Lattice Methods for Hadron Spectroscopy: new problems and challenges

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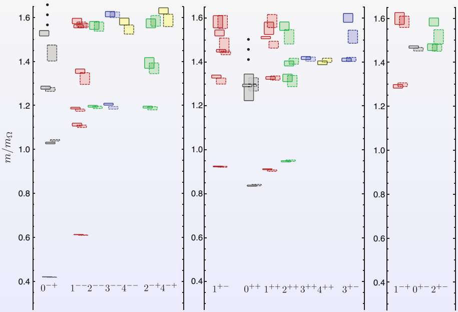
Plan

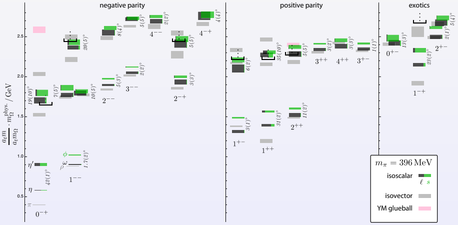
- A look at the spectrum of single-particle states
- Resonances
- Two particles in a box
- Accessing resonance information from finite volume calculations
- Toy models
- Recent simulations
- Spectrosopy at finite temperature
- Summary

The Lattice Hadron Frontier

The spectrum of light states

- Using the technology we have discussed the (light) spectrum of mesons and baryons can be determined precisely.





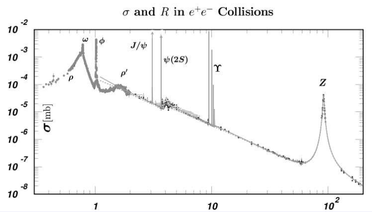
Is this everything?

Resonances and scattering states

- We have assumed that all particles in the spectrum are stable
- Many (the majority) are not.
- A resonance is a state that forms eg when colliding two particles and then decays quickly to scattering states.
- They respect conservation laws: if isospin of the colliding particles is 3/2, resonance must have isospin 3/2 (Δ resonance)
- Usually indicated by a sharp peak in a cross section as a function of c.o.m. energy of the collision.
- Can lattice qcd distinguish resonances and scattering states?

Resonances in $e^+e^- \to$ hadrons

$\sigma_{ee\to\text{hadrons}}(E_{ee})$

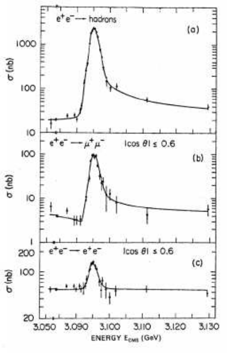


- Note the more or less sharp resonances on a comparably flat "continuum", coming from $e^+e^- \to q\bar{q}$

(We will discuss this in more detail!)

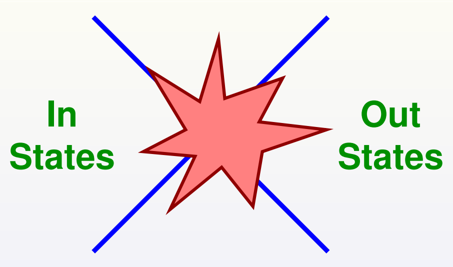
- They are (apart from the Z) all related to $q\bar{q}$-bound states.

Zoom into J/Ψ



- Note: Here width around 3 MeV completely determined by detector ($\Gamma_{J/\Psi} = 87$ keV)

Maiani-Testa no-go theorem

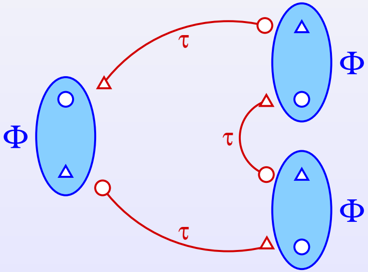


- Importance sampling Monte-carlo simulations rely on a path integral with positive definite probability measure: Euclidean space
- Maiani-Testa: scattering matrix (S-matrix) elements cannot be extracted from infinite-volume Euclidean-space correlation functions (except at threshold)

Michael 1989 and Maiani, Testa (1990)

Maiani-Testa (2)

- Can understand this since:
  - Minkowski space: S-matrix elements complex functions above kinematic thresholds
  - Euclidean space: S-matrix elements are real for all kinematics - phase information lost
- Lattice simulations with dynamical fermions admit strong decays eg for light-enough up and down dynamical quarks $\rho \to \pi\pi$



Maiani-Testa (3)

- Can the lattice get around the no-go theorem to extract the masses and widths of such unstable particles?
- Yes - use the finite volume.

For elastic two-body resonances (Lüscher): $M_1 M_2 \to R \to M_1 M_2$

$\longrightarrow$ Volume dependence of energy spectrum

$\longrightarrow$ Phase shift in infinite volume

$\longrightarrow$ Mass and width of resonance

Note: This is a rapidly developing field. I will add some refs for recent work or see Lattice Conf talks.

Particles in a box

- Spatial lattice of extent L with periodic boundary conditions
- Allowed momenta are quantized: $p = \frac{2\pi}{L}(n_x, n_y, n_z)$ with $n_i \in \{0, 1, 2, \ldots L-1\}$
- Energy spectrum is a set of **discrete** levels, classified by p: Allowed energies of a particle of mass m

$$E = \sqrt{m^2 + \left(\frac{2\pi}{L}\right)^2 N^2} \quad \text{with } N^2 = n_x^2 + n_y^2 + n_z^2$$

- Can make states with **zero total momentum** from pairs of hadrons with momenta $p, -p$.
- "Density of states" **increases** with energy since there are more ways to make a particular value of N^2 e.g. $\{3, 0, 0\}$ and $\{2, 2, 1\} \to N^2 = 9$

Avoided level crossings

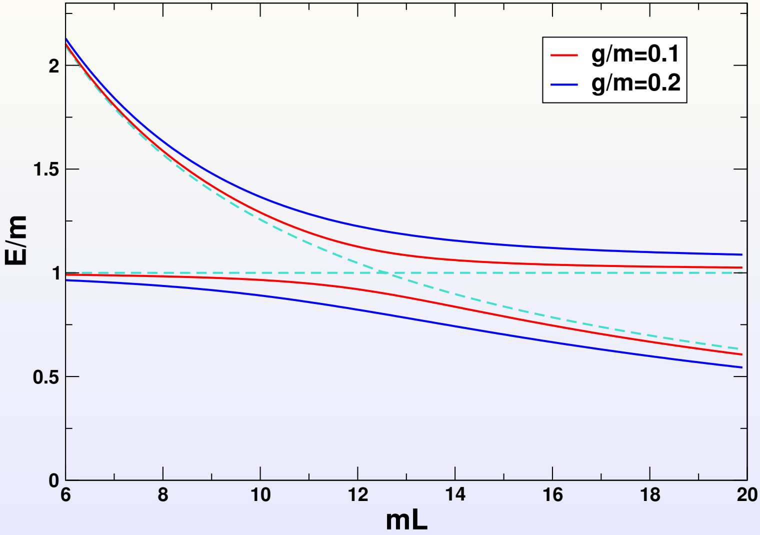
- Consider a toy model with two states (a resonance and a two-particle decay mode) in a box of side-length L
- Write a mixing hamiltonian:

$$H = \begin{pmatrix} m & g \\ g & \frac{4\pi}{L} \end{pmatrix}$$

- Now the energy eigenvalues of this hamiltonian are given by

$$E_{\pm} = \frac{(m + \frac{4\pi}{L}) \pm \sqrt{(m - \frac{4\pi}{L})^2 + 4g^2}}{2}$$

Avoided level crossings



Avoided level crossings

- **Ground-state** smoothly changes from resonance to two-particle state
- Need a large box. This example, levels cross at $mL = 4\pi \approx 12.6$
- Example: $m = 1$ GeV state, decaying to two massless pions - avoided level crossing is at $L = 2.5$fm.
- If the decay product pions have $m_\pi = 300$ MeV, this increases to $L = 3.1$fm

Lüscher's method

- Relates the spectrum in a finite box to the scattering phase shift (and so resonance properties)

Lüscher's formula

$$\delta(p) = -\phi(\kappa) + \pi n$$

$$\tan\phi(\kappa) = \frac{\pi^{3/2}\kappa}{Z_{00}(1;\kappa^2)}$$

$$\kappa = \frac{pL}{2\pi}$$

- p_n is defined for level n with energy E_n from the dispersion relation:

$$E_n = 2\sqrt{m^2 + p_n^2}$$

Lüscher's method

- Z_{00} is a generalised Zeta function:

$$Z_{js}(1, q^2) = \sum_{n \in Z^3} \frac{r^j Y_{js}(\theta, \phi)}{(n^2 - q^2)^s}$$

[Commun.Math.Phys.105:153-188,1986.]

- With the phase shift, and for a well-defined resonance, can fit a Breit-Wigner to extract the **resonance width** and **mass**.

$$\delta(p) \approx \tan^{-1}\left(\frac{4p^2 + 4m_\pi^2 - m_\sigma^2}{m_\sigma \Gamma \sigma}\right)$$

Lüscher (3): considering $\rho \to \pi\pi$

- For non-interacting pions, the energy levels of a 2 pion system in a periodic box of length L are

$$E = 2\sqrt{m_\pi^2 + p^2} \;\; p = 2\pi|\vec{n}|/L$$

and $\vec{n}$ has components $n_i \in \mathbb{N}$.

- In the interacting case the energy levels are shifted

$$E = 2\sqrt{m_\pi^2 + p^2} \;\; p = (2\pi/L)q$$

where q is no longer constrained to orginate from a quantised momentum mode.

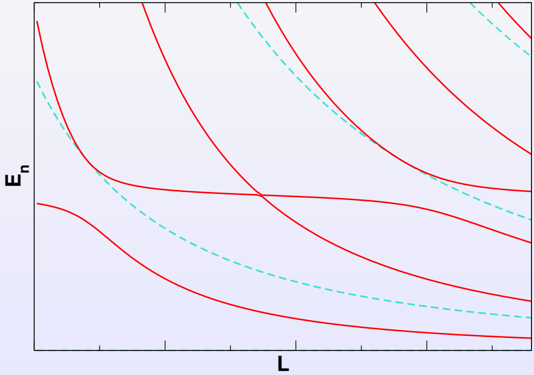
- In the presence of the interaction, energy eigenvalues deviate from the noninteracting case
- These deviations contain the information on the underlying strong interaction - yielding resonance information via Luscher formulism.

Schrödinger equation

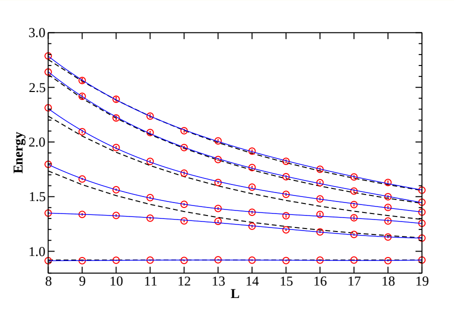
Exercise: find the phase shift for a 1-d potential

$$V(x) = V_0\delta(x-a) + V_0\delta(x+a)$$

- Now compute the spectrum in a finite box and use Lüscher's method to compare the two



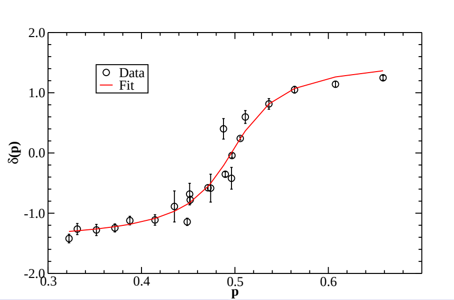
Test: O(4) Sigma model



M.Peardon and P. Giudice

Spectrum of $O(4)$ model in broken phase

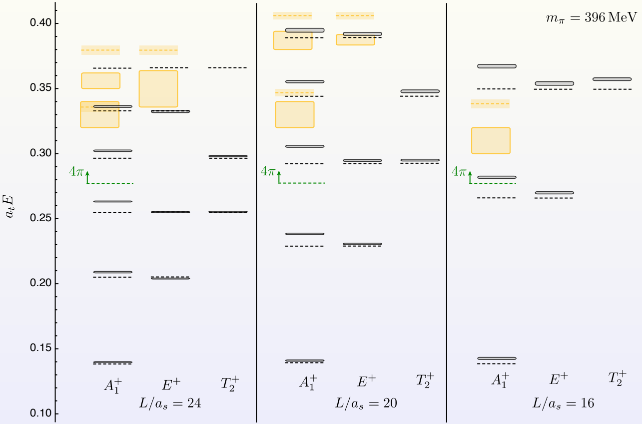
Phase shift inferred from Lüscher's method



$\rho \longrightarrow \pi\pi$

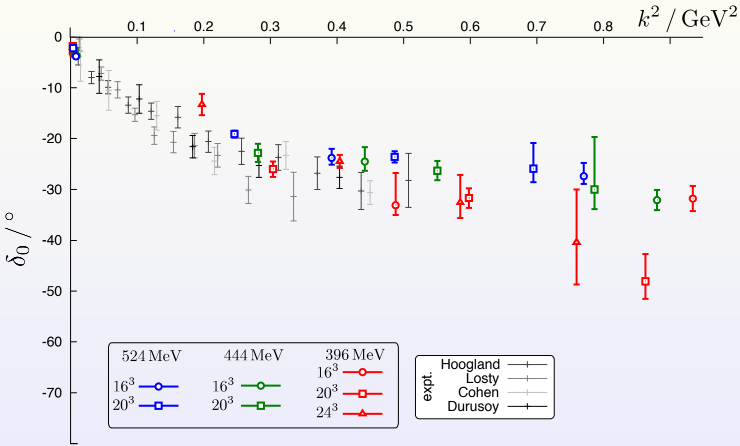
- In the real world $rho \rightarrow \pi\pi$ is in isospin $I = 1$
- This involves disconnected diagrams which is already a complication - although in principle doable.
- Start with an “easier” system, $I = 2\pi\pi$ and test methods there.
- $I = 1$ case is now studied (distillation has helped a lot here)

I=2 $\pi\pi$ scattering



- Resolve shifts in masses away from non-interacting values
- Orange boxes: possible $\pi\pi^*$ scattering states
- Dashed lines: non-interacting pion pairs

I=2 $\pi\pi$ scattering



- Non-resonant scattering in S-wave - compares well with experimental data

The inelastic threshold

- Lüscher's method is based on **elastic** scattering.
- Since m_π is small, most resonances are above this threshold
- Not clear how to proceed - perhaps a histogram approach will help us gain some expertise
- It will be crucial to ensure we have a comprehensive **basis of operators that create multi-hadron states**.

Summary: Measuring energies and widths

Requirements for measuring decay widths in QCD

- Light, dynamical quarks
- Accurate spectroscopy in appropriate channels
- Simulations in multiple box sizes (and/or momenta)
- Access to excited states in these channels
- Ability to create multi-hadron states

Next Generation lattice calculations

A different frontier: finite temperature spectroscopy

Spectroscopy at finite temperature

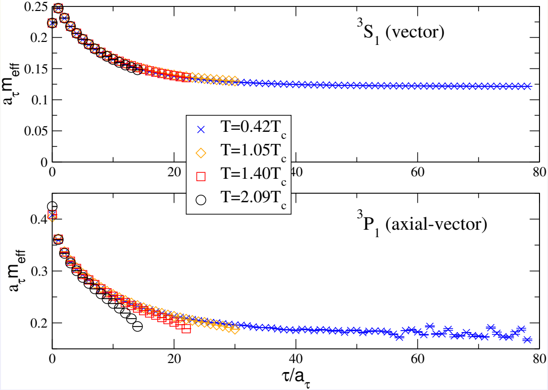
- You already learned a lot about finite temp LQCD.
- States made from heavy quarks are expected to act as a probe of dynamics of the QGP
- There are interesting results coming from RHIC and CERN for the melting and suppresion of such states.
- Can lattice say anything? It is a challenge!

- Remember, the thermal correlator is

$$C(\tau) \sim \int_0^\infty d\omega \rho(\omega, T) K(\omega, \tau, T), \quad (p = 0).$$

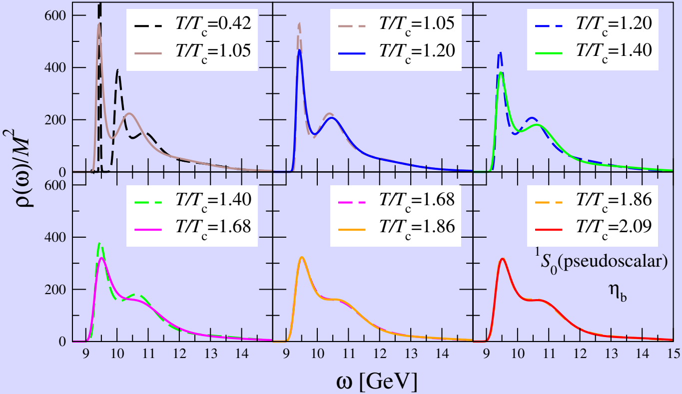
- $C(\tau)$ sampled discretely but ρ has values for continuous ω
- An ill-posed problem!
- Maximum entropy methods (MEM) can be used but can be unstable and model-dependent
- New ideas needed!

Effective masses at finite T



- Anisotropic lattices $\xi = 6$, $N_f = 2$, $a_s = 0.15$fm
- Note P wave behaviour at $T > T_c$
- Appears to rule out pure exponential decay at high T for P waves.

η_b MEM



- 1S survives to highest T examined.
- excited states not discernable at $1.4 \lesssim T/T_c \lesssim 1.68 \Rightarrow$ melting or suppression?

Executive Summary

- There is much that I did not cover in these lectures
- I chose to focus on methods, new and old, for the “basic” building blocks of spectroscopy
- ... and described their successful applications as well as some pitfalls
- Lattice hadron spectroscopy is moving rapidly at the moment as new techniques emerge
- There will be lots more experimental data in the near future and to keep pace will be challenging

Thanks for listening and enjoy the rest of the school!