Nuclear Astrophysics Topics II

Some neutrino physics of

- Sun
- supernovae
- nucleosynthesis

Notes: http://www.int.washington.edu/PHYS554/2011/2011.html

Introduction: why folks are interested in neutrino mass

Exercise: start with the Dirac equation, project out its four degrees of freedom

$$\psi_{R/L} = \frac{1}{2}(1 \pm \gamma_5)\psi \qquad\qquad C\ \psi_{R/L}\ C^{-1} = \psi^c_{R/L}$$

Allow for flavor mixing

$$L_m(x) \sim m_D \bar{\psi}(x)\psi(x) \Rightarrow M_D \bar{\Psi}(x)\Psi(x) \qquad\qquad \Psi_L \equiv \begin{pmatrix} \Psi^e_L \\ \Psi^\mu_L \\ \Psi^\tau_L \end{pmatrix}$$

To give the 4n by 4n mass matrix

$$(\bar{\Psi}^c_L, \bar{\Psi}_R, \bar{\Psi}_L, \bar{\Psi}^c_R) \begin{pmatrix} 0 & 0 & & M_D^T \\ 0 & 0 & M_D & \\ & M_D^\dagger & 0 & 0 \\ M_D^* & & 0 & 0 \end{pmatrix} \begin{pmatrix} \Psi^c_L \\ \Psi_R \\ \Psi_L \\ \Psi^c_R \end{pmatrix}$$

Because the neutrino lacks a conserved charge, one can add terms

$$L_m(x) \Rightarrow M_D \bar{\Psi}(x)\Psi(x) + (\bar{\Psi}^c_L(x) M_L \Psi_L(x) + \bar{\Psi}^c_R(x) M_R \Psi_R(x) + h.c.)$$

to give the more general matrix

$$(\bar{\Psi}^c_L, \bar{\Psi}_R, \bar{\Psi}_L, \bar{\Psi}^c_R) \begin{pmatrix} 0 & 0 & M_L & M_D^T \\ 0 & 0 & M_D & M_R^\dagger \\ M_L^\dagger & M_D^\dagger & 0 & 0 \\ M_D^* & M_R & 0 & 0 \end{pmatrix} \begin{pmatrix} \Psi^c_L \\ \Psi_R \\ \Psi_L \\ \Psi^c_R \end{pmatrix}$$

- the Majorana terms do not arise for other SM fermions, which carry additively conserved charges that distinguish particle, antiparticle

- the Dirac masses might naturally be typical of other SM fermions: this requires a RHed neutrino field

- now M_L can be introduced as a phenomenological term in the SM as it involves only ψ_L

- but this term is probed experimentally in the second-order weak process of neutrinoless double beta decay. Limits from ^{76}Ge $M_L \ll 1$ eV $\ll M_D$

- so this suggests a mass matrix that on diagonalization

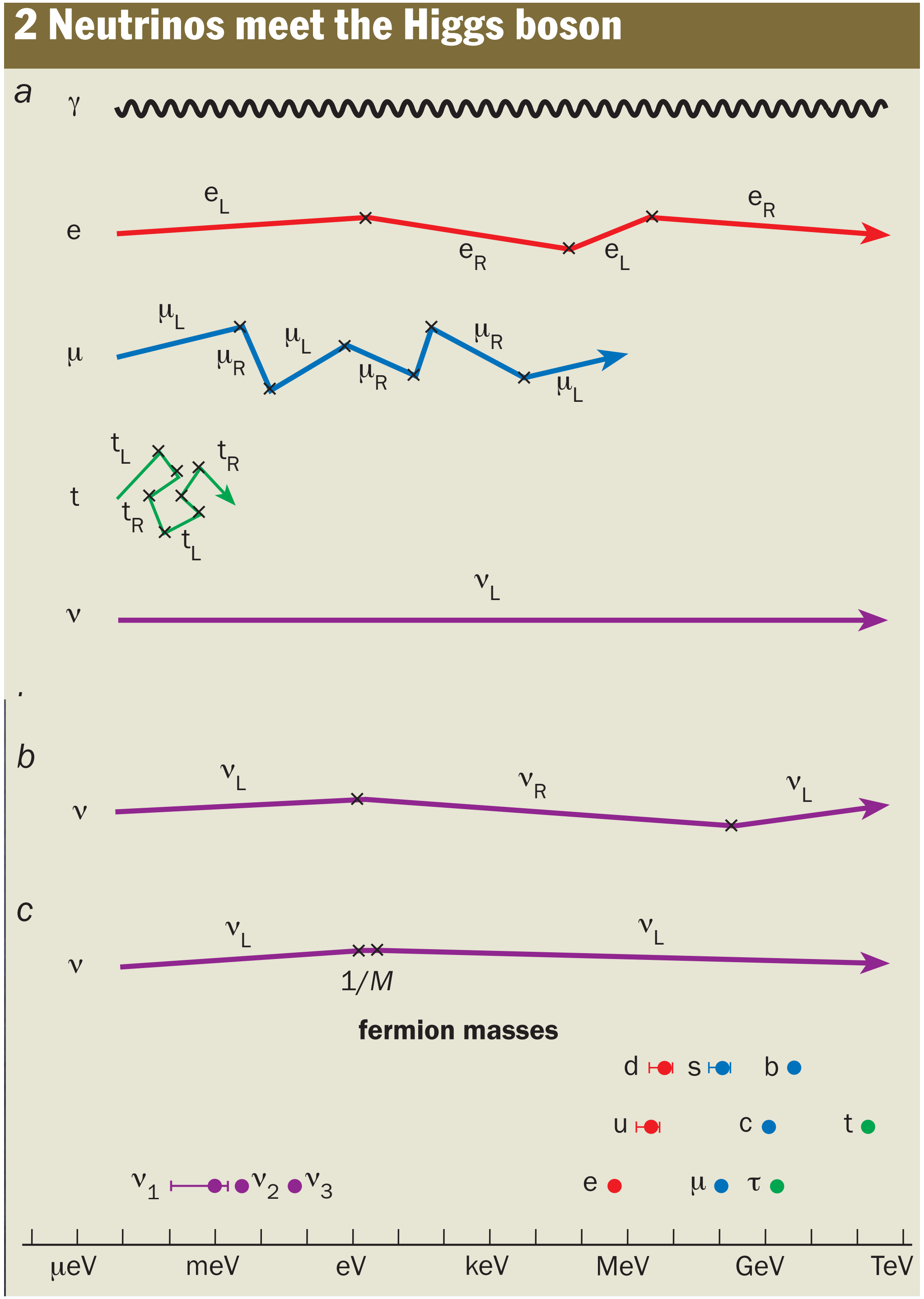
$$\begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \Rightarrow \; m_\nu^{\text{light}} \sim m_D \left(\frac{m_D}{m_R}\right)$$

- take $m_\nu \sim \sqrt{m^2{}_{23}} \sim 0.05$ eV and $m_D \sim m_{top} \sim 180$ GeV

$$\Rightarrow m_R \sim 0.3 \times 10^{15} \text{ GeV}$$

this connects small neutrino masses -- hard otherwise to explain -- with the high scale of the BSM RHed Majorana mass

Murayama's ν mass cartoon



standard model masses

light Dirac neutrino

quadratic coupling inducing M_L

← the anomalous ν mass scale

Most of what we now know about neutrinos

-- mass differences

-- mass patterns

-- oscillations between flavors

-- limits on the absolute mass

has come from astrophysics and cosmology

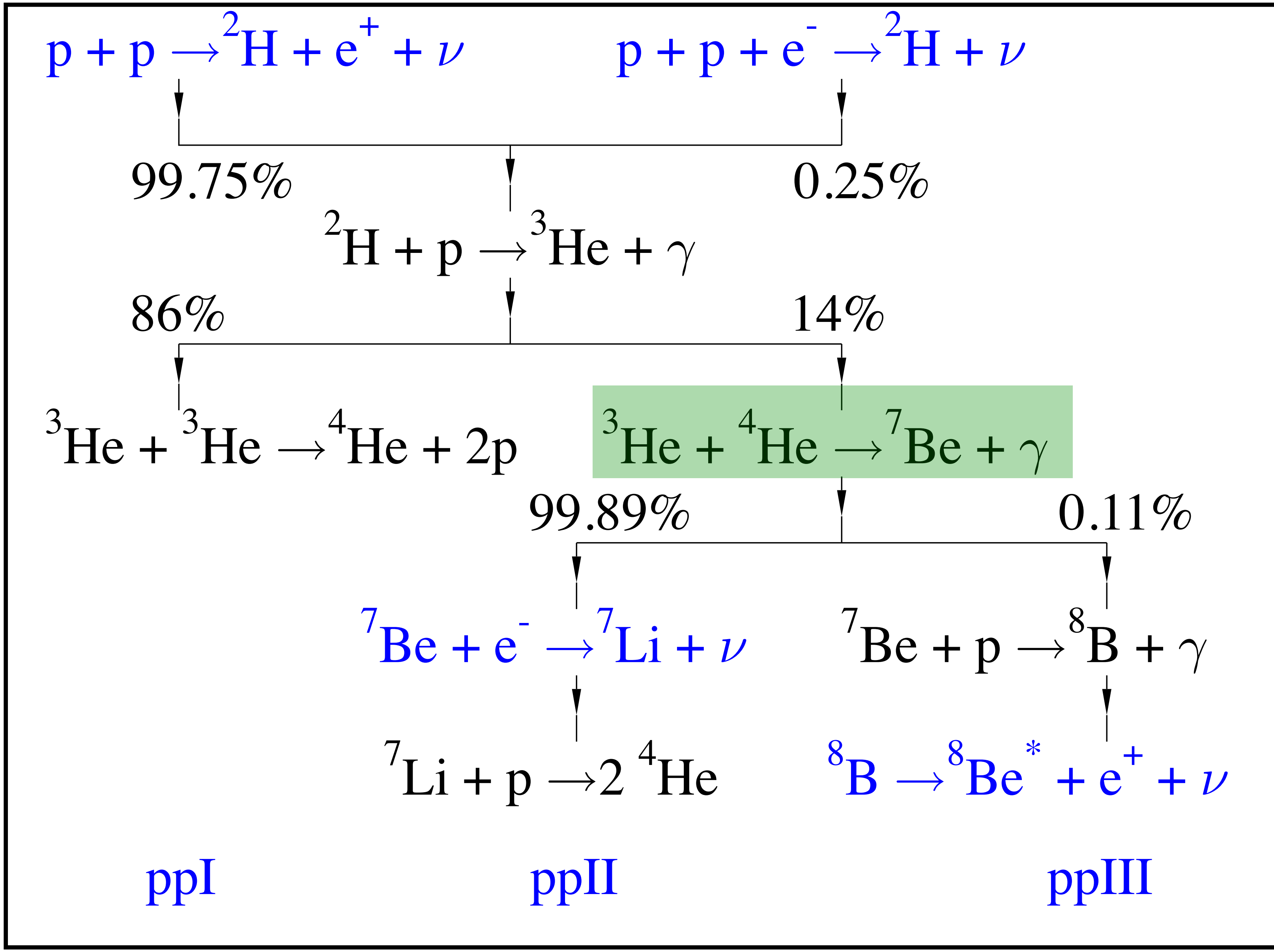
Solar Neutrinos

- a surprising nuclear physics discovery in 1959 -- a measured cross section for ^{3}He+^{4}He 1000 times larger than theory -- meant that ν detection techniques of Ray Davis could be used for solar νs

- this stimulated the development of the "standard solar model" to predict the core temperature of the Sun to the requisite precision, 1%

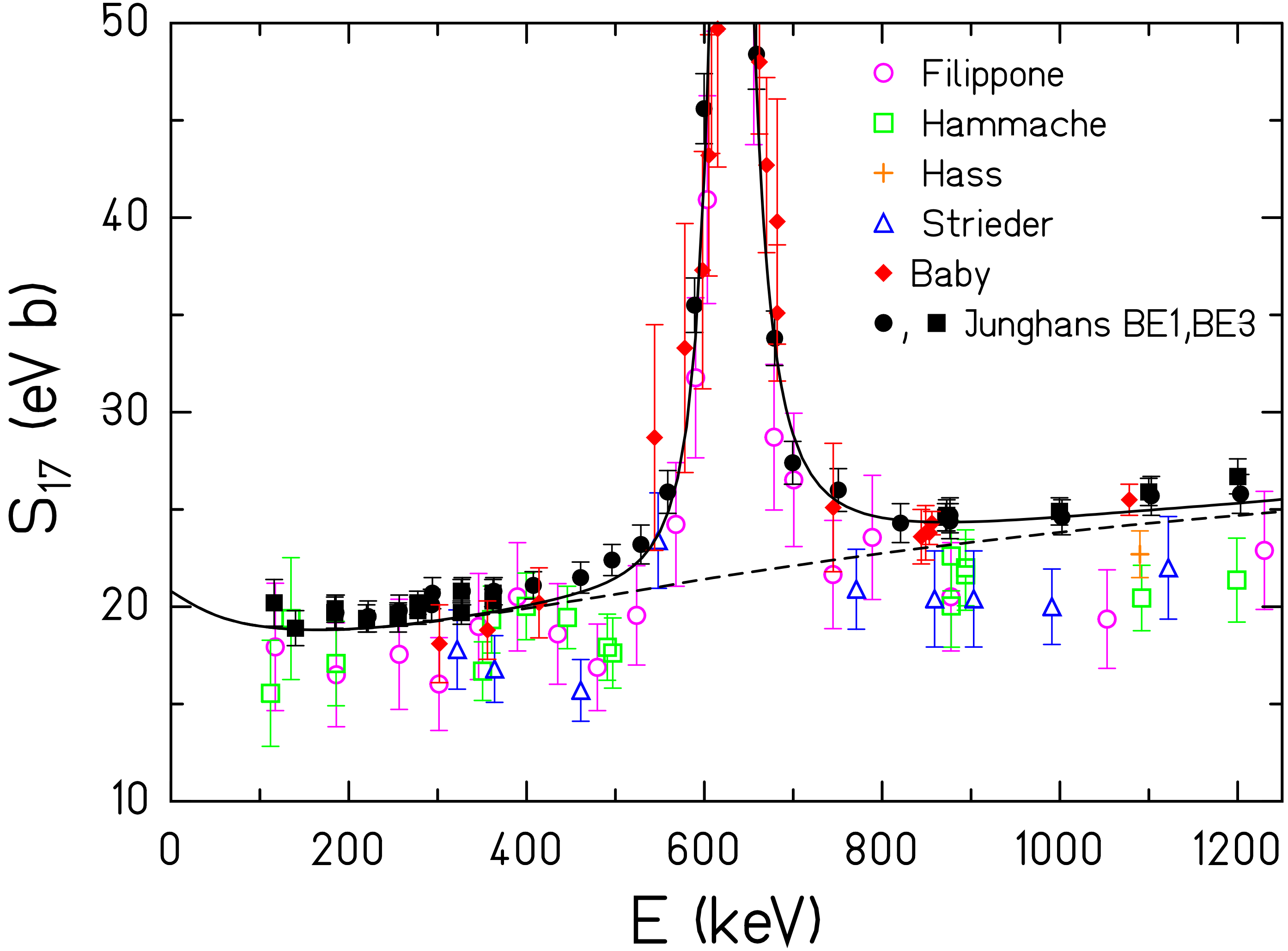
- the SSM assumes
  - ◇ local hydrostatic equilibrium: gas pressure gradient counteracting gravitational force
  - ◇ hydrogen burning, dominated by the pp chain
  - ◇ energy transport by radiation (within interior 70% by radius) and convection (convective envelope, outer 30%)
  - ◇ highly constrained by today's mass, radius, luminosity, and metal abundances (meteoritic, or measured from Sun's surface)

- the SSM is in fact our general theory for hydrogen-burning stars, about 80% of the stars we see in our galaxy



three competing branches $\Leftrightarrow$ three neutrino tags

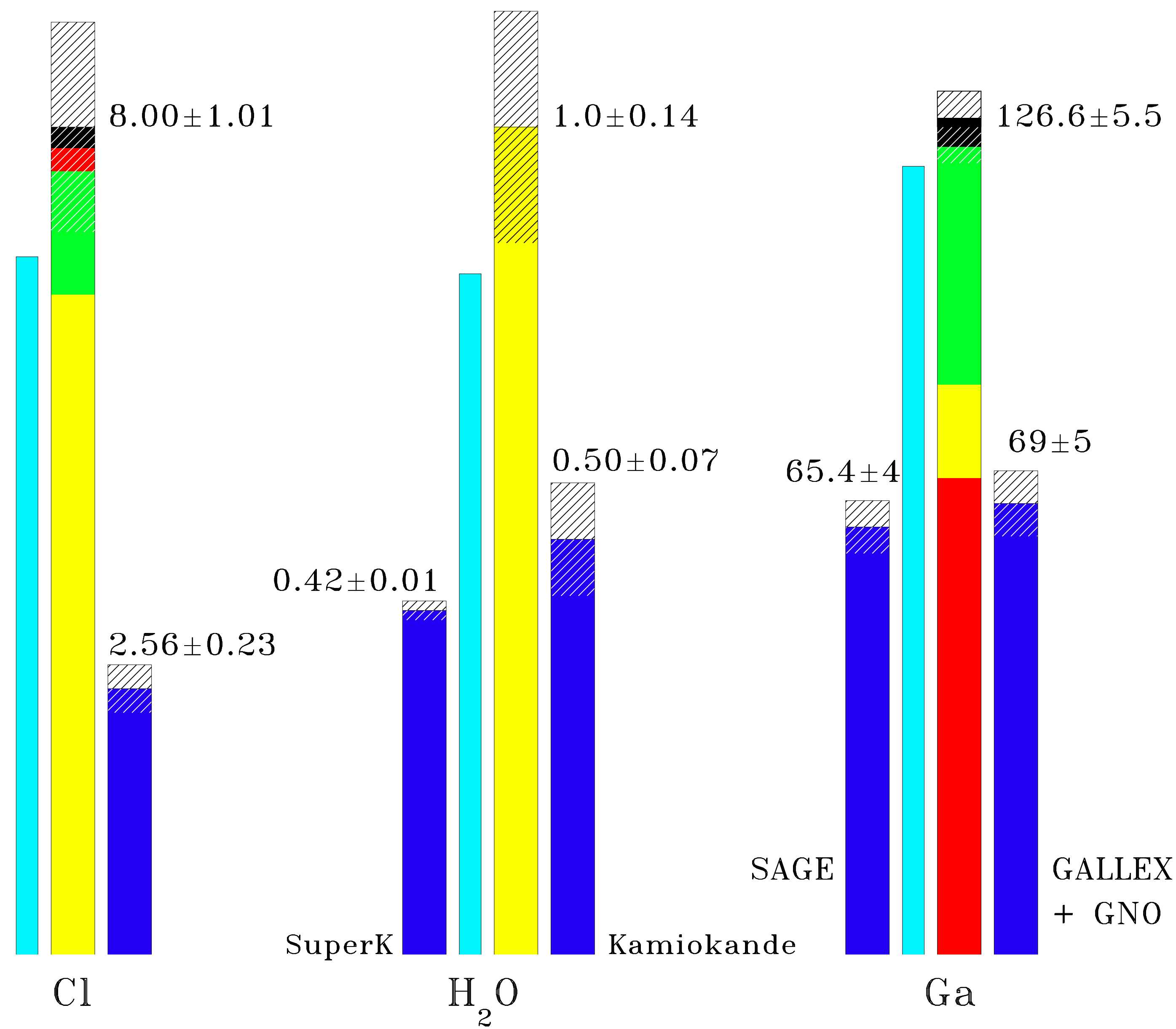
luminosity, pp νs $\propto T^4$ ^{8}B νs $\propto T^{22}$: a thermometer!



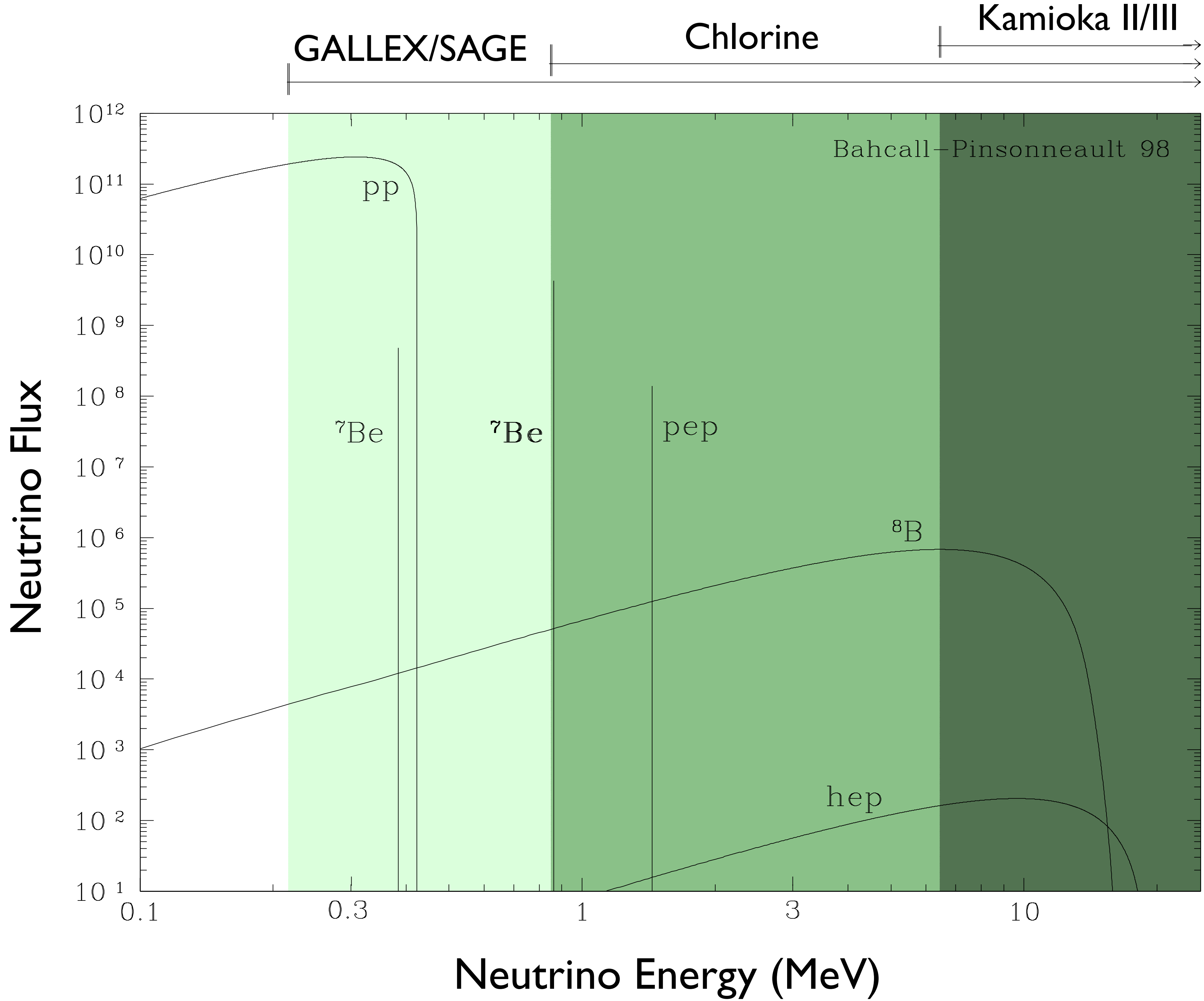
$^{7}Be(p,\gamma)^{8}B$: determines the ν flux measured by SNO, SuperK

The limiting uncertainties in the model are nuclear, and often theoretical

We lack a theory capable of extrapolating very precise data measured at feasible energies (here above 100 keV) to solar energies (here 20 keV) without degrading the experimental precision



By early 1990s three experiments -- using Cl, water (Kamiokande), and Ga -- had confirmed a solar neutrino deficit of a factor of 1/3-1/2



GALLEX/SAGE
Chlorine
Kamioka II/III
Bahcall–Pinsonneault 98
pp
^{7}Be
^{7}Be
pep
^{8}B
hep
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
0.1
0.3
1
3
10
Neutrino Flux
Neutrino Energy (MeV)

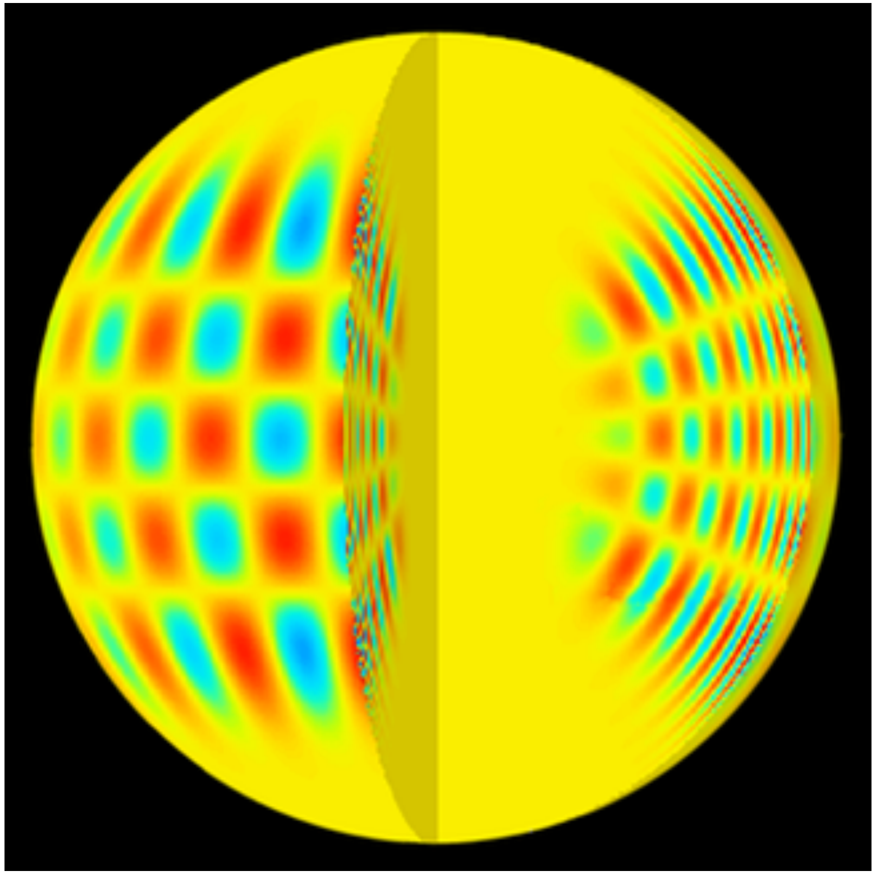
- The initial Cl result could be accounted for by stipulating that the SSM prediction of the solar core temperature was too high by 5%

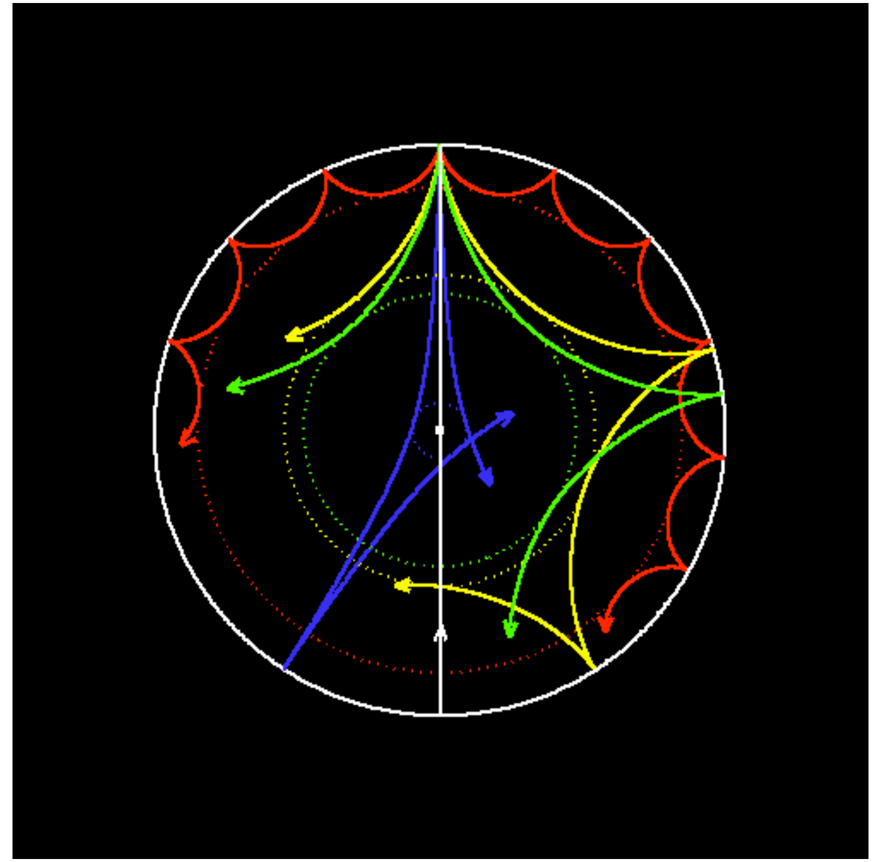
- Subsequent results from the Ga (sensitive to the lowest-energy pp neutrinos) and water (sensitive only to the ^{8}B νs) undercut this explanation: other observables required a hotter solar core

- Astronomy developed techniques to deduce the sound speed profile throughout most of the solar interior -- exceedingly precise (few 0.1%) test of solar density and temperature profiles via helioseismology

- The excellent agreement between the observed and SSM sound speed profiles strongly suggested that the solar neutrino puzzle was due to new neutrino physics

- The prospect of confirming BSM physics motivated two extraordinary experiments, SNO, Super-Kamiokande, and Borexino

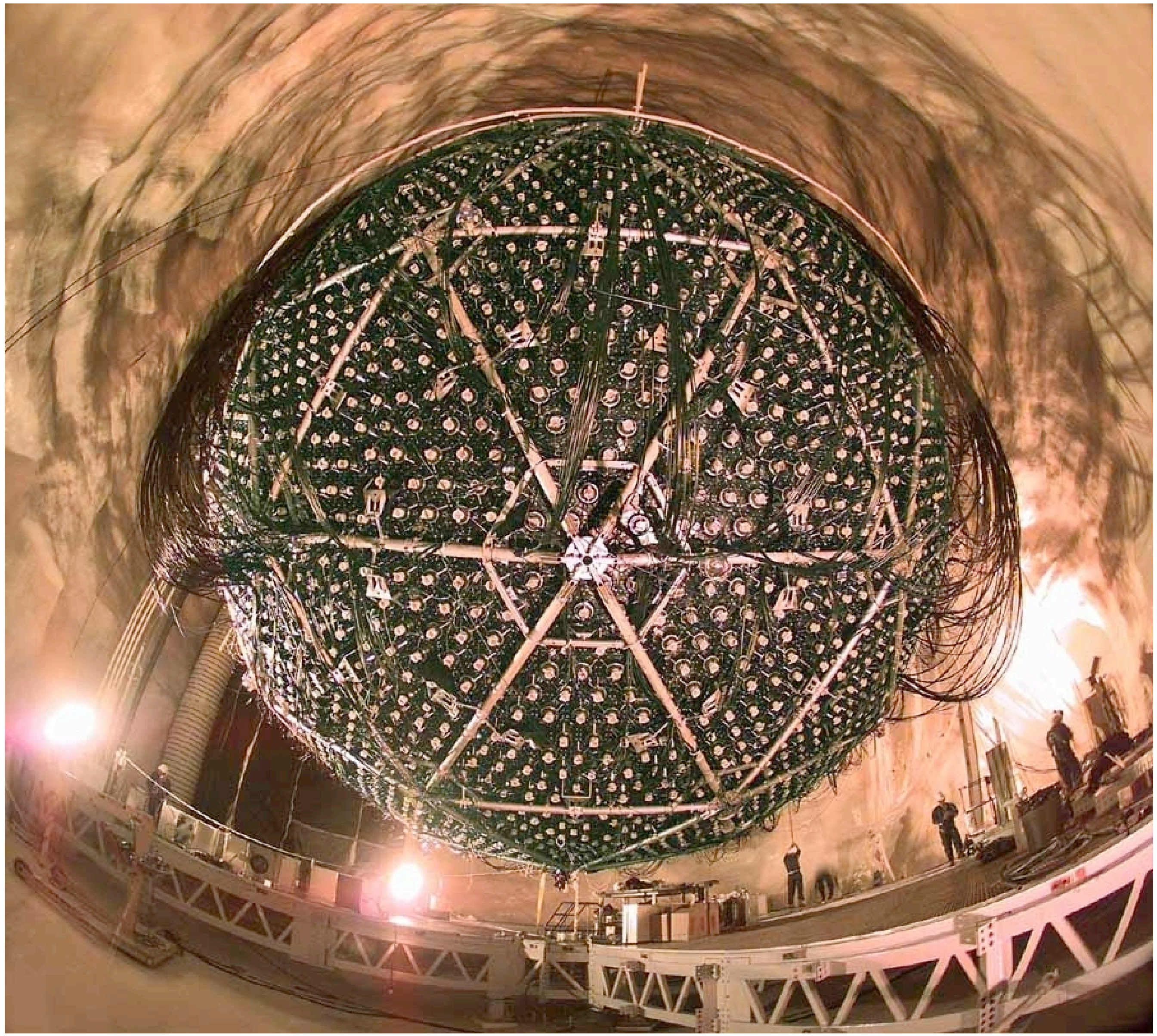




turbulence in the solar convective zone acts as a random driver

specific "normal modes" are excited that probe the Sun's sound speed to specific solar depths: Doppler shifts measured

SNO heavy-water Cerenkov detector constructed 2 km below ground in Sudbury nickel mine, Ontario



Vacuum flavor oscillations: mass and weak eigenstates

flavor states	$\|\nu_e>$	$\leftrightarrow$	$\|\nu_L> \quad m_L$	mass states
	$\|\nu_\mu>$		$\|\nu_H> \quad m_H$	

Noncoincident bases ⇒ oscillations down stream:

$$\begin{aligned} |v_e> &= \cos\theta|\nu_L> + \sin\theta|\nu_H> \\ |v_\mu> &= -\sin\theta|\nu_L> + \cos\theta|\nu_H> \end{aligned}$$

vacuum mixing angle

$$\begin{aligned} |\nu_e^k> &= |\nu^k(x=0,t=0)> \qquad E^2 = k^2 + m_i^2 \\ |\nu^k(x\sim ct,t)> &= e^{ikx}\left[e^{-iE_Lt}\cos\theta|\nu_L> + e^{-iE_Ht}\sin\theta|\nu_H>\right] \\ |<\nu_\mu|\nu^k(t)>|^2 &= \sin^2 2\theta\sin^2\left(\frac{\delta m^2}{4E}t\right), \qquad \delta m^2 = m_H^2 - m_L^2 \end{aligned}$$

ν_μ appearance downstream ⇔ vacuum oscillations

Can slightly generalize this

$$|\nu(0)\rangle \;\rightarrow\; a_e(0)|\nu_e\rangle + a_\mu(0)|\nu_\mu\rangle$$

with the subsequent evolution downstream governed by

$$i\frac{d}{dx}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\delta m^2 \cos 2\theta & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & \delta m^2 \cos 2\theta \end{pmatrix}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix}$$

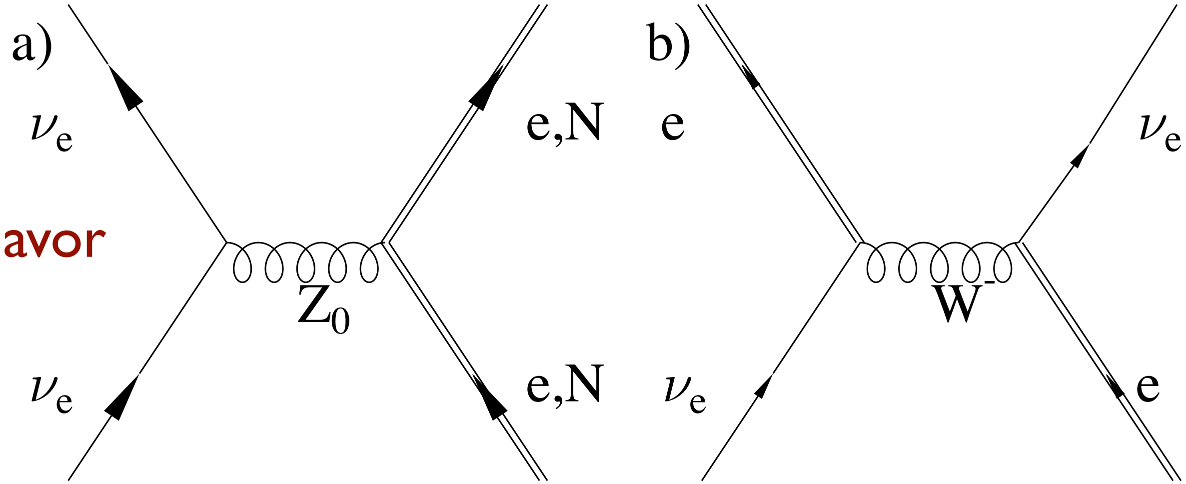
vacuum m_ν^2 matrix

This problem familiar from hadronic physics: the Cabibbo angle and CKM matrix.

But in astrophysics, we can use matter to control the mixing angle!

solar matter generates a flavor asymmetry

any ν flavor



only ν_es

- modifies forward scattering amplitude: flavor-dependent index of refraction
- the affect is proportional to the (changing) solar electron density
- makes the electron neutrino heavier at high density

$$m^2_{\nu_e} = 4E\sqrt{2}G_F\ \rho_e(x)$$

inserting this into mass matrix generates the 2-flavor MSW equation

$$i\frac{d}{dx}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\delta m^2\cos 2\theta + 4E\sqrt{2}G_F\rho_e(x) & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & \delta m^2 \cos 2\theta \end{pmatrix}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix}$$

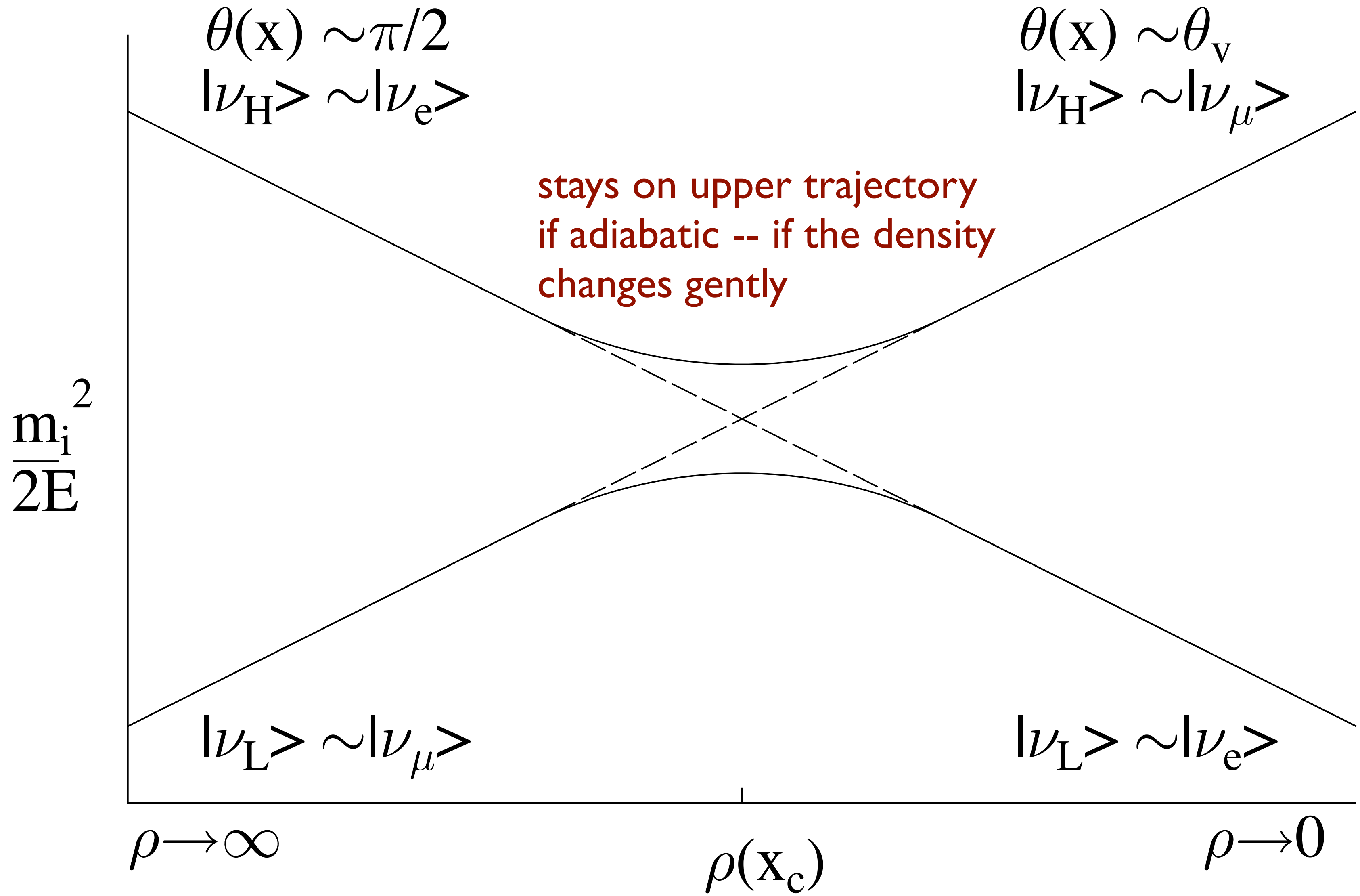
or equivalently

$$i\frac{d}{dx}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\delta m^2\cos 2\theta + 2E\sqrt{2}G_F\rho_e(x) & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & -2E\sqrt{2}G_F\rho_e(x) + \delta m^2 \cos 2\theta \end{pmatrix}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix}$$

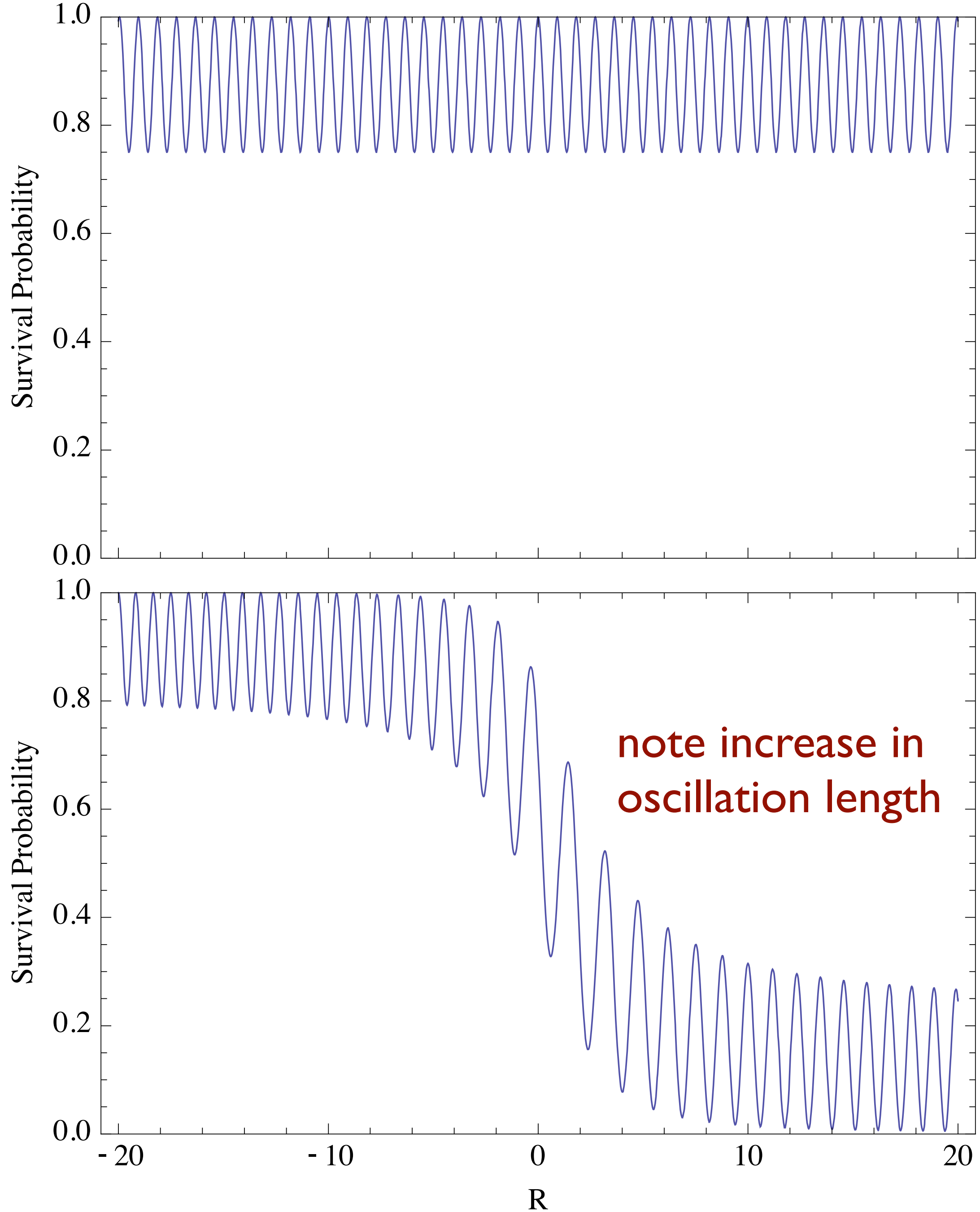
the m_ν^2 matrix's diagonal elements vanish at a critical density (that must mean maximal mixing of degenerate states)

$$\rho_c : \quad \delta m^2 \cos 2\theta \equiv 2E\sqrt{2}G_F\rho_c$$

Via a picture for the case of a small vacuum mixing angle



MSW mechanism



Mathematica HW problem

a) vacuum oscillations θ=15°
R from -20 to +20

R in units of $\dfrac{4E\cos 2\theta}{\delta m^2 \sin^2 2\theta}$

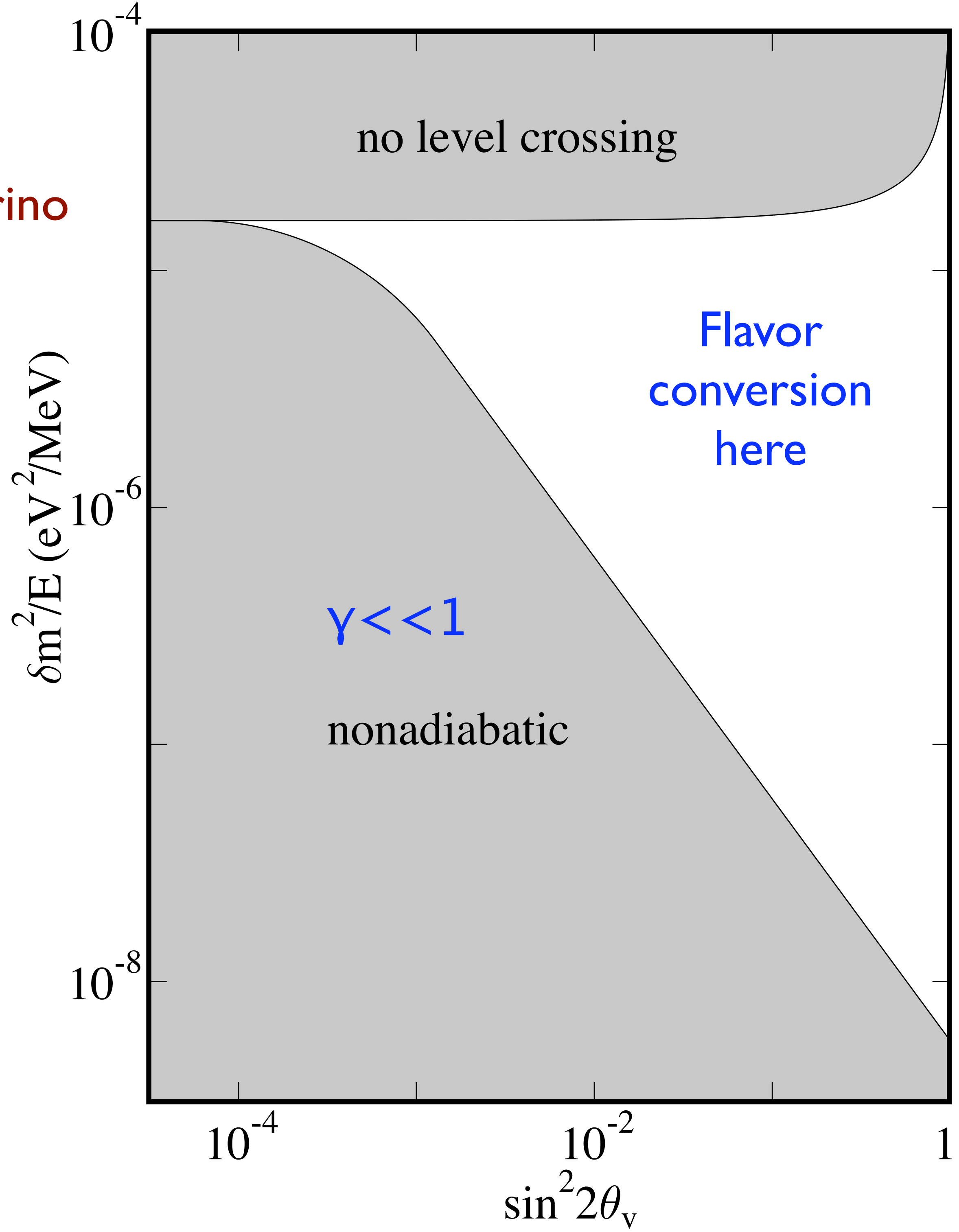
b) matter oscillations

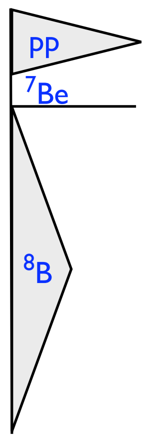
add $\rho_e(R) \propto 1 - \dfrac{2}{\pi}\arctan aR$

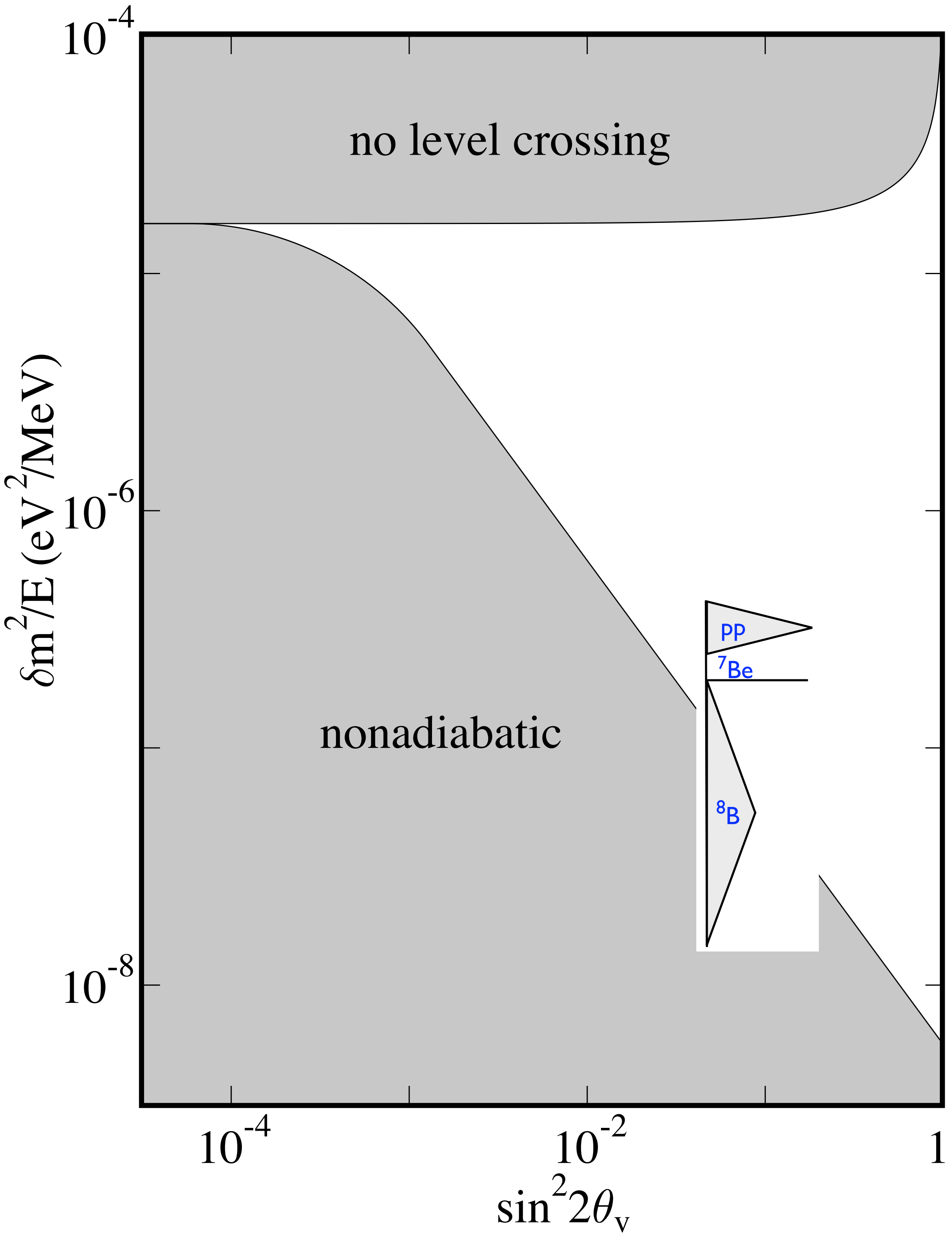
normalize so that crossing occurs at R = 0

note $\rho_e(R) \rightarrow 0$ as $R \rightarrow \infty$

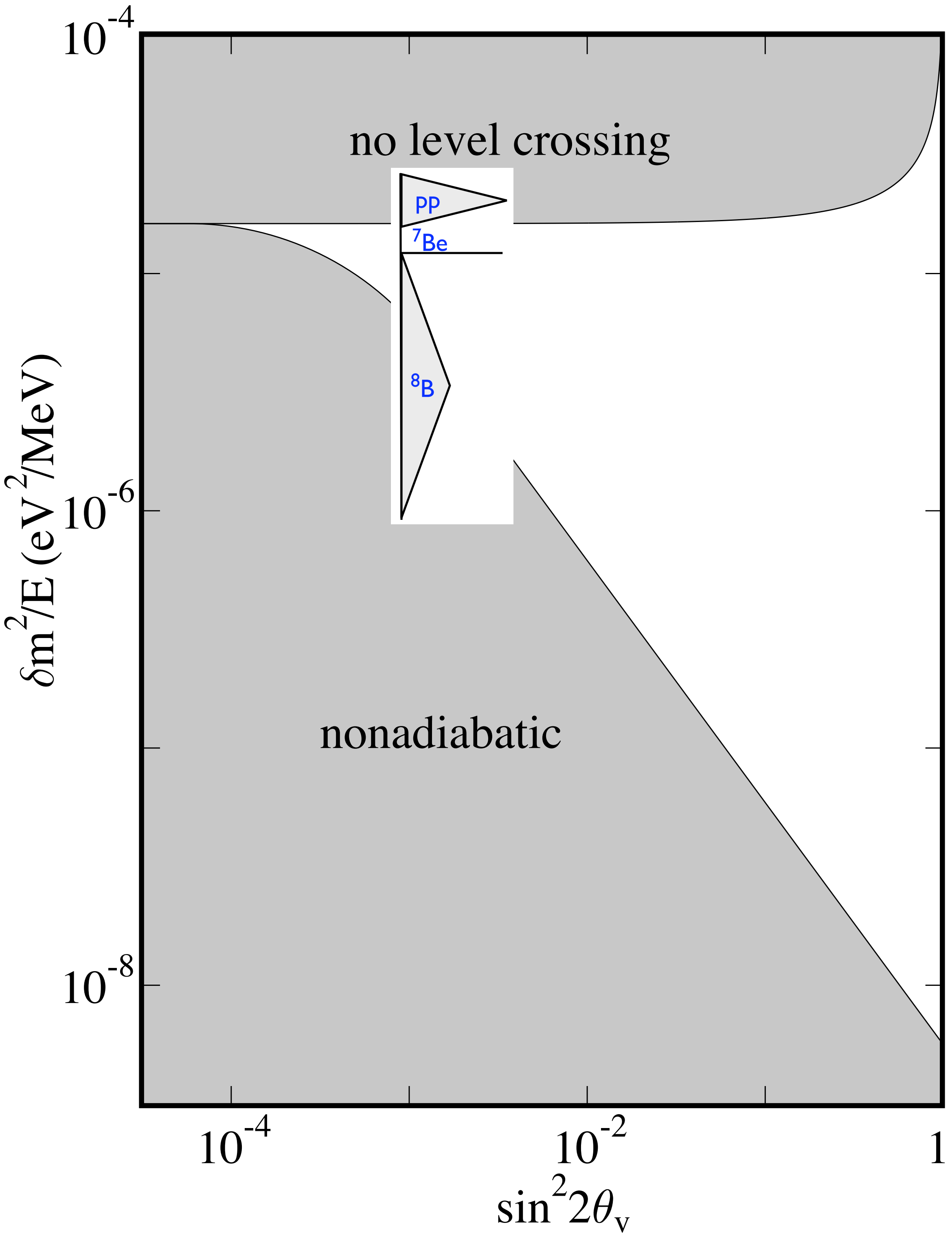
So ν_e is produced as a heavy eigenstate, then propagates toward the vacuum, where it is the light eigenstate solving the solar neutrino problem



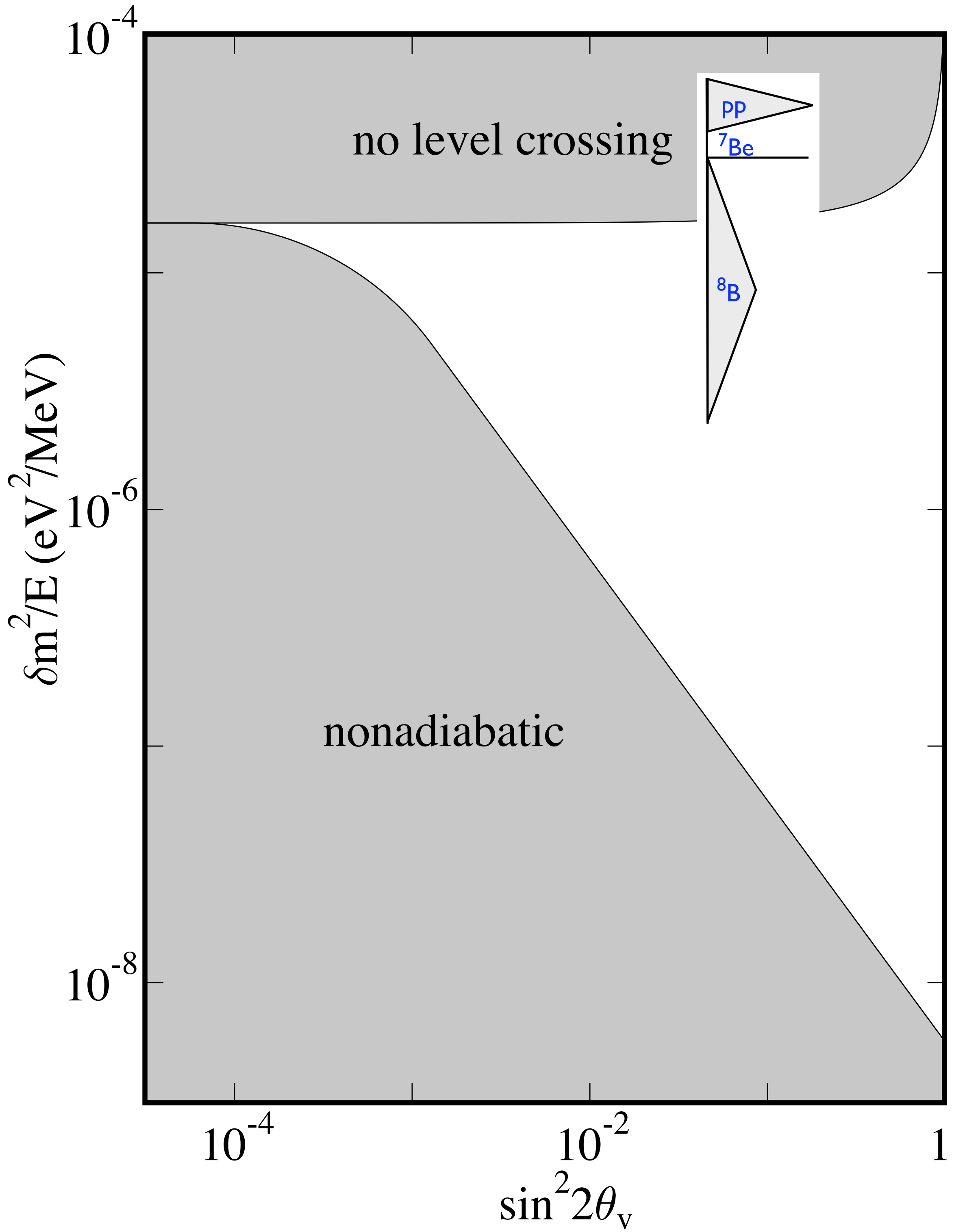




Low solution



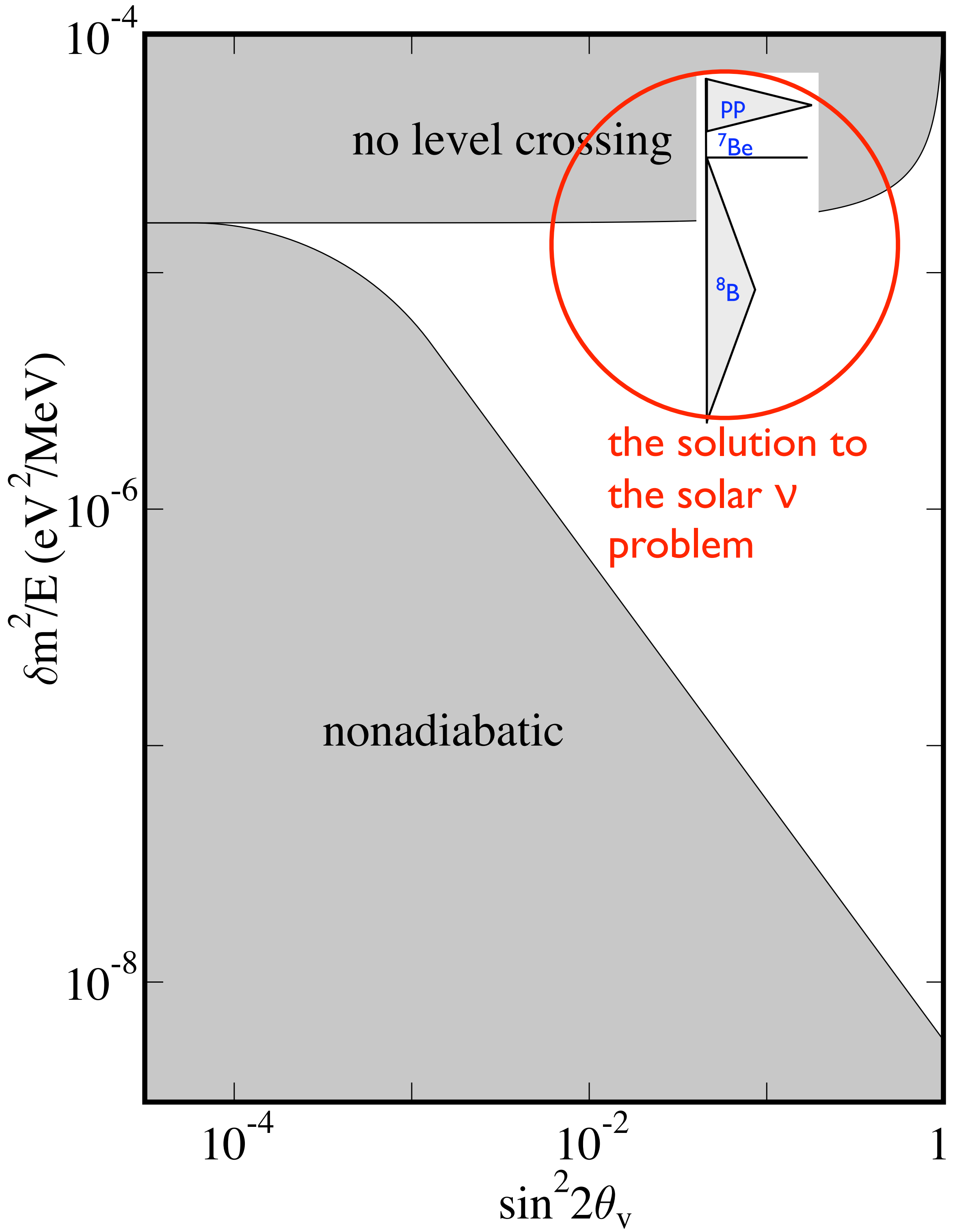
Small angle solution



Large angle solution

this is the solution matching SNO and SuperK results + Ga/Cl/KII

$\tan^2\theta_v \sim 0.40$



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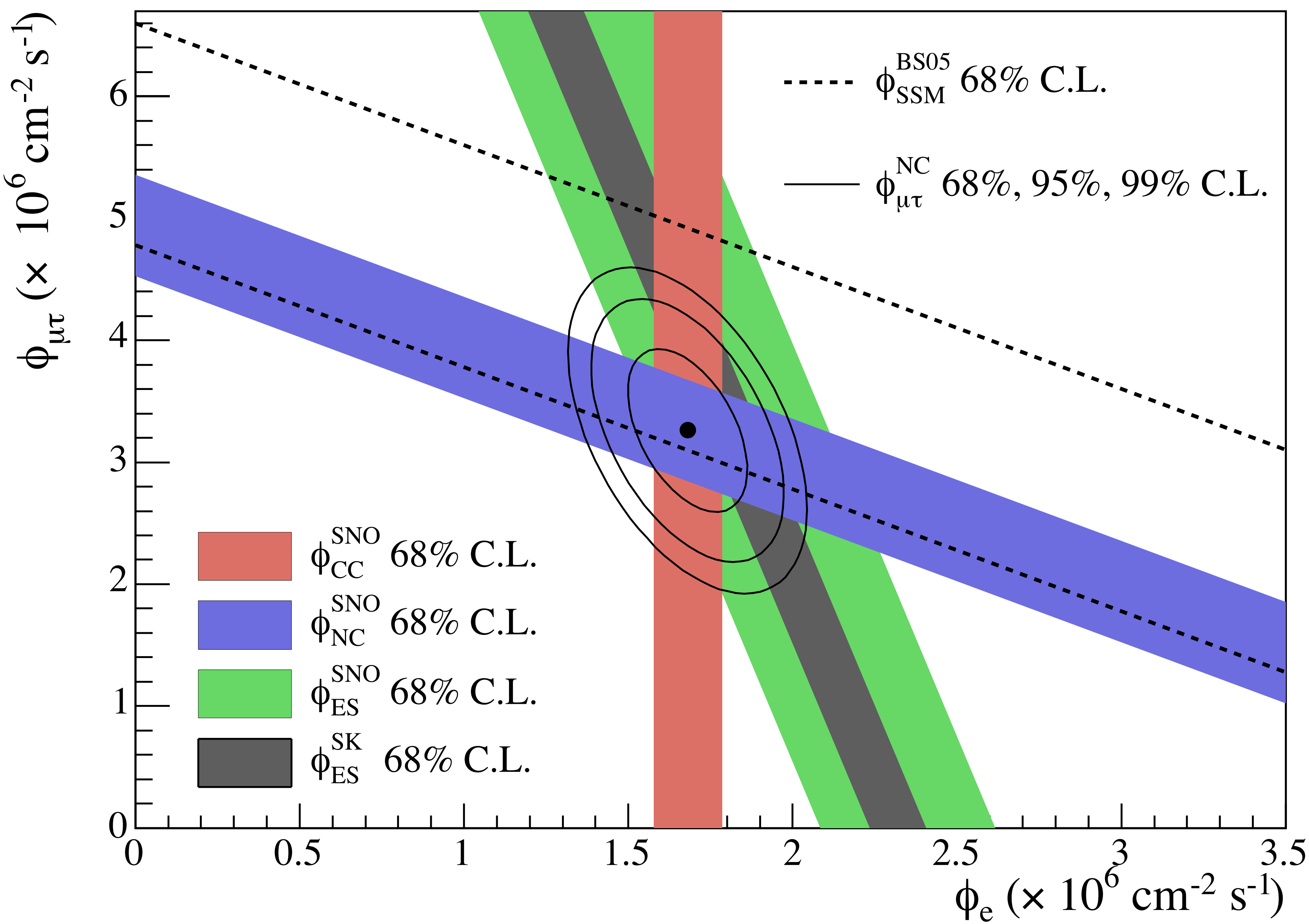
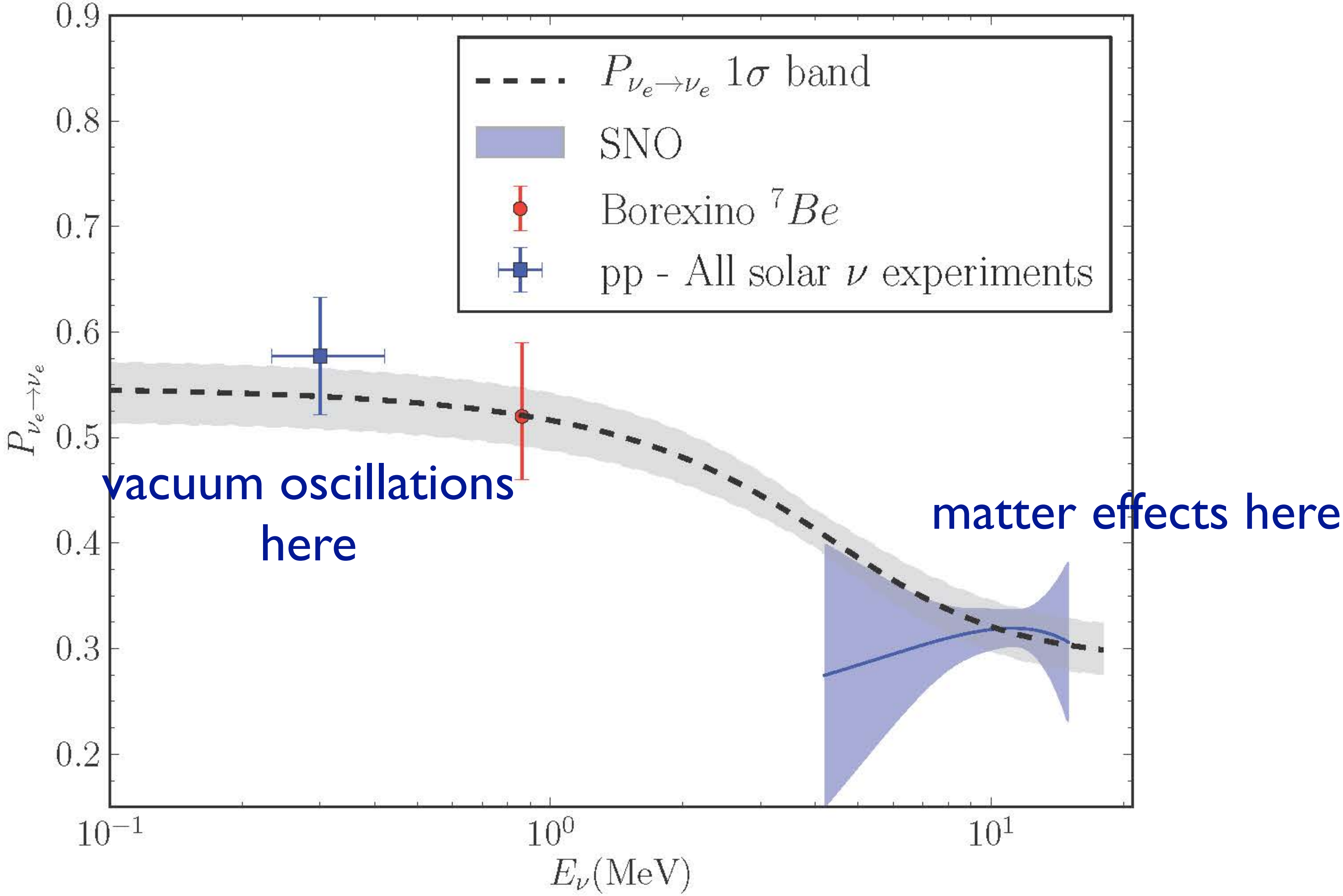


The definitive experiment was SNO, which measured the flux in three different flavor channels from Art McDonald



Matter effects produce a characteristic energy-dependence in the ν_e survival probability, in accord with experiments

- the solution corresponds so a $\Delta m^2_{12} \sim 10^{-5}\mathrm{eV}^2$
- because we use matter to alter this splitting in the Sun, the measurements determine the sign of this mass difference
- similar work done with atmospheric νs, probing $\Delta m^2_{23} \sim 10^{-3}\mathrm{eV}^2$, in vacuum
- this year the reactor experiments Daya Bay, RENO, and Double Chooz measured the third mixing, θ_{13}
- the neutrino analog of the three-generation CKM matrix is mostly in hand

The mixing matrix

(where we have additional blanks to fill in)

knowns: $\theta_{12}, \theta_{23}, \theta_{13}$ unknowns: δ, ϕ_1, ϕ_2

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} \nu_1 \\ e^{i\phi_1}\nu_2 \\ e^{i\phi_2}\nu_3 \end{pmatrix}$$

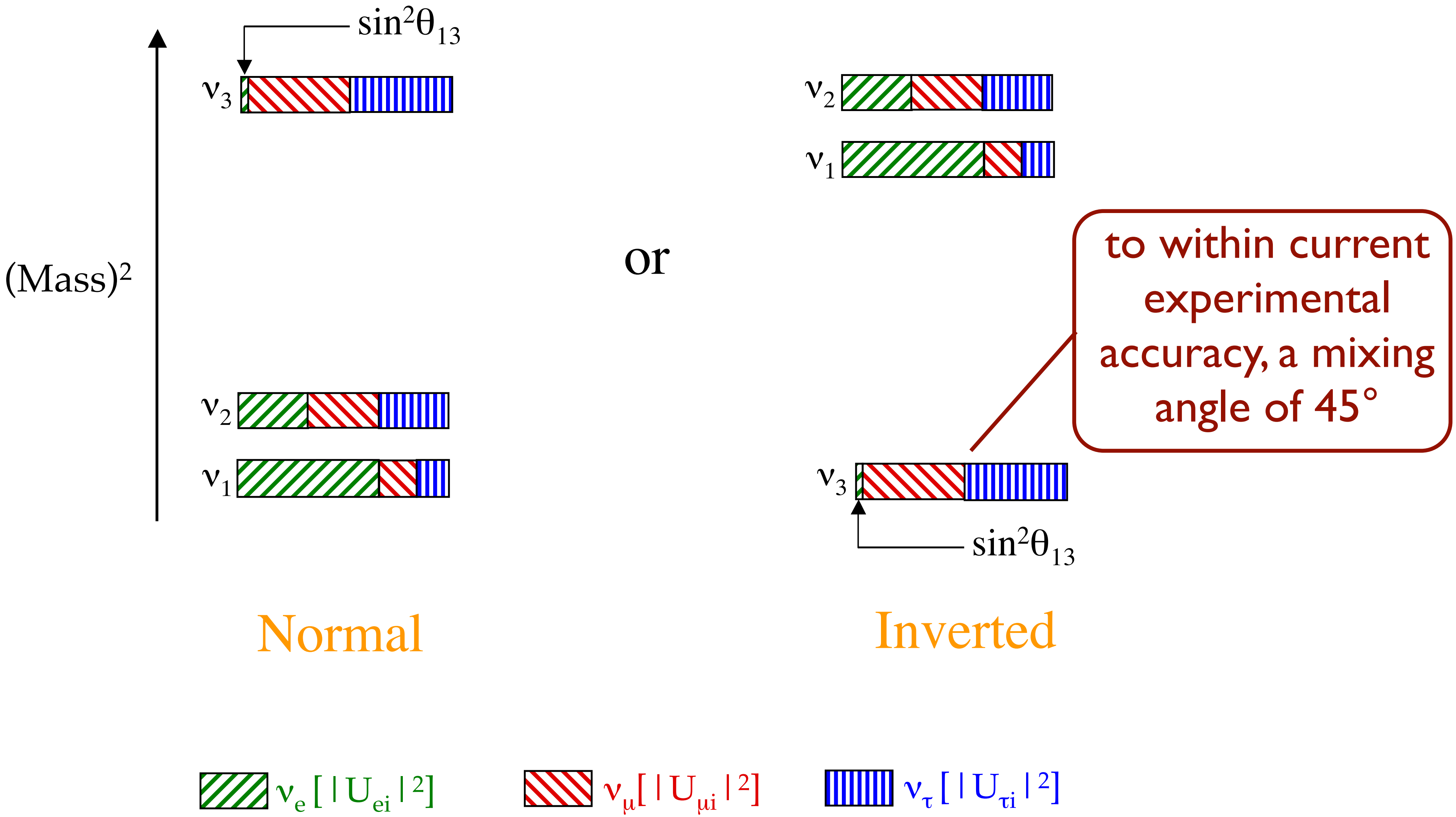
$$= \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{\boxed{-i\delta}} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \boxed{e^{i\phi_1}}\nu_2 \\ \boxed{e^{i\phi_2}}\nu_3 \end{pmatrix}$$

atmospheric ν_e disappearance   solar

results: $\theta_{23} \sim 45^\circ$ $\theta_{13} \sim 8.7^o$ $\theta_{12} \sim 30^\circ$

Δ_{12} $|\Delta_{23}|$ $\boxed{\mathrm{sign}[\Delta_{23}]}$ $\boxed{\text{absolute scale}}$

Hierarchy:



(artwork: Boris Kayser)

These results are important to the quest to understand baryogenesis. CP violating observables proportional to

$$\begin{aligned} J_{CP}^{\nu} &= \sin\theta_{12}\sin\theta_{23}\sin\theta_{13}\cos\theta_{12}\cos\theta_{23}\cos^2\theta_{13}\ \sin\delta \\ &\sim 0.03\ \sin\delta \end{aligned}$$

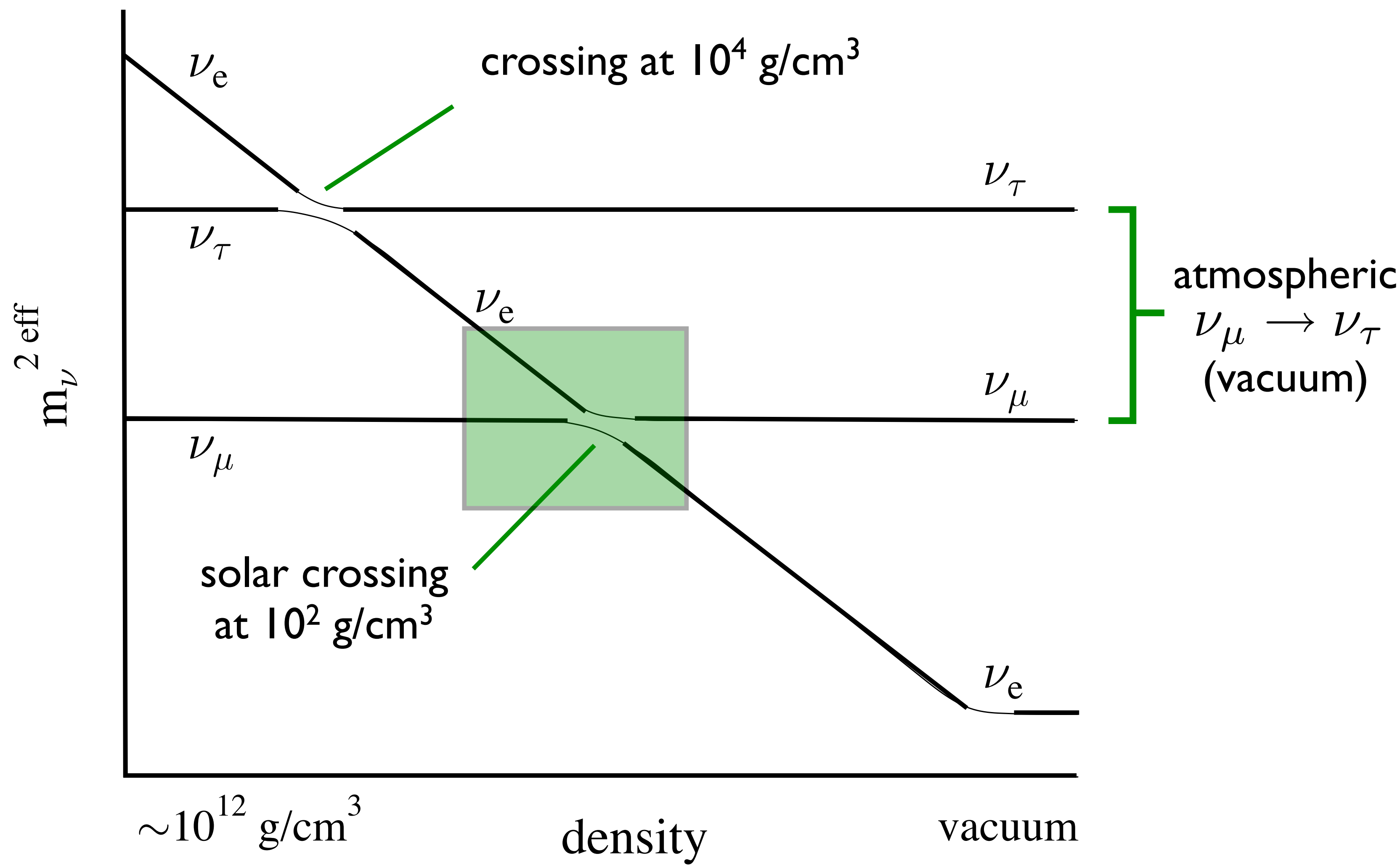
the coefficient is a factor of 1000 larger than in the case of the CKM matrix -- reflecting the generally large ν mixing angles

This has encouraged the long-baseline program to see leptonic CP violation, e.g.,

$$P(\nu_\mu \to \nu_e)\ \text{vs.}\ P(\bar{\nu}_\mu \to \bar{\nu}_e)$$

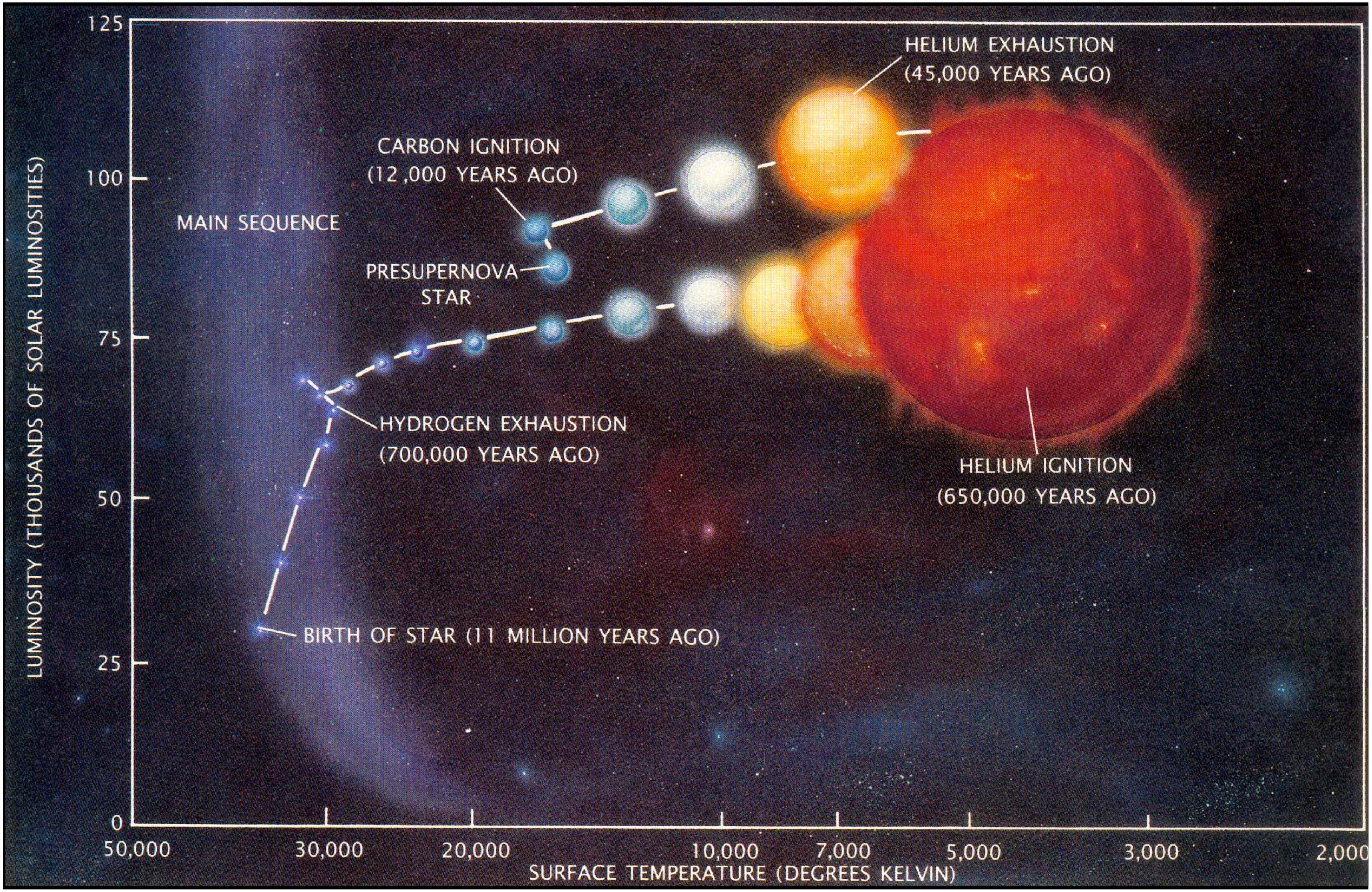
(and hopes to connect the results to processes like baryogenesis)

Higher densities needed to see matter effects in the 2-3 oscillations



This situation is found in core collapse supernovae

SNII progenitor evolution to collapse instability



Woosley and Weaver, 1987

- massive star evolves through burning cycles ... H, He, C, ..., to Si burning, which leads to a growing inert Fe core at its center

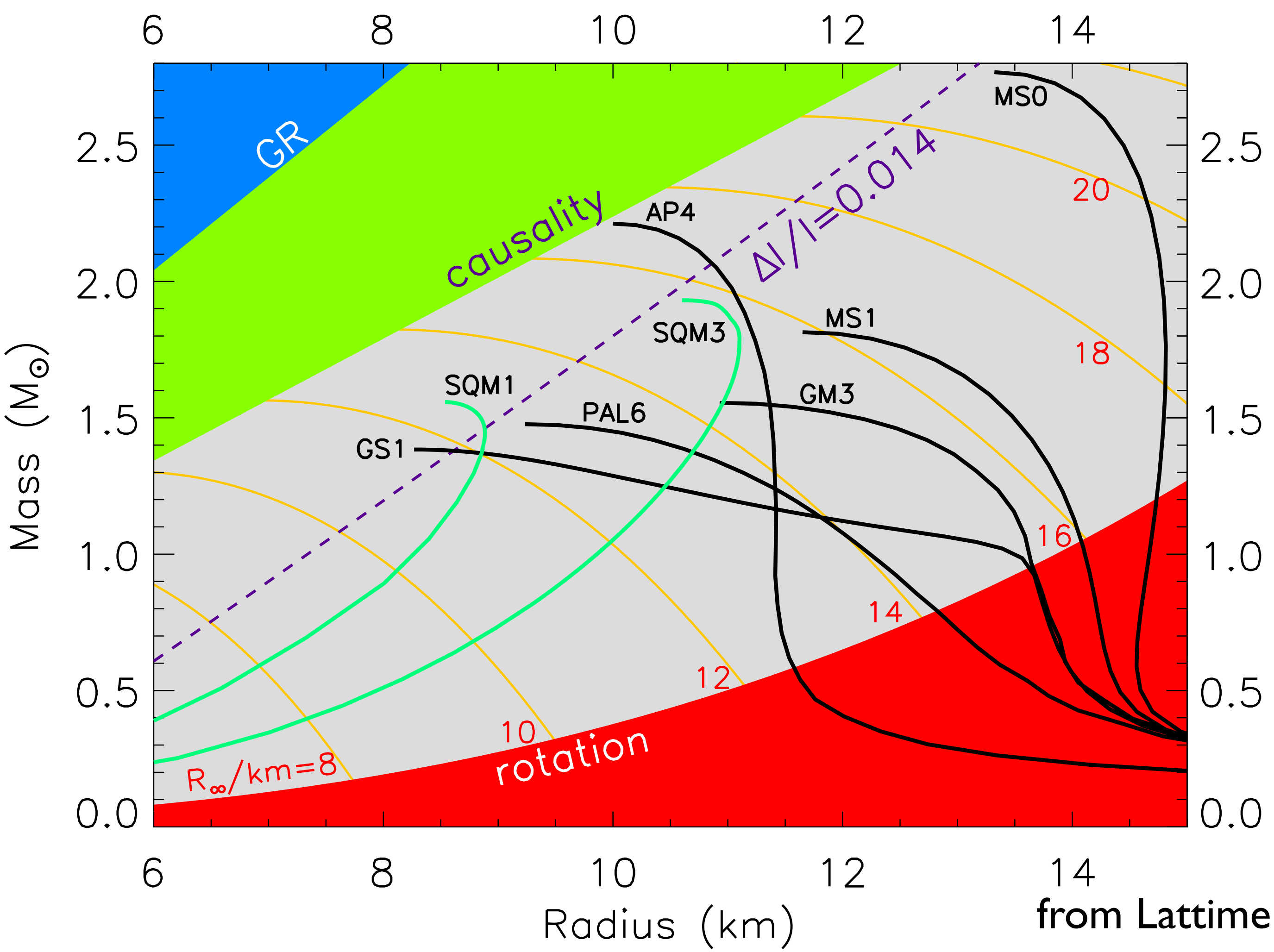
- when the core mass reaches ~ 1.4 $M_\odot$ it can do longer be stably supported by electron gas pressure, implodes

- the collapse is rapid, at nearly the free-fall velocity, and relatively little of the released gravitational energy is radiated away: not even neutrinos can escape the star on the collapse timescale

- collapse is halted at super-nuclear densities by the nuclear equation of state: a trampoline like rebound of the core produce pressure waves that collect at the sonic point, in the outer iron core, producing a shock wave that travels outward through the star's mantle

- modern simulations predict that a prompt hydrodynamic explosion fails, but that ν heating of the hot nucleon soup left in the shock's wake can revive the shock wave and successfully explode the star

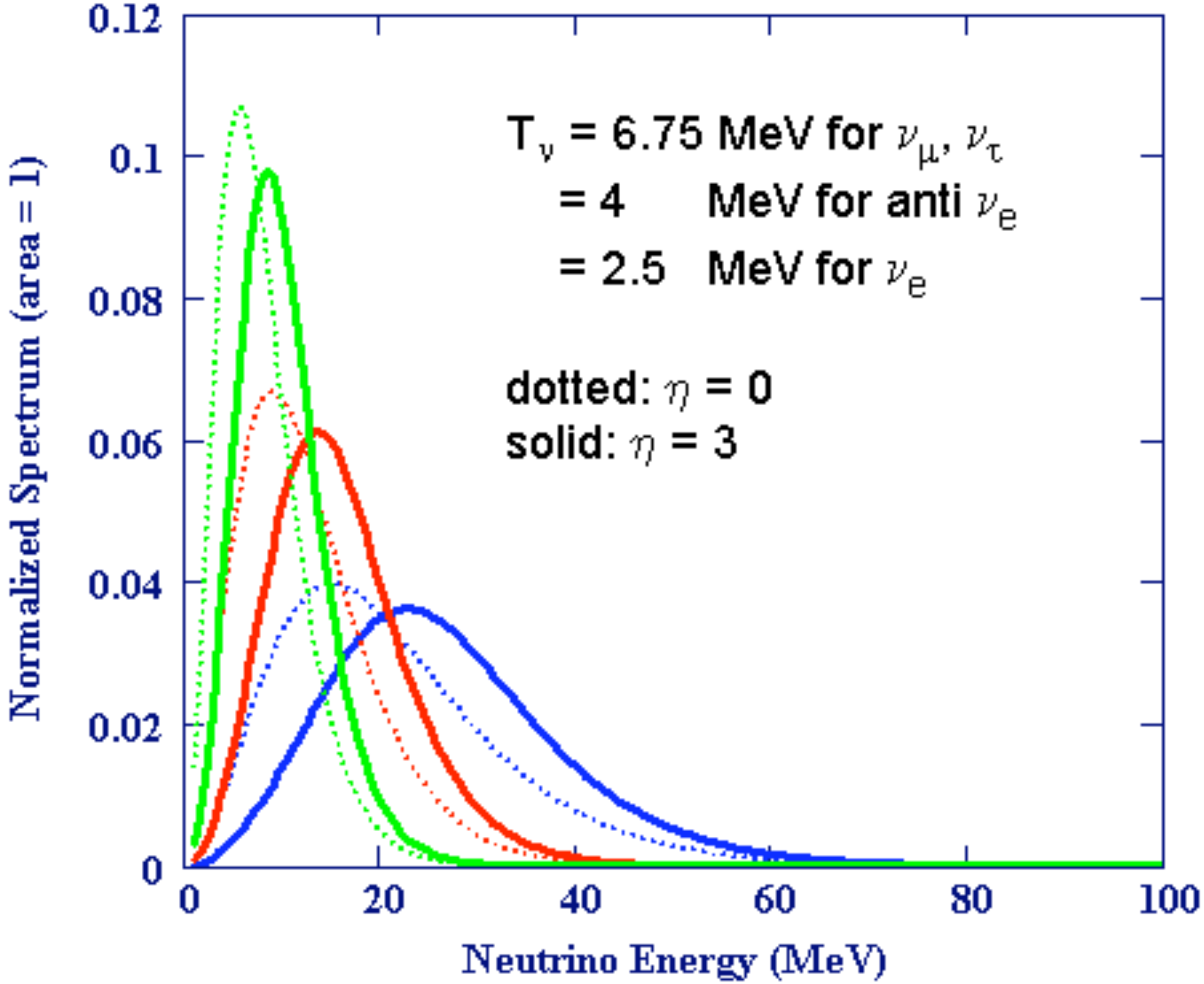
Significant nuclear EOS uncertainties here (hot, dense, high trapped lepton number) and in the corresponding neutron star problem (cooler, lepton number radiated away, neutron dominated)



from Lattimer and Prakash

Neutrino decoupling from star flavor-dependent: temperature hierarchy

$$(weakly\ coupled)\ \ T_{heavy flavor} > T_{\bar{\nu}_e} > T_{\nu_e}\ \ (neutron\ rich)$$



This neutrino environment is extraordinary

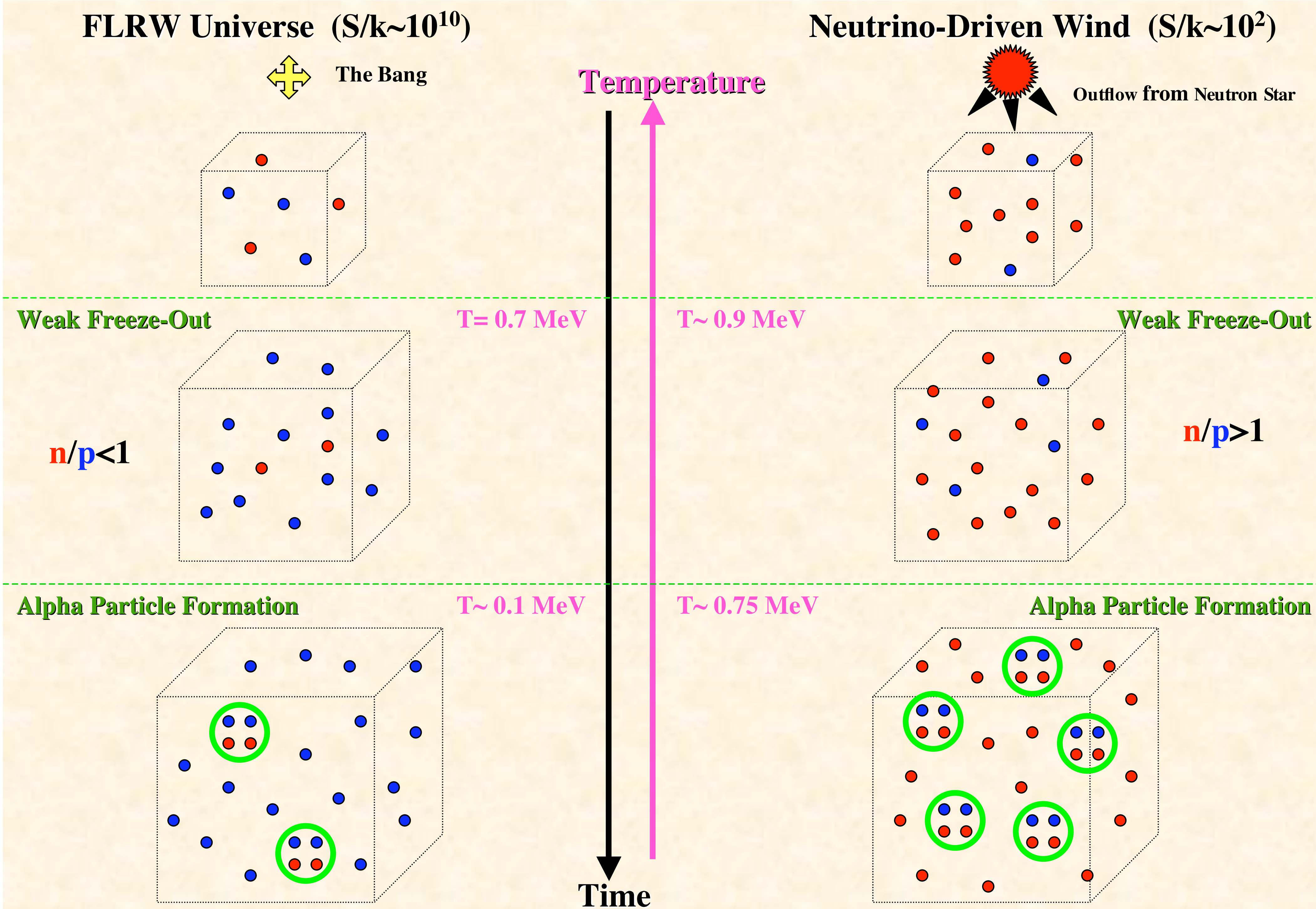
- 99% of the gravitational collapse energy radiated in neutrinos over the ~ 3 sec proto-neutron star cooling time: 10^{57}/flavor

- the ν heating of the nucleon gas lifts and drives that material off the star: a neutrino wind

- oscillations can enhance this coupling

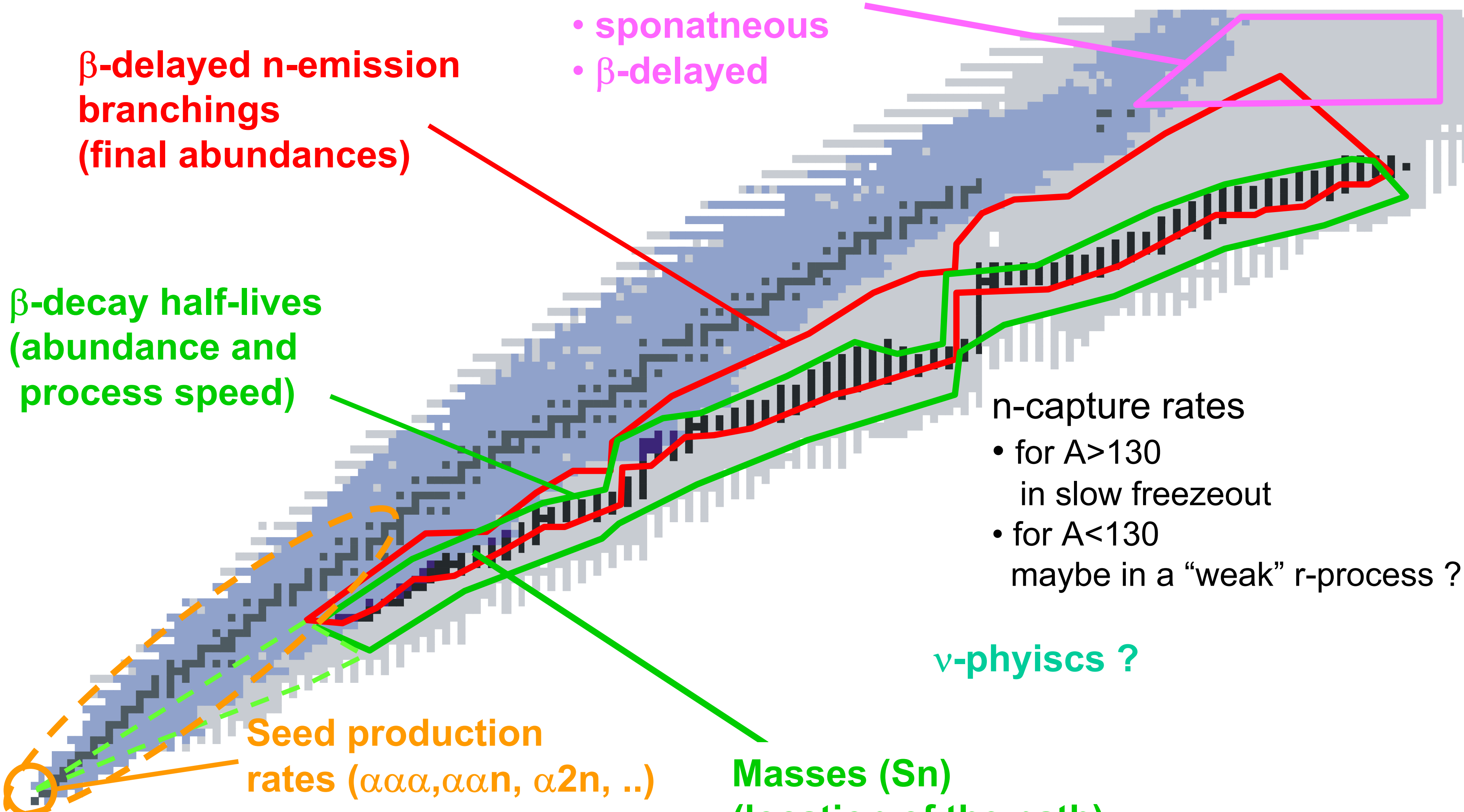
- novel new MSW mechanisms operate: neutrinos scattering off other trapped neutrinos can dominate the effective ν mass

- these effects create environments where the many nuclei not created in BBN may be synthesized -- under thermodynamic conditions remarkable similar to BBN



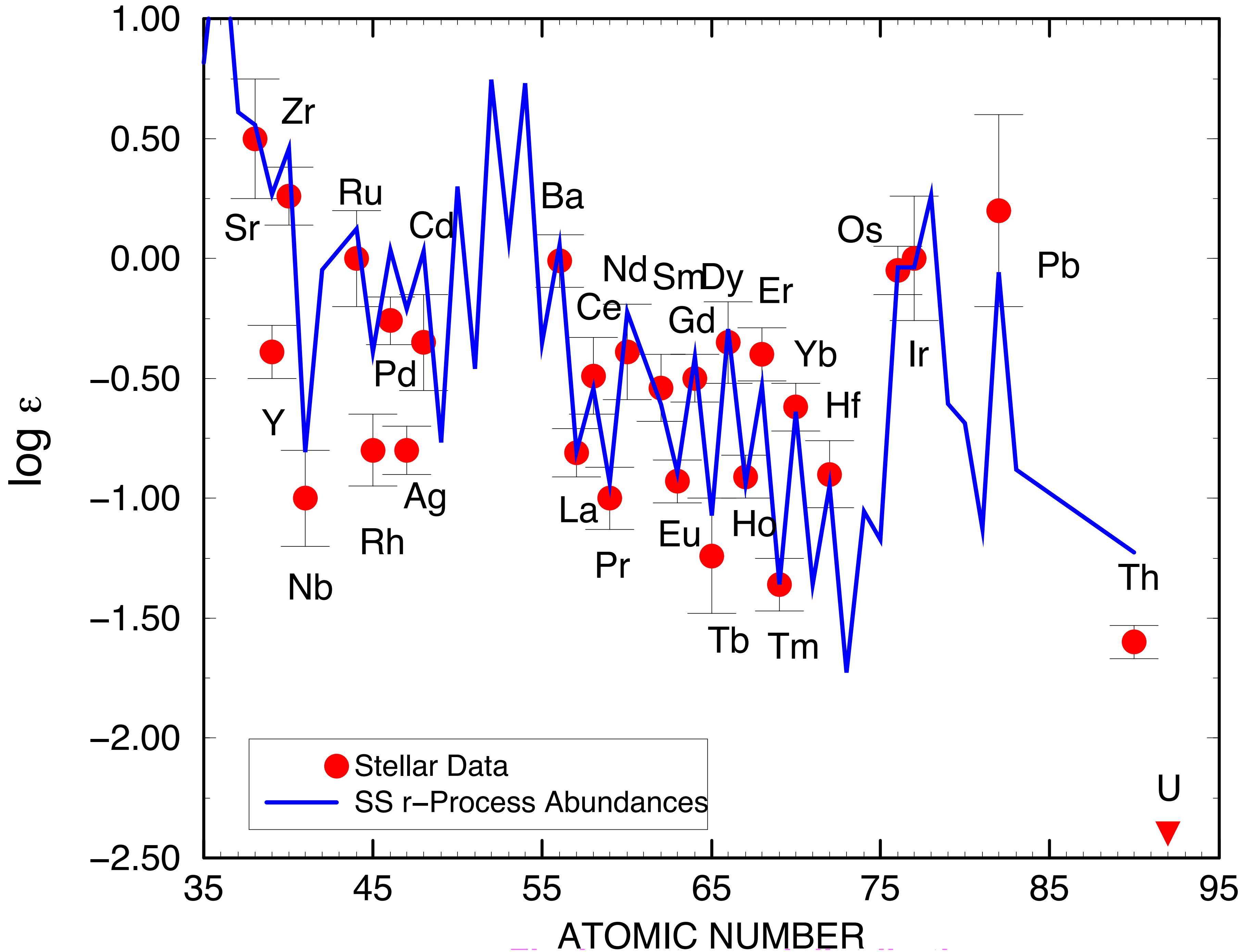
a neutron rich big bang: figure by George Fuller

From Schatz



Fission rates and distributions:
• n-induced
• sponatneous
• β-delayed
β-delayed n-emission
branchings
(final abundances)
β-decay half-lives
(abundance and
process speed)
n-capture rates
• for A>130
in slow freezeout
• for A<130
maybe in a “weak” r-process ?
ν-phyiscs ?
Seed production
rates (ααα,ααn, α2n, ..)
Masses (Sn)
(location of the path)

From Cowan et al.



Summary: theme here has been deep connection between weak interactions in astrophysics and strong interactions

-- the baryon number of the universe is known because BBN allows us to compare a weak interactions clock with the cosmological expansion scale

-- the dominance of protons in our universe is connected with the anomalously small binding energy of the deuteron

-- an essential requirement for the existence of baryons, CP violation, could originate among the νs: the large mixing angles discovered in the past several years potentially magnify low-energy CP violation by 3 orders of magnitude

-- the conditions needed to synthesize neutron-rich nuclei are found in neutrino winds of SN, and may be enhanced by oscillations