Lattice Methods for Hadron Spectroscopy: introduction and basics

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Housekeeping

- Web page for these lectures at http://www.maths.tcd.ie/~ryan/INT2012
- lectures, homework problems and data as well as references will all be there

Plan

- Motivation - why is hadron spectroscopy interesting?
- Lattice hadron spectroscopy
- Quark model and notation
- Path integrals and correlation functions: the building blocks
- Extracting the spectrum in a lattice calculation
- Why do we need good methods and precision results: a brief overview of experiments.

Spectroscopy: why?

- Many recently discovered hadrons have unexpected properties.
- Understand the hadron spectra to separate EW physics from strong-interaction effects
- Techniques for non-perturbative physics useful for physics at LHC energies.
- Understanding EW symmetry breaking may require nonperturbative techniques at TeV scales, similar to spectroscopy at GeV.
- Better techniques may help understand the nature of masses and transitions

Lattice Hadron Spectroscopy: an overview

Lattice Hadron Spectrosopy

Goal: to determine the low-energy hadron spectrum of quarks and gluons from $\mathcal{L}_{QCD}$.

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_f \bar{\Psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \Psi_f, \quad f = u, d, s, c, b, t$$

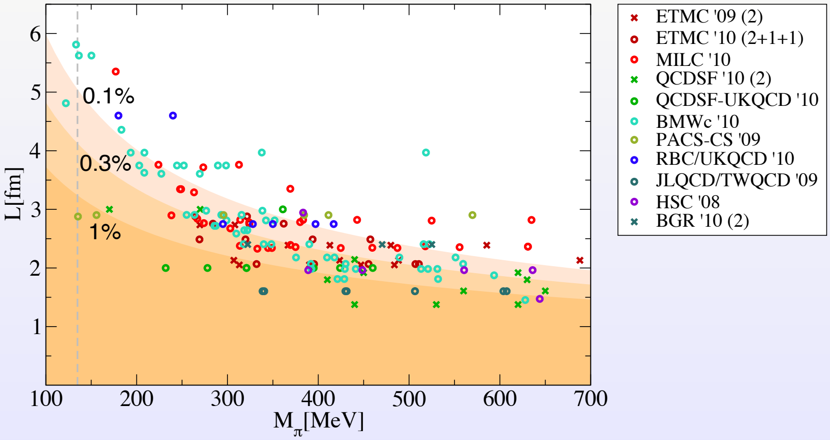
and $D_\mu = \partial_\mu - ig \left(\frac{1}{2} \lambda^a \right) A^a_\mu$

Recall the only inputs are the coupling g_0 and m_q so continuum QCD is recovered for

- simulation at or extrapolation to $m_\pi = m_{phys}$
- $a \to 0$ and $V \to \infty$

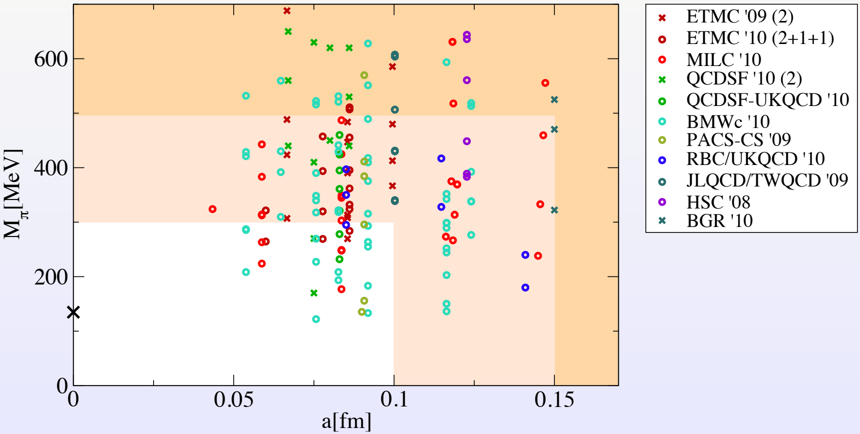
How are we doing?

C. Hoelbling, arXiV.1102.0410



How are we doing (2)?

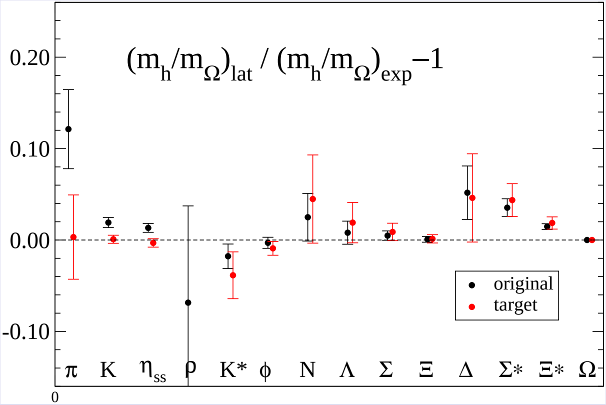
C. Hoelbling, arXiV.1102.0410



$N_f = 2 + 1$ simulations at the physical point

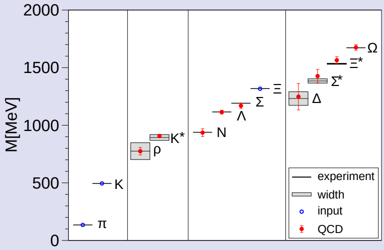
- First $N_f = 2 + 1$ simulations at physical quark mass.
- PACS-CS computer, U Tsukuba. 14.3 Tflops peak
- Lattice spacing: $a = 0.08995(40)$fm (from m_Ω).

PACS-CS Collaboration [arXiv:0911.2561]

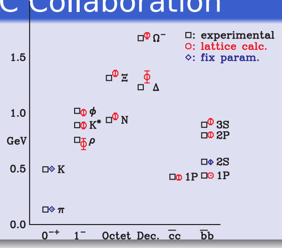


Convergence through universality

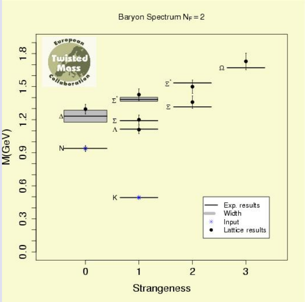
BMW Collaboration



MILC Collaboration

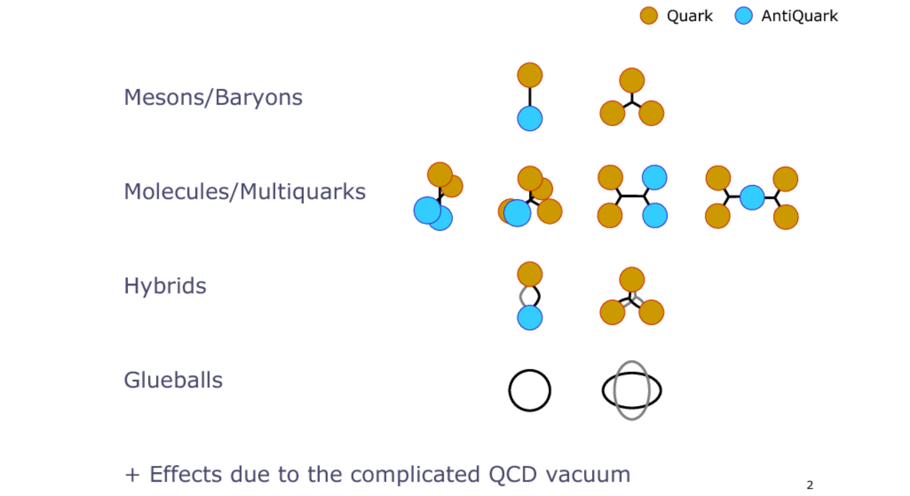


ETMC Collaboration



- **BMW:** SW-Wilson
[Science 322:1224-1227,2008.]
- **ETMC:** Twisted Mass
[arXiv:0910.2419,0803.3190]
- **MILC:** Staggered
[arXiv:0903.3598]

Objects of interest



A constituent model

- QCD has fundamental objects: quarks (in 6 flavours) and gluons
- Fields of the lagrangian are combined in colorless combinations: the mesons and baryons. Confinement.

quark model object	structure
meson	$3 \otimes \bar{3} = 1 \oplus 8$
baryon	$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$
hybrid	$\bar{3} \otimes 8 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 8 \oplus 10 \oplus 10$
glueball	$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus 10$
$\vdots$	$\vdots$

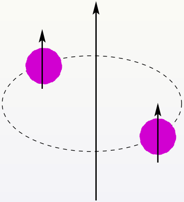
- This is a model. QCD does not always respect this constituent picture! There can be strong mixing.

Classifying states: mesons

- Recall that continuum states are classified by J^{PC} multiplets (representations of the poincare symmetry):
  - Recall the naming scheme: $n^{2S+1}L_J$ with $S = \{0, 1\}$ and $L = \{0, 1, \ldots\}$
  - J, hadron angular momentum, $|L - S| \leq J \leq |L + S|$
  - $P = (-1)^{(L+1)}$, parity
  - $C = (-1)^{(L+S)}$, charge conjugation. Only for $q\bar{q}$ states of same quark and antiquark flavour. So, not a good quantum number for eg heavy-light mesons $(D_{(s)}, B_{(s)})$.

Mesons

- two spin-half fermions $^{2S+1}L_J$
- $S = 0$ for antiparallel quark spins and $S = 1$ for parallel quark spins;



- States in the natural spin-parity series have $P = (-1)^J$ then $S = 1$ and $CP = +1$:
  - $J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 2^{--}, 2^{-+}, \ldots$ allowed
- States with $P = (-1)^J$ but $CP = -1$ forbidden in $q\bar{q}$ model of mesons:
  - $J^{PC} = 0^{+-}, 0^{--}, 1^{-+}, 2^{+-}, 3^{-+}, \ldots$ forbidden (by quark model rules)
  - These are **EXOTIC** states: not just a $q\bar{q}$ pair ...

Baryons

Baryon number $B = 1$: three quarks in colourless combination

- J is half-integer, C not a good quantum number: states classified by J^P
- spin-statistics: a baryon wavefunction must be antisymmetric under exchange of any 2 quarks.
- totally antisymmetric combinations of the colour indices of 3 quarks
- the remaining labels: flavour, spin and spatial structure must be in totally symmetric combinations

$$|qqq\rangle_A = |\text{color}\rangle_A \times |\text{space, spin, flavour}\rangle_S$$

With three flavours, the decomposition in flavour is

$$3 \otimes 3 \otimes 3 = 10_S \oplus 8_M \oplus 8_M \oplus 1_A$$

Many more states predicted than observed: missing resonance problem

Path integrals and correlation functions

Field theory on a Euclidean lattice

- Monte Carlo simulations are only practical using **importance sampling**
- Need a non-negative weight for each field configuration on the lattice

Minkowski $\longrightarrow$ Euclidean

- **Benefit:** can isolate lightest states in the spectrum (as we will see!).
- **Problem:** direct information on scattering is lost and must be inferred indirectly.
- To access radial and orbital excitations and resonances need a **variational method.**

Correlators in an EFT

- In EFT physical observables $\mathcal{O}$ is determined from

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\Psi \mathcal{D}\bar{\Psi} \mathcal{O} e^{-S_{QCD}}$$

- Analytically integrate Grassman fields $(\Psi, \bar{\Psi}) \rightarrow$ factors of $\det M$ the fermion mx.

$$\langle \mathcal{O} \rangle \overset{N_f=2}{=} \frac{1}{Z} \int \mathcal{D}U \det M^2 \mathcal{O} e^{-S_G}$$

  The expectation value is calculated by importance sampling of gauge fields and averaging over ensembles.
- We are interested in two-point correlation functions built from interpolating operators (functions of Ψ):
  - Eg the local meson operator $\mathcal{O}(x) = \bar{\Psi}_a(x) \Gamma \Psi_b(x)$
  - Γ an element of the Dirac algebra with possible displacements; a and b flavour indices

- The two-point function is then

$$C(t) = \langle \mathcal{O}(x)\mathcal{O}^\dagger(0)\rangle = \langle \bar{\Psi}_a(x)\Gamma\Psi_b(x)\bar{\Psi}_b(0)\Gamma^\dagger\Psi_a(0)\rangle$$

  where $x \equiv (t, \mathbf{x}); t \geq 0$
- Using Wick's theorem to contract quark fields replaces fields $\rightarrow$ quark propagators

$$\begin{aligned} C(t,\mathbf{x}) &= -\langle \mathrm{Tr}(M_a^{-1}(0,x)\Gamma M^{-1}{}_b(x,0)\Gamma^\dagger)\rangle \\ &\quad +\delta_{ab}\langle \mathrm{Tr}(\Gamma M^{-1}{}_a(x,x))\mathrm{Tr}(\Gamma^\dagger M^{-1}{}_a(0,0))\rangle \end{aligned}$$

  where the trace is over spin and colour.
- For flavour non-singlets ($a \neq b$) this leads to

$$C(t,\mathbf{x}) = \langle \mathrm{Tr}(\gamma_5 M_a^{-1}(x,0)^\dagger \gamma_5 \Gamma M_b^{-1}(x,0)\Gamma^\dagger)\rangle$$

- We consider the correlation function in momentum space at zero momentum

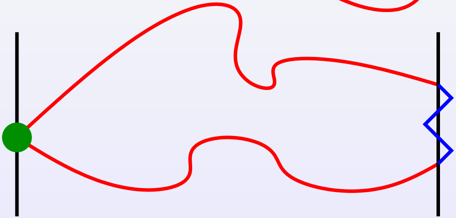
$$C(\vec{p},t) = \int d^3x e^{i\vec{p}\cdot\vec{x}} C(\vec{x},t,0,0) \text{ and } C(0,t) = C(t) \sim \sum_{\vec{x}} C(\vec{x},t,0,0)$$

Notes

- Fermions in lagrangian → fermion determinant
- Fermions in measurement → propagators
- The integral over gauge fields is done using importance sampling.
- γ_5 hermiticity: $M^{-1}(x,y) = \gamma_5 M^{-1}(y,x)^\dagger \gamma_5$ allows us to rewrite the correlator in terms of propagators from origin to all sites. Point (to-all) propagators
- practically: $M(x,0:U)^{-1}$ compute a singe column (in space-time indices) with linear solvers
- for flavour singlets $a=b$ terms like $M^{-1}(x,x)$ - requires the inverse of the **full** fermion mx on each config. More on this later

propagator cartoon

The most general operator.



A restricted correlation function accessible to one point-to-all computation.

Wick's Theorem

We used Wick's theorem to contract quark fields and replace with propagators ...

- Example — four field insertions: $\langle \psi_i \bar{\psi}_j \psi_k \bar{\psi}_l \rangle$
- the pairwise contraction can be done in two ways: $\psi_i \bar{\psi}_j \psi_k \bar{\psi}_l$ and $\psi_i \bar{\psi}_j \psi_k \bar{\psi}_l$
- giving the propagator combination $M^{-1}_{ij} M^{-1}_{kl} - M^{-1}_{jk} M^{-1}_{il}$
- minus-sign from the anti-commutation in second term.
- More fields means more combinations. Important in (eg.) isoscalar meson spectroscopy. We will see this again later

Warm Up Exercise!

For a system with six degrees of freedom, $\{\bar{\eta}_i, \eta_i\}, i = 1, 2, 3$, evaluate the grassmann integral

$$I_4 = \int \prod_{i=1}^{3} d\bar{\eta}_i d\eta_i \ \eta_1 \bar{\eta}_2 \eta_2 \bar{\eta}_1 \ e^{-\bar{\eta} M \eta}$$

and compare this answer to the prediction of Wick's theorem.

- See problem sheet for a related problem.

The QCD spectrum

- Want to extract the energy of (colourless) states of QCD.
- This information is encoded in the 2-point correlation functions

$$C(t) = \langle \phi_i(t) | \phi^\dagger(0) \rangle$$

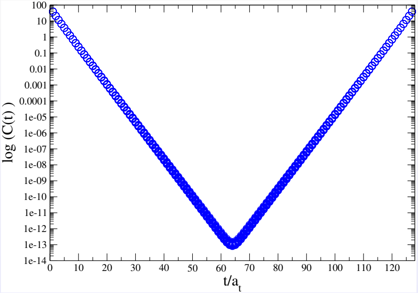
  where $\phi^\dagger$ and ϕ are operators acting on the quark fields to create a state at $t = 0$ and annihilate at $t = t$.
- Euclidean time evolution: $\phi(t) = e^{Ht} \phi e^{-Ht}$ and inserting a complete set of states

$$C(t) = \sum_{n=0}^{\infty} \frac{|\langle \phi | n \rangle|^2}{2m_n} e^{-E_n t}$$

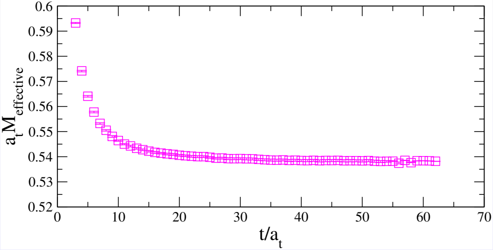
  we work in the low-temp limit ie $\beta = 1/kT = L_t$ large.
- Now as $t \to \infty C(t) = Ze^{-E_0 t}$
- At large times the exponential fall off of $C(t)$ gives the ground state energy.

From correlators to energies

- In general works well for extracting ground states
- Higher excitation energies hard to extract by just fitting to exponentials.



$$a_t m_{effective} = -log\left(\frac{C(t)}{C(t-1)}\right)$$



- The correlator and effective mass of the J/Ψ meson.
- For $\mathcal{O}_i = \mathcal{O}_j$ the correlation function is positive definite and $m_{effective}$ converges monotonically from above.

The QCD spectrum (2)

- The lattice has finite extent - impose (anti)-periodic boundary conditions. Then meson correlators are symmetric about the midpoint of the lattice ie $e^{-mt} \rightarrow e^{-mt} + e^{-m(T-t)}$ where T is the time extent.
- Want to optimise $\mathcal{O}$ to get a large overlap with the wavefunction of the state of interest ie make

$$\mathcal{Z}_n(\vec{p}) \equiv \frac{|\langle 0|\mathcal{O}_i|n\rangle|^2}{2E_n(\vec{p})}$$

the spectral weight of the n^{th} state large for state of interest and small for the rest.

What about excitations?

One approach: variational method

If we can measure $C_{ij}(t) = \langle 0|\phi_i(t)\phi_j^\dagger(0)|0\rangle$ for all i, j and solve generalised eigenvalue problem:

$$\mathbf{C}(t)\,\underline{v} = \lambda \mathbf{C}(t_0)\,\underline{v}$$

then

$$\lim_{t-t_0\to\infty} \lambda_k = e^{-E_k t} + \mathcal{O}\left(e^{-\Delta E_n t}\right)$$

For this to be practical, we need:

- a ‘good’ basis set that **resembles the states** of interest
- **all elements** of this correlation matrix measured

[see Blossier et.al. JHEP 0904 (2009) 094]

Current and Future Experiments: motivating lattice spectroscopy

the light unflavoured mesons

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from the 2010 Review of Particle Physics.
Please use this **CITATION**: K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).

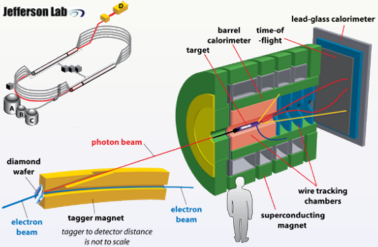
back to contents

LIGHT UNFLAVORED MESONS ($S = C = B = 0$)

For $I=1$ (π, b, ρ, a): $u\bar{d}$, $(u\bar{u}-d\bar{d})/\sqrt{2}$, $d\bar{u}$;
for $I=0$ (η, η', h, h', ω, φ, f, f'): $c_1(u\bar{u}+d\bar{d})+c_2(s\bar{s})$

$\pi^\pm$	$1^-(0^-)$	$\eta(1475)$	$0^+(0^{-+})$	• $f_2(1910)$	$0^+(2^{++})$
π^0	$1^-(0^{-+})$	$f_0(1500)$	$0^+(0^{++})$	$f_2(1950)$	$0^+(2^{++})$
η	$0^+(0^{-+})$	• $f_1(1510)$	$0^+(1^{++})$	• $\rho_3(1990)$	$1^+(3^{--})$
$f_0(600)$ or σ	$0^+(0^{++})$	$f_2'(1525)$	$0^+(2^{++})$	$f_2(2010)$	$0^+(2^{++})$
$\rho(770)$	$1^+(1^{--})$	• $f_2(1565)$	$0^+(2^{++})$	• $f_0(2020)$	$0^+(0^{++})$
$\omega(782)$	$0^-(1^{--})$	• $\rho(1570)$	$1^+(1^{--})$	$a_4(2040)$	$1^-(4^{++})$
$\eta'(958)$	$0^+(0^{-+})$	• $h_1(1595)$	$0^-(1^{+-})$	$f_4(2050)$	$0^+(4^{++})$
$f_0(980)$	$0^+(0^{++})$	$\pi_1(1600)$	$1^-(1^{-+})$	• $\pi_2(2100)$	$1^-(2^{-+})$
$a_0(980)$	$1^-(0^{++})$	• $a_1(1640)$	$1^-(1^{++})$	• $f_0(2100)$	$0^+(0^{++})$
$\varphi(1020)$	$0^-(1^{--})$	• $f_2(1640)$	$0^+(2^{++})$	• $f_2(2150)$	$0^+(2^{++})$
$h_1(1170)$	$0^-(1^{+-})$	$\eta_2(1645)$	$0^+(2^{-+})$	• $\rho(2150)$	$1^+(1^{--})$
$b_1(1235)$	$1^+(1^{+-})$	$\omega(1650)$	$0^-(1^{--})$	$\varphi(2170)$	$0^-(1^{--})$
$a_1(1260)$	$1^-(1^{++})$	$\omega_3(1670)$	$0^-(3^{--})$	• $f_0(2200)$	$0^+(0^{++})$
$f_2(1270)$	$0^+(2^{++})$	$\pi_2(1670)$	$1^-(2^{-+})$	• $f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
$f_1(1285)$	$0^+(1^{++})$	$\varphi(1680)$	$0^-(1^{--})$	• $\eta(2225)$	$0^+(0^{-+})$
$\eta(1295)$	$0^+(0^{-+})$	$\rho_3(1690)$	$1^+(3^{--})$	• $\rho_3(2250)$	$1^+(3^{--})$
$\pi(1300)$	$1^-(0^{-+})$	$\rho(1700)$	$1^+(1^{--})$	$f_2(2300)$	$0^+(2^{++})$
$a_2(1320)$	$1^-(2^{++})$	• $a_2(1700)$	$1^-(2^{++})$	• $f_4(2300)$	$0^+(4^{++})$
$f_0(1370)$	$0^+(0^{++})$	$f_0(1710)$	$0^+(0^{++})$	• $f_0(2330)$	$0^+(0^{++})$
• $h_1(1380)$	$?^-(1^{+-})$	• $\eta(1760)$	$0^+(0^{-+})$	$f_2(2340)$	$0^+(2^{++})$
$\pi_1(1400)$	$1^-(1^{-+})$	$\pi(1800)$	$1^-(0^{-+})$	• $\rho_5(2350)$	$1^+(5^{--})$
$\eta(1405)$	$0^+(0^{-+})$	• $f_2(1810)$	$0^+(2^{++})$	• $a_6(2450)$	$1^-(6^{++})$
$f_1(1420)$	$0^+(1^{++})$	• $X(1835)$	$?^?(?^{-+})$	• $f_6(2510)$	$0^+(6^{++})$
$\omega(1420)$	$0^-(1^{--})$	$\varphi_3(1850)$	$0^-(3^{--})$		
• $f_2(1430)$	$0^+(2^{++})$	• $\eta_2(1870)$	$0^+(2^{-+})$	• — OMITTED FROM SUMMARY TABLE	
$a_0(1450)$	$1^-(0^{++})$	$\pi_2(1880)$	$1^-(2^{-+})$		
$\rho(1450)$	$1^+(1^{--})$	• $\rho(1900)$	$1^+(1^{--})$		

The GlueX experiment at JLab



- 12 GeV upgrade to CEBAF ring
- New experimental hall: Hall D
- New experiment: GlueX

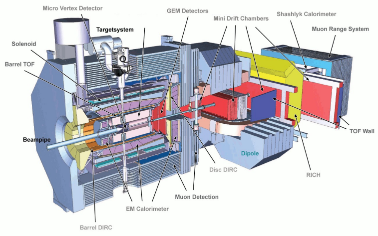
- Aim: photoproduce mesons, in particular the hybrid meson (with intrinsic gluonic excitations)
- Expected to start taking data 2014

Panda@FAIR, GSI

- Extensive new construction at GSI Darmstadt
- Expected to start operation 2014

PANDA: Anti-Proton ANnihilation at DArmstadt

- Anti-proton beam from FAIR on fixed-target.
- Physics goals include searches for hybrids and glueballs (as well as charm and baryon spectroscopy).



A renaissance in charmonium spectroscopy

- Early in the noughties, new narrow structures were seen by Belle and BaBar above the open-charm threshold.
- This led to substantial renewed interest in spectroscopy. Were these more quark-anti-quark states, or something more?
  - $X(3872)$: very close to $D\bar{D}$ threshold - a molecule?
  - $Y(4260)$: a 1^{--} hybrid?
  - $Z^{\pm}(4430)$: charged, can't be $\bar{c}c$.
- Very little is known and no clear picture seems to be emerging...
- Lattice calculations have a role to play

Lattice Hadron Spectroscopy

- Significant experimental effort hoping to understand light hadron and charm spectroscopy
  - Are there resonances that don't fit in the quark model?
  - Are there gluonic excitations in this spectrum?
  - What structure does confinement lead to?
  - How do resonances decay?
- To use LQCD to address these questions means:
  - identifying continuum properties of states
  - going beyond precision ground state spectroscopy to compute scattering and resonance widths
- To achieve this we need new tools
  - Techniques that give statistical precision
  - Methods for operator construction and spin identification on lattice
  - New methods for resonance and isoscalar physics
  - Control over extrapolations ($m_q \to 0, V \to \infty, a \to 0$).

Tools not interdependent but should work well together

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