Future Applications of Lattice QCD for High-Energy Physics

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Lecture at INT summer school, August 10, 2012

Outline

- Summary of present status
  - ✶ Focus on weak matrix elements
- Future directions and challenges
  - ✶ $K\rightarrow\pi\pi$ decays ($\Delta I=1/2$ rule & CP violation) & ΔM_K
  - ✶ $D\rightarrow\pi\pi$, KK-bar---is a lattice calculation possible?
  - ✶ $\pi^0\rightarrow\gamma\gamma$ (pushing the limits of Euclidean-space calculations)
  - ✶ Predicting hadronic contributions to muonic g-2
- Outlook

Strengths & Weaknesses of LQCD

- ✓ Lattice QCD (LQCD) provides, at present, the only first principles method for calculating quantities in the non-perturbative (low energy) realm of QCD
- ✓ Thus we can test that QCD is indeed the correct theory of strong interactions
- ✓ We can use LQCD to understand the physics of confinement
- ✓ We can use LQCD to remove the non-perturbative QCD "background" in searches for rare processes induced by new physics
- ▶ LQCD is an essential component of searches for new physics at the "intensity frontier"

Strengths & Weaknesses of LQCD

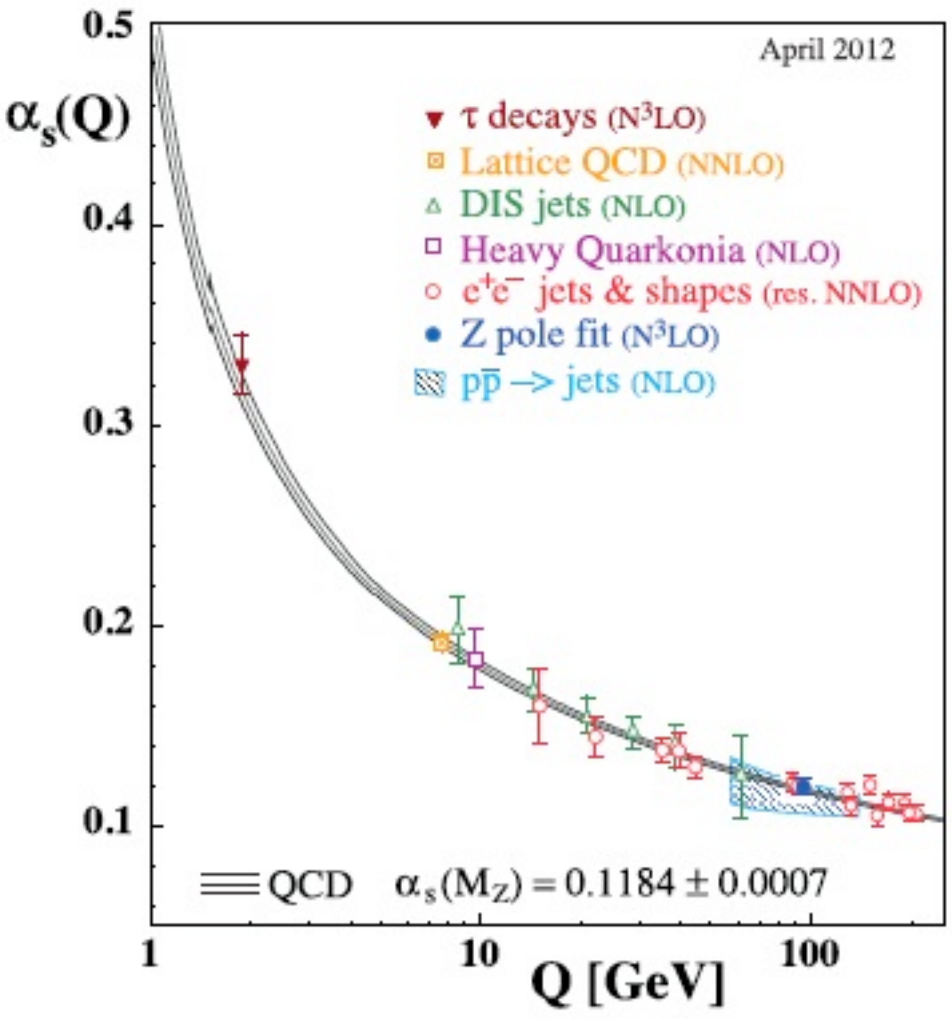
- ✓ Can simulate at or near physical quark masses for u,d,s, & c
- ✓ Discretization (a^2) errors typically small & controlled (a ~ 0.05 - 0.1 fm)
- ✓ Volumes large enough that most finite-volume effects are small (L ~ 4 - 6 fm)
- ✓ Non-perturbative matching of lattice and continuum operators (e.g. H_W) routine
- ✓ Efficient methods exist for extracting few excited state energies and many J^{PC}'s
- ✓ Efficient methods exist for calculating quark-disconnected Wick contractions
- ✓ Computers have come a long way (we are in the Petaflops era)!
- We are stuck in Euclidean space---real-time phenomena not directly accessible (e.g. hadronization in jet physics, light-cone distributions, transport properties)
- We are stuck in a finite volume---cannot directly calculate scattering processes
- We are stuck with discrete rotation group---hard to study high ang. mom.
- Most simulations of b quark use NRQCD or HQET, introducing truncation errors
- We are stuck with a finite range of scales---L/a < 100---making calculations in nearly conformal theories very challenging
- We are restricted to relatively small N_{color}; hinders connections to string theory

Examples of present status

✓ Spectrum of stable hadrons agrees with nature [Sinead Ryan's lectures]

- Crucial test of QCD in non-perturbative regime

✓ Strong-coupling constant obtained from LQCD agrees with high-energy results



[PDG 2012 (web)]
PDG now includes
a review of LQCD
[Hashimoto, Laiho, SRS]

Examples of present status

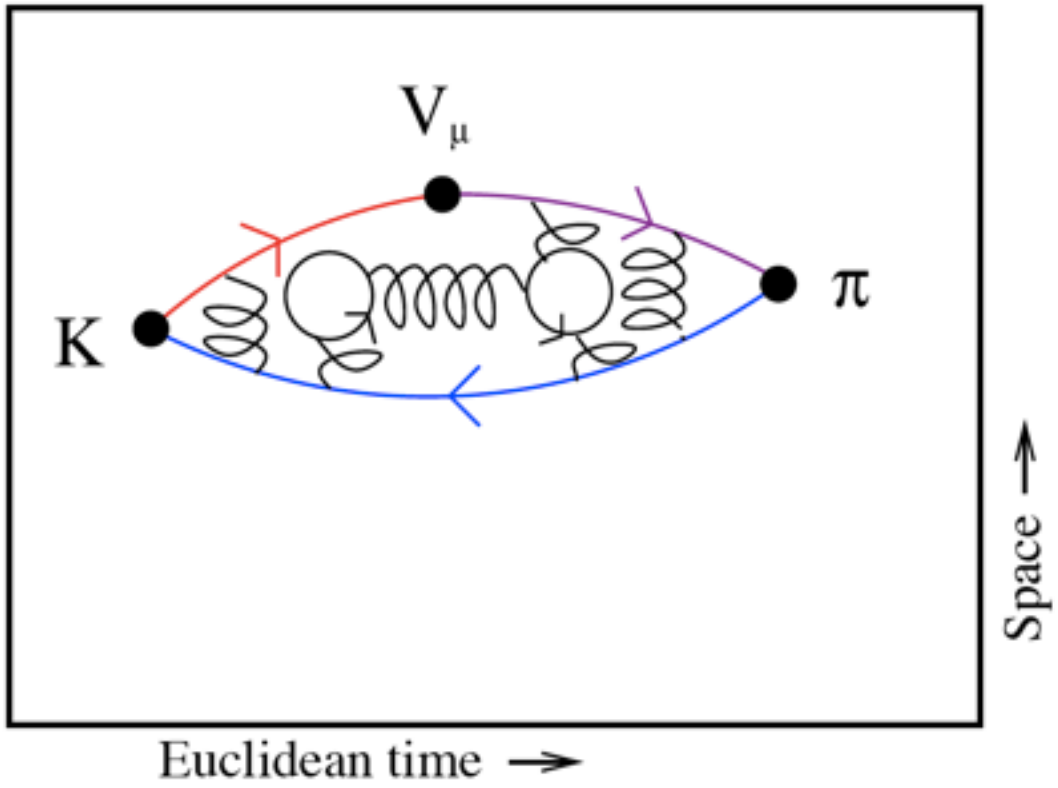
★ "Gold-plated" processes involving single ground-state hadron (or vacuum) in initial & final states connected by a local operator

- Generically called "weak matrix elements"
- Examples: f_π, f_K, K→π form factor (⇒ K→πeν decay), D→K, B→π & B→$D^{(*)}$ form factors, B_K (⇒ CP-violation in K-Kbar mixing)

K→π form factor

$$\langle \pi(\vec{p}_2) | V_\mu(0) | K(\vec{p}_1) \rangle$$

$$\Rightarrow f_+(0)$$



Examples of present status

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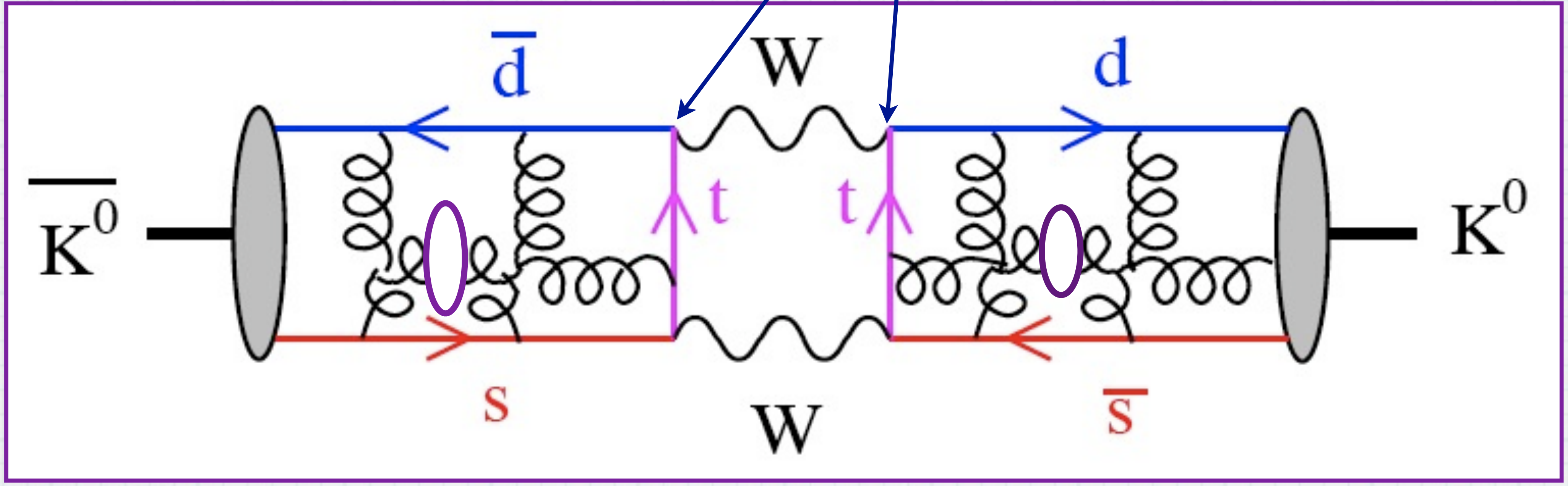
    - Generically called “weak matrix elements”
    - Examples: f_π, f_K, K→π form factor (⇒ K→πeν decay),D→K, B→π & B→$D^{(*)}$ form factors, B_K (⇒ CP-violation in K-Kbar mixing)
- Why do we care about these quantities?
    - Because, combined with experiment, they allow an (over)determination of the parameters of the CKM matrix---fundamental params of the SM

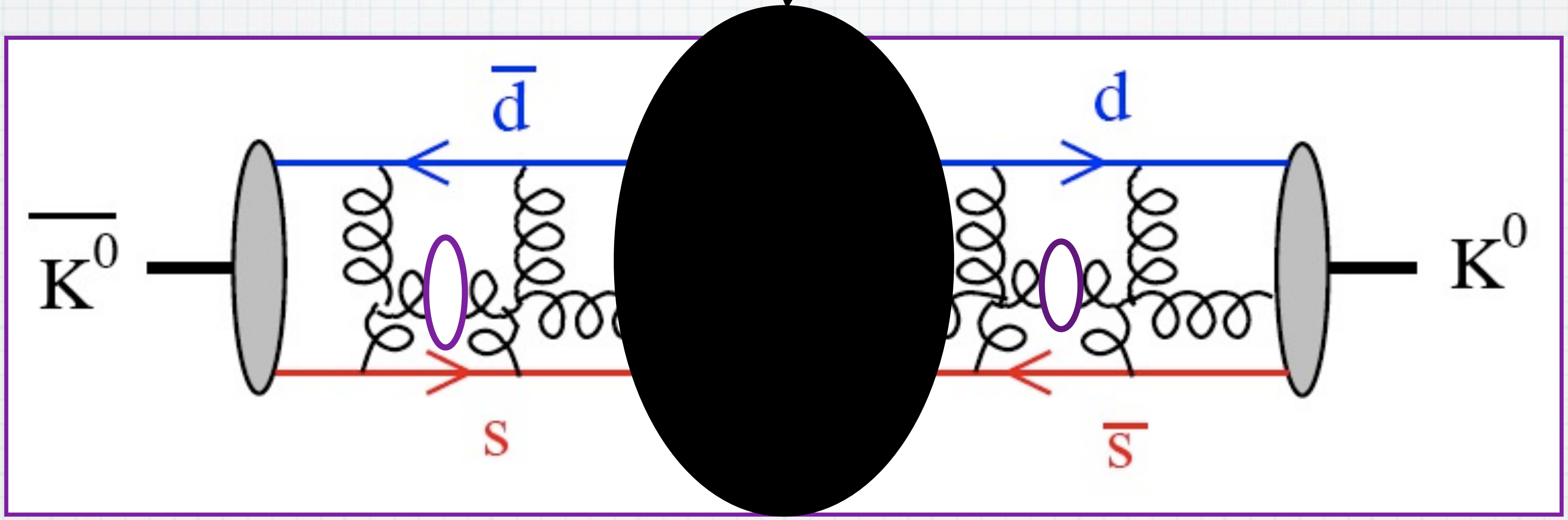
Example: B_K

V_{td} contains
CP violating phase



Example: B_K

Known local four-fermion operator



$$\epsilon_K = (\text{known factors})\text{Im}(V_{td}^2)B_K$$

Measured (in 1964!)

Fundamental parameter we wish to determine

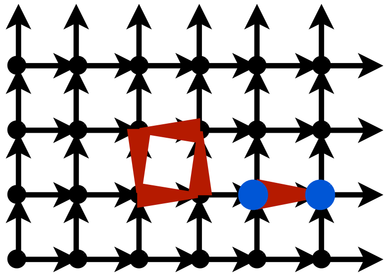
Weak matrix element calculated in LQCD

(Over)determining V_{CKM}

- Examples of processes needing QCD input

$$\left(\begin{array}{c|c|c}
\mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\
\pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\
 & K \to \pi\ell\nu & \\
\hline
\mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\
D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\
D \to \pi\ell\nu & D \to K\ell\nu & B \to D^{*}\ell\nu \\
\hline
\mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\
B_d \leftrightarrow \overline{B}_d & B_s \leftrightarrow \overline{B}_s &
\end{array}\right)$$

ε_K ε_K



Overdetermining V_{CKM}

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}.$$

What we know:

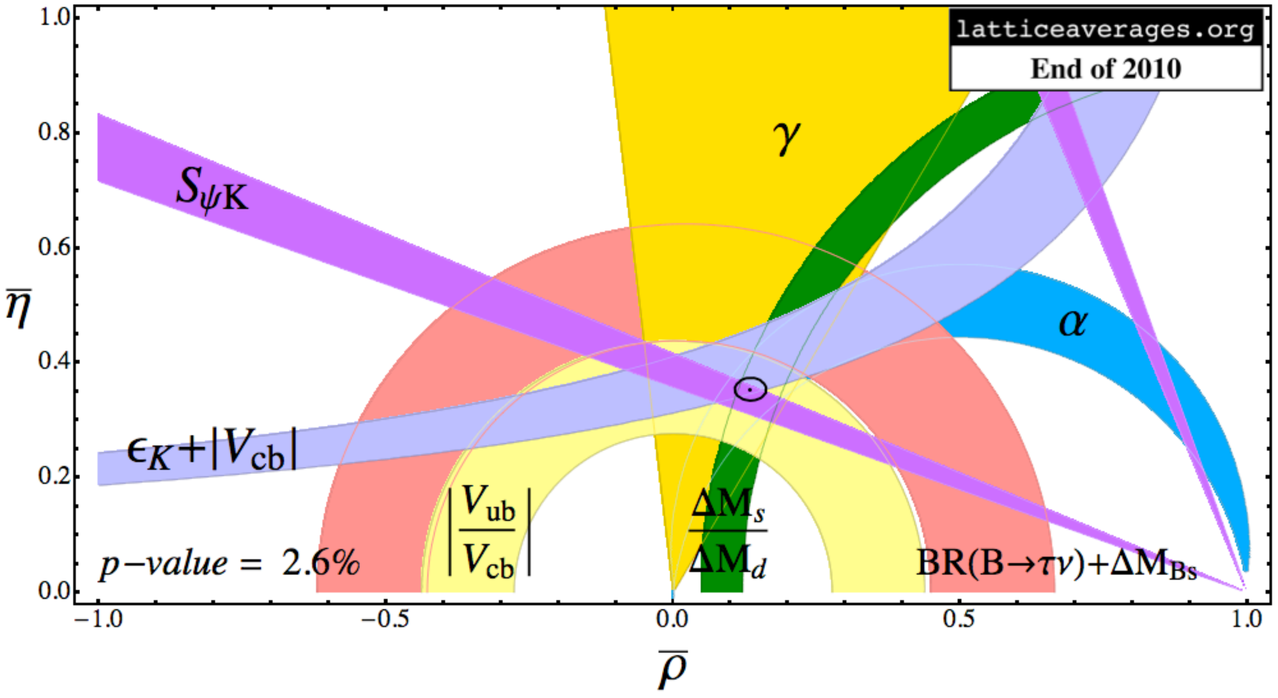
$$\boxed{\lambda = 0.2253 \pm 0.0007\,, \qquad A = 0.808^{+0.022}_{-0.015}}$$

PDG, 2010

$\eta \neq 0$

⇔

CP violation



Laiho, Lunghi & Van de Water

Overdetermining V_{CKM}

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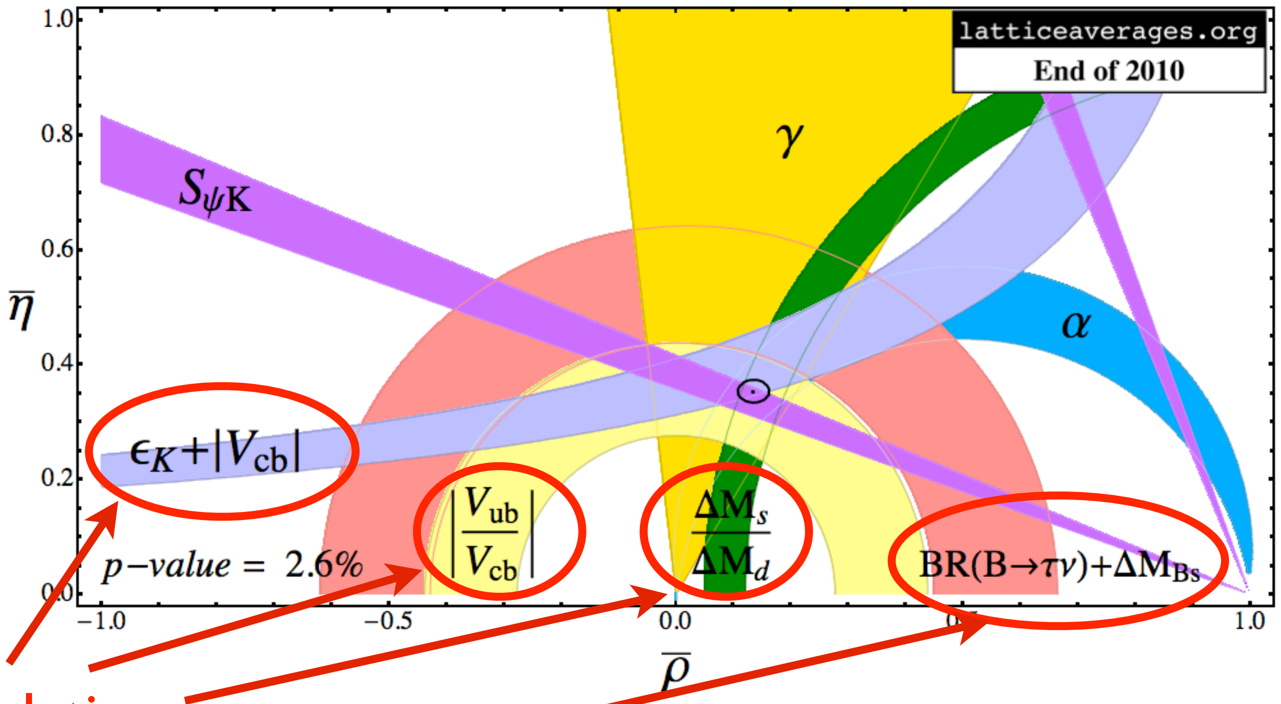


Laiho, Lunghi & Van de Water

Lattice calculation required

Lattice vs. experimental errors

[USQCD collaboration SciDac3 proposal]

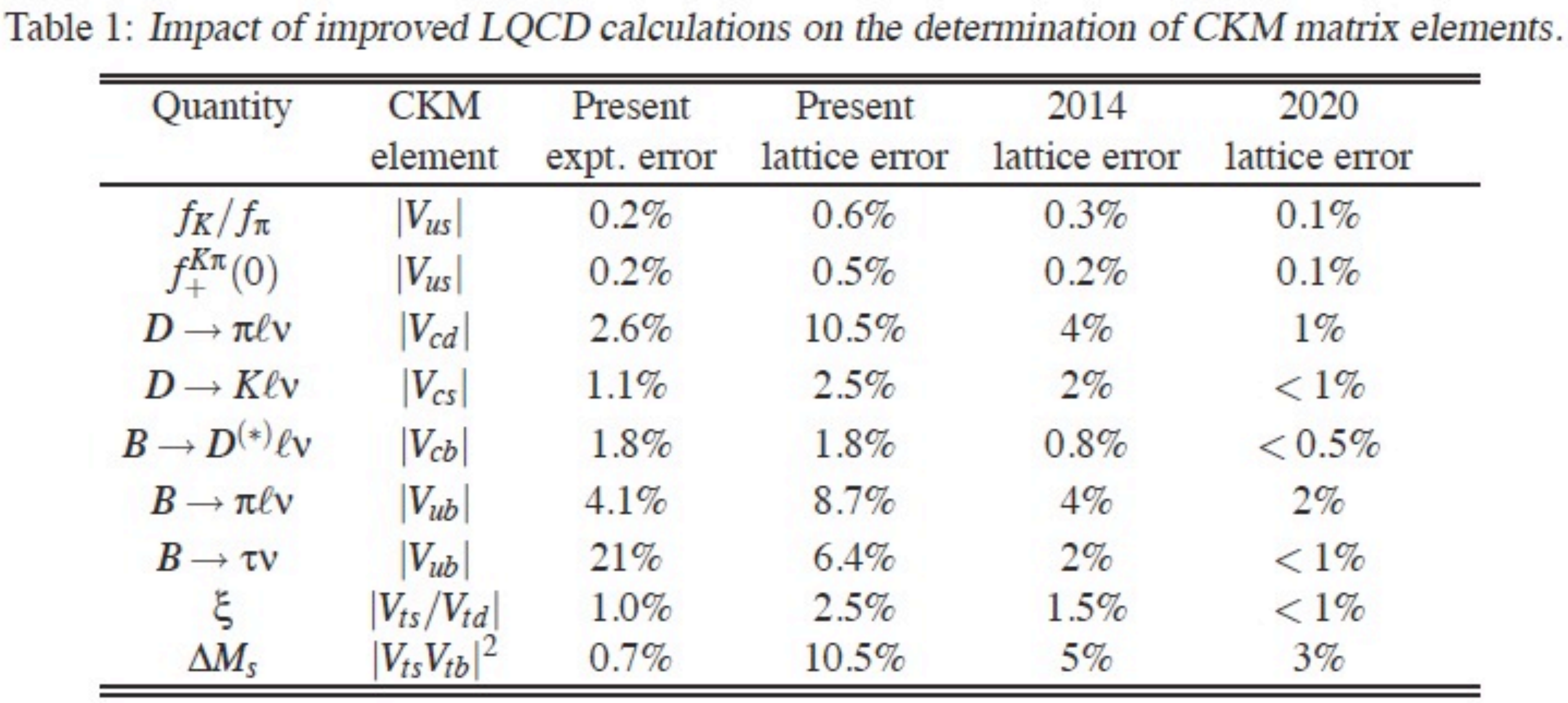


Table 1: *Impact of improved LQCD calculations on the determination of CKM matrix elements.*

Quantity	CKM element	Present expt. error	Present lattice error	2014 lattice error	2020 lattice error
f_K/f_π	$\lvert V_{us}\rvert$	0.2%	0.6%	0.3%	0.1%
$f_+^{K\pi}(0)$	$\lvert V_{us}\rvert$	0.2%	0.5%	0.2%	0.1%
$D \to \pi\ell\nu$	$\lvert V_{cd}\rvert$	2.6%	10.5%	4%	1%
$D \to K\ell\nu$	$\lvert V_{cs}\rvert$	1.1%	2.5%	2%	$< 1\%$
$B \to D^{(*)}\ell\nu$	$\lvert V_{cb}\rvert$	1.8%	1.8%	0.8%	$< 0.5\%$
$B \to \pi\ell\nu$	$\lvert V_{ub}\rvert$	4.1%	8.7%	4%	2%
$B \to \tau\nu$	$\lvert V_{ub}\rvert$	21%	6.4%	2%	$< 1\%$
ξ	$\lvert V_{ts}/V_{td}\rvert$	1.0%	2.5%	1.5%	$< 1\%$
ΔM_s	$\lvert V_{ts}V_{tb}\rvert^2$	0.7%	10.5%	5%	3%

partial list!

Expt = (known) V_{CKM} (matrix element from LQCD)

★ Present lattice error typically larger than experimental error

- One future direction of LQCD is to improve the errors
- We think this can be done using improved methods & faster CPUs

Examples of present status

★ Flavianet Lattice Averaging Group (FLAG) and its successor Flavo(u)r Lattice Averaging Group (FLAG2) aim to provide "world averages" of well calculated lattice quantities along with a critical appraisal

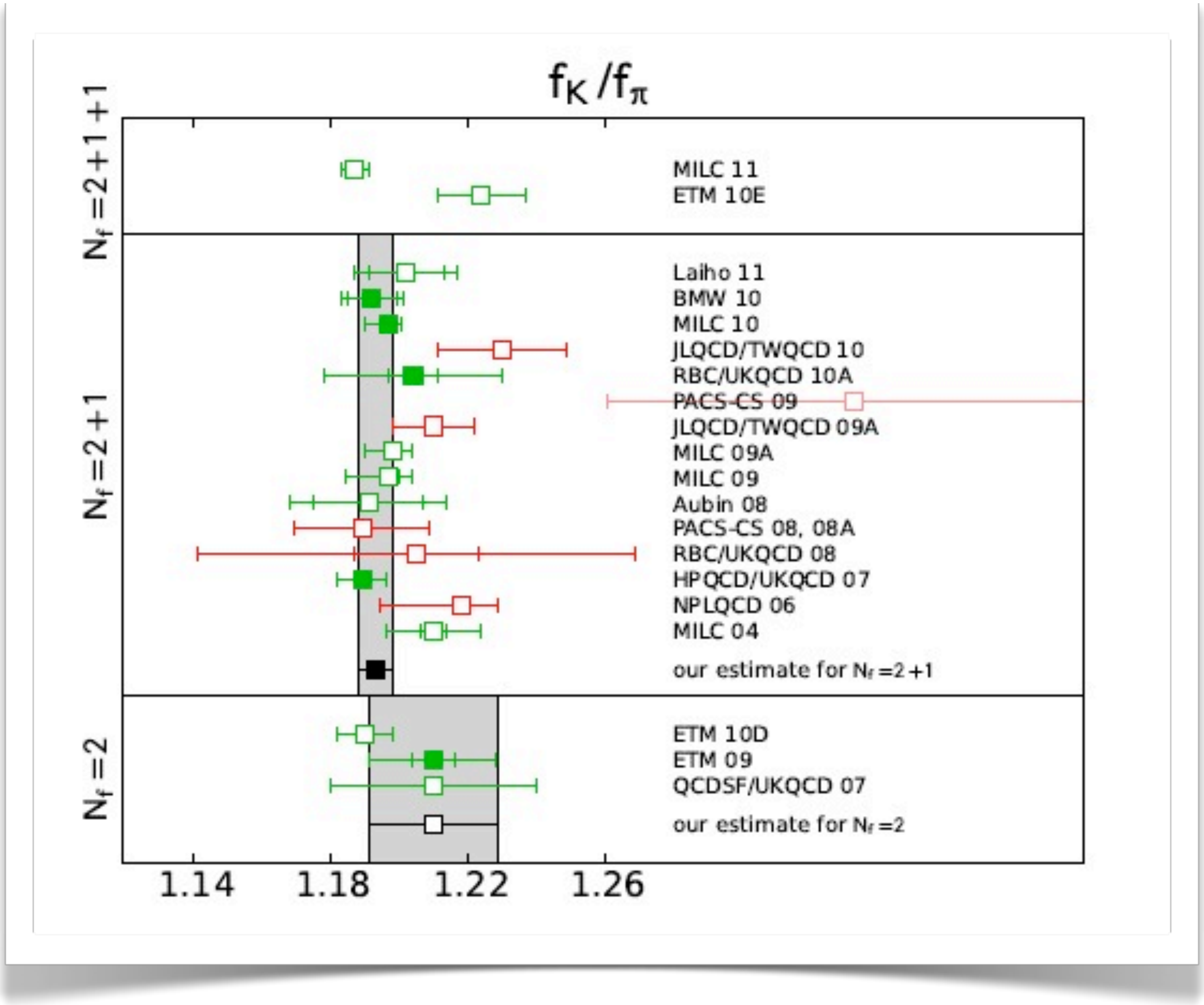
- FLAG2 is a world-wide organization with 28 members
- subsumes FLAG and latticeaverages.org [Laiho, Lunghi & Van de Water]

- Advisory Board:
  S. Aoki, C. Bernard, C. Sachrajda
- Editorial Board:
  GC, H. Leutwyler, T. Vladikas, U. Wenger
- Working Groups

Quark masses	L. Lellouch, T. Blum, V. Lubicz
V_{us}, V_{ud}	A. Jüttner, T. Kaneko, S. Simula
LEC	S. Dürr, H. Fukaya, S. Necco
B_K	H. Wittig, J. Laiho, S. Sharpe
α_s	R. Sommer, T. Onogi, J. Shigemitsu
f_B, B_B	A. El Khadra, Y. Aoki, M. Della Morte
$B \to H\ell\nu$	R. Van de Water, E. Lunghi, C. Pena

[Colangelo, Lattice 2012]

Example of FLAG2 averages

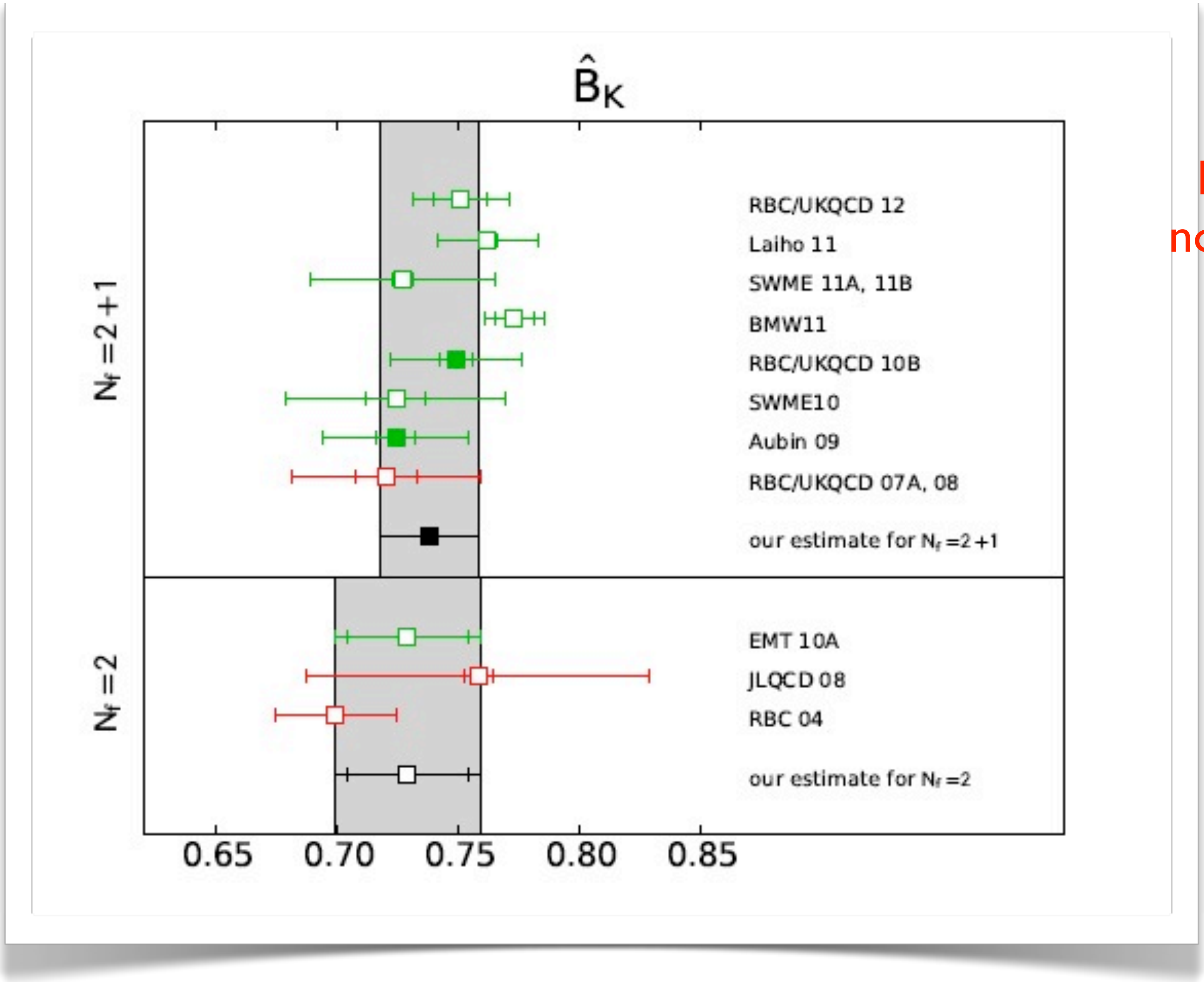


Preliminary!

[Colangelo, Lattice 2012]

- ★ Subpercent-level accuracy
- ★ Agreement between different fermion discretizations is crucial cross-check

Example of FLAG2 averages

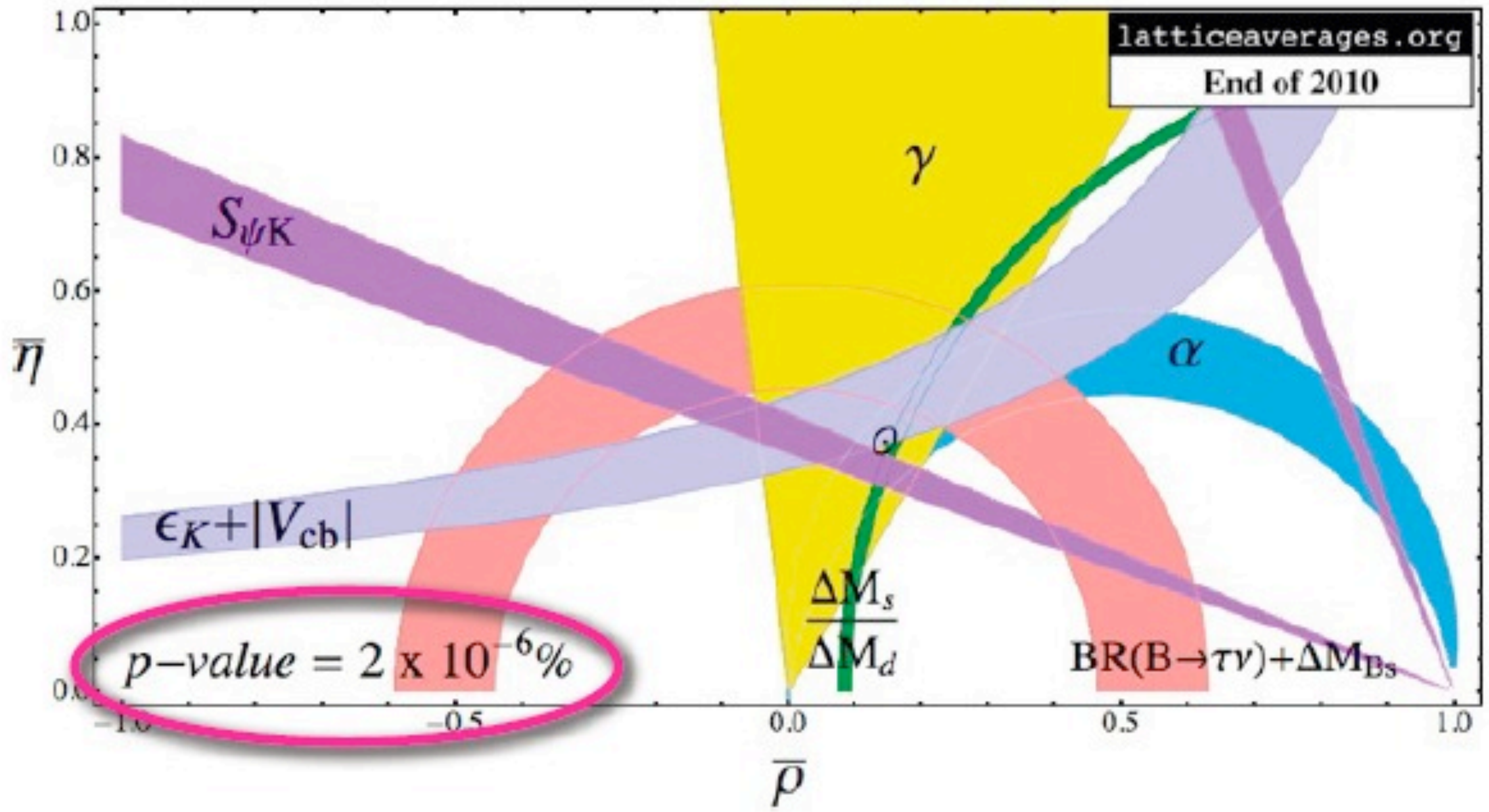


Note: average not yet updated!

[Colangelo, Lattice 2012]

Possible future of CKM constraints

- Currently the constraints from ε_K, $\Delta M_s/\Delta M_d$, and $|V_{ub}/V_{cb}|$ are **limited by uncertainties in the lattice QCD calculations** of $|V_{cb}|_{excl.}$, ξ, and $|V_{ub}|_{excl.}$, respectively
- To illustrate the potential impact of future lattice calculations, reduce the lattice uncertainties to 1% with central values fixed, but keep experimental uncertainties fixed



- Lattice QCD is poised to play a key role in discovering new physics in the flavor sector!

[Van de Water, 2012]

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K→ππ decays and kaon mixing

- Can SM explain "ΔI=1/2 rule"? $$\frac{A[K^0\to\pi\pi(I=0)]}{A[K^0\to\pi\pi(I=2)]}\sim 22$$
- Can SM explain CP violation in Kaon decays ($\varepsilon'_K/\varepsilon_K$)? Is new physics needed?
- Can SM explain the K_L-K_S mass difference, ΔM_K? Is new physics needed?

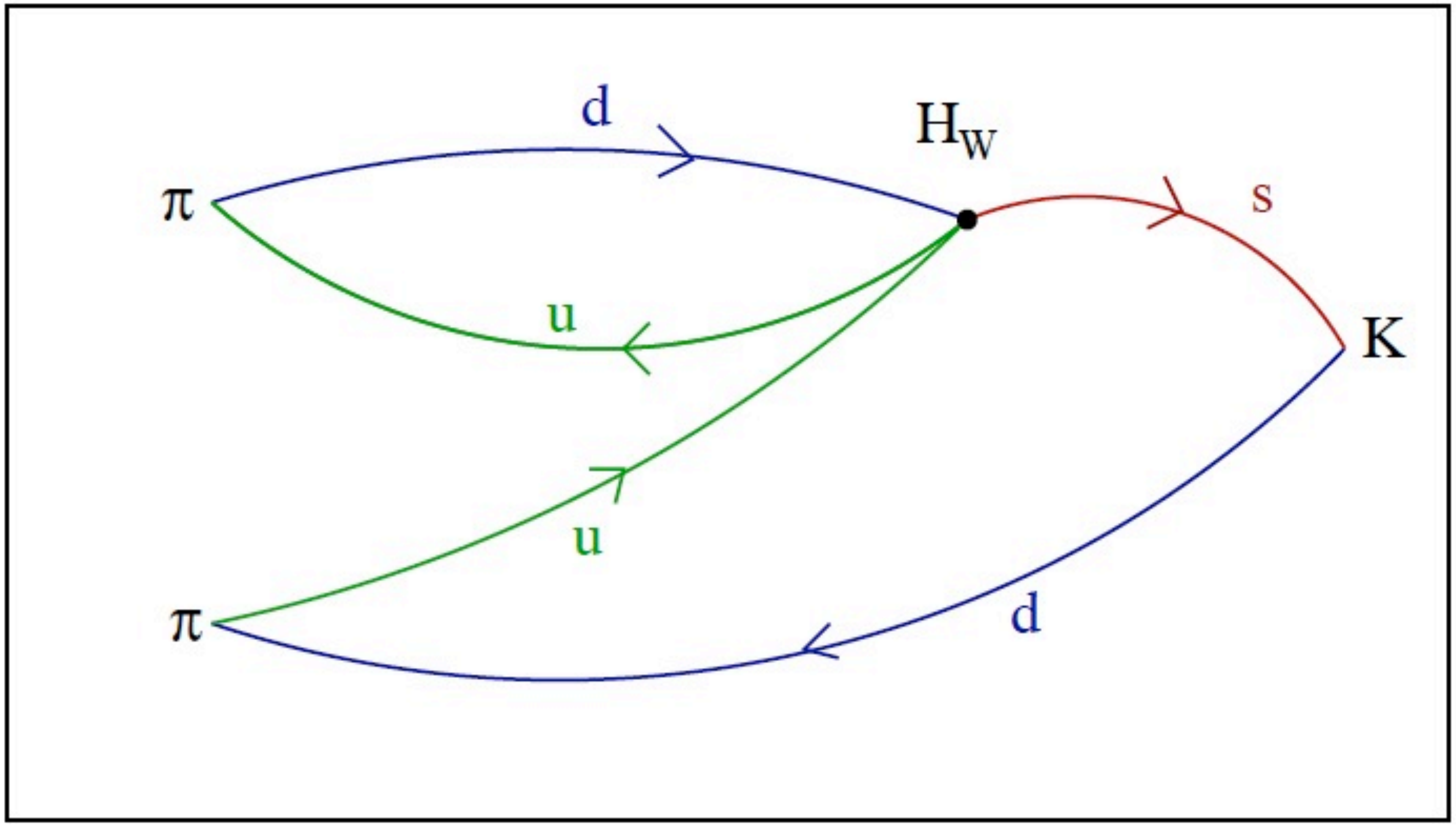
✶ All three quantities require extension of lattice methods beyond those needed for "gold-plated" quantities

✶ Kaon decays require two-particle final states, which are inevitably affected by the finite box

✶ Kaon mixing requires the insertion of time-ordered product of two H_W's

➡ LQCD methods exist in principle for all three quantities, but challenging to implement in practice: hope for significant progress over next 5 years

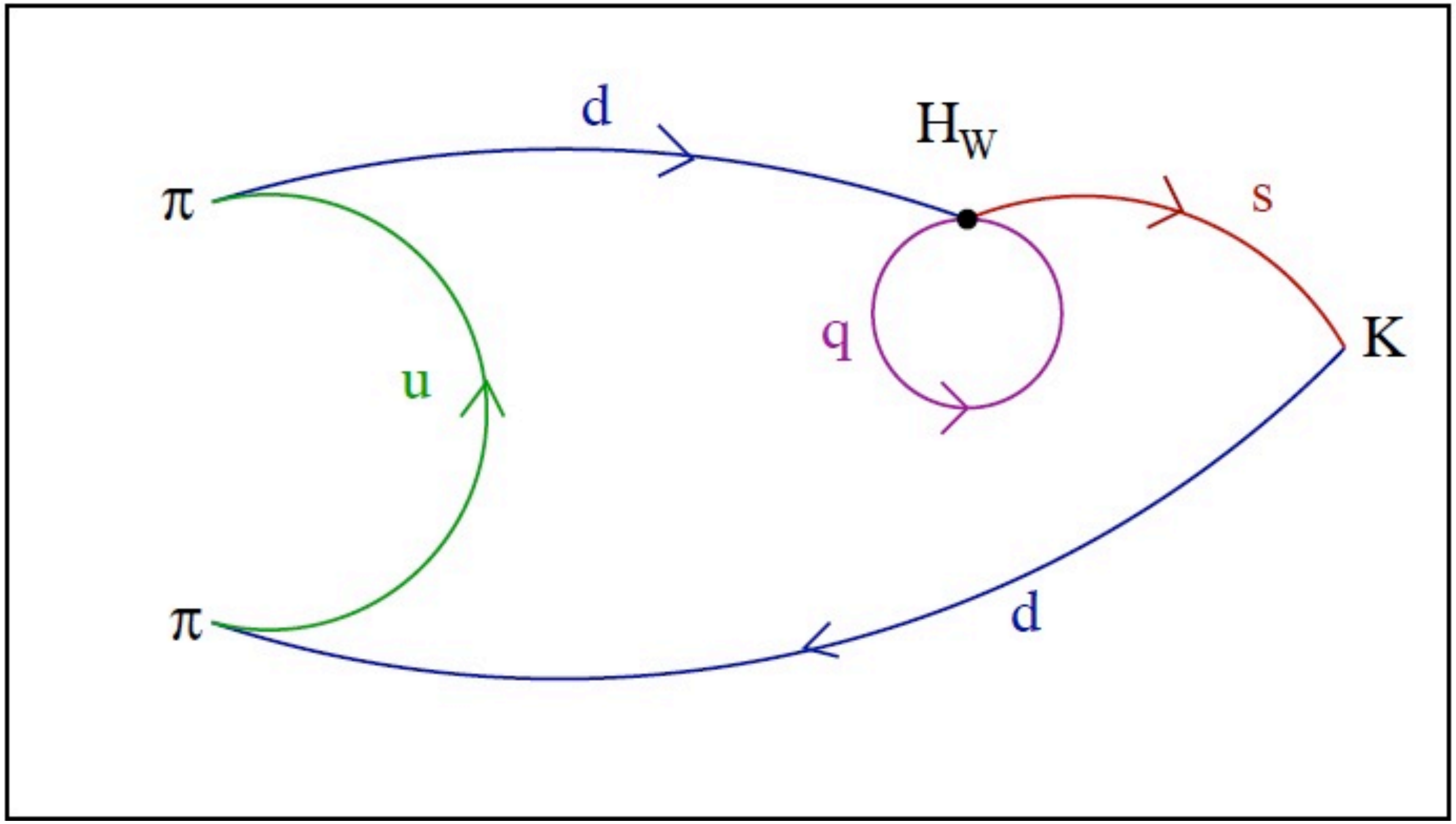
Ingredients for K→ππ



$$\Rightarrow \quad {}_L\langle \pi\pi | \mathcal{H}_W | K \rangle_L$$

- Many Wick contractions: disconnected ones particularly challenging, but distillation and diluted noise appear to make problem tractable [RBC 2012]

Ingredients for K→ππ



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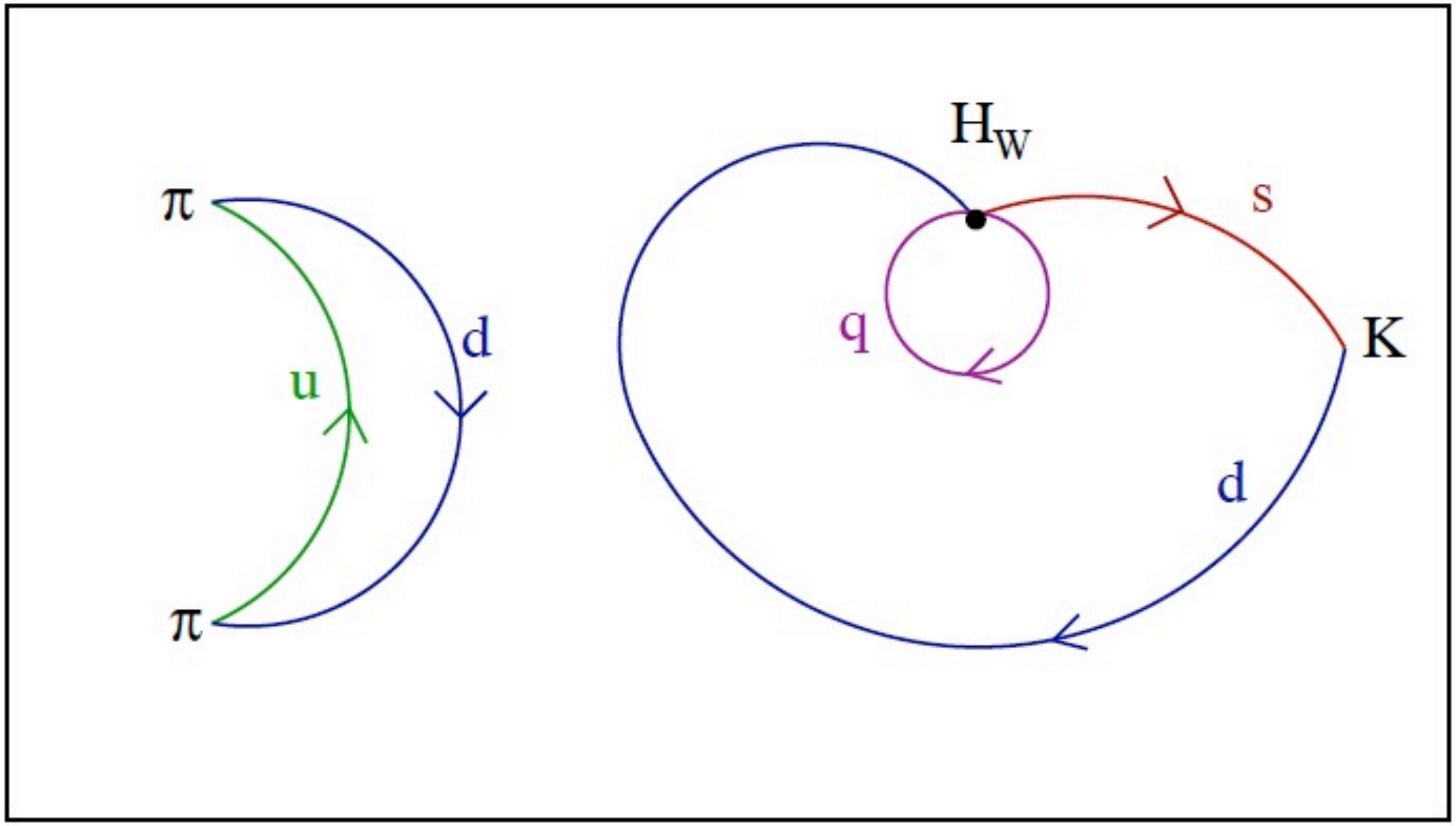


$$\Rightarrow \quad {}_L\langle \pi\pi | \mathcal{H}_W | K \rangle_L$$

- Many Wick contractions: disconnected ones particularly challenging, but distillation and diluted noise appear to make problem tractable [RBC 2012]

Ingredients for K→ππ

- Key theoretical issues arise from use of finite volume
  - Two-particle states have discrete spectrum: need to choose L so $E_{\pi\pi} = M_K$ (L~6fm, unless use clever BC)

$$\Rightarrow \quad {}_L\langle \pi\pi | \mathcal{H}_W | K \rangle_L$$

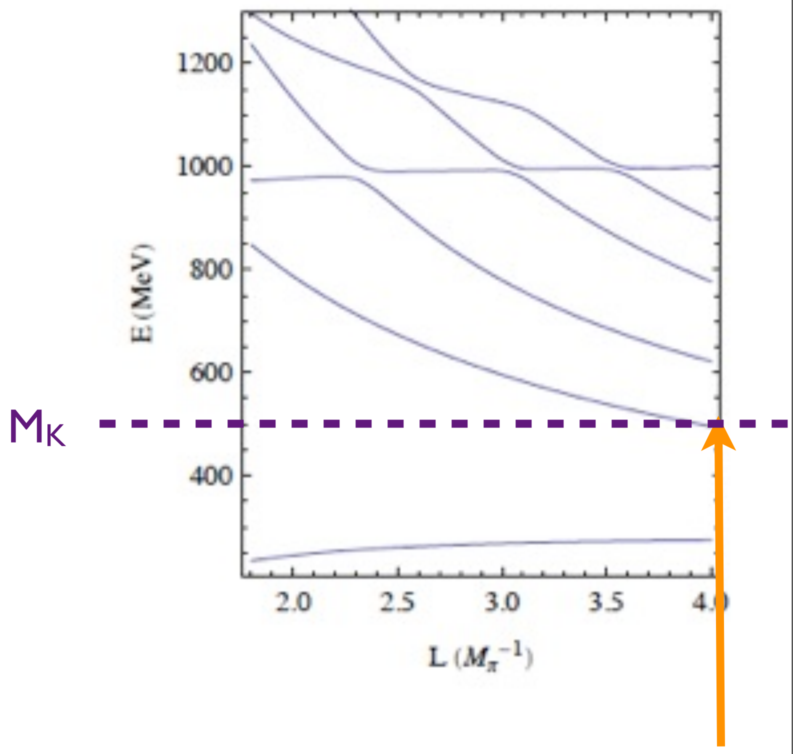
  - Two-particle states distorted significantly compared to "out-states" of QFT

$$_L\langle \pi\pi| = c_0 \langle \pi\pi(\ell=0), \mathrm{out}| + c_4 \langle \pi\pi(\ell=4), \mathrm{out}| + \ldots$$

  - Combining methods of Luscher and Lellouch & Luscher one can determine c_0 and $\delta(M_K)$ from E(L) & dE/dL:

$$\langle \pi\pi(\ell=0), \mathrm{out}|\mathcal{H}_W|K\rangle = \frac{e^{i\delta(M_K)}}{c_0}\, {}_L\langle \pi\pi|\mathcal{H}_W|K\rangle_L$$

  - Successful calculation for easier I=2 case; pilot calculation for I=0 [RBC 2012]

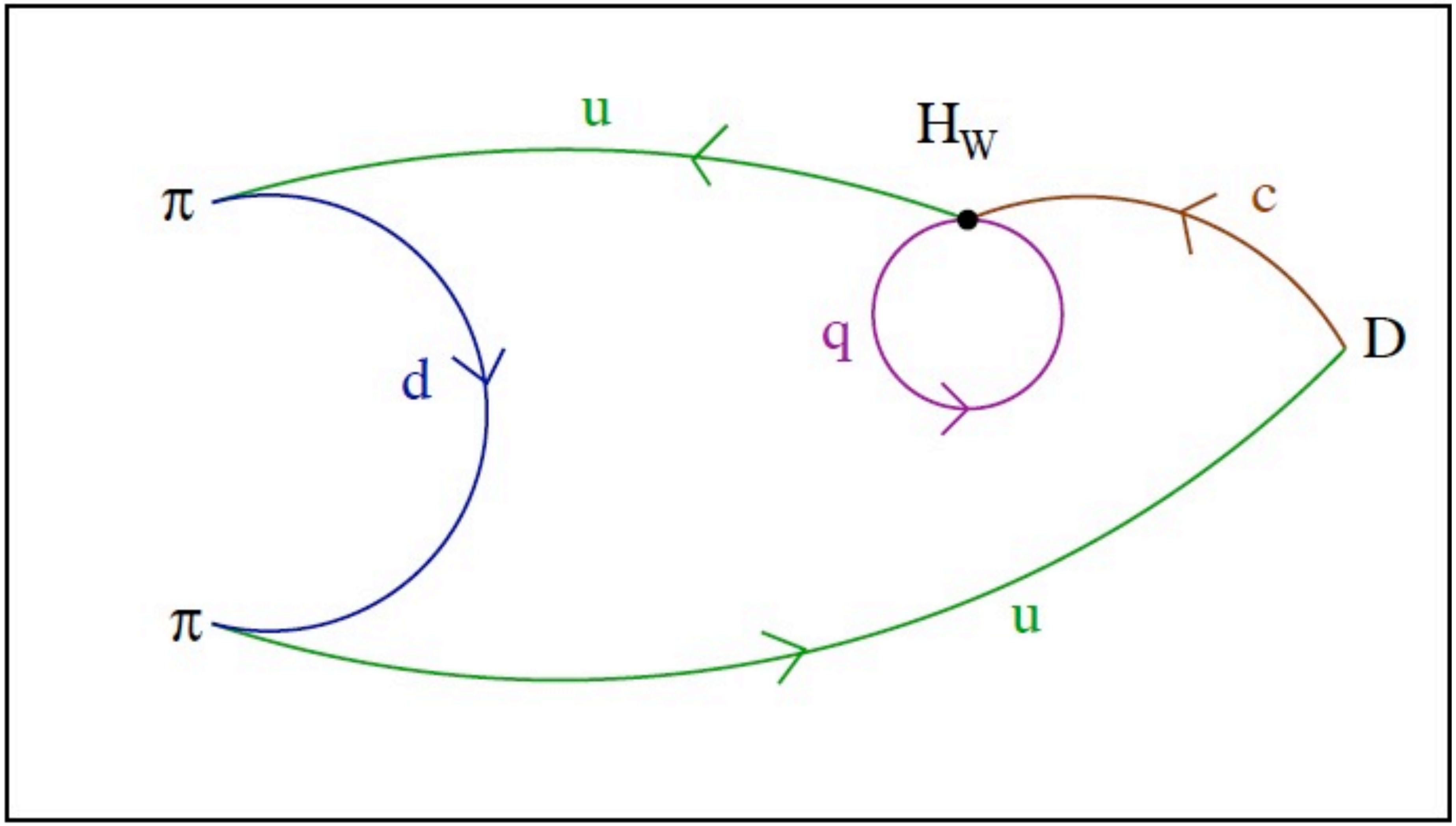


P=0, I=0 spectrum from UChPT with PBC (ignoring 4π etc.) [Hansen, Lat2012]

Generalize to D→ππ,KK?

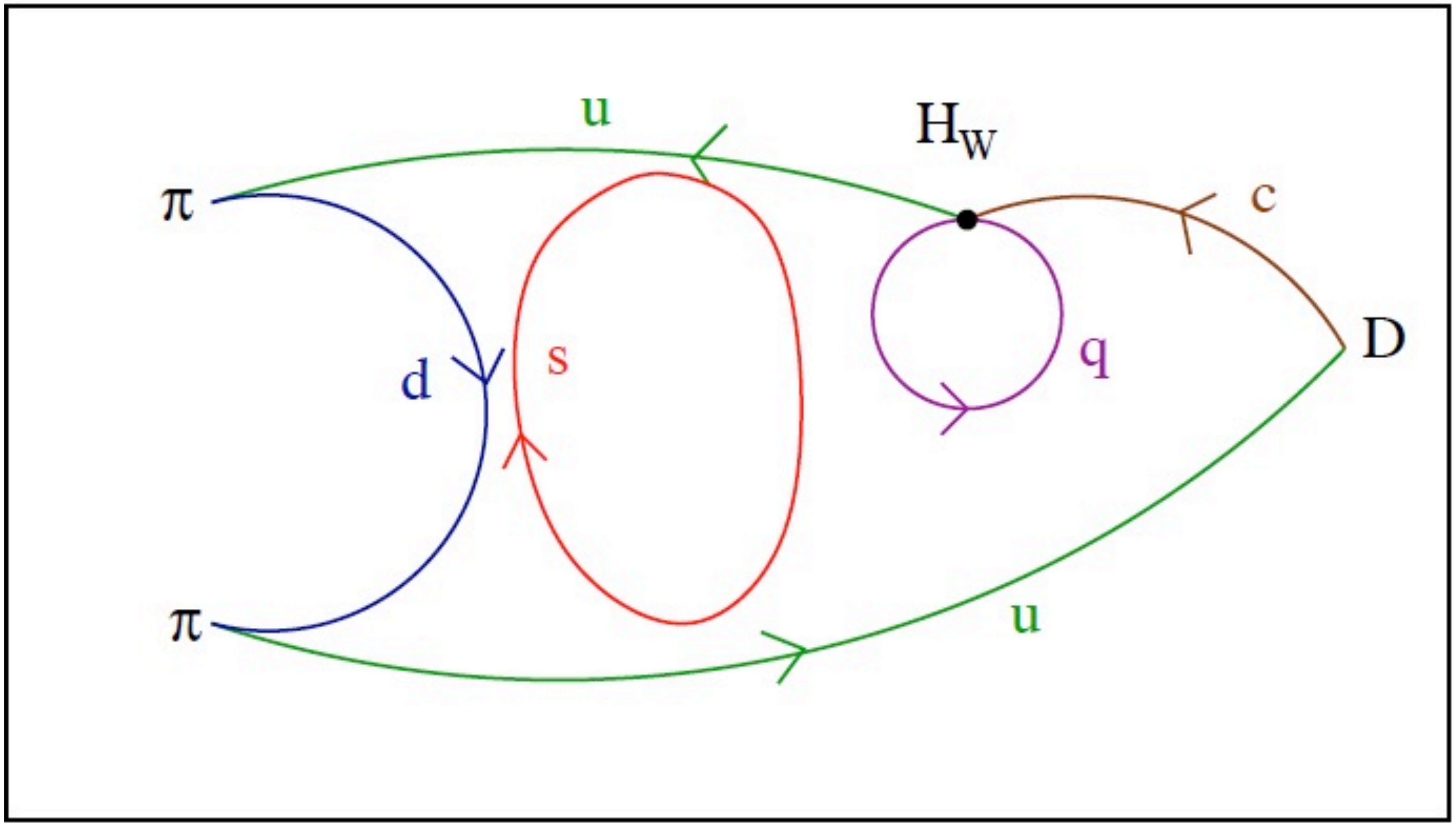
- LHCb recently presented evidence for CP-violation in D→ππ, KK decays
  - Larger rate than (naively) expected in SM, but large hadronic uncertainties in estimates
  - Is a LQCD calculation possible?

Similar to K→ππ?



$$\Rightarrow \quad {}_L\langle n|\mathcal{H}_W|D\rangle_L$$

Key problem: inelasticity



- Even if create two-particle state with 2 pion operator, strong interactions will mix it with K-antiK, with 4 pions, etc
- FV states are mixtures!

Ingredients for D→ππ,KK

- Need to generalize Luscher's quantization formula and Lellouch & Luscher's relation between finite and infinite volume states to multiple channels
    - Possible if keep only two-particle channels (ππ, KK, ηη) [Bernard, Lage, Meissner & Rusetsky, Liu et al., Lage et al., Doring et al., Aoki et al., Hansen & SS, Briceno & Davoudi]
    - Not yet generalized to include 4π, 6π, etc.
- Completing the theoretical framework and carrying out a numerical calculation are challenging and very interesting future problems
- Other related interesting (and challenging) applications, e.g.

$$\Omega^- \to \Lambda K^-,\ \Xi^0\pi^-,\ \Xi^-\pi^0$$

Selected References

- M. Luscher, Comm. Math. Phys. 104 (1986) 177, ibid. 105 (1986) 153
- L. Lellouch and M. Luscher, Comm. Math. Phys. 219 (2001) 31
- V. Bernard et al., "Scalar Mesons in a Finite Volume," arXiv:1010.6018
- M. Doring et al., "Unitarized Chiral Perturbation Theory in a finite volume: Scalar meson sector," arXiv:1107.3988
- M.T. Hansen and S.R. Sharpe, "Multiple-channel generalization of Lellouch-Luscher formula," arXiv:1204.0826
- R.A. Briceno and Z. Davoudi, "Moving Multi-Channel Systems in a Finite Volume," arXiv:1204.1110
- T. Blum et al., "The K→ππ (I=2) Decay Amplitude from Lattice QCD," arXiv:1111.1699
- T. Blum et al., "K to ππ Decay Amplitudes from Lattice QCD," arXiv:1108.2714

Pushing the Euclidean frontier

- How can we use LQCD to calculate two-photon decays of hadrons?
  - $\pi^0 \to \gamma\gamma$: Predicted by ABJ anomaly; test of chiral symmetry of lattice fermion formulation
  - $\pi^0 \to \gamma^* \gamma^*$: Needed as part of model for "light-by-light" contributions to muonic g-2
  - $\eta \to \gamma\gamma$, $\eta' \to \gamma\gamma$, $a_0 \to \gamma\gamma$: test LQCD calculations in new domain
  - $\eta_c \to \gamma\gamma$, $\chi_c \to \gamma\gamma$: shed light on models for charmonium states
- Problem: photons are not strong-interaction eigenstates
- Solution [Ji & Jung]: integrate out photons by hand (in perturbation theory) & perform analytic continuation to obtain a (weighted) Euclidean correlator
- Appears practical with modern methods, and results obtained by [Dudek & Roberts; Cohen, Lin, Dudek & Richards; Feng et al.]

How does this work?

- Use LSZ reduction in Minkowski space

$$\langle\gamma(q_1,\lambda_1)\gamma(q_2,\lambda_2)|M(p)\rangle = -\lim_{\substack{q_1'\to q_1\\ q_2'\to q_2}} \epsilon^*_\mu(q_1,\lambda_1)\epsilon^*_\nu(q_2,\lambda_2)q_1'^2 q_2'^2 \int d^4x d^4y e^{iq_1'\cdot y+iq_2'\cdot x}\langle 0|T\{A^\mu(y)A^\nu(x)\}|M(p)\rangle,$$

- Use leading order QED perturbation theory to rewrite in terms of

$j_\mu = \sum_f Q_f \bar{q}_f \gamma_\mu q_f$ QCD EM current

$$M_{\mu\nu}(p_1,p_2) = i\int d^4x\, e^{ip_1 x}\langle\Omega|T\{j_\mu(x)j_\nu(0)\}|\pi^0(q)\rangle$$

[Apologies: q₁ has become p₁ !]

$$= \varepsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta \mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2)$$

off-shell photon amplitude

- Relation to decay rate $\Gamma_{\pi^0\gamma\gamma} = \frac{\pi\alpha_e^2 m_\pi^3}{4}\mathcal{F}^2_{\pi^0\gamma\gamma}(m_\pi^2,0,0)$

- Anomaly/Chiral PT prediction: $\mathcal{F}_{\pi^0\gamma\gamma}(0,0,0) = \frac{1}{4\pi^2 F_0}$

How does this work?

- Analytically continue from Minkowski to Euclidean time

$$M_{\mu\nu}(p_1,p_2) \;=\; i\int d^4x\; e^{ip_1x}\langle\Omega|T\{j_\mu(x)j_\nu(0)\}|\pi^0(q)\rangle$$

$$\downarrow$$

pick out pion

grows for $t_1 > t_2$!

$$M_{\mu\nu}(p_1,p_2) = \lim_{t_{1,2}-t_\pi\to\infty} \frac{1}{\frac{\phi_{\pi,\vec{q}}}{2E_{\pi,\vec{q}}}e^{-E_{\pi,\vec{q}}(t_2-t_\pi)}} \int dt_1\; e^{\omega(t_1-t_2)}C_{\mu\nu}(t_1,t_2,t_\pi)\;,$$

$$\boxed{C_{\mu\nu}(t_1,t_2,t_\pi) \equiv \int d^3\vec{x}\; e^{-i\vec{p}_1\cdot\vec{x}}\int d^3\vec{z}\; e^{i\vec{q}\cdot\vec{z}}\langle\Omega|T\{j_\mu(\vec{x},t_1)j_\nu(\vec{y},t_2)\pi^0(\vec{z},t_\pi)\}|\Omega\rangle}$$

$$\uparrow$$

Something LQCD can calculate!

How does this work?

$$M_{\mu\nu}(p_1,p_2) = \lim_{t_{1,2}-t_\pi\to\infty} \frac{1}{\frac{\phi_{\pi,\vec{q}}}{2E_{\pi,\vec{q}}} e^{-E_{\pi,\vec{q}}(t_2-t_\pi)}} \int dt_1\, e^{\omega(t_1-t_2)} C_{\mu\nu}(t_1,t_2,t_\pi)\,,$$

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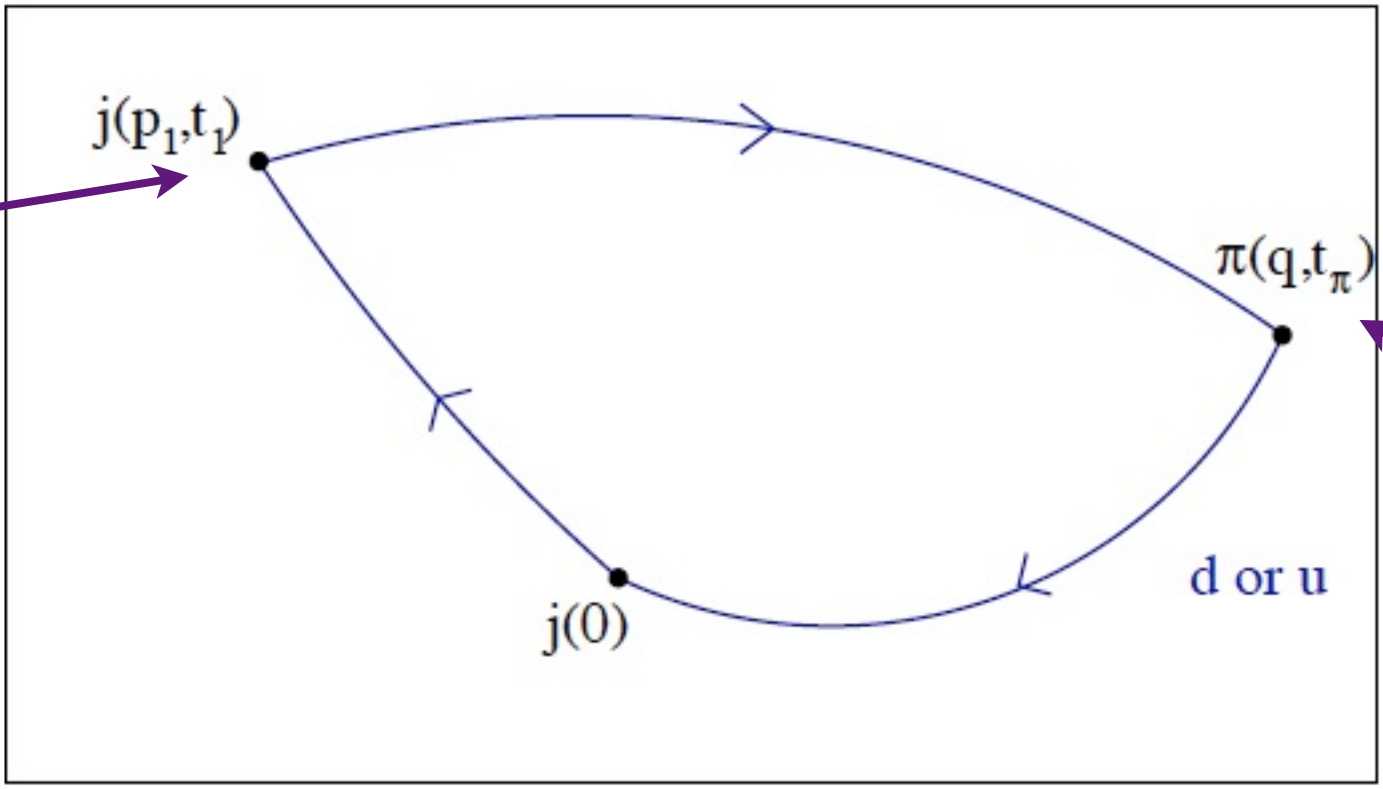


Sum over t_l multiplying by $\exp(\omega\ t_l)$

Sum over t_π multiplying by $\exp(-E_\pi\ t_\pi)$

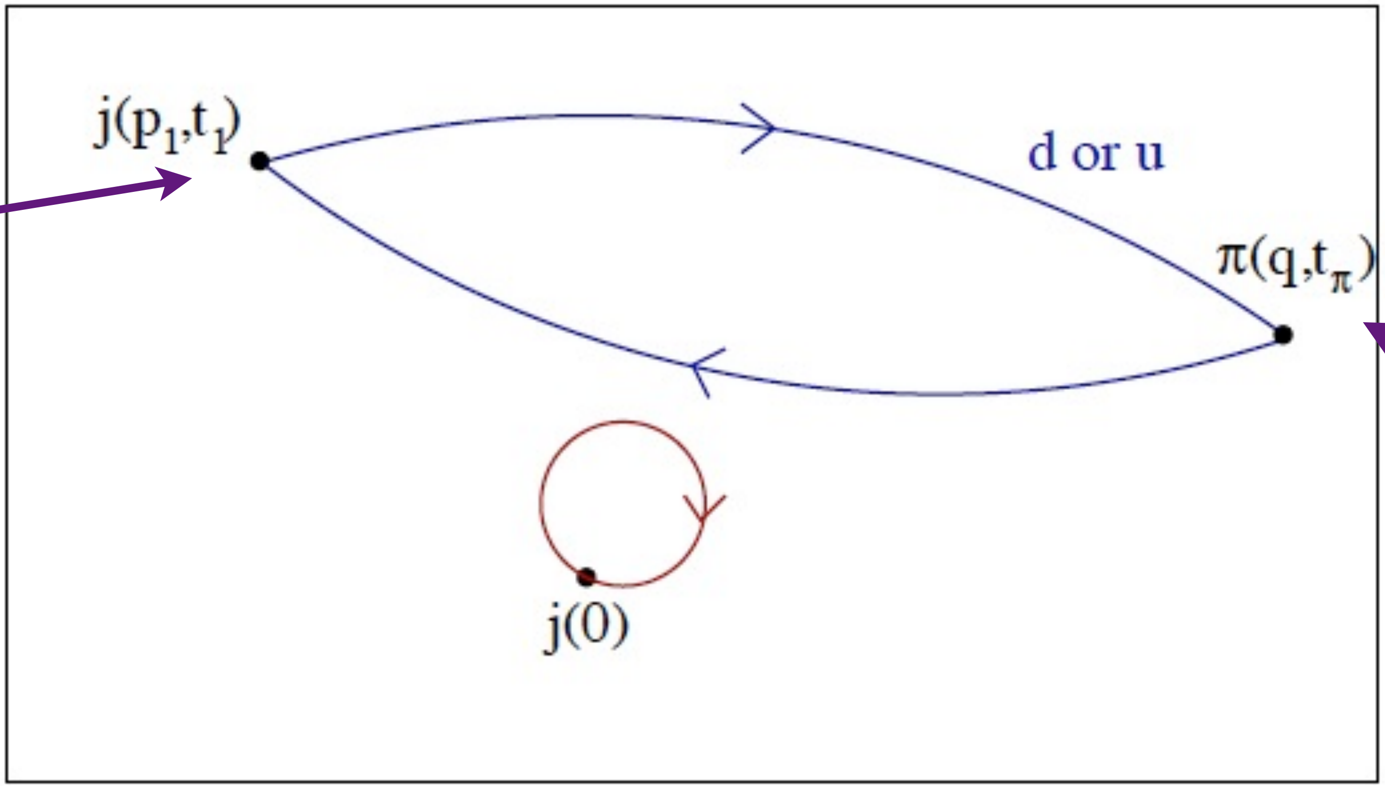
← increasing Euclidean time

- Valid for $\omega < E(\pi\pi)$
- Works in practice

How does this work?

$$M_{\mu\nu}(p_1,p_2) = \lim_{t_{1,2}-t_\pi\to\infty} \frac{1}{\frac{\phi_{\pi,\vec{q}}}{2E_{\pi,\vec{q}}} e^{-E_{\pi,\vec{q}}(t_2-t_\pi)}} \int dt_1\; e^{\omega(t_1-t_2)} C_{\mu\nu}(t_1,t_2,t_\pi)\;,$$

$$C_{\mu\nu}(t_1,t_2,t_\pi) \equiv \int d^3\vec{x}\; e^{-i\vec{p}_1\cdot\vec{x}} \int d^3\vec{z}\; e^{i\vec{q}\cdot\vec{z}} \langle\Omega|T\{j_\mu(\vec{x},t_1)j_\nu(\vec{y},t_2)\pi^0(\vec{z},t_\pi)\}|\Omega\rangle$$



Sum over t_l multiplying by $\exp(\omega\, t_l)$

Sum over t_π multiplying by $\exp(-E_\pi\, t_\pi)$

← increasing Euclidean time

- Disconnected contractions harder, but make small contribution and can be handled with distillation etc.

Results

- Only unquenched calculation to date is for $\pi^0 \rightarrow \gamma\gamma$ by [Feng et al.]
    - Use overlap fermions and (as expected) reproduce anomaly prediction to 2% accuracy
    - Important test of new method

Selected References

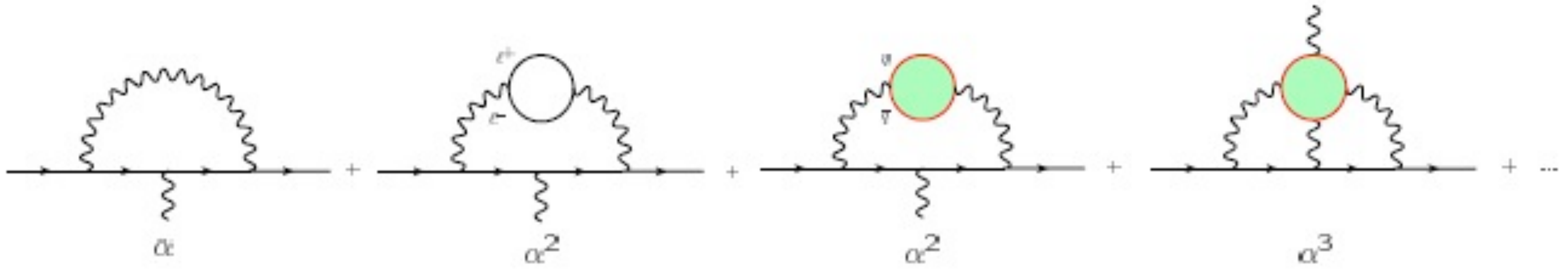
- X. Ji & C. Jung, "Studying Hadronic Structure of the Photon in Lattice QCD," hep-lat/0101014
- J.J. Dudek & R.G. Edwards, "Two-Photon Decays of Charmonia from Lattice QCD," hep-ph/0607140
- S.D. Cohen, H.-W. Lin, J. Dudek & R.G. Edwards, "Light-Meson Two-Photon Decays in Full QCD," arXiv:0810.5550
- X. Feng et al., "Two-photon decay of the neutral pion in lattice QCD," arXiv:1206.1375

Hadronic contributions to g-2

Good reference: [T. Blum, plenary talk at Lattice 2012]

- Magnetic moment of muon is proportional to its spin

$$\vec{\mu} = g\left(\frac{e}{2m}\right)\vec{S} \qquad g = 2 + \frac{a_\mu}{2} \qquad a_\mu = \frac{\alpha_{EM}}{2\pi} + \ldots$$



Schwinger

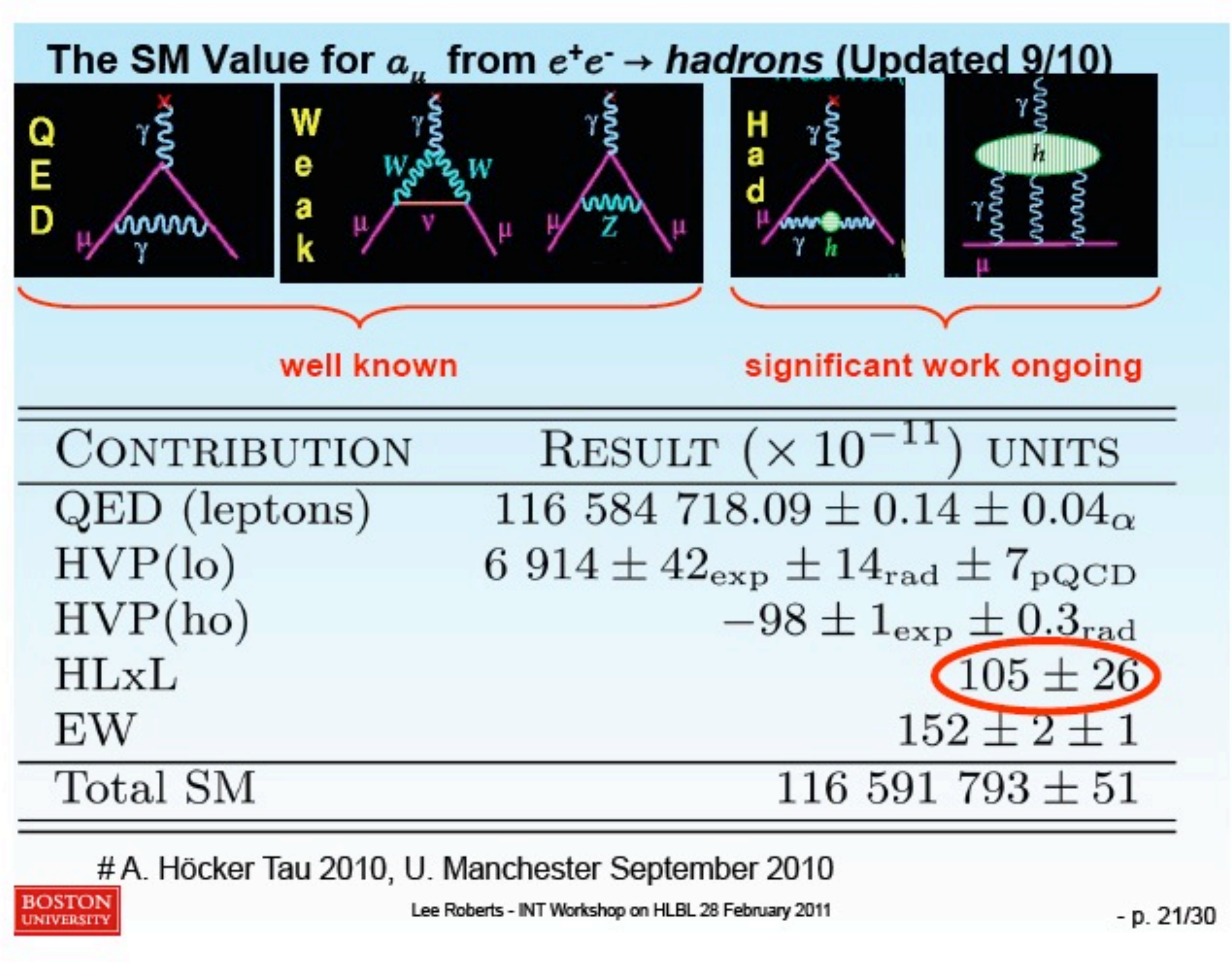
Higher order
QED

hadronic vacuum
polarization

hadronic
light-by-light
scattering

- Present & proposed experimental accuracy requires calculation of both hadronic contributions
- Vacuum pol. can be obtained from experiment, with LQCD starting to compete
- Light-by-light requires non-perturbative methods

Theory vs. experiment



The SM Value for a_μ from $e^+e^- \to$ *hadrons* (Updated 9/10)

well known

significant work ongoing

CONTRIBUTION	RESULT ($\times 10^{-11}$) UNITS
QED (leptons)	$116\ 584\ 718.09 \pm 0.14 \pm 0.04_\alpha$
HVP(lo)	$6\ 914 \pm 42_{\text{exp}} \pm 14_{\text{rad}} \pm 7_{\text{pQCD}}$
HVP(ho)	$-98 \pm 1_{\text{exp}} \pm 0.3_{\text{rad}}$
HLxL	$\boxed{105 \pm 26}$
EW	$152 \pm 2 \pm 1$
Total SM	$116\ 591\ 793 \pm 51$

A. Höcker Tau 2010, U. Manchester September 2010

Lee Roberts - INT Workshop on HLBL 28 February 2011

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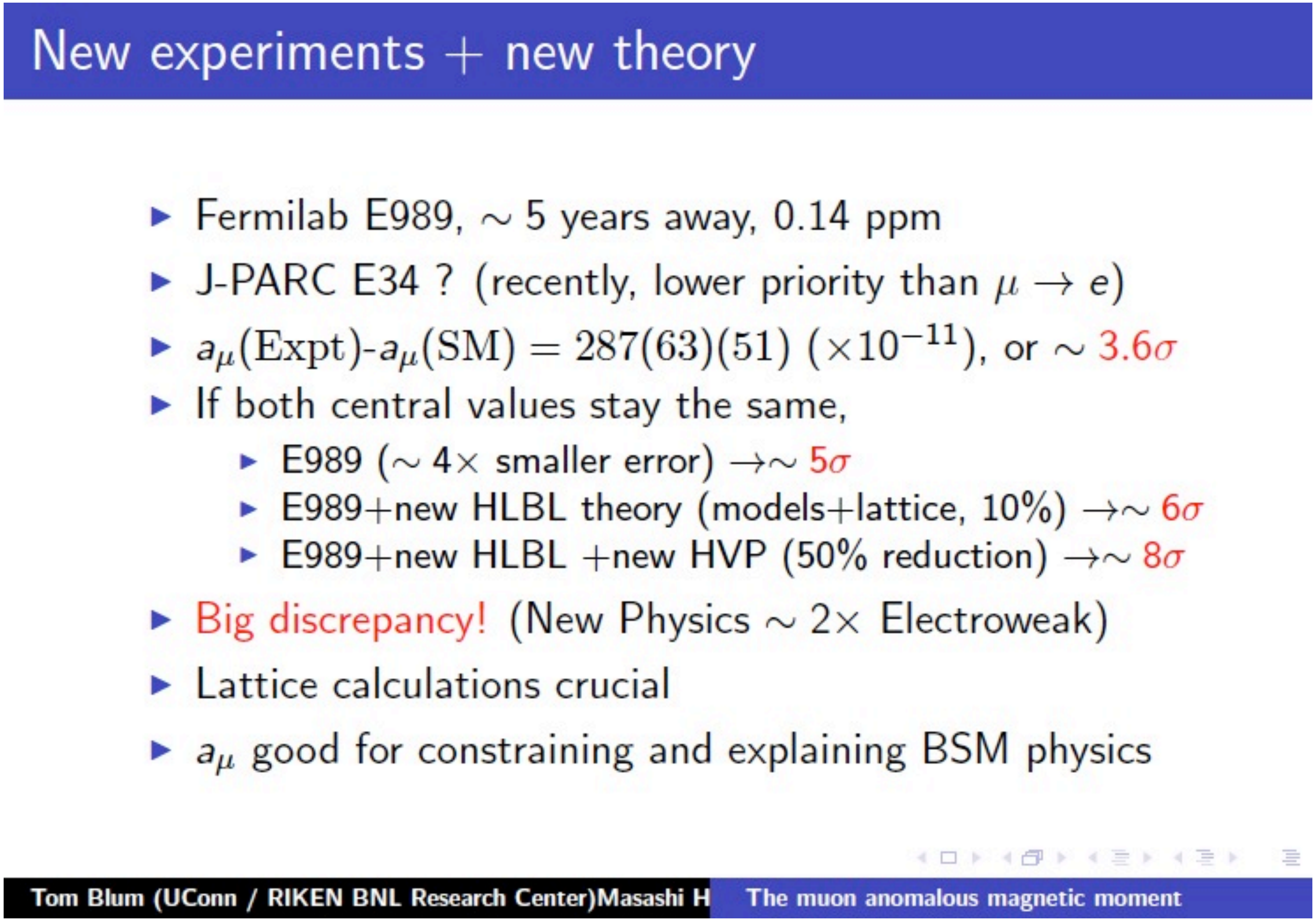
$$a_\mu^{exp} = 116\,592\,089(63) \times 10^{-11} \ (0.54\ \text{ppm})$$
$$\Delta a_\mu \equiv a_\mu^{exp} - a_\mu^{SM} = (287 \pm 80) \times 10^{-11}$$

Difference is ~3.6 σ

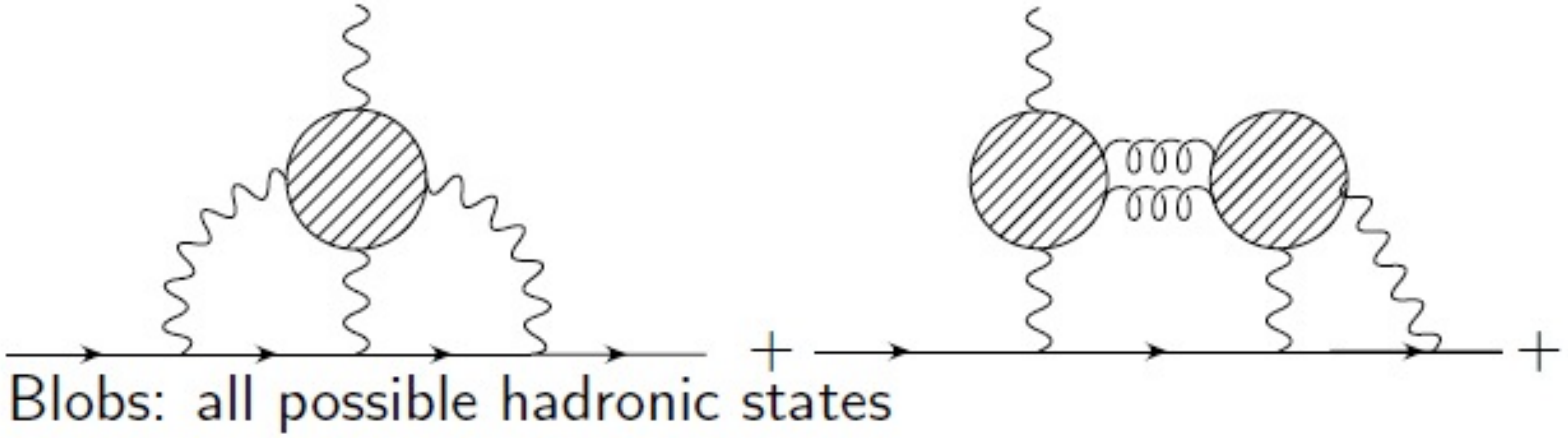
Theory vs. experiment



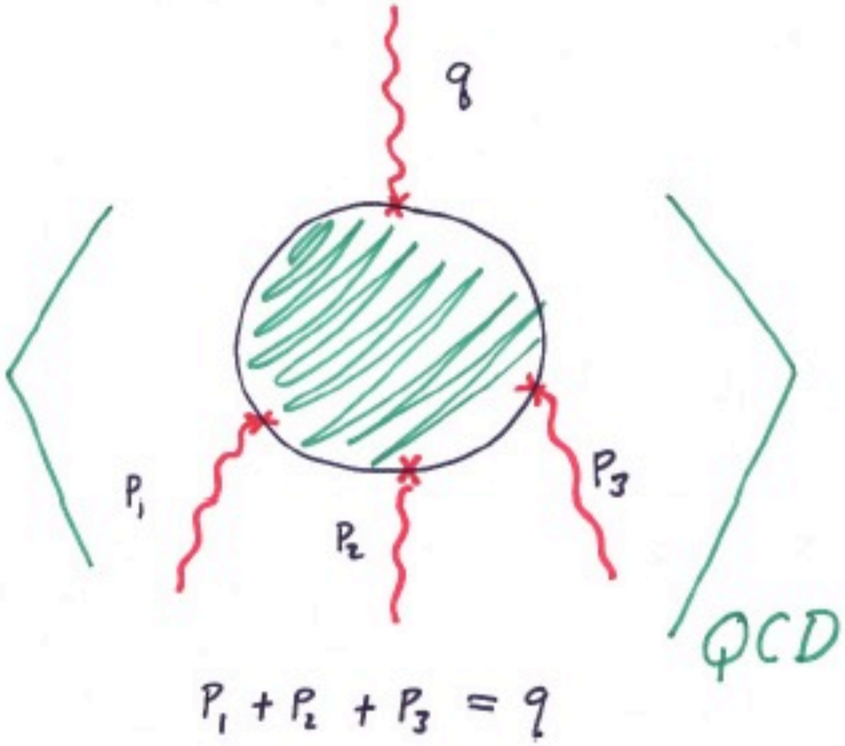
New experiments + new theory

- Fermilab E989, ~ 5 years away, 0.14 ppm
- J-PARC E34 ? (recently, lower priority than $\mu \to e$)
- a_μ(Expt)-a_μ(SM) $= 287(63)(51)$ $(\times 10^{-11})$, or ~ 3.6σ
- If both central values stay the same,
  - E989 (~ 4× smaller error) →~ 5σ
  - E989+new HLBL theory (models+lattice, 10%) →~ 6σ
  - E989+new HLBL +new HVP (50% reduction) →~ 8σ
- Big discrepancy! (New Physics ~ 2× Electroweak)
- Lattice calculations crucial
- a_μ good for constraining and explaining BSM physics

Hadronic light-by-light contribution



Conventional approach



Correlation of 4 EM currents $\Pi^{\mu\nu\rho\sigma}(q, p_1, p_2)$

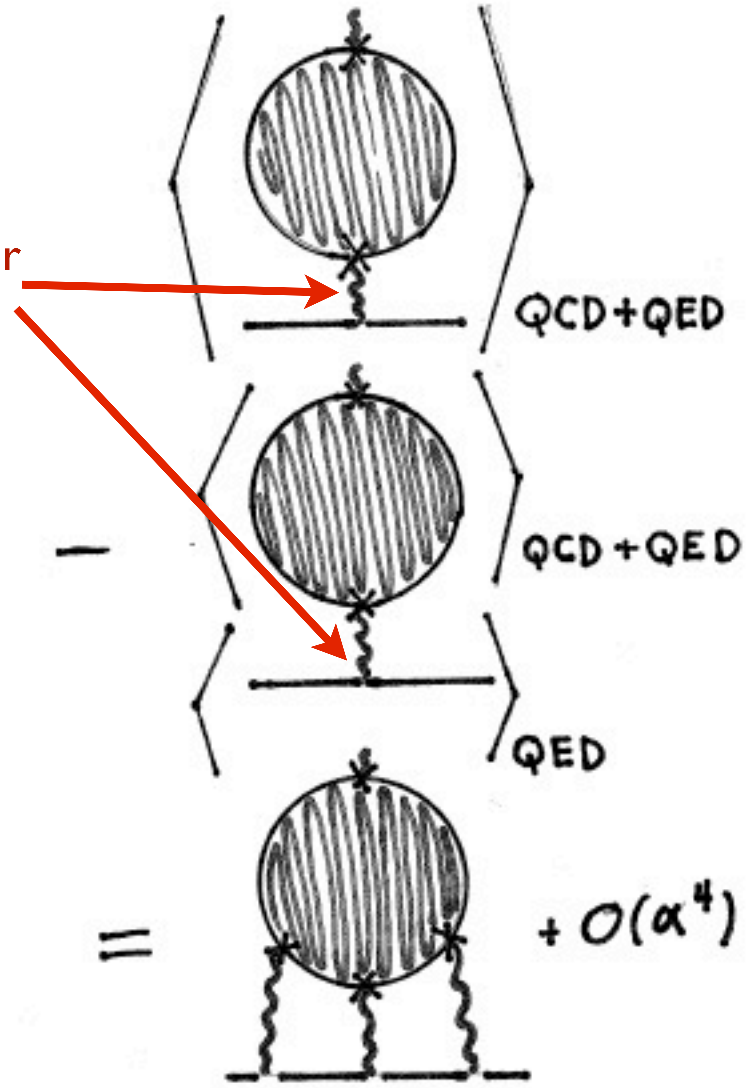
Two independent momenta +external mom q

Compute for all possible values of p_1 and p_2, $(O(V^2))$ four index tensor (32 Lorentz structures for g-2!)

several q,(extrap $q \to 0$), fit, plug into perturbative QED two-loop integrals

Hadronic light-by-light contribution

New approach [Blum et al.]



Photon propagator added by hand

Subtraction term is product of separate averages of the loop and line

Gauge configurations identical in both, so two are highly correlated

In PT, correlation function and subtraction have same contributions except the light-by-light term which is absent in the subtraction

First results look promising---but ~5 year project

Outline

- Summary of present status
  - ✳ Focus on weak matrix elements
- Future directions and challenges
  - ✳ K→ππ decays (ΔI=1/2 rule & CP violation) & ΔM_K
  - ✳ D→ππ, KK-bar---is a lattice calculation possible?
  - ✳ $\pi^0 \to \gamma\gamma$ (pushing the limits of Euclidean-space calculations)
  - ✳ Predicting hadronic contributions to muonic g-2
- **Outlook**

Outlook

- ★ LQCD is a mature method when applied single, stable hadrons
- ★ LQCD plays an essential role in the search for new physics at the intensity frontier
- ★ Many opportunities to extend this role both by improving "gold-plated" calculations and by generalizing methods to new quantities (requiring theoretical, algorithmic and computational work)
- ★ Many other new directions relevant to high-energy physics not discussed here
  - Including isospin breaking and QED effects [see Taku Izubuchi's lectures]
  - Using LQCD to test & determine constants in chiral perturbation theory [see Brian Tiburzi's lectures]
  - Using LQCD as a "sand box" to understand physics of confinement
  - Studying nearly conformal QCD-like theories as potential models of dynamical electroweak symmetry breaking ("walking technicolor")
  - Extending to large N_c to make contact with AdS/QCD approaches---possibly using Eguchi-Kawai reduction so as to allow one to work on a single site
- ★ I look forward to your exciting contributions in the years to come!