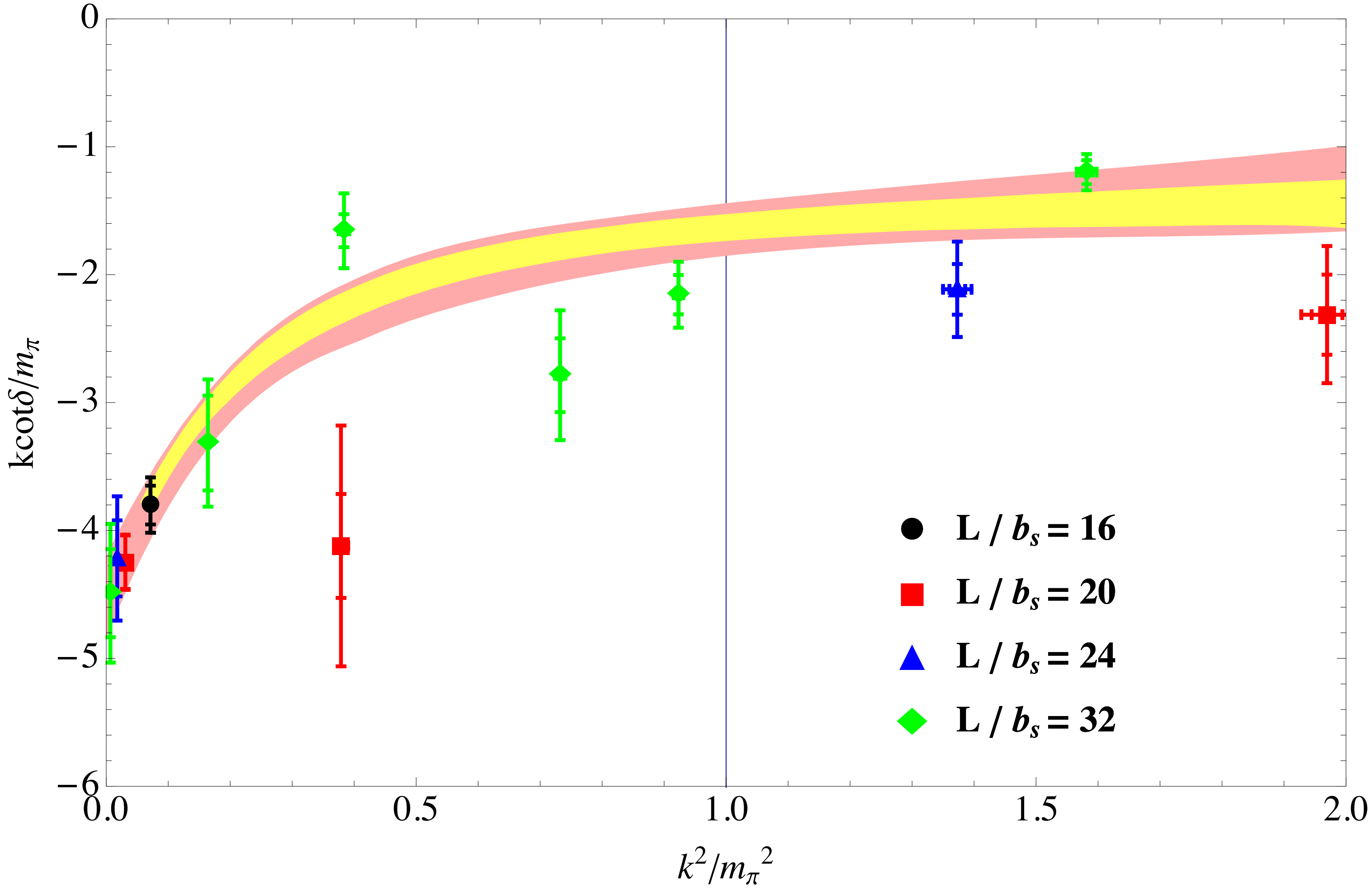
Dashed lines are non-interacting energy levels

@ m_π = 390 MeV

1107.5023 [prd]

Example: I=2 ππ

- Input into Lüscher eigenvalue equation
- Allows phase shift to be extracted at multiple energies

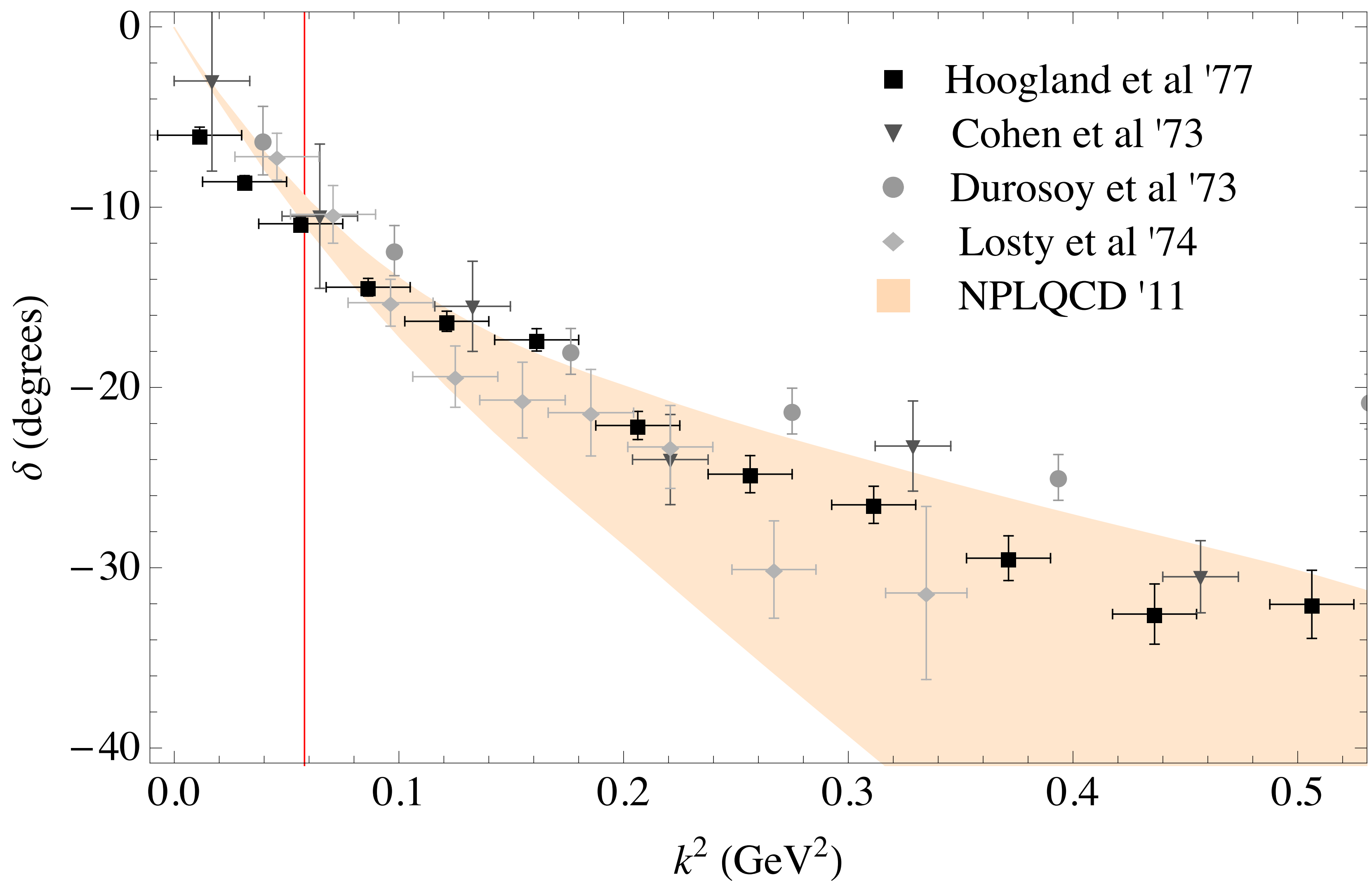


@ m_π = 390 MeV

1107.5023 [prd]

Example: I=2 ππ

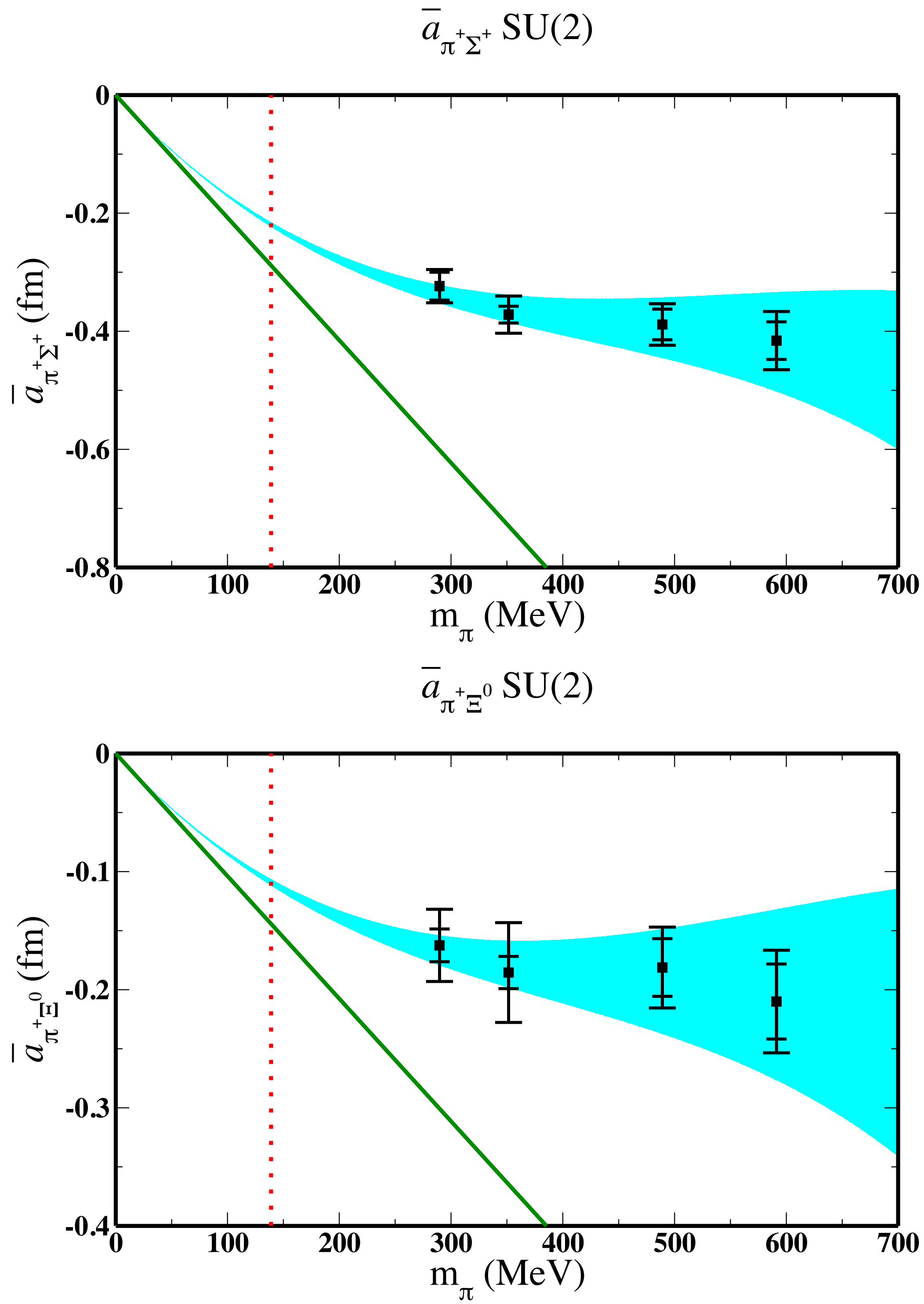
- Combine with chiral perturbation theory (low-momentum interactions turn off in the chiral limit) to interpolate to physical pion mass

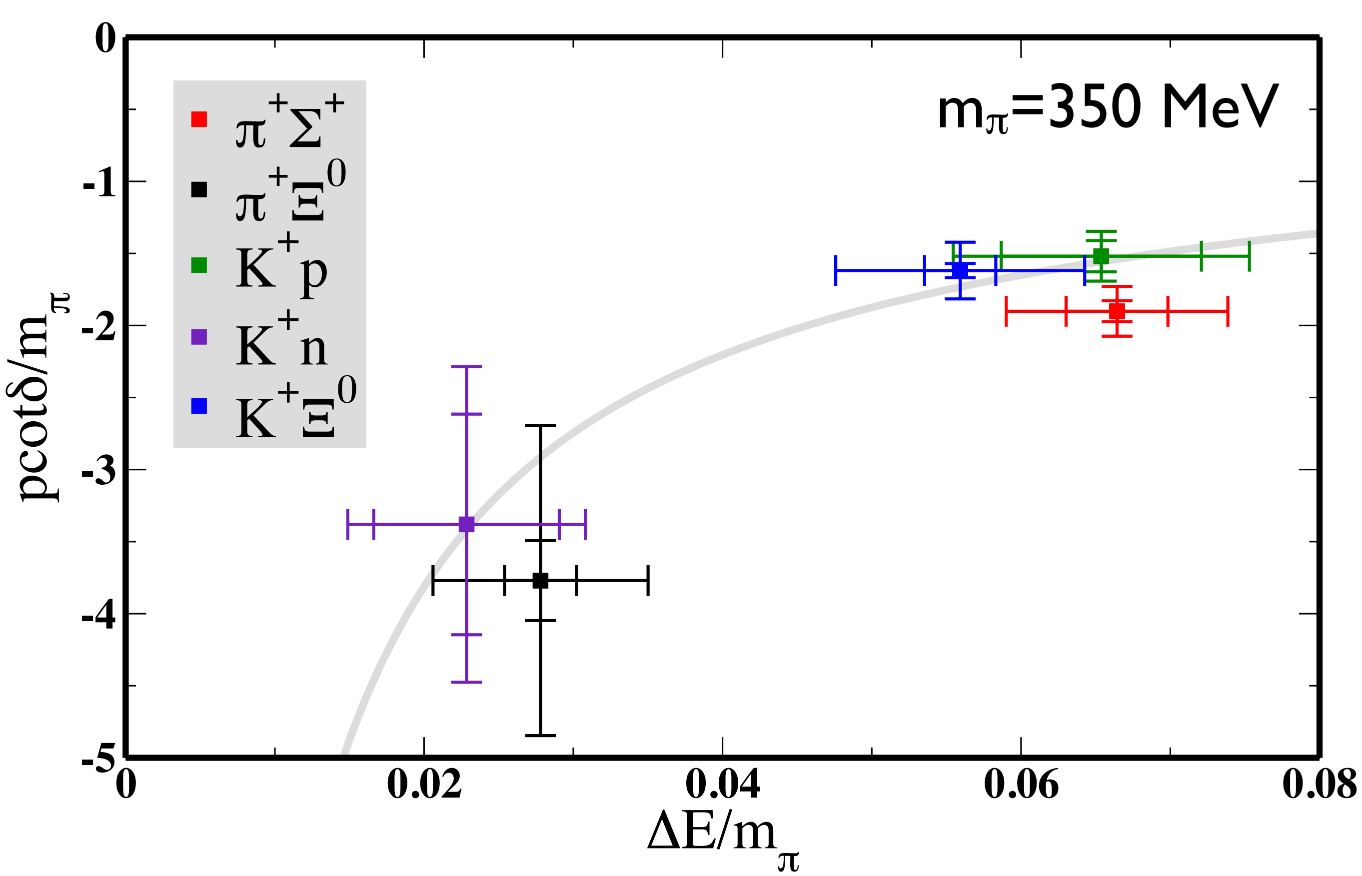


1107.5023 [prd]

Meson-baryon scattering

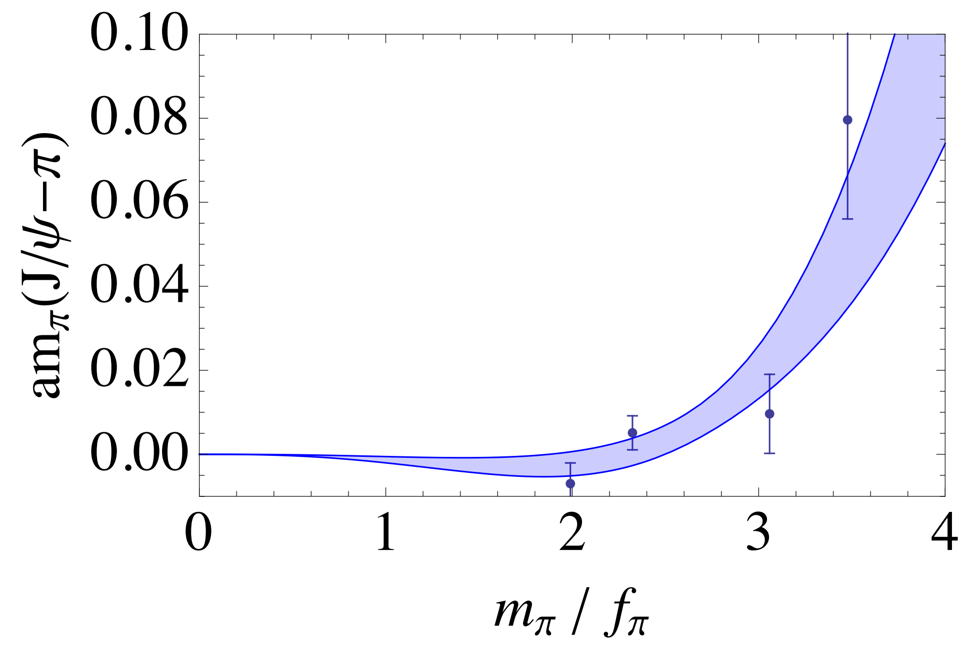
- Also constrained in the chiral limit
- Study of channels with no annihilation using DWF on MILC lattices



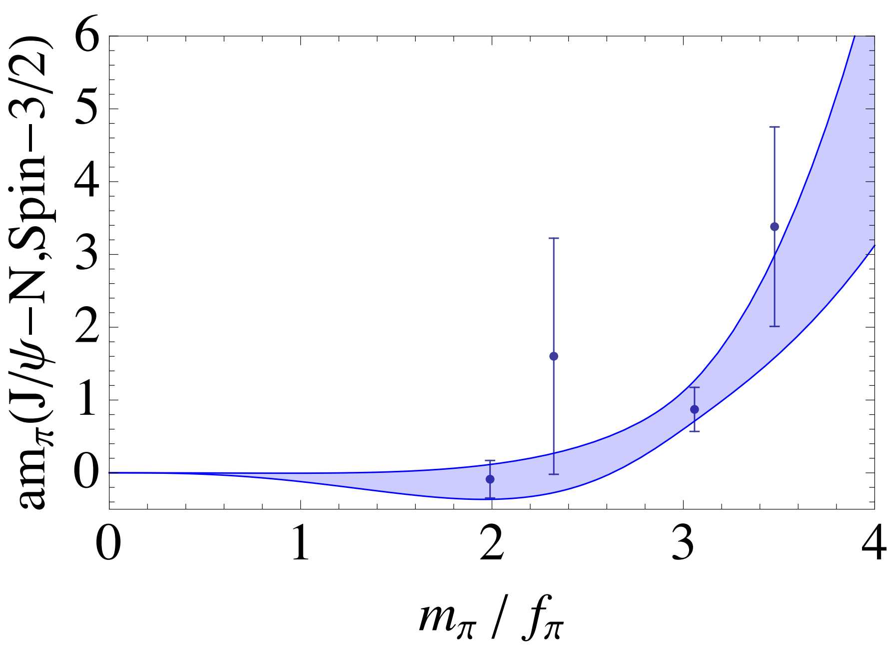


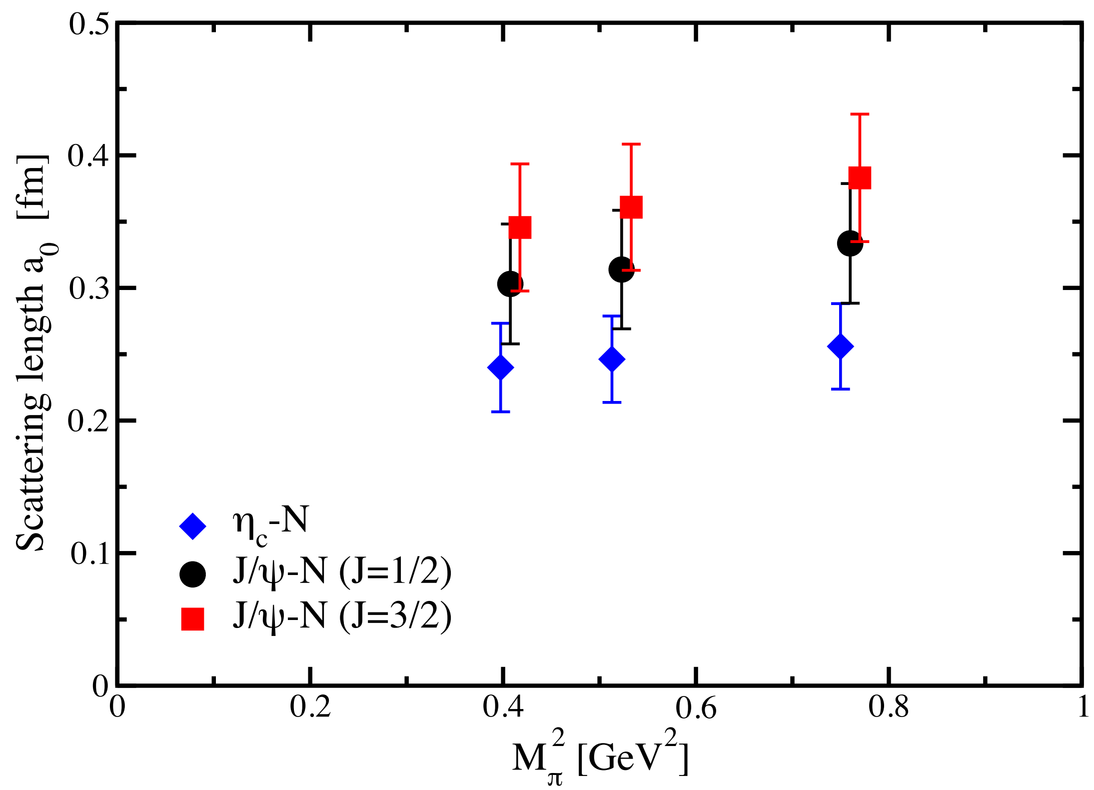
[Torok et al. 0907.1913]

J/Ψ-h scattering



- Studies by L Liu,; Kawanai &Sasaki
- Interactions are purely gluon/sea quark effects
- Phenomenological studies suggest J/Ψ might bind in a nucleus through attraction of multiple nucleons
- Interactions are small



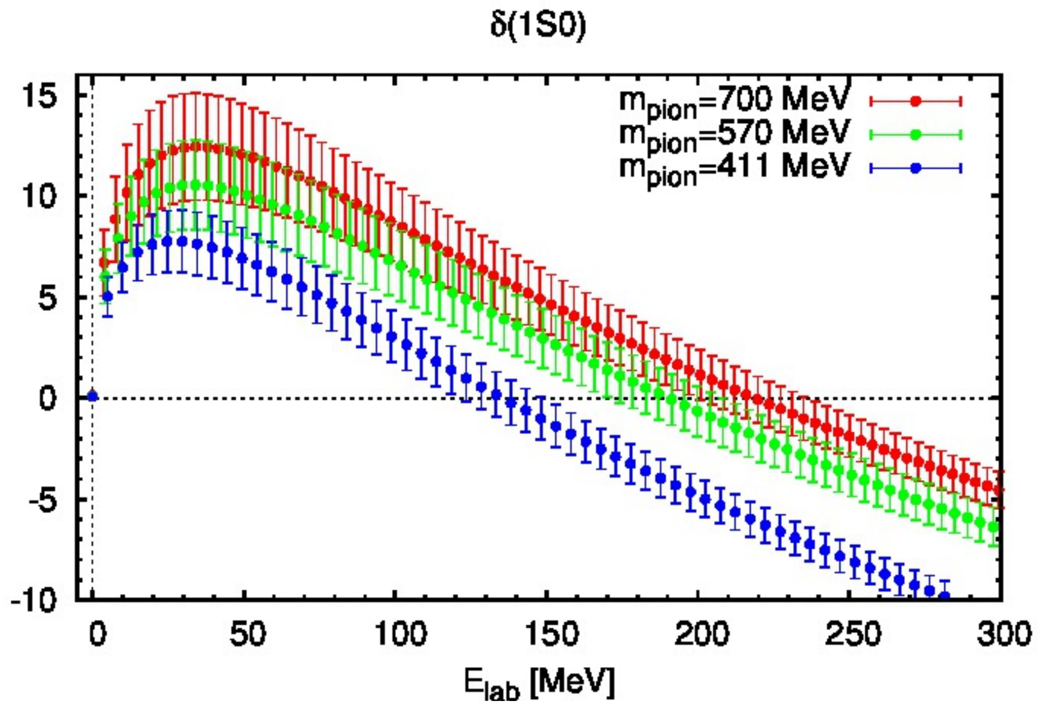


NN phase shifts

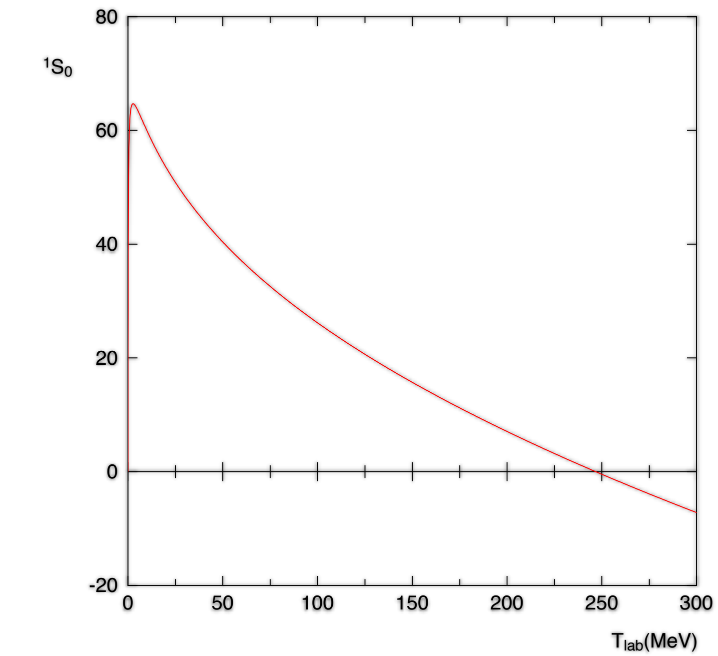
- Potential extracted for BS wave function method at E~0, use to evaluate phase shift
  - NB: energy dependence of potential neglected

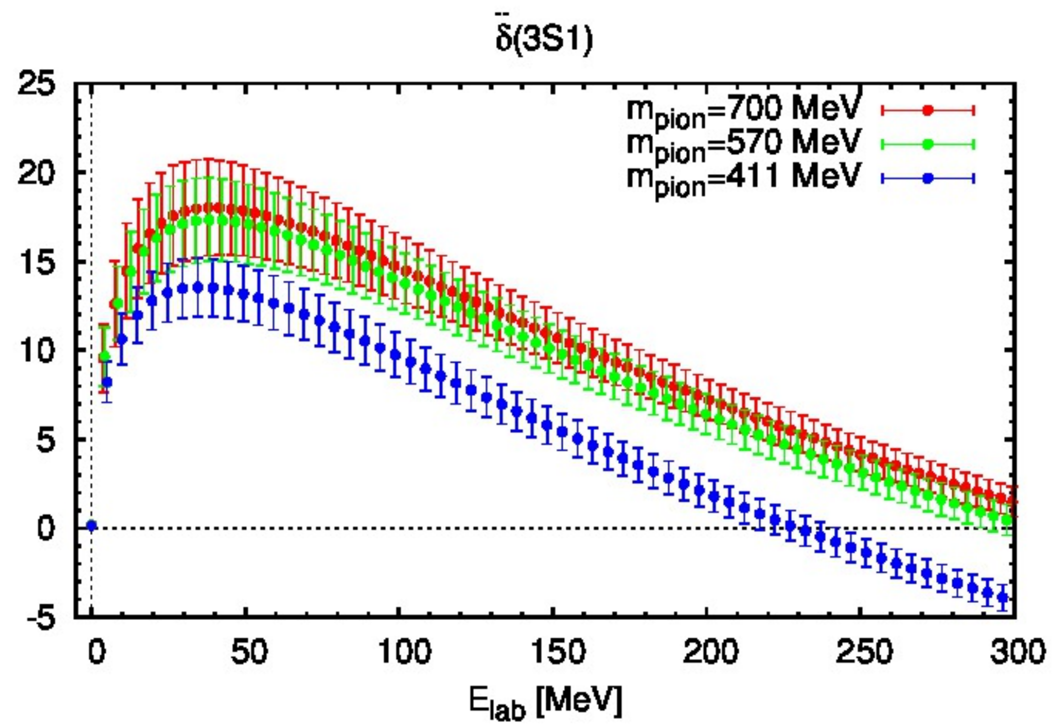
Hadrons to Atomic nuclei
HAL
from Lattice QCD

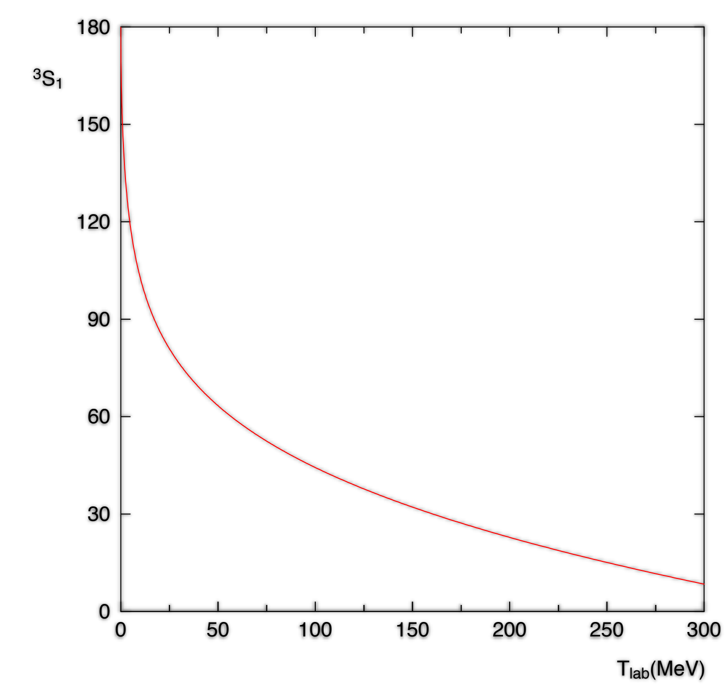
[N Ishii, Chiral Dynamics 2012]



NN-online ~ expt

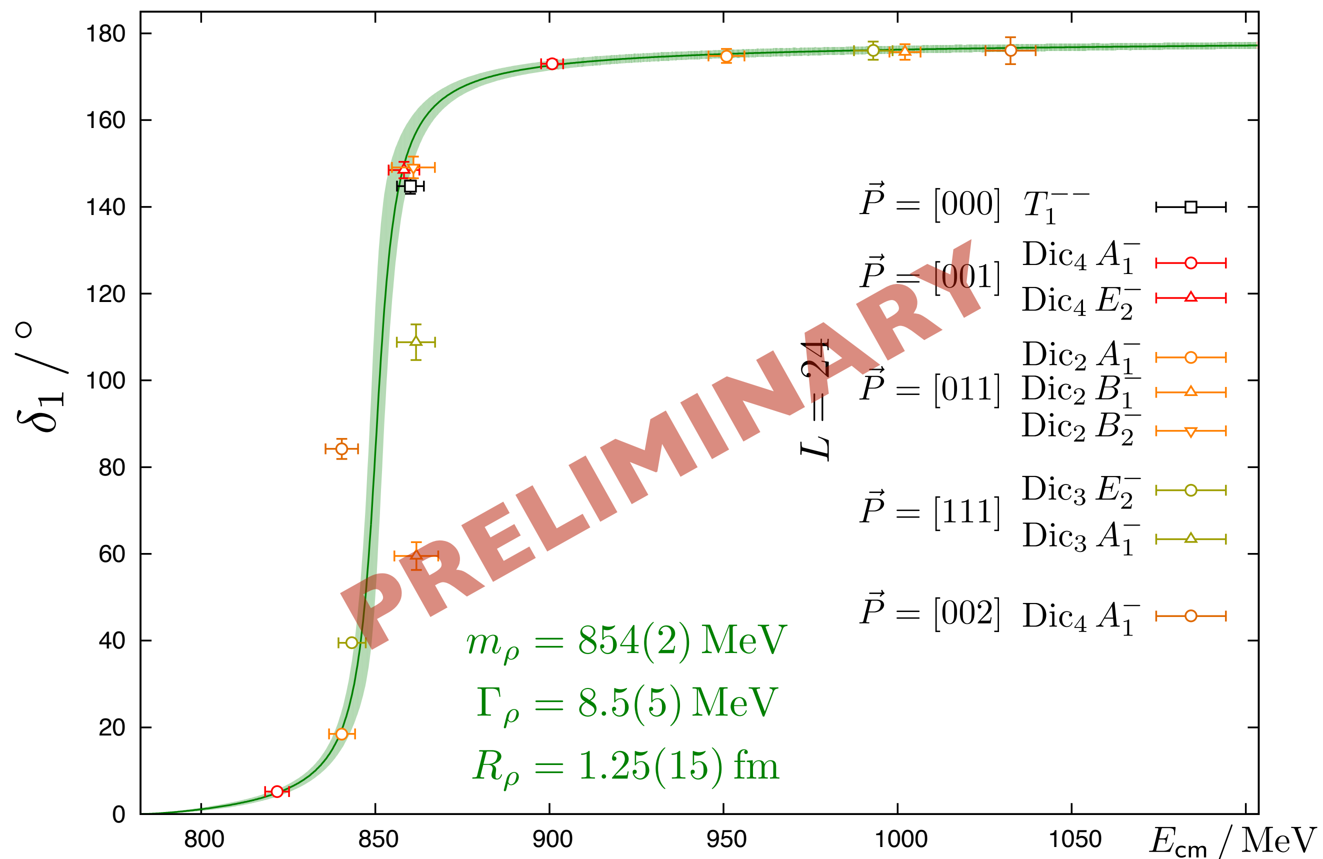




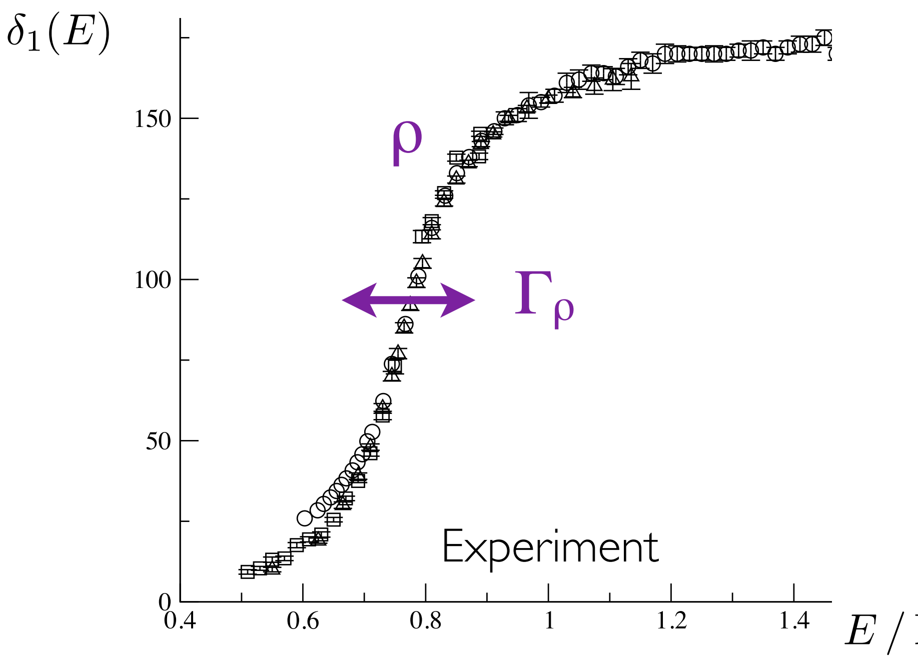


Resonance studies: rho

- Map out phase shift to identify a resonance
  - Having multiple P_{CM} frames is crucial
- Phase shift can be well determined at heavy quark masses



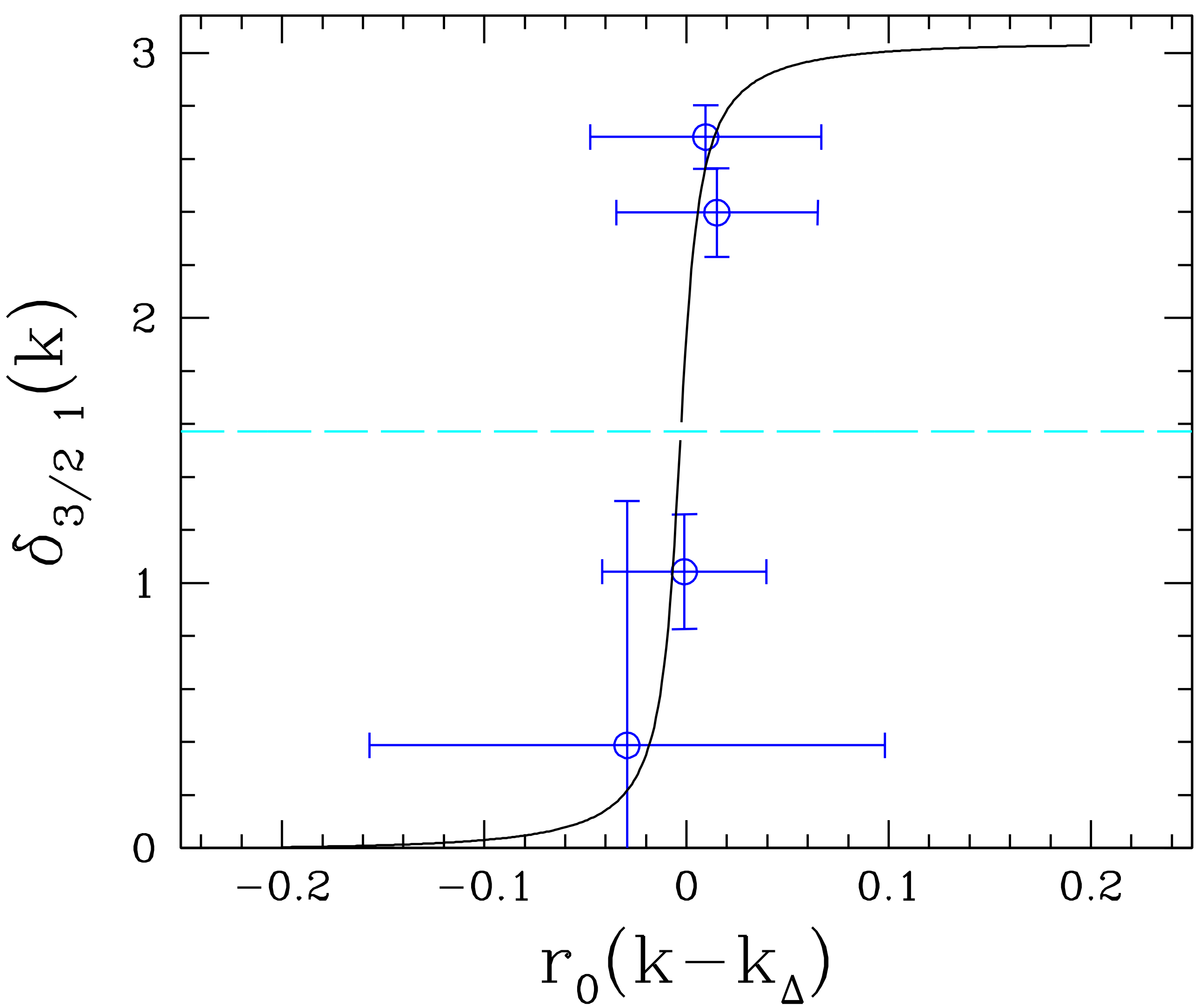
[HadSpec Collaboration]



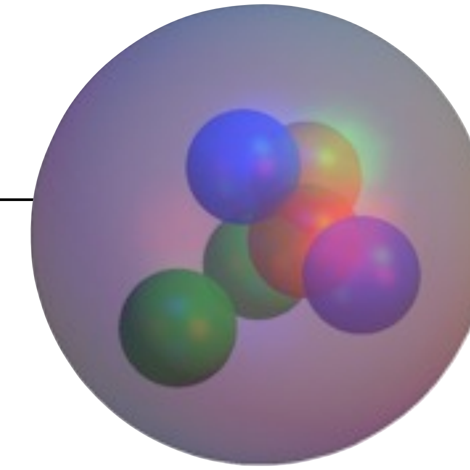
- Properties of resonance requires modelling– eg fit with modified Breit-Wigner
- Much current work addressing best way to get the most information

Resonance studies: Delta(1232)

- Delta baryon studied by QCDSF-Bonn-Jülich collaboration [Meißner QNP12] – a more challenging case [see Schierholz talk tomorrow at INT program]
- Multiple volumes at light quark masses



H-dibaryon



- First QCD bound state observed in LQCD
  – NPLQCD [PRL 106, 162001 (2011)] and HALQCD [PRL 106, 162002 (2011)]

- Jaffe [1977]: chromo-magnetic interaction between quarks

$$\langle H_m \rangle \sim \frac{1}{4}N(N-10) + \frac{1}{3}S(S+1) + \frac{1}{2}C_c^2 + C_f^2$$

  most attractive for spin, colour, flavour singlet

  - H-dibaryon (uuddss) J=I=0, s=-2 most stable

$$\Psi_H = \frac{1}{\sqrt{8}}\left(\Lambda\Lambda + \sqrt{3}\Sigma\Sigma + 2\Xi N\right)$$

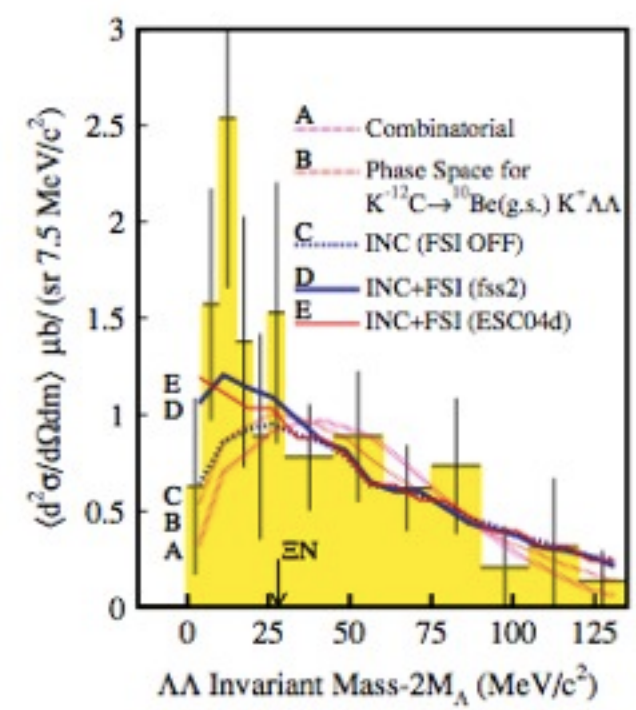
  - Bound in a many hadronic models

- Experimental searches

  - Emulsion expts, heavy-ion, stopped kaons

  - No conclusive evidence for or against

KEK-ps (2007)
$K^- \, {}^{12}C \rightarrow K^+ \, \Lambda\Lambda \, X$



H dibaryon in QCD

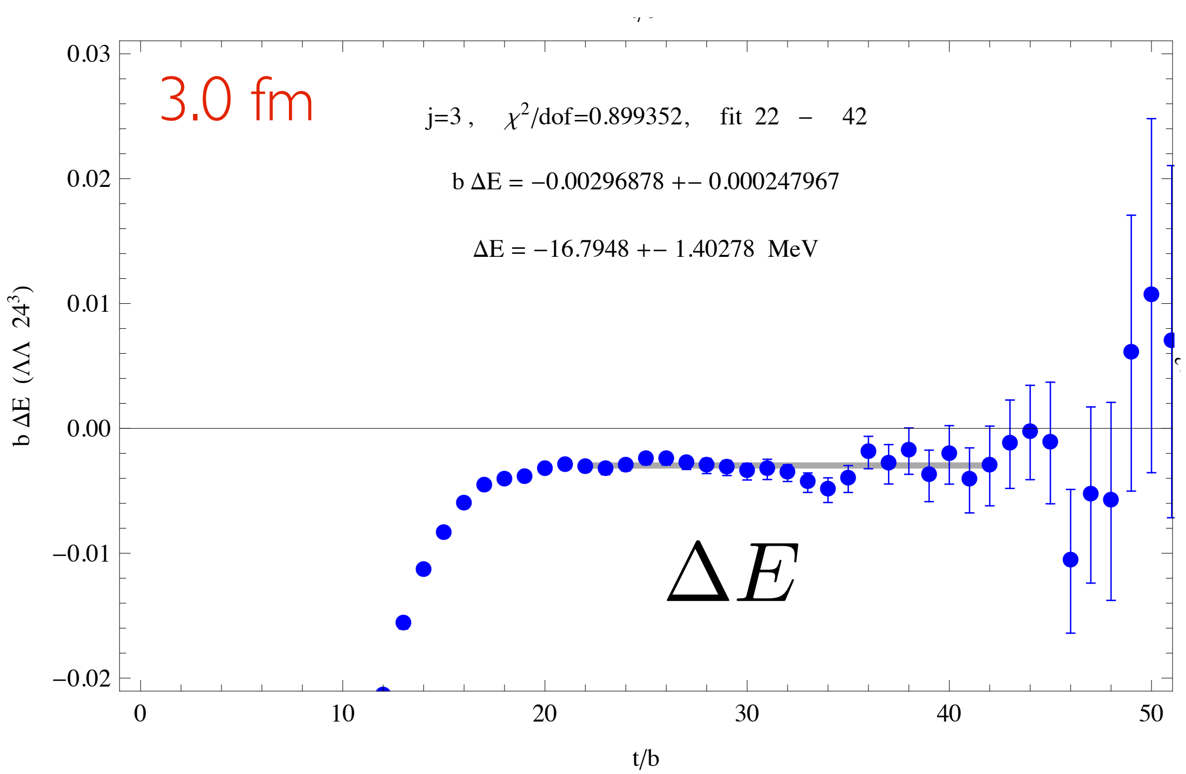
- Extract energy eigenstates from large Euclidean time behaviour of two-point correlators

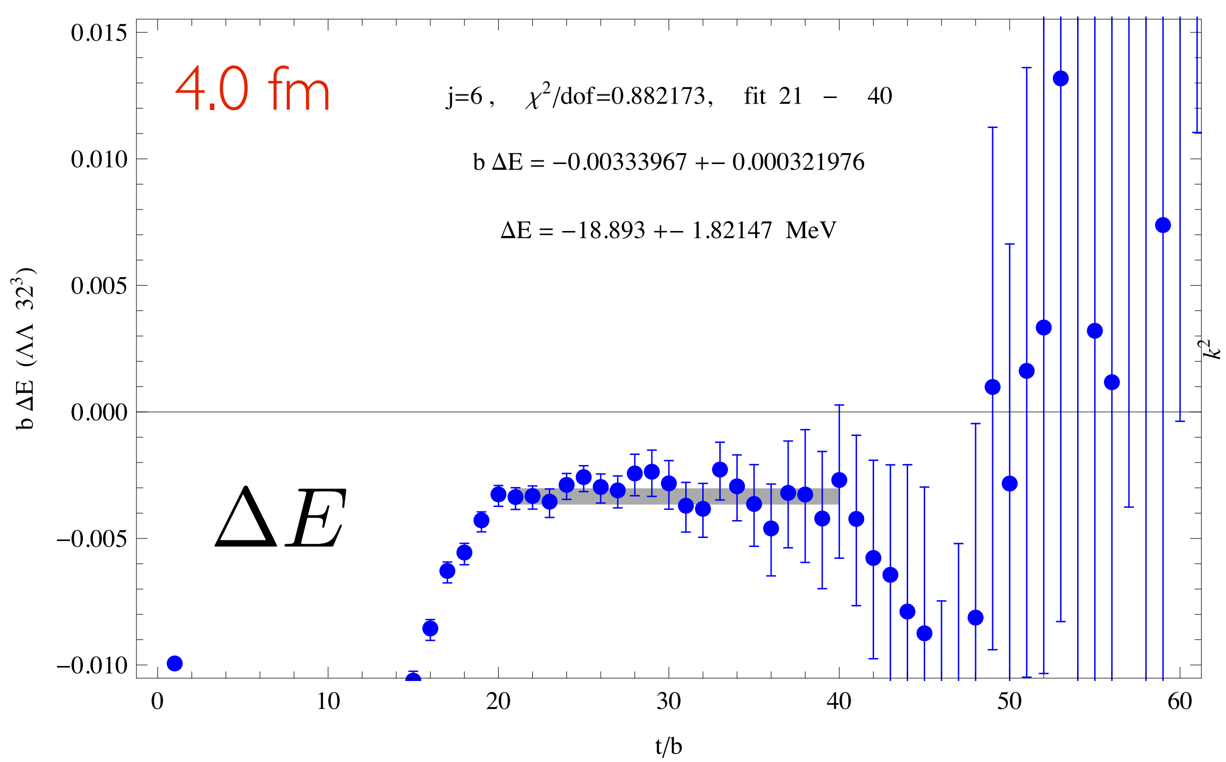
$$C_\Lambda(t) = \sum_{\mathbf{x}} \langle 0|\chi(\mathbf{x},t)\overline{\chi}(0)|0\rangle \stackrel{t\to\infty}{\longrightarrow} Z_\Lambda e^{-M_\Lambda t}$$

$$C_{\Lambda\Lambda}(t) = \sum_{\mathbf{x}} \langle 0|\phi(\mathbf{x},t)\overline{\phi}(0)|0\rangle \stackrel{t\to\infty}{\longrightarrow} Z_{\Lambda\Lambda} e^{-E_{\Lambda\Lambda} t}$$

$$R(t) = \frac{C_{\Lambda\Lambda}(t)}{C_\Lambda^2(t)} \stackrel{t\to\infty}{\longrightarrow} \widetilde{Z} e^{-\Delta E_{\Lambda\Lambda} t}$$

- Correlator ratio allows direct access to energy shift





Simple extrapolations

- After volume extrapolation
  H bound at unphysical quark masses

- Quark mass extrapolation is uncertain and unconstrained

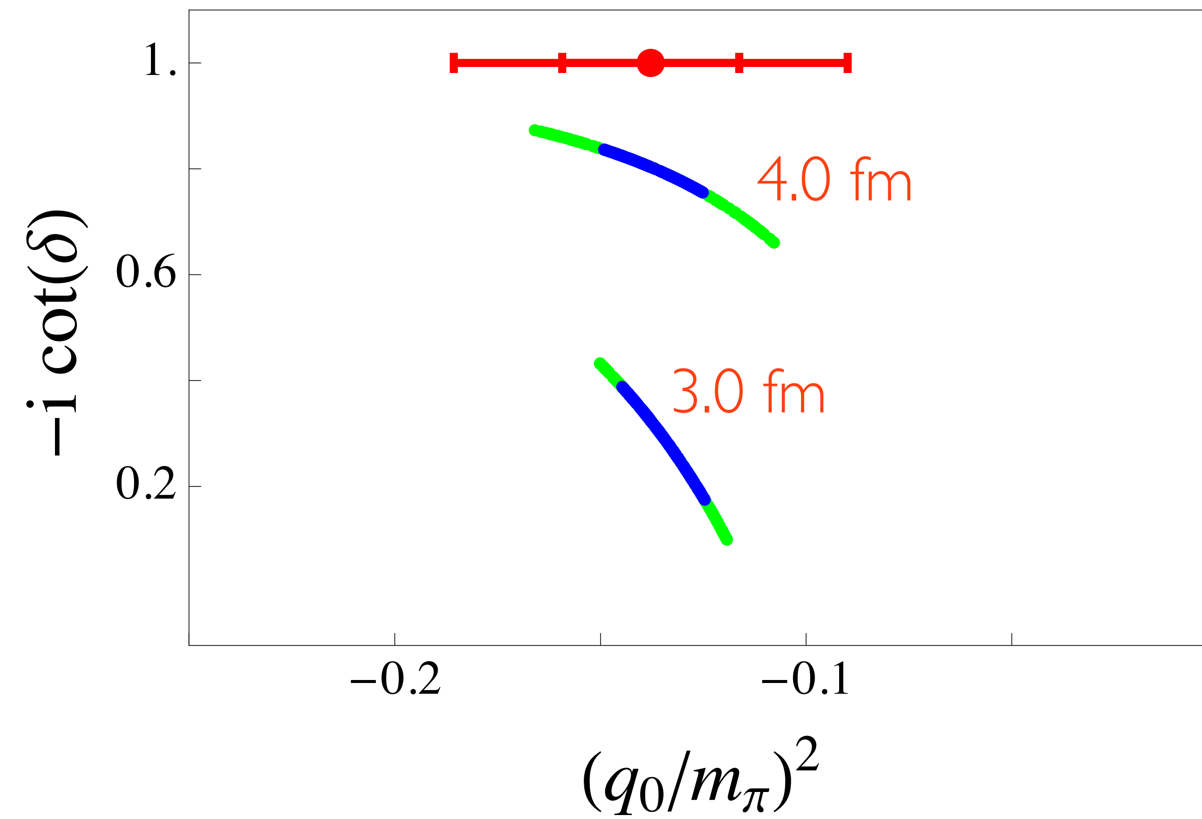
$$B_H^{\text{quad}} = +11.5 \pm 2.8 \pm 6.0 \text{ MeV}$$

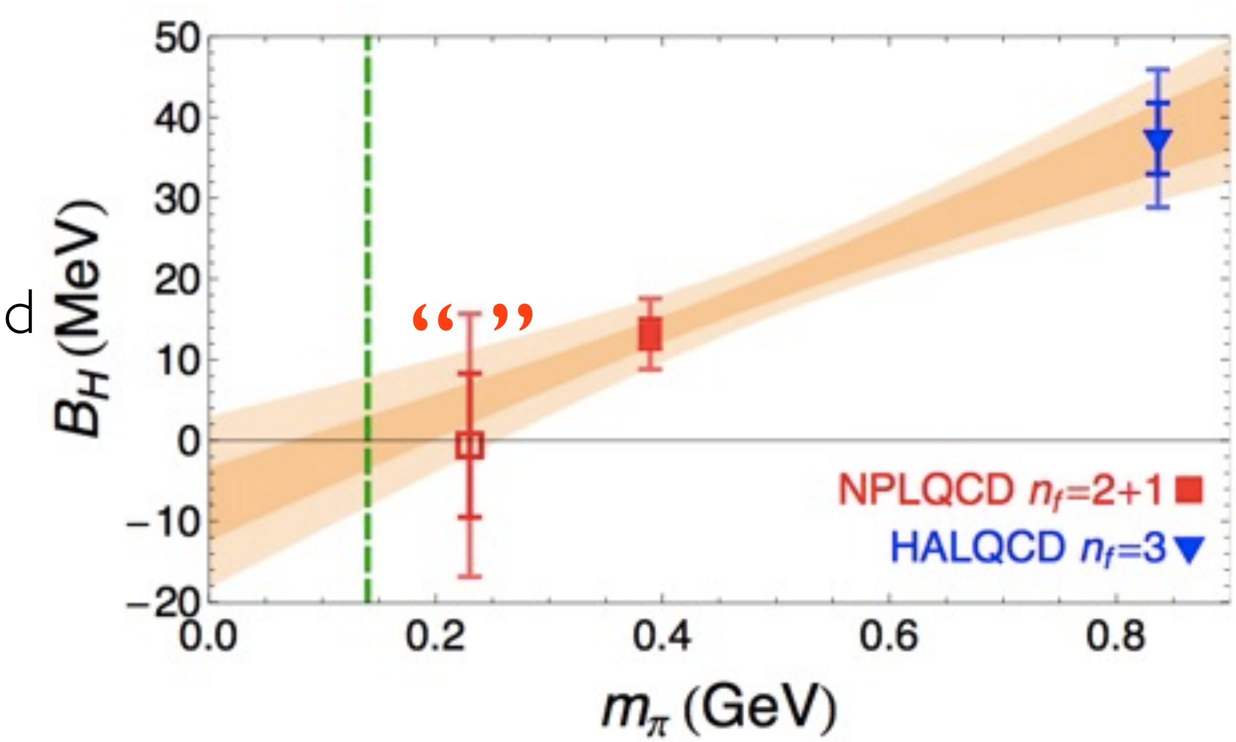
$$B_H^{\text{lin}} = +4.9 \pm 4.0 \pm 8.3 \text{ MeV}$$

- Other extrapolations, see
  [Shanahan, Thomas & Young PRL. 107 (2011) 092004, Haidenbauer & Meissner 1109.3590]

- Suggests H is weakly bound or just unbound

- More study required at light masses



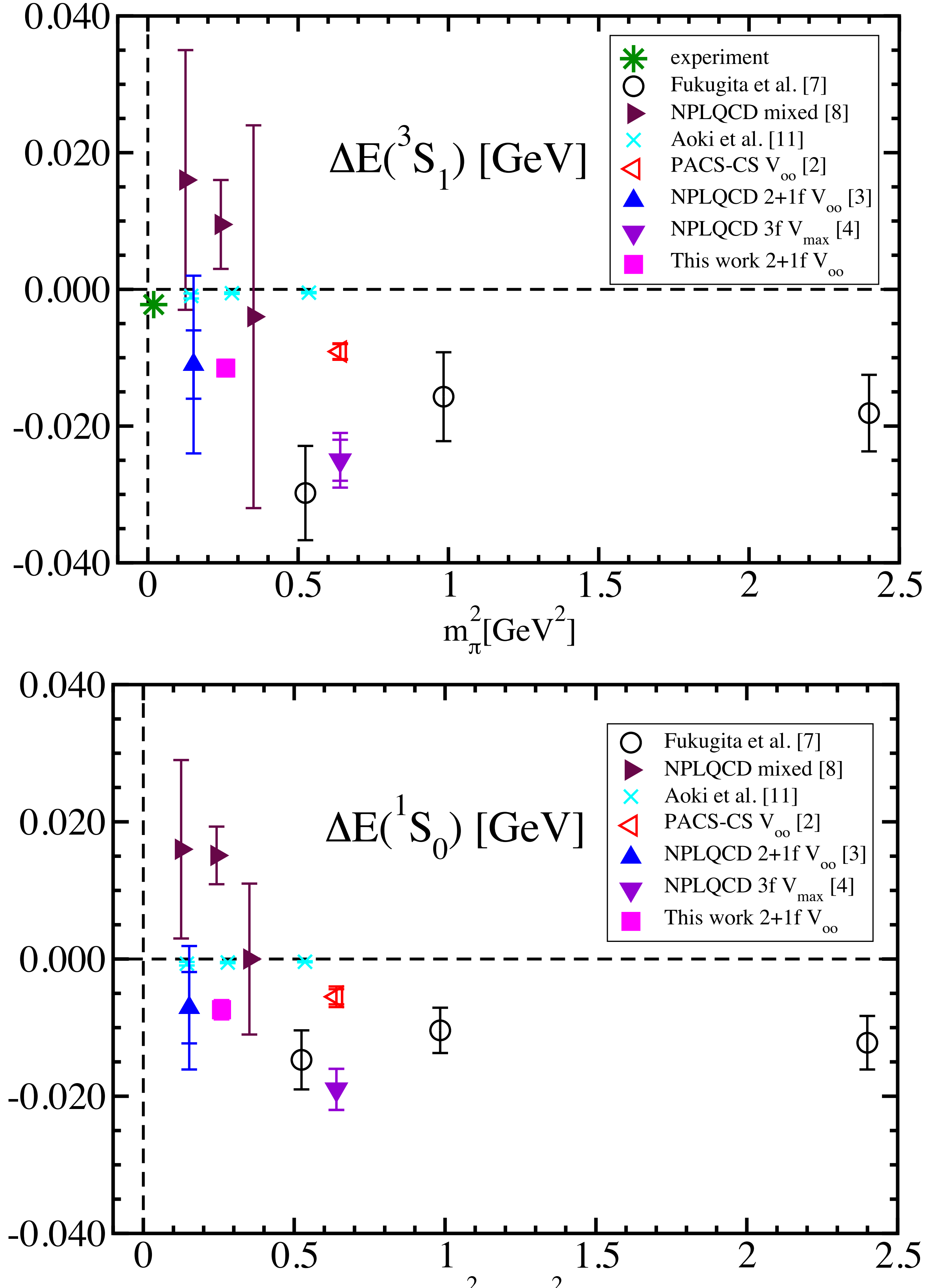


* 230 MeV point preliminary (one volume)

Deuteron and Dineutron

- Deuteron, di-neutron also investigated
  - NPLQCD
  - PACS-CS
- More work needed at lighter masses
- Also s=-4 ΞΞ dibaryon is found to be bound

[Yamazaki et al. 1207.4277]



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